Wiskunde-bijles - functions

The INFDEV team

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Functions

Functions

Specification

- A function f is a mathematical entity with the following attributes:

F-1 f has a **domain** and a **codomain**, each of which be a set.

F-2 For every element x of the domain, f has a **value** at x, which is an element of the codomain and is denoted $f(x)$.

F-3 The domain, the codomain, and the value $f(x)$ for each x in the domain are all determined completely by the function.

F-4 Conversely, the data consisting of the domain, the codomain, and the value $f(x)$ for each element x of the domain completely determine the function f.

Functions

Arrow notation

- $f : S \longrightarrow T$ is a succint way of saying that the function f has domain S and codomain T. It could also be read as 'Let f be a function from S to T'.
- One also says that f is of type 'S arrow T'.

Barred arrow notation

- Used to provide an anonymous notation for functions. For example the function from $\mathbb{R}$ to $\mathbb{R}$ that squares its input can be denoted $x \longmapsto x^2 : \mathbb{R} \longrightarrow \mathbb{R}$.
- The barred arrow goes from data to da, whereas the long arrow goes from domain to codomain.

Functions

Examples of functions

(i) $x \longmapsto x^2 : \mathbb{R} \longrightarrow \mathbb{R}^+$

(ii) $x \longmapsto x^2 : \mathbb{R} \longrightarrow \mathbb{R}$

(iii) $x \longmapsto x^2 : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$

(iv) $x \longmapsto x^2 : \mathbb{R}^+ \longrightarrow \mathbb{R}$

Functions

The graph of a function

- $f : S \longrightarrow T$ is the set of ordered pairs: $\{(x, f(x))|x \in S\}$.
- The graph of a function from S to T is a relation from S to T with the restriction that it must have the **functional property** that for all $s \in S$, there is one and only one $t \in T$ such that (s, t) is in the graph.

The image of a function

- Is its set of values; that is, the image of $f : S \longrightarrow T$ is $\{t \in T | \exists s \in S \text{ for which } f(s) = t\}$.
- The image of the previous example is the set of non-negative reals.

Injective (one-to-one)

- A function $f : S \longrightarrow T$ is **injective** if whenever $s \neq s'$ in S, then $f(s) \neq f(s')$ in T.

Example

- The function $x \longmapsto x^2 : \mathbb{R} \longrightarrow \mathbb{R}$ is not injective since it takes 2 and -2 and returns the same value, namely 4.
- On the other hand $x \longmapsto x^3 : \mathbb{R} \longrightarrow \mathbb{R}$ is injective.

Functions

Surjective

- A function $f : S \longrightarrow T$ if its image is T.
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Examples

- The identity function is a surjective function.
- $x \longmapsto x^2 : \mathbb{R} \longrightarrow \mathbb{R}^+$ is a surjective function.
- $x \longmapsto x^2 : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ is a surjective function.

Bijective

- A function is bijective if it is surjective and injective. It is also **called one to one correspondence**.

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Coordinate functions

- If S and T are sets, the cartesian product $S \times T$ is equipped with two **coordinate** or **projection** functions $proj_1 : S\times \longrightarrow S$ and $proj_2 : S\times \longrightarrow T$.
- The coordinate functions are surjective if S and T are both nonempty.

Functions

Notation - $\langle f, g \rangle$

- If X, S and T are sets, and $f : X \longrightarrow S$ and $g : X \longrightarrow T$ are functions, then the function $\langle f, g \rangle : X \longrightarrow S \times T$ is defined by $\langle f, g \rangle(x) = (f(x), g(x))$ for all $x \in X$

Functions

Cartesian product

- If X, Y, S and T are sets, and $f : X \longrightarrow S$ and $g : Y \longrightarrow T$ are functions, then the function $f\ timesg : X \times Y \longrightarrow S \times T$ is defined by $(f, g)(x, y) = (f(x), g(y))$
- It is called the **cartesian product** of the functions f and g.

Functions

Composite function

- If $f : S \longrightarrow T$ and $g : T \longrightarrow U$, then the **composite** function $g \circ f : S \longrightarrow U$ is defined to be the unique function with domain S and codomain U for which $(g \circ f)(x) = g(f(x))$ for all $x \in S$.
- In computer science literatures $f; g$ is often used for $g \circ f$
- It is neccessay to insist that the codomain of f be the domain of g for the composite $g \circ f$ to be defined

Restriction

- If $f : S \longrightarrow T$ and $A \subseteq S$, then the **restriction** of f to A is the composity $f \circ i$, where $i : A \longrightarrow S$ is the inclusion function.
- The square squaring function in $x \longmapsto x^2 : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ is the restriction to $\mathbb{R}^+$ of the squaring function in $x \longmapsto x^2 : \mathbb{R} \longrightarrow \mathbb{R}^+$

Functions

Functions in theory and practice

- The concept of a function can be explicitly defined in terms of its domain, codomain, and graph.
- Precisely, a function $f : S \longrightarrow T$ could be defined as an ordered triple (S, Γ, T) with the property that Γ is a subset of the cartesian $S \times T$ with the functional property (Γ is the graph of f).
- For $x \in S$, $f(x)$ is the unique element $y \in T$ for which $(x, y) \in \Gamma$

Functions

Homomorphism

- Let S and T e sets, and let $Hom(S,T)$ denote the set of all functions with domain S and codomain T. Let $f : T \longrightarrow V$ be a function. The function

$$Hom(S,f) : Hom(S,T) \longrightarrow Hom(S,V)$$

is defined by

$$Hom(S,f)(g) = f \circ g$$

This is it!

The best of luck, and thanks for the attention!