

# Lab Nr. 5, Numerical Calculus

## Linear Systems II

### Iterative Methods: Jacobi, Gauss-Seidel, SOR; Conditioning of Linear Systems

Implement the Jacobi and Gauss-Seidel iterative methods, in the form

$$[x, nit] = \text{Name}(A, b, x0, err, maxnit),$$

where

$A$  is the system matrix,

$b$  is the right-hand side of the system,

$x0$  is the initial solution,

$err$  is the desired error,

$maxnit$  is the maximum number of iterations allowed (in case the method does not converge),

$x$  is the iterative solution of the system,

$nit$  is the number of iterations.

#### Applications

1. Use both iterative methods to approximate the solution of the linear system (Problem 1, Lab Nr. 4)

$$\begin{cases} 5x_1 - x_2 & = 4 \\ -x_{j-1} + 5x_j - x_{j+1} & = 3, \quad j = \overline{2, n-1}, \\ -x_{n-1} + 5x_n & = 4 \end{cases}$$

for  $n = 1000$ . Compare the number of iterations needed for an error of  $10^{-5}$ .

2. (Wilson's Example) Consider the system  $Ax = b$ , where

$$A = \begin{bmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{bmatrix}, \quad b = \begin{bmatrix} 32 \\ 23 \\ 33 \\ 31 \end{bmatrix}.$$

- a) Solve the system.

- b) Perturb  $b$  to  $\tilde{b} = \begin{bmatrix} 32.1 \\ 22.9 \\ 33.1 \\ 30.9 \end{bmatrix}$ . Solve the system  $Ax = \tilde{b}$ . What are the input and output relative errors?

- c) The same question for the perturbed system matrix  $\tilde{A} = \begin{bmatrix} 10 & 7 & 8.1 & 7.2 \\ 7.8 & 5.04 & 6 & 5 \\ 8 & 5.98 & 9.89 & 9 \\ 6.99 & 4.99 & 9 & 9.98 \end{bmatrix}$ .

- d) Explain the phenomenon.

### Optional

3. Implement the successive overrelaxation (SOR) method, in the form

$$[x, nit] = Name(A, b, \omega, x0, err, maxnit),$$

where  $\omega$  is the optimal relaxation parameter. Then use it to approximate the solution of the system (Problem 3, Lab Nr. 4)

$$\begin{bmatrix} 5 & -1 & 0 & -1 & \dots & \dots & 0 \\ -1 & 5 & -1 & 0 & -1 & \dots & \dots & \vdots \\ 0 & -1 & 5 & -1 & \ddots & \ddots & \vdots & \vdots \\ -1 & & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -1 & 5 & -1 & 0 & -1 \\ 0 & \dots & \ddots & 0 & -1 & 5 & -1 & 0 \\ 0 & \dots & & -1 & 0 & -1 & 5 & -1 \\ 0 & \dots & & & -1 & 0 & -1 & 5 \end{bmatrix} x = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 1 \\ \vdots \\ 1 \\ 2 \\ 2 \\ 3 \end{bmatrix},$$

for  $n = 10$  and  $n = 1000$ . Compare the number of iterations needed in all three iterative methods, for a precision of  $10^{-5}$ .