Lab Nr. 4, Numerical Calculus

Linear Systems I

Direct Methods: Gaussian Elimination, Factorizations

1. Implement back substitution for solving an upper triangular linear system and forward substitution for solving a lower triangular linear system.

2. Implement Gaussian elimination with partial pivoting.

3. Implement a routine that solves the system $Ax = b$ using

- LUP factorization;
- Cholesky factorization;
- QR factorization.

Applications

1. Use both methods (Gaussian elimination and factorizations, when possible) to solve the linear system

$$\begin{bmatrix} 2 & 1 & -1 & -2 \\ 4 & 4 & 1 & 3 \\ -6 & -1 & 10 & 10 \\ -2 & 1 & 8 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -5 \\ 1 \end{bmatrix}.$$

2. Use both methods (Gaussian elimination and factorizations, when possible) to solve the following general linear system, that can be generated of any order $n \geq 3$:

$$\begin{cases} 5x_1 - x_2 = 4 \\ -x_{j-1} + 5x_j - x_{j+1} = 3, \quad j = \overline{2, n-1} \\ - x_{n-1} + 5x_n = 4 \end{cases}$$

Optional

3. Use both methods (Gaussian elimination and factorizations, when possible) to solve the following general linear system, that can be generated of any order $n \geq 7$:

$$\begin{bmatrix} 5 & -1 & 0 & -1 & \dots & & \dots & 0 \\ -1 & 5 & -1 & 0 & -1 & \dots & \dots & \vdots \\ 0 & -1 & 5 & -1 & \ddots & \ddots & \vdots & \vdots \\ -1 & & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -1 & 5 & -1 & 0 & -1 \\ 0 & \dots & \ddots & 0 & -1 & 5 & -1 & 0 \\ 0 & \dots & & -1 & 0 & -1 & 5 & -1 \\ 0 & \dots & & & -1 & 0 & -1 & 5 \end{bmatrix} x = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 1 \\ \vdots \\ 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}.$$