MATHS NOTES

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Contents

Chapter 1

Logarithm

1.1 Logarithm

With $a, b > 0, a \neq 1$:

$$\log_a xy = \log_a x + \log_a y \tag{1.1}$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y \tag{1.2}$$

$$\log_a \frac{1}{x} = -\log_a x \tag{1.3}$$

$$\log_{a^\beta} x^\alpha = \frac{\alpha}{\beta} \log_a x \tag{1.4}$$

$$\log_a a^\alpha = \alpha \tag{1.5}$$

$$\log_a 1 = 0 \tag{1.6}$$

$$\log_a a = 1 \tag{1.7}$$

$$a^{\log_a b} = b \tag{1.8}$$

$$\log_a a^\alpha = \alpha \tag{1.9}$$

$$\log_a \sqrt[n]{b} = \frac{1}{n} \log_a b \tag{1.10}$$

1.2 Natural Logarithm

$$e = \lim_{n \to +\infty} \left(1 + \frac{1}{n}\right)^n \tag{1.11}$$

$$e^{\ln x} = x, \qquad (x > 0) \tag{1.12}$$

$$\ln e^x = x, \qquad (x > 0) \tag{1.13}$$

$$\ln u^r = r \ln u \tag{1.14}$$

$$a^b = e^{b \ln a} \tag{1.15}$$

Chapter 2

Derivative

$$(u \pm v)' = u' \pm v' \tag{2.1}$$

$$(uv)' = u'v + uv' \tag{2.2}$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \tag{2.3}$$

$$(kx)' = k, (k \text{ is a constant}) \tag{2.4}$$

$$(x^n)' = nx^{n-1} \tag{2.5}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2} \tag{2.6}$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \tag{2.7}$$

$$(\sin x)' = \cos x \tag{2.8}$$

$$(\cos x)' = -\sin x \tag{2.9}$$

$$(\tan x)' = 1 + \tan^2 x = \frac{1}{\cos^2 x} \tag{2.10}$$

$$(\cot x)' = -(1 + \cot^2 x) = -\frac{1}{\sin^2 x} \tag{2.11}$$

$$(e^x)' = e^x \tag{2.12}$$

$$(a^x)' = a^x \ln a \tag{2.13}$$

$$(\ln x)' = \frac{1}{x} \tag{2.14}$$

$$(\log_a x)' = \frac{1}{x \ln a} \tag{2.15}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1 - x^2}} \tag{2.16}$$

$$(\arccos x)' = \frac{-1}{\sqrt{1 - x^2}} \tag{2.17}$$

$$(\arctan x)' = \frac{1}{x^2 + 1} \tag{2.18}$$

$$\left(\frac{ax + b}{cx + d}\right)' = \frac{ad - bc}{(cx + d)^2} \tag{2.19}$$

$$(ku)' = ku', (k \text{ is a constant}) \tag{2.20}$$

$$(u^n)' = nu^{n-1}u' \tag{2.21}$$

$$\left(\frac{1}{u}\right)' = -\frac{u'}{u^2} \tag{2.22}$$

$$(\sqrt{u})' = \frac{u'}{2\sqrt{u}} \tag{2.23}$$

$$(\sin u)' = \cos u u' \tag{2.24}$$

$$(\cos u)' = -\sin u u' \tag{2.25}$$

$$(\tan u)' = (1 + \tan^2 u)u' = \frac{u'}{\cos^2 u} \tag{2.26}$$

$$(\cot u)' = -(1 + \cot^2 u)u' = -\frac{u'}{\sin^2 u} \tag{2.27}$$

Chapter 3

Antiderivative

$$\int 0dx = c \tag{3.1}$$

$$\int dx = x + c \tag{3.2}$$

$$\int kdx = kx + c \tag{3.3}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, (n \neq -1) \tag{3.4}$$

$$\int \frac{1}{x} dx = \ln|x| + c, (x \neq 0) \tag{3.5}$$

$$\int \frac{1}{x^2} dx = -\frac{1}{x} + c \tag{3.6}$$

$$\int \frac{1}{x^a} dx = -\frac{1}{(a-1)x^{a-1}} + c \tag{3.7}$$

$$\int e^x dx = e^x + c \tag{3.8}$$

$$\int a^x dx = \frac{a^x}{\ln a} + c, (0 < a \neq 1) \tag{3.9}$$

$$\int \cos x dx = \sin x + c \tag{3.10}$$

$$\int \sin x dx = -\cos x + c \tag{3.11}$$

$$\int \frac{1}{\cos^2 x} dx = \int 1 + \tan^2 x dx = \tan x + c \tag{3.12}$$

$$\int \frac{1}{\sin^2 x} dx = \int 1 + \cot^2 x dx = \cot x + c \tag{3.13}$$

$$\int \frac{1}{2\sqrt{x}} dx = \sqrt{x} + c \tag{3.14}$$

$$\int (ax+b)^\alpha dx = \frac{1}{a} \frac{(ax+b)^{\alpha+1}}{\alpha+1} + c, (\alpha \neq -1) \tag{3.15}$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b| + c \tag{3.16}$$

$$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c \tag{3.17}$$

$$\int m^{ax+b} dx = \frac{1}{a \ln m} m^{ax+b} + c \tag{3.18}$$

$$\int \ln(ax+b) dx = \left(x + \frac{b}{a}\right) \ln(ax+b) - x + c \tag{3.19}$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + c, (a > 0) \tag{3.20}$$

$$\int \frac{dx}{\sqrt{a + x^2}} = \ln|x + \sqrt{x^2 + a}| + c \tag{3.21}$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + c, (a > 0) \tag{3.22}$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + c \tag{3.23}$$

$$\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + c \tag{3.24}$$

$$\int \sin(ax+b) dx = \frac{-1}{a} \cos(ax+b) + c \tag{3.25}$$

$$\int \tan(ax+b) dx = \frac{-1}{a} \ln|\cos(ax+b| + c \tag{3.26}$$

$$\int \cot(ax+b) dx = \frac{1}{a} \ln|\sin(ax+b| + c \tag{3.27}$$

$$\int \frac{dx}{\sin^2(ax+b)} = \frac{-1}{a} \cot(ax+b) + c \tag{3.28}$$

$$\int \frac{dx}{\cos^2(ax+b)} = \frac{1}{a} \tan(ax+b) + c \tag{3.29}$$

Chapter 4

Trigonometry

$$\sin^2 x + \cos^2 x = 1 \tag{4.1}$$
$$\sin 2x = \sin x \cos x \tag{4.2}$$
$$\sin 3x = 3\sin x - 4\sin^3 x \tag{4.3}$$
$$\sin^2 x = \frac{1-\cos 2x}{2} \tag{4.4}$$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x\\ &= 2\cos^2 - 1\\ &= 1 - \sin^2 x\end{aligned} \tag{4.5}$$

$$\cos^2 x = \frac{1+\cos 2x}{2} \tag{4.6}$$
$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)] \tag{4.7}$$
$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)] \tag{4.8}$$
$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)] \tag{4.9}$$

$$\sin 2a = 2\sin a\cos a \tag{4.10}$$

$$\begin{aligned}\cos 2a &= \cos^2 a - \sin^2 a\\ &= 2\cos^2 a - 1\\ &= 1 - 2\sin^2 a\end{aligned} \tag{4.11}$$

$$\tan 2a = \frac{2\tan a}{1-\tan^2 a} \tag{4.12}$$

$$\sin 3a = 3\sin a - 4\sin^3 a \tag{4.13}$$
$$\cos 3a = 4\cos^3 a - 3\cos a \tag{4.14}$$
$$\tan 3a = \frac{3\tan a - \tan^3 a}{1-3\tan^2 a} \tag{4.15}$$

$$\sin^2 a = \frac{1-\cos 2a}{2} \tag{4.16}$$
$$\sin^3 a = \frac{3\sin a - \sin 3a}{4} \tag{4.17}$$
$$\sin^4 a = \frac{\cos 4a - 4\cos 2a + 3}{8} \tag{4.18}$$
$$\tan^2 a = \frac{1-\cos 2a}{1+\cos 2a} \tag{4.19}$$

$$\cos^2 a = \frac{1+\cos 2a}{2} \tag{4.20}$$
$$\cos^3 a = \frac{3\cos a + \cos 3a}{4} \tag{4.21}$$
$$\cos^4 a = \frac{\cos 4a + 4\cos 2a + 3}{8} \tag{4.22}$$

Chapter 5

Series

$$\sum_{n=1}^{\infty} \frac{4(-1)^{n+1}}{2n-1} = \pi \tag{5.1}$$

$$\sum_{n=0}^{\infty} \frac{1}{n!} = e \tag{5.2}$$