Converting Point Clouds to Meshes with Poisson Surface Reconstruction

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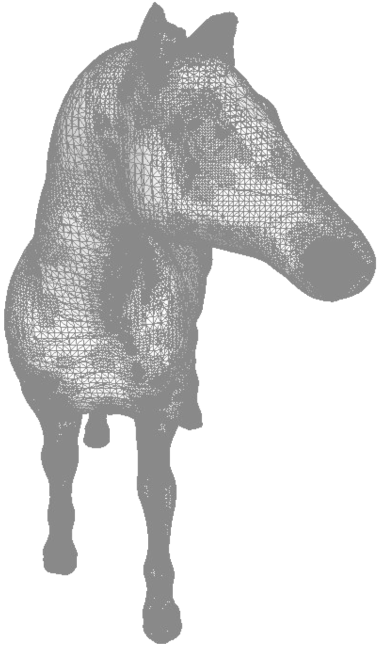
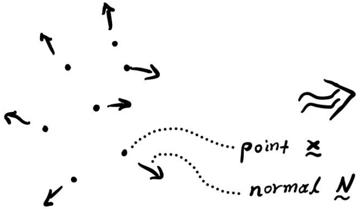
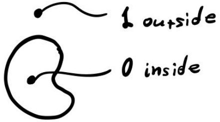


Fundamentals

- Kazhdan, M., Bolitho, M., & Hoppe, H. (2006, June). **Poisson surface reconstruction**. In *Proceedings of the fourth Eurographics symposium on Geometry processing* (Vol. 7).
- Kazhdan, Michael, and Hugues Hoppe. **Screened poisson surface reconstruction.** *ACM Transactions on Graphics (ToG)* 32.3 (2013): 1-13.
- Wilhelms, Jane, and Allen Van Gelder. **Octrees for faster isosurface generation**. *ACM Transactions on Graphics (TOG)* 11.3 (1992): 201-227.

MIchael Kazhdan

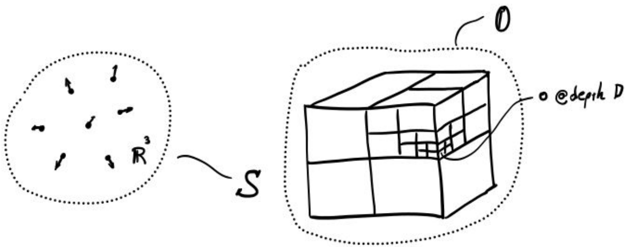




$$\chi = \arg\min_{\chi} \int \| \nabla \chi(x) - N(x) \|_2^2 \, d\chi$$

$$\Delta \chi = \nabla \cdot N$$

Fundamentals



(any x,y,z position)

$$F_o(\underset{\sim}{q}) = F\left(\frac{1}{o.w}(\underset{\sim}{q} - \underset{\sim}{o.c})\right) \cdot \frac{1}{(o.w)^3}$$

$$F(\underset{\sim}{q}) = F(x, y, z) = \left(B(x)\, B(y)\, B(z) \right)^{*n=3}$$

3rd convolution ..

$$B(t) = \begin{cases} 1 & |t| < 0.5 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathcal{F}_{\mathcal{O},F} \equiv \operatorname{span}(F_o)$$

Continuous approximation of the normal field $\underset{\sim}{V}$:

$$\underset{\sim}{V}(\underset{\sim}{q}) = \sum_{s \in \mathcal{S}} \left\{ \sum_{o \in Ngbr_D(s)} \left\{ \alpha_{o,s}\, F_o(\underset{\sim}{q})\, \underset{\sim}{s.N} \right\} \right\}$$

Issue: Even though $\tilde{\chi}(\underset{\sim}{q}) \in \mathcal{F}_{\mathcal{O},F}$ & $\underset{\sim}{V}(\underset{\sim}{q}) \in \mathcal{F}_{\mathcal{O},F}$ by their definitions,

$\nabla^2 \tilde{\chi}(\underset{\sim}{q})$ and $\nabla \cdot \underset{\sim}{V}(\underset{\sim}{q})$ might $\notin \mathcal{F}_{\mathcal{O},F}$

Workaround: minimize the difference of their projections (which $\in \mathcal{F}_{\mathcal{O},F}$ ...)

$$\tilde{\chi}(\underset{\sim}{q}) = \underset{\tilde{\chi}}{\operatorname{argmin}} \sum_{o \in \mathcal{O}} \left\{ \left\| \left\langle \nabla^2 \tilde{\chi} - (\nabla \cdot \underset{\sim}{V}),\ F_o \right\rangle \right\|^2 \right\}$$

$$= \underset{\tilde{\chi}}{\operatorname{argmin}} \sum_{o \in \mathcal{O}} \left\{ \left\| \left\langle \nabla^2 \tilde{\chi}, F_o \right\rangle - \left\langle (\nabla \cdot \underset{\sim}{V}),\ F_o \right\rangle \right\|^2 \right\}$$

$$\tilde{\chi} = \sum_{o \in \mathcal{O}} \left\{ x_o\, F_o \right\}$$

$$\underset{\sim}{x} = \{x_1, x_2, \dots\}^T = \underset{\underset{\sim}{x}}{\operatorname{argmin}} \left\| \underline{L}\, \underset{\sim}{x} - \underset{\sim}{v} \right\|_2^2$$

$$L_{ij} = \left\langle \frac{\partial^2 F_i}{\partial x^2}, F_j \right\rangle + \left\langle \frac{\partial^2 F_i}{\partial y^2}, F_j \right\rangle + \left\langle \frac{\partial^2 F_i}{\partial z^2}, F_j \right\rangle$$

$$v_i = \left\langle \nabla \cdot \underset{\sim}{V},\ F_i \right\rangle$$

→ we can then compute $\tilde{\chi}(\underset{\sim}{q})$

$$\partial \tilde{\mathcal{M}} \triangleq \left\{ \underset{\sim}{q} \in \mathbb{R}^3 \,\middle|\, \tilde{\chi}(\underset{\sim}{q}) = \gamma \right\} \text{ with } \gamma = \frac{1}{|S|} \sum_{s \in \mathcal{S}} \left\{ \tilde{\chi}(\underset{\sim}{s.P}) \right\}$$

Clustering with Poisson Reconstruction

Poisson surface reconstruction can produce undesired results with unclosed point clouds

Clustering points based on their position before performing the reconstruction will create the intended mesh