

Linked List, Stack, and Queue

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Weekly Objectives

- This week, we learn the first set of data structures: linked list, stack, and queue.
- Objectives are
 - Understanding the definition of abstract data types
 - Firmly understanding how references work
 - Understanding various linked list, stack, and queue structures
 - Singly linked list, doubly linked list, circular linked list...
 - Able to implement a stack and a queue with a list
 - Understanding the procedures of linked list, stack, and queue management
 - Insert, delete, search...
 - Should be able to estimate the number of steps for inserts, deletes, and searches

ARRAY FOR LIST

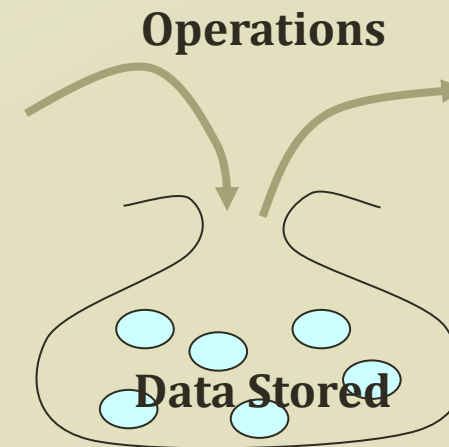
Scenario for List

- You are looking for Koh in the mass public
 - You are going to ask one by one “Are you Ms. Koh?”
 - Sometimes, you ask the question to a person multiple times
 - You realize that this is not going to work!
 - So, you line up the people and again ask the question one-by-one
- You are looking for a restroom on the floor
 - The floor has a long corridor, and every room has an entrance to the corridor
 - You only need to follow the corridor
- You have a dump of information of customers
 - How to store, search, and manipulate the information
- You line them up as a ***list***!



Abstract Data Types

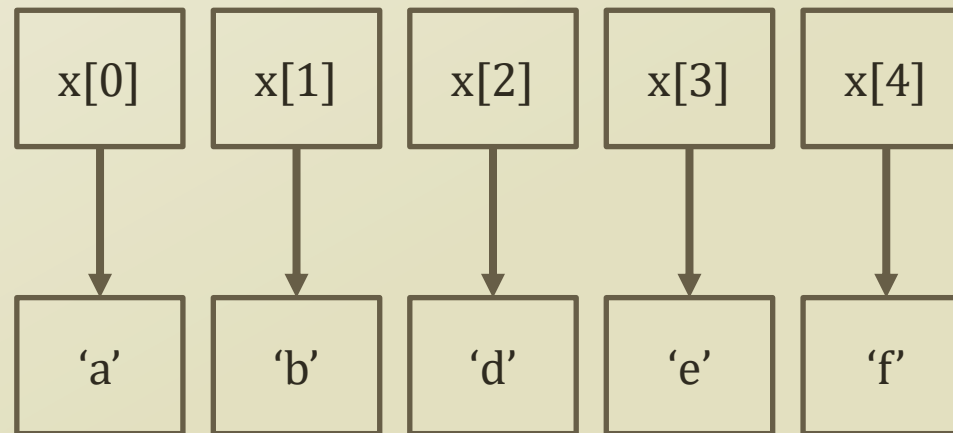
- An abstract data type (ADT) is an abstraction of a data structure
 - An ADT specifies:
 - Data stored
 - Operations on the data
 - Error conditions associated with operations
 - Example: ADT modeling a simple stock trading system
 - The data stored are buy/sell orders
 - The operations supported are
 - `order buy(stock, shares, price)`
 - `order sell(stock, shares, price)`
 - `void cancel(order)`
 - Error conditions:
 - Buy/sell a nonexistent stock
 - Cancel a nonexistent order



Creating a List by Array

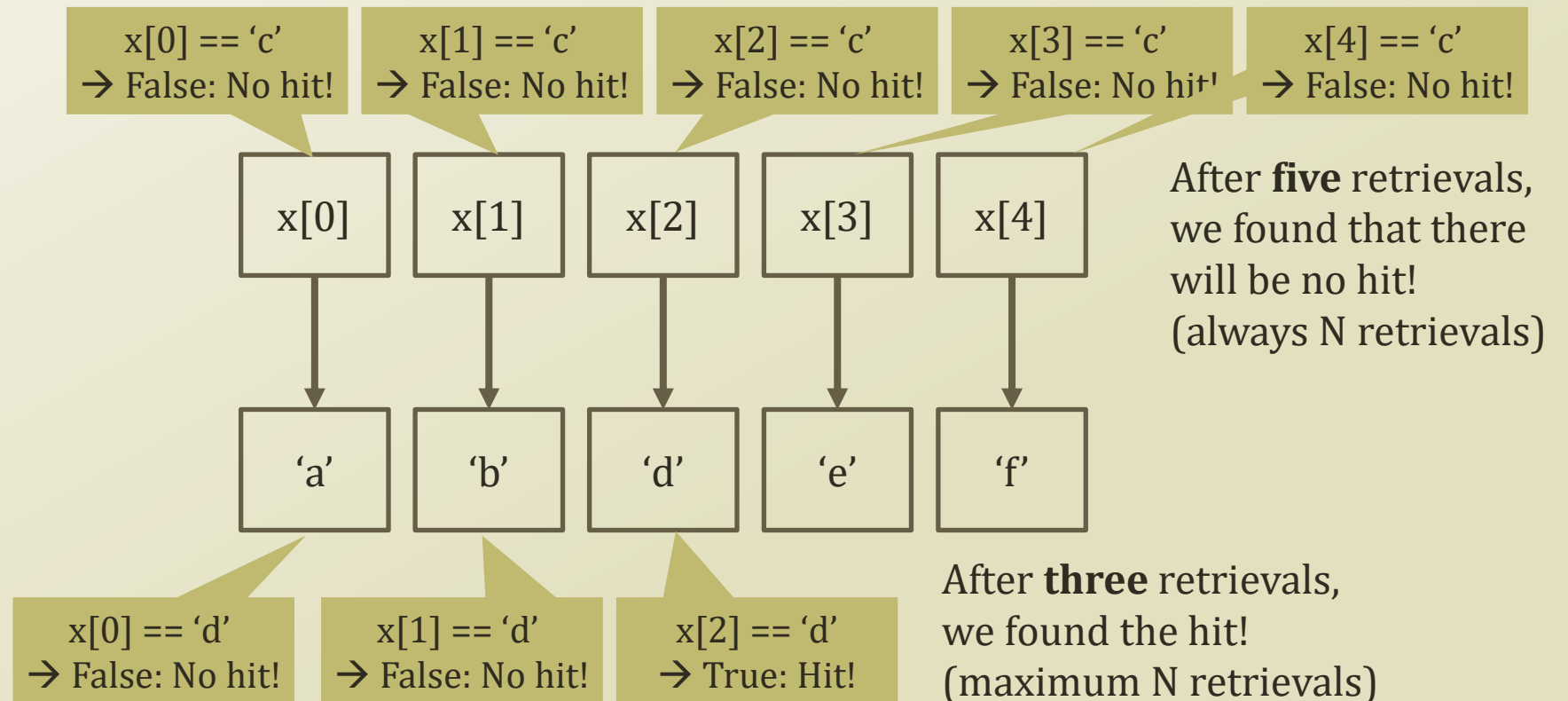
- Array (in our Python example, List, yet we will use only its index function)
 - Each element is accessible by index
 - Index is typically zero or a positive integer
 - Very simple creation
 - That's why people use it

```
x = ['a', 'b', 'd', 'e', 'f']
```



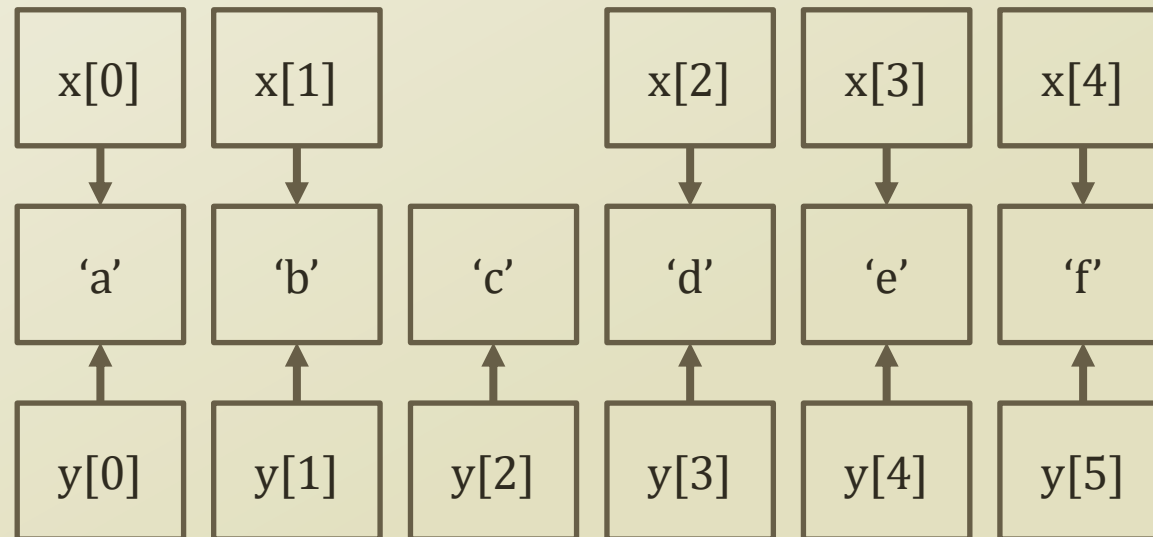
Search procedure in array

- Let's find 'd' and 'c' from the list in an array
 - Of course, you can use *'in'*, but we commit ourselves to use indexes only
- Then, navigating from the first to the last until hit is the only way



Insert procedure in array

- Let's insert 'c' between 'b' and 'd' in the list ($a = \text{insert position index}$)
 - First, make new list, or y, with six cells
 - Second, copy the reference links of $x[0:a-1]$ to $y[0:a-1]$ (*retrieval cnt.: a*)
 - Third, put a reference link to 'c' in $y[a]$ (*retrieval cnt.: 1*)
 - Fourth, copy the reference links of $x[a:]$ to $y[a+1:]$ (*retrieval cnt.: n-a-1*)
 - Fifth, change x's reference to y's reference
 - Total count of retrievals = $a + 1 + n - a - 1 = n$



```
x = ['a', 'b', 'd', 'e', 'f']
idxInsert = 2
valInsert = 'c'

y = list(range(6))

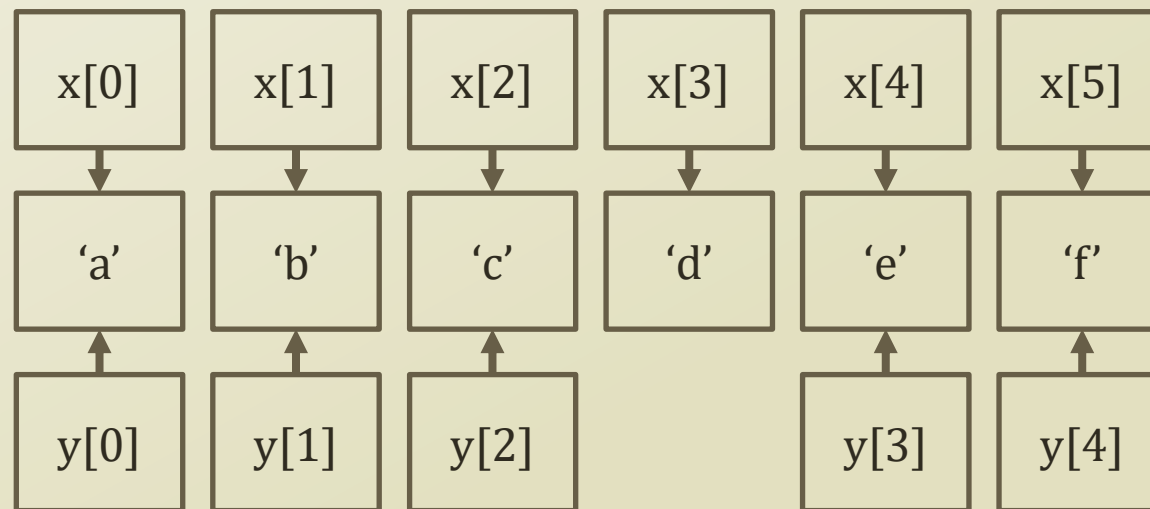
for itr in range(0, idxInsert):
    y[itr] = x[itr]

y[idxInsert] = valInsert

for itr in range(idxInsert, len(x)):
    y[itr+1] = x[itr]
```


Delete procedure in array

- Let's remove 'd' in the list ($a = \text{delete position index}$)
 - First, make new list, or y, with five cells
 - Second, copy the reference links of $x[0:a-1]$ to $y[0:a-1]$ (*retrieval cnt.: a*)
 - Third, copy the reference links of $x[a+1:]$ to $y[a:]$ (*retrieval cnt.: n-a-1*)
 - Fifth, change x's reference to y's reference
 - Total count of retrievals = $a + n - a - 1 = n - 1$



```
"""
delete example
"""
idxDelete = 3

y = list(range(5))

for itr in range(0, idxDelete):
    y[itr] = x[itr]

for itr in range(idxDelete+1, len(x)):
    y[itr-1] = x[itr]

x = y
```

Problems in Array

- Whenever you put something in or get something out
 - You have to perform line-wise retrievals
 - Which is N retrievals
(by assuming $100,000 - 1 \approx 100,000$)
 - This process is just like that
 - There is a line of airline passengers
 - You want to put a passenger in the middle of the line because his flight is about to leave
 - You are moving all the passengers one step back
 - Then, you put the customer in the line
- What-if we have a magic to create a space in the middle of the line?
 - Array \rightarrow you are bounded to the 1-dimension that you have
 - Linked list \rightarrow you are bounded no more!



LINKED LIST

Detour: Assignment and Equivalence

```
x = [1, 2, 3]
y = [100, x, 120]
z = [x, 'a', 'b']
```

```
print('x : ', x)
print('y : ', y)
print('z : ', z)
```

```
x[1] = 1717
```

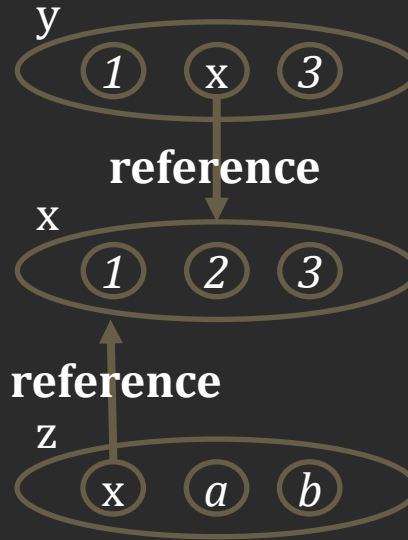
```
print('\nx : ', x)
print('y : ', y)
print('z : ', z)
```

```
x[1] = 2
x2 = [1, 2, 3]
```

```
if x == x2:
    print("Values are equivalent")
else:
    print("Values are not equivalent")
```

```
if x is x2:
    print("Values are stored at the same place")
else:
    print("Values are not stored at the same place")
```

```
if x[1] is y[1][1]:
    print("Values are stored at the same place")
else:
    print("Values are not stored at the same place")
```



```
x : [1, 2, 3]
y : [100, [1, 2, 3], 120]
z : [[1, 2, 3], 'a', 'b']
```

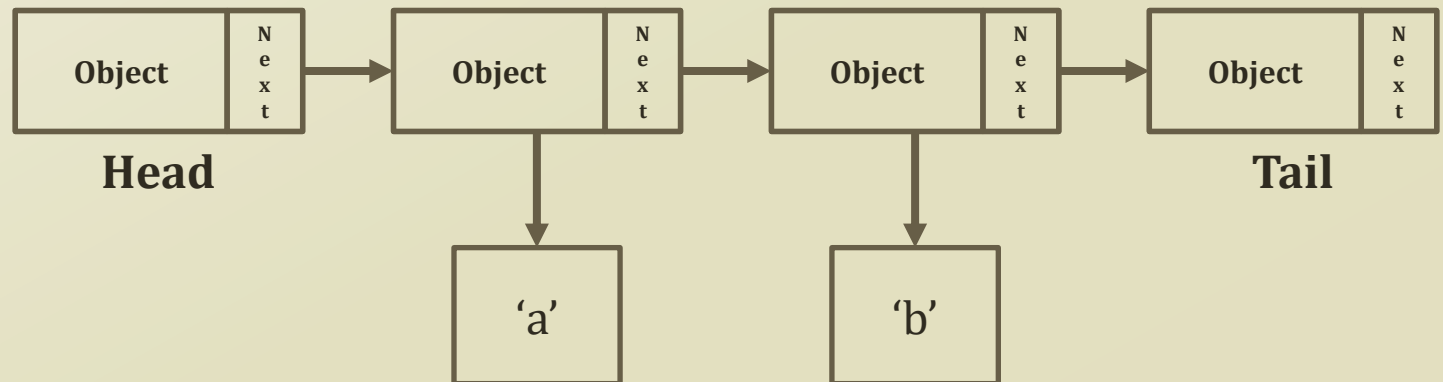
```
x : [1, 1717, 3]
y : [100, [1, 1717, 3], 120]
z : [[1, 1717, 3], 'a', 'b']
```

Values are equivalent
Values are not stored at the same place
Values are stored at the same place

- One variable's value is changed
- But, you see three changes
- Why this happened?
 - Because of references
 - x has two references from y and z
 - The values of y and z are determined by x, and x is changed
 - See the ripple effects
- **==**
 - Checks the equivalence of two referenced values
- **is**
 - Checks the equivalence of two referenced objects' IDs

Basic Structure: Singly linked list

- Construct a singly linked list with nodes and references
 - A node consists of
 - A variable to hold a reference to its next node
 - A variable to hold a reference to its value object
 - Special nodes: Head and Tail
 - You can construct the singly linked list without them
 - But, using them makes search, insert and delete more convenient
 - Generally, requires more coding than array



Implementation of Node class

- Member variables
 - Variable to reference the next node
 - Variable to hold a value object
 - (Optional) Variable to check whether it is a head or not
 - (Optional) Variable to check whether it is a tail or not
- Member functions
 - Various set/get methods

```
class Node:
    nodeNext = None
    nodePrev = ''
    objValue = ''
    binHead = False
    binTail = False

    def __init__(self, objValue = '', nodeNext = None, binHead = False, binTail = False):
        self.nodeNext = nodeNext
        self.objValue = objValue
        self.binHead = binHead
        self.binTail = binTail

    def getValue(self):
        return self.objValue

    def setValue(self, objValue):
        self.objValue = objValue

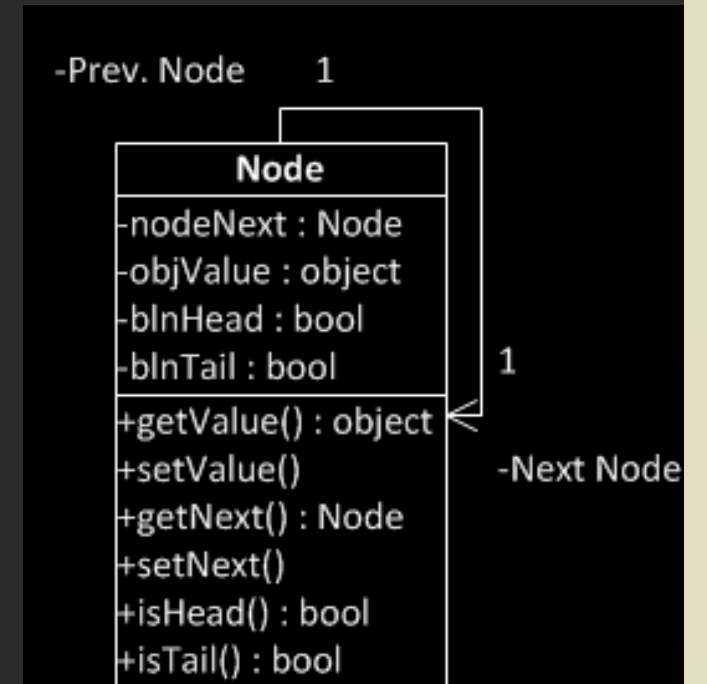
    def getNext(self):
        return self.nodeNext

    def setNext(self, nodeNext):
        self.nodeNext = nodeNext

    def isHead(self):
        return self.binHead

    def isTail(self):
        return self.binTail

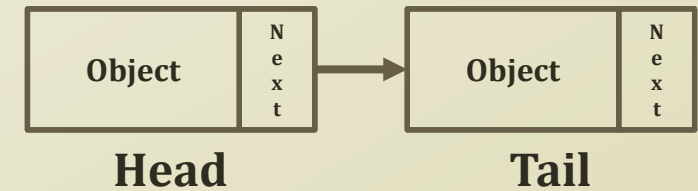
node1 = Node(objValue = 'a')
nodeTail = Node(binTail = True)
nodeHead = Node(binHead = True, nodeNext = node1)
```



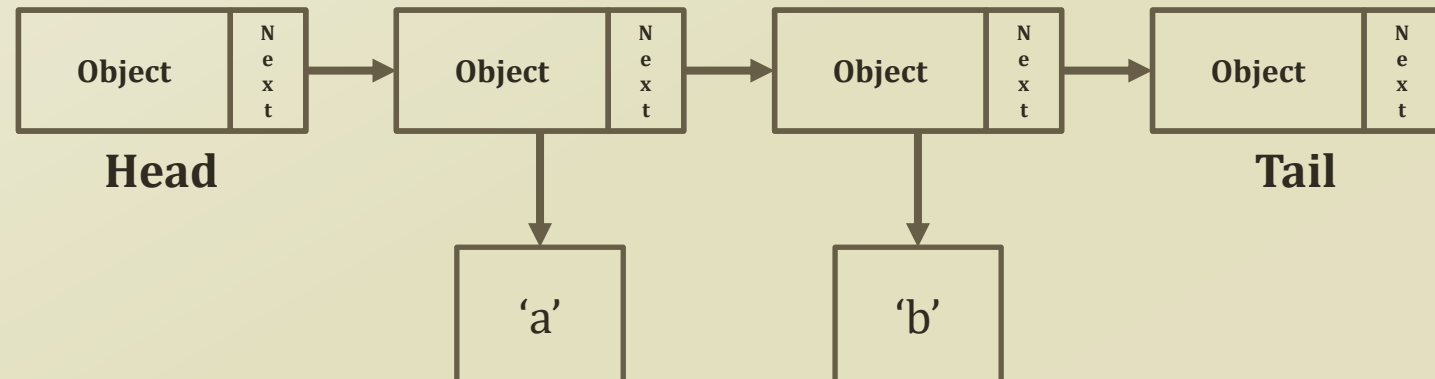
Head and Tail

- Specialized node
 - Head : Always at the first of the list
 - Tail : Always at the last of the list
 - These are the two corner stone showing the start and the end of the list
- These are optional nodes.
 - Linked list works okay without these
 - However, having these makes implementation very convenient
 - Any example?

Empty Linked List

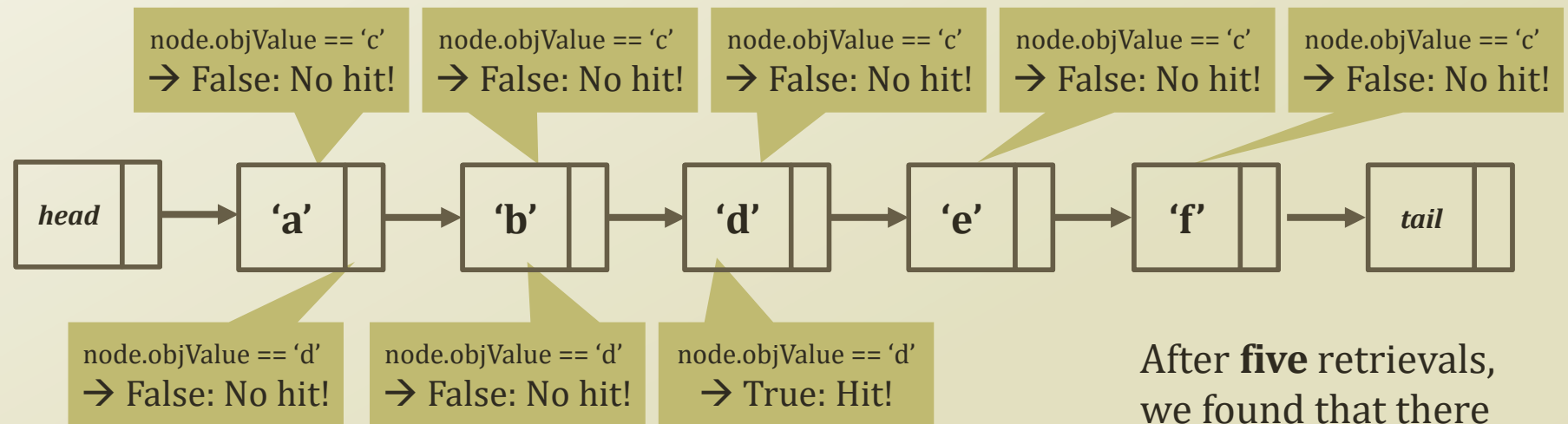


Linked List with Two Nodes



Search procedure in singly linked list

- Again, let's find 'd' and 'c' from the list
- Just like an array, navigating from the first to the last until hit is the only way
- No difference in the search pattern, though you cannot use index any further!
 - Your list implementation may include the index function, but it is not required in the linked list

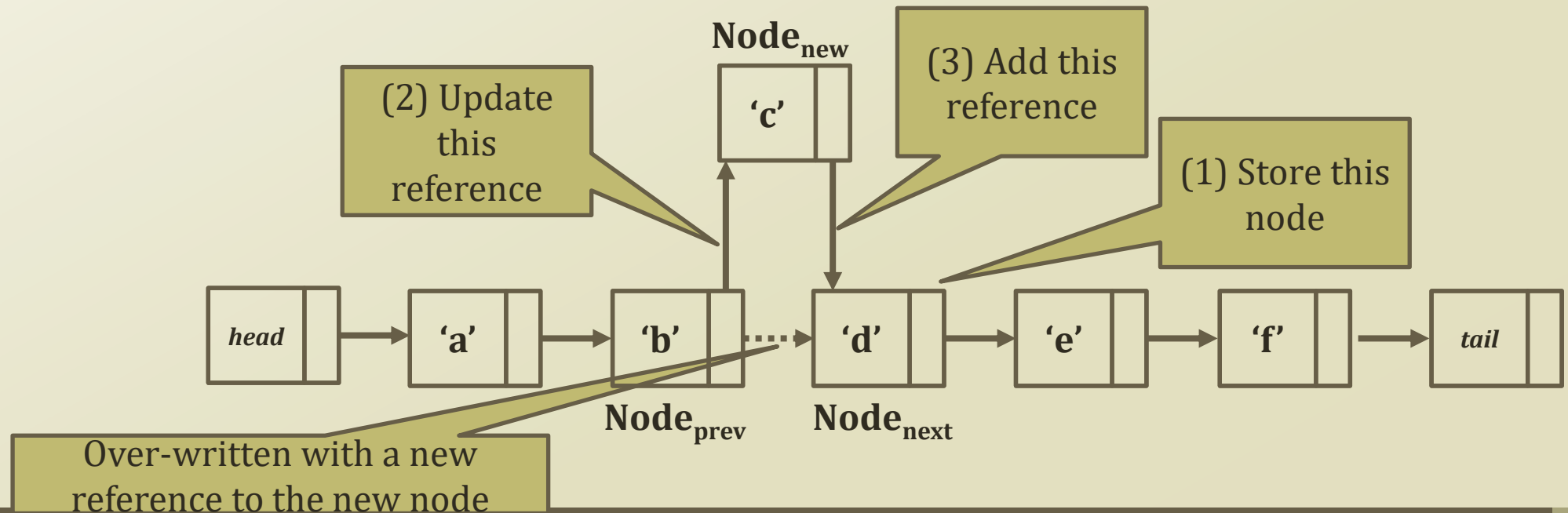


After **three** retrievals,
we found the hit!
(maximum N retrievals)

After **five** retrievals,
we found that there
will be no hit!
(always N retrievals)

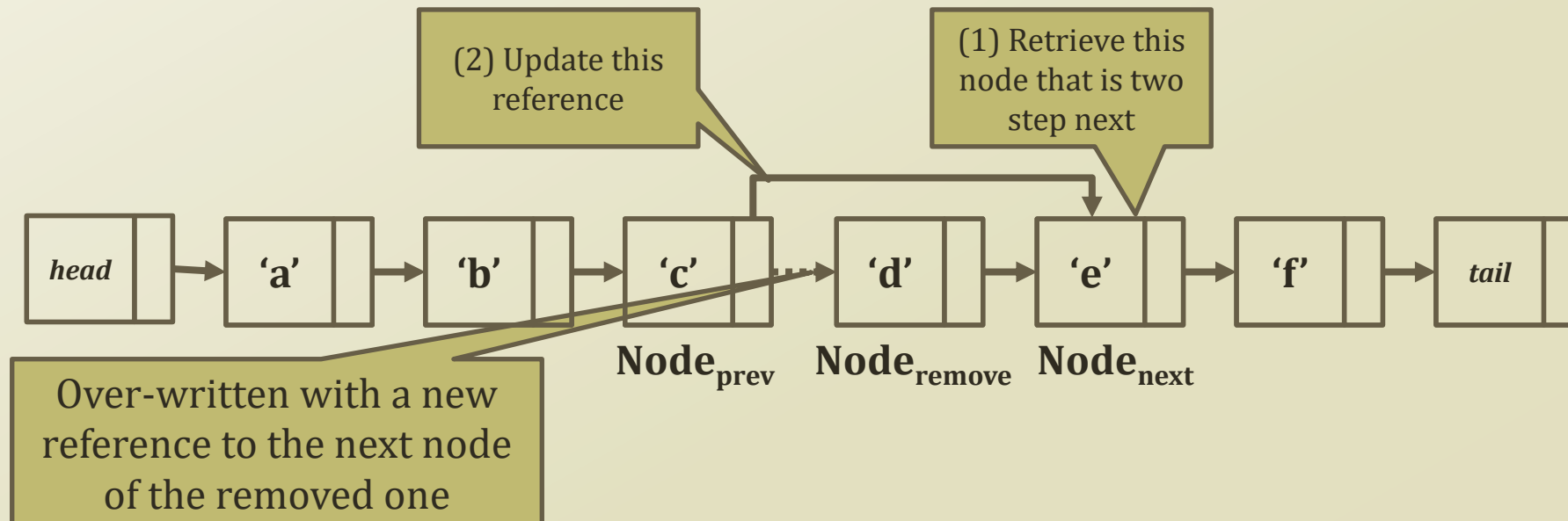
Insert procedure in singly linked list

- This is the moment that you see the power of a linked list
- Last time, you need N retrievals to insert a value in the array list
- This time, you need only three operations
 - With an assumption that you have a reference to the node, $\text{Node}_{\text{prev}}$ that you want to put your new node next
 - First, you store a Node, or a $\text{Node}_{\text{next}}$, pointed by a reference from $\text{Node}_{\text{prev}}$'s `nodeNext` member variable
 - Second, you change a reference from $\text{Node}_{\text{prev}}$'s `nodeNext` to Node_{new}
 - Third, you change a reference from Node_{new} 's `nodeNext` to $\text{Node}_{\text{next}}$



Delete procedure in singly linked list

- This is the another moment that you see the power of a linked list
- Last time, you need N retrievals to delete a value in the array list
- This time, you need only three operations
 - With an assumption that you have a reference to the node, $\text{Node}_{\text{prev}}$ that you want to remove the node next
 - First, you retrieve $\text{Node}_{\text{next}}$ that is two steps next from $\text{Node}_{\text{prev}}$
 - Second, you change a reference from $\text{Node}_{\text{prev}}$'s `nodeNext` to $\text{Node}_{\text{next}}$
- The node will be removed because there is no reference to $\text{Node}_{\text{remove}}$



Implementation of Singly linked list

```
class SinglyLinkedList:
    nodeHead = ''
    nodeTail = ''
    size = 0

    def __init__(self):
        self.nodeTail = Node(binTail=True)
        self.nodeHead = Node(binHead=True, nodeNext=self.nodeTail)

    def insertAt(self, objInsert, idxInsert):
        nodeNew = Node(objValue=objInsert)
        nodePrev = self.get(idxInsert - 1)
        nodeNext = nodePrev.getNext()
        nodePrev.setNext(nodeNew)
        nodeNew.setNext(nodeNext)
        self.size = self.size + 1

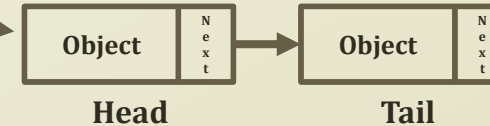
    def removeAt(self, idxRemove):
        nodePrev = self.get(idxRemove - 1)
        nodeRemove = nodePrev.getNext()
        nodeNext = nodeRemove.getNext()
        nodePrev.setNext(nodeNext)
        self.size = self.size - 1
        return nodeRemove.getValue()

    def get(self, idxRetrieve):
        nodeReturn = self.nodeHead
        for itr in range(idxRetrieve + 1):
            nodeReturn = nodeReturn.getNext()
        return nodeReturn

    def printStatus(self):
        nodeCurrent = self.nodeHead
        while nodeCurrent.getNext().isTail() == False:
            nodeCurrent = nodeCurrent.getNext()
            print(nodeCurrent.getValue(), end=" ")
        print("")

    def getSize(self):
        return self.size
```

Empty Linked List



```
list1 = SinglyLinkedList()
list1.insertAt('a', 0)
list1.insertAt('b', 1)
list1.insertAt('d', 2)
list1.insertAt('e', 3)
list1.insertAt('f', 4)
list1.printStatus()

list1.insertAt('c', 2)
list1.printStatus()

list1.removeAt(3)
list1.printStatus()
```

```
a b d e f
a b c d e f
a b c e f
```

STACK AND QUEUE

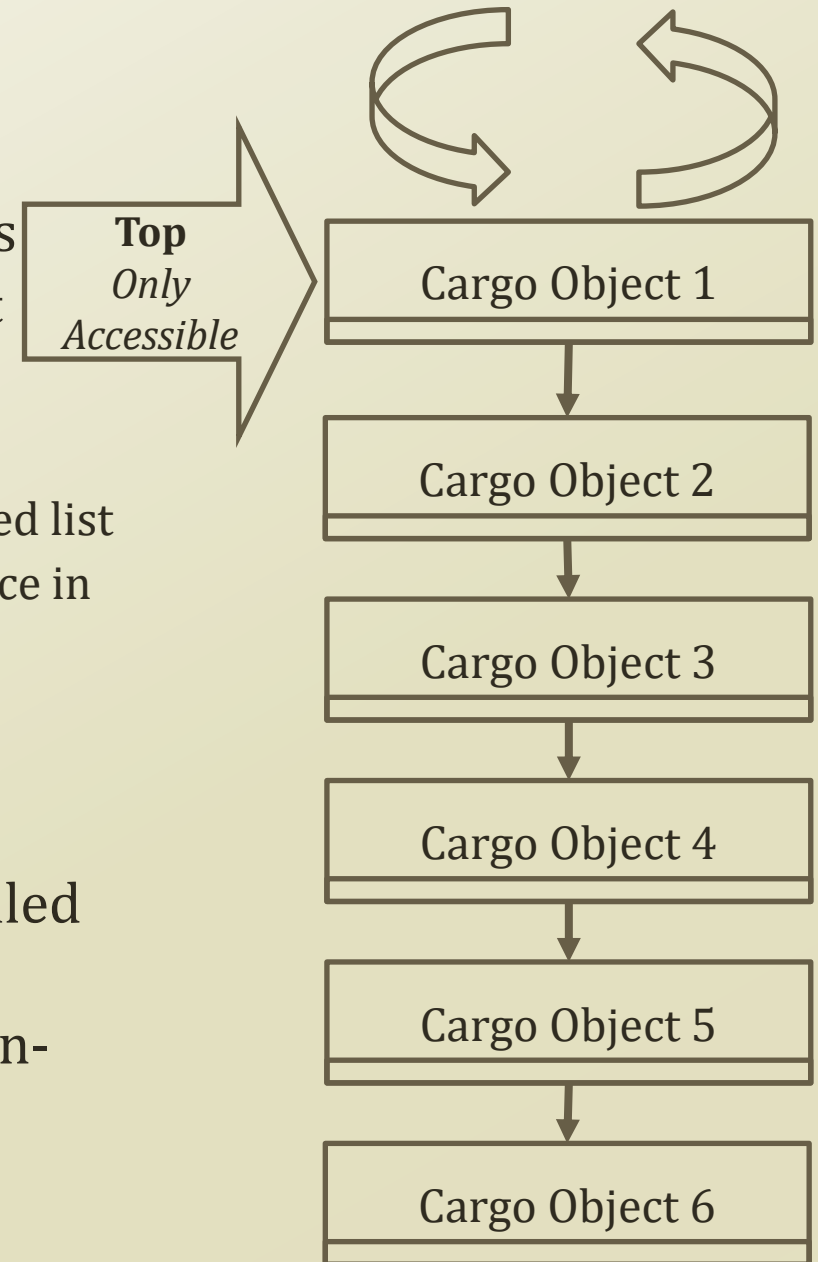
Scenario for Stack

- Back seat of a taxi
 - One way in and out
 - We are traveling from the source to our destinations
 - Who should seat first and last?
- A scenario for a taxi
= A scenario for SCM
 - A cargo plane with one way in and out
 - How to organize the cargo loading to the plane?



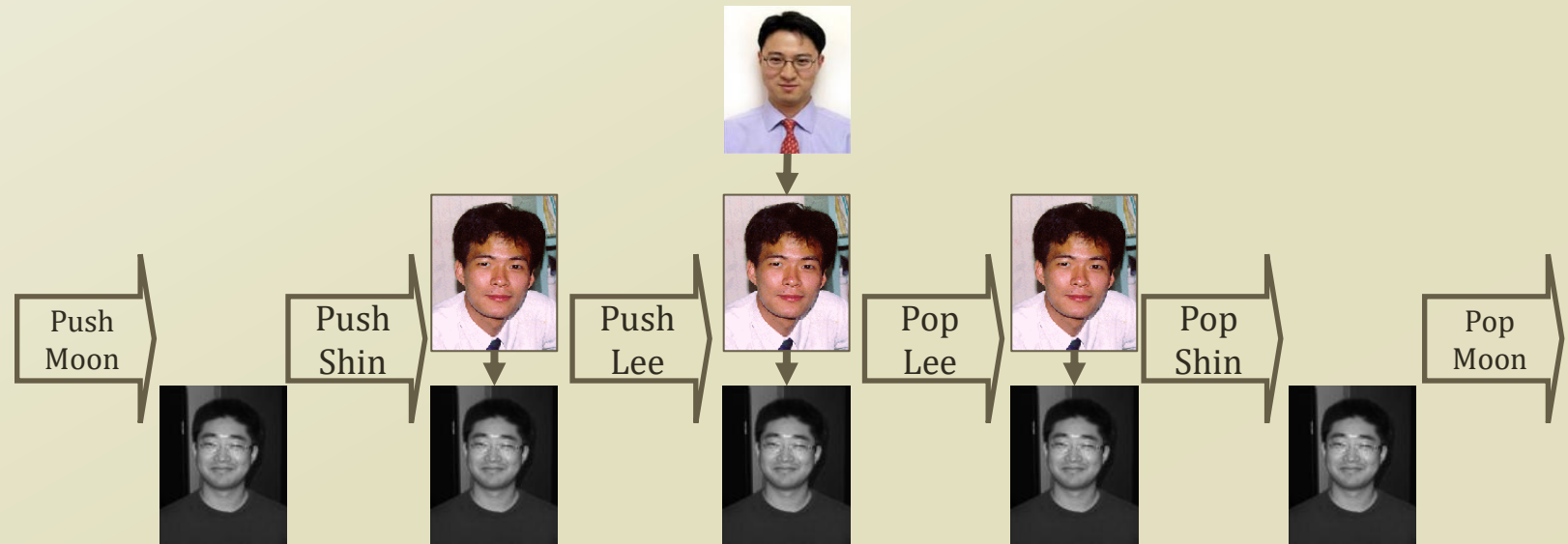
Structure of Stack

- Stacks are linear like linked lists
 - A variation of a singly linked list
- Difference
 - Voluntarily giving up
 - Access to the middle in the linked list
 - Only accesses to the first instance in the list
 - The first instance in the list
= The top instance in the stack
- An item is inserted or removed from the stack from one end called the “top” of the stack.
- This mechanism is called Last-In-First-Out (LIFO).



Operation of Stack

- Stack operation
 - Push
 - = Insert an instance at the first in the linked list
 - = Put an instance at the top in the stack
 - Pop
 - = Remove and return an instance at the first in the linked list
 - = Remove and return an instance at the top in the stack



Implementation of Stack

- Python code of a stack
 - Utilizing a singly linked list
 - To pop an instance
 - 1 retrieval count
 - To push an instance
 - 1 retrieval count

```
from src.edu.kaist.seslab.ie362.week3.SinglyLinkedList import SinglyLinkedList

class Stack(object):
    lstInstance = SinglyLinkedList()
    def pop(self):
        return self.lstInstance.removeAt(0)
    def push(self, value):
        self.lstInstance.insertAt(value, 0)

stack = Stack()
stack.push("a")
stack.push("b")
stack.push("c")

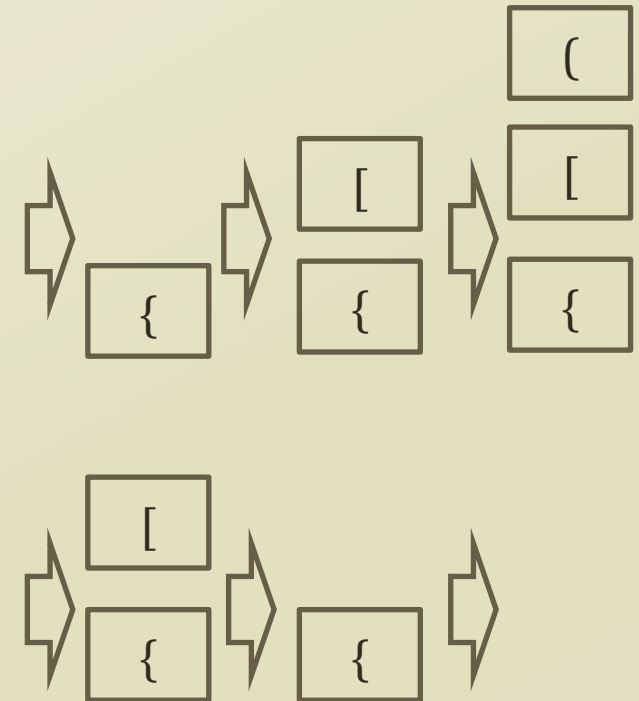
print(stack.pop())
print(stack.pop())
print(stack.pop())
```

c
b
a

Example: Balancing Symbols

- Balancing symbols?
 - $[2+(1+2)]-3 \rightarrow$ Symbols are balanced
 - $[2+(1+2)]-3 \rightarrow$ Symbols are not balanced
 - Then, just counting opening and closing symbols?
 - What if? $[2+(1)+2]-3$
- Algorithm for the balanced symbol checking
 - Make an empty stack
 - read symbols until end of formula
 - if the symbol is an opening symbol push it onto the stack
 - if it is a closing symbol do the following
 - If the stack is empty report an error
 - Otherwise pop the stack.
 - If the symbol popped does not match the closing symbol report an error
 - At the end of the of formula if the stack is not empty report an error

$7+\{[2+(1+2)]*3\}$



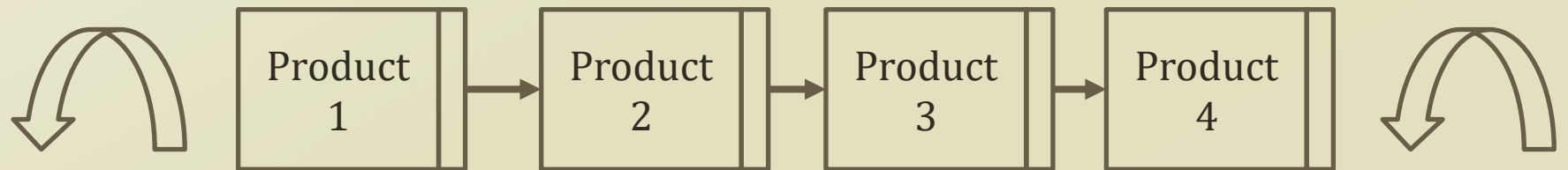
Scenario for Queue

- Line at an airport
 - A way for going in
 - At the end of the line
 - Another way for going out
 - At the first of the line
 - No one gets in the middle of the line
- A scenario of a line at the airport
= A scenario of a production line at a factory
 - The first product out is the first product in
 - How to track the production line?



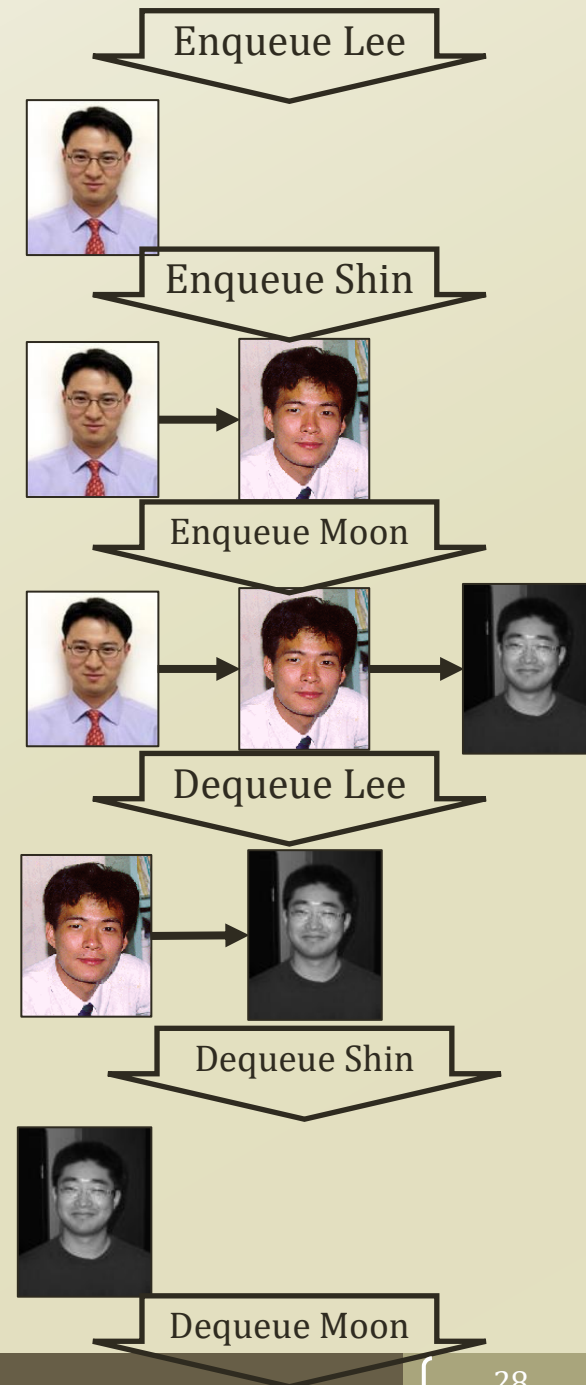
Structure of Queue

- Queues are linear like linked lists
 - A variation of a singly linked list
- Difference
 - Voluntarily giving up
 - Access to the middle in the linked list == Same to the stacks
 - Only accesses to the first and the last instances in the list
 - The first instance in the list
= The front instance in the queue
 - The last instance in the list
= The rear instance in the queue
- An item is inserted at the last
- An item is removed at the front
- This mechanism is called First-In-First-Out (FIFO)



Operation of Queue

- Queue operation
 - Enqueue
 - = Insert an instance at the last in the linked list
 - = Put an instance at the rear in the queue
 - Dequeue
 - = Remove and return an instance at the first in the linked list
 - = Remove and return an instance at the front in the queue



Implementation of Queue

- Python code of a queue
 - Utilizing a singly linked list
 - To enqueue an instance
 - 1 retrieval count
 - To dequeue an instance
 - 1 retrieval count

```
from src.edu.kaist.seslab.ie362.week3.SinglyLinkedList import SinglyLinkedList

class Queue(object):
    lstInstance = SinglyLinkedList()

    def dequeue(self):
        return self.lstInstance.removeAt(0)

    def enqueue(self, value):
        self.lstInstance.insertAt(value, self.lstInstance.getSize())

queue = Queue()
queue.enqueue("a")
queue.enqueue("b")
queue.enqueue("c")

print(queue.dequeue())
print(queue.dequeue())
print(queue.dequeue())
```

a
b
c