Roadmap

What controls to include?

Proof of Unconfoundedness given propensity score

Which of these variables would you control for? Think identification and efficiency



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 X_1 Necessary for identification



X₂ Bad for efficiency (but may be good for robustness)



 X_3 Good for efficiency



 X_4 "Collider," generates bias (but may serve as proxy for U_2 if U_2 affects D)

Which of these variables would you control for? Think identification and efficiency

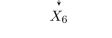






Which of these variables would you control for? Think identification and efficiency







 X_5 Bad control, generate bias

 $X_{
m 6}$ Bad control, generate bias

 X_7 Exercise for you

Roadmap

What controls to include?

Proof of Unconfoundedness given propensity score

Proof

This proof is from Imbens (2005, RESTAT) (may skip in class).

Show $\mathbb{P}(D_i = 1 \mid Y_i(0), Y_i(1), \pi(\mathbf{X}_i)) = \mathbb{P}(D_i = 1 \mid \pi(\mathbf{X}_i)) = \pi(\mathbf{X})_i$ which implies that D_i and the potential outcomes are independent conditional on the propensity score

Proof

$$\mathbb{P}(D_{i} = 1 \mid Y_{i}(0), Y_{i}(1), \pi(\boldsymbol{X}_{i})) = \mathbb{E}[D_{i} = 1 \mid Y_{i}(0), Y_{i}(1), \pi(\boldsymbol{X}_{i})] \\
= \mathbb{E}[\mathbb{E}[D_{i} = 1 \mid Y_{i}(0), Y_{i}(1), \pi(\boldsymbol{X}_{i}), \boldsymbol{X}_{i}] \mid Y_{i}(0), Y_{i}(1), \pi(\boldsymbol{X}_{i})] \\
= \mathbb{E}[\mathbb{E}[D_{i} = 1 \mid Y_{i}(0), Y_{i}(1), \boldsymbol{X}_{i}] \mid Y_{i}(0), Y_{i}(1), \pi(\boldsymbol{X}_{i})] \\
= \mathbb{E}[\mathbb{E}[D_{i} = 1 \mid \boldsymbol{X}_{i}] \mid Y_{i}(0), Y_{i}(1), \pi(\boldsymbol{X}_{i})] \\
= \mathbb{E}[\pi(\boldsymbol{X}_{i}) \mid Y_{i}(0), Y_{i}(1), \pi(\boldsymbol{X}_{i})] = \pi(\boldsymbol{X}_{i}),$$

where 1st equality comes from D_i being a 0/1 variable, 2nd from LIE, 4th from CIA (conditional on X), and fifth from definition of the propensity score