Topic 4: Regression Discontinuity

ECON 5783 — University of Arkansas

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Regression Discontinuity Design (RDD)

Example 1

Lee (2008, JOE) sutdies the “incumbency advantage”, the hypothesis that being a serving elected official improves future election outcomes

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- e.g. are more charming, in a more one-sided district, have a better resume

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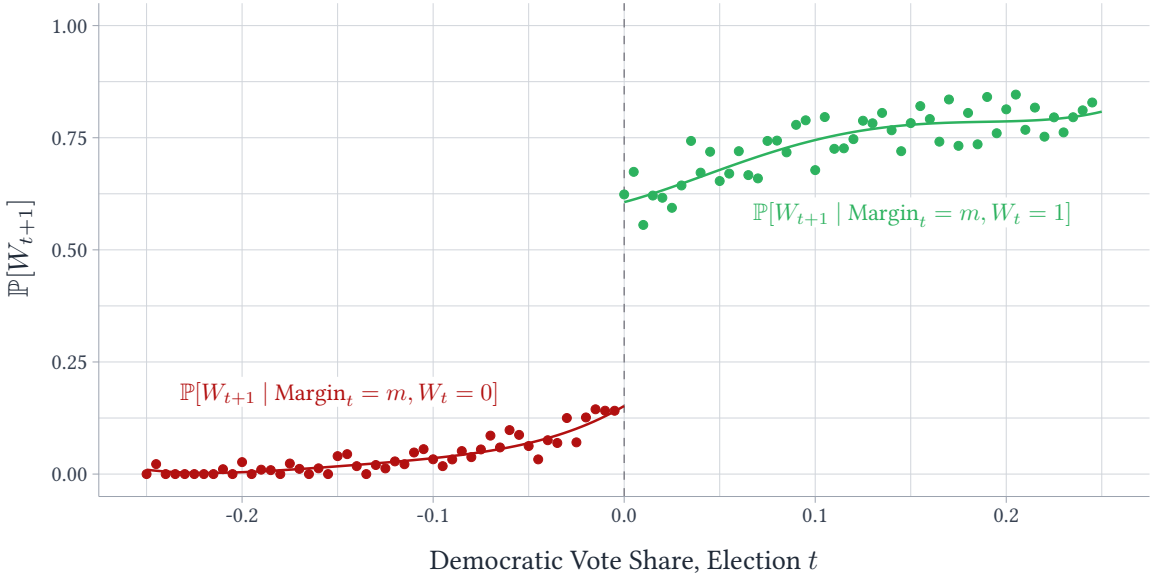
The problem, of course, is that candidates who won their election usually are different than those that do not

- e.g. are more charming, in a more one-sided district, have a better resume

Lee’s idea is to compare candidates who narrowly lost to those that narrowly won

- By having similar vote percentage, Lee hopes, to be comparing candidates with similar unobservables


1.00
0.75
0.50
0.25
0.00
$\mathbb{P}[W_{t+1}]$
-0.2
-0.1
0.0
0.1
0.2
Democratic Vote Share, Election t
$\mathbb{P}[W_{t+1} \mid \text{Margin}_t = m, W_t = 0]$
$\mathbb{P}[W_{t+1} \mid \text{Margin}_t = m, W_t = 1]$

Regression Discontinuity Design (RDD)

Example 1

There is a clear jump from candidates that narrowly lost to candidates that narrowly won

- The main concern is that candidates that just lost look different in terms of unobservables to those that just won

Regression Discontinuity Design (RDD)

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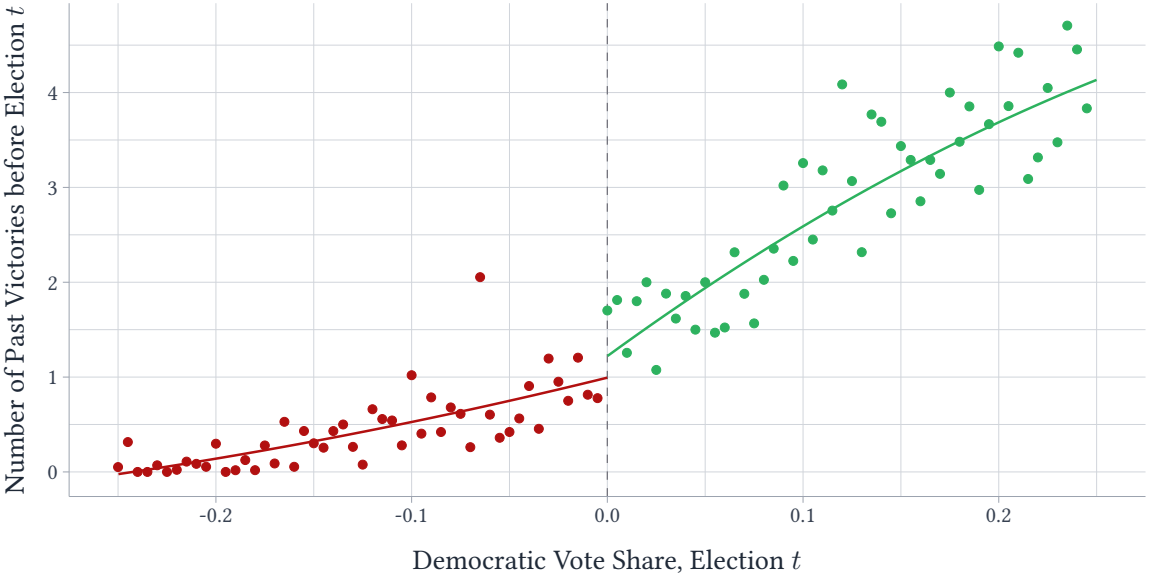
- The main concern is that candidates that just lost look different in terms of unobservables to those that just won

To try and alleviate this concern, Lee checks if other observed variables ‘jump’ at the cutoff

- A jump in other ‘pre-determined’ variables at the cutoff would be problematic to our story


Number of Past Victories before Election t
4
3
2
1
0
-0.2
-0.1
0.0
0.1
0.2
Democratic Vote Share, Election t

Regression Discontinuity Design (RDD) Terminology

In the RDD literature, we will have a 'running' variable ('score' variable), X_i, and a 'cutoff' value c

- Units with $X_i < c$ do not receive the 'policy' and units with $X_i \geq c$ do

The treatment variable is defined as $D_i = \mathbb{1}[X_i \geq c]$

Regression Discontinuity Design (RDD)

Example 2

Bleemer and Mehta (2022, AEJ Applied) study the returns to being an economics major by leveraging a requirement of a 2.8 GPA threshhold in Econ 1 and 2 at UCSC:

- Comparing students just above a 2.8 GPA to those just below helps address selection into economics major
- A potential concern is that highly-motivated students near the threshold might ask for higher grade, extra credit, etc.

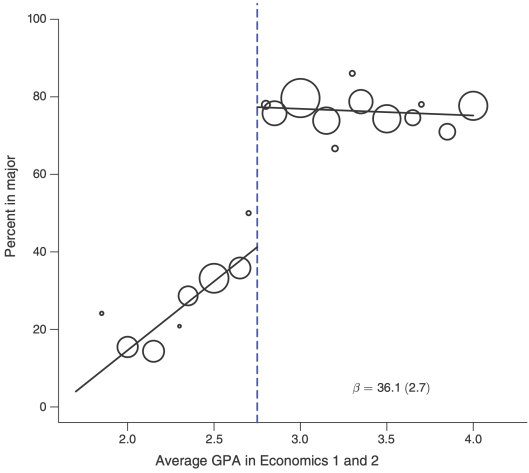


FIGURE 1. THE EFFECT OF THE UCSC ECONOMICS GPA THRESHOLD ON MAJORING IN ECONOMICS

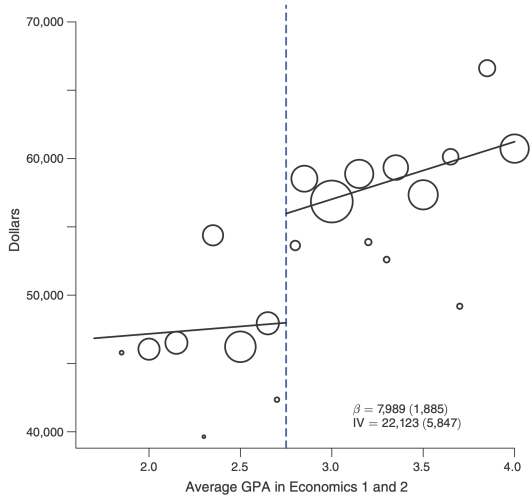


FIGURE 2. THE EFFECT OF THE UCSC ECONOMICS GPA THRESHOLD ON ANNUAL WAGES

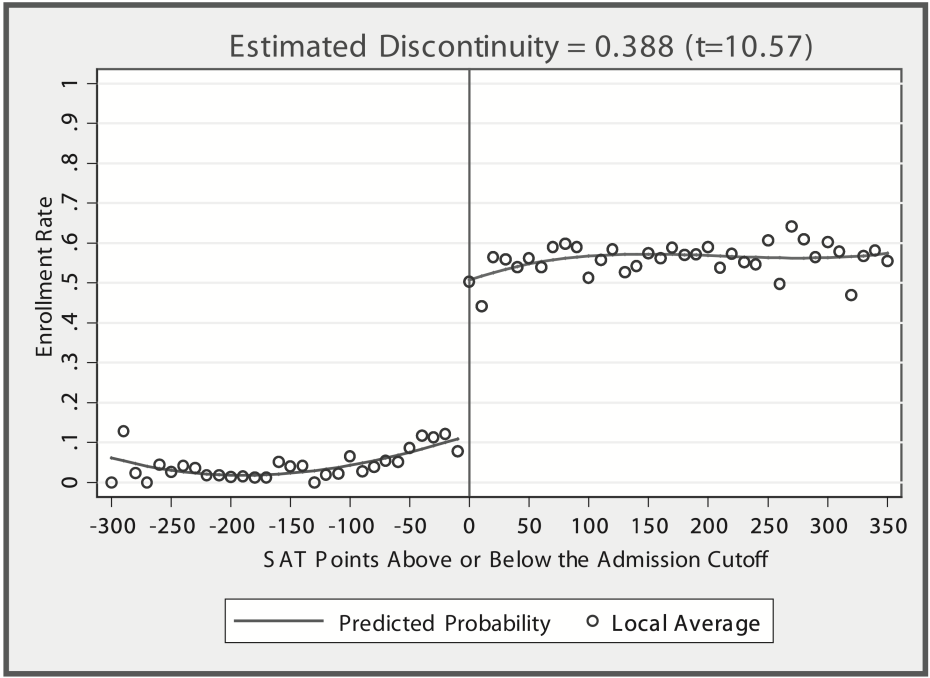
Regression Discontinuity Design (RDD)

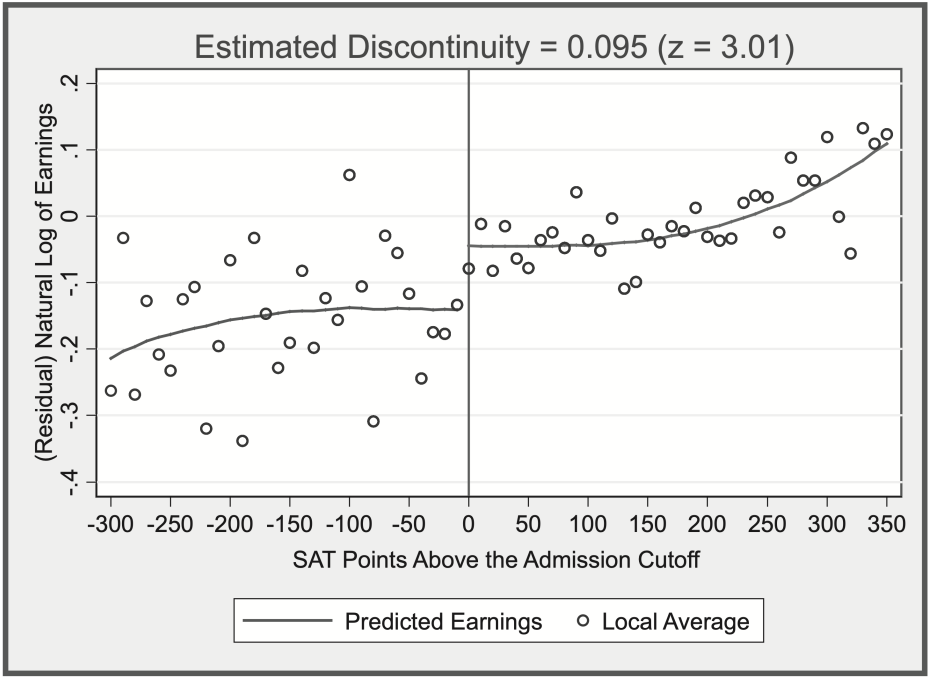
Example 3

Hoekstra (2009, RESTAT) estimates the impact of attending a ‘flagship university’ on future earnings

- There was an internal (secret) SAT cutoff that had a large increase in the probability of acceptance

That this cutoff was secret helps with concerns over retaking SAT, e.g.


Estimated Discontinuity = 0.388 (t=10.57)
Enrollment Rate
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
-300
-250
-200
-150
-100
-50
0
50
100
150
200
250
300
350
SAT Points Above or Below the Admission Cutoff
Predicted Probability
Local Average


Estimated Discontinuity = 0.095 (z = 3.01)
(Residual) Natural Log of Earnings
.2
.1
0
-.1
-.2
-.3
-.4
-300 -250 -200 -150 -100 -50 0 50 100 150 200 250 300 350
SAT Points Above the Admission Cutoff
Predicted Earnings
Local Average

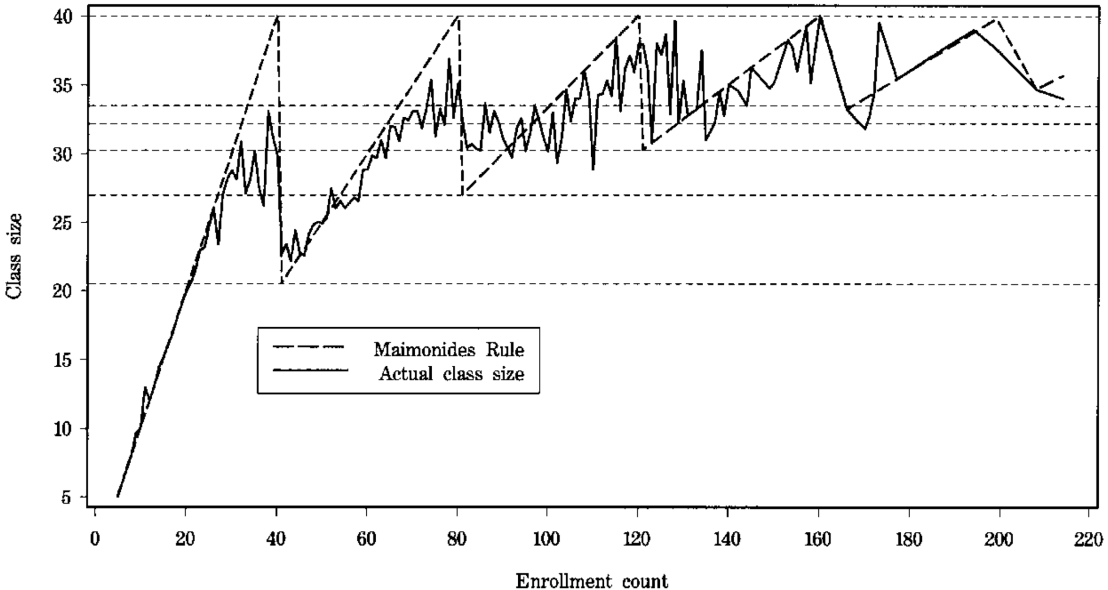
Regression Discontinuity Design (RDD)

Example 4

Angrist and Lavy (1999, QJE) study the effect of class size on kid's learning. They use a rule in Israel that requires all classes to have 40 or fewer students

- This creates sharp drops in class size at 41, 81, 121, 161 students

a. Fifth Grade



Regression Discontinuity Design (RDD)

Example 5

Anderson and Magruder (2011, Economics Journal) study the impact of restaurant Yelp ratings on restaurant business outcomes

- Yelp takes the average rating (e.g. 3.27) and shows the nearest number of stars rounding/up or down. E.g. 3.24 rounds down to 3 and 3.25 rounds up to 3.5

Regression Discontinuity Design (RDD)

Example 5

Anderson and Magruder (2011, Economics Journal) study the impact of restaurant Yelp ratings on restaurant business outcomes

- Yelp takes the average rating (e.g. 3.27) and shows the nearest number of stars rounding/up or down. E.g. 3.24 rounds down to 3 and 3.25 rounds up to 3.5

Their argument is that the score, average rating. is a noisy measure of the true quality and so restaurants look the same on either side of the cutoff

- Their main concern is 'manipulation' of running variable (we'll come back to this)

Regression Discontinuity Design (RDD)

Example 6

Turner et. al. (2014, ECTA) study the impacts of land-use regulations on the value of land

- They compare homes on one side of a zoning border to homes on the others
- The assumption is that the neighborhood doesn't 'abruptly' change when crossing the zoning boundary

This is an example of a spatial RDD which we'll discuss more about later

Difficulties with RDD

We can not do our usual strategy of comparing treated and untreated individuals with the same X_i

- Either everyone is treated or no one is

But, there is one point where we *kind of* have treated and untreated units: the cutoff c

- This is our intuition for using ‘just above’ versus ‘just below’ the cutoff.

Formalizing RDD (Take 1)

"Noisy" running variable

Let's use a canonical example of students taking a test (X_i) and students above a certain cutoff, c, are treated (e.g. put in honor's class).

Students will vary in terms of their expected score, $\sigma_i = \mathbb{E}[X_i]$. We think that future outcomes Y_i vary systematically based on σ_i

- Comparing students above and below the cutoff will be biased

“Noisy” running variable creates an experiment

For random reasons (a weird question, skipped breakfast, etc.) students score is given by $X_i = \sigma_i + \varepsilon_i$, where ε is random noise.

- For students with σ_i close to c, ε_i will make them above or below the cutoff

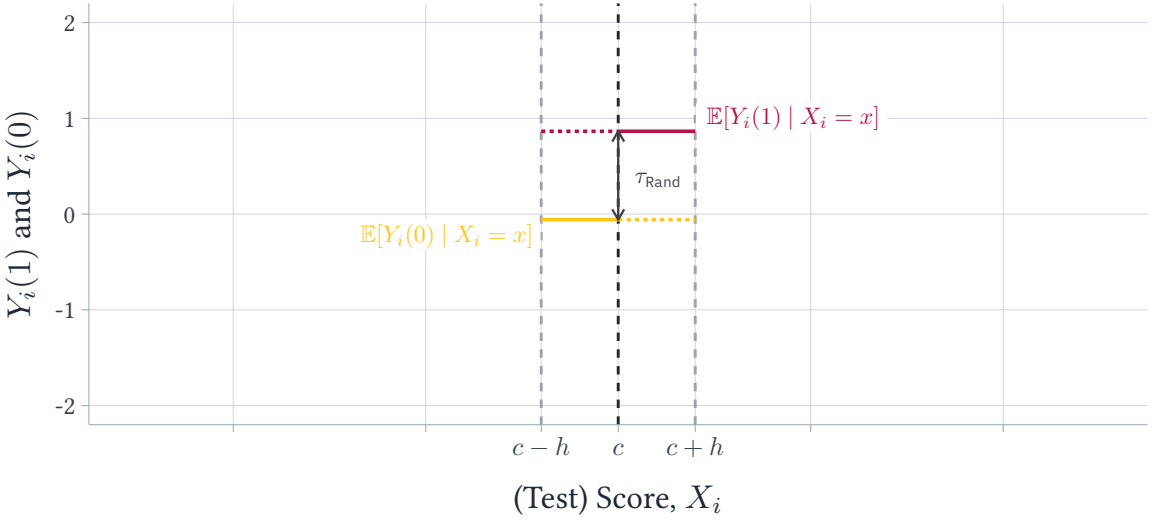
“Noisy” running variable creates an experiment

For random reasons (a weird question, skipped breakfast, etc.) students score is given by $X_i = \sigma_i + \varepsilon_i$, where ε is random noise.

- For students with σ_i close to c, ε_i will make them above or below the cutoff

This means for σ_i close to c, we have a *natural experiment*

- Of course, we don’t observe σ_i, so instead we take students within some range $c \pm h$.
- If h is “small enough”, we can do a difference in means between students below and above the cutoff


2
1
0
-1
-2
$Y_i(1)$ and $Y_i(0)$
$\mathbb{E}[Y_i(1) \mid X_i = x]$
τ_{Rand}
$\mathbb{E}[Y_i(0) \mid X_i = x]$
$c - h$
c
$c + h$
(Test) Score, X_i

“Local” treatment effect

In essence, we throw out all the data outside of $[c-h, c+h]$ and compare students with $\sigma_i \approx c$

This means we only learn about what the impact of treatment is for students with $\sigma_i \approx c$

- E.g. if the cutoff is an SAT score of 1950, then we estimate the impact of treatment for students with around that score

“Local” treatment effect

That is, we compare the following:

$$\begin{aligned}
&\mathbb{E}[Y_i \mid c < \sigma_i < c + h] - \mathbb{E}[Y_i \mid c - h < \sigma_i < c] \\
&\quad = \mathbb{E}[Y_i(1) \mid c < \sigma_i < c + h] - \mathbb{E}[Y_i(0) \mid c - h < \sigma_i < c]
\end{aligned}$$

“Local” treatment effect

That is, we compare the following:

$$\begin{aligned}
&\mathbb{E}[Y_i \mid c < \sigma_i < c + h] - \mathbb{E}[Y_i \mid c - h < \sigma_i < c] \\
&= \mathbb{E}[Y_i(1) \mid c < \sigma_i < c + h] - \mathbb{E}[Y_i(0) \mid c - h < \sigma_i < c] \\
&\approx \mathbb{E}[Y_i(1) \mid \sigma_i = c] - \mathbb{E}[Y_i(0) \mid \sigma_i = c]
\end{aligned}$$

That is, we estimate the CATE for people with $\sigma_i \approx c$

How much ‘noise’ is there in the running variable?

How do we know how large to make the cut-off h?

- The gold-star answer is to have some application-specific understanding to know how much noise there is and take h to be half that noise

Otherwise, we are left to trying to determine a ‘reasonable’ h

How much ‘noise’ is there in the running variable?

There is a trade-off between using a smaller or larger h

- On the one hand, a smaller h makes it more likely that units in $(c - h, c)$ look similar to units in $(c, c + h)$
- On the other, a larger h uses more observations for estimation. A larger h runs a risk of including units that differ systematically from the $\sigma_i = c$ units

When data on attributes of units are available, then we can use those to help with determining h

Estimation of h using extra covariates

Cattaneo, Frandsen and Titiunik (2015, Journal of Causal Inference) recommend a procedure (available in `rdlocrand`) where:

- Start with a very small h_1 and test if the mean of $\boldsymbol{X}_i$ are the same in $(c - h_1, c)$ and $(c, c + h_1)$
- If you fail to reject the null of no difference in means, then expand to h_2
- Continue until you reject the null

The idea being, select the largest h where units 'look the same' on both sides of the cutoff.

`rdlocrand` basic syntax

```
library(rdlocrand)

# Estimate effect and get p-values
rdrandinf(
  Y = df$y, R = df$score, cutoff = 0, wl = 0.025, wr = 0.025
)

# Estimate optimal h
rdwinselect(
  R = df$score, X = cbind(df$x1, df$x2, df$x3), obsmin = 10, wobs = 5
)
```

Local randomization argument

This approach has a nice intuition: local to the cutoff we have a quasi-experiment due to noise in the running variable

Despite this, this approach is less popular than the next approach we discuss

- In part, sometimes we don’t believe there is noise in the running variable
- The plots, like in Lee (2008), are the bread and butter of the RDD
  - → These are not "difference-in-means" but instead rely on trying to identify a "jump" in smoothed lines

Formalizing RDD (Take 2)

Continuity of outcomes

The second approach we will discuss is the older (and more commonly used) approach to RDD estimation

- Instead of assumping the score is randomly assigned, this method will rely on a 'continuity assumption' of potential outcomes

Formalizing RDD (Take 2)

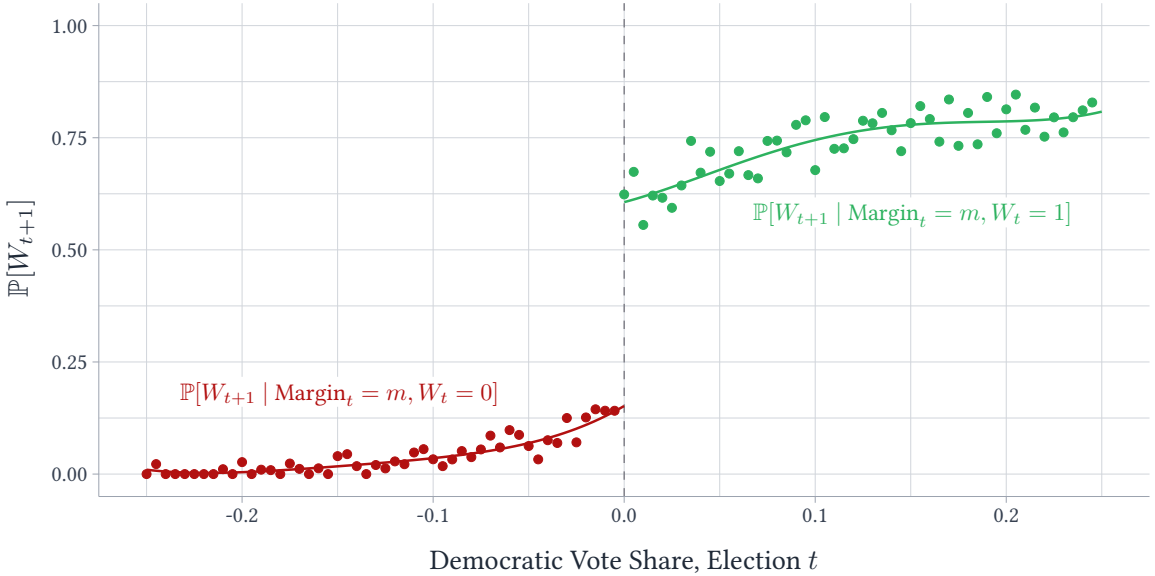
Continuity of outcomes

The second approach we will discuss is the older (and more commonly used) approach to RDD estimation

- Instead of assumping the score is randomly assigned, this method will rely on a 'continuity assumption' of potential outcomes

The continuity assumption says that both the treated and untreated potential outcomes evolve 'smoothly' and do not have an abrupt 'jump' (discontinuity) at the cutoff

- This is what our mind 'tells us to do' when we see these RDD plots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0.25
0.00
$\mathbb{P}[W_{t+1}]$
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-0.1
0.0
0.1
0.2
Democratic Vote Share, Election t
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$\mathbb{P}[W_{t+1} \mid \text{Margin}_t = m, W_t = 1]$

Formalizing continuity

The goal of our treatment effect estimator is to identify the following:

$$\mathbb{E}[Y_i(0) \mid X_i = c] \text{ and } \mathbb{E}[Y_i(1) \mid X_i = c]$$

We don't typically observe anyone with X_i exactly equal to 0 (assuming continuous X_i)

- So, necessarily we need to extrapolate to the cutoff using observations away from the cutoff

Formalizing continuity

Simliar to the selection on observables, define:

$$\mu_d(x) = \mathbb{E}[Y_i(d) \mid X_i = x]$$

to be the conditional expectation of $Y_i(0)/Y_i(1)$ conditional on the running variable being equal to x

We will fit $\mu_0(x)$ using obseravtions with $X_i < c$ and $\mu_1(x)$ using $X_i > c$

- E.g. fit a linear model of X_i with observations in $(c-h, c)$ / $(c, c+h)$

Extrapolation as limits

We are able to learn about the relationship between Y_i and X_i away from the cutoff. We are going to need to take out model and extrapolate it to the cutoff

- Imagine taking averages of Y_i from $(c, c+h)$ to estimate $\mathbb{E}[Y_i(1) \mid X_i = c]$. Assuming infinite data, as $h \to 0$ our average should get closer to closer to the true CEF.

More formally, under 'continuity' we have

$$\mathbb{E}[Y_i(1) \mid X_i = c] = \lim_{s \downarrow c} \mathbb{E}[Y_i(1) \mid X_i = x]$$

- Note here, we are taking the limit from above to only use obs. with $Y_i = Y_i(1)$

RDD Estimand

The regression discontinuity estimand is formed as follows:

$$\begin{aligned}\tau_{\texttt{RD}} &= \mathbb{E}[Y_i(1) - Y_i(0) \mid X_i = c] \\ &= \lim_{s \downarrow c} \mathbb{E}[Y_i \mid X_i = x] - \lim_{s \uparrow c} \mathbb{E}[Y_i \mid X_i = x]\end{aligned}$$

- In words, take the difference between the right-hand limit of $\mu_1(x)$ and the left-hand limit of $\mu_0(x)$

Why ‘continuity’?

Since we are fitting a model using observations away from the cutoff c, we are extrapolating the estimated $\mu_0(x)$ from the range of X_i we used for estimation to the cutoff $X_i = c$

- For this to work, we need the $\mu_d(x)$ to be continuous in a neighborhood around c

Why might continuity fail?

One of the main concerns people have with an RDD empirical application is that units are 'sorting' near the cutoff:

- E.g. if there is an SAT cutoff for a scholarship, some kinds of students who were close to making the threshold might retake the SAT multiple times

If students who retook look different than those that don't, $\implies$ at a jump in $Y_i(0)$ at the cutoff

- $c + \varepsilon$ is 'contaminated' by the retakers

Why might continuity fail?

In general, the main intuition is that we want there to be no ‘sorting’ around the threshold

- If we think units’ characteristics (observable and unobservable) are smooth over the cutoff, then it’s reasonable to assume the potential outcomes are smooth too

Why might continuity fail?

In general, the main intuition is that we want there to be no ‘sorting’ around the threshold

- If we think units’ characteristics (observable and unobservable) are smooth over the cutoff, then it’s reasonable to assume the potential outcomes are smooth too

For the covariates you do observe, can show that these do not jump at the cutoff

- Hopefully, the unobservables do not as well

Using observations ‘near’ the cutoff

Returning to the notion of continuity, to estimate $\mu_d(x)$ at the cutoff $X_i = c$, we want to use observations very close to the cutoff

- So that we extrapolate as little as possible

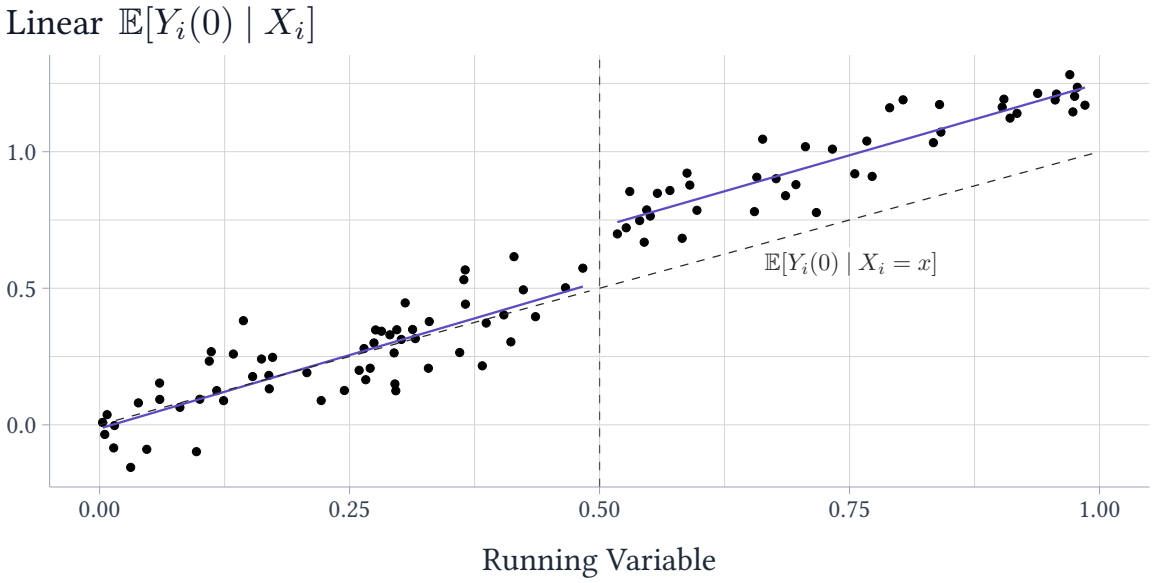
In finite samples, there might be very few observations near the cutoff

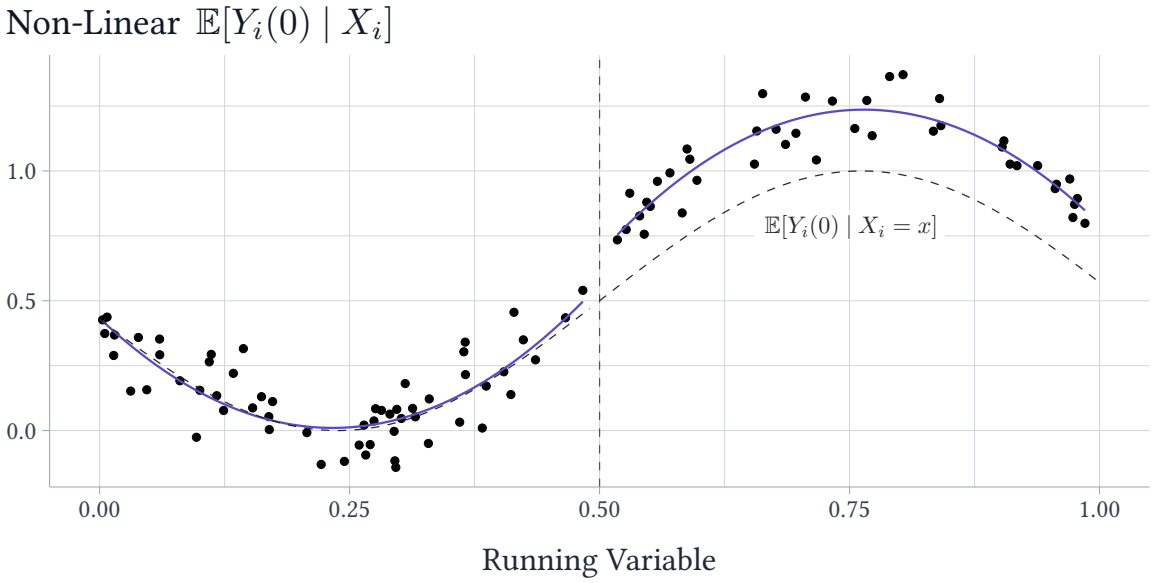
- In these settings, noise in the data can make it hard to learn information about the data-generating process

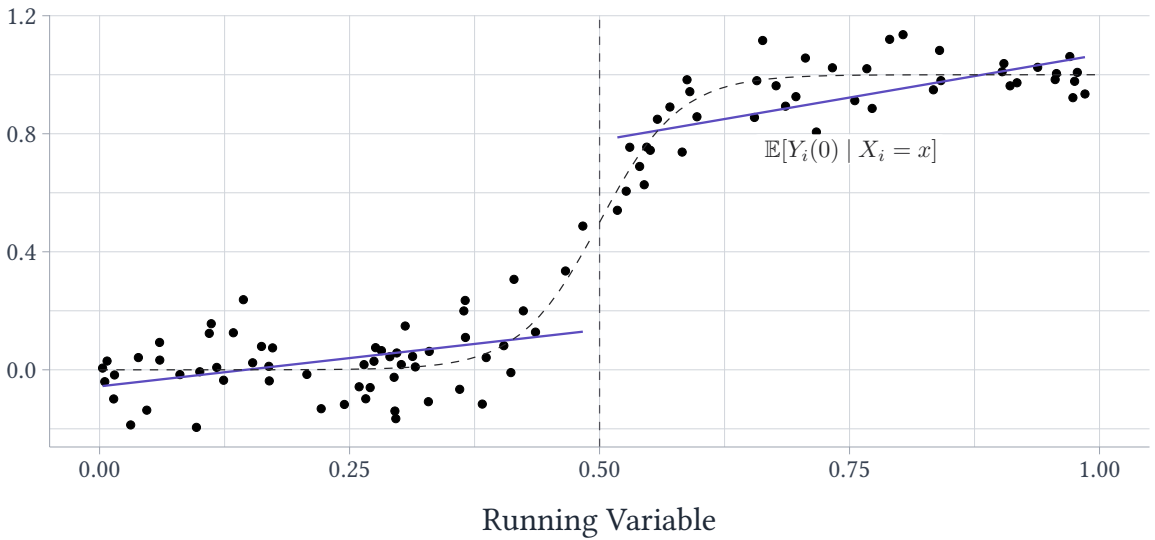
Estimation of RDD

There are *many* different approaches to estimation of the RD coefficient, but there are a few common questions:

1. How large of a window around the cutoff should we use?
   → Smaller window relies on less 'extrapolation' but is 'noisier' (bias-variance trade-off)
2. How should we fit the model of $\mathbb{E}[Y_i(d) \mid X_i = x]$
   → Simpler models are more robust to extrapolation, but may get the functional form wrong


Linear $\mathbb{E}[Y_i(0) \mid X_i]$
1.0
0.5
0.0
0.00
0.25
0.50
0.75
1.00
$\mathbb{E}[Y_i(0) \mid X_i = x]$
Running Variable


Non-Linear $\mathbb{E}[Y_i(0) \mid X_i]$
1.0
0.5
0.0
0.00
0.25
0.50
0.75
1.00
$\mathbb{E}[Y_i(0) \mid X_i = x]$
Running Variable


Non-Linear mistaken for linear
1.2
0.8
0.4
0.0
0.00
0.25
0.50
0.75
1.00
Running Variable
$\mathbb{E}[Y_i(0) \mid X_i = x]$

Difference-in-means estimation

For now, take the window to be $(c-h, c+h)$; we will return to the choice of h later in the slides

The simplest estimator is the *locally constant estimator*

$$\mu_0(x) = \hat{\mathbb{E}}[Y_i \mid X_i \in (c-h, c)] \text{ and } \mu_1(x) = \hat{\mathbb{E}}[Y_i \mid X_i \in (c-h, c)]$$

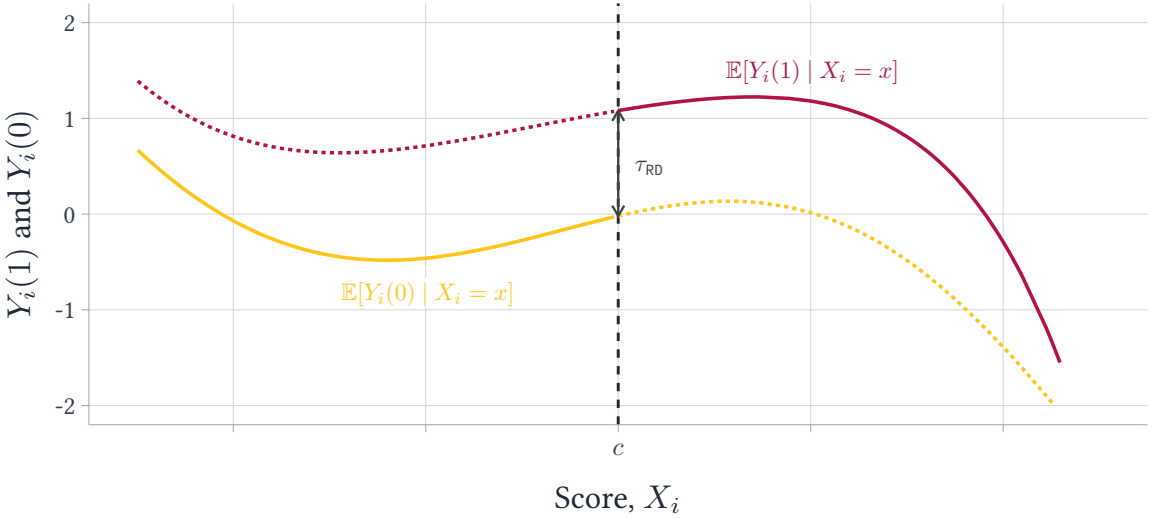
- This is our difference-in-means estimator

Estimation via Regression

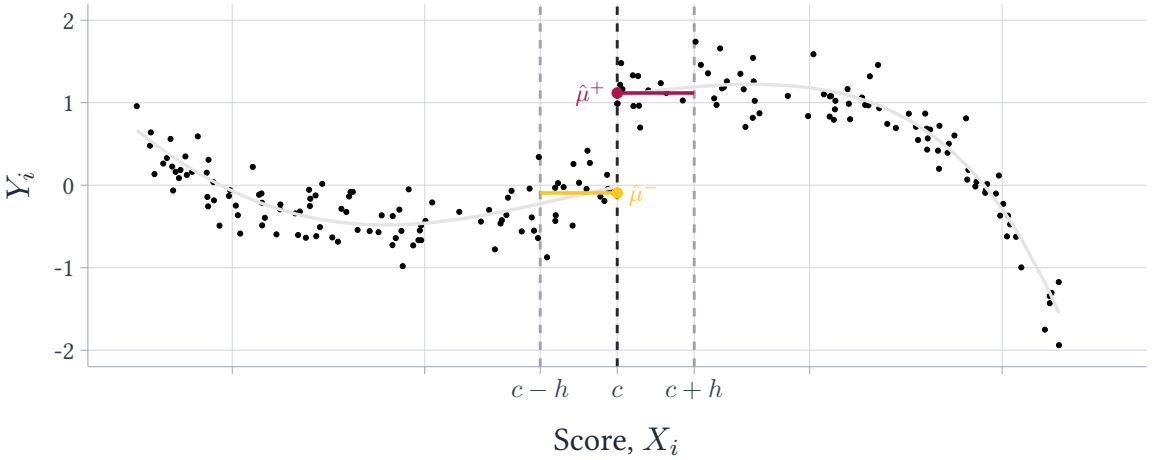
This can be estimated with the simple regression:

$$Y_i = \alpha_0 + \alpha_1 D_i + u_i$$

on the subsample with $X_i \in (c - h, c + h)$


2
1
0
-1
-2
$Y_i(1)$ and $Y_i(0)$
$\mathbb{E}[Y_i(1) \mid X_i = x]$
τ_{RD}
$\mathbb{E}[Y_i(0) \mid X_i = x]$
c
Score, X_i

$\hat{\tau}_{\mathsf{RD}} = \hat{\mu}^+ - \hat{\mu}^-$



Locally-linear estimation

Now, let's use a linear-model $\mu_d(x) = \alpha_d + \beta_d X_i$

- Note we let the slope vary for $Y_i(0)$ and $Y_i(1)$

Take our estimated models and form the regression adjustment estimator as:

$$\hat{\tau} = \left(\hat{\alpha}_1 + \hat{\beta}_1 c\right) - \left(\hat{\alpha}_0 + \hat{\beta}_0 c\right)$$

- This looks like our regression adjustment estimator!

Estimation via Regression

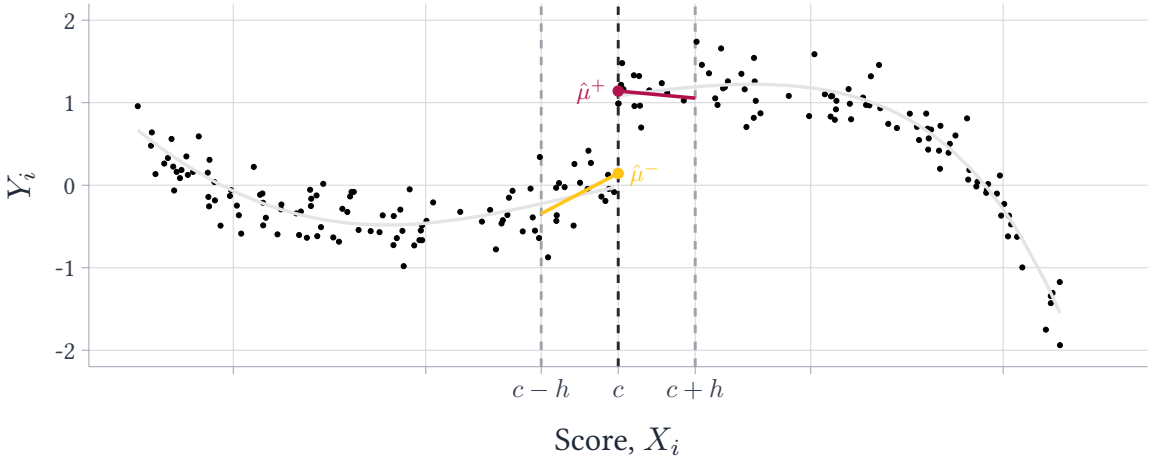
The locally linear estimator can be estimated with an interacted regression (like with regression adjustment):

$$Y_i = \alpha_0 + \alpha_1 D_i + \beta_0 (X_i - c) + \beta_1 D_i (X_i - c) + u_i$$

on the subsample with $X_i \in (c - h, c + h)$

- Note we recenter, $X_i - c$, so that $\hat{alpha}_1$ is the RDD estimate (e.g. "margin of victory" instead of "vote-share" in Lee, 2008)

$\hat{\tau}_{\mathsf{RD}} = \hat{\mu}^+ - \hat{\mu}^-$



Why linear if our data looks wiggly?

There is a lingering question, if we think our data is very ‘wiggly’ why are we assuming linearity in X_i?

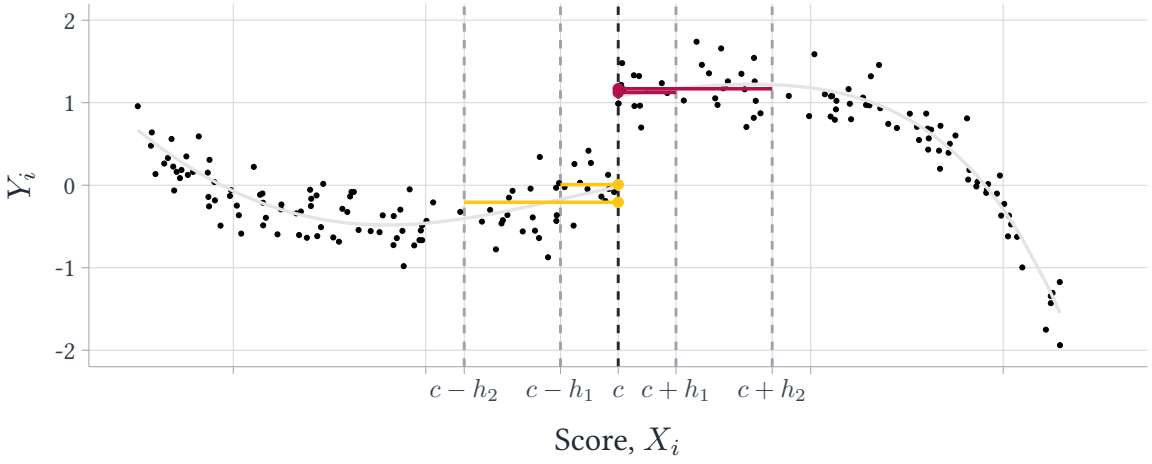
- Remember we are using only observations *local* to the cutoff, so (at least asymptotically) wiggly functions are approximately linear on small bandwidths
- Essentially we are using the logic of the Taylor expansion that we can approximate a function locally using a lienar function

Different Bandwidths

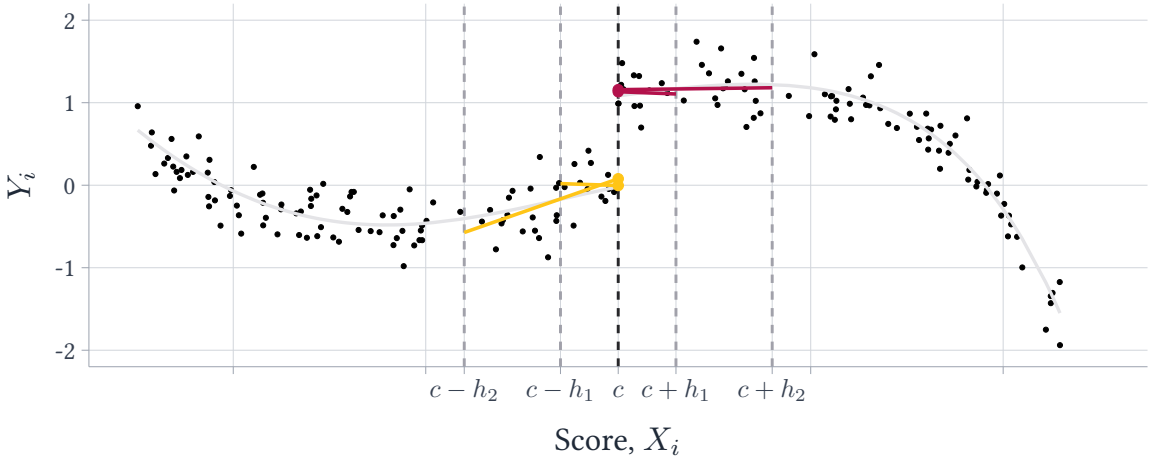
Our estimator, implicitly depends on our choice of bandwidth h

- Larger h uses more observations so should help with precision of our estimate
- but relies more on functional form for extrapolation (Taylor approximation only holds locally)

$\hat{\tau}_{\mathsf{RD}} = \hat{\mu}^{+} - \hat{\mu}^{-}$



$\hat{\tau}_{\mathsf{RD}} = \hat{\mu}^{+} - \hat{\mu}^{-}$



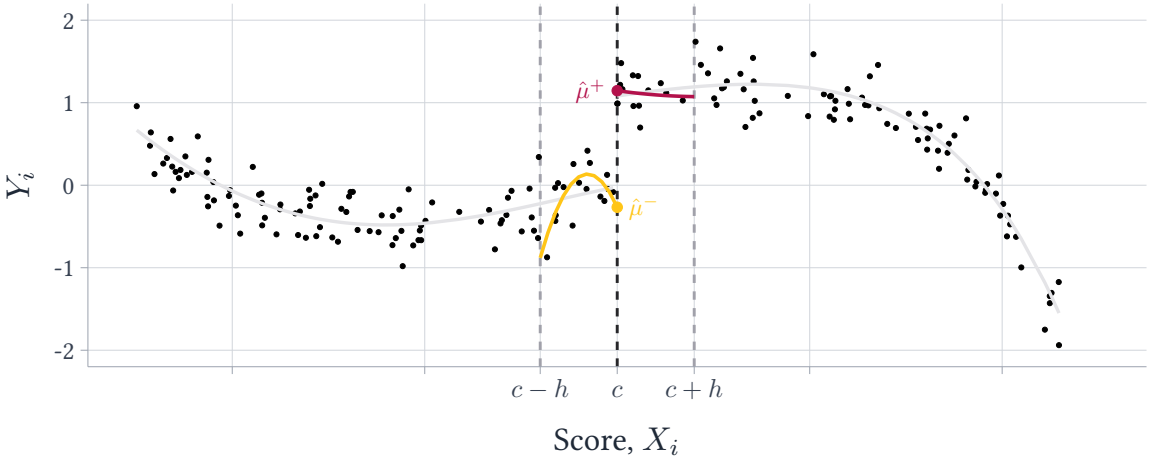
Local polynomial estimation

We can extend our logic to higher-order polynomials:

$$Y_i = \alpha_0 + \alpha_1 D_i \sum_{p=1}^{\rho} \beta_{0,p}(X_i - c)^p + \sum_{p=1}^{\rho} \beta_{1,p}(X_i - c)^p + u_i$$

Still, $\hat{\alpha_1}$ is our RDD estimator

$\hat{\tau}_{\mathsf{RD}} = \hat{\mu}^+ - \hat{\mu}^-$



RDD By Hand

The most 'straight forward' way to estimate the RDD is to do two regressions:

1. Regress Y_i on a k-th order polynomial of $X_i - c$ for $c - h < X_i < c$
2. Regress Y_i on a k-th order polynomial of $X_i - c$ for $c < X_i < c + h$

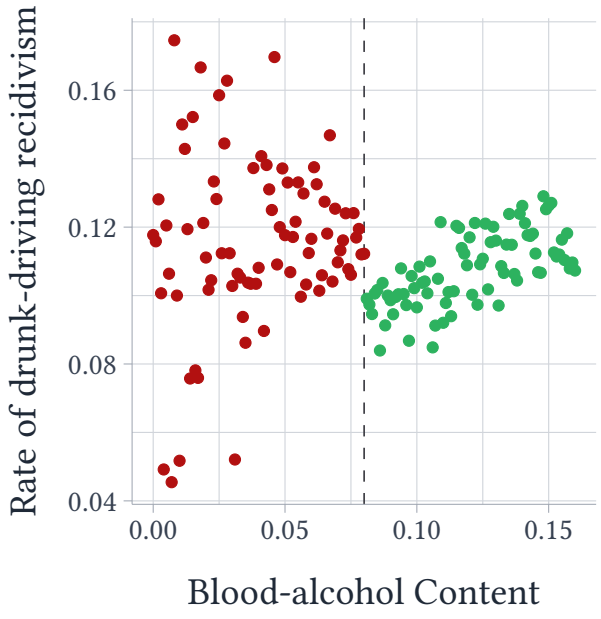
Predict Y at $X_i = 0$ for both models. These are your estimates for $\hat{\mu}^-$ and $\hat{\mu}^+$ respectively. Then our RD estimate can be formed as

$$\hat{\tau}_{\texttt{RD}} = \hat{\mu}^+ - \hat{\mu}^-$$

Example: Punishment and Deterrence: Evidence from Drunk Driving

Hansen (2015, AER) considers the impact of getting a DUI (driving while drunk) has on future drinking behavior

- Uses the ‘legal limit’ of a blood-alcohol content of 0.08 as the RDD ‘cutoff’


Rate of drunk-driving recidivism
0.16
0.12
0.08
0.04
0.00
0.05
0.10
0.15
Blood-alcohol Content

RDD by hand

```
feols(
  recidivism ~ 1 + over_limit + bac1_centered + over_limit * bac1_centered,
  data = subset(hansen, bac1 >= 0.08 - 0.04 & bac1 <= 0.08 + 0.04)
)
```

RDD using `rdrobust`

```
rdplot(y = hansen$recidivism, x = hansen$bac1, c = 0.08, h = bw)
rdrobust(
  y = hansen$recidivism, x = hansen$bac1, c = 0.08
)
```

Maniuplation of the running variable

Check 1: Balance

Balance Check By Hand

Balance Check using `rdrobust`

Check 2: McCrary Density

Real example

Camacho and Conover (2011, AEJ: EP)

Camacho and Conover (2011, AEJ: EP) discuss a real example of sorting

- Towns in Colombia receive social programs if their poverty index score is below a cutoff
- The poverty index algorithm becomes public in 1997


1996
1997
1998
1999
Percent
0 1 2 3 4 5 6
0 7 14 21 28 35 42 49 56 63 70 77 84 91 98
Poverty index score

Real example

Camacho and Conover (2011, AEJ: EP)

The authors show in their paper that the towns that corruptly gamed their score looked different than those that did not

- Those that gamed their scores had more political competition

In this setting, if we saw a big jump at the poverty index cutoff, then we can't know if it's from the social programs or from the higher-level of political competition

Density Check using `rddensity`

Spatial RDD

Spatial RDD

Complications

A lot of things change at a border