PRELIMINARY RESULTS ON EFFICIENT, ROBUST LIMIT CYCLE SYNTHESIS

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Define P as:

$$\begin{aligned} \sup \quad & \int_{X\times A} E(\alpha) d\mu_T && (P) \\ \text{s.t.} \quad & \mathcal{L}'\mu = \delta_T \otimes \mu_T - \delta_0 \otimes \mu_0, \\ & \mu_0 = \hat{\mu}_0 + \int_X d\mu_T \otimes \mathbb{1}_R(x)\lambda, \\ & \int_{X\times A} d\mu_T = 1, \\ & \mu, \mu_0, \mu_T, \hat{\mu}_0 \geq 0 \end{aligned}$$

where the given data are f, X, A, X_T, A_T and the supremum is taken over a tuple of measures $(\mu, \mu_0, \hat{\mu}_0, \mu_T) \in (\mathcal{M}\big([0,T]\times X\times A\big)\times\mathcal{M}(X\times A)\times\mathcal{M}(X\times A)\times\mathcal{M}(X_T\times A_T))$. Define the dual program to P denoted D as:

$$\begin{aligned} \inf \quad & p && (D) \\ \text{s.t.} \quad & w(x,\alpha) \leq 0 && \forall (x,\alpha) \in X \times A, \\ & \mathcal{L}v(t,x,\alpha) \leq 0 && \forall (t,x,\alpha) \in [0,T] \times X \times A \\ & w(x,\alpha) - v(0,x,\alpha) \geq 0 && \forall (x,\alpha) \in X \times A, \\ & p + v(T,x,\alpha) - \int_X w(\tilde{x},\alpha)\mathbb{1}_R(\tilde{x},\alpha)d\lambda(\tilde{x}) - E(\alpha) \geq 0 && \forall (x,\alpha) \in X_T \times A_T \end{aligned}$$

where the given data are f, X, A, X_T, A_T and the infimum is taken over $(v, w, p) \in (C^1([0,T]\times X\times A)\times C(X\times A)\times\mathbb{R})$.

Theorem 1. *If either P or D is feasible, then both are feasible and have the same value.*

Theorem 2. *If P is feasible, then the marginal of the solution μ_T in the A-coordinate is a Dirac delta.*

Theorem 3. *Suppose (v, w, p) are solutions to D and α_T is the location of the Dirac delta for the μ_T that solves P, then $w(\cdot,\cdot) \geq$*

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$\int_X w(\tilde{x}, \alpha_T) \mathbb{1}_R(\tilde{x}, \alpha) d\lambda(\tilde{x})$ *on the backwards reachable set to the limit cycle at* α_T.

Proof. Suppose $x : [0, T] \to X \times A$ is a solution to the ODE, beginning at $x_0 \times \alpha_0 \in X \times A$, and $x(T) = (x_T \times \alpha_T) \in X_T \times A_T$, then:

$$v(T, x_T, \alpha_T) \leq v(0, x_0, \alpha_0) \leq w(x_0, \alpha_0). \tag{1}$$

Notice also that $p = E(\alpha_T)$.The desired result follows immediately. □

Theorem 4. *Suppose* (v, w, p) *are solutions to* D *and* α_T *is the location of the Dirac delta for the* μ_T *that solves* P*, then* $w(x, \alpha_T) \geq \int_X w(\tilde{x}, \alpha_T) \mathbb{1}_R(\tilde{x}, \alpha) d\lambda(\tilde{x})$ *for almost every* $x \in \{x \in X | \mathbb{1}_R(x, \alpha_T) = 1\}$.

Proof. This follows from straightforward measure theory arguments given that $w \leq 0$. □