Peergrade #2: Proofs, Sets, Relations and Functions

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Exercise 1. *1. Let* $A = \{1, 3, 5\}, B = \{1, 2, 3, 4\}, C = \{x \in \mathbb{N} \mid \exists k \in \mathbb{N}(x = 2 \cdot k + 1)\}$. *Which of the following relations holds?*

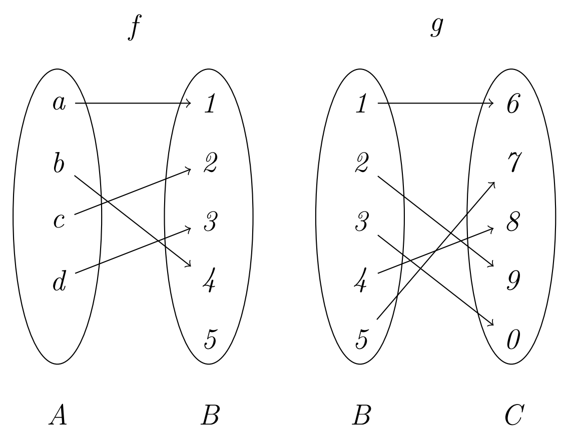
(a) $A - B = \{4\}$

(b) $A \subseteq B \cap C$

(c) $A - B \subseteq C$

(d) $(A \times B) \cup (A \times C) \subset A \times \mathbb{N}$

2. Let $f : A \to B$ *and* $g : B \to C$ *be the two functions defined below:*



Is $g \circ f$*:*

(a) one-to-one (injective)?

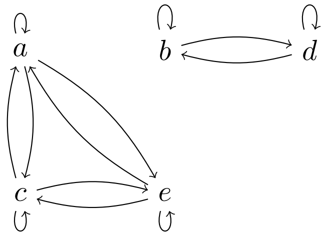
(b) onto (surjective)?

(c) one-to-one correspondence (bijective)?

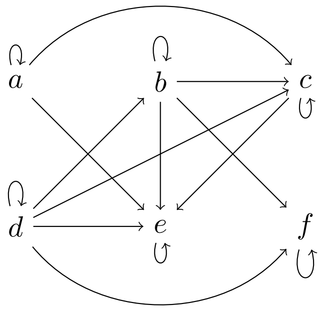
3. Which of the following graphs defines a partial order *or an* equivalence relation*?*

In case of a partial order, show a Hasse diagram representing the relation, and find the maximal, minimal, greatest and least elements, if they exist.

In case of an equivalence relations, find the partitions of the elements that are in the same equivalence class.



(a)



(b)

For the next two exercises, you will have to construct "informal" proofs on sets, relations and functions. Use the style and proof techniques discussed in week 3, and the cheat sheet and tips for proving statements provided in the section for week 4.

Exercise 2. *Provide a proof by contradiction of this statement: Let $f : \emptyset \to A$ be a function with domain the empty set and an arbitrary set A as co-domain. Then f is one-to-one (injective).*

Exercise 3. *Provide a direct proof of this statement: Let A and B be two arbitrary sets such that $B \subseteq A$, and let R be an equivalence relation on A. Consider the relation $S = \{(a, b) \in R \mid a, b \in B\}$ on the set B. Then S is an equivalence relation on B (that is, reflexive, symmetric and transitive).*