

## Peergrade #4: Mathematical induction

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October 26, 2019

**Exercise 1.** Let  $n \in \mathbb{N}$  and  $n > 6$ . Prove by simple induction that  $3^n < n!$ .  
(Remember that  $n!$  is called the factorial of  $n$ , and is defined as  $n! = \prod_{i=1}^n i = 1 \cdot 2 \cdot \dots \cdot n$ )

**Exercise 2.** Prove that  $f_0^2 + f_1^2 + f_2^2 + \dots + f_n^2 = f_n \cdot f_{n+1}$ .

That is, for any  $n \in \mathbb{N}$ , the sum of the squares of the first  $n$  Fibonacci numbers is equal to the product of  $f_n \cdot f_{n+1}$ .

**Exercise 3.** Give iterative and recursive algorithms for the  $n$ th term of the sequence defined by

$$a_0 = 1$$

$$a_1 = 3$$

$$a_2 = 5$$

$$a_n = a_{n-1} \cdot a_{n-2}^2 \cdot a_{n-3}^3$$

Which one is more efficient?