Informal proofs

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1 Notes

1.1 Proof the sum of two even numbers is even.

Let $n, m \in \mathbb{Z}$, such that n is even and m is even.
By definition there exists $k, l \in \mathbb{Z}$ such that $n = 2 \cdot k$ and $m = 2 \cdot l$.
Therefore, $n + m = 2 \cdot k + 2 \cdot l$
$n + m = 2(k + l)$
Hence, by definition $n + m$ is even.

1.2 Proof the sum of two Rational numbers is even.

Let $r, q \in \mathbb{R}$, such that r and q is rational.
By definition there are $a, b, c, d, \in \mathbb{Z}$ such that $b \neq 0, d \neq 0$ and $r = \frac{a}{b}, q = \frac{c}{d}$.
Then, $r + q = \frac{a}{b} + \frac{c}{d} = \frac{ad+cb}{bd}$
Hence, $r + q$ is rational.

1.3 Proof if the square of an integer is even, then the integer itself is even.

if n is not even, then n is odd.
$\neg Even(n) \rightarrow Odd(n) \equiv Even(n) \vee Odd(n)$
Let $n, \in Z$, such that n is not even.
By the Parity Theorem, n is odd.
By definition there exists $k \in Z$ such that $n = 2k + n$.
$n^2 = (2k + n)^2 = (4k)^2 + 2 \cdot 2k \cdot n + n^2$
$2 \cdot 2k^2 + 2 \cdot 2k + n$
$2(2k^2 + 2k) + n$
By definition x^2 is odd.
Hence, by the Parity Theorem, n^2 is not even.

1.4

Propostion:

The negative of any irrational number is rational
For all real numbers r, if r is irrational then $\neg r$ is irrational,
or

$$\forall r \in \mathbb{R}. r \notin \mathbb{Q} \to \neg r \notin \mathbb{Q}$$

Contraposition

$$\forall r \in \mathbb{R}. \neg(\neg r \notin \mathbb{Q}) \to \neg(r \notin Q)$$

$$\equiv \forall r \in \mathbb{R}. \neg r \in \mathbb{Q} \to r \in \mathbb{Q}$$

Let $r \in \mathbb{R}$ such that $-r \in \mathbb{Q}$
By def. there are $a, b \in \mathbb{Z}$ such that $b \neq 0$ and $-r = \frac{a}{b}$

$$(-r)(-1) = \frac{a}{b}(-1)$$

$$r = \frac{\neg a}{b}$$

Hence by def. r is rational.

1.5 Proof by contradiction

Proposition:

There is no greatest integer.
$N \geq n$
$\neg \exists N \in \mathbb{Z}. \forall n \in \mathbb{N}. N \geq n$
Assume $\exists N \in \mathbb{Z}. \forall n \in \mathbb{N}. N \geq n$
Let $N \in \mathbb{Z}$ such that $\forall n \in \mathbb{Z}. N \geq n$
substitute n with $N + 1$
In particular, this means $N \geq N + 1$
Contradiction. Therefore $\neg \exists N \in \mathbb{Z}. \forall n \in \mathbb{Z}. N \geq n$

1.6 Proof by cases

Propostion

For any integer n, $n^2 - n + 3$ is odd.
Let $n \in \mathbb{Z}$, By the Parity Theorem,
n is even or n is odd. We proceed by cases
1. n is even
By def. there is some $k \in \mathbb{Z}$ such that $n = 2k$.
Hence,

$n^2 - n + 3 = (2k)^2 - 2k + 3$
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$= 2 \cdot (2k^2 - k + 1) + 1$
Hence, $n^2 - n + 3$ odd.
2. n is odd
$\exists k$ such that $n = 2k + 1$
$n^2 - n + 3 = 2 \cdot 2k^2 + 2 \cdot 2k + 1 - (2k + 1)$
$= 2(2k^2 + 2k + 1 - k) + 1$

1.7 Proof by exhaustion

1.8 Disproving a Universal Statement

Disproving Universal $\rightarrow$ find counterexample

1.9 Disproving a Existential Statement

Disproving Existential $\rightarrow$ proof all