Peergrade assignment 4

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1

1.1

There are four suits (the domain).

The co-domain, to ensure at least two cards of the same suit, will be 4+1=5 cards

1.2

$$13 \cdot 3 + 1 = 40 \text{ cards}$$

2

S = snow, A = Arrive on time

Probability of all four outcomes:

$$P(S) \cdot P(\bar{A}|S) = 0.65 \cdot (1 - 0.72) = 0.182$$

$$P(S) \cdot P(A|S) = 0.65 \cdot 0.72 = 0.468$$

$$P(\bar{S}) \cdot P(\bar{A}|\bar{S}) = (1 - 0.65) \cdot (1 - 0.96) = 0.014$$

$$P(\bar{S}) \cdot P(A|\bar{S}) = (1 - 0.65) \cdot 0.96 = 0.336$$

2.1

$$P(A) = P(\bar{S}) \cdot (A|\bar{S}) + P(S) \cdot (A|S)$$

$$P(A) = 0.336 + 0.468 = 0.804 = 80.4\%$$

2.2

Probability of it snowing given not arive on time:

$$P(S|\bar{A}) = \frac{P(S) \cdot P(\bar{A}|S)}{P(\bar{A})} = \frac{0.182}{1 - 0.804} = 0.9286 = 92.86\%$$

3

3.1

unique dishes = 5 + 4 + 7 = 16

combinations of size 6 from set of size 16

$$\binom{16}{6} = \frac{16!}{(16-6)! \cdot 6!} = 8008$$

3.2

Product of combinations of 2 veg from 5, 2 fish from 4 and 2 meat from 7

Valid combinations =
$$\binom{5}{2} \cdot \binom{4}{2} \cdot \binom{7}{2} = 1260$$

The probability is the ratio between number of valid combinations and number of all combinations

$$\frac{1260}{8008} = 0.1573 = 15.73\%$$

3.3

The constant price for the four predetermined dishes:

$$2 \cdot 6eur + 1 \cdot 5eur + 1 \cdot 3eur = 20eur$$

Sample space (not including duplicates):

$$S = \{VV, VF, FM, FF, FM, MM\}$$

Let X(s) be the random variable that equals the sum of the prices of the dishes.

E.g.:
$$X(VF) = 3 + 5 = 8$$

$$X(VV) = 6$$

$$X(VF) = 8$$

$$X(VM) = 9$$

$$X(FF) = 10$$

$$X(FM) = 11$$
$$X(MM) = 12$$

Number of all combinations = ${12 \choose 2} = 66$

Number of valid combinations for $S_n=\{\binom{4}{2},4\cdot 3,4\cdot 5,\binom{3}{2},3\cdot 5,\binom{5}{2}\}$

$$= \{6, 12, 20, 3, 15, 10\}$$

divided by 66 to get probability:

$$P(X = S_n) = \{0.0909, 0.1818, 0.3030, 0.045, 0.2272, 0.1515\}$$

$$E(X) = 20eur + \sum P(s) \cdot Price(s) = 29.5eur$$