Part I. Answer the following multiple choice questions.

1. (2 pts) Which of the following statements is *true*?

☐ $\{(2,4),(2,6),(2,8),(3,6),(3,9),(4,8)\} \subset | \cap (\{2\ldots 9\} \times \{2\ldots 9\})$ where $|$ is the "divides" relation

0

☐ $\mathcal{P}(A \cup B) \subseteq \mathcal{P}(A) \cup \mathcal{P}(B)$ for arbitrary sets A, B

☑ $\{3, \sqrt{10}, 24 \,\mathbf{mod}\, 7\} \subseteq \{8 \,\mathbf{mod}\, 5\}$ $\sqrt{10} \approx 3.16$ $3.16 \neq 3$

☐ $(A - C) \cap (B - C) = \emptyset$ for arbitrary sets A, B, C

2. (2 pts) Which one is a valid representation of $(E74)_{16}$?

☐ $(3900)_{10}$

2

☑ $(111001110100)_2$

☐ $(7174)_8$

☐ $(11011011)_2$

3. (2 pts) Let $a_k = a_{k-1} + 2a_{k-2}, a_0 = -1, a_1 = 2$. Which of the following statements is *true*?

☐ $a_0 = a_3$

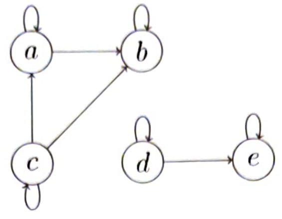
2

☐ $a_k + a_{k+1} = a_k - a_{k-1}$ for all k

☑ $a_k \cdot a_{k+1} = a_k^2 + 2 \cdot a_k \cdot a_{k-1}$ for all k

☐ $\sum_{i=1}^{4} a_k = a_5$

4. (2 pts) Consider the following directed graph representing the relation R on the set $\{a, b, c, d, e\}$.



Which of the following statements is *true*?

☐ R is antisymmetric and transitive, but not reflexive

2

☐ R is transitive, but neither reflexive nor antisymmetric

☐ R is antisymmetric, but neither reflexive nor transitive

☑ R is reflexive and antisymmetric and transitive

Part II. Answer the following questions. Be brief but precise, your correct use of mathematical notation is an important aspect of your answer.

5. (a) (3 pts) Construct a valid judgment (derivation) for the following predicate:

$$\exists x(\neg Q(x)) \to (\forall y(P(y) \to Q(y))) \to \neg\forall z(P(z))$$

1

$$\dfrac{\dfrac{}{\exists x(\neg Q x)}h_1 \quad \dfrac{}{\neg Q x}h_5}{\neg Q x}\exists E^{h_5} \quad \dfrac{}{Q x}h_4$$

$$\dfrac{\neg Q x \quad Q x}{F}\neg E$$

$$\dfrac{\dfrac{}{\exists x(\neg Q(x))}h_1 \quad \dfrac{\dfrac{F}{\neg Q(x)}\neg I^{h_4}}{}}{\neg Q(x)}\exists E^{h_4} \qquad \dfrac{\dfrac{\dfrac{}{\forall y(P(y)\to Q(y))}h_2}{P(y)\to Q(x)}\forall E \quad \dfrac{\dfrac{}{\forall z(P(z))}h_3}{P(y)}\forall E}{Q(x)}\to E$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\neg Q(x) \quad Q(x)}{F}\neg E}{\neg\forall z(P(z))}\neg I^{h_3}}{(\forall y(P(y)\to Q(y)))\to\neg\forall z(P(z))}\to I^{h_2}}{\exists x(\neg Q(x)) \to (\forall y(P(y)\to Q(y))) \to \neg\forall z(P(z))}\to I^{h_1}$$

$h_4 : Q(x)$

$h_5 : \neg Q x$

$h_1 : \exists x(\neg Q(x))$

$h_2 : \forall y(P(y) \to Q(y))$

$h_3 : \forall z(P(z))$

1

(b) (1 pt) Is this predicate valid in intuitionistic logic, classical logic, or both? Explain your answer.

I've not used LEM, $\neg\neg C$ or F^u_C therefore the predicate is valid in both intuitionistic and classical logic

6. (3 pts) Assume that A and B are non-empty sets and that $f : A \to B$ is a bijective function. Let $g : A \to \mathcal{P}(B)$ be the function defined as $g(a) = \{f(a)\}$ for all $a \in A$. One of the following statements is true, and one is false.

1. g is injective.
2. g is surjective.

Write which statement is true and prove it. Write which statement is false and find a counter example. (Just answering which is true and which is false gives no points.)

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7. (2 pts) Let A, B and C be sets. Show that if $A \subseteq B$ and $B \subseteq \overline{C}$, then $A \cap C = \emptyset$.

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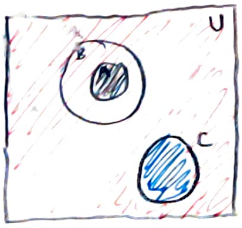

$\overline{C}$ C A

~~Assume $\exists$ set D : $D = \overline{C}$~~

~~that impues $B \subseteq D$ and $D \cap C = \emptyset$~~

~~since $A \subseteq B \subseteq D$ and $D \cap C = \emptyset \to A \cap C = \emptyset$~~

To prove $A \cap C = \emptyset$ … Assume $A \cap C \neq \emptyset \longrightarrow \exists x.\ x \in A \wedge x \in C$

$x \in A \to x \in B$ since $A \subseteq B$ by def. …

8. (2 pts) Let $a, b \in \mathbb{Z}$ be integers, and assume that $a \mid b$. Prove that $a \mid 5b + a$.

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… $x \in B \to x \in \overline{C}$ since $B \subseteq \overline{C}$ by def.

$x \in \overline{C} \to x \notin C$ since by def.

$\forall$ set C. $\overline{C} \cap C = \emptyset$.

Therefore $x \notin C$ and by prev. assumption $x \in C$.

This is a contradiction.

Therefore $A \cap C = \emptyset$ True.