Peergrade assignment 1

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Exercise 1

1.1. Prove in natural deduction...

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{h_1}
\UnaryInfC{$\neg A \lor \neg B$}
\AxiomC{}
\RightLabel{h_5}
\UnaryInfC{$\neg A$}
\AxiomC{}
\RightLabel{h_4}
\UnaryInfC{$A \land B$}
\RightLabel{$\land e_1$}
\UnaryInfC{A}
\RightLabel{$\neg e$}
\BinaryInfC{$\bot$}
\AxiomC{}
\RightLabel{h_6}
\UnaryInfC{$\neg B$}
\AxiomC{}
\RightLabel{h_4}
\UnaryInfC{$A \land B$}
\RightLabel{$\land e_1$}
\UnaryInfC{B}
\RightLabel{$\neg e$}
\BinaryInfC{$\bot$}
\RightLabel{$\lor e^{h_5, h_6}$}
\TrinaryInfC{$\bot$}
\RightLabel{$\neg i^{h_4}$}
\UnaryInfC{$\neg(A \land B)$}
\AxiomC{}
\RightLabel{h_3}
\UnaryInfC{C}
\AxiomC{}
\RightLabel{h_2}
\UnaryInfC{$C \to A \land B$}
\RightLabel{$\to E$}
\BinaryInfC{$A \land B$}
\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\RightLabel{$\neg I^{h_3}$}
\UnaryInfC{$\neg C$}
\RightLabel{$\to I^{h_2}$}
\UnaryInfC{$(C \to (A \land B)) \to \neg C$}
\RightLabel{$\to I^{h_1}$}
\UnaryInfC{$(\neg A \lor \neg B) \to (C \to (A \land B)) \to \neg C$}
\end{prooftree}
$$

Note: at $\neg i^{h_4}$ the proof for De Morgan's Law begins, the equivalence of:

$$\neg(A \land B) \equiv (\neg A \lor \neg B)$$

1.2. Construct a truth table...

A	B	C	$\neg C$	$A \land B$	$C \to (A \land B)$	$(C \to (A \land B)) \to \neg C$
T	T	T	F	T	T	F
T	T	F	T	T	T	T
T	F	T	F	F	F	T
T	F	F	T	F	T	T
F	T	T	F	F	F	T
F	T	F	T	F	T	T
F	F	T	F	F	F	T
F	F	F	T	F	T	T

$\neg A$	$\neg B$	$\neg A \lor \neg B$	$(\neg A \lor \neg B) \to (C \to (A \land B)) \to \neg C$
F	F	F	T
F	F	F	T
F	T	T	T
F	T	T	T
T	F	T	T
T	F	T	T
T	T	T	T
T	T	T	T

It seems that $(\neg A \lor \neg B) \to (C \to (A \land B)) \to \neg C$ is a Tautology.

$$\mathbf{T} \equiv (\neg A \lor \neg B) \to (C \to (A \land B)) \to \neg C$$

Exercise 2

2.1

Expression using quantifiers

T = table, L = has legs

$$\exists x(T(x) \to \neg L(x)) \tag{1}$$

Negation of the statement

T = table, L = has legs

$$\forall x(T(x) \to L(x)) \tag{2}$$

Expression using English

For all x, if x is a table then x has legs.

2.2

Expression using quantifiers

E = Even

$$\forall x \in Z(E(x) \to \exists y \in Z((y = x^2) \to E(y)) \tag{3}$$

Negation of the statement

E = Even

$$\exists x \in Z(E(x) \to \exists y \in Z((y = x^2) \to \neg E(y)) \tag{4}$$

Expression using English

There exists an x in the domain of all integers, if x is an even integer then there exists a y in the domain of all integers, if y is equal to the square of x, then y is NOT an even integer.

2.3

Expression using quantifiers

H = Is a house, P = Has pink doors

$$\exists x(H(x) \to P(x)) \tag{5}$$

Negation of the statement

H = Is a house, P = Has pink doors

$$\forall x(H(x) \to \neg P(x)) \tag{6}$$

Expression using English

For all x, if x is a house then x does not have pink doors.

2.4

Expression using quantifiers

P = Is a part of ”the pigs”, F = Can fly, D = Can dance

$$\forall x(P(x) \rightarrow F(x) \wedge D(x)) \tag{7}$$

Negation of the statement

P = Is a part of ”the pigs”, F = Can fly, D = Can dance

$$\exists x(P(x) \rightarrow \neg(F(x) \wedge D(x))) \tag{8}$$

Expression using English

There exists an x where, if x is a part of ”the pigs” then x can not fly and dance.

2.5

Expression using quantifiers

Q = Rational

$$\exists x \left(Q(x) \rightarrow \sqrt{Q(x)} = 2\right) \tag{9}$$

Negation of the statement

Q = Rational

$$\forall x \left(Q(x) \rightarrow \sqrt{Q(x)} \neq 2\right) \tag{10}$$

Expression using English

For all x, if x is a rational number then the square root of x is not equal to 2.

Exercise 3

Prove in natural deduction...

$$
\dfrac{\dfrac{\dfrac{\dfrac{
\dfrac{\dfrac{}{\exists y(\neg B(y))}\;h_2 \quad \dfrac{\dfrac{}{\neg B(y)}\;h_4 \quad \dfrac{\dfrac{\dfrac{}{\forall x(A(x) \to B(x))}\;h_1}{A(y) \to B(y)}\;\forall e \quad \dfrac{\dfrac{}{\forall z(A(z))}\;h_3}{A(y)}\;\forall e}{B(y)}\;\to e}{\neg B(a)}\;\neg e}{\neg B(a)}\;\exists e^{h_4}
\quad
\dfrac{\dfrac{\dfrac{}{\forall x(A(x) \to B(x))}\;h_1}{A(a) \to B(a)}\;\forall e \quad \dfrac{\dfrac{}{\forall z(A(z))}\;h_3}{A(a)}\;\forall e}{B(a)}
}{\bot}\;\neg e}{\forall z(A(z))}\;\neg I^{h_3}}{\exists y(\neg B(y)) \to \neg \forall z(A(z))}\;\to I^{h_2}}{\forall x(A(x) \to B(x)) \to \exists y(\neg B(y)) \to \neg \forall z(A(z))}\;\to I^{h_1}
$$