1 Exercises in Natural Deduction, Intuitionistic Fragment

Use the rules from page 11 in the notes, *excluding* the classical rules, to prove the following statements:

1. $((P \wedge Q) \wedge R) \to (S \wedge T) \to (Q \wedge S)$

2. $(P \wedge (Q \wedge S)) \to ((P \wedge Q) \wedge S)$

3. $(P \to (P \to Q)) \to P \to Q.$

4. $(P \wedge Q) \to Q$

5. $P \to Q \to P$

6. $(P \wedge Q) \vee (Q \wedge R) \to Q$

7. $(P \to Q) \to (R \to S) \to (P \wedge R) \to (Q \wedge S)$

8. $(P \vee Q) \to (Q \to R) \to (P \to R) \to R$

9. $P \to (Q \to (P \vee R))$

10. $(((S \wedge R) \vee (Q \wedge P)) \vee ((P \wedge Q) \vee (R \wedge S))) \to ((P \vee S) \vee (Q \vee R))$

2 Exercises in Natural Deduction, Classical Fragment

Use the rules from page 11 in the notes, *including* the classical rules, to prove the following statements:

1. $(\neg P \to P) \to P$

2. $(P \vee Q) \to \neg Q \to P$

3. $(P \to Q) \to (R \to S) \to (P \vee R) \to (Q \vee S)$

4. $Q \to ((P \wedge Q) \vee (\neg P \wedge Q))$

5. $(P \to Q) \to (\neg P \vee Q)$

6. $(\neg P \vee Q) \to P \to Q$

7. $(P \to Q) \to (P \to \neg Q) \to \neg P$

8. $(P \to \neg P) \to \neg P$

9. $(P \wedge (Q \vee R)) \to ((P \wedge Q) \vee (P \wedge R))$

10. $(P \to (Q \to R)) \to P \to (\neg R) \to \neg Q$

11. $(P \rightarrow Q) \rightarrow (\neg P \rightarrow R) \rightarrow (Q \vee R)$

12. $(P \rightarrow Q) \rightarrow (R \rightarrow \neg T) \rightarrow (Q \rightarrow R) \rightarrow (P \rightarrow \neg T)$

13. $((P \wedge \neg Q) \rightarrow R) \rightarrow (\neg R) \rightarrow P \rightarrow Q$