Peergrade assignment 3

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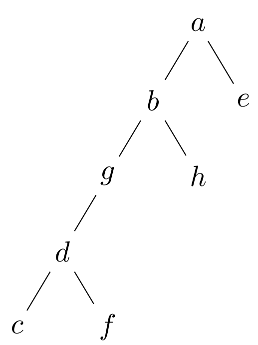
Exercise 1

1.1.

(a) **true**
Example: a, d is a walk without repeated edges and vertices.
(b) **false**
Counterexample.
c and d is not connected.
Therefore there is no simple path from c to d.
(c) **false**
Counterexample.
There is no path from c to d

1.2.

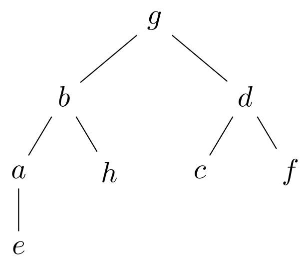
(a) **true**
The given graph is a tree, because the graph is connected and there doesn't exist a simple circuit.
(b) **true**
There exists a way to root the tree such that the graph is a binary tree.



If you root the tree by vertex b or d, the graph will not be a binary tree.

(c) **true**

There exists a way to root and arrange the graph such that the graph is a complete binary tree.



Exercise 2

2.1.

(a)

$a_{[2;9]} = 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2$
$a_{[2;9]} = 4, 9, 16, 25, 36, 59, 64, 81$

(b)

$b_{[2;9]} = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}$

(c)

$c_2 = a_1 \cdot b_1 + a_2 \cdot b_2 = 1 \cdot \frac{1}{1} + 4 \cdot \frac{1}{2} = 1 + 2 = 3$
apparently k can be described as $k = a_k \cdot b_k$ therefore c_k can be described as:
$c_k = c_{k-1} + k$ for $k > 2$
$c_3 = c_2 + 3 = 3 + 3 = 6$
$c_4 = 6 + 4 = 10$
$c_5 = 10 + 5 = 15$
$c_6 = 15 + 6 = 21$
$c_7 = 21 + 7 = 28$
$c_8 = 28 + 8 = 36$
$c_9 = 36 + 9 = 45$
$c_{[2;9]} = 3, 10, 15, 21, 28, 36, 45$

2.2.

(a)

$(763636 \cdot 437813 \cdot 936257) \mod 43$
$= ((763636 \mod 43) \cdot (437813 \mod 43) \cdot (936257 \mod 43)) \mod 43$
$= 19$ (b)
$(763636 \cdot (437813 + 936257)) \mod 43$
$= ((894461 \mod 59) \cdot ((206193 \mod 59) + (83218 \mod 59))) \mod 59$
$= 41$

2.3.

(a)

Octal to binary

Each character in octal can be represented by exactly 3 bits.

Therefore I'm just concatenating blocks of 3 bits starting with the least significant digits. Example: $(7)_8 = (111)_2$ and $(1)_8 = (001)_2$ therefore $(17)_8 = (001111)_2$.

$(73217)_8 = (111011010001111)_2$

Octal to decimal

$(73217)_8 = (7 \cdot 8^0 + 8 \cdot 8^1 + 2 \cdot 8^2 + 3 \cdot 8^3 + 7 \cdot 8^4)_{10} = (30351)_{10}$

(b)

$62290 = 8 \cdot 7786 + 2$
$7786 = 8 \cdot 973 + 2$
$973 = 8 \cdot 121 + 5$
$121 = 8 \cdot 15 + 1$
$5 = 8 \cdot 1 + 7$
$1 = 8 \cdot 0 + 1$
$(62290)_{10} = (171522)_8$

$62290 = 16 \cdot 3893 + 2$
$3893 = 16 \cdot 243 + 5$
$243 = 16 \cdot 15 + 3$
$15 = 16 \cdot 0 + 15$
$(62290)_{10} = (F352)_{16}$

2.4.

(a)
$a = bq + r$
$2574 = 1976 \cdot 1 + 598$
$1976 = 598 \cdot 3 + 182$
$598 = 182 \cdot 3 + 52$
$182 = 52 \cdot 3 + 26$
$52 = 26 \cdot 2 + 0$
$gcd(2574, 1976) = 26$
(b)
$1525 = 4405 \cdot 0 + 1525$
$4405 = 1525 \cdot 2 + 1355$
$1525 = 1355 \cdot 1 + 170$
$1355 = 170 \cdot 7 + 165$
$170 = 165 \cdot 1 + 5$
$165 = 5 \cdot 33 + 0$
$gcd(1525, 4405) = 5$
$lcm(1525, 4405) = \frac{1525 \cdot 4405}{5} = 1343525$

Exercise 3

Assumptions

A1: $x, y, z, m \in \mathbb{Z}$

A2: $x \equiv y(\mathbf{mod}\ m)$

A3: $m|z$

Theorems

T1: $a|b \wedge a|c \rightarrow a|(b + c)$

T2: $a \equiv b(\mathbf{mod}\ m) \iff m|a - b$

To prove $x + 2z \equiv y(\mathbf{mod}\ m)$

Proof.

By T2, $\exists m$ such that $m|x - y \wedge m|z$

By T1, $m|z \wedge m|x - y$ implies $m|(x - y) + z$ which you can rearrange to $m|(x + z) - y$

By T2, $m|(x - y) - z$ implies $x + z \equiv y(\mathbf{mod}\ m)$, where a from T2 corresponds to $(x + z)$ and b from T2 corresponds to y

By T1, $m|z \wedge m|z$ implies $m|z + z$ which means that adding another z wont affect the truth of the statement, therefore

$x + z \equiv y(\mathbf{mod}\ m)$ implies $x + 2z \equiv y(\mathbf{mod}\ m)$

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