Solution to Hand-ins for Discrete Mathematics '17
Assignment 6

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13.1.18

The grammar is always gonna be defined the same way, namely:
$G = (V, T, S, P)$. What changes is the variables we construct it from.

a)

$$\begin{aligned} V = \quad & \{0, 1, S, A\} \\ T = \quad & \{0, 1\} \\ P = \quad & S \to 0A \\ & A \to 11A \mid \lambda \end{aligned}$$

b)

$$\begin{aligned} V = \quad & \{0, 1, S\} \\ T = \quad & \{0, 1\} \\ P = \quad & S \to 0S11 \mid \lambda \end{aligned}$$

c)

$$\begin{aligned} V = \quad & \{0, 1, S, A\} \\ T = \quad & \{0, 1\} \\ P = \quad & S \to 0S0 \mid A \\ & A \to 1A \mid \lambda \end{aligned}$$

13.4.6

For this exercise, remember that there can be more than one way to describe a set of bitstrings using regular expressions. When you are giving feedback, if the person who solved the exercise has given answers that differs from the ones below, try to determine whether they are still describing the correct set. If not, give examples of strings that should be recognized be the regular expression but are not, or strings that are recognized by the regular expression but should not be.

a) $\lambda \cup 0 \cup 1 \cup 00 \cup 01 \cup 10 \cup 11$ (This is quite a long way to write it as you have only learnt regular expressions using * and $\cup$. It could be written more succinctly as $(0 \cup 1)\{0-2\}$ where $\{0-2\}$ indicates that we can have between 0 and 2 of what comes before or as $(0\cup 1)?(0\cup 1)?$ where ? means either 0 or 1 of what comes before)

b) 001^*0

c) As it's a bit unclear whether the exercise ask for *at least* or *exactly* two 0's after every 1, we give at solution for both:

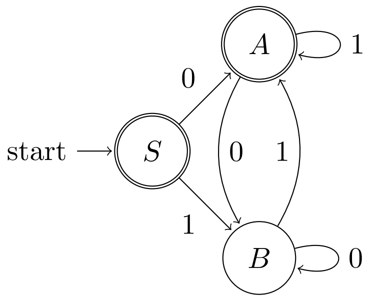
- At least two 0's after a 1: $(0^*(100)^*)^*$
- Exactly two 0's after a 1: $0^*(100)^*$

d) $(0 \cup 10)(0 \cup 10)^*0$ (As above, it is possible to write this shorter with other regular expression symbols. The same set could be described with $(0 \cup 10)^+0$ where $^+$ indicates 1 or more of what comes before)

e) $0^*(10^*10^*)^*$

13.4.16

If we name the states S, A and B as seen here:



we can construct the grammar $G = (V, T, S, P)$ where S is the start symbol and:

$$
\begin{aligned}
V =&\quad \{0, 1, S, A, B\} \\
T =&\quad \{0, 1\} \\
P =&\quad S \to 0A \mid 1B \mid \lambda \\
&\quad A \to 0B \mid 1A \mid \lambda \\
&\quad B \to 1A \mid 0B
\end{aligned}
$$