Intelligent Systems Programming (ISP)
BSc and MSc Exam

IT University of Copenhagen

29 May 2018

This exam consists of 8 numbered pages with 3 problems. Each problem is marked with the weight in percent it is given in the evaluation. You have 4 hours to complete the exam. **Notice that the first version of problem 3 only is to be solved by Bachelor (BSc) students, while the second version only is to be solved by Master (MSc) students**.

You can answer the questions in English or Danish.

Use notations and methods that have been discussed in the course.

Make appropriate assumptions when a problem has been incompletely specified.

Good luck!

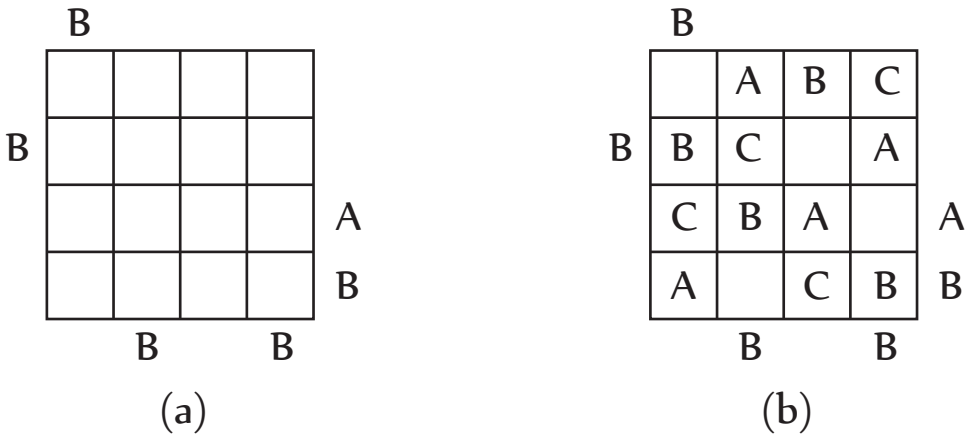
Problem 1: Constraint Programming (35%)

A letter puzzle consists of a board with $n \times n$ squares plus some clues appearing immediately next to the board at the beginning or end of a row or column (see figure (a) below). The clues are letters, and to solve the puzzle, each square must either be left empty or filled with a letter, such that:

(i) In each row and each column, the $n-1$ first letters in the alphabet appear exactly once in a square, while one square is left blank.

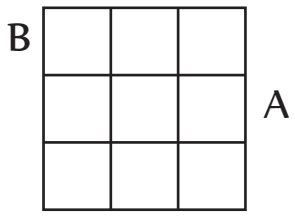
(ii) A clue next to a column/row specifies the first letter to appear in that column/row when the column/row is read in the direction of the clue. That is, if e.g. an A appears to the right of the third row, then the first letter in the third row when read from right to left, has to be an A.

An example of a letter puzzle (a) and a possible solution (b) is shown below.

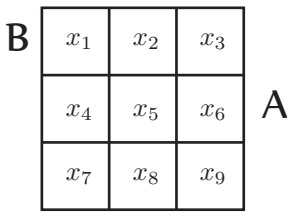


In the following we will look at the 3×3 letter puzzle shown in figure (c) below and formulate it as a CSP with 9 variables, $x_1, \ldots, x_9$, where x_i denotes the letter or blank space that should be put in the corresponding square shown in figure (d) below. Each variable x_i for $i \in \{1, 2, \ldots, 9\}$ has the domain $D_i = \{\circ, A, B\}$, where $\circ$ denotes an empty space.

a) Write binary constraints corresponding to the rules in (i) and (ii) above applied to the puzzle shown in figure (c) below. Note that AllDiff constraints should be rephrased as a number of binary constraints.



(c)



(d)

a) Answer:

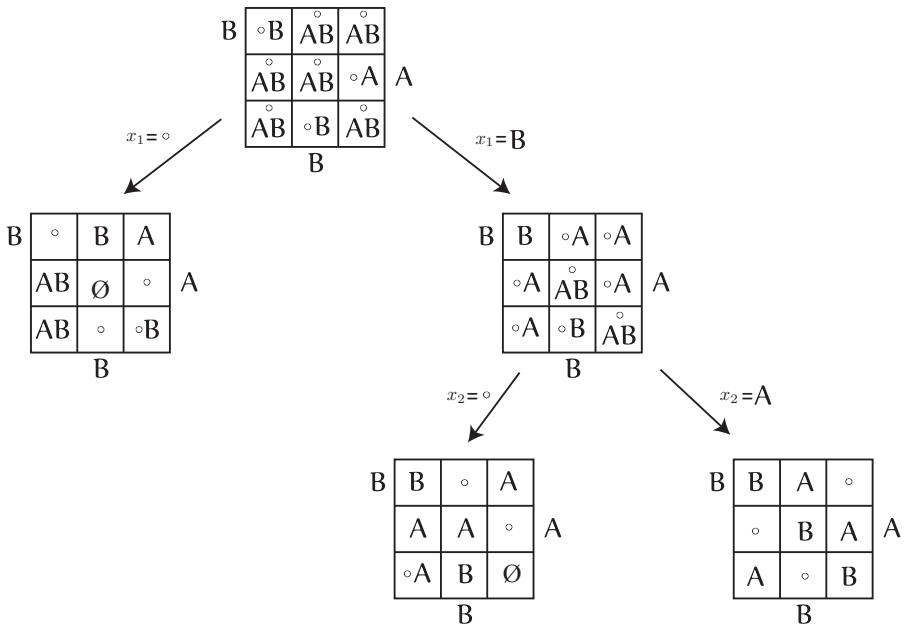
$$
\begin{aligned}
x_1 \neq x_2, x_1 \neq x_3, x_2 \neq x_3,\\
x_4 \neq x_5, x_4 \neq x_6, x_5 \neq x_6,\\
x_7 \neq x_8, x_7 \neq x_9, x_8 \neq x_9,\\
x_1 \neq x_4, x_1 \neq x_7, x_4 \neq x_7,\\
x_2 \neq x_5, x_2 \neq x_8, x_5 \neq x_8,\\
x_3 \neq x_6, x_3 \neq x_9, x_6 \neq x_9,\\
x_1 = B \vee x_1 = \circ \wedge x_2 = B,\\
x_6 = A \vee x_6 = \circ \wedge x_5 = A,\\
x_8 = B \vee v_8 = \circ \wedge x_5 = B.
\end{aligned}
$$

b) Make the CSP node consistent and write the domains for the variables, whose domains changed (if any). Make the CSP arc-consistent and write the domains for the variables, whose domains changed (if any).

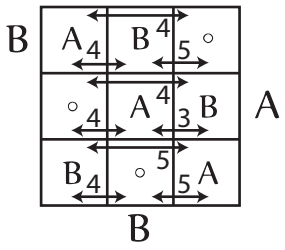
b) Answer: The CSP is already node-consistent since there are no unary constraints. Making the graph arc-consistent changes the following domains: $D_1 = \{\circ, B\}$, $D_6 = \{\circ, A\}$, $D_8 = \{\circ, B\}$.

c) Use the MAC algorithm to solve the puzzle. Draw the search tree with a node for each variable branched on. For each arc, indicate which variable is branched on and which value is assigned. In each node, write the remaining domains of each unassigned variable after arc-consistency is applied. Indicate failures and empty domains. You should use the MRV (minimum-remaining value) heuristic to choose the variable to branch on. If there is a tie, take the variables in numerical order of the subscript. Assign the domain values in the order $\circ \prec A \prec B$.

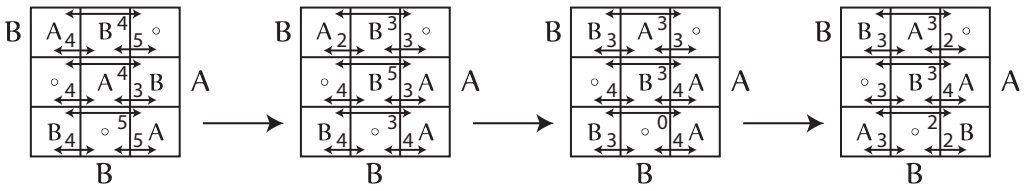
c) Answer:



d) Instead of MAC, use hill climbing to solve the problem. The initial state is shown below. The neighborhood of a state appears by swapping the content of two squares in the same row (thus, each state has a neighborhood of 9 other states). The cost of a state is the number of conflicts, where a conflict is a clue that is not satisfied or a column where A and B do not appear exactly once. You are allowed to take side-steps, and if so, you should not return immediately to the previous state. Indicate possible swaps for a state by arrows and write the cost of the resulting state above the arrow as shown for the initial state below.



d) Answer:



Problem 2: Heuristic Search and Logic (35%)

Prof. Smart is working on using search to prove logical equivalences in propositional logic. The problem has five rules. Each rule is defined by a precondition and an effect as shown in the table below:

Rule	Precondition	Effect
R1	$x \Rightarrow \text{False}$	$\neg x \vee \text{False}$
R2	$\neg x \vee \text{False}$	$\neg x$
R3	$\neg x \wedge y$	$y \wedge \neg x$
R4	$x \wedge \neg y$	$\neg y \wedge x$
R5	$\neg x \wedge x$	False

False represents the propositional constant *False*, while x and y represent propositional symbols. A rule is applicable if its precondition matches a sub-sentence of a sentence in propositional logic. The result of applying a rule is that the matched sub-sentence is substituted with the effect of the rule. Notice that we have required that x and y only can match a single propositional symbol. They cannot match general sentences. Four examples of rule application are shown below. Notice in the last example that a parenthesis around a sub-sentence disappear if it no longer is needed.

$$
\begin{aligned}
(A \Rightarrow \text{False}) \wedge \neg B &\xrightarrow{\text{Rule R1}} (\neg A \vee \text{False}) \wedge \neg B \\
\neg B \wedge A \wedge \neg A &\xrightarrow{\text{Rule R3}} A \wedge \neg B \wedge \neg A \\
\neg B \wedge A \wedge \neg A &\xrightarrow{\text{Rule R4}} \neg B \wedge \neg A \wedge A \\
(\neg A \vee \text{False}) \wedge B &\xrightarrow{\text{Rule R2}} \neg A \wedge B
\end{aligned}
$$

Assume that you are given a sentence S_0 in propositional logic and that you search for new sentences by using actions that correspond to applying these five rules.

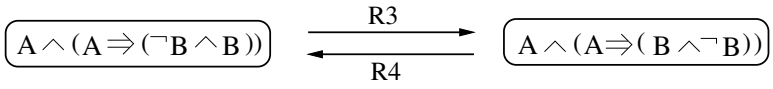
a) Would this approach be sound in the sense that all new sentences are logically equivalent to S_0 (why/why not)?

a) Answer: Yes, the approach is sound. The sub-sentences that the rules substitute are logically equivalent since the precondition and effect of each rule is a well-known logical equivalences (e.g., we can prove them with a truth table).

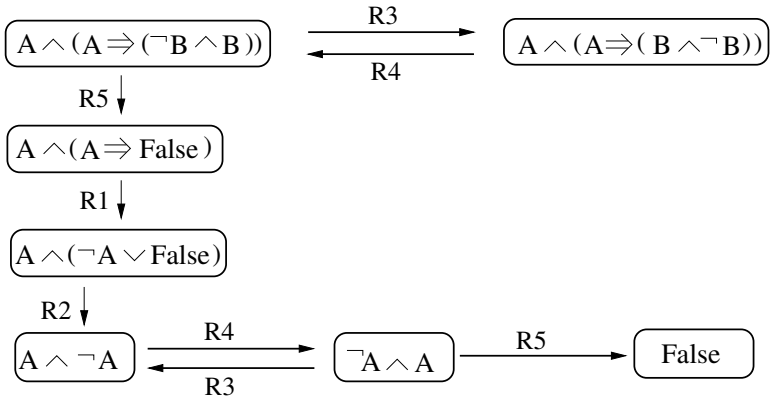
b) Would this approach be complete in the sense that any sentence that is logically equivalent to S_0 can be found in this way (why/why not)?

b) Answer: No, say S_0 is $\neg x \vee x$. No rule can be applied to this sentence, so for instance the sentence *True* that is logically equivalent to S_0 will never be found as required by completeness.

c) Draw the complete state space that can be reached by applying the rules from the initial sentence $A \wedge (A \Rightarrow (\neg B \wedge B))$. Annotate each edge with the rule it corresponds to. A part of this state space is shown below.



c) Answer:



Prof. Smart wants to use his search approach to determine if a sentence is logically equivalent to *False*. He claims that if the cost of all rule applications is one, a valid heuristic for this search problem is to count the number of propositional symbols and constants and subtract one. As an example, the heuristic value of $A \wedge (A \Rightarrow \text{False})$ is two since there are two propositional symbols in the sentence (A and A) and one constant (False).

d) Is the heuristic admissible (why/why not)?

d) Answer: Yes, a rule application can at most remove one propositional symbol or constant from a sentence. So the number of rule applications needed to reach the goal is larger than or equal to the heuristic value of the state.

e) Is the heuristic consistent (why/why not)?

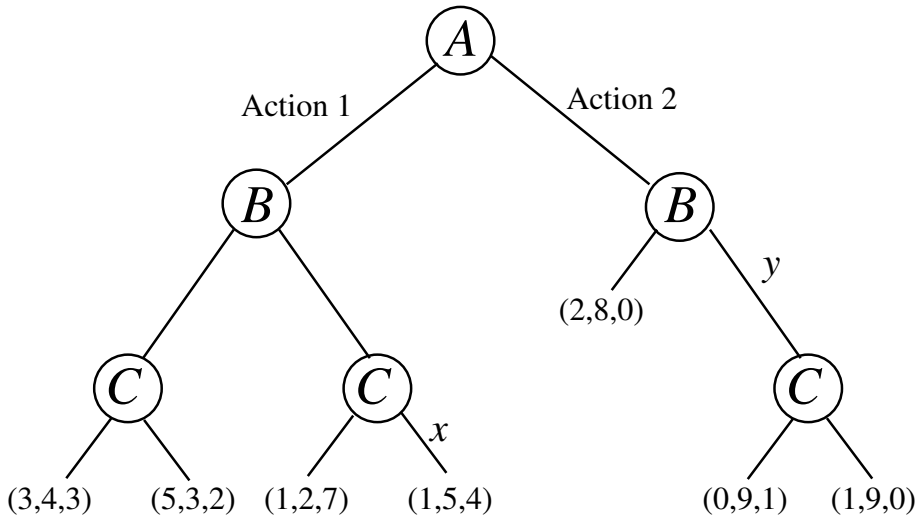
e) Answer: Yes, we need $h(s') + c(s, a, s') \geq h(s)$, where s' is the state resulting from applying a rule a in s and $c(s, a, s')$ is the cost of applying a. In our case this is equivalent to $h(s) - h(s') \leq 1$ since all rule applications have a cost of one. This inequality must hold, since a rule application at most can remove a single propositional symbol or constant.

f) Which sequence of rule applications corresponds to the solution returned by A* Tree Search given an initial state of the sentence $A \wedge (A \Rightarrow (\neg B \wedge B))$ and using Prof. Smarts heuristic?

f) Answer: $R5, R1, R2, R4, R5$, since this is the shortest path to False as seen in the state space graph.

Problem 3: FOR BSC STUDENTS ONLY (30%)

The figure below shows a game tree of a turn-taking game with three players A, B, and C.



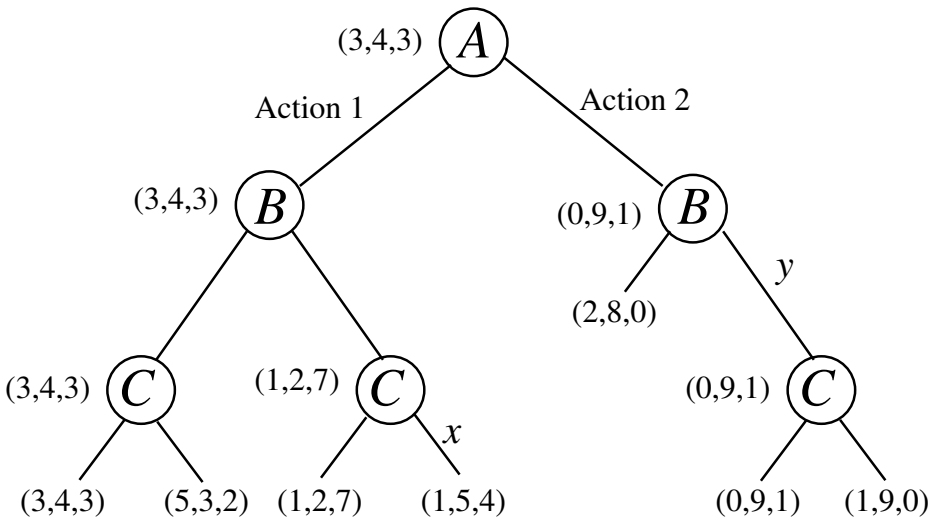
Each leaf-node is a terminal state. Its utility is given as a triple (a, b, c), where a, b, and c are the utility of player A, B, and C, respectively.

a) The game is zero-sum. How do the utilities in the game tree show this?

a) Answer: The sum of the utilities in each triple is a constant 10.

b) Write the multi-player minimax value next to each internal node of the tree.

b) Answer:



c) What is the minimax decision for player A in the root node and why?

c) Answer: It is action 1, since this action leads to the maximum utilization 3 for player A.

The MINIMAX function below can be used to calculate the minimax value of a state of this 3-player game.

```
function MINIMAX(state) returns the Minimax value of state
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← (−∞, −∞, −∞)
    for each a in ACTIONS(state) do
        w ← MINIMAX(RESULT(state,a))
        if VALUE(w,PLAYER(state)) > VALUE(v,PLAYER(state)) then
            v ← w
    return v
```

TERMINAL-TEST(s) returns true if s is a terminal state of the game. UTILITY(s) returns the utility triple of a terminal state s. ACTIONS(s) returns the applicable actions of the player that is to take a turn in state s. RESULT(s,a) returns the state resulting from executing action a in state s. VALUE(u,p) returns the utility of player p in utility triple u. Finally, PLAYER(s) returns the player that is to take a turn in state s.

d) Write pseudo-code of a function MINIMAX-DECISION(s) that returns the minimax decision of the player that is to take a turn in state s.

d) Answer:

function MINIMAX-DECISION(*state*) **returns** an action
  return $argmax_{a \in Actions(state)}$ VALUE(MINIMAX(RESULT(*state*,a)),PLAYER(*state*))

e) Prof. Smart claims that if the MINIMAX function explores actions from left to right in the tree, it can make cuts similar to α and β cuts at edge x and y. Is this correct (why/why not)?

e) Answer: Prof. smart is right. For edge x, C just learned that it can get 7. This means that A and B at most can get 3 at this node, and since B already has 4 in the node above, there is no need for further exploration. For edge y, B just learned that it can get 8. But this means that A and C at most can get 2 at this node, and since A already has 3 in the node above, there is no need for further exploration.

Problem 3: FOR MSC STUDENTS ONLY (30%)

Linear Programming

Assume we have an LP problem, where the dual LP problem is shown below.

$$\begin{array}{lrcrcr} \text{Minimize} & -y_1 & & & & \\ \text{Subject to} & y_1 & - & y_2 & \geq & 0 \\ & & & -y_2 & \geq & -4 \\ & & & y_1, y_2 & \geq & 0 \end{array}$$

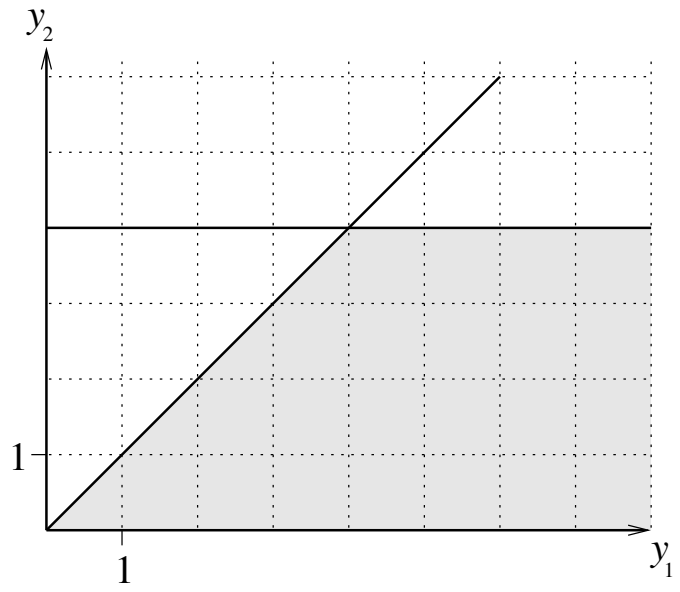
a) Write the LP problem shown above on standard form.

a) Answer:

$$\begin{array}{lrcrcr} \text{Maximize} & y_1 & & & & \\ \text{Subject to} & -y_1 & + & y_2 & \leq & 0 \\ & & & y_2 & \leq & 4 \\ & & & y_1, y_2 & \geq & 0 \end{array}$$

b) Draw the geometric interpretation of the LP problem shown above.

b) Answer:



c) Determine based on your answer to b) alone, whether the primal LP problem is feasible, infeasible, or unbounded. Explain your answer.

c) Answer: Since the dual LP problem is unbounded, the primal LP must be infeasible.

d) Write the primal LP problem.

d) Answer:

$$\begin{array}{lrcrcr} \text{Maximize} & & & -4x_2 & & \\ \text{Subject to} & x_1 & & & \leq & -1 \\ & -x_1 & - & x_2 & \leq & 0 \\ & & & x_1, x_2 & \geq & 0 \end{array}$$

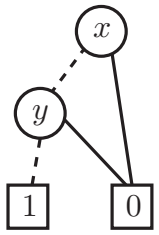
Binary Decision Diagrams

Consider the following Boolean expression

$$(x \Rightarrow y) \wedge (\neg x \wedge \neg y)$$

e) Draw the BDD for the expression using variable order $x \prec y$.

e) Answer:



f) Is the expression valid, unsatisfiable or neither? Give a reason for your answer.

f) Answer: Neither. If the expression is valid, the BDD is 1. If it is unsatisfiable, the BDD would be 0.