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Housing Habits and Their Implications for Life-Cycle Consumption and Investment*

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Abstract

We solve a rich life-cycle model of household decisions involving consumption of perishable goods and housing services, habit formation for housing consumption, stochastic labor income, stochastic house prices, home renting and owning, stock investments, and portfolio constraints. In line with empirical observations, the optimal decisions involve (i) stock investments that are low or zero for many young agents and then gradually increasing over life, (ii) an age- and wealth-dependent housing expenditure share, (iii) non-housing consumption being significantly more sensitive to wealth and income shocks than housing consumption, and (iv) non-housing consumption being hump-shaped over life.

JEL classification: G10, D14, D91, E21, R21

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1. Introduction

Residential real estate is important for households both as a consumption good, an investment asset, and as collateral facilitating borrowing.[1] Therefore, housing consumption and investment should be included in life-cycle models of household decisions but, due to modeling complexity, only few existing papers do so. We solve for the optimal life-cycle decisions in a rich model featuring consumption of perishable goods and housing services, stochastic labor income, stochastic house prices, renting/owning decisions, stock investments, and portfolio constraints. An innovative model feature is habit formation in the preferences for housing consumption. Calibrated to US data, our model is the first to jointly explain why households hold no or few stocks early in life, choose a hump-shaped non-housing consumption pattern and an increasing housing expenditure share over life, and adjust non-housing consumption much more than housing consumption in response to wealth and income shocks. Our results highlight the importance of collateralized borrowing for understanding investment and consumption decisions over the life cycle.

Habit formation finds empirical support (Browning and Collado, 2007; Ravina, 2007), and the consequences of perishable consumption habits for consumption and investment decisions are well-studied (Ingersoll, 1992; Munk, 2008). Housing habits are unexplored but plausible because homes serve as visible status symbols and because individuals build up affection for their home and neighborhood as well as social ties within the community.[2] Moreover, home transaction costs are likely to induce consumption and trading patterns similar to those caused by habit persistence. We assume an internal, additive, multi-period habit meaning that the agent's utility depends on the difference between her current housing consumption and a weighted average of her own past housing consumption.[3]

The housing habit significantly impacts the agent's optimal investment decisions. In a model without habit formation, Cocco (2005) finds that housing crowds out stock holdings which, together with a sizable stock market entry cost, can explain stock market non-participation early in life in contrast to standard models ignoring housing (e.g., Cocco, Gomes, and Maenhout, 2005). Young individuals are attracted by housing investments rather than stock investments mainly because of the associated access to credit via mortgages, which facilitate consumption smoothing over the life cycle. While Cocco's model ignores the renting/owning decision and assumes a perfect correlation between house prices and aggregate income shocks, we obtain early-life non-participation without these restrictive

1 Housing services (shelter) weigh 31.9% in the November 2013 US Consumer Price Index for All Urban Consumers, cf. http://www.bls.gov/cpi/. In the 2010 US Survey of Consumer Finances, residential property constituted 36% of total household wealth, see Tables 8 and 9.1 in Bricker *et al.* (2012). The home ownership rate was 65–70% in the USA in the period 1996–2013 and also well above 50% in most other developed countries, see the Census Bureau at http://www.census.gov/housing/hvs/ for the USA and Andrews and Sanchez (2011) for selected OECD countries. Moreover, 27% of US home sales in 2011 were purely for investment purposes, cf. Choi *et al.* (2016).

2 Solnick and Hemenway (2005) report empirical evidence supporting status concerns regarding housing.

3 Aydilek (2013) considers a one-period multiplicative housing habit in a two-good, discrete-time, life-cycle model that disregards stock investments and focuses on housing late in life. The habit has stronger implications in our setting because of both the multi-period and the additive specification of the habit, and we also consider the interaction between housing decisions and stock investments.

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assumptions and without entry costs. The housing habit induces a minimum future housing consumption which the investor finances by maintaining a wealth buffer invested in the housing market. The habit reduces early-life optimal stock investments—and thus postpones stock market entry—by lowering the wealth disposable for speculative investments and raising the importance of housing investments relative to stocks.

Also consistent with data, but unlike standard models, the stock weight is increasing in wealth since the habit-induced buffer then seizes a smaller share of wealth. Wachter and Yogo (2010) show that an "addilog" utility of basic consumption and luxury consumption can explain why the stock portfolio weight increases in wealth, but their model does not explain a zero or low stock weight of young investors. The habit-induced buffer investment in housing more than outweighs the reduction caused by the housing habit in both the speculative demand and the house-price hedging demand for housing investments, so the overall housing investment is larger with the habit than without.

Our analysis highlights that the access to mortgages is crucial for the optimal investment behavior. When the maximum house loan-to-value ratio is reduced, the optimal stock (house) investment increases (decreases). Furthermore, we find that optimal stock investments are increasing in risk aversion (in some range) as stocks are a less risky alternative to leveraged house investments. This finding questions the empirical practice of inferring risk attitudes only from observed stock investments.

The housing habit also substantially affects the agent's life-cycle consumption pattern. Housing consumption early in life is expensive since the individual then commits to a certain level of housing consumption in the remaining lifetime, which necessitates the above-mentioned wealth buffer. As the time horizon shrinks, the buffer decreases, and housing consumption effectively becomes less expensive. Early in life the individual thus tilts the consumption bundle toward perishable consumption, and less so as time passes.

By embedding the housing habit mechanism in a setting in which overall consumption tends to increase over life, the model generates a hump-shaped perishable consumption over the life cycle. Empirically, perishable consumption is hump shaped over life (Thurow, 1969), but frictionless one-good models lead to either increasing, decreasing, or flat consumption. A predominant explanation of the hump is borrowing constraints (Nagatani, 1972; Carroll, 1997; Gourinchas and Parker, 2002), but in reality most individuals do borrow, and in particular homeowners borrow via mortgages. Relaxing the borrowing constraint postpones the consumption hump. We show that housing habits induce a hump in perishable consumption without constraints or risks, and the habits move the hump to an earlier age when realistic constraints and risks are included.

The Cobb–Douglas utility assumed in standard two-good models implies a constant relation between expenditures on housing and expenditures on perishable goods, that is, a constant housing expenditure share. This contradicts empirical observations. First, the housing expenditure share varies counter cyclically. For example, Figure 1 shows the 1980–2013 annual US GDP growth rates and percentage changes in the housing expenditure share of US households reported in the National Income and Product Accounts (NIPA).[4]

4 See NIPA Table 2.3.5 published by the Bureau of Economic Analysis, the US Department of Commerce, at http://www.bea.gov/itable/. Following Piazzesi, Schneider, and Tuzel (2007), the housing expenditure share is the ratio of housing consumption expenditures to the sum of housing and non-housing consumption expenditures. Housing consumption expenditures are represented by the item "housing and utilities" and non-housing consumption is the sum of "nondurable goods"

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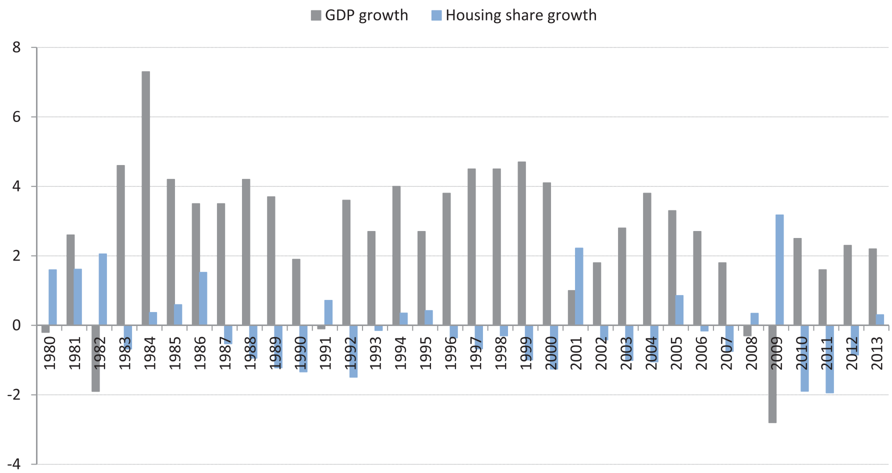

Figure 1. Growth rates of GDP and the housing expenditure share. The data are taken from the US National Income and Product Accounts published by the Bureau of Economic Analysis. GDP growth rates are in Table 1.1.1 (dated January 30, 2015), and the housing expenditure share is in Table 2.3.5 (dated October 30, 2014).

The correlation over the period shown is –0.51 (even –0.59 in 1990–2013), and the housing expenditure ratio has varied between 21.0% (in 2012) and 23.1% (1986) with an average of 22.0%. The Consumer Expenditure Survey (CEX) confirms this variation: in 1993–2013 the housing expenditure share for all age groups averaged 32.9% and varied between 31.4% (1993) and 34.4% (2009, 2010).[5] Across households, the housing expenditure share decreases in income. For example, in the 2013 CEX, the share in the five income quintiles (from lowest to highest) is 40.0%, 36.8%, 34.8%, 32.4%, and 31.1%. On a micro level, Chetty and Szeidl (2007) report from Panel Study of Income Dynamics (PSID) data that in the year following a job loss, home owners on average reduce food consumption much more than housing consumption. Our model with habit formation in housing explains these empirical findings: in times of declining (increasing) wealth or income, housing expenditures are reduced (increased) less than non-housing expenditures.

Secondly, the housing expenditure share varies with age. By analyzing CEX consumption data and housing ownership data from the Survey of Consumer Finances (SCF), Yang (2009) finds that non-housing consumption is hump shaped, whereas housing consumption per adult-equivalent increases throughout life, quite steeply early in life and then flattening out. Based on her data, Figure 2 shows the housing expenditure share over the life cycle for renters and owners as well as the average of the two.[6] The housing expenditure share

(less "clothing and footwear") and "services" (less "housing and utilities"). While NIPA includes a quantity index for housing consumption, Piazzesi, Schneider, and Tuzel (2007) argue that this measure is imprecise so, following their lead, we do not use it.

5 The CEX data were downloaded from the Bureau of Labor Statistics, the US Department of Labor, at http://www.bls.gov/cex/csxshare.htm. We use the expenditure share for the item "housing." We obtain very similar results by using "shelter" that subtracts various expenditures from "housing."

6 We are grateful to Fang Yang for sharing the data with us. The data include the housing stock of owners, and we assume that housing consumption is a constant fraction of the stock. The fraction

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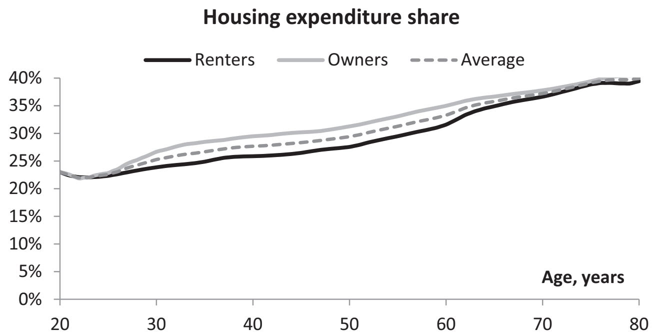

Figure 2. The housing expenditure share over the life cycle. The figure shows housing expenditure shares of renters and owners, as well as the average of the two, over the life cycle. The housing expenditure share is the ratio of housing consumption to the sum of housing and non-housing consumption. The data are based on CEX and SCF, and processed and supplied by Yang (2009).

increases with age from around 22% to 40%, with the steepest increases at age 25–35 years and age 60–70 years. Furthermore, the correlation between the housing expenditure share and GDP growth is more negative for young households than for old. For the 25–34 year olds the correlation is −0.22, whereas for households above 75 years the correlation is −0.03. Also these features are consistent with habit formation. Current housing consumption decisions are influenced by their impact on the housing habit in the remaining life, which obviously is less important for households with a shorter expected remaining lifetime.

As seen in the data, the housing expenditure share in our model depends on the house price level as well as the agent's age and wealth. Obvious alternative utility specifications do not share these properties. With Constant Elasticity of Substitution (CES) utility the share depends only on the house price. CES utility was assumed in the housing-extended Consumption-CAPM of Piazzesi, Schneider, and Tuzel (2007).[7] The addi-log utility specification of Wachter and Yogo (2010) implies an expenditure share depending on good prices and the level of consumption, but not directly on the agent's age.

Our paper relates to recent papers on housing decisions. Corradin, Fillat, and Vergara-Alert (2014) show that predictability in house prices causes house (stock) investments to be increasing (decreasing) in the current expected house price growth. Even with a high risk aversion, they report significant stock investments. By abstracting from the renting-owning decision, they force housing investment and consumption to be identical. Yao and Zhang (2005) find that home owners invest less in stocks than renters, supporting that housing risk crowds out stock risk, but early in life stock investments are large both for owners and renters. Chetty, Sandor, and Szeidl (2017) show empirically and in a stylized model that

is set to 8.22% since then 20-year-old owners have the same expenditure share as 20-year-old renters, but the life-cycle profile of the expenditure share is the same for a wide range of values for this constant. We use the mean housing and non-housing consumption for each age to compute the expenditure share.

7 Their main idea is that agents are especially concerned about states with low overall consumption when the share of housing consumption is low relative to perishable consumption. This holds for CES utility if the intratemporal elasticity of substitution between housing and non-housing consumption exceeds 1. In our model low housing consumption is particularly bad due to the habit.

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mortgage debt reduces stockholding while home equity increases it. In contrast to our paper, these papers do not address the life-cycle consumption patterns. In a life-cycle setting, Fischer and Stamos (2013) find that also home ownership rates and mortgage debt comove with the expected house price growth rate. In their baseline parameterization, zero stock investments are optimal only when expected house price growth is very high. The life-cycle consumption profile they report is steeply increasing until retirement where it flattens out, and they do not address the housing expenditure share.

The paper is organized as follows. Section 2 specifies our model and solution technique. Results for a baseline parameterization of the model are discussed in Section 3, whereas Section 4 considers alternative parameterizations. Section 5 concludes.

2. The Model

2.1 Consumption Goods and Preferences

Our economy features two consumption goods: perishable (or non-housing) consumption and housing consumption. The perishable good is the numeraire in the economy. The housing good is measured in a number of "units" reflecting size, quality, and location of the residential property. Think of one unit as one square foot residence of average quality and location. For convenience we refer to this good as "houses." Let c_t denote the perishable consumption rate at time t (i.e., the agent consumes $\int_t^{t+\mathrm{d}t} c_s \,\mathrm{d}s \approx c_t \,\mathrm{d}t$ goods over the time interval $[t, t+\mathrm{d}t]$), and let q_t denote the number of housing units the agent occupies at time t.

The agent develops habits for housing consumption. The habit level $\bar{q}_t$ satisfies

$$\bar{q}_t = \bar{q}_0 \mathrm{e}^{-\epsilon t} + \alpha \int_0^t \mathrm{e}^{-\epsilon(t-s)} q_s \,\mathrm{d}s \quad \Rightarrow \quad d\bar{q}_t = (\alpha q_t - \epsilon \bar{q}_t)\,\mathrm{d}t, \tag{1}$$

where the initial habit level $\bar{q}_0$, the persistence parameter ϵ, and the scaling parameter α are non-negative constants. The agent derives utility from the consumption at any given time according to the habit-extended Cobb–Douglas function

$$U(c, q, \bar{q}) = \frac{1}{1-\gamma}\left[c^b (q-\bar{q})^{1-b}\right]^{1-\gamma}, \tag{2}$$

where $\gamma > 1$ is a risk aversion parameter, and $b \in (0,1)$ is a weighting parameter. For later use, we define

$$k = \frac{(1-b)(\gamma-1)}{\gamma}, \qquad \hat{b} = b^{-kb/(1-b)}(1-b)^{-k}. \tag{3}$$

Note that $\gamma > 1$ implies that perishable and housing consumption are substitutes in the sense that $U_{cq} < 0$. The habit level $\bar{q}$ represents an endogenously determined subsistence level of housing consumption. If, from time t onward, the agent's housing consumption is exactly at the minimum, $q_s = \bar{q}_s$ for $s \geq t$, the future habit level is $\bar{q}_u = \bar{q}_t \mathrm{e}^{-(\epsilon-\alpha)(u-t)}$ housing units. The difference $\epsilon - \alpha$ indicates the strength of the habit by determining how much the current habit level restricts future decisions.

The model could be extended to allow for habit formation in non-housing consumption at the cost of increased computational complexity. The effects of non-durable consumption habits are already well studied in the literature, and as long as the housing habit is stronger than the non-housing habit, the results would qualitatively be similar to our results. A non-housing

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habit stronger than the housing habit would generate life-cycle patterns in perishable consumption and the housing expenditure share very different from the empirically observed patterns.

The agent lives until time T. The agent's objective at any time $t < T$ is to maximize $\mathrm{E}_t[\int_t^T \mathrm{e}^{-\delta(s-t)} U(c_s, q_s, \bar{q}_s)\, \mathrm{d}s]$ over the feasible consumption and investment strategies as will be specified in more detail. Here, δ is the agent's subjective time preference rate.

2.2 Investments

The agent can invest in a risk-free asset offering a continuously compounded rate of return r. We assume a constant r throughout to focus on the impact of housing decisions and habits on consumption and investment, and the effects of stochastic interest rates on life-cycle decisions are already well-studied.[8] The agent can also invest in a single stock—the market index—with the price S_t (including reinvested dividends) having dynamics

$$\mathrm{d}S_t = S_t[(r + \mu_S)\, \mathrm{d}t + \sigma_S\, \mathrm{d}W_{St}], \tag{4}$$

where W_S is a standard Brownian motion, and where the expected excess return μ_S and the volatility σ_S are assumed constant.

Furthermore, the agent can invest in housing units at the unit price H_t with dynamics

$$\mathrm{d}H_t = H_t[(r + \mu_H)\, \mathrm{d}t + \sigma_H(\rho_{HS}\, \mathrm{d}W_{St} + \sqrt{1 - \rho_{HS}^2}\, \mathrm{d}W_{Ht})], \tag{5}$$

where W_H is a standard Brownian motion independent of W_S, and where μ_H, σ_H, and the correlation ρ_{HS} are constant. To simplify the analysis, we let the agent continuously adjust the number of units owned without transaction costs. While changes in the physical ownership of homes seem rare and costly, the remodeling or the extension of a house also constitutes an increase in the number of housing units owned due to the higher quality or increased space. In addition, individuals can indirectly invest in housing units by purchasing shares in residential REITs, exchange-traded funds emulating the REIT market, or other financial assets linked to house prices such as the Case-Shiller derivatives.[9] Home owners can implement minor variations in the desired housing investment position through remodeling or REITs, whereas they can implement larger changes in both desired housing consumption and investment through infrequent physical house transactions.[10]

The agent can rent housing units at a rent proportional to the price of the rented property; renting q units over the time interval $[t, t + \mathrm{d}t]$ costs $\int_t^{t+\mathrm{d}t} \chi q H_s\, \mathrm{d}s \approx \chi q H_t\, \mathrm{d}t$, and we refer to $\chi \geq 0$ as the rental rate. The agent can disentangle housing investment from housing

8 See, for example, van Hemert (2010); Koijen, Nijman, and Werker (2010); Munk and Sørensen (2010); and Kraft and Munk (2011).

9 Well-developed REIT markets exist in many countries. Cotter and Roll (2015) study the risk and return characteristics of US REITs. As explained by (Ang, 2014, Chap. 11), REIT returns exhibit only a low short-run correlation with returns on directly owned real estate (and a higher correlation with common stocks), but longer-term correlations are significantly higher, cf., for example, Hoesli and Oikarinen (2012). Pagliari, Scherer, and Monopoli (2005) argue that after various relevant adjustments REIT returns and direct real-estate returns are much more highly correlated even in the short run, and Lee, Lee, and Chiang (2008) and others report that REITs behave more and more like real estate and less and less like ordinary stocks.

10 The results of Kraft and Munk (2011) indicate that household welfare is reduced very little if housing consumption and investments are only infrequently adjusted suggesting that, without exogenously triggered forced moves, transaction costs have modest effects on the overall housing position.

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consumption by simultaneously owning and renting (out). A pure housing investment position is obtained by owning housing units and renting them out, either directly or through REITs. Ownership entails maintenance costs (including property taxes) equal to a constant fraction $m \geq 0$ of the property value. Hence, the return on a pure investment in a housing unit is $\mathrm{d}H_t + (\chi - m)H_t\,\mathrm{d}t$.

2.3 Labor Income

Throughout life the agent receives an income stream from non-financial sources at a rate of Y_t. The individual retires at a predetermined time $\tilde{T} \leq T$. At retirement the income drops to a known fraction Υ of the income immediately before,

$$Y_{\tilde{T}+} = \Upsilon Y_{\tilde{T}-}. \tag{6}$$

This reflects the wide-spread final-salary pension schemes and is a common assumption in the literature (e.g., Cocco, Gomes, and Maenhout, 2005; Lynch and Tan, 2011). Before and after retirement the income rate follows:

$$\mathrm{d}Y_t = Y_t[\mu_Y(t)\,\mathrm{d}t + \sigma_Y(t)(\rho_{YS}\,\mathrm{d}W_{St} + \hat{\rho}_{HY}\,\mathrm{d}W_{Ht} + \hat{\rho}_Y\,\mathrm{d}W_{Yt})], \tag{7}$$

where $W_Y = (W_{Yt})$ is a standard Brownian motion independent of W_S and W_H. Unless $\hat{\rho}_Y = 0$, the income risk is unspanned by traded assets so the individual faces an incomplete market. The income-house and income-stock correlations ρ_{HY} and ρ_{YS} are constant, and

$$\hat{\rho}_{HY} = \frac{\rho_{HY} - \rho_{YS}\rho_{HS}}{\sqrt{1-\rho_{HS}^2}}, \qquad \hat{\rho}_Y = \sqrt{1-\rho_{YS}^2 - \hat{\rho}_{HY}^2}. \tag{8}$$

Both the expected growth rate and volatility of income may depend on the individual's age. Where most life-cycle papers assume a constant retirement income, we allow for risk. This is motivated by (i) some retirees continue to earn income from proprietary businesses or other non-traded assets; (ii) uncertainty about medical expenses implies that the disposable income is risky (De Nardi, French, and Jones, 2010); (iii) because of mortality risk, the individual may miss retirement payments and, when we do not model mortality formally, retirement income risk captures this effect parsimoniously.

2.4 Wealth, Portfolios, and Constraints

The agent's tangible wealth X_t is the sum of the values of her investments in the available assets. Let Π_{St} be the fraction of tangible wealth invested in the stock, and let ϕ_{ot} and ϕ_{rt} denote the housing units owned and rented, respectively. Then the units of housing consumed are $q_t \equiv \phi_{ot} + \phi_{rt}$, and the fraction of tangible wealth invested in housing is $\Pi_{Ht} \equiv \phi_{ot}H_t/X_t$. The wealth invested in the risk-free asset is residually determined as $M_t = X_t(1 - \Pi_{St} - \Pi_{Ht})$. The dynamics of tangible wealth are

$$\begin{aligned}
\mathrm{d}X_t &= \Pi_{St}X_t\frac{\mathrm{d}S_t}{S_t} + M_t r\,\mathrm{d}t + \phi_{ot}(\mathrm{d}H_t - mH_t\,\mathrm{d}t) - \phi_{rt}\chi H_t\,\mathrm{d}t - c_t\,\mathrm{d}t + Y_t\,\mathrm{d}t \\
&= (X_t[r + \Pi_{St}\mu_S + \Pi_{Ht}(\mu_H + \chi - m)] + Y_t - c_t - q_t\chi H_t)\,\mathrm{d}t \\
&\quad + X_t(\Pi_{St}\sigma_S + \Pi_{Ht}\sigma_H\rho_{HS})\,\mathrm{d}W_{St} + X_t\Pi_{Ht}\sigma_H\sqrt{1-\rho_{HS}^2}\,\mathrm{d}W_{Ht}.
\end{aligned} \tag{9}$$

Since income contains unspanned risk and is not bounded from below by a positive level, non-negative terminal wealth can only be insured by keeping tangible wealth

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non-negative throughout, that is, $X_t \geq 0$ for all $t \in [0, T]$. Furthermore, we impose the constraints

$$\Pi_S \geq 0, \quad \Pi_H \geq 0, \quad \Pi_S + \kappa \Pi_H \leq 1, \tag{10}$$

which rule out short-selling and limit borrowing to a fraction $(1 - \kappa)$ of the current value of the housing investment, where $\kappa \in [0, 1]$.

2.5 The Utility Maximization Problem

The agent has to choose the perishable consumption rate c_t, the housing consumption q_t, the stock portfolio weight Π_{St}, and the housing portfolio weight Π_{Ht}. Let $\mathcal{A}$ denote the set of all admissible strategies, that is, strategies (c, q, Π_S, Π_H) satisfying standard integrability conditions, the wealth constraint $X_t \geq 0$, the portfolio constraints (10), and the consumption constraints $c_t \geq 0$, $q_t \geq \bar{q}_t \geq 0$. The value function is defined as

$$J(t, x, y, h, \bar{q}) = \sup_{(c,q,\Pi_S,\Pi_H)\in\mathcal{A}} E_t\left[\int_t^T e^{-\delta(s-t)} U(c_s, q_s, \bar{q}_s)\, ds\right], \tag{11}$$

where the expectation is conditional on $X_t = x$, $Y_t = y$, $H_t = h$, and $\bar{q}_t = \bar{q}$.

Because of incomplete markets and portfolio constraints, we cannot solve the problem in closed form, but resort to the numerical technique outlined in the next subsection. In the Online Appendix, we derive closed-form solutions for two simpler, unconstrained, complete market problems that share many of the properties of the full problem and illustrate these properties transparently. In both models, the value function is of the form

$$J(t, x, h, y, \bar{q}) = \frac{1}{1-\gamma}(\chi h)^{\gamma k} G(t)^{\gamma} (x + yF(t) - \bar{q}\chi h B(t))^{1-\gamma}, \tag{12}$$

where G, F, and B are deterministic functions of time. Here, $yF(t)$ is the human capital—the present value of the future income stream—which is uniquely determined in complete markets. The term $\bar{q}\chi h B(t)$ is the housing habit buffer, that is, the costs of ensuring that housing consumption exceeds the housing habit level through the remaining lifetime.

The first model disregards stocks, uncertainty, and frictions, and therefore focuses on the effects of housing habits on life-cycle consumption. Defining the disposable wealth as $\hat{X}_t = X_t + Y_t F(t) - \bar{q}_t \chi H_t B(t)$, the optimal consumption strategy in this model is

$$c_t = b\hat{b}\frac{(1 + \alpha B(t))^k}{G(t)}\hat{X}_t, \qquad q_t = \bar{q}_t + (1 - b)\hat{b}\,\frac{(1 + \alpha B(t))^{k-1}}{\chi H_t G(t)}\hat{X}_t. \tag{13}$$

Optimal perishable and housing consumption satisfy

$$c_t = \frac{b}{1-b}(q_t - \bar{q}_t)\chi H_t(1 + \alpha B(t)), \tag{14}$$

so that the housing expenditure share is

$$\frac{\chi q_t H_t}{c_t + \chi q_t H_t} = \frac{1 - b}{1 - b + b\left(1 - \frac{\bar{q}_t}{q_t}\right)(1 + \alpha B(t))}. \tag{15}$$

Without habit formation ($\bar{q}_t = 0, \alpha = 0$), the housing expenditure share is the constant $1 - b$, and perishable consumption grows at a constant positive (for reasonable parameter values) rate throughout life. The housing habit changes these conclusions. Equation (14)

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shows that $\chi H_t(1+\alpha B(t))$ is the effective time t unit price of housing consumption. Since B is decreasing over time, so is the effective price of housing, other things equal. Housing consumption is more expensive for young agents: the consumption of one extra unit of housing costs not only the current rent but also future required rents due to the increase in the habit level. The optimal composition of consumption is tilted toward perishable goods when young and toward housing when old. Embedding this mechanism in a setting where overall consumption is optimally increasing over life can lead to a hump-shaped life-cycle pattern in perishable consumption as seen in the data. With housing habits, the housing expenditure share is time dependent, decreases with q_t and thus with disposable wealth $\hat{X}_t$, and is thus higher in hard times than in good times, as seen in the data. Note that these effects cannot be generated by habit formation in non-housing consumption.

The second model adds risky investments and uncertainty about both house prices and labor income, but assumes that income risk is fully spanned by traded assets. The optimal unconstrained investment strategy is of the form

$$\Pi_{St} = \frac{1}{\gamma}\frac{\lambda_S - \rho_{HS}\lambda_H}{\sigma_S(1-\rho_{HS}^2)}\frac{\hat{X}_t}{X_t} - \frac{\sigma_Y(t)}{\sigma_S}(\rho_{YS} - \rho_{HS}\tilde{\rho}_{HY})\frac{Y_t F(t)}{X_t}, \tag{16}$$

$$\Pi_{Ht} = \frac{1}{\gamma}\frac{\lambda_H - \rho_{HS}\lambda_S}{\sigma_H(1-\rho_{HS}^2)}\frac{\hat{X}_t}{X_t} - \frac{\sigma_Y(t)}{\sigma_H}\tilde{\rho}_{HY}\frac{Y_t F(t)}{X_t} + k\frac{\hat{X}_t}{X_t} + \bar{q}_t\chi B(t)\frac{H_t}{X_t}. \tag{17}$$

where $\lambda_S = \mu_S/\sigma_S$, $\lambda_H = (\mu_H + \chi - m)/\sigma_H$, and $\tilde{\rho}_{HY} = \sqrt{1-\rho_{YS}^2}/\sqrt{1-\rho_{HS}^2}$. The optimal stock investment in Equation (16) is the sum of a speculative demand determined by the current risk-return tradeoff and an income-adjustment term. The first two terms of the optimal housing investment have a similar interpretation. The third term in Equation (17) hedges against increases in the price of housing consumption, which is accomplished by having a higher wealth exposure to house prices, cf. Sinai and Souleles (2005). The final term ensures that the agent can reach at least the minimum housing consumption in the remaining life time. As the costs of ensuring that the minimum is achieved vary with house prices, this calls for an increased wealth exposure to house prices. Both the hedge term and the habit-insurance term are thus positive. The income-adjustment terms undo the extent to which the human capital resembles an investment in the stock or housing units.

Habit formation lowers the disposable wealth $\hat{X}_t$ through the buffer $\bar{q}_t\chi H_t B(t)$, which is typically slightly increasing early in life (since $\bar{q}_t$ increases) and then decreasing as the remaining life time shrinks ($B(t)$ is decreasing). Therefore, the habit reduces the optimal stock investment, in particular for young investors. The habit also reduces the speculative housing demand and the house price hedge demand but, on the other hand, induces a positive habit-insurance term so that the net effect is parameter dependent. Note that habit formation in non-housing consumption would lead to a wealth buffer invested in the (real) risk-free asset and thus reduce both stock and housing investments.

2.6 Numerical Solution Approach

The full model is solved numerically. Due to the high number of state variables, grid-based methods are cumbersome to implement and lead to high computation times with the grid sizes required for high precision. Instead we apply an adapted version of the approach introduced by Bick, Kraft, and Munk (2013); for details see the Online Appendix. The method exploits that in each of various artificial markets a closed-form solution to the utility maximization problem exists. In any of the artificial markets the agent is better off than

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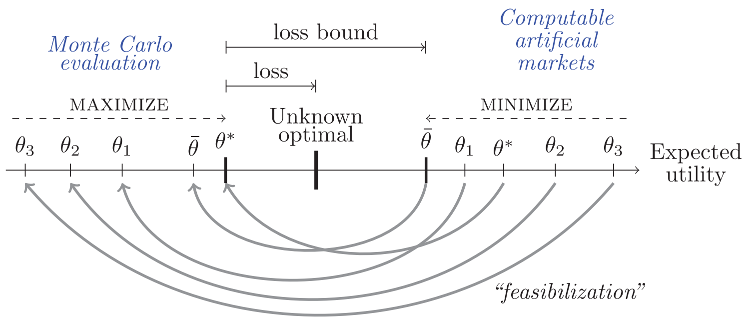

Figure 3. Our solution technique. The axis shows the agent's expected utility. "Unknown optimal" represents the indirect utility in the true market, that is, the expected utility generated by the unknown optimal consumption-investment strategy. Each point to the right corresponds to the indirect utility in an artificial market with deterministic modifiers characterized by some parameter set θ. The corresponding strategy is transformed into a feasible strategy in the true market which generates an expected utility on the left part of the axis. The best of these strategies is derived from the optimal strategy in an artificial market for some θ^*. The arrows above the axis indicate the unknown utility loss and a computable upper bound on the loss the agent suffers by following the best of the considered feasible strategies instead of the unknown optimal strategy.

in the true market, since she is unconstrained, has access to the same assets (with identical or higher returns), plus an additional asset completing the market. In Figure 3, the points marked to the right on the axis indicate the maximal utility in different artificial markets denoted by θ_1, θ_2, etc. The lowest expected utility among these artificial markets—indicated by $\bar{\theta}$ on the axis—is still at least as large as the unknown maximum in the true market.

The explicit, optimal strategy in an artificial market is infeasible in the true market, but can be feasibilized—transformed into a feasible strategy—and its expected utility in the true market can be evaluated by Monte Carlo simulation. In this way we can generate the points on the left part of the axis in Figure 3. Maximizing over these feasibilized strategies, we obtain the expected utility indicated by θ^* in the figure. The corresponding near-optimal strategy is the strategy suggested by the SAMS approach.

As with other numerical methods, the suggested strategy is unlikely to be identical to the truly optimal strategy so the agent suffers a welfare loss by applying the suggested strategy. We derive an upper bound on the welfare loss by comparing the expected utility generated by the near-optimal strategy to the expected utility in the worst of the artificial markets considered. This upper bound represents a measure of precision of the approach. In wealth-equivalent terms, the loss bound in our baseline case below is only 1.1% of the agent's wealth. In the examples of Bick, Kraft, and Munk (2013), the true loss is significantly smaller than the loss bound and typically even smaller than the loss suffered from implementing a standard grid-based approach.

3. Results with Baseline Parameter Values

3.1 Parameter Values

Table I summarizes our baseline parameter values. We first estimate the parameter values for the stock, house, and income dynamics using quarterly US data from 1953q1 to 2010q4

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Table I. Baseline parameter values Monetary quantities are measured in thousands of USD. The text explains how the parameter values are determined. For the no-habit case we set the utility weight parameter b to 0.65 in order to obtain the same average housing expenditure share as in the case with habits

Parameter	Description	Value
δ	Time preference rate	0.05
γ	Risk aversion coefficient	3
b	Perishable/housing utility weight	0.69
α	Habit scaling parameter	0.8
ϵ	Habit persistence parameter	0.9
$\bar{q}_0$	Initial housing habit level	200
T	Remaining life time	50
$\tilde{T}$	Time until retirement	35
X_0	Initial financial wealth	20
r	Risk-free rate	0.01
μ_S	Excess expected stock return	0.04
σ_S	Stock volatility	0.17
h_0	Initial unit house price	0.25
μ_H	Excess expected house price growth	−0.01
σ_H	House volatility	0.12
χ	Rental rate	0.067
m	Maintenance cost	0.035
κ	Collateral parameter	0.2
y_0	Initial income per year	20
μ_Y	Expected income growth	0.01
σ_Y	Income volatility	0.1
Υ	Replacement ratio	0.6
ρ_{HS}	House-stock correlation	0.25
ρ_{YS}	Income-stock correlation	0.22
ρ_{HY}	Income-House correlation	0.16

on the stock market index, the national house price index, and aggregate labor income. Subsequently some values are adjusted to be more representative of individual house prices and labor income, and other values are slightly rounded. For stocks we use the returns on the CRSP value-weighted market portfolio inclusive of the NYSE, AMEX, and NASDAQ markets. The risk-free rate is the 3-month Treasury bill yield from the Risk-Free File on CRSP Bond tape. The house price is represented by the national Case–Shiller home price index with data taken from Robert Shiller's homepage.[11] We obtain quarterly US data for aggregate disposable personal income from the NIPA tables and divide by the population size to compute the disposable labor income per capita. All time series are deflated using the CPI taken from the website of the Bureau of Labor Statistics.

The stock price volatility is estimated at 17%, a standard value. We reduce the estimated equity premium from 5.3% to 4% to account for the survivorship bias (Brown, Goetzmann, and Ross, 1995) and the decline in discount rates and the implied unexpected

11 http://www.econ.yale.edu/~shiller/data.htm.

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capital gains over the sample period (Fama and French, 2002). Furthermore, closely related papers (e.g., Cocco, Gomes, and Maenhout, 2005) also assume an equity premium of 4%.

For the house price, the estimated expected real growth rate is $r + \mu_H = 0$ as there has been virtually no growth in real house prices over the full sample period, and the same value was used, for example, by Yao and Zhang (2005). We increase the volatility of the house series from 6.1% to 12%, in line with Flavin and Yamashita (2002) and Yao and Zhang (2005). We report monetary quantities (wealth, income, prices) in thousands of USD so the initial house price $H_0 = 0.25$ corresponds to \$250 per square foot, that is, \$250,000 for a home of 1000 square feet of average quality and location.[12] We use a rental rate of 6.7% as estimated by Fischer and Stamos (2013) and a maintenance rate of 3.5% which includes property taxes that constitute 1–2% in many US states. We let $\kappa = 0.2$ so that homeowners can borrow up to 80% of the home value (at the risk-free rate).

The average growth rate of the aggregate income series is 1.7% per year, but this does not reflect the income growth that an individual can expect. As our benchmark we assume an expected growth rate of 1% throughout the working life. Over the 35-year working period the income is then expected to grow by a factor $\exp(0.01 \times 35) \approx 1.42$, which seems reasonable and is close to the 38% reported as the median individual's income growth by Guvenen *et al.* (2015). We consider age- and education-dependent income in Section 4.6. We adjust the income volatility from 2.1% to 10% in line with Cocco, Gomes, and Maenhout (2005) and others. As in Yao and Zhang (2005) and Kraft and Munk (2011), the replacement ratio is 0.6, implying a 40% income drop at retirement. We allow income volatility in retirement as motivated earlier and assume the same volatility as before retirement.

The pairwise correlations between stock prices, house prices, and labor income are all slightly positive and close to values used, for example, by Cocco (2005), Yao and Zhang (2005), and Fischer and Stamos (2013). Other studies find income-stock correlations closer to zero, but obviously with some variation across individuals (e.g., Cocco, Gomes, and Maenhout, 2005; Heaton and Lucas (2000)).[13] In Section 4.7 we consider the effect of setting each of these correlation parameters to zero.

We focus on the decisions of a 30-year old who retires at 65 and dies at 80. Initial wealth and (after-tax) income are set to 20—representing \$20,000—in line with the median net worth and before-tax income statistics for young individuals according to the 2013 SCF.[14]

12 Home prices vary a lot across states and regions. According to www.zillow.com the March 2015 median sale price per square foot was \$260 in California so a housing unit in our model is roughly corresponding to a square foot of housing of an average Californian location and quality. In Illinois, for example, the median sale price was \$122 per square foot so one housing unit is about two square foot of housing of an average Illinois location and quality.

13 Benzoni, Collin-Dufresne, and Goldstein (2007) argue that the instantaneous correlation underestimates the longer-run stock-income correlation which is more important for long-term household decisions.

14 See http://www.federalreserve.gov/econresdata/scf/scfindex.htm. Summary results are presented in "Changes in US Family Finances from 2010 to 2013: Evidence from the Survey of Consumer Finances" published in the Federal Reserve Bulletin (September 2014) and available on the above homepage. According to Table I of this document, the *before*-tax median *family* income was \$35.300 for age (of family head) less than 35 years and \$60.900 for age 35–44 years. From Table II, the median net worth per family was \$10.400 for age less than 35 years and \$46.700 for age 35–44 years. Our numbers reflect after-tax income and wealth per adult.

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The initial housing habit is set at 200 housing units, which might represent a reasonable minimum home for a 30-year old individual. The habit parameters are $\alpha = 0.8$ and $\epsilon = 0.9$, which implies a relatively weak habit in the sense that whenever the agent consumes housing at the minimum, this minimum decreases by $\epsilon - \alpha = 10\%$ per year.

We assume $\gamma = 3$. In our simplified models with the known value function (12), we let $W_t = X_t + Y_t F(t)$ denote total wealth and compute the relative risk aversion as

$$\mathrm{RRA} = -\frac{W J_{WW}}{J_W} = \frac{\gamma W}{W - \bar{q}\chi h B(t)}, \tag{18}$$

in line with other habit settings, for example, Campbell and Cochrane (1999). In the full model, we can calculate a relative risk aversion in the same way, since our near-optimal strategy is derived from an artificial market, where the value function is of the same form as Equation (12). With our baseline values for initial wealth, income, and habit, this relative risk aversion is 3.28, which is lower than values often used in this literature, but more in line with empirical estimates (Meyer and Meyer, 2005). The time preference rate of 5% is a standard choice in the literature. The utility weight parameter b is set to 0.69 (0.65) with (without) habit formation in order to obtain the same average housing expenditure share in the two cases.

3.2 Results

Our numerical solution approach generates consumption and investments as functions of time and the state variables. Applying that consumption and investment strategy in the wealth dynamics, we simulate 10,000 paths of the dynamic system forward using 50 time steps per year. The results presented below are age-by-age averages over these paths and can thus be considered as expected life-cycle patterns.

Figure 4 illustrates various aspects of the optimal investments over the life cycle. First, we focus on the case with habit formation as illustrated by the black curves. Initially the agent borrows 80 and invests that amount plus her initial wealth of 20 in housing units, and nothing in stocks. This corresponds to a portfolio weight of 500% for housing, 0% for stocks, and −400% for the risk-free asset. She enters the stock market later and gradually increases her position (both amount and portfolio weight) until retirement, after which the amount invested in stocks decrease toward zero through retirement but at a roughly constant share.[15] Note that this is obtained for a low relative risk aversion. This pattern is in stark contrast to life-cycle models without housing and housing habits. For example, Cocco, Gomes, and Maenhout (2005) find that even with a risk aversion of 10, the agent typically starts out investing 100% of wealth in the stock market and after some years, the stock weight is gradually reduced to around 50% around retirement. Our model thus provides an explanation for the observed non-participation in stock markets of many young individuals without relying on stock market entry costs. The housing habit contributes to this result by creating a need for an extra investment in housing. Thereby, the habit reduces the wealth available for speculative investments and makes housing investments more attractive compared with stocks. For a constrained investor, this can lead to a zero stock position being optimal.

15 We interpret the difference between the maximum and actual loan size as an amount invested in the risk-free asset. The stock weight in the financial portfolio is thus the amount in stocks divided by the sum of the amounts invested in stocks and the risk-free asset.

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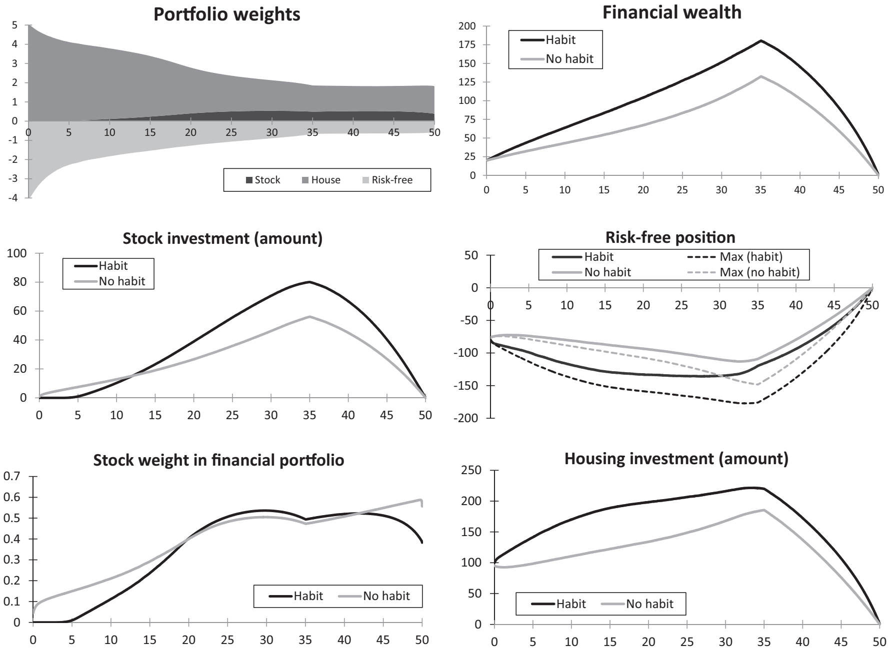

Figure 4. Investments over the life cycle. The graphs show averages across 10,000 simulations in which the consumption and investment strategy determined by our numerical method is used. The black curves are for the case with a housing habit, the grey curves are for the case without. The baseline parameters listed in Table I are used.

The portfolio is dominated by housing investments, partly funded by collateralized borrowing, throughout life. When we plug the baseline parameter values into the unconstrained portfolio formulas (16)–(17) for the simple case with spanned income risk, the speculative demands for stocks and housing are of the same order. The house price hedge and habit buffer components add to the importance of housing investments. Furthermore, housing investments can be levered through collateralized borrowing, whereas stock investments cannot. The housing investment peaks at around \$200,000 at retirement. As shown in the mid-right panel, the individual borrows throughout life, but only in the early years at the maximum amount possible (i.e., 80% of the housing investment).

By comparing with the case without habit formation (grey curves in the figure), we see that the housing habits induce the individual to save more primarily through increased housing investments, partly financed by borrowing. The habit-forming agent enters the stock market at a later age and holds a somewhat lower fraction of wealth in stocks both early and very late in life. Interestingly, the housing habit leads the agent to invest a larger amount in stocks except for the first 10–15 years.

Figure 5 depicts optimal consumption over the life cycle. The left panel shows that perishable consumption in the no-habit model increases through life. With housing habits, perishable consumption is hump shaped and peaks around retirement. Cocco, Gomes, and Maenhout (2005), among others, report a consumption hump around retirement and explain it by a mix of mortality risk, borrowing constraints, and a hump-shaped income profile. We obtain the consumption hump without mortality risk or a hump-shaped income,

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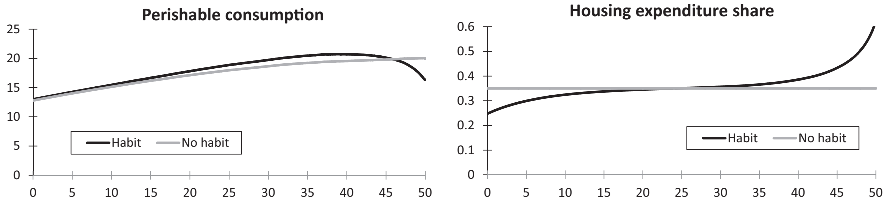

Figure 5. Consumption over the life cycle. The graphs show averages across 10,000 simulations in which the consumption and investment strategy determined by our numerical method is used. The black curves are for the case with a housing habit, the grey curves are for the case without. The baseline parameters listed in Table I are used.

and with much less tight borrowing constraints due to the 80% loan-to-value mortgage access; the determinants of the consumption hump is discussed further in Section 4. The right panel shows that the housing expenditure share is constant in the no-habit model, but with habits it exhibits the shape seen in the data, cf. Figure 2. The intuition behind these patterns is that housing consumption is effectively more expensive early in life because of the commitments to future consumption caused by habit formation. The housing habit reduces housing consumption and increases housing investment in early adult life.

Habits also impact the sensitivity of consumption with respect to wealth or income shocks. We focus on the ratio $\frac{\partial c_t}{\partial \hat{X}_t} / \frac{\partial(\chi q_t H_t)}{\partial \hat{X}_t}$ of the marginal propensities to consume out of disposable wealth. In the simple deterministic model this ratio equals $(1 + \alpha B(t))b/(1 - b)$ as can be derived from Equation (13). The ratio is constant in the no-habit case ($\alpha = 0$). With habits, the ratio is bigger and decreasing over life. In the full model, we can compute the same ratio using the B-function in the artificial market which the near-optimal strategy is derived from. With the baseline parameter values, the ratio is 1.9 in the no-habit case, whereas it declines from 14.0 initially to 2.2 at the end of life. If hit by a negative wealth or income shock, a young habit-forming agent reduces perishable consumption expenditures much more than housing expenditures, as seen in the data, cf. the "Introduction" section.

4. Robustness of Results

Sections 4.1–4.7 illustrate the sensitivity of our results to the selected key parameters. Section 4.8 summarizes the results of an extension to mortality risk and bequest.

4.1 The Habit Strength

Recall that the habit dynamics involve the two parameters ϵ and α with $\epsilon - \alpha$ representing the strength of the habit: if $\epsilon - \alpha$ is small, the habit only declines very slowly even with minimum housing consumption and therefore restricts the agent more. Figure 6 illustrates the importance of the habit strength by fixing $\epsilon = 0.9$ and considering $\alpha = 0.7$ (weak habit) and $\alpha = 0.88$ (strong habit) in addition to the baseline $\alpha = 0.8$.

The upper graphs confirm that stronger habits imply more savings mainly by larger housing investments needed to sustain future minimum housing needs. The amount invested in the stock is also increased, especially in the years leading up to retirement, as shown in the lower-left panel. The lower-right panel reveals that the life-cycle pattern in perishable consumption is similar for the different habit strengths, but with the strong habit the pattern is more curved and the peak appears earlier in life.

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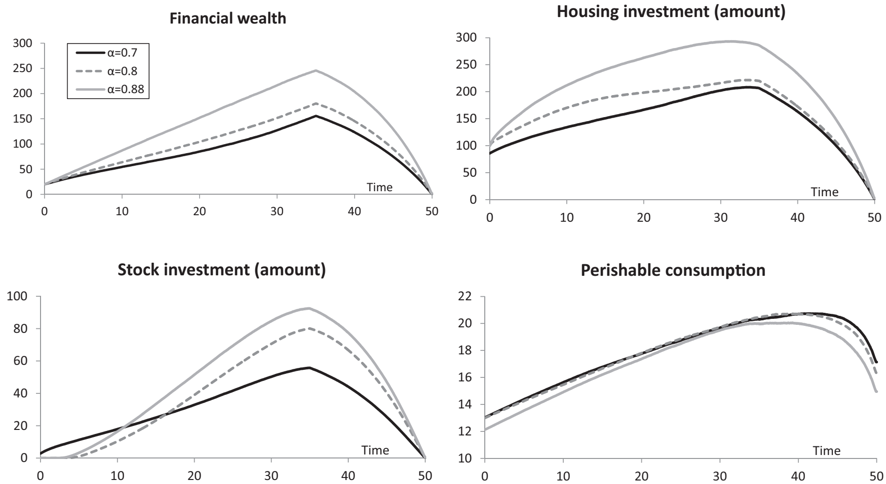

Figure 6. Consumption and investments for different habit strengths. The case of $\alpha = 0.8$ is our baseline case, whereas $\alpha = 0.7$ represents a weaker habit and $\alpha = 0.88$ a stronger habit. For all other parameters the baseline values in Table I are used.

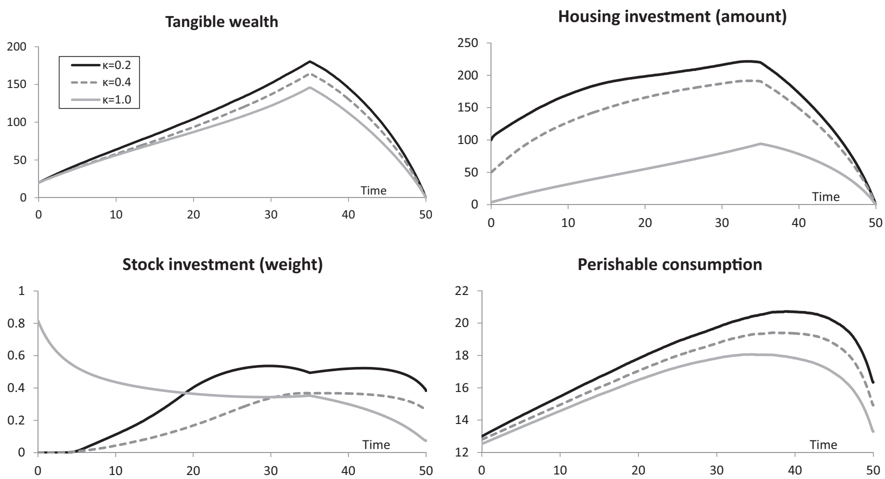

Figure 7. Consumption and investments for different borrowing limits. The case of $\kappa = 0.2$ is our baseline case, whereas $\kappa = 0.4$ limits the loan-to-value ratio to 60%, and $\kappa = 1$ prohibits any borrowing. For all other parameters, the baseline values in Table I are used.

4.2 The Access to Collateralized Borrowing

In the baseline case the agent can borrow up to 80% of the value of her investment position in the housing market, corresponding to $\kappa = 0.2$. In Figure 7 we compare this with $\kappa = 0.4$ (max 60% loan-to-value) and $\kappa = 1$ (no borrowing).

In most situations, the agent optimally exploits the borrowing capacity early in life so that access to more borrowing also generates more borrowing in combination with a higher

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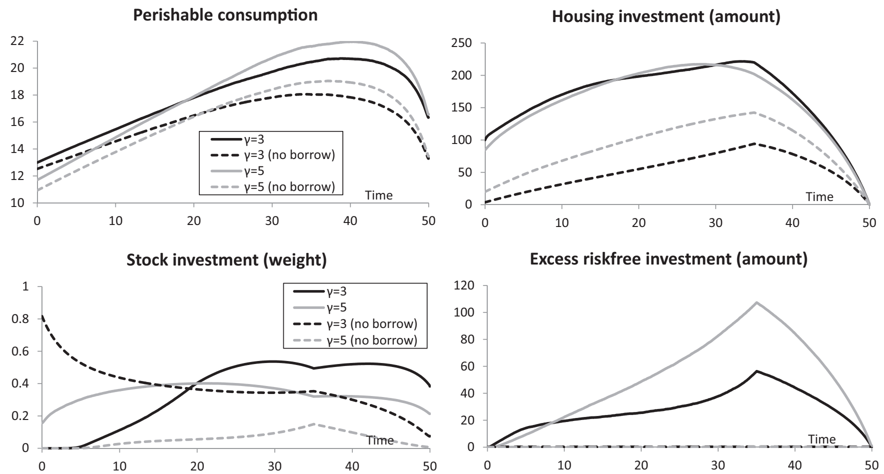

Figure 8. Consumption and investments for different degrees of risk aversion. Each panel considers the four combinations arising from (i) $\gamma = 3$ or $\gamma = 5$ and (ii) $\kappa = 0.2$ or $\kappa = 1$ (no borrowing). For all other parameters, the baseline values in Table I are used.

house investment. The upper-right panel confirms that the attractiveness of housing as an investment asset is to a large extent driven by the associated access to credit. If the agent cannot borrow at all, she optimally invests a significant share of wealth in the stock market already in the early years as illustrated in the lower-left panel.

Perishable consumption is significantly affected by the access to collateralized borrowing. First, the consumption level throughout life increases when borrowing is possible. Secondly, perishable consumption peaks later when borrowing is allowed. Various papers explain the consumption hump by borrowing constraints and have formulated and calibrated models in which optimal consumption is indeed hump shaped typically with a peak around retirement, cf., for example, Gourinchas and Parker (2002) and Cocco, Gomes, and Maenhout (2005). However, these models restrict the agent from any borrowing, whereas real-life homeowners typically have mortgages. The lower-right panel shows that the access to borrowing postpones the peak age of consumption significantly. The housing habit in our model generates the hump-shaped perishable consumption with or without borrowing. As discussed earlier, a stronger habit leads to an earlier peak in consumption. Mortality risk can also contribute to a relatively early peak as will be discussed in Section 4.8.

4.3 Risk Aversion

Next we investigate how the risk aversion coefficient γ affects the results. As this depends on the access to borrowing, Figure 8 considers $\gamma = 3$ (baseline value) and $\gamma = 5$ together with $\kappa = 0.2$ (baseline value, 80% loan-to-value) and $\kappa = 1$ (no borrowing). A higher risk aversion coefficient induces the agent to increase precautionary savings and consume less in the early years, whereas later in life the higher savings finance a larger consumption. Therefore, the perishable consumption profile is steeper and peaks later when the risk aversion coefficient is increased from 3 to 5. These observations hold with (solid curves) or without borrowing (dotted curves).

The impact on investments of an increase in risk aversion is intriguing. The lower-left panel shows that, with access to borrowing, the stock investment increases with risk

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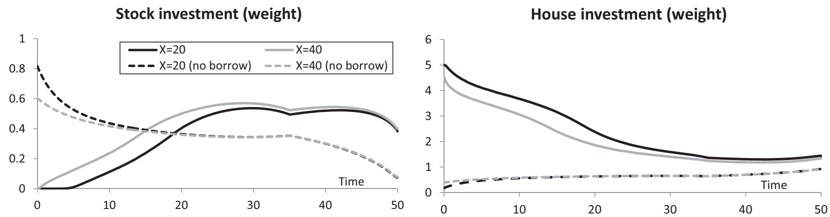

Figure 9. Portfolio weights and initial wealth. The black curves assume an initial financial wealth of $X_0 = 20$ as in the baseline case, whereas the grey curves assume an initial wealth of $X_0 = 40$. The solid curves are for the baseline case of $\kappa = 0.2$ (80% borrowing of house value), whereas the dotted curves are for the case $\kappa = 1$ (no borrowing). For all other parameters, the baseline values in Table I are used.

aversion early in life. The reason is that stocks are less risky than the mortgage-leveraged house investment. Hence, when increasing the risk aversion, the agent moves from a levered house investment to stocks and then eventually to the risk-free asset.

As shown by the lower-right panel, the difference between the portfolio weight of the risk-free asset and its minimum value [defined by $-(1-\kappa)$ times the weight of the housing investment] increases with the risk aversion except very early life, so in that sense borrowing is reduced. Without borrowing, the same increase in risk aversion leads to a significantly higher housing investment and a significantly lower stock investment, both in absolute and relative terms. Here, stock investments are replaced by the less risky non-leveraged housing investments. Note how sensitive the optimal portfolio is to both the access to borrowing and the risk aversion coefficient.

4.4 Initial Wealth

The left panel of Figure 9 shows that when collateralized borrowing is allowed, an increase in the initial wealth leads to an increase in the optimal stock weight. This relation is seen in the data (Wachter and Yogo, 2010), but is inconsistent with the basic Samuelson–Merton model and various extensions thereof. The optimal house weight is decreasing in initial wealth. These effects can be explained by the fact that, when wealth is increased, the habit-induced wealth buffer seizes a smaller share of the sum of financial and human wealth. Notably, initial wealth has the opposite effect on the portfolio if collateralized borrowing is impossible as shown by the dotted curves in the figure. Higher wealth raises optimal future housing consumption, and hedging considerations may rationalize a higher housing investment.

4.5 The Initial Housing Habit Level

In the baseline analysis we assumed an initial housing habit level of $\bar{q}_0 = 200$ housing units. Figure 10 shows how life-cycle consumption and investments are affected if the initial habit is changed to either 100 or 300. An increase in the initial housing habit level shifts consumption motives to housing and prompts a larger housing consumption in the early years with the quantitative effect being reduced over the life cycle. The additional housing expenditures demand a reduction in early-life perishable consumption and investments. Housing investments are reduced less than stock investments since the larger habit necessitates a larger buffer in the housing market. Due to the binding portfolio constraint, the higher housing habit postpones the stock market entry of young individuals. The lower investments imply that less wealth is accumulated. Perishable consumption is thus reduced

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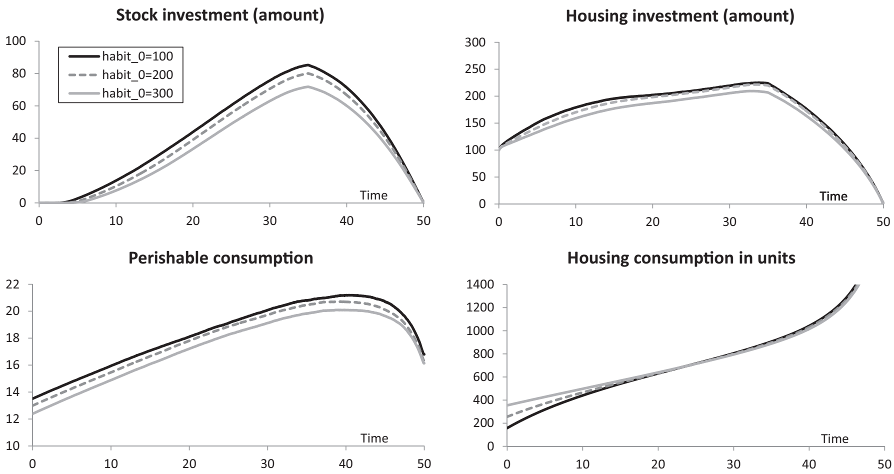

Figure 10. Consumption and investments for different initial habit levels. The dashed curves assume an initial housing habit of $\bar{q}_0 = 200$ as in our baseline case, whereas the black curves are for $\bar{q}_0 = 100$ and the grey curves for $\bar{q}_0 = 300$. For all other parameters, the baseline values in Table I are used.

throughout the life cycle, and in the final years even housing consumption becomes inversely related to the initial habit level.

4.6 Labor Income Depending on Age and Education

Above we assumed a constant expected income growth rate in the working phase, but income is known to grow faster for young adults than for more mature adults. We follow Cocco, Gomes, and Maenhout (2005) and use their estimated income profiles for three different educational levels labeled "no high school," "high school," and "college." Their estimated profiles determine $\mu_Y(t)$ in our model, cf. Munk and Sørensen (2010). In addition, the initial income is set to 20 for the agent with no high school, 25 with high school, and 29 with a college degree. These values roughly correspond to the average income of a 30-year old in the different educational groups according to Cocco, Gomes, and Maenhout (2005). The other parameters are unchanged compared with the baseline case.

Figure 11 depicts the average income profiles in the upper-left panel. The other panels show wealth, consumption, and investments for the three profiles, both in the case where the agent can borrow up to 80% of house value ($\kappa = 0.2$) and the no-borrowing case ($\kappa = 1$). The overall shape of the perishable consumption curve is the same for all three educational levels, but of course the level of consumption increases with income and thus education. The level of housing consumption also increases with education, but the housing expenditure share is initially lowest for college graduates and highest for individuals without high school education, consistent with the shares across income quintiles reported in the "Introduction" section. This can be understood from Equation (15), which shows that the expenditure share in the deterministic version of the model increases in the ratio $\bar{q}_t/q_t$ of the habit level to current consumption. We assume the same initial habit level for all three educational groups, and since optimal housing consumption increases with education, this ratio and hence the housing expenditure share decrease with education. Eventually, the expenditure shares for the three groups converge.

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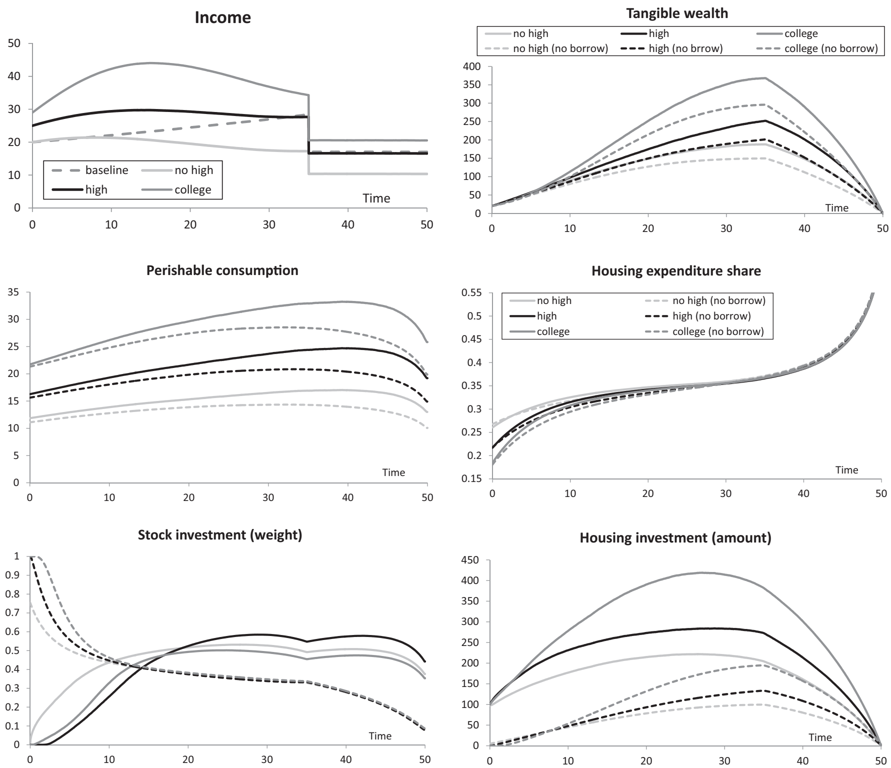

Figure 11. Consumption and investments for different educational levels. The average income profiles estimated by Cocco, Gomes, and Maenhout (2005) for three different educational groups are used. The solid curves are for the baseline case of $\kappa = 0.2$ (80% borrowing of house value), whereas the dotted curves are for the case $\kappa = 1$ (no borrowing). For all other parameters, the baseline values in Table I are used.

The young college graduate saves very little and wants to borrow substantial amounts due to his large human capital, so mortgage-financed housing investments are particularly attractive to him. In fact, early in life the college graduate invests less in stocks than the agent with no high school degree. The share of wealth invested in the stock is similar for the three educational groups except in early adulthood. Note again that the investment pattern is very different if borrowing is prohibited. In this case, all three individuals participate in the stock market from the beginning with the college graduate holding the largest share of wealth in stocks due to his higher human capital. Overall, with or without access to borrowing, the optimal portfolio weights vary only modestly with education level.

4.7 Correlations

In the baseline case we used the pairwise stock-house-income correlations estimated from the aggregate data in the solution of the individual's problem. Now we consider the effect of setting each of these correlations to zero. In Figure 12, the dashed curves show the results of our model for $\rho_{YS} = 0$, a value often used in the literature, instead of the baseline value of 0.22. Since income is now less stock-like, the portfolio is more tilted toward stocks and away from real estate. Perishable consumption is somewhat lower and peaks a little earlier

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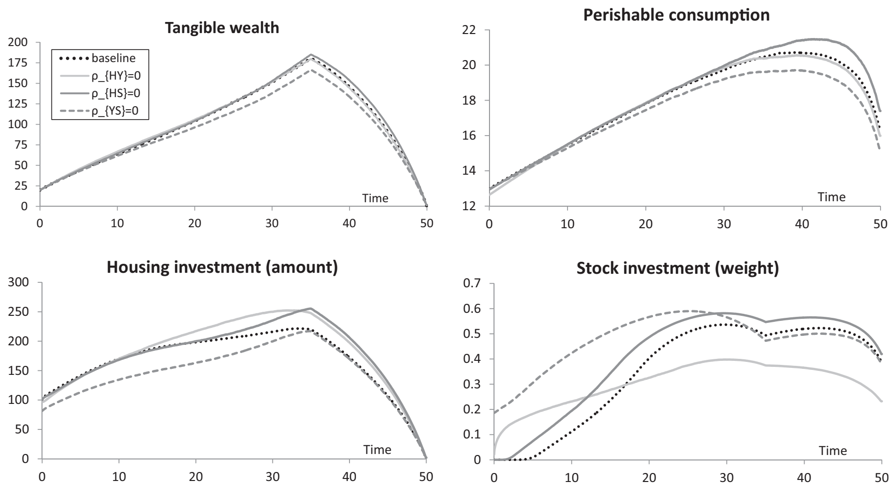

Figure 12. Consumption and investments for different correlations. Each panel compares the baseline case with the three cases where each of the correlation parameters ρ_{HY}, ρ_{HS}, and ρ_{YS} have been set to zero, but all other parameters assume the baseline values listed in Table I. The dotted curve for the baseline case is hardly visible in the upper panels as it is almost indistinguishable from the light grey curve.

than in the benchmark case. If the baseline income-house correlation ρ_{HY} of 0.16 is replaced by zero, the portfolio is naturally tilted somewhat to houses and thus away from stocks. Finally, replacing the baseline house-stock correlation ρ_{HS} of 0.25 by zero improves diversification so the individual generally invests slightly more in both stocks and houses.

4.8 An Extension to Mortality Risk and Bequests

Next, we consider an extension of our model to mortality risk and bequest. The bequest utility is $w^{\gamma}\frac{1}{1-\gamma}X_{\tau}^{1-\gamma}$, where τ is the time of death, X_{τ} is the bequeathed wealth, and w is the preference weight of bequest. We use a deterministic mortality intensity $\zeta(t)$ derived from the life tables for the total US population as of 2009 with an imposed maximum age of 100 years.[16] Following Richard (1975), Blanchard (1985), and others, we assume the agent has access to a life insurance contract that pays a continuous flow at the rate of $\Gamma\zeta(t)N_t$ per year to the agent. Upon death the agent pays the amount $N_{\tau-}$ to the contract issuer (formally the bequest is $X_{\tau} = X_{\tau-} - N_{\tau-}$). Here, N_t is chosen by the agent. With a moderate or high bequest motive, the optimal N_t is typically negative. With a low bequest motive, the optimal N_t tends to be positive except when the agent is relatively young or very close to 100 years. We let $\Gamma = 0.8$, corresponding to a 20% margin to the insurance company. The agent's optimal strategy is similar to the no-mortality case but basically with the mortality intensity $\zeta(t)$ added to the constant time preference δ, so that the agent effectively becomes more impatient with age. The Online Appendix presents a closed-form solution to the extended problem in a set of artificial markets. In conjunction with the numerical approach of Bick, Kraft, and Munk (2013), this leads to the near-optimal strategies in the true market.

16 Published at the Centers for Disease Control and Prevention under the US Department of Health and Human Services, see http://www.cdc.gov/nchs/data/nvsr/nvsr62/nvsr62_07.pdf.

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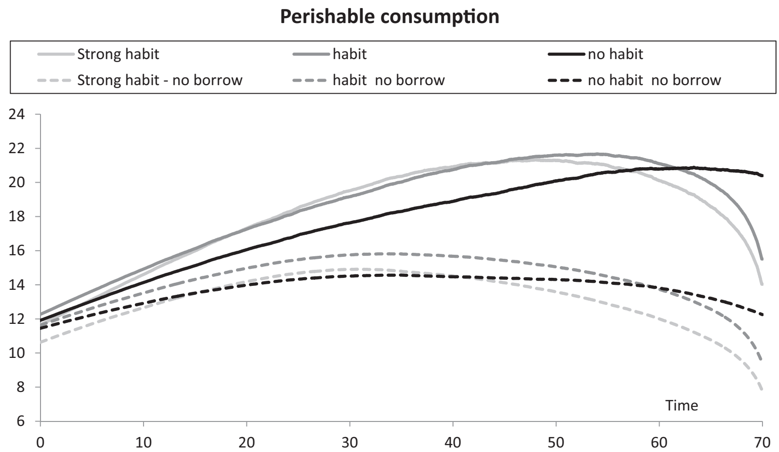

Figure 13. Perishable consumption over the life cycle with mortality risk and bequest. The graphs show averages across 10,000 simulations in which the consumption and investment strategy determined by our numerical method is used. The light grey curves are for a strong habit ($\alpha = 0.88$), the grey curves for the baseline habit strength ($\alpha = 0.8$), and the black curves for the case without habit formation. For each shade the solid curve is for the case with collateralized borrowing ($\kappa = 0.2$) and the dashed curve for the case without borrowing ($\kappa = 1$). The mortality rates are derived from the 2009 life table of the total US population. We assume an insurance price parameter of $\Gamma = 0.8$ and a preference weight on bequest of $w = 10$. For other parameters the baseline values listed in Table I are used.

Mortality risk and bequest turn out to have a limited impact on optimal investments. With a preference for bequest, more wealth needs to be built up and maintained even late in retirement, which leads to larger amounts invested in both stocks and housing units, but the portfolio weights are similar to the baseline case in Figure 4. Mortality risk and bequest preferences have more interesting effects on life-cycle consumption. Figure 13 shows six curves of perishable consumption over life. The three solid curves are for the case with collateralized borrowing ($\kappa = 0.2$) and the corresponding dotted curves for the no-borrowing case ($\kappa = 1$). The black curves are for no habit, the grey curves for the baseline habit strength ($\alpha = 0.8$), and the light grey curves for a stronger habit ($\alpha = 0.88$). We assume a bequest preference weight of $w = 10$, which leads to a sizeable bequest; for example, in the standard case with $\alpha = 0.8$ and $\kappa = 0.2$, the average bequest is 327.5 (thousand USD) should the agent survive until age 100 years. With access to borrowing, the consumption of the no-habit agent has a very late hump. Comparing the three solid curves, we see again how a stronger housing habit leads to an earlier peak in consumption. This is also true if borrowing is disallowed as revealed by a comparison of the dashed curves. The figure also confirms that a relatively early hump can be obtained without habits if borrowing is impossible which, however, is a questionable premise. With access to borrowing, the housing habit can restore a hump around the retirement age.

5. Conclusion

We have studied the optimal life-cycle consumption and investment decisions in a rich model with many realistic features. In particular, we include preferences for both perishable consumption and consumption of housing services with habit formation for housing. Our

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model can generate various stylized facts that seem puzzling through the lens of standard life-cycle models. First, optimal stock investments are low or zero for many young agents even without imposing exogenous stock market entry costs. Housing investments crowd out stock investments in particular because of the associated access to credit. The housing habit increases the importance of house investments relative to stock investments since the agent can only ensure the future habit-generated minimum housing consumption by holding more of the housing asset. Secondly, our model generates an age- and wealth-dependent housing expenditure share consistent with the data instead of the counter-factually constant share found in standard models. Thirdly, in our model perishable consumption is much more sensitive to wealth and income shocks than housing consumption, in particular for young agents. Finally, the housing habit provides a plausible explanation of the observed life-cycle pattern in non-housing consumption.

Supplementary Material

Supplementary data are available at *Review of Finance* online.

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