

Welfare Effects of Individualizing Life-Cycle Investments for Households in Turkey

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Abstract

In this paper, we perform an in-depth welfare comparison of the most common life-cycle investment strategies provided by retirement funds or suggested by classical portfolio theory in the case of households in Turkey. To perform our benchmarking, we constructed heterogeneous agents who worked and invested throughout their lifetime, using parameters calibrated from the historical data. We find that to households with upper-to-middle income, individually customized portfolios result in considerable welfare gains, while “off-the-shelf” life-cycle portfolio allocations perform better for households with lower income. We also show that life-cycle investment options outperform “fixed over the lifetime” options.

Keywords: Life-cycle portfolio decisions, human capital, housing, stock market participation

1. Introduction

The field of financial economics has gone through big changes since its foundation by Markowitz [1] and Tobin [2]. They pioneered the mean-variance analysis, which, given some assumptions, suggested that if investors cared about maximizing returns (mean) and minimizing risks (variance), then the optimal ratio of stocks to bonds in a single-period portfolio would be fixed for everyone, the share of former being equal to:

$$\alpha = \frac{E[R] - R_f}{\gamma\sigma^2} \quad (1)$$

Merton [3] generalized the problem to multiple periods using dynamic programming and found that it is optimal for all households to repeat the same fixed mean-variance solution every period.

These results were inconsistent with the popular financial advice suggesting that younger investors should have higher share of stocks in portfolio, and older investors — higher share of bonds. This advice was summarized by the famous rule of thumb:

$$\alpha_t = (100 - t)\% \quad (2)$$

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Although Samuelson [4] denied that risk-aversion changes by age, dismissing this advice would question the rationality of investors and “constitute *prima facie* evidence that people do not optimize” (Canner et al. [5]).

Bodie et al. [6] solved this problem by adding human capital into the Merton [3]’s dynamic model and found that for complete markets and constant risk-free labor income, the optimal share of stocks in a portfolio is:

$$\alpha_t = \frac{\mu - R_f}{\gamma \sigma^2} \left(\frac{W_t + H_t}{W_t} \right) \quad (3)$$

Steady depletion of human capital through life explained the higher share of stocks in younger people. Cocco et al. [7] extended this idea to the case of variable labor income, to find a recursive solution which could be approximated by the following rule of thumb:

$$\alpha_t = \begin{cases} 100\% & t < 40 \\ (200 - 2.5t)\% & t \in [40, 60] \\ 50\% & t > 60 \end{cases} \quad (4)$$

However, the hump-shaped lifetime stock share graph, observed by Chang et al. [8] of Richmond Fed (see Figure 1) instead of expected downward sloping one, suggested the presence of an opposing force.

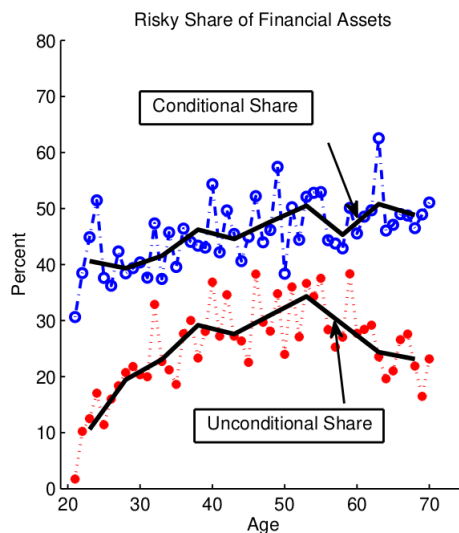


Figure 1: Hump-shaped unconditional lifetime stock share. Source: Chang et al. [8]

Cocco [9] found that this force was housing investment, which, due to its large size, crowded out all stocks from younger investors’ portfolios. Flavin and Yamashita [10] supported this view by showing that younger people, who already own the house, tend to invest more aggressively, as expected by Bodie et al. [6].

Munk [11] found the same patterns using a series of one-period mean-variance optimizations without any dynamic stochastic modeling tools. He stated that in presence of housing, the optimal

stock (π) and housing (π_h) shares can be solved analytically as follows:

$$\pi_{t+1} = \frac{1}{\gamma(1 - \rho_{SH}^2)\sigma_S} \cdot \frac{W_t}{F_t} \left(\frac{\mu_s - r_f}{\sigma_S} - \rho_{SH} \frac{\mu_h - r_f}{\sigma_h} \right) - \frac{L_t}{F_t} \cdot \frac{\sigma_L}{\sigma_S} \frac{\rho_{SL} - \rho_{SH}\rho_{HL}}{1 - \rho_{SH}^2} \quad (5a)$$

$$\pi_{h,t+1} = \frac{1}{\gamma(1 - \rho_{SH}^2)\sigma_H} \cdot \frac{W_t}{F_t} \left(\frac{\mu_h - r_f}{\sigma_h} - \rho_{SH} \frac{\mu_s - r_f}{\sigma_s} \right) - \frac{L_t}{F_t} \cdot \frac{\sigma_L}{\sigma_h} \frac{\rho_{HL} - \rho_{SH}\rho_{SL}}{1 - \rho_{SH}^2} \quad (5b)$$

Finally, Ascheberg et al. [12] illustrated the existence of long-term cointegration among house prices, stock prices, and labor income, the fact often omitted by previous portfolio researches for simplicity. Therefore, in our analysis we will not neglect the correlations as being equal to zero.

In this paper, we calculate the wealth accumulated from lifetime investments given by all the strategies above and suggested by current Turkish banks. We benchmark the resulting welfare for heterogeneous agents in Turkey with varying sectors, levels of education and risk-aversion. We conclude with the separate investment advice for all types of households mentioned above.

The remainder of the paper is organized as follows: Section 2 presents the theoretical framework we will use in our analysis. Section 3 contains detailed information about our data sources, parameter calibration and solves for the optimal investment strategies. Section 4 does the welfare benchmarking of every investment option for every agent, and Section 5 concludes.

2. Theoretical framework

2.1. House prices, stock prices and labor income series

In accordance with Campbell et al. [13] and Olear [14] we model the labor income process as a function of individual characteristics $f(t, Z_{it})$ plus idiosyncratic shocks v_{it} . Upon reaching the retirement age T , an individual receives a certain percentage λ of his/her last wage:

$$Y_{i,t+1} = \begin{cases} Y_{it}(1 + \Delta f(t+1, Z_{i,t+1}) + v_{it}), & t \leq T \\ \lambda(1 + f(T, Z_{iT}) + v_{iT}), & t > T \end{cases} \quad (6)$$

We model house prices and stock prices as time series satisfying the discrete version of Ascheberg et al. [12]'s stock, housing and labor income correlation structure, mentioned in [Appendix A](#):

2.2. Calibration

We calibrate the above parameters using data obtained from the repeated cross-sectional study by TUIK Household Budget Survey [15]. In line with Aktug et al. [16] we aggregate these studies to obtain a pseudo-panel, which we will use to summarize labor income dynamics.

In summarizing the series we will use simple medians and means. In determining the expected values of time-series parameters, we will use simple arithmetic mean and standard deviation.

. In constructing the simulated labor income series, we used kinked ordinary least-squares (OLS) estimations with kinks at ages of 35 and 45 separately for every education level i :

$$\Delta \log(\text{wage}_{it}) = \alpha_0 + \alpha_1 \cdot d_{35} + \alpha_2 \cdot d_{45} + \alpha_3 \cdot t$$

2.3. Welfare measurement

Similarly to Cocco et al. (2005) we use CRRA utility function in our model:

$$E_1[U(c)] = E_1 \left[\sum_{t=1}^T \delta^{t-1} \prod_{j=0}^{t-1} p_j \cdot \frac{c_{it}^{1-\gamma}}{1-\gamma} \right]$$

where p_k is the probability of survival from time $k-1$ to time k . Note that we omitted the bequest motives from the original formulation, thus retired person consumes all of his income at any given time.

2.4. Individualization

To derive the individual lifecycle portfolios for every wealth type we use a special case of Munk (2016) with housing investment and human capital included. The similar approach and its solution has been done by Olear (2016), but she included housing capital as a pre-existing wealth bought for an outside mortgage, independent of the investment decision. In this paper, we consider housing as a purely financial investment, in line with Munk (2016). Munk provides the following optimal asset allocation:

$$\pi_{t+1} = \frac{\mu_s - r_f}{\gamma \sigma_s^2} + \frac{L_t}{F_t} \cdot \left(\frac{\mu_s - r_f}{\gamma \sigma_s^2} - \frac{\rho_{SL} \sigma_L}{\sigma_S} \right)$$

without housing investment, and:

$$\begin{aligned} \pi_{t+1} &= \frac{1}{\gamma(1-\rho_{SH}^2)\sigma_S} \cdot \frac{W_t}{F_t} \left(\frac{\mu_s - r_f}{\sigma_S} - \rho_{SH} \frac{\mu_h - r_f}{\sigma_h} \right) - \frac{L_t}{F_t} \cdot \frac{\sigma_L}{\sigma_S} \frac{\rho_{SL} - \rho_{SH}\rho_{HL}}{1-\rho_{SH}^2} \\ \pi_{h,t+1} &= \frac{1}{\gamma(1-\rho_{SH}^2)\sigma_H} \cdot \frac{W_t}{F_t} \left(\frac{\mu_h - r_f}{\sigma_h} - \rho_{SH} \frac{\mu_s - r_f}{\sigma_s} \right) - \frac{L_t}{F_t} \cdot \frac{\sigma_L}{\sigma_h} \frac{\rho_{HL} - \rho_{SH}\rho_{SL}}{1-\rho_{SH}^2} \end{aligned}$$

when housing is considered as a second risky financial investment. In line with Munk (2016), we calculate the risk free asset share as $(1 - \pi - \pi_h)$ and only then impose constraints. If any of the above shares is negative, we equate it to zero, and if the sum of remaining shares exceeds 1, we divide all of the shares by their sum to obtain a proportionate asset allocation.

2.5. Retirement income

The funds invested in retirement are modeled to be paid back in annuities. Thus, we ignore the case when some share of matured fund can be withdrawn immediately. Further, in order for our individualization analysis (based on house possessions) to have an effect, we also use reverse mortgages proposed. In theory, reverse mortgages pay retired individuals fixed annuity in return for inheriting the house after she dies. This is a plausible tool, because we ignore all bequest motives, and it liquidifies the housing possessions, although it is not yet available for Turkish investors.

. So, at the age of 57, the price of owned house is calculated and is added to the matured pension amount (MP) to obtain total wealth:

$$W_{57} = H_{57} + MP$$

All of the W_{57} is used to buy an annuity which will repay an individual $\frac{W_{57}}{1 + \sum_{t=58}^{100} \frac{p_t}{1+r_f}}$ annually. The calculation details are available in Appendix D.

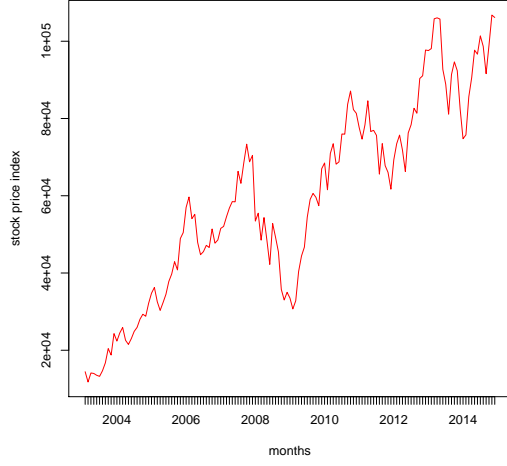


Figure 2: BIST30 Turkish stock market performance index

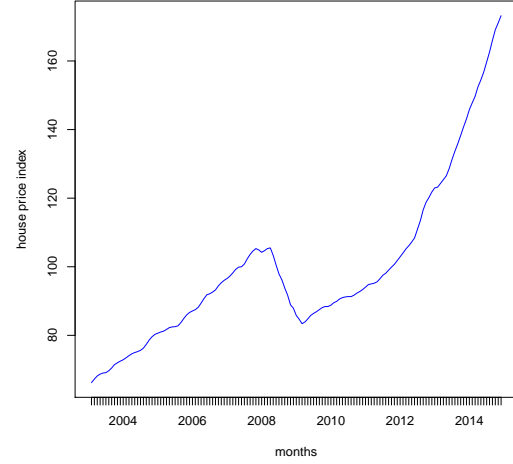


Figure 3: Reidin Turkish house price index

3. Data and simulation

In our simulation we will compare welfare effects of default lifecycles and individualized lifecycles defined below. The magnitudes and sources of parameters are given in the next section. Default lifecycles are taken from the ones mentioned in our Literature Review chapter and from the real investment strategies of the largest Turkish pension fund provider Anadolu Hayat Emeklilik. Individualized lifecycles will be calculated using Munk's optimal portfolio formula from previous chapter taking idiosyncracies into consideration.

3.1. Data

To measure the stock series we used BIST 30 index which measures the aggregate performance of 30 best companies in Turkey. The monthly data is taken from Borsa Istanbul. We can see from Figure 4.1 the general upward trend with collapse during 2008 crisis.

. We used Reidin AEINDEXF index to obtain data on house prices in Istanbul. The historical dynamics of this index are illustrated in Figure 4.2.

. We used TUIK's Household Budget Survey Data and regression results of Aktug, Kuzubas, Torul (2017). We have 55 thousand panel data points for 170 households for 2001 to 2014 years. We can observe the hump-shaped lifetime income distribution in Figure 4.3. This corresponds well to an established literature in this field.

3.1.1. Default parameters

In our simulation we start with a 28 years old individual who invests in her retirement for 30 years until she reaches retirement at 57. We set the default coefficient of relative risk aversion at 5, but, in line with Torul et al. (2018), we check the sensitivity for 1.5 and other values. Also, according to Torul et al. (2018), we set the subjective discount rate at 0.89.

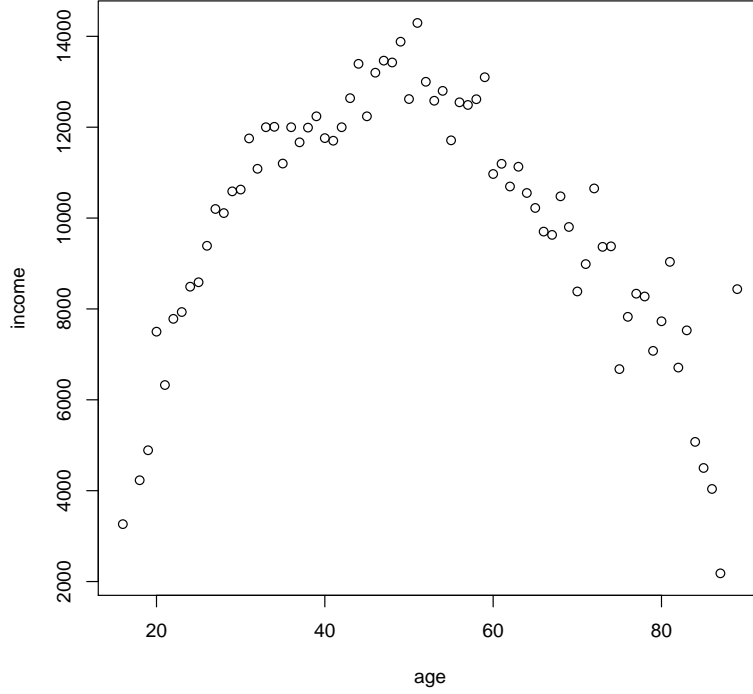


Figure 4: Median Turkish salaries by age

- . Stock returns for Turkey were estimated from historical data to be equal to 23.2% annually, with standard deviation 36%. Risk-free rate forecasts are obtained from OECD Data Bank (2018) and are equal to 10.8% per annum.
- . Housing capital appreciation averaged at 8.3% with 9.5% standard deviation. But in our simulation we have excluded the temporary housing price drop during 2008 crisis to clear from macroeconomic effects. Therefore, in our simulation, housing capital appreciation averaged at 11.3% with 5.2% standard deviation.
- . Aggregate wage growth series showed 5.5% standard deviation.
- . The correlations between house and stock prices, and house prices and wages gave 0.24 and 0.37 respectively.
- . The data on survival probability for all ages has been taken from Turkish Statistical Institute's (TUIK) database and illustrated in Figure 4.4. All of these findings have been summarized in Table 4.1.

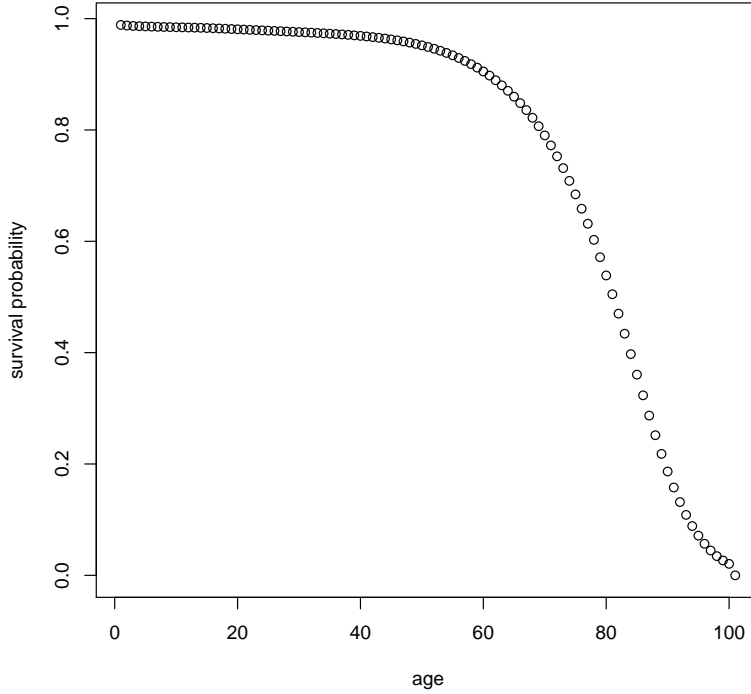


Figure 5: Survival probabilities by age

3.1.2. Heterogeneity parameters

In the same manner as Olear(2014) we will use wage growth rate, stock-income correlation and idiosyncratic labor income risk to model heterogeneities in education and sector. Let's now consider the heterogeneities one at a time:

Heterogeneity in education. In line with Olear's (2016) approach we model the heterogeneity in education using the differences in wage growth rates. Indeed, we expect the salaries for higher education level to grow faster than for the lower education level. Some of this expectation comes from the fact that people with lower education are restricted in their career ladders and cannot rise very high in a workplace. Another intuition is that while college dropouts start working immediately, college graduates and graduate students continue to study and thus report zero income. When they graduate, their salary immediately rises from zero to the average salary, and this constitutes a steeper wage growth curve at the beginning of their lives. Figure 4.5 shows the wage series for different levels of education. Note that the curves for the lowest education levels are practically flat and the those for the highest education levels have varying non-zero slopes.

Performing kinked regressions of log wages on ages, as described in the previous chapter, yielded best linear fits for three different education levels of our choice: postgraduate, high school, and no schooling. The corresponding labor income growth rates can be characterized as steep, moderate,

Table 1: Benchmark Parameters

Parameter	Description	Value
Y	Beginning age	28
R	Retirement age	57
T	Lifespan (years)	100
γ	Risk aversion	5
β	Discount rate	0.89
r_f	Risk-free rate	0.108
π	Average inflation rate	0.084
μ_s	Expected stock returns	0.232
μ_h	Expected housing returns	0.113
σ_s	Stock returns volatility	0.36
σ_h	Housing returns volatility	0.052
σ_w	Wage growth volatility	0.056
ρ_{hs}	Housing-stock correlation	0.24
ρ_{hw}	Housing-wage correlation	0.37
p_{28}	Survival probability at age 28	0.977
p_{57}	Survival probability at age 57	0.924
p_{100}	Survival probability at age 100	0

Table 2: Estimated Benchmark Wage Growth Rates μ_w

Age	Flat	Moderate	Steep
0-35	0%	3.5%	6.5%
36-45	0%	3%	2%
46-60	0%	0%	0%

and flat respectively. We considered wage data until 60 years, as Turkish retirement age is at 57, and have added empirically best kinks at ages 35 and 45. The results are summarized in Table 4.2 and illustrated in Figure 4.6. In this figure red lines represent the actual wages dynamics for postgraduate, high school, and no schooling, and blue lines are plots of the percentage changes proposed in Table 4.2, where the starting points all correspond to the actual starting points from the data. Figure 4.7 illustrates the same parameterized curves without actual wage curves.

Heterogeneity in sectors of work. As we explained in Chapter 3, we model our labor income series as functions of age, gender, education, sector of work etc. We already showed in previous section how we incorporated the heterogeneity in education. We also stated at the beginning of this chapter that we consider only male individuals as heads of households; age is implicit in our analysis. In this section we will model the differences in sectors of work. Again, similar to Olear's (2016) approach we model these differences using correlations of wage growth rates with stock market returns. Indeed, it is expected that sectoral wages would be proportional to the stock returns only to the extent of their exposure to stock markets. In that sense, the stock-wage correlations are expected to be zero for public sector, where wages are fixed regardless the markets, and high in financial institutions. When we juxtaposed wages by sector and stock prices we obtained very high correlations, which was expected due to the economic growth over the years represented in Figure 4.8. Therefore we

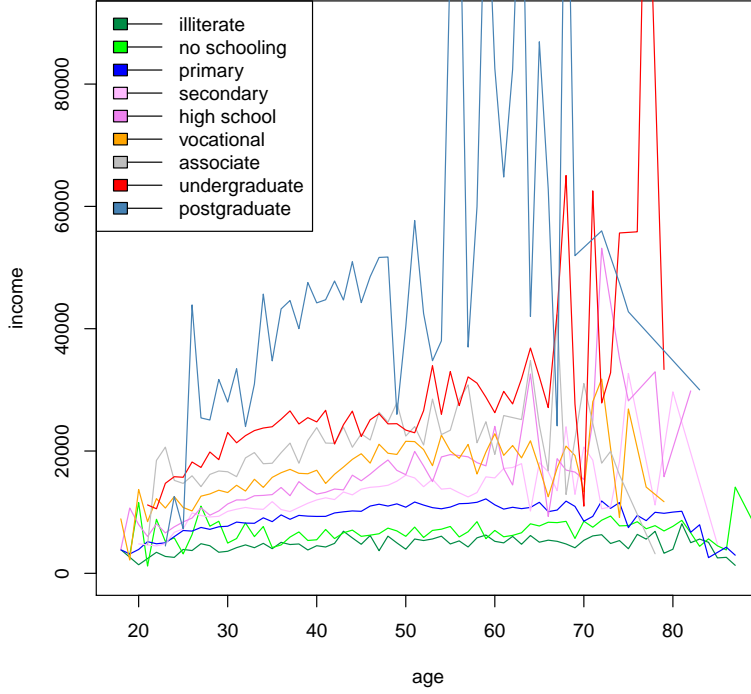


Figure 6: Lifetime wage dynamics by education level

Table 3: Benchmark Wage to Stock Correlations

	Low	Moderate	High
ρ_{sw}	0	0.2	0.4

divided the wages by price levels to obtain the real wages, and, as expected, we obtained more realistic correlations. The financial sector's correlations were as high as 0.44 and public sector, social service and education's were as low as 0.08. Thus, our expectations that different sectors have different wage-stock correlation ρ_{sw} were confirmed and we decided on three measures of ρ_{sw} as our simulation benchmarks: 0, 0.2 and 0.4 as stated in Table 4.3.

Individual heterogeneity. We check for individual heterogeneity using a sensitivity analysis on different risk aversion coefficients, summarized in Table 4.4.

3.2. Capital series

3.2.1. Human capital

Human capital at all ages has been calculated as a discounted sum of all future wages until retirement with the discount factor r_f . To construct the individualized capital we used steep,

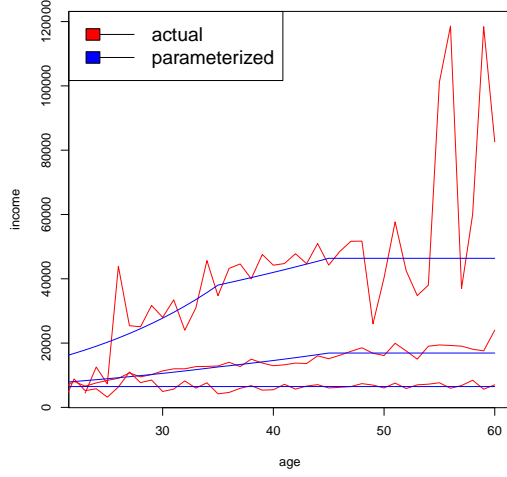


Figure 7: Actual and parameterized benchmark wage dynamics by age

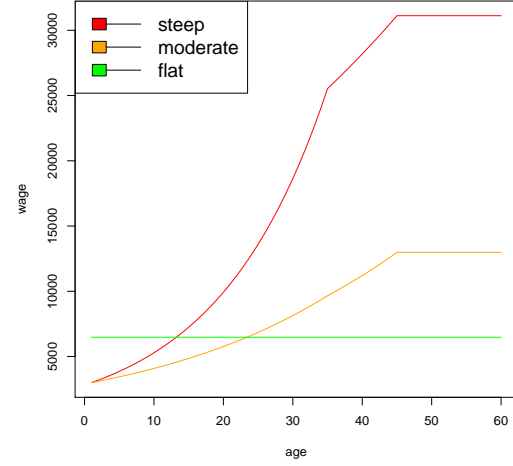


Figure 8: Parameterized wage dynamics by age

Table 4: Coefficients of Risk Aversion

Values	Torul	low	default	high
γ	1.5	3	5	10

moderate and flat wage series mentioned in the previous section. It can be seen in Figure 4.9 that human capital is declining for flat wages and hump-shaped for moderate and steep wages.

3.2.2. Financial capital

Financial capital evolves according to dynamic investment illustrated in Figure 4.11. Every period t a certain percentage c (equal to 3% in Turkey) of that period's wage w_t is invested in a retirement portfolio. At the same time, the previously invested amount accrues interest. We started with 28 years old individual who invests for 30 years until retirement, as has been mentioned previously numerous times.

. Note that the shape of the financial capital depends on r_p — the rate of return on portfolio, which itself depends on the risky-to-riskless asset ratio. Different such ratios are listed and analyzed in detail in the next section. Figure 4.10 demonstrates the evolution of financial capital for two investment options, provided by Anadolu Hayat Emeklilik — 30% in stocks and 70% in bonds, and by solving Markowitz's formula — 63% in stocks and 37% in bonds.

. It is important to notice here that since human capital is declining by age and financial capital is increasing, the ratio $\frac{L_t}{F_t}$ is declining in t . Recalling Merton's formula for α_t from Chapter 2, $\frac{\mu - R_f}{\gamma \sigma^2} (1 + \frac{L_t}{F_t})$, it should be now clear how the above fraction creates a lifecycle effect — different ratios for different ages.

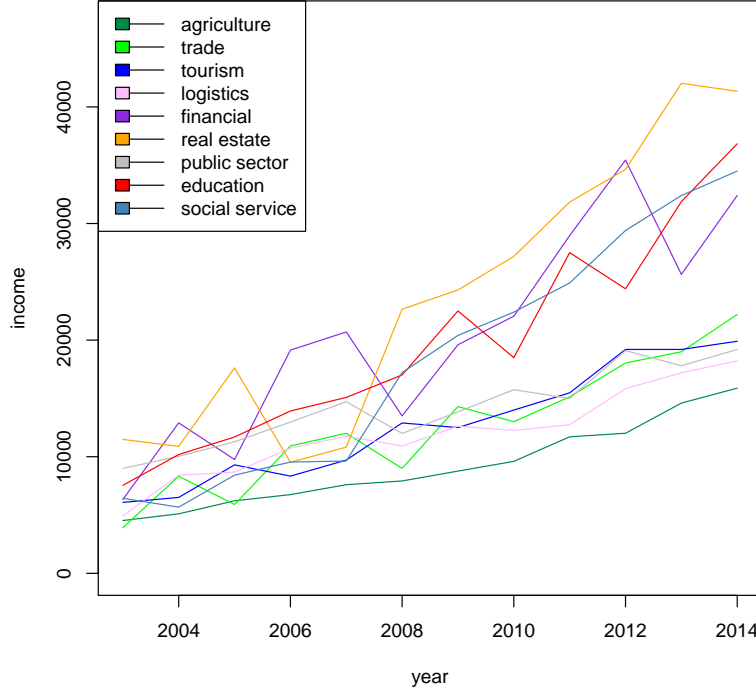


Figure 9: Historical wage dynamics by sector

RESULTS HERE START In this chapter we present the results of our simulation. In the first section we will present the default and individualized lifecycle investment strategies, the former being provided by real retirement funds and the latter being solved by Merton and Munk. We will plug in the heterogeneous parameters, described in detail in the previous chapter. In the next section, we will calculate the resulting financial capital flows and compare their welfare effects using their expected utilities, as mentioned in Chapter 3.

3.3. Investment strategies

3.3.1. Default lifecycles

As was discussed in the previous chapter, our individual decides between investing in risky (stocks) or risk-free (bonds) assets. The default allocations for share of risky asset are given by:

- $100 - t$, for all t
- $\begin{cases} 100\%, & t < 40 \\ (200 - 2.5t)\%, & t \in [40, 57] \end{cases}$
- 63%, for all t

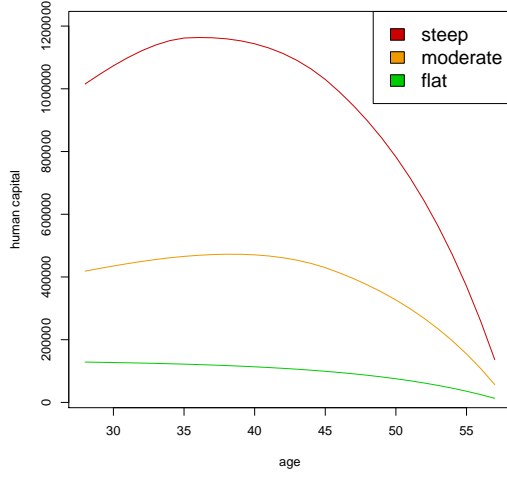


Figure 10: Human capital by age

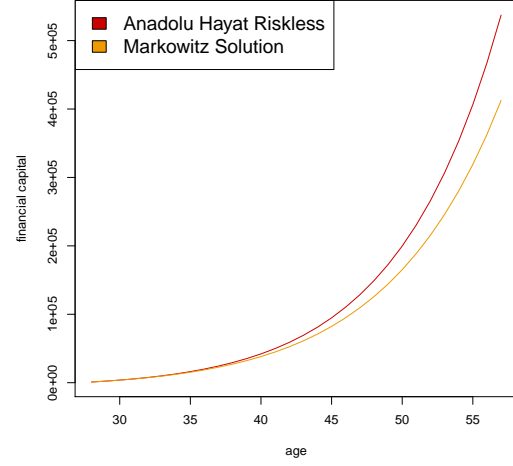


Figure 11: Financial capital by age

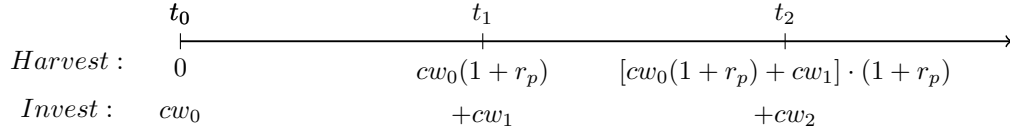


Figure 12: Law of motion of financial capital

- 30%, for all t

where the latter two are Markowitz's solution and Anadolu Hayat's moderate investment option respectively. Since we are only interested in age span between 28 and 57, Figure 5.1 shows the risky asset share π_t only for that interval.

3.3.2. Individualized lifecycles

To derive individualized lifecycle strategies, we used Merton's (1971) and Munk's (2016) optimal portfolio allocation formulas mentioned in chapters 2 and 3. Since these formulas depended on intratemporal amounts of capital, we have constructed three human capital series corresponding to flat, moderate and steep wage growth curves mentioned in the previous chapter. Figure 5.2 illustrates the risky asset shares given by Merton and Munk without housing for various levels of labor income growth and risk aversion. The figure shows that the steeper the wage growth curves get or the lower their risk aversion is, the more aggressive investors are. However, as a whole, they follow the similar pattern. The figure also shows that for small correlations between labor income and stock prices, Munk's solution converges to Merton's solution, as was shown in Chapter 3.

. Figure 5.3 illustrates the stock and housing asset shares determined by Munk with housing for flat, moderate and steep labor income curves and low, moderate and high stock-labor income correlations

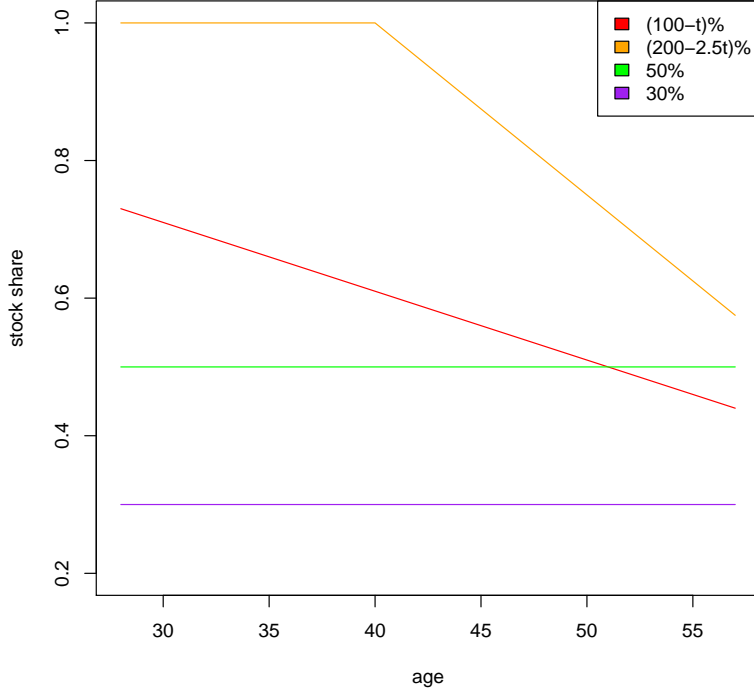


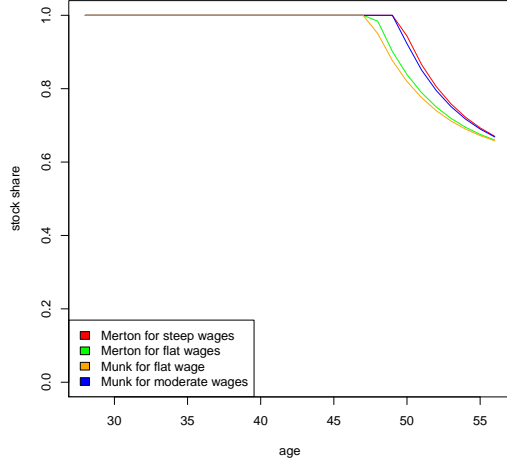
Figure 13: Default portfolio allocations of stock investments

for different levels of risk aversion, to capture heterogeneity, as mentioned in the previous chapter. The figure confirms that steeper labor income curves result in larger share of stocks in portfolio and less of bonds. Around age of 45, the sum of optimal stock and housing investments falls below 1 and optimal bond investment becomes positive. Also, the higher the risk aversion, the less the individuals want to invest in both housing and stocks — for $\gamma = 10$, the optimal Munk solutions are negative or add up to less than 1, meaning that they should sell all stocks and housing to invest in risk-free long term bonds. In our analysis we do not allow for negative investing, since this is not a primary focus of our thesis.

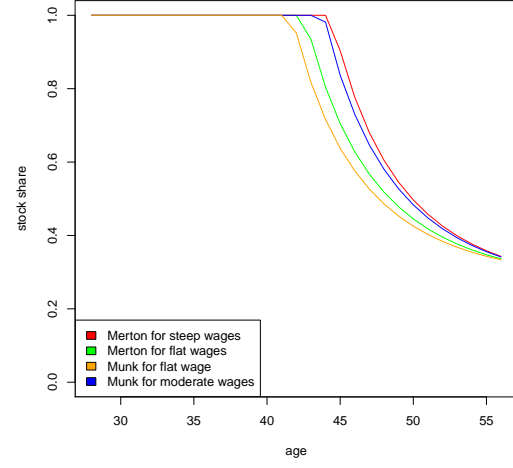
. Full tables with actual solutions are available in Tables E.1 and E.2 of Appendix E for models without housing and in Table E.3 for a model with housing.

4. Welfare comparison

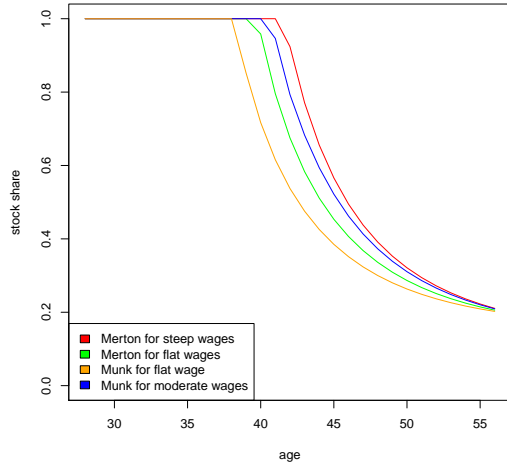
In order to compare welfare we use CRRA expected utility function, mentioned in Chapter 3. The probabilities of survival are taken from TUIK as mentioned in the previous chapter. The consumption is calculated in numbers of consumption baskets which cost exactly 1 CPI — consumer price index at that period, which is increasing with inflation rate annually, and is equal to 100 at



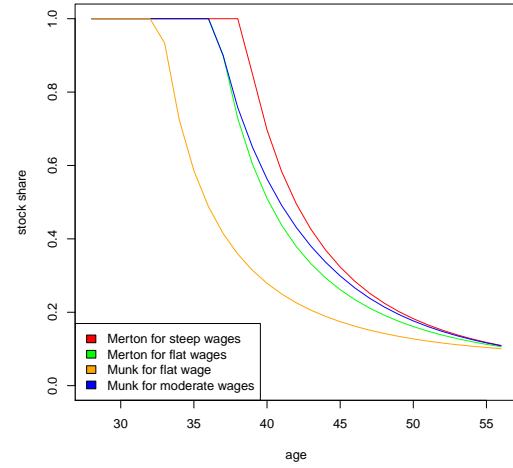
(a) $\gamma = 1.5$



(b) $\gamma = 3$



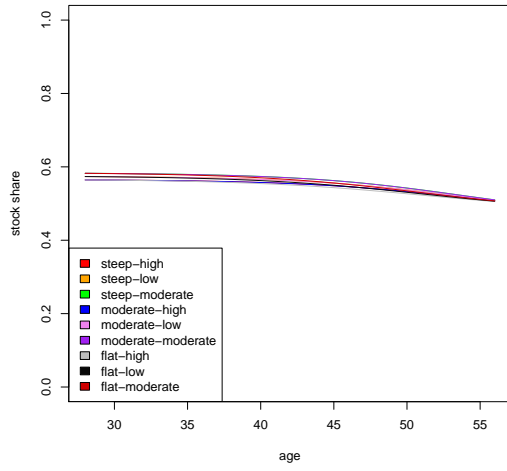
(c) $\gamma = 5$



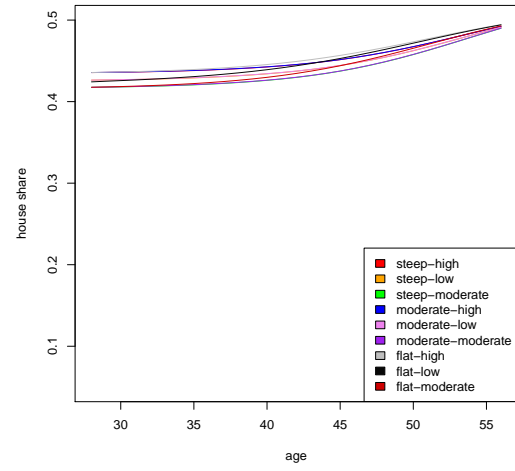
(d) $\gamma = 10$

Figure 14: Merton and Munk's solution without housing for different wage growth and risk aversion levels

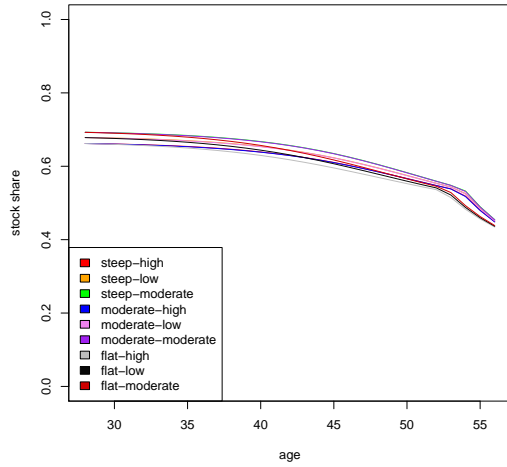
retirement age 57. The wealth for which consumption baskets are purchased is a total accumulated wealth, including stock returns, bond returns and housing returns, annuitized according to the formula described in Chapter 3. Discount rate is 0.89, as mentioned in the previous chapter. We compare expected utilities for different levels of risk aversion.



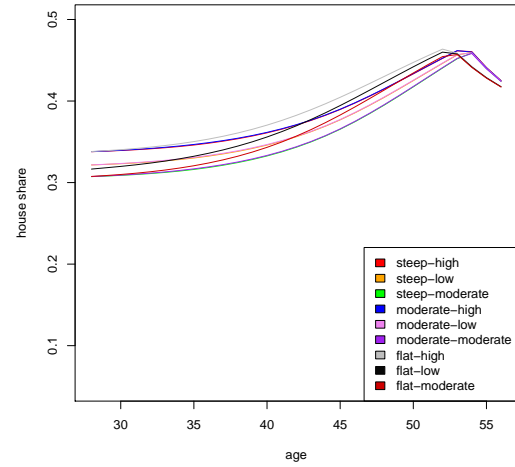
(a) Stocks for $\gamma = 1.5$



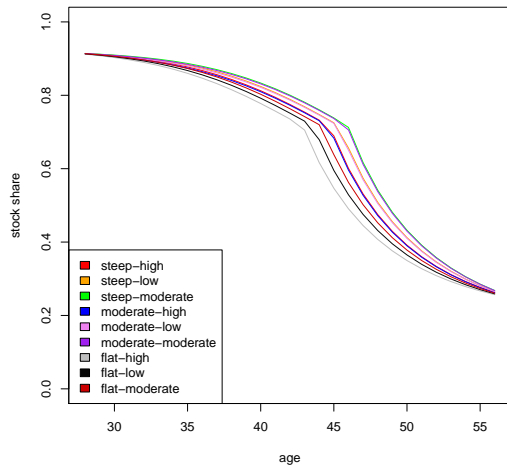
(b) Housing for $\gamma = 1.5$



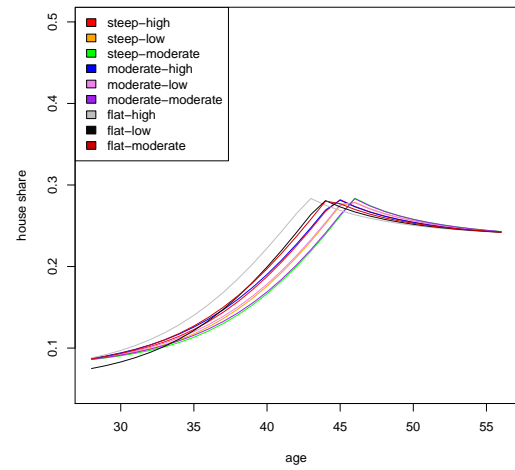
(c) Stocks for $\gamma = 3$



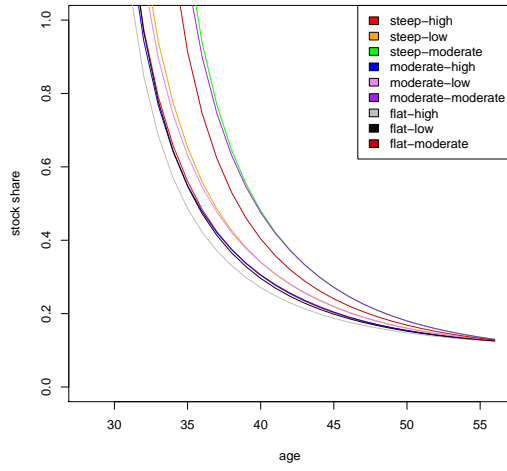
(d) Housing for $\gamma = 3$



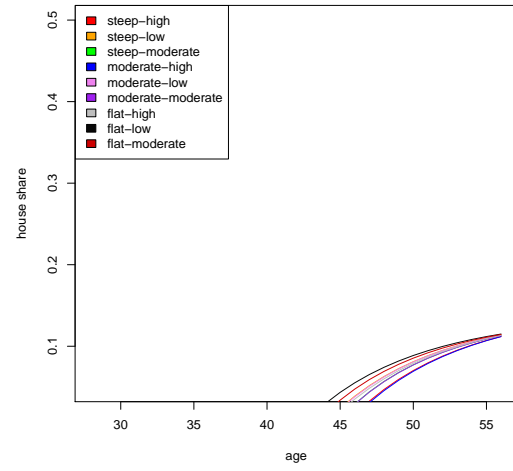
(e) Stocks for $\gamma = 5$



(f) Housing for $\gamma = 5$



(g) Stocks for $\gamma = 10$



(h) Housing for $\gamma = 10$

Figure 15: Munk's stock and housing shares for different wage growth, stock-wage correlation and risk aversion levels

4.0.1. Early results

After a lifetime of investing in the above portfolios, the household accumulates different levels of wealth, summarized in Table 5.1. Since the consumption preferences are monotonic, we can make early conclusions even without calculating expected utilities — the higher the accumulated wealth, the better:

- Naively considering lifecycles does not guarantee the better investment. So, in our case, $(100 - age)\%$ is worse than fixed Markowitz.
- Different stock-wage correlations don't make much difference in the outcome without housing, and make considerable difference in models with housing
- Stock-wage correlations are negatively correlated with total wealth
- Both too low and too high risk aversion results in less accumulated capital than moderate risk aversion for models with housing and only too high risk aversion — for models without housing (because of monotonicity).
- Default options are better for people with flat wage growth rate and worse for people with moderate or steep wage curves

4.0.2. Annuitization

To formalize the observations made in the previous section, we will construct annuities and plug them into expected utility functions. As described in detail in Chapter 3, we define annuities by dividing the total wealth before retirement by the discount factor $1 + \sum_{t=58}^{100} \frac{p_t}{1+r_f}$. Using survival probabilities, obtained from TUIK, and risk-free rate of return, described in the previous chapter,

we calculate the discount factor as 205.29. The annuities are obtained by dividing all the values in the Table 5.1 by 205.29. The resulting values are presented in Table 5.2.

4.0.3. Consumption during retirement

We model that the individuals spend their annuity returns to consume baskets which cost 100 TL during the year, when our agent is 58 years old, and increase by inflation rate $\pi = 8.4\%$. Thus, every period $t > 57$ our agent consumes $\frac{\text{annuity}}{100 \cdot (1.084)^{t-58}}$. We do not provide the separate table with the value streams, as they will be implicitly included in utility values.

4.0.4. Expected utilities

Finally, we will plug the consumption streams, calculated in the previous section, into the constant relative risk-aversion expected utility functions to compare the welfare effects. The resulting expected utility values are presented in Table 5.3.

Table 5: Total Accumulated Wealth by Investment Option

Option	$\gamma = 5$ (default)	$\gamma = 1.5$	$\gamma = 3$	$\gamma = 10$
Defaults				
Anadolu Hayat (riskless)	614,195 TL	614,195 TL	614,195 TL	614,195 TL
(100 - age)%	1,160,977 TL	1,160,977 TL	1,160,977 TL	1,160,977 TL
Cocco et al.	1,599,852 TL	1,599,852 TL	1,599,852 TL	1,599,852 TL
Markowitz	1,300,187 TL	1,300,187 TL	1,300,187 TL	1,300,187 TL
Individualized (no housing)				
Merton (steep)	3,812,053 TL	4,212,735 TL	3,104,746 TL	2,920,892 TL
Merton (moderate)	1,629,904 TL	1,813,821 TL	1,447,951 TL	1,248,330 TL
Merton (flat)	543,602 TL	612,548 TL	639,866 TL	398,235 TL
Munk (steep-lo)	3,547,442 TL	4,168,823 TL	3,829,218 TL	2,085,883 TL
Munk (steep-mod)	3,543,435 TL	4,164,850 TL	3,825,221 TL	2,081,872 TL
Munk (steep-hi)	3,539,425 TL	4,160,872 TL	3,821,219 TL	2,077,854 TL
Munk (moderate-lo)	1,513,198 TL	1,794,772 TL	1,689,491 TL	868,955 TL
Munk (moderate-mod)	1,511,527 TL	1,793,113 TL	1,687,822 TL	867,281 TL
Munk (moderate-hi)	1,509,853 TL	1,791,452 TL	1,686,152 TL	865,604 TL
Munk (flat-lo)	508,757 TL	606,410 TL	657,762 TL	262,161 TL
Munk (flat-mod)	508,371 TL	606,025 TL	657,376 TL	261,774 TL
Munk (flat-hi)	507,984 TL	605,640 TL	656,990 TL	261,386 TL
Individualized (no housing)				
Munk (steep-lo)	8,937,159 TL	3,087,377 TL	4,718,332 TL	3,920,863 TL
Munk (steep-mod)	7,645,621 TL	2,966,467 TL	4,371,031 TL	2,900,022 TL
Munk (steep-hi)	6,353,327 TL	2,846,577 TL	4,017,054 TL	1,872,799 TL
Munk (moderate-lo)	3,652,223 TL	1,304,014 TL	1,998,234 TL	1,605,470 TL
Munk (moderate-mod)	3,125,930 TL	1,252,432 TL	1,850,278 TL	1,189,240 TL
Munk (moderate-hi)	2,599,301 TL	1,201,297 TL	1,700,235 TL	770,295 TL
Munk (flat-lo)	1,026,845 TL	430,353 TL	614,929 TL	455,296 TL
Munk (flat-mod)	881,934 TL	435,168 TL	598,789 TL	341,199 TL
Munk (flat-hi)	736,896 TL	397,829 TL	527,533 TL	226,188 TL

Table 6: Annual Pensions by Investment Option

Option	$\gamma = 5$ (default)	$\gamma = 1.5$	$\gamma = 3$	$\gamma = 10$
Defaults				
Anadolu Hayat (riskless) TL	2,992 TL	2,992 TL	2,992 TL	2,992 TL
$(100 - age)\%$ TL	5,655 TL	5,655 TL	5,655 TL	5,655 TL
Cocco et al. TL	7,793 TL	7,793 TL	7,793 TL	7,793 TL
Markowitz TL	6,333 TL	6,333 TL	6,333 TL	6,333 TL
Individualized (no housing)				
Merton (steep) TL	18,569 TL	20,521 TL	15,124 TL	14,228 TL
Merton (moderate) TL	7,940 TL	8,835 TL	7,053 TL	6,081 TL
Merton (flat) TL	2,648 TL	2,984 TL	3,117 TL	1,940 TL
Munk (steep-lo) TL	17,280 TL	20,307 TL	18,653 TL	10,161 TL
Munk (steep-mod) TL	17,261 TL	20,288 TL	18,633 TL	10,141 TL
Munk (steep-hi) TL	17,241 TL	20,268 TL	18,614 TL	10,122 TL
Munk (moderate-lo) TL	7,371 TL	8,743 TL	8,230 TL	4,233 TL
Munk (moderate-mod) TL	7,363 TL	8,735 TL	8,222 TL	4,225 TL
Munk (moderate-hi) TL	7,355 TL	8,726 TL	8,214 TL	4,216 TL
Munk (flat-lo) TL	2,478 TL	2,954 TL	3,204 TL	1,277 TL
Munk (flat-mod) TL	2,476 TL	2,952 TL	3,202 TL	1,275 TL
Munk (flat-hi) TL	2,474 TL	2,950 TL	3,200 TL	1,273 TL
Individualized (housing)				
Munk (steep-lo) TL	43,534 TL	15,039 TL	22,984 TL	19,099 TL
Munk (steep-mod) TL	37,243 TL	14,450 TL	21,292 TL	14,126 TL
Munk (steep-hi) TL	30,948 TL	13,866 TL	19,568 TL	9,123 TL
Munk (moderate-lo) TL	17,791 TL	6,352 TL	9,734 TL	7,820 TL
Munk (moderate-mod) TL	15,227 TL	6,101 TL	9,013 TL	5,793 TL
Munk (moderate-hi) TL	12,662 TL	5,852 TL	8,282 TL	3,752 TL
Munk (flat-lo) TL	5,002 TL	2,096 TL	2,995 TL	2,218 TL
Munk (flat-mod) TL	4,296 TL	2,120 TL	2,917 TL	1,662 TL
Munk (flat-hi) TL	3,590 TL	1,938 TL	2,570 TL	1,102 TL

4.0.5. Conclusions

Now, we can observe the final results:

- Naive lifecycle investment portfolios, such as $(100 - age)\%$ don't overperform fixed-ratio Markowitz, because they don't take the risk aversion into consideration.
- Cocco et al.'s $(200 - 2.5 \cdot age)\%$ approximation is the best default portfolio. It is easy to interpret and captures lifecycle effect.
- All models perform better for higher risk aversion and worse for lower risk aversion — for everyone except flat-wagers.
- Higher stock-wage correlation considerably decreases the utility for moderate and flat wages, and doesn't affect much for steep wages.
- Merton's solution outperforms Munk's solution without housing for low levels of risk aversion, and performs same for high level of risk aversion ($\gamma = 10$).
- Munk's solution with housing outperforms every other solution for high levels of risk aversion ($\gamma = 10$) — for everyone except flat-wagers.
- Munk's solution with housing outperforms Munk's solution without housing for $\gamma = 5, 10$.
- Markowitz's solution outperforms both Merton's and Munk's solutions for flat wages and low risk aversion.

- Individualizing lifecycles by wage growth rate and stock-wage correlation increases welfare for steep wagers and decreases welfare for flat wagers.
- . We did not provide numerical conclusions, since they are trivial — they can be obtained by calculating percentage differences in Table 5.3.

Table 7: Expected Utilities by Investment Option

Option	$\gamma = 5$ (default)	$\gamma = 1.5$	$\gamma = 3$	$\gamma = 10$
Defaults				
Anadolu Hayat (riskless)	-0.0008389	-3.6004130	-0.0247934	-0.0000509
(100-age)%	-0.0000657	-2.6187470	-0.0069390	-0.0000002
Cocco et al.	-0.0000182	-2.2308240	-0.0036542	0.0000000
Markowitz	-0.0000418	-2.4745850	-0.0055327	-0.0000001
Individualized (no housing)				
Merton (steep)	-0.0000006	-1.3747480	-0.0009703	0.0000000
Merton (moderate)	-0.0000169	-2.0951160	-0.0044611	-0.0000001
Merton (flat)	-0.0013672	-3.6052480	-0.0228439	-0.0025138
Munk (steep-lo)	-0.0000008	-1.3819700	-0.0006379	0.0000000
Munk (steep-mod)	-0.0000008	-1.3826290	-0.0006392	0.0000000
Munk (steep-hi)	-0.0000008	-1.3832900	-0.0006405	0.0000000
Munk (moderate-lo)	-0.0000228	-2.1062050	-0.0032767	-0.0000022
Munk (moderate-mod)	-0.0000229	-2.1071790	-0.0032832	-0.0000023
Munk (moderate-hi)	-0.0000230	-2.1081550	-0.0032897	-0.0000023
Munk (flat-lo)	-0.0017820	-3.6234500	-0.0216177	-0.1082611
Munk (flat-mod)	-0.0017874	-3.6245990	-0.0216431	-0.1097095
Munk (flat-hi)	-0.0017929	-3.6257500	-0.0216686	-0.1111816
Individualized (housing)				
Munk (steep-lo)	0.0000000	-1.6058700	-0.0004201	0.0000000
Munk (steep-mod)	0.0000000	-1.6382700	-0.0004895	0.0000000
Munk (steep-hi)	-0.0000001	-1.6724140	-0.0005796	0.0000000
Munk (moderate-lo)	-0.0000007	-2.4709510	-0.0023424	0.0000000
Munk (moderate-mod)	-0.0000013	-2.5213210	-0.0027320	-0.0000001
Munk (moderate-hi)	-0.0000026	-2.5744240	-0.0032354	-0.0000066
Munk (flat-lo)	-0.0001074	-4.3012340	-0.0247341	-0.0007532
Munk (flat-mod)	-0.0001973	-4.2773710	-0.0260855	-0.0101044
Munk (flat-hi)	-0.0004049	-4.4735990	-0.0336084	-0.4086488

5. Conclusion

In this thesis we have reviewed the current state of pension investments in Turkey and the history and recent developments of a field of financial economics. We have reviewed the concept of "lifecycle investment" and summarized the common models and heuristics.

. We have presented our model as an application of Munk's (2016) recent findings and Olear's (2016) simulation techniques into Turkish retirement market. We have collected historical data to calibrate and estimate the best parameters to be used in our simulation.

. Using these parameters, we have constructed heterogeneous agents, who worked and invested throughout their lifetime. We considered different investment models that our hypothetical agents would use and calculated the resulting investment capitals.

. Finally, we have calculated and compared the welfare effects of all popular models to the individualized Munk's solutions. We have concluded that for rich-to-middle class citizens, the individualized strategies considerably increase their welfare. Moreover, the solutions we proposed can be solved analytically without complex dynamic optimizations, and therefore are easy to interpret to households. We also found that even naive lifecycle investments perform better than fixed lifetime investment.

. We propose these models to Turkish pension providers and to Turkish working-age households belonging to middle-to-upper class, as these options will increase their retirement welfare considerably.

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Appendix A. Ascheberg's correlation structure

The structure that returns desired correlation coefficients ρ_{sy} , ρ_{hs} and ρ_{hy} is as follows:

$$\frac{\Delta S_{t+1}}{S_t} = \mu_s + \sigma_s \cdot \epsilon_{st} \quad (\text{A.1a})$$

$$\frac{\Delta H_{t+1}}{H_t} = \mu_h + \sigma_h \cdot \left(\rho_{hs} \epsilon_{st} + (\sqrt{1 - \rho_{hs}^2}) \epsilon_{ht} \right) \quad (\text{A.1b})$$

$$\frac{\Delta Y_{t+1}}{Y_t} = \mu_v + \sigma_v \cdot \left(\rho_{ys} \epsilon_{st} + \left(\frac{\rho_{yh} - \rho_{sh} \rho_{sy}}{\sqrt{1 - \rho_{sh}^2}} \right) \epsilon_{ht} + \left(\sqrt{1 - \rho_{ys}^2} - \left(\frac{\rho_{yh} - \rho_{sh} \rho_{sy}}{\sqrt{1 - \rho_{sh}^2}} \right)^2 \right) \epsilon_{vt} \right) \quad (\text{A.1c})$$

To derive this, let Σ be a correlation matrix of a vector $X = (x_1, x_2, \dots, x_K)$. Also, let $\Sigma = LL'$ be a Cholesky decomposition of this matrix.

Notice that the variance-covariance matrix of an i.i.d. random vector $\Omega = (\epsilon_1, \epsilon_2, \dots, \epsilon_K)$ with variances equal to 1, is an identity matrix. Thus, the product $L\Omega$ has the same correlation structure as X :

$$\begin{aligned} \text{cov}(L\Omega) &= E[(L\Omega)(L\Omega)'] = E[L\Omega\Omega'L'], \\ \text{cov}(L\Omega) &= L \cdot E[\Omega\Omega'] \cdot L' = L \cdot \text{var}(\Omega) \cdot L', \\ \text{cov}(L\Omega) &= L \cdot I \cdot L' = LL' = \Sigma \end{aligned}$$

The conclusion comes from the fact that the Cholesky decomposition of a correlation matrix R :

$$R = \begin{bmatrix} 1 & \rho_{sh} & \rho_{sy} \\ \rho_{hs} & 1 & \rho_{hy} \\ \rho_{ys} & \rho_{yh} & 1 \end{bmatrix}$$

can be easily calculated to be equal to Q :

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ \rho_{hs} & \sqrt{1 - \rho_{hs}^2} & 0 \\ \rho_{ys} & \frac{\rho_{yh} - \rho_{sh} \rho_{sy}}{\sqrt{1 - \rho_{sh}^2}} & \sqrt{1 - \rho_{ys}^2} - \left(\frac{\rho_{yh} - \rho_{sh} \rho_{sy}}{\sqrt{1 - \rho_{sh}^2}} \right)^2 \end{bmatrix}$$