Welfare Comparison of Different Life-Cycle Investment Strategies for Turkey

Master's Thesis

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Introduction

Introduction i

- Retirement is one of the most important investment decisions we face in our lives.
- Current investment menus are either too simplistic and inefficient or too complicated and unintuitive.
- Most individuals find active involvement in investment too complex.
- Lifecycle investments — investments that are designed for different age profiles, as opposed to fixed-over-lifetime investments.
- Naive investments — asset allocations that do not consider individual characteristics.

Introduction ii

Table 1: Largest Turkish Pension Funds

Fund name	Fund size
AvivaSA Emeklilik ve Hayat	14.8 bln
Anadolu Hayat Emeklilik	14.1 bln
Garanti Emeklilik ve Hayat	11.1 bln
Allianz Yasam ve Emeklilik	10.4 bln
Vakif Emeklilik	6.1 bln

Source: Pension Monitoring Center (2018)

Literature Review

Literature Review i

- Markowitz's Mean-Variance Analysis — maximize return while minimizing volatility:

  $$\max_{\alpha}\{E[R_p] - \frac{\gamma}{2}\sigma_p^2\}$$

  solution:

  $$\alpha = \frac{E[R]-R_f}{\gamma\sigma^2}$$

  where α is risky asset share in portfolio.
- Markowitz derived fixed one-period solution.

Literature Review ii

- Merton (1971) generalized the problem to multiple periods, found it optimal to repeat Markowitz every period.
- Contradicted financial advice and rationality $\alpha_t = (100 - t)\%$
- Bodie (1992) added human capital (discounted sum of future fixed wage) into the model:

$$\alpha_t = \frac{\mu - R_f}{\gamma\sigma^2}\left(1 + \frac{L_t}{F_t}\right)$$

- L_t/F_t changed over time and captured lifecycle effect. Young people would be more aggressive than Markowitz, and old people would converge to Markowitz.

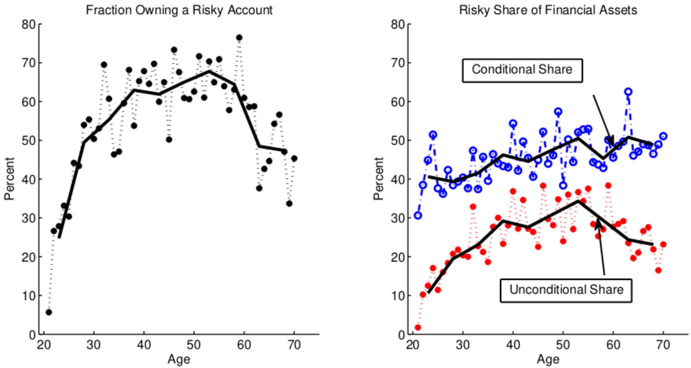
Literature Review iii

- Cocco et al. (2005) did similar analysis and added the following heuristic:

$$\alpha_t = \begin{cases} 100\% & t < 40 \\ (200 - 2.5t)\% & t \in [40, 60] \\ 50\% & t > 60 \end{cases},$$

- Did not explain hump-shaped stock share (Chang (2014))

Figure 1: Risky Share over the Life Cycle



Literature Review v

- Cocco and Flavin and Yamashita (2002) found that if individuals possessed housing, they would be even more aggressive, using dynamic optimization.

- Munk (2016) reinvented analytical solution to the problem.

$$\max_{\pi}\{E[\frac{W_1}{W_0}] - \frac{\gamma}{2}var(\frac{W_1}{W_0})\}$$

where total wealth is a sum of financial and human wealth: $W_t = F_t + L_t$ and human capital has returns $r_L \sim (\mu_L, \sigma_L)$.

$$\pi^* = \frac{1}{\gamma}\frac{W_0}{F_0} \cdot \Sigma^{-1}(\mu - r_f \cdot 1) - \frac{L_0}{F_0} \cdot \Sigma^{-1} cov(r, r_L)$$

- We used Munk's solution without housing:

$$\pi_{t+1} = \frac{\mu_s - r_f}{\gamma\sigma_s^2} + \frac{L_t}{F_t} \cdot \left(\frac{\mu_s - r_f}{\gamma\sigma_s^2} - \frac{\rho_{SL}\sigma_L}{\sigma_S} \right)$$

and with housing:

$$\pi_{t+1} = \frac{1}{\gamma(1-\rho_{SH}^2)\sigma_S} \cdot \frac{W_t}{F_t} \left(\frac{\mu_s - r_f}{\sigma_S} - \rho_{SH}\frac{\mu_h - r_f}{\sigma_h} \right) - \frac{L_t}{F_t} \cdot \frac{\sigma_L}{\sigma_S} \frac{\rho_{SL} - \rho_{SH}\rho_{HL}}{1-\rho_{SH}^2}$$

$$\pi_{h,t+1} = \frac{1}{\gamma(1-\rho_{SH}^2)\sigma_H} \cdot \frac{W_t}{F_t} \left(\frac{\mu_h - r_f}{\sigma_h} - \rho_{SH}\frac{\mu_s - r_f}{\sigma_s} \right) - \frac{L_t}{F_t} \cdot \frac{\sigma_L}{\sigma_h} \frac{\rho_{HL} - \rho_{SH}\rho_{SL}}{1-\rho_{SH}^2}$$

$$\pi_{R_f} = (1 - \pi - \pi_h)$$

Model

Model i

- We use Olear's (2016) approach to model labor income:

$$Y_{i,t+1} = \begin{cases} Y_{it}(1 + g_{i,t+1} + \xi_t + \omega_{it}), & t \leq T \\ \lambda(1 + f(T, Z_{iT}) + v_{iT}), & t > T \end{cases}$$

- We model labor income, house prices, and stock prices as Geometric Brownian Motions with drifts μ_L, μ_H, μ_S and volatilities σ_L, σ_H, σ_S,

Model ii

- Welfare measurement — we use CRRA utility:

$$E_1[U(c)] = \sum_{t=1}^{T} \delta^{t-1} \prod_{j=0}^{t-1} p_j \cdot \frac{c_{it}^{1-\gamma}}{1-\gamma}$$

  where p_k is the probability of survival from time $k-1$ to time k.

- We omitted the bequest motives from the original formulation, thus retired person consumes all of his income at any given time.

Model iii

- Retirement income — accumulated financial wealth is repaid back in annuities.
- Reverse mortgages — housing wealth is reinvested for annuities in return of inheriting a house to the payer (no bequest motives)

$$W_{65} = H_{65} + MP$$

- Annuity is equal to: $A_t = W_{65} \cdot \left(1 + \sum_{t=66}^{100} \frac{\prod_{j=66}^{t} p_j}{(1+r_f)^{t-65}}\right)^{-1}$
- Welfare calculation — we convert annuity stream into consumption (considering the inflation) and plug into CRRA expected utility function.

Data Structure and Sources

- Stock rates of return are obtained from Borsa Istanbul BIST30 index:

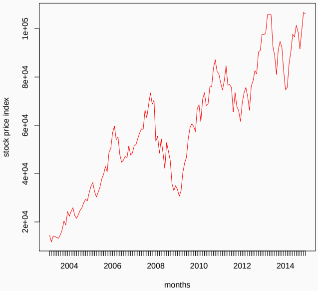


Figure 1: BIST30 Turkish stock market performance index

Data Structure and Sources ii

- Housing returns are obtained from Reidin AEINDEXF index:

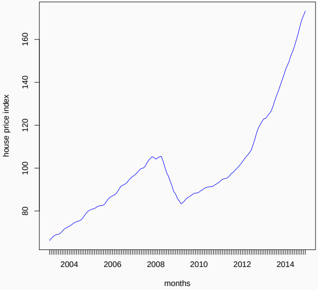


Figure 2: Reidin Turkish house price index

Data Structure and Sources iii

- Wage dynamics are obtained from TUIK Houshold Budget Survey (HBS) and Aktug, Kuzubas, Torul (2017) (notice the hump shape):

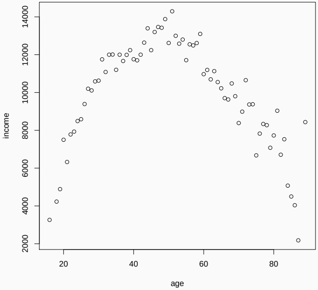


Figure 3: Median Turkish salaries by age

Data Structure and Sources iv

- We start with 25 years old individual, who invests for 40 years until retirement at 65.
- The investments are done from 3% of every wage from 25 to 64 years.
- In line with Torul et al. (2018), $\delta = 0.89$ and $\gamma = 1.5$ for Turkey is chosen as a default.

Data Structure and Sources v

Table 2: Benchmark Parameters

Parameter	Description	Value
Y	Beginning age	25
R	Retirement age	65
T	Lifespan (years)	100
γ	Risk aversion	1.5
β	Discount rate	0.89
r_f	Risk-free rate	0.03
μ_s	Expected stock returns	0.0669
μ_h	Expected housing returns	0.0067
σ_s	Stock returns volatility	0.3844
σ_h	Housing returns volatility	0.0542
σ_w	Wage growth volatility	0.036
ρ_{hs}	House-stock correlation	0.24

Data Structure and Sources vi

- The data on survival probability for all ages is obtained from TUIK database

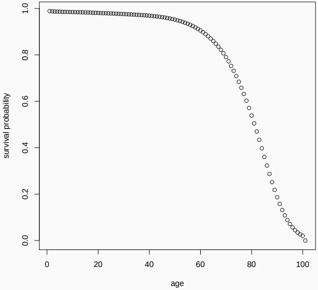


Figure 4: Survival probabilities by age

Data Structure and Sources vii

- We consider heterogeneity of agents as follows:
- Heterogeneity in education — defined as difference in wage curve steepness

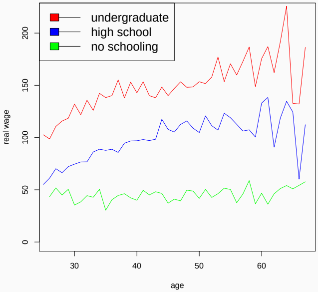


Figure 5: Lifetime wage dynamics by education level

Data Structure and Sources viii

- We use undergraduate, high school, and no schooling, to model “steep”, “moderate”, and “flat” wages.
- Performing regressions of wages on age, with kinks at $t = 40$ and $t = 55$:

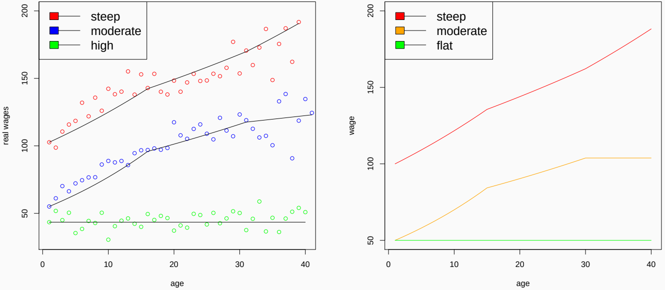
$$\Delta \log(wage_{it}) = \alpha_0 + \alpha_1 \cdot d_{40} + \alpha_2 \cdot d_{55} \tag{1}$$

Table 3: Estimated Benchmark Wage Growth Rates μ_w

Age	Flat	Moderate	Steep
25-40	0%	3.8%	2.2%
41-55	0%	1.4%	1.2%
56-65	0%	0%	1.5%

Data Structure and Sources ix

- Parameterized and actual wage curves



- Starting salary: 100 for steep wages, 50 for moderate and flat.

Data Structure and Sources x

- Heterogeneity in sectors of work — it is captured by differing stock-wage correlations
- Zero for agricultural sector / teaching
- As high as 0.4 for financial sector
- 0.2 in the middle
- Notice movements during 2008 crisis

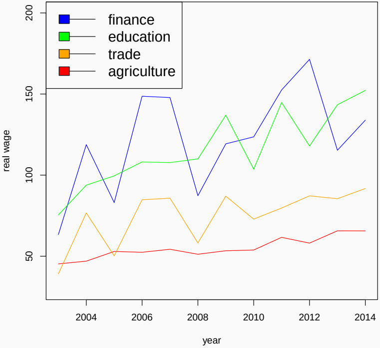


Figure 6: Historical wage dynamics by sector

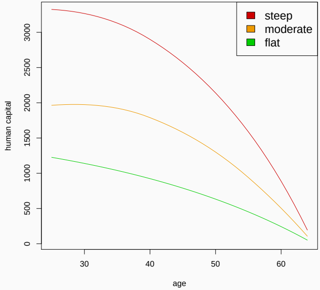
- Individual heterogeneity — it is captured by different risk aversion levels:

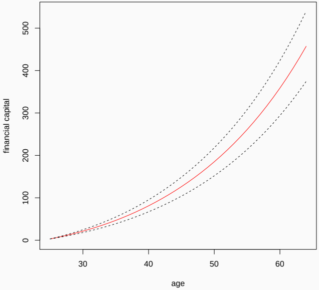
Table 4: Coefficients of Risk Averion

Values	default	low	moderate	high
γ	1.5	3	5	10

Data Structure and Sources xiii

- We constructed human capital and financial capital series taking the heterogeneities into consideration:
- L_t/F_t is declining in t — optimal risky asset share is declining





Data Structure and Sources xiv

	t_0	t_1	t_2
H :	0	$cw_0(1+r_p)$	$[cw_0(1+r_p)+cw_1]\cdot(1+r_p)$
I :	cw_0	$+cw_1$	$+cw_2$

Figure 7: Law of motion of financial capital. Every period, a certain percentage c of the wage w_t is invested in a retirement portfolio, while the previously invested amount accrues interest at portfolio rate of return r_p.

All Strategies

Strategies i

- First, we list default and derived investment strategies
- Then we calculate the capital movements using these strategies
- We obtain total wealth before retirement and annuitize it
- We convert annuities into consumption levels considering inflation
- We plug consumption levels into expected utilities
- We compare resulting utilities and conclude

Strategies ii

- Homogeneous strategies

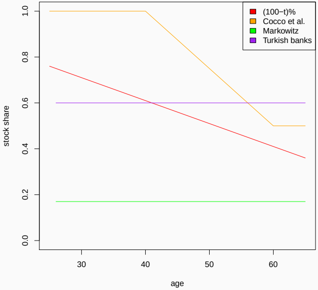
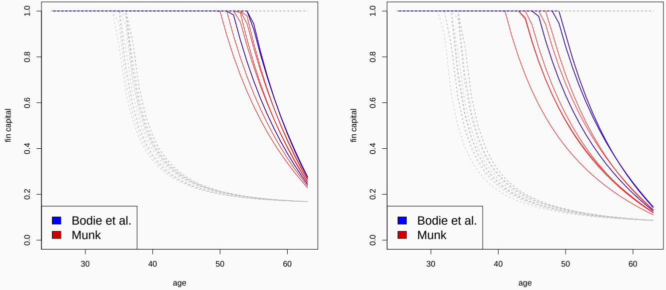
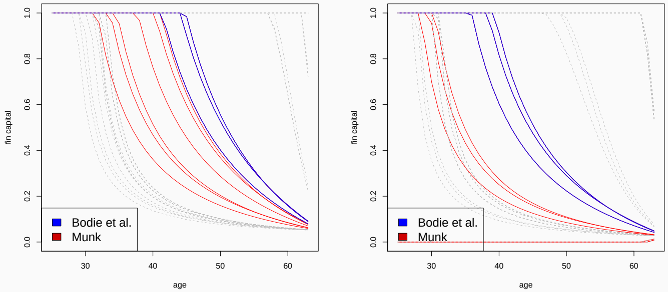


Figure 8: Default portfolio allocations of stock investments

- Several individualized solutions ($\gamma = 1.5, 3, 5, 10$):



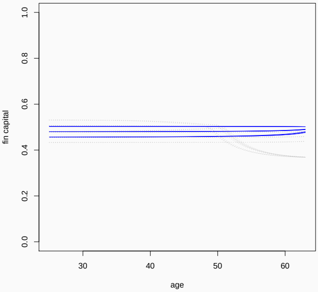
Strategies iv

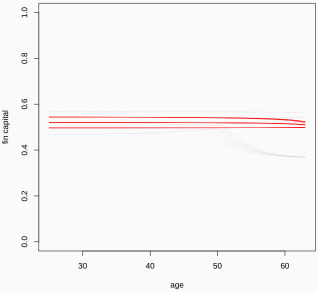


Strategies v

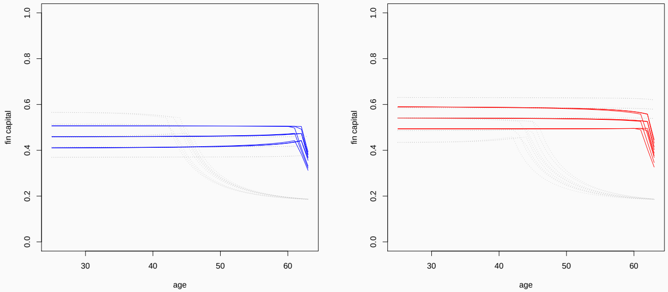
- Note that for small stock-wage correlations, Munk's solution without housing is equivalent to Merton's solution
- Note that for smaller risk aversion, households invest more aggressively
- Note that flat wagers are less aggressive than steeper wagers
- Munk's solution with housing are presented below.
- Left graph is optimal stock share and right graph is optimal housing share
- Graphs are done for $\gamma = 1.5, 3, 5, 10$

Strategies vi

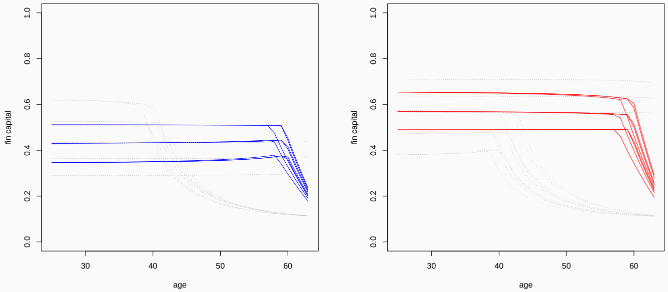




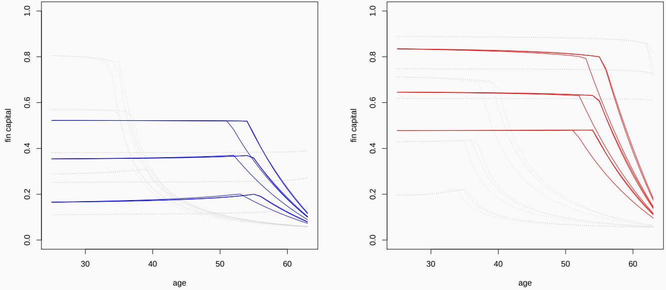
Strategies vii



Strategies viii



Strategies ix



Strategies x

- In line with Munk, the stock-house allocation is done as follows:
  - If optimal stock and housing allocations sum up to a number greater than 1, then we allocate our wealth proportionately between the two
  - If optimal stock and housing allocations sum up to a number less than 1, then we allocate those very shares and invest the rest into risk-free bonds
- Note that as risk aversion coefficeint increases, the kink happens earlier
- Note that the steeper the wage curve is, the more aggressive the individual is
- Note that stock-wage correlation does not influence steep and flat wagers much

Results i

Table 5.1. Monte Carlo Results of Accumulated Wealth for Homogeneous Investment Strategies

wages	steep	moderate	flat
Markowitz	345	203	131
$100 - age$	458	266	179
Cocco et al.	567	327	226
Turkish banks	494	287	193

Results ii

Table 5.2. Monte Carlo Results of Accumulated Wealth for Heterogeneous Investment Strategies

wages	steep			moderate			flat		
ρ_{ws}	high	moderate	low	high	moderate	low	high	moderate	low
$\gamma = 1.5$									
Bodie et al.	563	563	563	327	327	327	214	214	214
Munk (no housing)	539	547	554	313	317	321	204	208	210
Munk (housing)	413	427	441	241	249	257	158	163	169
$\gamma = 3$									
Bodie et al.	498	498	498	290	290	290	187	187	187
Munk (no housing)	457	478	494	267	279	287	171	179	185
Munk (housing)	532	553	566	309	321	328	200	209	214
$\gamma = 5$									
Bodie et al.	455	455	455	266	266	266	170	170	170
Munk (no housing)	380	426	454	223	249	265	141	159	169
Munk (housing)	509	523	535	296	304	311	191	197	201
$\gamma = 10$									
Bodie et al.	408	408	408	239	239	239	151	151	151
Munk (no housing)	380	426	454	223	249	265	141	159	169
Munk (housing)	509	523	535	296	304	311	191	197	201

Results iii

Table 5.3. A: Summary of Expected Utilities from Simulation for $\gamma = 1.5$

wages	steep			moderate			flat		
ρ_{ws}	high	moderate	low	high	moderate	low	high	moderate	low
Markowitz	-2.456	-2.456	-2.456	-3.202	-3.202	-3.202	-3.986	-3.986	-3.986
$100 - age$	-2.132	-2.132	-2.132	-2.798	-2.798	-2.798	-3.410	-3.410	-3.410
Cocco et al.	-1.916	-1.916	-1.916	-2.523	-2.523	-2.523	-3.035	-3.035	-3.035
Turkish banks	-2.053	-2.053	-2.053	-2.693	-2.693	-2.693	-3.284	-3.284	-3.284
Bodie et al.	-1.923	-1.923	-1.923	-2.523	-2.523	-2.523	-3.119	-3.119	-3.119
Munk (no housing)	-1.965	-1.951	-1.938	-2.579	-2.563	-2.547	-3.194	-3.164	-3.149
Munk (housing)	-2.245	-2.208	-2.173	-2.939	-2.891	-2.846	-3.630	-3.574	-3.510

Results iv

Table 5.3. B: Summary of Expected Utilities from Simulation for $\gamma = 3$

wages	steep			moderate			flat		
ρ_{ws}	high	moderate	low	high	moderate	low	high	moderate	low
Markowitz	-0.0049	-0.0049	-0.0049	-0.0141	-0.0141	-0.0141	-0.0338	-0.0338	-0.0338
$100 - age$	-0.0028	-0.0028	-0.0028	-0.0082	-0.0082	-0.0082	-0.0181	-0.0181	-0.0181
Cocco et al.	-0.0018	-0.0018	-0.0018	-0.0054	-0.0054	-0.0054	-0.0114	-0.0114	-0.0114
Turkish banks	-0.0024	-0.0024	-0.0024	-0.0070	-0.0070	-0.0070	-0.0156	-0.0156	-0.0156
Bodie et al.	-0.0023	-0.0023	-0.0023	-0.0069	-0.0069	-0.0069	-0.0166	-0.0166	-0.0166
Munk (no h.)	-0.0028	-0.0025	-0.0024	-0.0081	-0.0075	-0.0070	-0.0198	-0.0181	-0.0170
Munk (h.)	-0.0021	-0.0019	-0.0018	-0.0061	-0.0056	-0.0054	-0.0145	-0.0133	-0.0127

Results v

Table 5.3. C: Summary of Expected Utilities from Simulation for $\gamma = 5$

wages	steep			moderate			flat		
ρ_{ws}	high	moderate	low	high	moderate	low	high	moderate	low
Markowitz	-4e-06	-4e-06	-4e-06	-0.00003	-0.00003	-0.00003	-0.00019	-0.00019	-0.00019
$100 - age$	-1e-06	-1e-06	-1e-06	-0.00001	-0.00001	-0.00001	-0.00005	-0.00005	-0.00005
Cocco et al.	-1e-06	-1e-06	-1e-06	-0.00001	-0.00001	-0.00001	-0.00002	-0.00002	-0.00002
Turkish banks	-1e-06	-1e-06	-1e-06	-0.00001	-0.00001	-0.00001	-0.00004	-0.00004	-0.00004
Bodie et al.	-1e-06	-1e-06	-1e-06	-0.00001	-0.00001	-0.00001	-0.00007	-0.00007	-0.00007
Munk (no h.)	-3e-06	-2e-06	-1e-06	-0.00002	-0.00001	-0.00001	-0.00014	-0.00004	-0.00004
Munk (h.)	-1e-06	-1e-06	-1e-06	-0.00001	-0.00001	-0.00001	-0.00004	-0.00004	-0.00004

Results vi

Table 5.3. D: Summary of Expected Utilities from Simulation for $\gamma = 10$

wages	steep			moderate			flat		
ρ_{ws}	high	moderate	low	high	moderate	low	high	moderate	low
Markowitz	-1.7e-13	-1.7e-13	-1.7e-13	-2e-11	-2e-11	-2e-11	-1.0e-09	-1.0e-09	-1.0e-09
$100 - age$	-1.0e-14	-1.0e-14	-1.0e-14	-1.8e-12	-1.8e-12	-1.8e-12	-6.3e-11	-6.3e-11	-6.3e-11
Cocco et al.	0	0	0	-2.8e-13	-2.8e-13	-2.8e-13	-7.7e-12	-7.7e-12	-7.7e-12
Turkish banks	-1.0e-14	-1.0e-14	-1.0e-14	-9e-13	-9e-13	-9e-13	-3.2e-11	-3.2e-11	-3.2e-11
Bodie et al.	-4.0e-14	-4.0e-14	-4.0e-14	-4.7e-12	-4.7e-12	-4.7e-12	-2.9e-10	-2.9e-10	-2.9e-10
Munk (no h.)	-5.7e-13	-1.7e-13	-4.0e-14	-6.6e-11	-1.9e-11	-4.8e-12	-4e-09	-1.2e-09	-2.9e-10
Munk (h.)	-3.0e-14	-1.0e-14	-1.0e-14	-3.3e-12	-1.8e-12	-1.2e-12	-1.9e-10	-1e-10	-6.6e-11

Results vii

Table 7: A: Pairwise comparisons for $\gamma = 1.5$

$L \geq R$	Markowitz	$100 - age$	Cocco et al.	Turkish banks	Bodie et al.	Munk (no h.)	Munk (h.)
Markowitz	1	0.5129	0.6169	0.5489	0.5474	0.5564	0.6384
$100 - age$		1.0000	0.7364	0.6094	0.5860	0.5982	0.8017
Cocco et al.			1.0000	0.2925	0.5429	0.5557	0.6705
Turkish banks				1.0000	0.6469	0.6602	0.9447
Bodie et al.					1.0000	0.9305	0.4727
Munk (no h.)						1.0000	0.4584
Munk (h.)							1.0000

Table 7: B: Pairwise comparisons for $\gamma = 3$

$L \geq R$	Markowitz	$100 - age$	Cocco et al.	Turkish banks	Bodie et al.	Munk (no h.)	Munk (h.)
Markowitz	1	0.5129	0.6169	0.5489	0.4893	0.4947	0.5714
$100 - age$		1.0000	0.7364	0.6094	0.5717	0.5786	0.6141
Cocco et al.			1.0000	0.2925	0.4949	0.5019	0.5776
Turkish banks				1.0000	0.5838	0.5910	0.6749
Bodie et al.					1.0000	0.9353	0.5477
Munk (no h.)						1.0000	0.5455
Munk (h.)							1.0000

Results viii

Table 7: C: Pairwise comparisons for $\gamma = 5$

$L \geq R$	Markowitz	$100 - age$	Cocco et al.	Turkish banks	Bodie et al.	Munk (no h.)	Munk (h.)
Markowitz	1	0.5129	0.6169	0.5489	0.4482	0.4513	0.5231
$100 - age$		1.0000	0.7364	0.6094	0.5594	0.5631	0.5784
Cocco et al.			1.0000	0.2925	0.4439	0.4475	0.5288
Turkish banks				1.0000	0.5258	0.5295	0.6234
Bodie et al.					1.0000	0.9930	0.4914
Munk (no h.)						1.0000	0.4910
Munk (h.)							1.0000

Table 7: D: Pairwise comparisons for $\gamma = 10$

$L \geq R$	Markowitz	$100 - age$	Cocco et al.	Turkish banks	Bodie et al.	Munk (no h.)	Munk (h.)
Markowitz	1	0.5129	0.6169	0.5489	0.4217	0.4234	0.4737
$100 - age$		1.0000	0.7364	0.6094	0.5148	0.5164	0.5789
Cocco et al.			1.0000	0.2925	0.3984	0.3988	0.4820
Turkish banks				1.0000	0.4731	0.4740	0.5703
Bodie et al.					1.0000	0.9960	0.4512
Munk (no h.)						1.0000	0.4502
Munk (h.)							1.0000

Results ix

- After a lifetime of investing, the household accumulated various levels of wealth, summarized in the Table 5.1 of our thesis
- Looking at these total wealth levels, we can make early conclusions even before calculating utilities:
  - Cocco et al.'s $(200 - 2.5 \cdot age)\%$ performs better, on average, than any other portfolio.
  - Even a naive life-cycle investment portfolio $(100 - age)\%$ overperforms fixed-over-lifetime Markowitz.
  - All models perform better for higher risk aversion and worse for lower risk aversion.
  - Munk's solution performs worse for flat wages than for steep wages.

Results x

- Munk's solution with housing is better than without housing when $\gamma > 1.5$.
- When $\rho_{ws} = 0$, Bodie's solution is almost equal to Munk's solution without housing, with the former performing slightly better than the latter.
- Munk's solution with housing performs better for sectors with low stock-wage correlation, being a low-risk investment.

- Expected utilities are summarized in Table 5.3 of our thesis.

Conclusion