Welfare Effects of Individualizing Life-Cycle Pension Investments to Households in Turkey

Master's Thesis

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Introduction I

- Retirement is one of the most important investment decisions we face in our lives.
- Current investment menus are either too simplistic and inefficient or too complicated and unintuitive.
- Most individuals find active involvement in investment too complex.
- Lifecycle investments — investments that solve for asset allocations separately for every age, as opposed to fixed investments.
- Naive investments — asset allocations that do not consider indivdual characteristics.

Introduction II

Table: Largest Turkish Pension Funds

Fund name	Fund size
Anadolu Hayat Emeklilik	8.7 bln
Garanti Emeklilik ve Hayat	7.4 bln
AvivaSA Emeklilik ve Hayat	9.1 bln
Allianz Yasam ve Emeklilik	6.8 bln
Vakif Emeklili	3.5 bln

Source: Pension Monitoring Center (2016)

Literature Review I

- Markowitz's Mean-Variance Analysis — maximize return while minimizing volatility:

$$\max_{\alpha}\{E[R_p] - \frac{\gamma}{2}\sigma_p^2\}$$

solution:

$$\alpha = \frac{E[R]-R_f}{\gamma\sigma^2}$$

where α is risky asset share in portfolio.

- Markowitz derived fixed solution and didn't capture lifecycle differences.

Literature Review II

- Merton (1971) added human capital (discounted sum of future fixed wage) into the model:

$$\alpha_t = \frac{\mu - R_f}{\gamma\sigma^2}\left(1 + \frac{H_t}{W_t}\right)$$

- H_t/W_t changed over time and captured lifecycle effect. Moreover, since young people had higher human capital, they would be more aggressive than Markowitz, and old people would converge to Markowitz.
- Bodie et al. (1992) solved Merton for variable wage and used dynamic optimization to solve it.

Literature Review III

- Cocco et al. (2005) did similar analysis and added the following heuristic:

$$\alpha_t = \begin{cases} 100\% & t < 40 \\ (200 - 2.5t)\% & t \in [40, 60], \\ 50\% & t > 60 \end{cases}$$

- Flavin and Yamashita (2002) added housing capital as well and concluded that if individuals possessed housing, they would be even more aggressive. Solution was still done using dynamic optimization.

Literature Review IV

- Munk (2016) reinvented analytical solution to the problem.

$$\max_{\pi}\{E[\frac{W_1}{W_0}] - \frac{\gamma}{2} var(\frac{W_1}{W_0})\}$$

where total wealth is a sum of financial and human wealth: $W_t = F_t + L_t$ and human capital has returns $r_L \sim (\mu_L, \sigma_L)$.

$$\pi^* = \frac{1}{\gamma}\frac{W_0}{F_0} \cdot \Sigma^{-1}(\mu - r_f \cdot 1) - \frac{L_0}{F_0} \cdot \Sigma^{-1} cov(r, r_L)$$

Model I

- We use Olear's (2016) approach to model labor income:

$$Y_{i,t+1} = \begin{cases} Y_{it}(1 + g_{i,t+1} + \xi_t + \omega_{it}), & t \leq T \\ \lambda(1 + f(T, Z_{iT}) + v_{iT}), & t > T \end{cases}$$

- We model two risky assets and a wage as correlated discrete series:

$$\frac{\Delta S_{t+1}}{S_t} = \mu_s + \sigma_s \cdot \epsilon_{st}$$

$$\frac{\Delta H_{t+1}}{H_t} = \mu_h + \sigma_h \cdot \left(\rho_{hs}\epsilon_{st} + (\sqrt{1-\rho_{hs}^2})\epsilon_{ht}\right)$$

$$\frac{\Delta Y_{t+1}}{Y_t} = \mu_v + \sigma_v \cdot \left(\rho_{ys}\epsilon_{st} + \left(\frac{\rho_{yh}-\rho_{sh}\rho_{sy}}{\sqrt{1-\rho_{sh}^2}}\right)\epsilon_{ht} + \left(\sqrt{1-\rho_{ys}^2-(\frac{\rho_{yh}-\rho_{sh}\rho_{sy}}{\sqrt{1-\rho_{sh}^2}})^2}\right)\epsilon_{vt}\right)$$

Model II

- Welfare measurement — we use CRRA utility:

$$E_1[U(c)] = E_1\left[\sum_{t=1}^{T} \delta^{t-1} \prod_{j=0}^{t-1} p_j \cdot \frac{c_{it}^{1-\gamma}}{1-\gamma}\right]$$

  where p_k is the probability of survival from time $k-1$ to time k.

- We omitted the bequest motives from the original formulation, thus retired person consumes all of his income at any given time.

Model III

- We used Munk's solution without housing:

$$\pi_{t+1} = \frac{\mu_s - r_f}{\gamma\sigma_s^2} + \frac{L_t}{F_t} \cdot \left(\frac{\mu_s - r_f}{\gamma\sigma_s^2} - \frac{\rho_{SL}\sigma_L}{\sigma_S} \right)$$

and with housing:

$$\pi_{t+1} = \frac{1}{\gamma(1-\rho_{SH}^2)\sigma_S} \cdot \frac{W_t}{F_t} \left(\frac{\mu_s - r_f}{\sigma_S} - \rho_{SH} \frac{\mu_h - r_f}{\sigma_h} \right) - \frac{L_t}{F_t} \cdot \frac{\sigma_L}{\sigma_S} \frac{\rho_{SL} - \rho_{SH}\rho_{HL}}{1-\rho_{SH}^2}$$

$$\pi_{h,t+1} = \frac{1}{\gamma(1-\rho_{SH}^2)\sigma_H} \cdot \frac{W_t}{F_t} \left(\frac{\mu_h - r_f}{\sigma_h} - \rho_{SH} \frac{\mu_s - r_f}{\sigma_s} \right) - \frac{L_t}{F_t} \cdot \frac{\sigma_L}{\sigma_h} \frac{\rho_{HL} - \rho_{SH}\rho_{SL}}{1-\rho_{SH}^2}$$

$$\pi_{R_f} = (1 - \pi - \pi_h)$$

Model IV

- Retirement income — accumulated financial wealth is repaid back in annuities.
- Reverse mortgages — housing wealth is reinvested for annuities in return of inheriting a house to the payer (no bequest motives)

$$W_{57} = H_{57} + MP$$

- Annuity is equal to: $\frac{W_{57}}{1+\sum_{t=58}^{100} \frac{p_t}{1+r_f}}$
- Welfare calculation — we convert annuity stream into consumption (considering the inflation) and plug into CRRA expected utility function.

Data Structure and Sources I

- Stock rates of return are obtained from Borsa Istanbul BIST30 index:

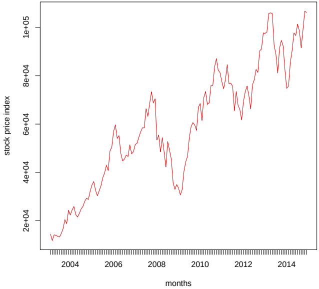


Figure: BIST30 Turkish stock market performance index

Data Structure and Sources II

- Housing returns are obtained from Reidin AEINDEXF index:

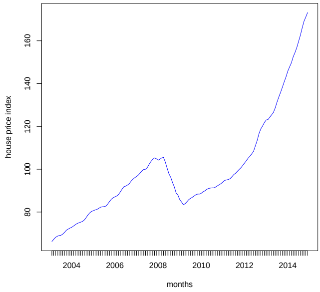


Figure: Reidin Turkish house price index

Data Structure and Sources III

- Wage dynamics are obtained from TUIK Houshold Budget Survey (HBS) and Aktug, Kuzubas, Torul (2017) (notice the hump shape):

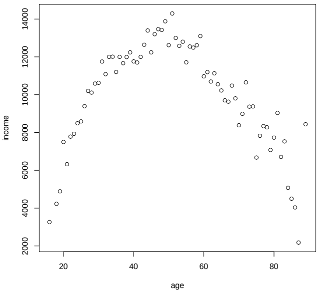


Figure: Median Turkish salaries by age

Data Structure and Sources IV

- We start with 28 years old individual, who invests for 30 years until retirement at 57.
- In line with Torul et al. (2018), $\delta = 0.89$
- Stock returns: 23.2% with standard deviation 36%
- Bond returns obtained from OECD Data Bank (2018) and are equal to 10.8%
- Housing capital appreciation: 11.3% with standard deviation 5.2%

Data Structure and Sources V

Table: Benchmark Parameters

Parameter	Description	Value
Y	Beginning age	28
R	Retirement age	57
T	Lifespan (years)	100
γ	Risk aversion	5
β	Discount rate	0.89
r_f	Risk-free rate	0.108
π	Average inflation rate	0.084
μ_s	Expected stock returns	0.232
μ_h	Expected housing returns	0.113
σ_s	Stock returns volatility	0.36
σ_h	Housing returns volatility	0.052
σ_w	Wage growth volatility	0.056
ρ_{hs}	Housing-stock correlation	0.24
ρ_{hw}	Housing-wage correlation	0.37

Data Structure and Sources VI

- The data on survival probability for all ages is obtained from TUIK database

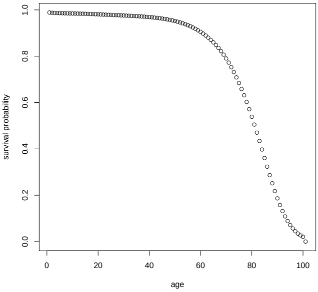


Figure: Survival probabilities by age

Data Structure and Sources VII

- We consider heterogeneity of agents as follows:
- Heterogeneity in education — defined as difference in wage curve steepness

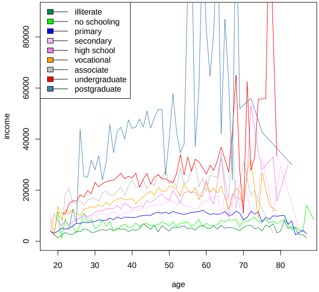


Figure: Lifetime wage dynamics by education level

Data Structure and Sources VIII

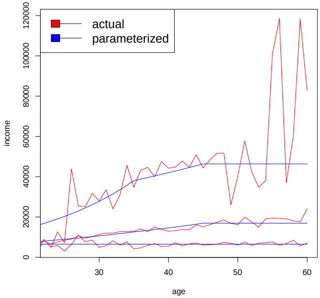
- Performing kinked regressions results in the following three wage growth rates:

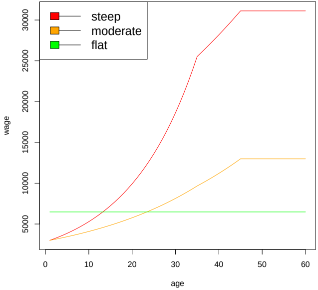
Table: Estimated Benchmark Wage Growth Rates μ_w

Age	Flat	Moderate	Steep
0-35	0%	3.5%	6.5%
36-45	0%	3%	2%
46-60	0%	0%	0%

Data Structure and Sources IX

- Parameterized and actual wage curves





Data Structure and Sources X

- The investments are done from 0.03% of every wage from 28 to 56 years.
- Heterogeneity in sectors of work — it is captured by differing stock-wage correlations
- Zero for agricultural sector / teaching
- As high as 0.4 for financial sector
- 0.2 in the middle
- Notice movements during 2008 crisis

Data Structure and Sources XI

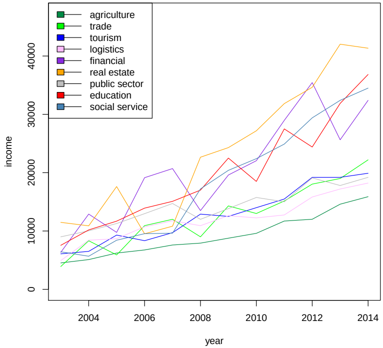


Figure: Historical wage dynamics by sector

Data Structure and Sources XII

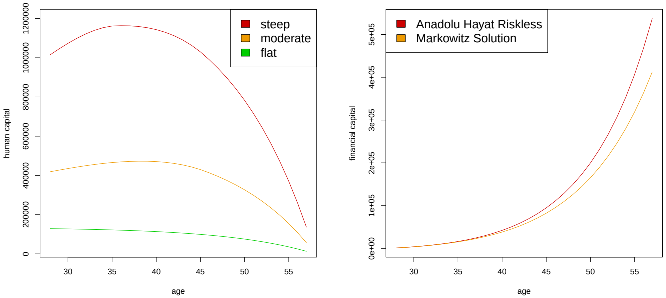
- Individual heterogeneity — it is captured by different risk aversion levels:

Table: Coefficients of Risk Averion

Values	Torul	low	default	high
γ	1.5	3	5	10

Data Structure and Sources XIII

- We constructed human capital and financial capital series taking the heterogeneities into consideration:
- H_t/F_t is declining in t — optimal risky asset share is declining



Results I

- First, we list default and derived investment strategies
- Then we calculate the capital movements using these strategies
- We obtain total wealth before retirement and annuitize it
- We convert annuities into consumption levels considering inflation
- We plug consumption levels into expected utilities
- We compare resulting utilities and conclude

Results II

- Default strategies

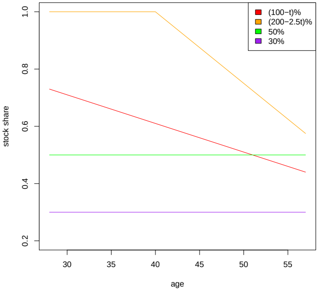
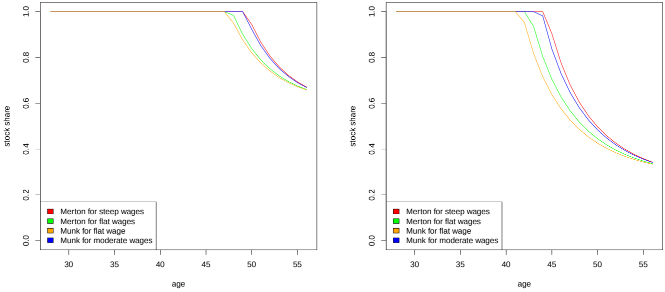


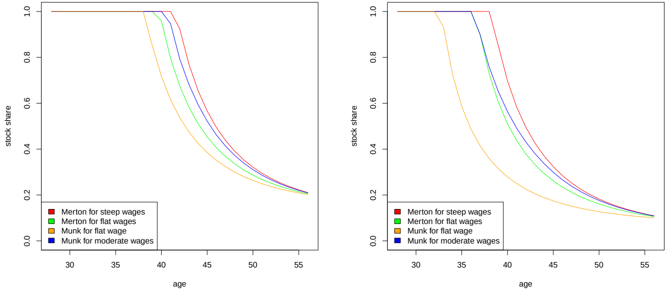
Figure: Default portfolio allocations of stock investments

Results III

- Several individualized solutions ($\gamma = 1.5, 3, 5, 10$):



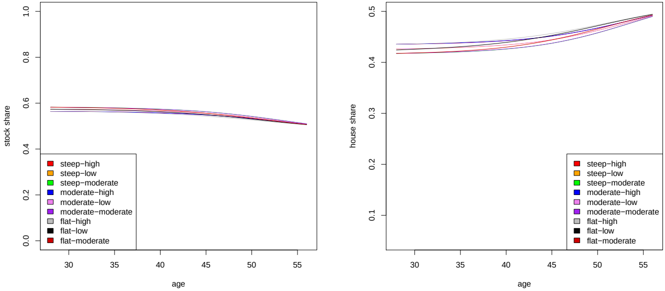
Results IV


stock share
age
Merton for steep wages
Merton for flat wages
Munk for flat wage
Munk for moderate wages
stock share
age
Merton for steep wages
Merton for flat wages
Munk for flat wage
Munk for moderate wages

Results V

- Note that for small stock-wage correlations, Munk's solution without housing is equivalent to Merton's solution
- Note that for smaller risk aversion, households invest more aggressively
- Note that flat wagers are less aggressive than steeper wagers
- Munk's solution with housing are presented below.
- Left graph is optimal stock share and right graph is optimal housing share
- Graphs are done for $\gamma = 1.5, 3, 5, 10$

Results VI


stock share
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0.6
0.4
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0.0
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age
steep−high
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steep−moderate
moderate−high
moderate−low
moderate−moderate
flat−high
flat−low
flat−moderate
house share
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0.2
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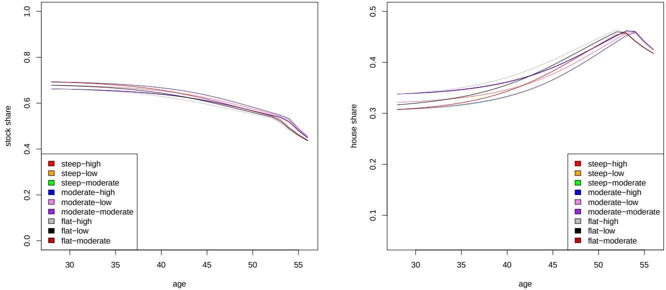
Results VII



stock share
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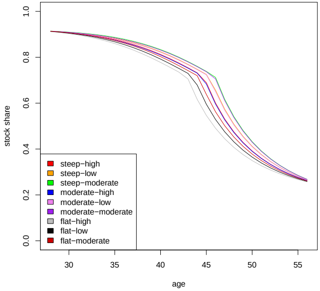
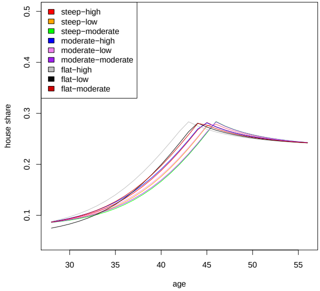
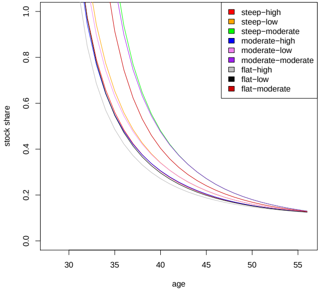
Results VIII





Results IX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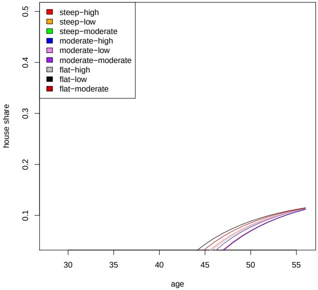
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flat−moderate

Results X

- In line with Munk, the stock-house allocation is done as follows:
  - If optimal stock and housing allocations sum up to a number greater than 1, then we allocate our wealth proportionately between the two
  - If optimal stock and housing allocations sum up to a number less than 1, then we allocate those very shares and invest the rest into risk-free bonds
- The detailed tables with results are presented in Appendix E of our thesis
- Note that around age of 45 this sum falls below 1 and kink happens. Bond share becomes nonnegative.
- Note that as risk aversion coefficeint increases, the kink happens earlier

Results XI

- Note that for $\gamma = 10$ the optimal housing is mostly negative and the sum is less than 1, causing more allocation into risk-free bonds.
- Note that the steeper the wage curve is, the more aggressive the individual is
- Note that stock-wage correlation does not influence steep and flat wagers much

Results XII

- After a lifetime of investing, the household accumulated various levels of wealth, summarized in the Table 5.1 of our thesis
- Looking at these total wealth levels, we can make early conclusions even before calculating utilities:
  - Considering lifecycles, even naively, like $(100 - age)\%$, is better than investing a fixed amount throughout lifetime
  - Different stock-wage correlations don't make much difference in the outcome without housing, and make considerable difference in models with housing
  - Stock-wage correlations are negatively correlated with total wealth
  - The risk aversion is negatively correlated with total wealth for models without housing and positively — for models with housing

Results XIII

- Default options are better for people with flat wage growth rate and worse for people with moderate or steep wage curves

Results XIV

- Annuitization — is done using the formula mentioned earlier,
- Consumption levels — are calculated as amounts of consumption baskets that cost exactly 1 CPI_r, which is equal to $100\,TL$ at age 58.
- Consumption baskets increase in price every year by an inflation level $\pi = 8.4\%$
- Expected Utilities — are calculated by plugging in survival probabilities, discount rate and corresponding risk aversion defined earlier and consumption levels just defined.
- Expected utilities are summarized in Table 5.3 of our thesis.

Conclusion I

- Naive lifecycle investment portfolios, such as $(100 - age)\%$ don't overperform fixed-ratio Markowitz, because they don't take the risk aversion into consideration.
- Cocco et al.'s $(200 - 2.5 \cdot age)\%$ approximation is the best default portfolio. It is easy to interpret and captures lifecycle effect.
- All models perform better for higher risk aversion and worse for lower risk aversion.
- Higher stock-wage correlation considerably decreases the utility for moderate and flat wages, and doesn't affect much for steep wages.
- Merton's solution outperforms Munk's solution without housing for low levels of risk aversion, and performs save for high level of risk aversion ($\gamma = 10$).

Conclusion II

- Munk's solution with housing outperforms every other solution for high levels of risk aversion ($\gamma = 10$).
- Munk's solution with housing outperforms Munk's solution without housing for $\gamma = 5, 10$.
- Markowitz's solution outperforms both Merton's and Munk's solutions for flat wages.
- Individualizing lifecycles by wage growth rate and stock-wage correlation increases welfare for steep wagers and decreases welfare for flat wagers.