WELFARE EFFECTS OF INDIVIDUALIZING LIFE-CYCLE PENSION INVESTMENTS TO HOUSEHOLDS IN TURKEY

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Welfare Effects of Individualizing Life-Cycle Pension Investments

to Households in Turkey

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May 10, 2018

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- this is a true copy of the thesis approved by my advisor and thesis committee at Boğaziçi University, including final revisions required by them.

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ABSTRACT

Welfare Effects of Individualizing Life-Cycle Pension Investments
to Households in Turkey

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ÖZET

Kişiselleştirilmiş Emeklilik Yatırımının Türkiye’deki Hanehalkının Refahına Etkileri

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CHAPTER 1

INTRODUCTION

1.1 Theory and Heuristics of Life-cycle Investments

One of the most important investment decisions individuals face in their lives is investment in retirement portfolio. Since the birth of concept of retirement over a century ago, the various advisors have been ubiquitous. They tried to consult people on best ways to invest their money to afford a good standard of living during their old ages. The field of financial economics, however, started analyzing this type of investment decades later, starting from Markowitz's Modern Portfolio Theory (1950 CHECK THE DATE). Therefore, this field of theoretical economics has been heavily intertwined with empirical findings of non-academic financial consultants.

Very soon financial economists have found inefficiencies in portfolio allocations suggested by finacial advisors (Campbell & and Viceira, 2002) and came up with quantitative solutions that would increase investors' welfare. The rapid adoption of Defined Contribution (DC) pension plans, where individuals choose their own pension investment funds and amounts freely, has made it even easier to adopt portfolio decisions described in formulas by economists. At the same time this shifted the whole responsibility on the individuals, and this opened a room for confusion among non-experts, who then decided to naively allocate 50% of their money to risky assets and the other 50% to riskless assets.

To address this issue, lifecycle investment strategies have been introduced by some institutions. They constituted the predefined percentages of risky and riskless fund investments for all ages until retirement. Such portfolios would be in compliance with theory that younger people should invest more in risky assets because this would increase expected earnings and in case of fault, they will be able to reallocate before getting older, and older people should invest more conservatively in less risky assets because they won't have enough time to recover from potential losses. Such "investment menus" were designed to help laymen make their decisions easier while still complying with complex theory. Alas, Turkish consulting firms have not included easy-to-comprehend lifecycle strategies (investment menus) in their bulletins and didn't try to spread transparent information. We will fill this important gap in our paper, but firstly we will recap the state of Turkish pension system.

1.2 Turkish Pension System

The main pension funds in Turkey have been public for a long time: three main options existed: SSK for public and private sector workers, ES for civil servants, and Bag-Kur for self-employed workers and farmers. In 2006 they all merged into SGK. Private pensions have gained pace recently. As of January, 2017 a new clause of Turkish Labor Law came into action, that automatically enrolled every wage earner younger than 45 years into "Individual Retirement Scheme" - a private pension fund. To further incentivize people not to opt out, the government promised to subsidize 25% of their monthly contributions (as long as this wouldn't exceed 25% of minimal wage). According to PwC research, this has tremendously increased fund sizes. Under the current system individuals may retire after contributing to a pension fund for at least 10 years and at least reaching the age of 56.

The largest retirement funds in Turkey are listed in the table 1.1. All of them offer 3-4 default investment options with varying degrees of riskiness but they are not lifecycle investment strategies mentioned above. They also provide flexible investment options with ability to change portfolio allocation up to six times a year, but they are not very popular as they assume active involvement in their own portfolio and require a certain level of financial literacy. Not much academic research has been done on Turkish pension systems, and the existing research doesn't provide easy solutions. A recent

Table 1.1. Largest Turkish Pension Funds

Fund name	Fund size
Anadolu Hayat Emeklilik	8.7 bln
Garanti Emeklilik ve Hayat	7.4 bln
AvivaSA Emeklilik ve Hayat	9.1 bln
Allianz Yasam ve Emeklilik	6.8 bln
Vakif Emeklili	3.5 bln

Source: Pension Monitoring Center (2016)

example of this is Iscanoglu-Cekic's (2016) paper which doesn't consider life cycles and uses dynamic programming in the solution, which is also not accessible to wide audience.

1.3 Focus of this Thesis

In this thesis we will consider the general framework of lifecycle investments and its historical evolution within the field of financial economics. We will take a look at standard heuristics suggested by financial consultancies and compare their welfare outcomes with those of optimal solutions given by theory. We will use the latest theoretical findings by Munk (2016) and show that optimal solutions can be both efficient and easy to comprehend without use of complex dynamic optimization results.

Next chapter will review all the relevant literature in this field and show the theoretical developments. Chapter 3 will summarize the theoretical framework and model we will use in our simulation. Chapter 4 will explain the data sources and the structure of our simulation. Chapter 5 will present the results of the simulation and Chapter 6 will conclude our findings. The used sources will be listed in References chapter. All the relevant proofs will be available in Appendices.

CHAPTER 2

LITERATURE REVIEW

2.1 Beginnings of financial economics

The financial economics is generally thought to be started with Modern Portfolio Theory (MPT) by Markowitz (1952). He pioneered the mean-variance analysis and was followed by Mutual Funds Separation Theorem of Tobin (1958). The premise of the model was, that if investors care only about the return and the volatility (modelled by mean and variance(or standard deviation) respectively) over a single period, then there is a straight line representing a fixed ratio of risky assets in the optimal portfolio. We summarize their model below.

2.1.1 Mean-variance analysis

Let there be two assets, risky and risk-free with returns R and R_f respectively. Let α be the ratio of total wealth invested in a risky asset. Then the portfolio return is:

$$R_p = \alpha E[R] + (1-\alpha)R_f$$

We want to choose α that maximizes the expected return and minimizes the volatility of the portfolio. Markowitz solves the following unconstrained optimization problem:

$$\max_{\alpha}\{E[R_p] - \frac{\gamma}{2}\sigma_p^2\}$$

where γ is risk-aversion coefficient. The classical solution is:

$$\alpha = \tfrac{E[R]-R_f}{\gamma\sigma^2}$$

This is a crucial result used a lot in industry, academia and MBA courses but most importantly, revived recently by Munk (2016) which we will explore later in this chapter. The issue with this basic model was that it demanded fixed risky asset ratio for everybody and thus could not explain (i) why younger investors take more risks than older ones and (ii) why aggressive investors invest more in stocks than bonds compared to the conservative ones. The model also didn't include loss aversion of people, because although this solution performs best on average in the long run, when it underperforms, it does so in a high way and investors are willing to sacrifice possible gains for loss avoidance.

Following Markowitz, Merton (1970) and Samuelson (1969) introduced a framework to understand long term portfolio investments using changes of investment opportunities during the life. Their result was to repeat the Markowitz's myopic choice in every period. Formally he stated that whenever the relative risk aversion does not depend on wealth, the time horizon is not important for an investor. The Merton solution was not adopted outside academia because it failed to justify the financial rules of thumb like "young should invest more aggresively" and because it used a dynamic programming approach without general closed form solution, which finance analysts found complicated (Campbell and Viceira 2000).

2.2 Advancements in thought

Merton (1971) added labor into the model and found that when the markets are complete and labor income is constant and risk-free, the optimal portfolio choice is:

$$\alpha_t = \frac{\mu - R_f}{\gamma\sigma^2}\left(\frac{W_t + H_t}{W_t}\right)$$

Which meant that adding labor income into the model increased the risky asset ratio in the portfolio choice. The idea of considering labor income was further advanced with the collaboration of Merton and Samuelson with Zvi Bodie in their paper Bodie et al. (1992) where they introduced a notion of human capital to the problem. They formalized the view that labor income is a divident on individual's lifelong human wealth. It is non-tradeable because of moral hazard problem (any future claims of immediate salary for the promise of working for years to come are not enforcable as they constitute some form of slavery). Introducing human capital came as follows: they let individuals to solve two problems simultaneously each period - (a) the decision between consumption and leisure and (b) the decision of allocating portfolio between risky and riskless assets. The framework is that individuals maximize their lifetime utility from consumption and leisure:

$$E_t\left[\int_0^T e^{-\delta s}u(C(s),L(s))ds\right]$$

The risky asset and labor income both follow the Ito's process:

$$\frac{dP}{P} = E_t[R]dt + \sigma dz,$$
$$\frac{dw}{w} = E_t[ROR_w]dt + \sigma^* dz^*$$

For convenience, only special cases of $\sigma^* \in \{0, k\sigma\}$ are considered. Bodie et al. summarized their method in a very accessible way. The timing is in 5 steps:

1. At the beginning of time t the individual calculates the present value of his future earnings and finds its risk characteristics. This value is called a human capital at time t and is denoted by $H(t)$.

2. The total wealth at time t is defined as a sum of human wealth and financial wealth: $W(t) = F(t) + H(t)$.

3. The individual determines the optimal wealth allocated to consumption of a numeraire good and leisure.

4. The individual solves for $\hat{x}(t)$ - the percentage of total wealth $W(t)$ invested in a risky asset.

5. Subtract the wage's implicit exposure to risk from Step 1 from the total amount to be invested in a risky asset $\hat{x}(t) \cdot W(t)$.

The result is that since the optimal risky investment ratio is calculated from the total wealth, the outside observer trained using classical theory, who only sees the financial wealth and not the human wealth, cannot explain why it is rational to invest such a large portion of a financial wealth in a risky asset. According to this model, though, the ratio invested in risky asset is not very high, because it considers the total wealth. This also means that when people get older, their human capital, which is calculated as a present value of the future labor income streams, gradually depletes. Therefore, as an investor gets older, $H(t)$ approaches 0, the total wealth $W(t)$ approaches the financial wealth $F(t)$, which means that older people are advised to invest a smaller percentage of their financial wealth into risky assets than younger people. The paper claimed that this result holds under so-called "normal circumstances" but empirics did not confirm that view because many young people didn't invest in risky assets at all.

Cocco et al. (2005) solve the same problem numerically and simulate the investment processes using calibrated and/or conventional parameters. They introduce the heterogeneity in the model and consider different educational levels, marital status and family sizes of investors. Their results are complementary to Bodie et al. in a sense that they study incomplete markets. These results were complex but a referee of the paper suggested the following simplification (which was then included in the paper):

$$\alpha_t = \begin{cases} 100\% & t < 40 \\ (200 - 2.5t)\% & t \in [40, 60] \\ 50\% & t > 60 \end{cases},$$

where α_t is the investment share in risky assets and t is investor's age.

Flavin and Yamashita (2002) used mean-variance analysis to study how including housing as a separate investment instrument will affect life-cycle investment behavior. They simplified the above model by excluding human capital and risk-free asset from the model. Nevertheless, the results were substantial. They found that due to the large magnitude of housing investment (which is both investment and consumption good) in utility function, when consumers are forced to satisfy particular housing constraint, it exceeds their risk capacity and they tend to not invest in risky assets at all. This explained why young people don't invest in stocks empirically. They also found that introducing housing added individualization into model: similar households would invest different amounts depending on their housing wealth. Ascheberg et al. (2013) found the long-term cointegration among housing, stocks and labor income. They found similar results explaining young people's non-participation in stock markets.

2.3 Reinvention of analytical solution

Munk (2016) showed how simple one-period mean-variance analysis can be expanded to capture all the life-cycle effects mentioned above without any need for dynamic programming and numerical solutions. His model resembles Bodie et al. (1992) but does not use numerical approach. Munk considered a decision between single risk-free asset with return r_f and a vector of risky assets (including housing investment) with return $r \sim (\mu, \Sigma)$. The control variable is π - a vector of shares of total wealth invested in each of risky assets with return r. The Markowitz's optimization problem is transformed to capture dynamics:

$$\max_{\pi}\{E[\frac{W_1}{W_0}] - \frac{\gamma}{2}var(\frac{W_1}{W_0})\}$$

where total wealth is a sum of financial and human wealth: $W_t = F_t + L_t$ and human capital has returns $r_L \sim (\mu_L, \sigma_L)$. The solution is easily obtained from calculus:

$$\pi^* = \frac{1}{\gamma}\frac{W_0}{F_0} \cdot \Sigma^{-1}(\mu - r_f \cdot 1) - \frac{L_0}{F_0} \cdot \Sigma^{-1}cov(r, r_L)$$

The solution captures all the results of the previous papers and is a lot more intuitive. So, for example, if human capital is very correlated with stocks, then the risky asset investment π^* is crowded out. Or, when a person gets old, her human capital depletes and the second term goes to zero and the solution replicates the Merton solution.

2.4 Relevant research

Olear (2016) uses Munk (2016) to study the welfare gains of individualized life-cycle retirement investments as opposed to standardized ones. She uses Ascheberg's human capital, stocks and labor income correlation structure and Cocco et al. (2005)'s labor income process to model the standardized life-cycles and Munk's solution to model the individualized profiles. She finds positive welfare gains when individualized investments are used for retirement on the data for Netherlands.

CHAPTER 3

MODEL

3.1 Labor income process

We model the labor income process as a regression prediction plus aggregate and idiosyncratic shocks for working people and as the percentage λ of the last received wage for the retired. This is summarized in Cocco et al. (2005):

$$\log(Y_{it}) = \begin{cases} f(t, Z_{it}) + v_{it} + \epsilon_{it}, & t \leq T \\ log(\lambda) + f(T, Z_{iT}) + v_{iT}, & t > T \end{cases}$$

where T is the retirement age and $f(t, Z_{it})$ is the log-wage regression outcome for individual i at time t. The error terms are decomposed as:

$$v_{it} = v_{i,t-1} + u_{it},$$
$$u_{it} = \xi_t + \omega_{it}$$

and distributed as:

$$u_i \sim N(0, \sigma_u^2),$$
$$\xi \sim N(0, \sigma_\xi^2),$$
$$\omega_i \sim N(0, \sigma_\omega^2).$$

We use Olear's (2016) approach (Appendix B) to transform the main equation into the following:

$$Y_{i,t+1} = \begin{cases} Y_{it}(1 + g_{i,t+1} + \xi_t + \omega_{it}), & t \leq T \\ \lambda(1 + f(T, Z_{iT}) + v_{iT}), & t > T \end{cases}$$

where $g_{i,t+1} = f(t+1, Z_{i,t+1}) - f(t, Z_{it})$, ξ is the aggregate shock and ω_i is idiosyncratic shock.

3.1.1 Correlations

To derive the above equation, we must construct the aggregate labor income shock ξ. Following the Approach of Ascheberg et al. (2013) we want labor income series to be correlated with both stock series and housing series. To do that we first create three uncorrelated standard normally distributed random series ϵ_{st}, ϵ_{ht}, ϵ_{yt} and multiply them by the Cholesky decomposition Q of the correlation matrix R, i.e. $R = QQ'$, where:

$$R = \begin{bmatrix} 1 & \rho_{sh} & \rho_{sy} \\ \rho_{hs} & 1 & \rho_{hy} \\ \rho_{ys} & \rho_{yh} & 1 \end{bmatrix}$$

Stock, housing and labor income series then take the form of expected rate of return plus the volatility multiplied by the modified error terms. For details please refer to Appendix A:

$$\frac{\Delta S_{t+1}}{S_t} = \mu_s + \sigma_s \cdot \epsilon_{st}$$
$$\frac{\Delta H_{t+1}}{H_t} = \mu_h + \sigma_h \cdot \left(\rho_{hs}\epsilon_{st} + (\sqrt{1-\rho_{hs}^2})\epsilon_{ht}\right)$$
$$\frac{\Delta Y_{t+1}}{Y_t} = \mu_v + \sigma_v \cdot \left(\rho_{ys}\epsilon_{st} + \left(\frac{\rho_{yh}-\rho_{sh}\rho_{sy}}{\sqrt{1-\rho_{sh}^2}}\right)\epsilon_{ht} + \left(\sqrt{1-\rho_{ys}^2 - (\frac{\rho_{yh}-\rho_{sh}\rho_{sy}}{\sqrt{1-\rho_{sh}^2}})^2}\right)\epsilon_{vt}\right)$$

3.2 Welfare measurement

Similarly to Cocco et al. (2005) we use CRRA utility function in our model:

$$E_1[U(c)] = E_1\left[\sum_{t=1}^{T} \delta^{t-1} \prod_{j=0}^{t-1} p_j \cdot \frac{c_{it}^{1-\gamma}}{1-\gamma}\right]$$

where p_k is the probability of survival from time $k-1$ to time k. Note that we omitted the bequest motives from the original formulation, thus retired person consumes all of his income at any given time.

Following Olear (2016) we use certainty equivalent consumptions (CEC) instead of expected utilities to compare the welfare effects between different lifecycle choices. Appendix C shows the calculation of the following formula for CEC:

$$CEC = \left(\frac{E[U(c)] \cdot (1-\gamma)}{\sum_{t=1}^{T} \delta^{t-1} \prod_{j=0}^{t-1} p_j} \right)^{1/(1-\gamma)}$$

3.3 Individualization

To derive the individual lifecycle portfolios for every wealth type we use a special case of Munk (2016) with housing and human capital included. This approach and its solution has been done by Olear (2016). First, note that total wealth at any time consists of financial, housing and human capital: $W_t = F_t + H_t + L_t$. Financial wealth corresponds to actual financial asset holdings at time t, HOUSING HERE EXPLAINED, and human capital corresponds to the present value of all the future wages until retirement. This gives the mean:

$$E[\frac{W_1}{W_0}] = \frac{F_1 + H_1 + L_1}{W_0} = \frac{F_0}{W_0}(1 + r_p) + \frac{H_0}{W_0}(1 + \mu_H) + \frac{L_0}{W_0}(1 + \mu_L)$$

where $r_p = r_f(1-\pi) + \pi \cdot \mu_s$ is a portfolio return, π is share of total wealth invested in stocks, r_f is a rate of return on a risk-free asset (bond) and $\mu_s = E[r]$ is expected stock return. The volatility is given by:

$$var(\frac{W_1}{W_0}) = (\frac{F_0}{W_0})^2 \pi^2 \sigma_s^2 + (\frac{H_0}{W_0})^2 (\sigma_H^2) + (\frac{L_0}{W_0})^2 (\sigma_L^2) + 2 \cdot X$$

where:

$$X = \frac{F_0 L_0}{W_0} \cdot \pi \cdot cov(S, L) + \frac{F_0 H_0}{W_0} \cdot \pi \cdot cov(S, H) + \frac{H_0 L_0}{W_0} \cdot cov(L, H)$$

Substituting this to the mean-variance maximization and solving yields:

$$\pi_{t+1} = \frac{\mu_s - r_f}{\gamma \sigma_s^2} \cdot \frac{W_t}{F_t} - \frac{L_t}{F_t} \cdot \frac{corr(S,Y)}{\sigma_s^2} - \frac{H_t}{F_t} \cdot \frac{corr(S,H)}{\sigma_s^2}$$

3.4 Retirement income

The funds invested in retirement are modeled to be paid back in annuities. Thus, we ignore the case when some share of matured fund can be withdrawn immediately. Further, in order for our individualization analysis (based on house posessions) to have an effect, we also use reverse mortgages proposed. In theory, reverse mortgages pay retired individuals fixed annuity in return for inheriting the house after she dies. This is a plausible tool, because we ignore all bequest motives, and it liquidifies the housing posessions, although it is not yet available for Turkish investors.

So, at the age of 57, the price of owned house is calculated and is added to the matured pension amount (MP) to obtain total wealth:

$$W_{57} = H_{57} + MP$$

All of the W_{57} is used to buy an annuity which will repay an individual $\frac{W_{57}}{1+\sum_{t=58}^{100} \frac{p_t}{1+r_f}}$ annually. The calculation details are available in Appendix D.

CHAPTER 4

DATA AND RESEARCH DESIGN

In our simulation we will compare welfare effects of default lifecycles and individualized lifecycles defined below. The magnitutes and sources of parameters are given in the next section. Default lifecycles are taken from the ones mentioned in our Literature Review chapter and from the real investment strategies of the largest Turkish pension fund provider Anadolu Hayat Emeklilik. Individualized lifecycles will be calculated using Munk's optimal portfolio formula from previous chapter taking idiosyncracies into consideration.

4.1 Data

To measure the stock series we used BIST 30 index which measures the aggregate performance of 30 best companies in Turkey. The monthly data is taken from Borsa Istanbul. We can see from Figure 4.1 the general upward trend with collapse during 2008 crisis.

We used Reidin AEINDEXF index to obtain data on house prices in Istanbul. The historical dynamics of this index are illustrated in Figure 4.2.

We used TUIK's Household Budget Survey Data and regression results of Aktug, Kuzubas, Torul (2017). We have 55 thousand panel data points for 170 households for 2001 to 2014 years. We can observe the hump-shaped lifetime income distribution in Figure 4.3. This corresponds well to an established literature in this field.

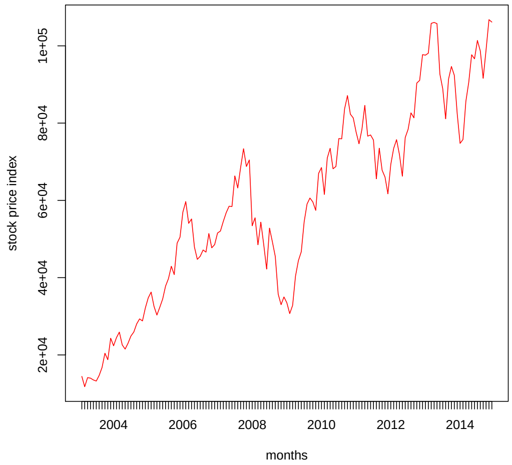

Figure 4.1. BIST30 Turkish stock market performance index

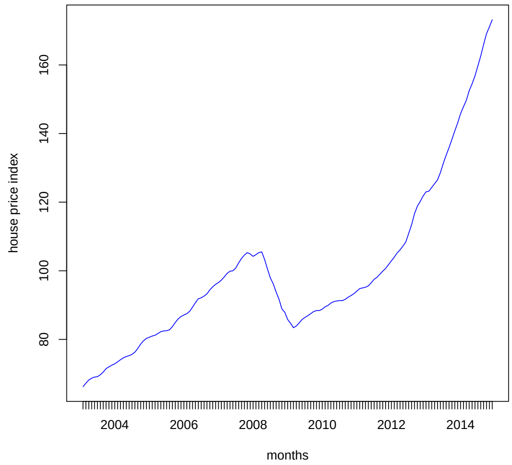

Figure 4.2. Reidin Turkish house price index

4.1.1 Default parameters

In our simulation we start with a 28 years old individual who invests in her retirement for 30 years until she reaches retirement at 57. In line with Torul et al. (2018) we set the coefficient of relative risk aversion for Turkey at 1.5, and the subjective discount rate at 0.89.

Stock returns for Turkey were estimated from historical data to be equal to 23.2% annually, with variance 36%. Risk-free rate forecasts are obtained from OECD Data Bank (2018) and are equal to 10.8% per annum.

Housing capital appreciation averaged at 8.3% with 9.5% variance. Aggregate wage growth series showed 5.5% standard deviation.

The correlations between house and stock prices, and house prices and wages gave 0.24 and 0.37 respectively.

The data on survival probability for all ages has been taken from Turkish Statistical Institute's (TUIK) database and illustrated in Figure 4.4. All of these findings have been summarized in Table 4.1.

Table 4.1. Benchmark Parameters

Parameter	Description	Value
Y	Beginning age	28
R	Retirement age	57
T	Lifespan (years)	100
γ	Risk aversion	1.5
β	Discount rate	0.89
r_f	Risk-free rate	0.108
π	Average inflation rate	0.084
μ_s	Expected stock returns	0.232
μ_h	Expected housing returns	0.083
σ_s	Stock returns volatility	0.36
σ_h	Housing returns volatility	0.095
σ_w	Wage growth volatility	0.056
ρ_{hs}	Housing-stock correlation	0.24
ρ_{hw}	Housing-wage correlation	0.37
p_{28}	Survival probability at age 28	0.977
p_{57}	Survival probability at age 57	0.924
p_{100}	Survival probability at age 100	0

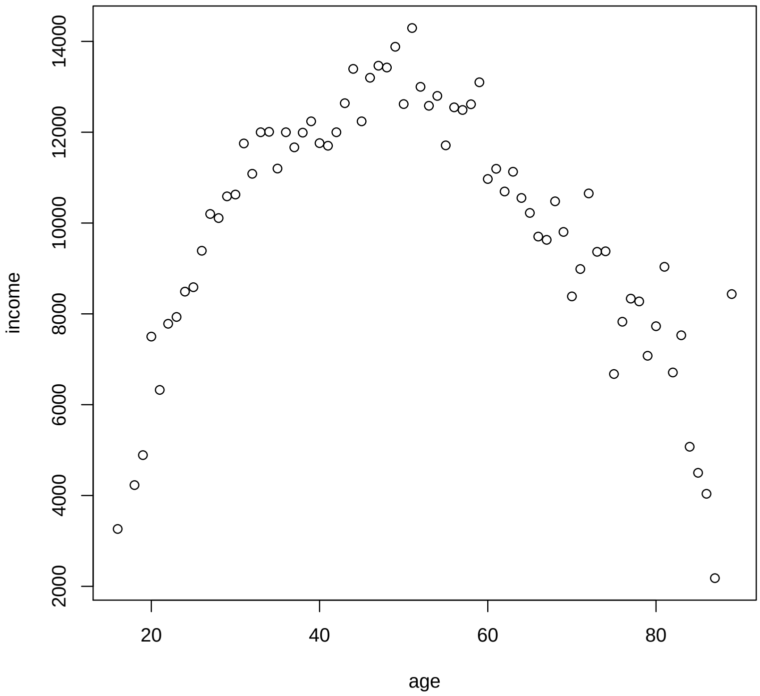

Figure 4.3. Median Turkish salaries by age

4.1.2 Heterogeneity parameters

In the same manner as Olear(2014) we will use wage growth rate, stock-income correlation and idiosyncratic labor income risk to model heterogeneities in education and sector. Let's now consider the heterogeneities one at a time:

Heterogeneity in education

In line with Olear's (2016) approach we model the heterogeneity in education using the differences in wage growth rates. Indeed, we expect the salaries for higher education level to grow faster than for the lower education level. Some of this expectation comes from the fact that people with lower education are restricted in their career ladders and cannot rise very high in a workplace. Another intuition is that while college dropouts start working immediately, college graduates and graduate students continue to study and thus report zero income. When they graduate, their salary immediately rises from zero to the average salary, and this constitutes a steeper wage growth curve

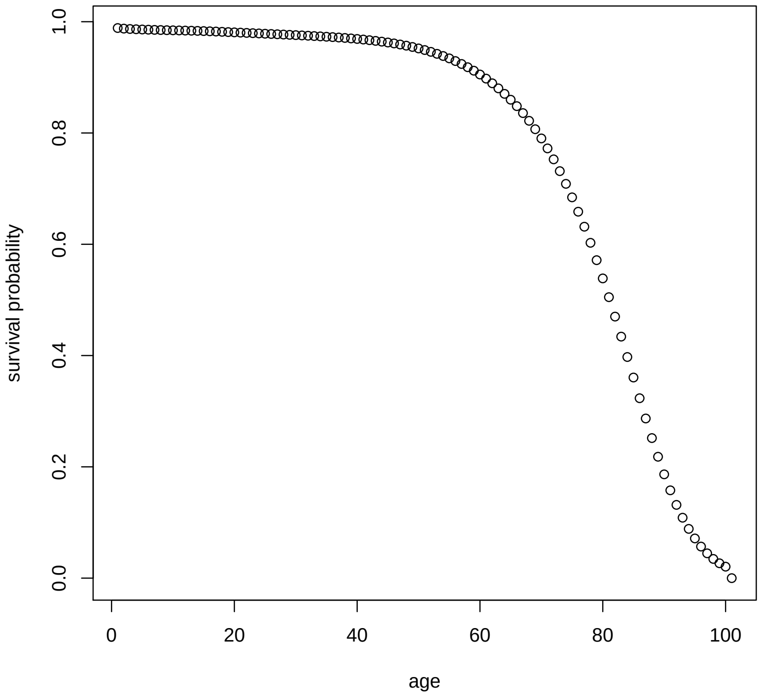

Figure 4.4. Survival probabilities by age

at the beginning of their lives. Figure 4.5 shows the wage series for different levels of education. Note that the curves for the lowest education levels are practically flat and the those for the highest education levels have varying non-zero slopes.

Performing kinked regressions of log wages on ages yielded best linear fits for three different education levels of our choice: postgraduate, high school, and no schooling. The corresponding labor income growth rates can be characterized as steep, moderate, and flat respectively. We considered wage data until 60 years, as Turkish retirement age is at 57, and have added empirically best kinks at ages 35 and 45. The results are summarized in Table 4.2 and illustrated in Figure 4.6. In this figure red lines represent the actual wages dynamics for postgraduate, high school, and no schooling, and blue lines are plots of the percentage changes proposed in Table 4.2, where the starting points all correspond to the actual starting points from the data. Figure 4.7 illustrates the same parameterized curves without actual wage curves.

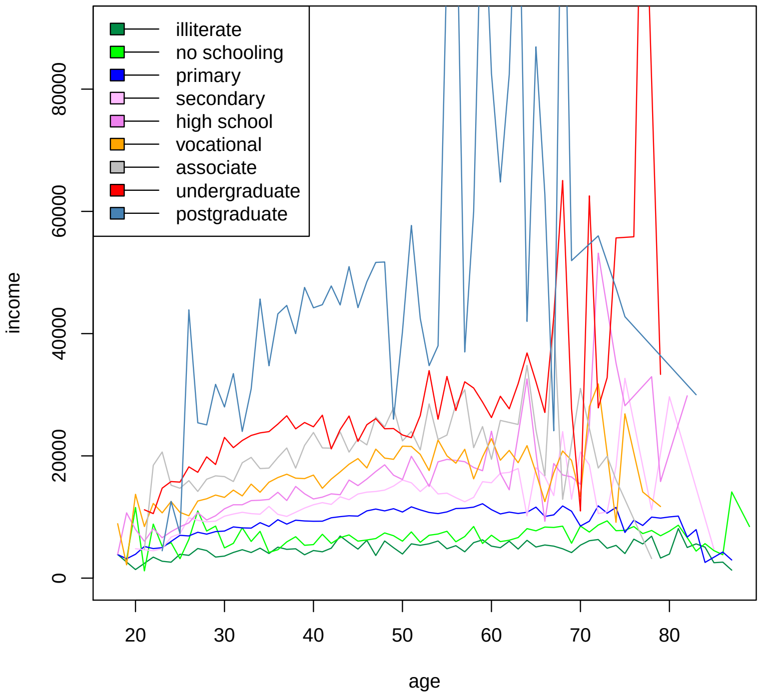

Figure 4.5. Lifetime wage dynamics by education level

Table 4.2. Estimated Benchmark Wage Growth Rates μ_w

Age	Flat	Moderate	Steep
0-35	0%	3.5%	6.5%
36-45	0%	3%	2%
46-60	0%	0%	0%

Heterogeneity in sectors of work

As we explained in Chapter 3, we model our labor income series as functions of age, gender, education, sector of work etc. We already showed in previous section how we incorporated the heterogeneity in education. We also stated at the beginning of this chapter that we consider only male individuals as heads of households; age is implicit in our analysis. In this section we will model the differences in sectors of work. Again, similar to Olear's (2016) approach we model these differences using correlations of wage growth rates with stock market returns. Indeed, it is expected that sectoral wages would be proportional to the stock returns only to the extent of their

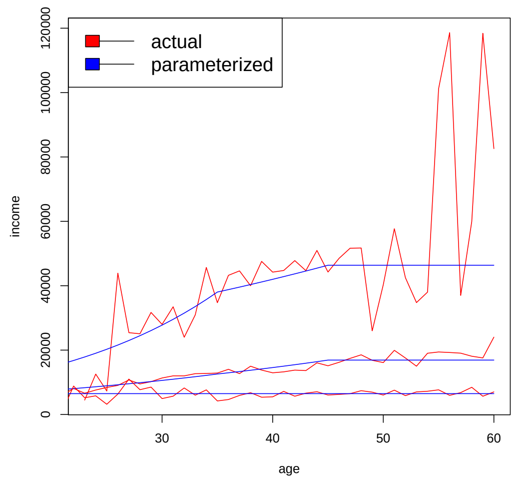

Figure 4.6. Actual and parameterized benchmark wage dynamics by age

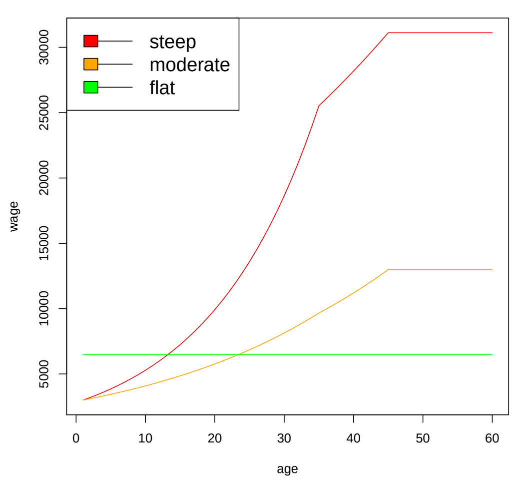

Figure 4.7. Parameterized wage dynamics by age

Table 4.3. Benchmark Wage to Stock Correlations

	Low	Moderate	High
ρ_{sw}	0	0.2	0.4

exposure to stock markets. In that sense, the stock-wage correlations are expected to be zero for public sector, where wages are fixed regardless the markets, and high in financial institutions. When we juxtaposed wages by sector and stock prices we obtained very high correlations, which was expected due to the economic growth over the years represented in Figure 4.8. Therefore we divided the wages by price levels to obtain the real wages, and, as expected, we obtained more realistic correlations. The financial sector's correlations were as high as 0.44 and public sector, social service and education's were as low as 0.08. Thus, our expectations that different sectors have different wage-stock correlation ρ_{sw} were confirmed and we decided on three measures of ρ_{sw} as our simulation benchmarks: 0, 0.2 and 0.4 as stated in Table 4.3.

Individual heterogeneity

We model individual heterogeneity using idiosyncratic labor income shocks σ_ϵ. We consider three different values 0.03,0.05 and 0.07. Total variance is calculated, as

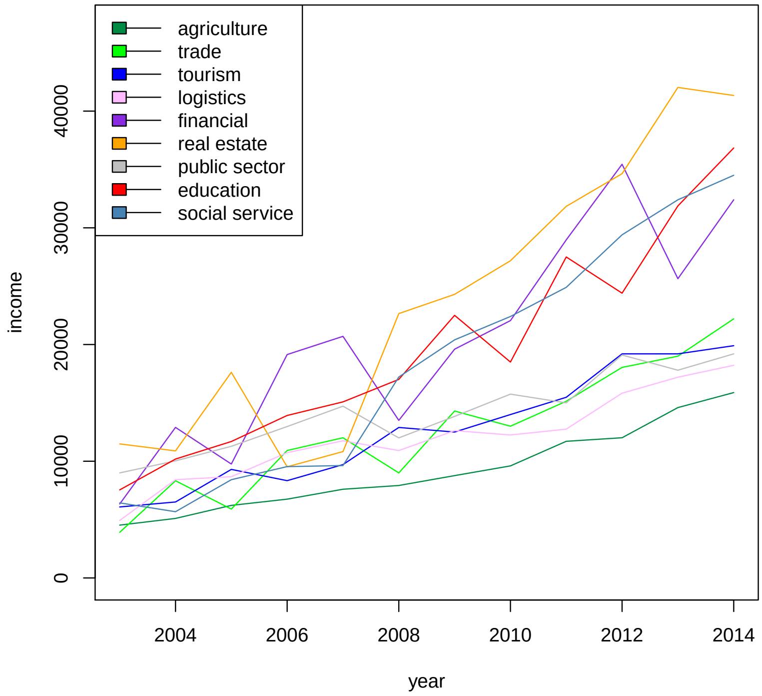

Figure 4.8. Historical wage dynamics by sector

Table 4.4. Benchmark Wage Volatilities and Their Sum of Squares

	Low	Moderate	High
σ_w	0.056	0.056	0.056
σ_ϵ	0.03	0.05	0.07
σ_W^2	0.004	0.0056	0.008

stated in previous chapter, as a sum of squares of aggregate and idiosyncratic shocks $\sigma_W^2 = \sigma_w^2 + \sigma_\epsilon^2$. This heterogeneity is summarized in Table 4.4.

Housing wealth heterogeneity

Apart from the above three heterogeneities, all related to labor income process, we also consider differences in housing wealth to match Munk's (2016) approach completely. We have SOMETHING ABOUT H ZERO ET CETERA...

4.2 Capital series

4.2.1 Human capital

Human capital at all ages has been calculated as a discounted sum of all future wages until retirement with the discount factor r_f. To construct the individualized capital we used steep, moderate and flat wage series mentioned in the previous section. It can be seen in Figure 4.9 that human capital is declining for flat wages and hump-shaped for moderate and steep wages.

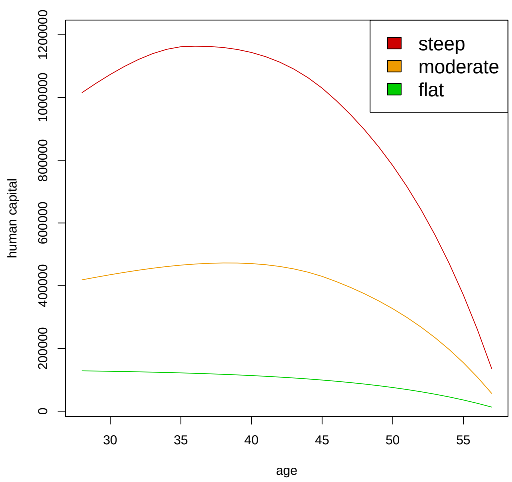

Figure 4.9. Human capital by age

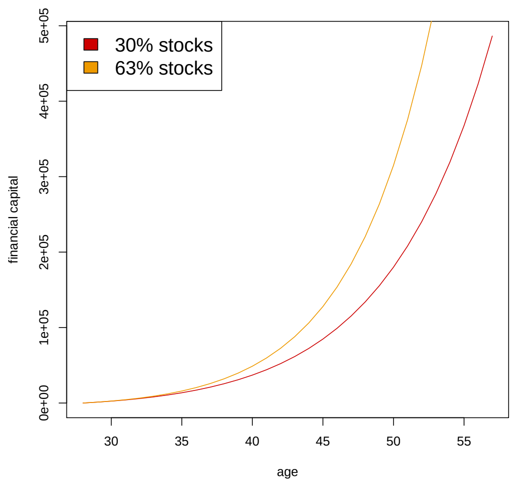

Figure 4.10. Financial capital by age

4.2.2 Financial capital

Financial capital evolves according to dynamic investment illustrated in Figure 4.11. Every period t a certain percentage c (equal to 3% in Turkey) of that period's wage w_t is invested in a retirement portfolio. At the same time, the previously invested amount accrues interest. We started with 28 years old individual who invests for 30 years until retirement, as has been mentioned previously numerous times.

Note that the shape of the financial capital depends on r_p - the rate of return on portfolio, which itself depends on the risky-to-riskless asset ratio. Different such ratios are listed and analyzed in detail in the next section. Figure 4.10 demonstrates the

$$
\begin{array}{lcccc}
 & t_0 & t_1 & t_2 & \\
 & \vdash\!\!\!\longrightarrow & & & \longrightarrow \\
\mathit{Harvest}: & 0 & cw_0(1+r_p) & [cw_0(1+r_p)+cw_1]\cdot(1+r_p) & \\
\mathit{Invest}: & cw_0 & +cw_1 & +cw_2 &
\end{array}
$$

Figure 4.11. Law of motion of financial capital

evolution of financial capital for two investment options, provided by Anadolu Hayat Emeklilik - 30% in stocks and 70% in bonds, and by solving Markowitz's formula - 63% in stocks and 37% in bonds.

It is important to notice here that since human capital is declining by age and financial capital is increasing, the ratio $\frac{L_t}{F_t}$ is declining in t. Recalling Merton's formula for α_t from Chapter 2, $\frac{\mu - R_f}{\gamma\sigma^2}(1 + \frac{L_t}{F_t})$, it should be now clear how the above fraction creates a lifecycle effect - different ratios for different ages.

4.2.3 Housing capital

4.3 Investment strategies

4.3.1 Default lifecyccles

As was discussed in the previous chapter, our individual decides between investing in risky (stocks) or risk-free (bonds) assets. The default allocations for share of risky asset are given by:

- $100 - t$, for all t
- $\begin{cases} 100\%, & t < 40 \\ (200 - 2.5t)\%, & t \in [40, 57] \end{cases}$
- 63%, for all t
- 30%, for all t

where the latter two are Markowitz's solution and Anadolu Hayat's moderate investment option respectively. Since we are only interested in age span between 28 and 57, Figure 4.9 shows the risky asset share π_t only for that interval.

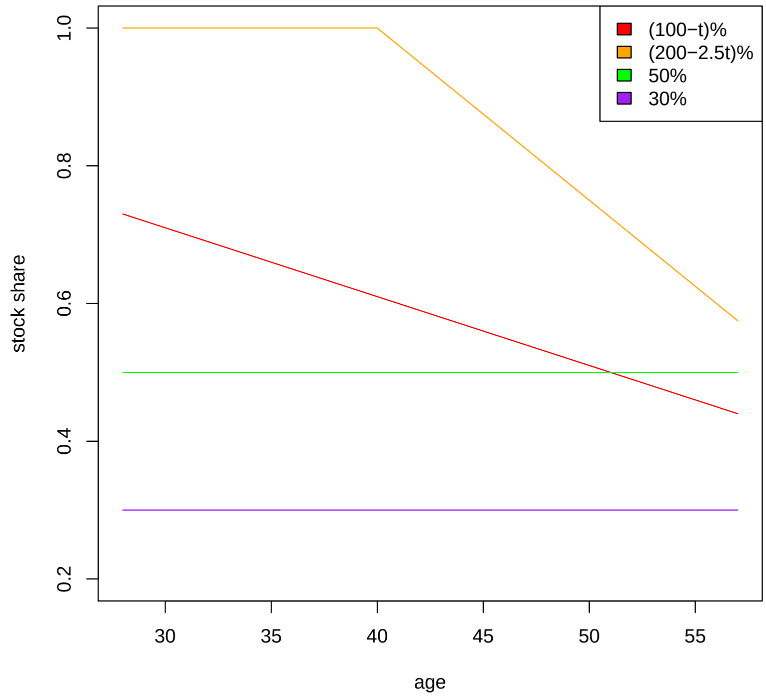

Figure 4.12. Default portfolio allocations of stock investments

4.3.2 Individualized lifecycles

To derive individualized lifecycle strategies, we used Merton's (1971) and Munk's (2016) optimal portfolio allocation formulas mentioned in chapters 2 and 3. Since these formulas depended on intratemporal amounts of capital, we have constructed three human capital series corresponding to flat, moderate and steep wage growth curves mentioned in the previous section. Figure 4.10 illustrates the risky asset shares given by Merton and Munk.

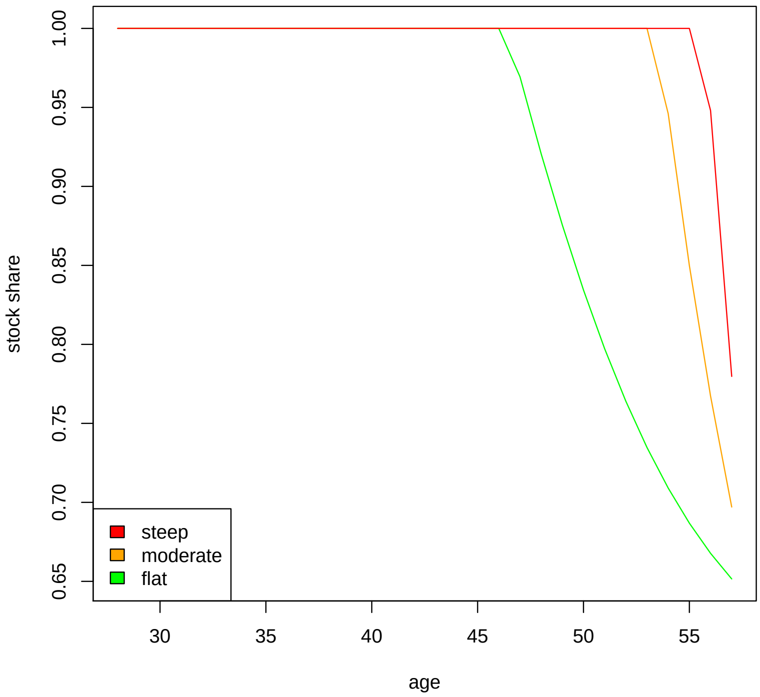

Figure 4.13. Individual portfolio allocations of stock investments

CHAPTER 5

RESULTS

5.1 Welfare comparison

CHAPTER 6

CONCLUSION

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APPENDIX A

CHOLESKY DECOMPOSED ERRORS

In order to create K random series which are correlated exactly like K deterministic series we have in mind, we can multiply independent random variables with the Cholesky decomposed part of the deterministic series. To illustrate this, let Σ be a correlation matrix of matrix X consisting of variables $x_1, x_2, ..., x_K$. Obviously the matrix is symmetric and the diagonal consists of 1s. Let $\Sigma = LL'$ be a Cholesky decomposition of this matrix. Now, let Ω be a vector of K independent random variables $\epsilon_1, \epsilon_2, ..., \epsilon_K$ with variance 1. Consequently, the variance-covariance matrix of Ω is an identity matrix. Then we claim that the product $L\Omega$ has the same correlation structure as X. The proof is below:

$$cov(L\Omega) = E[(L\Omega)(L\Omega)'] = E[L\Omega\Omega' L'],$$

$$cov(L\Omega) = L \cdot E[\Omega\Omega'] \cdot L' = L \cdot var(\Omega) \cdot L',$$

$$cov(L\Omega) = L \cdot I \cdot L' = LL' = \Sigma$$

Now consider the correlation matrix:

$$R = \begin{bmatrix} 1 & \rho_{sh} & \rho_{sy} \\ \rho_{hs} & 1 & \rho_{hy} \\ \rho_{ys} & \rho_{yh} & 1 \end{bmatrix}$$

Basic algebra gives its Cholesky decomposition as:

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ \rho_{hs} & \sqrt{1-\rho_{hs}^2} & 0 \\ \rho_{ys} & \frac{\rho_{yh}-\rho_{sh}\rho_{sy}}{\sqrt{1-\rho_{sh}^2}} & \sqrt{1-\rho_{ys}^2-(\frac{\rho_{yh}-\rho_{sh}\rho_{sy}}{\sqrt{1-\rho_{sh}^2}})^2} \end{bmatrix}$$

APPENDIX B

LOGARITHMS AS PERCENTAGE CHANGE

The first order Taylor series approximation of $\log(x)$ around 1 gives $\log(x) \approx x - 1$. Thus, for any x_t, x_{t+1} such that $\frac{x_{t+1}}{x_t}$ is close to 1, we have:

$$\log(x_{t+1}) - \log(x_t) = \log(\tfrac{x_{t+1}}{x_t}) \approx \tfrac{x_{t+1}}{x_t} - 1,$$

which is a percentage change in x_t between time periods t and $t + 1$.

APPENDIX C

DERIVATION OF CEC

Certainty equivalent consumption is a level of consumption which corresponds to deterministic utility which is exactly equal to the expected utility. We have:

$$U(CEC) = E[U(c)]$$

Substituting the formulas gives:

$$\sum_{t=1}^{T} \delta^{t-1} \prod_{j=0}^{t-1} p_j \cdot \frac{CEC^{1-\gamma}}{1-\gamma} = E[U(c)]$$

Leaving CEC alone gives the desired outcome:

$$CEC = \left(\frac{E[U(c)] \cdot (1-\gamma)}{\sum_{t=1}^{T} \delta^{t-1} \prod_{j=0}^{t-1} p_j} \right)^{1/(1-\gamma)}$$

APPENDIX D

ANNUITIZATION OF WEALTH

Let W_{57} be a total wealth and let x be a constant amount to be repaid annually. Then, at each age k, a firm will pay x with probability p_k and pay nothing with probability $1 - p_k$, where p_k is the survival probability at age k (assuming $p_{57} = 1$). To calculate the present value of the annuity, every payment will be discounted by riskless bond return rate r_f, resulting in the following discounted sum:

$$PV = x + p_{58} \cdot \frac{x}{1+r_f} + p_{59} \cdot \frac{x}{(1+r_f)^2} + ... + p_{100} \cdot \frac{x}{(1+r_f)^{100-57}}$$

Since the present value of annuity is equal to its price, and all of the total wealth will be used to buy such annuity, we set $W_{57} = PV$. Factoring out x from the right-hand side of the above equation gives the desired formula for annual payment amount:

$$x = \frac{W_{57}}{1 + \sum_{t=58}^{100} \frac{p_t}{1+r_f}}$$