WELFARE EFFECTS OF INDIVIDUALIZING LIFE-CYCLE PENSION INVESTMENTS TO HOUSEHOLDS IN TURKEY

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Welfare Effects of Individualizing Life-Cycle Pension Investments

to Households in Turkey

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June 2, 2018

DECLARATION OF ORIGINALITY

I, Ravshanbek Khodzhimatov, certify that

- I am the sole author of this thesis and that I have fully acknowledged and documented in my thesis all sources of ideas and words, including digital resources, which have been produced or published by another person or institution;
- this thesis contains no material that has been submitted or accepted for a degree or diploma in any other educational institution;
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ABSTRACT

Welfare Effects of Individualizing Life-Cycle Pension Investments
to Households in Turkey

We review the current state of Turkish pension system and the history and developments of financial economics. We apply Munk's (2016) individualized lifecycle investment model to simulate Turkish retirement process, and compare the welfare effects with default retirement portfolio options provided by retirement funds or suggested by classical portfolio theory. We find that for upper-to-middle class citizens, individualizing portfolios in Munk's sense results in considerable welfare increases.

ÖZET

Kişiselleştirilmiş Emeklilik Yatırımının Türkiye'deki Hanehalkın Refahına Etkileri

Bu tezde, önce Türkiye'deki emeklilik sistemini ve finansal ekonominin tarihçesini ve gelişmelelerini özetledik. Munk'un (2016) kişiselleştirilmiş yaşamboyu yatırım modelini, Türkiye'deki emeklilik sürecini simule etmeye, ve gerçek emeklilik fonların veya klasik portföy teorisinin tavsiye ettiği yatırım opsiyonlarıyla kıyaslamaya kullandık. Üst ve orta sınıf yatırımcıların, bu kişiselleştirilmiş yatırımları kullandığı takdirde, refah seviyesinin artacağını gösterdik.

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CHAPTER 1

INTRODUCTION

1.1 Theory and heuristics of life-cycle investments

One of the most important investment decisions individuals face in their lives is investment in retirement portfolio. Since the birth of concept of retirement over a century ago, the various advisors have been ubiquitous. They tried to consult people on best ways to invest their money to afford a good standard of living during their old ages. The field of financial economics, however, started analyzing this type of investment decades later, starting from Markowitz's Modern Portfolio Theory (1952). Therefore, this field of theoretical economics has been heavily intertwined with empirical findings of non-academic financial consultants.

In time, financial economists found inefficiencies in portfolio allocations suggested by finacial advisors (Campbell & Viceira, 2002) and came up with quantitative solutions that would increase investors' welfare. The rapid adoption of Defined Contribution (DC) pension plans, where individuals choose their own pension investment funds and amounts freely, has made it even easier to adopt portfolio decisions described in formulas by economists. At the same time this shifted the whole responsibility on the individuals, and this opened a room for confusion among non-experts, who then decided to naively allocate 50% of their money to risky assets and the other 50% to riskless assets.

To address this issue, lifecycle investment strategies have been introduced by some institutions. They constituted the predefined percentages of risky and riskless fund investments for all ages until retirement. Such portfolios would be in compliance with theory that younger people should invest more in risky assets because this would increase expected earnings and in case of fault, they will be able to reallocate before getting older, and older people should invest more conservatively in less risky assets because they won't have enough time to recover from potential losses. Such "investment menus" were designed to help laymen make their decisions easier while still complying with complex theory. Alas, Turkish consulting firms have not included easy-to-comprehend lifecycle strategies (investment menus) in their bulletins and didn't try to spread transparent information. We will fill this important gap in our paper, but firstly we will recap the state of Turkish pension system.

1.2 Turkish pension system

The main pension funds in Turkey have been public for a long time: three main options existed: SSK for public and private sector workers, ES for civil servants, and Bag-Kur for self-employed workers and farmers. In 2006 they all merged into SGK. Private pensions have gained pace recently. As of January, 2017 a new clause of Turkish Labor Law came into action, that automatically enrolled every wage earner younger than 45 years into "Individual Retirement Scheme" — a private pension fund. To further incentivize people not to opt out, the government promised to subsidize 25% of their monthly contributions (as long as this wouldn't exceed 25% of minimal wage). According to PwC research, this has tremendously increased fund sizes. Under the current system individuals may retire after contributing to a pension fund for at least 10 years and at least reaching the age of 56.

The largest retirement funds in Turkey are listed in the table 1.1. All of them offer 3-4 default investment options with varying degrees of riskiness but they are not lifecycle investment strategies mentioned above. They also provide flexible investment options with ability to change portfolio allocation up to six times a year, but they are not very popular as they assume active involvement in their own portfolio and require a certain level of financial literacy. Not much academic research has been done on Turkish pension systems, and the existing research doesn't provide easy solutions. A recent

Table 1.1. Largest Turkish Pension Funds

Fund name	Fund size
Anadolu Hayat Emeklilik	8.7 bln
Garanti Emeklilik ve Hayat	7.4 bln
AvivaSA Emeklilik ve Hayat	9.1 bln
Allianz Yasam ve Emeklilik	6.8 bln
Vakif Emeklili	3.5 bln

Source: Pension Monitoring Center (2016)

example of this is Iscanoglu-Cekic's (2016) paper which doesn't consider life cycles and uses dynamic programming in the solution, which is also not accessible to wide audience.

1.3 Focus of this thesis

In this thesis we will consider the general framework of lifecycle investments and its historical evolution within the field of financial economics. We will take a look at standard heuristics suggested by financial consultancies and compare their welfare outcomes with those of optimal solutions given by theory. We will use the latest theoretical findings by Munk (2016) and show that optimal solutions can be both efficient and easy to comprehend without use of complex dynamic optimization results.

Next chapter will review all the relevant literature in this field and show the theoretical developments. Chapter 3 will summarize the theoretical framework and model we will use in our simulation. Chapter 4 will explain the data sources and the structure of our simulation. Chapter 5 will present the results of the simulation and Chapter 6 will conclude our findings. The used sources will be listed in References chapter. All the relevant proofs will be available in Appendices.

CHAPTER 2

LITERATURE REVIEW

2.1 Beginnings of financial economics

The financial economics is generally thought to be started with Modern Portfolio Theory (MPT) by Markowitz (1952). He pioneered the mean-variance analysis and was followed by Mutual Funds Separation Theorem of Tobin (1958). The premise of the model was, that if investors care only about the return and the volatility (modelled by mean and variance (or standard deviation) respectively) over a single period, then there is a straight line representing a fixed ratio of risky assets in the optimal portfolio. We summarize their model below.

2.1.1 Mean-variance analysis

Let there be two assets, risky and risk-free with returns R and R_f respectively. Let α be the ratio of total wealth invested in a risky asset. Then the portfolio return is:

$$R_p = \alpha E[R] + (1 - \alpha)R_f$$

We want to choose α that maximizes the expected return and minimizes the volatility of the portfolio. Markowitz solves the following unconstrained optimization problem:

$$\max_{\alpha}\{E[R_p] - \frac{\gamma}{2}\sigma_p^2\}$$

where γ is risk-aversion coefficient. The classical solution is:

$$\alpha = \tfrac{E[R]-R_f}{\gamma\sigma^2}$$

This is a crucial result used a lot in industry, academia and MBA courses but most importantly, revived recently by Munk (2016) which we will explore later in this chapter. The issue with this basic model was that it demanded fixed risky asset ratio for everybody and thus could not explain (i) why younger investors take more risks than older ones and (ii) why aggressive investors invest more in stocks than bonds compared to the conservative ones. The model also didn't include loss aversion of people, because although this solution performs best on average in the long run, when it underperforms, it does so in a high way and investors are willing to sacrifice possible gains for loss avoidance.

Following Markowitz, Merton (1970) and Samuelson (1969) introduced a framework to understand long term portfolio investments using changes in investment opportunities during the life. Their result was to repeat the Markowitz's myopic choice in every period. Formally he stated that whenever the relative risk aversion does not depend on wealth, the time horizon is not important for an investor. The Merton solution was not adopted outside academia because it failed to justify the financial rules of thumb like "young should invest more aggresively" and because it used a dynamic programming approach without general closed form solution, which finance analysts found complicated (Campbell and Viceira 2000).

2.2 Advancements in thought

Merton (1971) added labor into the model and found that when the markets are complete and labor income is constant and risk-free, the optimal portfolio choice is:

$$\alpha_t = \frac{\mu - R_f}{\gamma\sigma^2}\left(\frac{W_t + H_t}{W_t}\right)$$

Which meant that adding labor income into the model increased the risky asset ratio in the portfolio choice. The idea of considering labor income was further advanced with the collaboration of Merton and Samuelson with Zvi Bodie in their paper Bodie et al. (1992) where they introduced a notion of human capital to the problem. They formalized the view that labor income is a divident on individual's lifelong human wealth. It is non-tradeable because of moral hazard problem (any future claims of immediate salary for the promise of working for years to come are not enforcable as they constitute some form of slavery). Introducing human capital came as follows: they let individuals solve two problems simultaneously each period — (a) the decision between consumption and leisure and (b) the decision of allocating portfolio between risky and riskless assets. The framework is that individuals maximize their lifetime utility from consumption and leisure:

$$E_t \left[\int_0^T e^{-\delta s} u(C(s), L(s)) ds \right]$$

The risky asset and labor income both follow the Ito's process:

$$\begin{aligned} \tfrac{dP}{P} &= E_t[R]dt + \sigma dz, \\ \tfrac{dw}{w} &= E_t[ROR_w]dt + \sigma^* dz^* \end{aligned}$$

For convenience, only special cases of $\sigma^* \in \{0, k\sigma\}$ are considered. Bodie et al. summarized their method in a very accessible way. The timing is in 5 steps:

1. At the beginning of time t, the individual calculates the present value of his future earnings and finds its risk characteristics. This value is called a human capital at time t and is denoted by $H(t)$.

2. The total wealth at time t is defined as a sum of human wealth and financial wealth: $W(t) = F(t) + H(t)$.

3. The individual determines the optimal wealth allocated to consumption of a numeraire good and leisure.

4. The individual solves for $\hat{x}(t)$ — the percentage of total wealth $W(t)$ invested in a risky asset.

5. Subtract the wage's implicit exposure to risk from Step 1 from the total amount to be invested in a risky asset $\hat{x}(t) \cdot W(t)$.

The result was that since the optimal risky investment ratio was calculated from the total wealth, the outside observer trained using classical theory, who only sees the financial wealth and not the human wealth, could not explain why it is rational to invest such a large portion of a financial wealth in a risky asset. According to this model, though, the ratio invested in risky asset was not very high, because it considered the total wealth. This also meant that when people get older, their human capital, which is calculated as a present value of the future labor income streams, gradually depletes. Therefore, as an investor gets older, $H(t)$ approaches 0, the total wealth $W(t)$ approaches the financial wealth $F(t)$, which means that older people are advised to invest a smaller percentage of their financial wealth into risky assets than younger people. The paper claimed that this result holds under so-called "normal circumstances" but empirics did not confirm that view because many young people didn't invest in risky assets at all.

Cocco et al. (2005) solved the same problem numerically and simulated the investment processes using calibrated and conventional parameters. They introduced the heterogeneity in the model and considered different educational levels, marital status and family sizes of investors. Their results were complementary to Bodie et al. in a sense that they studied incomplete markets. These results were complex but a referee of this paper suggested the following simplification (which was then incorporated in the paper):

$$\alpha_t = \begin{cases} 100\% & t < 40 \\ (200 - 2.5t)\% & t \in [40, 60] \\ 50\% & t > 60 \end{cases},$$

where α_t is the investment share in risky assets and t is investor's age.

Flavin and Yamashita (2002) used mean-variance analysis to study how including housing as a separate investment instrument will affect life-cycle investment behavior. They simplified the above model by excluding human capital and risk-free asset from the model. Nevertheless, the results were substantial. They found that due to the large magnitude of housing investment (which is both investment and consumption good) in utility function, when consumers are forced to satisfy particular housing constraint, it exceeds their risk capacity and they tend to not invest in risky assets at all. This explained why young people don't invest in stocks empirically. They also found that introducing housing added individualization into model: similar households would invest different amounts depending on their housing wealth. Ascheberg et al. (2013) found the long-term cointegration among housing, stocks and labor income. They found similar results explaining young people's non-participation in stock markets.

2.3 Reinvention of analytical solution

Munk (2016) showed how simple one-period mean-variance analysis can be expanded to capture all the life-cycle effects mentioned above without any need for dynamic programming and numerical solutions. His model resembled Bodie et al. (1992) but did not use numerical approach. Munk considered a decision between single risk-free asset with return r_f and a vector of risky assets (including housing investment) with return $r \sim (\mu, \Sigma)$. The control variable was π — a vector of shares of total wealth invested in each of risky assets with return r. Munk transformed Markowitz's optimization problem to capture dynamics:

$$\max_{\pi}\{E[\frac{W_1}{W_0}] - \frac{\gamma}{2}var(\frac{W_1}{W_0})\}$$

where total wealth is a sum of financial and human wealth: $W_t = F_t + L_t$ and human capital has returns $r_L \sim (\mu_L, \sigma_L)$. Munk derived the following solution (which can be obtained from calculus):

$$\pi^* = \tfrac{1}{\gamma}\tfrac{W_0}{F_0} \cdot \Sigma^{-1}(\mu - r_f \cdot 1) - \tfrac{L_0}{F_0} \cdot \Sigma^{-1}cov(r, r_L)$$

The solution captured all the results of the previous papers and was a lot more intuitive. So, for example, if human capital is very correlated with stocks, then the risky asset investment π^*_{risky} is crowded out. Or, when a person gets old, her human capital depletes and the second term goes to zero and the solution replicates the Merton solution.

It is worth noting that Munk added housing as purely financial investment and not a tool for heterogeneity as other papers below tried to do.

2.4 Relevant research

Olear (2016) used Munk (2016) to study the welfare gains of individualized life-cycle retirement investments as opposed to standardized ones. She used Ascheberg's human capital, stocks and labor income correlation structure and Cocco et al. (2005)'s labor income process to model the standardized life-cycles and Munk's solution to model the individualized profiles. Unlike Munk, she added housing capital as a pre-existing wealth with mortgage loan independent of other financial investments. She found positive welfare gains when individualized investments were used for retirement on the data for Netherlands.

CHAPTER 3

MODEL

3.1 Labor income process

We model the labor income process as a function of individiual characteristics plus aggregate and idiosyncratic shocks for working people, and as the percentage λ of the last received wage for the retired. This is summarized in Cocco et al. (2005) as follows:

$$\log(Y_{it}) = \begin{cases} f(t, Z_{it}) + v_{it} + \epsilon_{it}, & t \leq T \\ log(\lambda) + f(T, Z_{iT}) + v_{iT}, & t > T \end{cases}$$

where T is the retirement age and $f(t, Z_{it})$ is the log-wage regression outcome for individual i at time t. The error terms are decomposed as:

$$v_{it} = v_{i,t-1} + u_{it},$$
$$u_{it} = \xi_t + \omega_{it}$$

and distributed as:

$$u_i \sim N(0, \sigma_u^2),$$
$$\xi \sim N(0, \sigma_\xi^2),$$
$$\omega_i \sim N(0, \sigma_\omega^2).$$

We use Olear's (2016) approach (Appendix B) to transform the main equation into the following:

$$Y_{i,t+1} = \begin{cases} Y_{it}(1 + g_{i,t+1} + \xi_t + \omega_{it}), & t \leq T \\ \lambda(1 + f(T, Z_{iT}) + v_{iT}), & t > T \end{cases}$$

where $g_{i,t+1} = f(t+1, Z_{i,t+1}) - f(t, Z_{it})$, ξ is the aggregate shock and ω_i is idiosyncratic shock.

3.1.1 Correlations

To derive the above equation, we must construct the aggregate labor income shock ξ. Following the Approach of Ascheberg et al. (2013) we want labor income series to be correlated with both stock series and housing series. To do that we first create three uncorrelated standard normally distributed random series ϵ_{st}, ϵ_{ht}, ϵ_{yt} and multiply them by the Cholesky decomposition Q of the correlation matrix R, i.e. $R = QQ'$, where:

$$R = \begin{bmatrix} 1 & \rho_{sh} & \rho_{sy} \\ \rho_{hs} & 1 & \rho_{hy} \\ \rho_{ys} & \rho_{yh} & 1 \end{bmatrix}$$

Stock, housing and labor income series then take the form of expected rate of return plus the volatility multiplied by the modified error terms. For details please refer to Appendix A:

$$\frac{\Delta S_{t+1}}{S_t} = \mu_s + \sigma_s \cdot \epsilon_{st}$$
$$\frac{\Delta H_{t+1}}{H_t} = \mu_h + \sigma_h \cdot \left(\rho_{hs}\epsilon_{st} + (\sqrt{1-\rho_{hs}^2})\epsilon_{ht}\right)$$
$$\frac{\Delta Y_{t+1}}{Y_t} = \mu_v + \sigma_v \cdot \left(\rho_{ys}\epsilon_{st} + \left(\frac{\rho_{yh}-\rho_{sh}\rho_{sy}}{\sqrt{1-\rho_{sh}^2}}\right)\epsilon_{ht} + \left(\sqrt{1-\rho_{ys}^2-(\frac{\rho_{yh}-\rho_{sh}\rho_{sy}}{\sqrt{1-\rho_{sh}^2}})^2}\right)\epsilon_{vt}\right)$$

3.2 Welfare measurement

Similarly to Cocco et al. (2005) we use CRRA utility function in our model:

$$E_1[U(c)] = E_1\left[\sum_{t=1}^{T}\delta^{t-1}\prod_{j=0}^{t-1}p_j \cdot \frac{c_{it}^{1-\gamma}}{1-\gamma}\right]$$

where p_k is the probability of survival from time $k-1$ to time k. Note that we omitted the bequest motives from the original formulation, thus retired person consumes all of his income at any given time.

3.3 Individualization

To derive the individual lifecycle portfolios for every wealth type we use a special case of Munk (2016) with housing investment and human capital included. The similar approach and its solution has been done by Olear (2016), but she included housing capital as a pre-existing wealth bought for an outside mortgage, independent of the investment decision. In this paper, we consider housing as a purely financial investment, in line with Munk (2016). First, note that total wealth at any time consists of financial, housing and human capital: $W_t = F_t + L_t$. Financial wealth corresponds to actual financial and housing asset holdings at time t, and human capital corresponds to the present value of all the future wages until retirement. This gives the mean:

$$E[\frac{W_1}{W_0}] = \frac{F_1+L_1}{W_0} = \frac{F_0}{W_0}(1+r_p) + \frac{L_0}{W_0}(1+\mu_L)$$

where $r_p = r_f(1-\pi-\pi_h)+\pi\cdot\mu_s+\pi_h\cdot\mu_h$ is a portfolio return, π and π_h are shares of total wealth invested in stocks and housing respectively, r_f is a rate of return on a risk-free asset (bond), and $\mu_s = E[r]$ and μ_h are expected stock and housing returns respectively. The volatility is given by:

$$var(\frac{W_1}{W_0}) = (\frac{F_0}{W_0})^2\boldsymbol{\pi}\Sigma\boldsymbol{\pi} + (\frac{L_0}{W_0})^2(\sigma_L^2) + 2\cdot\frac{F_0L_0}{W_0^2}\cdot\pi\cdot cov(\boldsymbol{r},\mu_L)$$

where: $\boldsymbol{\pi} = (\pi,\pi_h)$ and $\boldsymbol{r} = (\mu_s,\mu_h)$. Substituting this into the unconstrained mean-variance maximization and solving yields:

$$\pi_{t+1} = \frac{\mu_s-r_f}{\gamma\sigma_s^2} + \frac{L_t}{F_t}\cdot\left(\frac{\mu_s-r_f}{\gamma\sigma_s^2} - \frac{\rho_{SL}\sigma_L}{\sigma_S}\right)$$

without housing investment, and:

$$\begin{aligned}
\pi_{t+1} &= \frac{1}{\gamma(1-\rho_{SH}^2)\sigma_S}\cdot\frac{W_t}{F_t}\left(\frac{\mu_s-r_f}{\sigma_S} - \rho_{SH}\frac{\mu_h-r_f}{\sigma_h}\right) - \frac{L_t}{F_t}\cdot\frac{\sigma_L}{\sigma_S}\frac{\rho_{SL}-\rho_{SH}\rho_{HL}}{1-\rho_{SH}^2}\\
\pi_{h,t+1} &= \frac{1}{\gamma(1-\rho_{SH}^2)\sigma_H}\cdot\frac{W_t}{F_t}\left(\frac{\mu_h-r_f}{\sigma_h} - \rho_{SH}\frac{\mu_s-r_f}{\sigma_s}\right) - \frac{L_t}{F_t}\cdot\frac{\sigma_L}{\sigma_h}\frac{\rho_{HL}-\rho_{SH}\rho_{SL}}{1-\rho_{SH}^2}
\end{aligned}$$

when housing is considered as a second risky financial investment. In line with Munk (2016), we calculate the risk free asset share as $(1-\pi-\pi_h)$ and only then impose constraints. If any of the above shares is negative, we equate it to zero, and if the sum of remaining shares exceeds 1, we divde all of the shares by their sum to obtain a proportionate asset allocation.

3.4 Retirement income

The funds invested in retirement are modeled to be paid back in annuities. Thus, we ignore the case when some share of matured fund can be withdrawn immediately. Further, in order for our individualization analysis (based on house posessions) to have an effect, we also use reverse mortgages proposed. In theory, reverse mortgages pay retired individuals fixed annuity in return for inheriting the house after she dies. This is a plausible tool, because we ignore all bequest motives, and it liquidifies the housing posessions, although it is not yet available for Turkish investors.

So, at the age of 57, the price of owned house is calculated and is added to the matured pension amount (MP) to obtain total wealth:

$$W_{57} = H_{57} + MP$$

All of the W_{57} is used to buy an annuity which will repay an individual $\frac{W_{57}}{1+\sum_{t=58}^{100} \frac{p_t}{1+r_f}}$ annually. The calculation details are available in Appendix D.

CHAPTER 4

DATA STRUCTURE AND SOURCES

In our simulation we will compare welfare effects of default lifecycles and individualized lifecycles defined below. The magnitutes and sources of parameters are given in the next section. Default lifecycles are taken from the ones mentioned in our Literature Review chapter and from the real investment strategies of the largest Turkish pension fund provider Anadolu Hayat Emeklilik. Individualized lifecycles will be calculated using Munk's optimal portfolio formula from previous chapter taking idiosyncracies into consideration.

4.1 Data

To measure the stock series we used BIST 30 index which measures the aggregate performance of 30 best companies in Turkey. The monthly data is taken from Borsa Istanbul. We can see from Figure 4.1 the general upward trend with collapse during 2008 crisis.

We used Reidin AEINDEXF index to obtain data on house prices in Istanbul. The historical dynamics of this index are illustrated in Figure 4.2.

We used TUIK's Household Budget Survey Data and regression results of Aktug, Kuzubas, Torul (2017). We have 55 thousand panel data points for 170 households for 2001 to 2014 years. We can observe the hump-shaped lifetime income distribution in Figure 4.3. This corresponds well to an established literature in this field.

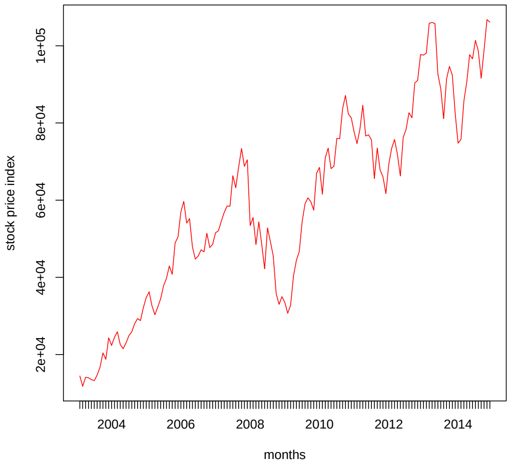


Figure 4.1. BIST30 Turkish stock market performance index

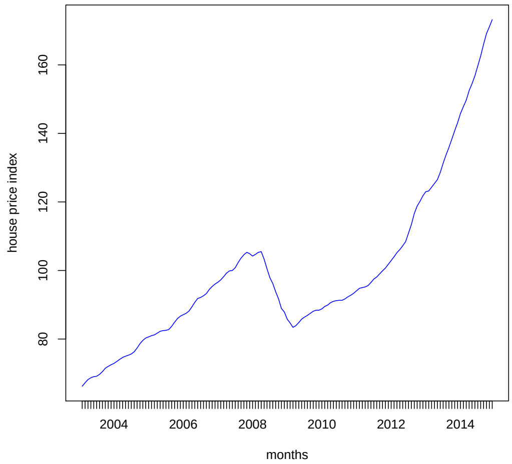


Figure 4.2. Reidin Turkish house price index

4.1.1 Default parameters

In our simulation we start with a 28 years old individual who invests in her retirement for 30 years until she reaches retirement at 57. We set the default coefficient of relative risk aversion at 5, but, in line with Torul et al. (2018), we check the sensitivity for 1.5 and other values. Also, according to Torul et al. (2018), we set the subjective discount rate at 0.89.

Stock returns for Turkey were estimated from historical data to be equal to 23.2% annually, with variance 36%. Risk-free rate forecasts are obtained from OECD Data Bank (2018) and are equal to 10.8% per annum.

Housing capital appreciation averaged at 8.3% with 9.5% standard deviation. But in our simulation we have excluded the temporary housing price drop during 2008 crisis to clear from macroeconomic effects. Therefore, in our simulation, housing capital appreciaton averaged at 11.3% with 5.2% standard deviation.

Aggregate wage growth series showed 5.5% standard deviation.

The correlations between house and stock prices, and house prices and wages gave 0.24 and 0.37 respectively.

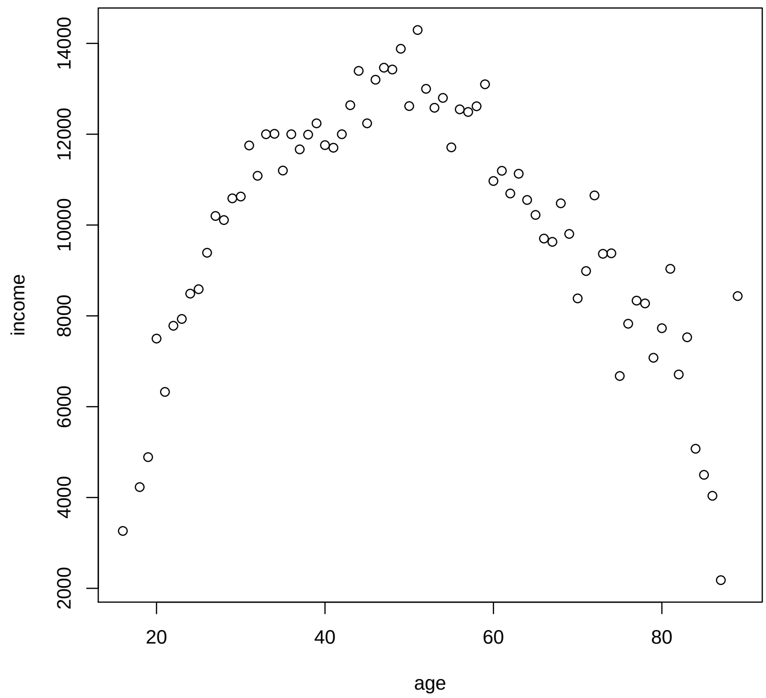


Figure 4.3. Median Turkish salaries by age

The data on survival probability for all ages has been taken from Turkish Statistical Institute's (TUIK) database and illustrated in Figure 4.4. All of these findings have been summarized in Table 4.1.

4.1.2 Heterogeneity parameters

In the same manner as Olear(2014) we will use wage growth rate, stock-income correlation and idiosyncratic labor income risk to model heterogeneities in education and sector. Let's now consider the heterogeneities one at a time:

Heterogeneity in education

In line with Olear's (2016) approach we model the heterogeneity in education using the differences in wage growth rates. Indeed, we expect the salaries for higher education level to grow faster than for the lower education level. Some of this expectation

Table 4.1. Benchmark Parameters

Parameter	Description	Value
Y	Beginning age	28
R	Retirement age	57
T	Lifespan (years)	100
γ	Risk aversion	5
β	Discount rate	0.89
r_f	Risk-free rate	0.108
π	Average inflation rate	0.084
μ_s	Expected stock returns	0.232
μ_h	Expected housing returns	0.113
σ_s	Stock returns volatility	0.36
σ_h	Housing returns volatility	0.052
σ_w	Wage growth volatility	0.056
ρ_{hs}	Housing-stock correlation	0.24
ρ_{hw}	Housing-wage correlation	0.37
p_{28}	Survival probability at age 28	0.977
p_{57}	Survival probability at age 57	0.924
p_{100}	Survival probability at age 100	0

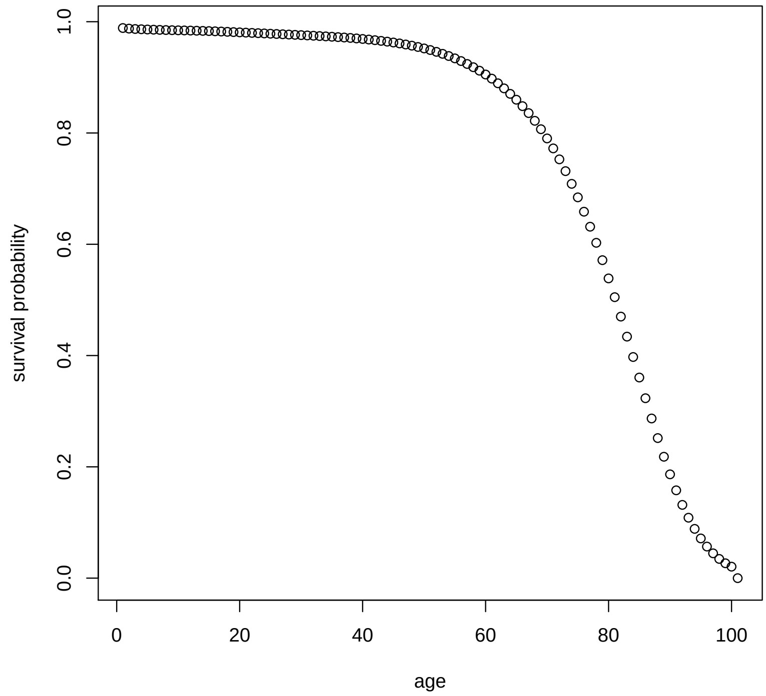


Figure 4.4. Survival probabilities by age

comes from the fact that people with lower education are restricted in their career ladders and cannot rise very high in a workplace. Another intuition is that while college dropouts start working immediately, college graduates and graduate students continue to study and thus report zero income. When they graduate, their salary immediately rises from zero to the average salary, and this constitutes a steeper wage growth curve at the beginning of their lives. Figure 4.5 shows the wage series for different levels of education. Note that the curves for the lowest education levels are practically flat and the those for the highest education levels have varying non-zero slopes.

Performing kinked regressions of log wages on ages yielded best linear fits for three different education levels of our choice: postgraduate, high school, and no schooling. The corresponding labor income growth rates can be characterized as steep, moderate, and flat respectively. We considered wage data until 60 years, as Turkish retirement age is at 57, and have added empirically best kinks at ages 35 and 45. The results are summarized in Table 4.2 and illustrated in Figure 4.6. In this figure red lines represent the actual wages dynamics for postgraduate, high school, and no schooling, and blue

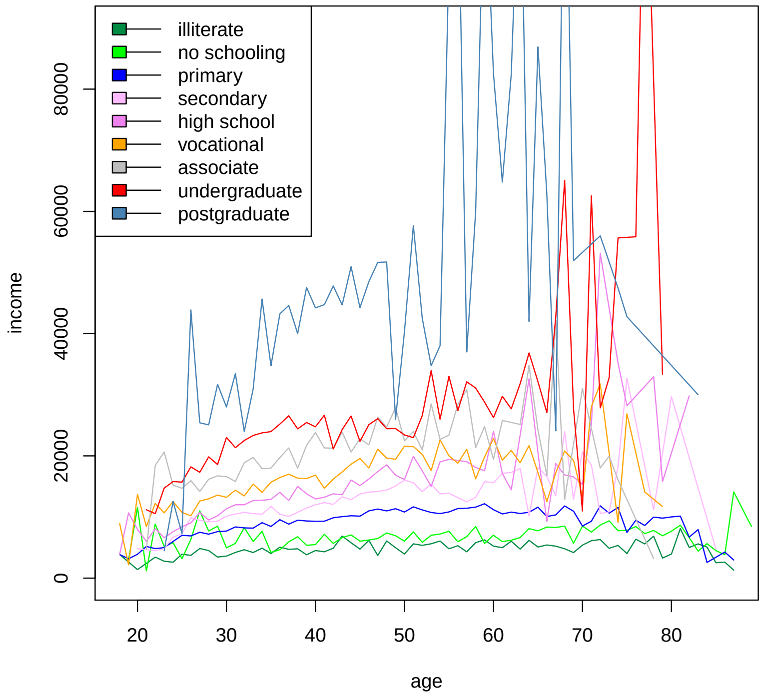


Figure 4.5. Lifetime wage dynamics by education level

Table 4.2. Estimated Benchmark Wage Growth Rates μ_w

Age	Flat	Moderate	Steep
0-35	0%	3.5%	6.5%
36-45	0%	3%	2%
46-60	0%	0%	0%

lines are plots of the percentage changes proposed in Table 4.2, where the starting points all correspond to the actual starting points from the data. Figure 4.7 illustrates the same parameterized curves without actual wage curves.

Heterogeneity in sectors of work

As we explained in Chapter 3, we model our labor income series as functions of age, gender, education, sector of work etc. We already showed in previous section how we incorporated the heterogeneity in education. We also stated at the beginning of this chapter that we consider only male individuals as heads of households; age is

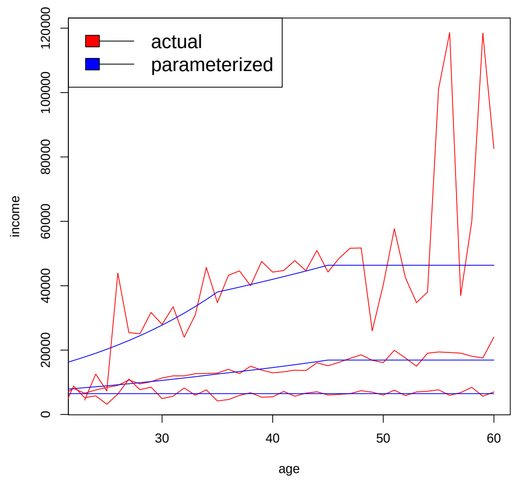


Figure 4.6. Actual and parameterized benchmark wage dynamics by age

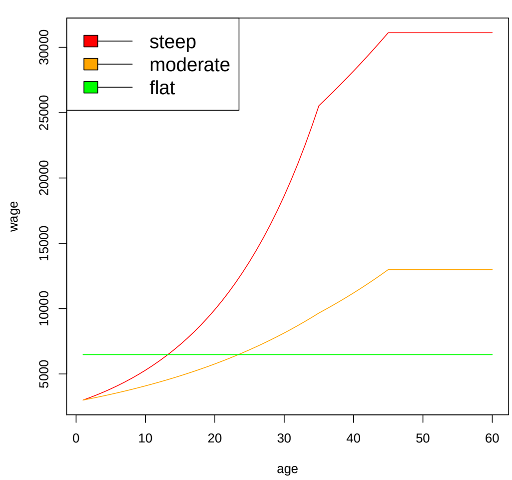


Figure 4.7. Parameterized wage dynamics by age

Table 4.3. Benchmark Wage to Stock Correlations

	Low	Moderate	High
ρ_{sw}	0	0.2	0.4

implicit in our analysis. In this section we will model the differences in sectors of work. Again, similar to Olear's (2016) approach we model these differences using correlations of wage growth rates with stock market returns. Indeed, it is expected that sectoral wages would be proportional to the stock returns only to the extent of their exposure to stock markets. In that sense, the stock-wage correlations are expected to be zero for public sector, where wages are fixed regardless the markets, and high in financial institutions. When we juxtaposed wages by sector and stock prices we obtained very high correlations, which was expected due to the economic growth over the years represented in Figure 4.8. Therefore we divided the wages by price levels to obtain the real wages, and, as expected, we obtained more realistic correlations. The financial sector's correlations were as high as 0.44 and public sector, social service and education's were as low as 0.08. Thus, our expectations that different sectors have different wage-stock correlation ρ_{sw} were confirmed and we decided on three measures of ρ_{sw} as our simulation benchmarks: 0, 0.2 and 0.4 as stated in Table 4.3.

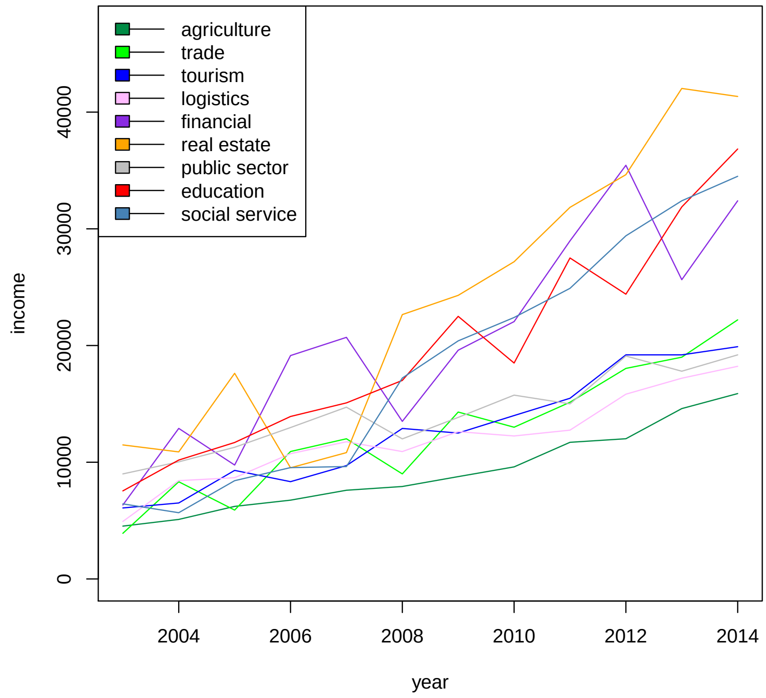


Figure 4.8. Historical wage dynamics by sector

Table 4.4. Coefficients of Risk Averion

Values	Torul	low	default	high
γ	1.5	3	5	10

Individual heterogeneity

We check for individual heterogeneity using a sensitivity analysis on different risk aversion coefficients, summarized in Table 4.4.

4.2 Capital series

4.2.1 Human capital

Human capital at all ages has been calculated as a discounted sum of all future wages until retirement with the discount factor r_f. To construct the individualized capital we used steep, moderate and flat wage series mentioned in the previous section. It can be seen in Figure 4.9 that human capital is declining for flat wages and hump-shaped for moderate and steep wages.

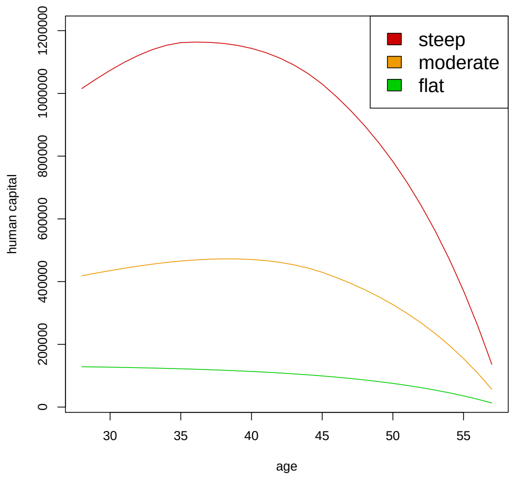


Figure 4.9. Human capital by age

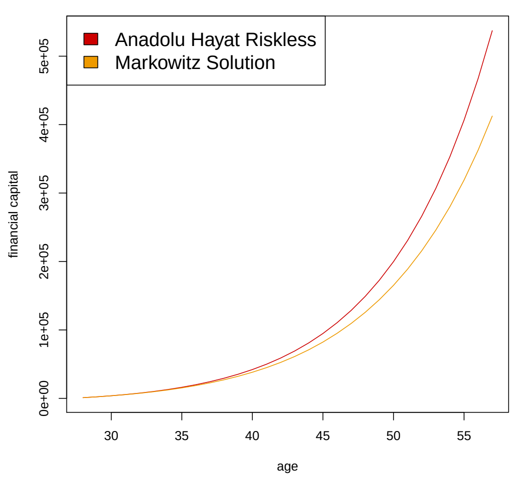


Figure 4.10. Financial capital by age

4.2.2 Financial capital

Financial capital evolves according to dynamic investment illustrated in Figure 4.11. Every period t a certain percentage c (equal to 3% in Turkey) of that period's wage w_t is invested in a retirement portfolio. At the same time, the previously invested amount accrues interest. We started with 28 years old individual who invests for 30 years until retirement, as has been mentioned previously numerous times.

$$
\begin{array}{rccc}
 & t_0 & t_1 & t_2 \\
Harvest: & 0 & cw_0(1+r_p) & [cw_0(1+r_p)+cw_1]\cdot(1+r_p) \\
Invest: & cw_0 & +cw_1 & +cw_2
\end{array}
$$

Figure 4.11. Law of motion of financial capital

Note that the shape of the financial capital depends on r_p — the rate of return on portfolio, which itself depends on the risky-to-riskless asset ratio. Different such ratios are listed and analyzed in detail in the next section. Figure 4.10 demonstrates the evolution of financial capital for two investment options, provided by Anadolu Hayat

Emeklilik — 30% in stocks and 70% in bonds, and by solving Markowitz's formula — 63% in stocks and 37% in bonds.

It is important to notice here that since human capital is declining by age and financial capital is increasing, the ratio $\frac{L_t}{F_t}$ is declining in t. Recalling Merton's formula for α_t from Chapter 2, $\frac{\mu - R_f}{\gamma\sigma^2}(1 + \frac{L_t}{F_t})$, it should be now clear how the above fraction creates a lifecycle effect — different ratios for different ages.

CHAPTER 5

RESULTS

In this chapter we present the results of our simulation. In the first section we will present the default and individualized lifecycle investment strategies, the former being provided by real retirement funds and the latter being solved by Merton and Munk. We will plug in the heterogeneous parameters, described in detail in the previous chapter. In the next section, we will calculate the resulting financial capital flows and compare their welfare effects using their expected utilities, as mentioned in Chapter 3.

5.1 Investment strategies

5.1.1 Default lifecyccles

As was discussed in the previous chapter, our individual decides between investing in risky (stocks) or risk-free (bonds) assets. The default allocations for share of risky asset are given by:

- $100 - t$, for all t
- $\begin{cases} 100\%, & t < 40 \\ (200 - 2.5t)\%, & t \in [40, 57] \end{cases}$
- 63%, for all t
- 30%, for all t

where the latter two are Markowitz's solution and Anadolu Hayat's moderate investment option respectively. Since we are only interested in age span between 28 and 57, Figure 5.1 shows the risky asset share π_t only for that interval.

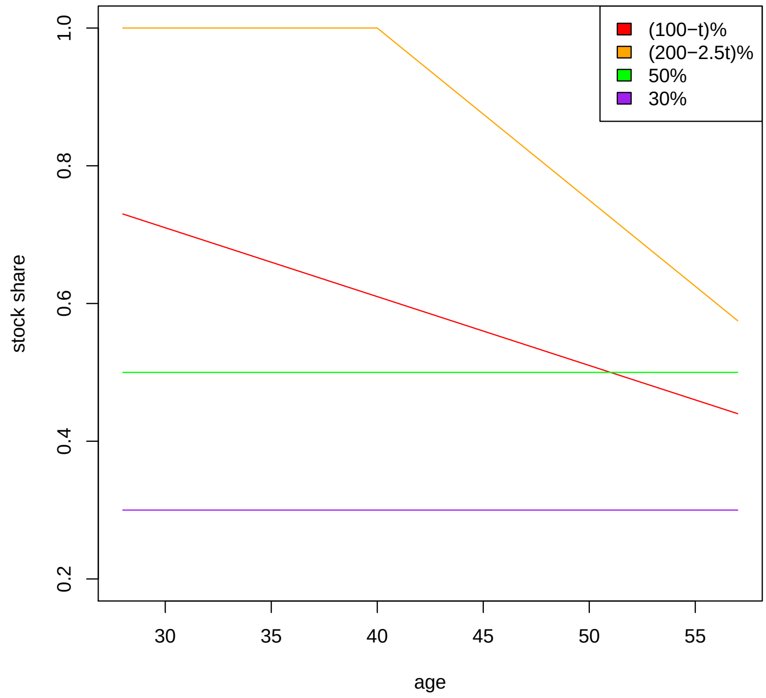
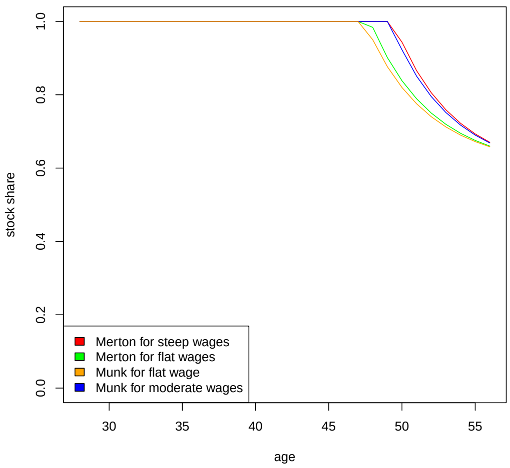


Figure 5.1. Default portfolio allocations of stock investments

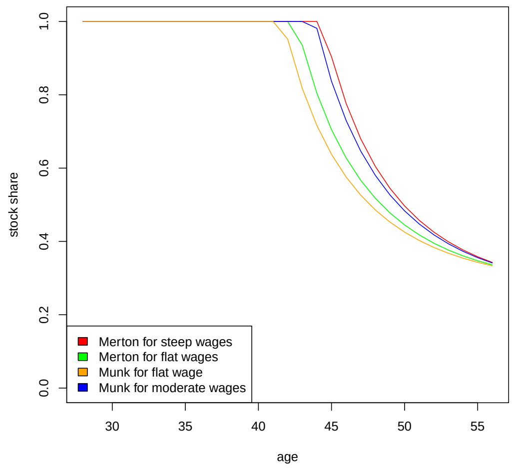
5.1.2 Individualized lifecycles

To derive individualized lifecycle strategies, we used Merton's (1971) and Munk's (2016) optimal portfolio allocation formulas mentioned in chapters 2 and 3. Since these formulas depended on intratemporal amounts of capital, we have constructed three human capital series corresponding to flat, moderate and steep wage growth curves mentioned in the previous chapter. Figure 5.2 illustrates the risky asset shares given by Merton and Munk without housing for various levels of labor income growth and risk aversion. The figure shows that the steeper the wage growth curves get or the lower their risk aversion is, the more aggressive investors are. However, as a whole, they follow the similar pattern. The figure also shows that for small correlations between labor income and stock prices, Munk's solution converges to Merton's solution, as was shown in Chapter 3.

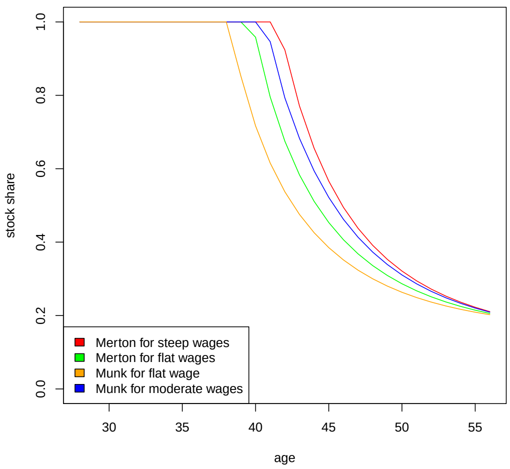
Figure 5.3 illustrates the stock and housing asset shares determined by Munk with housing for flat, moderate and steep labor income curves and low, moderate and high



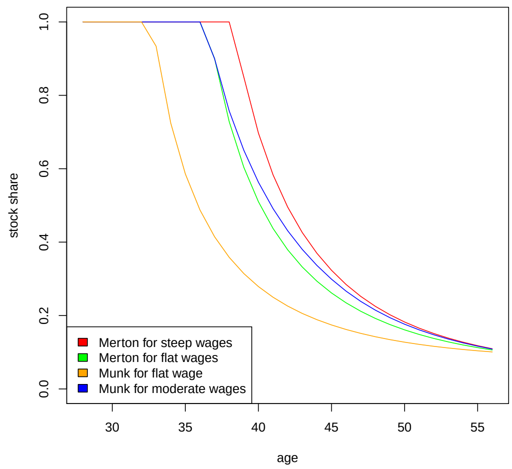
(a) $\gamma = 1.5$



(b) $\gamma = 3$



(c) $\gamma = 5$



(d) $\gamma = 10$

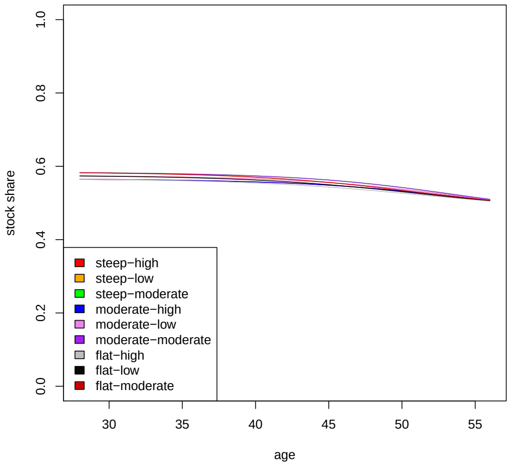
Figure 5.2. Merton and Munk's solution without housing for different wage growth and risk aversion levels

stock-labor income correlations for different levels of risk aversion, to capture heterogeneity, as mentioned in the previous chapter. The figure confirms that steeper labor income curves result in larger share of stocks in portfolio and less of bonds. Around age of 45, the sum of optimal stock and housing investments falls below 1 and optimal bond investment becomes positive. Also, the higher the risk aversion, the less the individuals want to invest in both housing and stocks — for $\gamma = 10$, the optimal Munk solutions are negative or add up to less than 1, meaning that they should sell all stocks and housing to invest in risk-free long term bonds. In our analysis we do not allow for negative investing, since this is not a primary focus of our thesis.

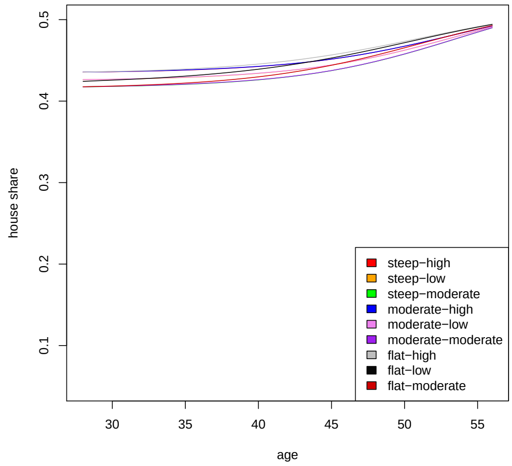
Full tables with actual solutions are available in Tables E.1 and E.2 of Appendix E for models without housing and in Table E.3 for a model with housing.

5.2 Welfare comparison

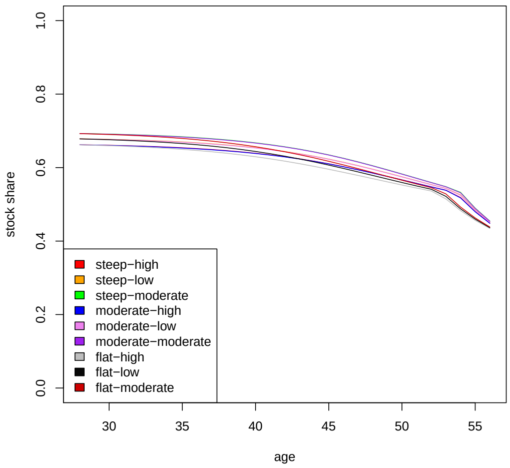
In order to compare welfare we use CRRA expected utility function, mentioned in Chapter 3. The probabilities of survival are taken from TUIK as mentioned in the previous chapter. The consumption is calculated in numbers of consumption baskets which cost exactly 1 CPI — consumer price index at that period, which is increasing with inflation rate annually, and is equal to 100 at retirement age 57. The wealth for which consumption baskets are purchased is a total accumulated wealth, including stock returns, bond returns and housing returns, annuitized according to the formula described in Chapter 3. Discount rate is 0.89, as mentioned in the previous chapter. We compare expected utilities for different levels of risk aversion.



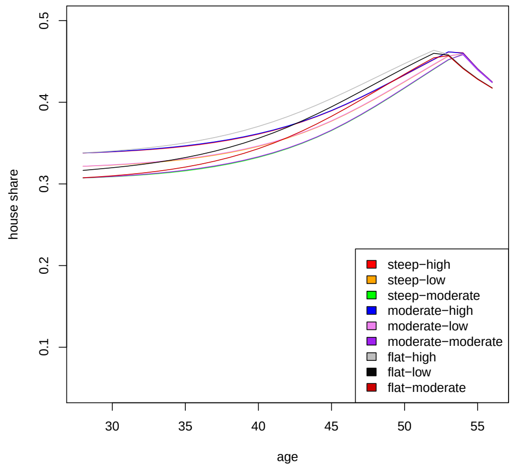
(a) Stocks for $\gamma = 1.5$



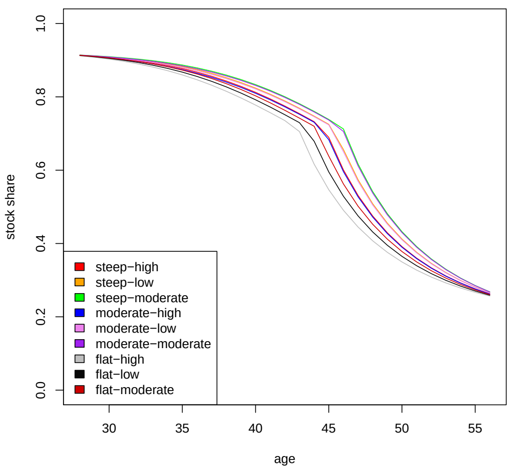
(b) Housing for $\gamma = 1.5$



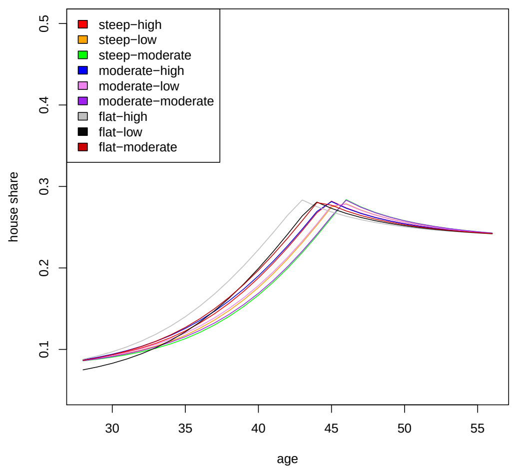
(c) Stocks for $\gamma = 3$



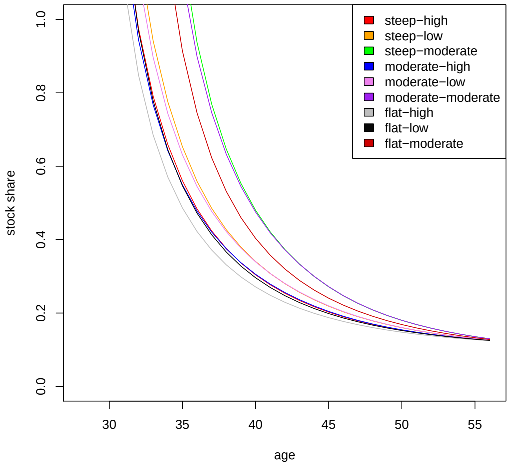
(d) Housing for $\gamma = 3$



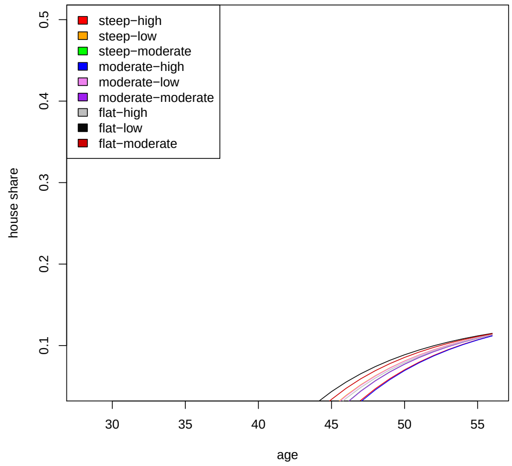
(e) Stocks for $\gamma = 5$



(f) Housing for $\gamma = 5$



(g) Stocks for $\gamma = 10$



(h) Housing for $\gamma = 10$

Figure 5.3. Munk's stock and housing shares for different wage growth, stock-wage correlation and risk aversion levels

5.2.1 Early results

After a lifetime of investing in the above portfolios, the household accumulates different levels of wealth, summarized in Table 5.1. Since the consumption preferences are monotonic, we can make early conclusions even without calculating expected utilities — the higher the accumulated wealth, the better:

- Considering lifecycles, even naively, like $(100 - age)\%$, is better than investing a fixed amount throughout lifetime
- Different stock-wage correlations don't make much difference in the outcome without housing, and make considerable difference in models with housing
- Stock-wage correlations are negatively correlated with total wealth
- The risk aversion is negatively correlated with total wealth for models without housing and positively — for models with housing
- Default options are better for people with flat wage growth rate and worse for people with moderate or steep wage curves

5.2.2 Annuitization

To formalize the observations made in the previous section, we will construct annuities and plug them into expected utility functions. As described in detail in Chapter 3, we define annuities by dividing the total wealth before retirement by the discount factor $1 + \sum_{t=58}^{100} \frac{p_t}{1+r_f}$. Using survival probabilities, obtained from TUIK, and risk-free rate of return, described in the previous chapter, we calculate the discount factor as 205.29. The annuities are obtained by dividing all the values in the Table 5.1 by 205.29. The resulting values are presented in Table 5.2.

Table 5.1. Total Accumulated Wealth by Investment Option

Option	$\gamma = 5$ (default)	$\gamma = 1.5$	$\gamma = 3$	$\gamma = 10$
		Defaults		
Anadolu Hayat (riskless)	537,309 TL	537,309 TL	537,309 TL	537,309 TL
$(100 - age)\%$	1,039,812 TL	1,039,812 TL	1,039,812 TL	1,039,812 TL
Cocco et al.	2,147,019 TL	2,147,019 TL	2,147,019 TL	2,147,019 TL
Markowitz	412,501 TL	1,206,147 TL	550,465 TL	325,428 TL
		Individualized (no housing)		
Merton (steep)	2,938,506 TL	6,078,824 TL	3,986,493 TL	2,023,847 TL
Merton (moderate)	1,242,367 TL	2,583,202 TL	1,689,263 TL	853,121 TL
Merton (flat)	379,516 TL	860,653 TL	529,776 TL	255,587 TL
Munk (steep-hi)	2,613,595 TL	6,021,186 TL	3,783,643 TL	1,443,410 TL
Munk (steep-mod)	2,614,594 TL	6,022,182 TL	3,784,641 TL	1,444,411 TL
Munk (steep-lo)	2,615,592 TL	6,023,178 TL	3,785,638 TL	1,445,411 TL
Munk (moderate-hi)	1,102,873 TL	2,558,560 TL	1,602,748 TL	603,991 TL
Munk (moderate-mod)	1,103,290 TL	2,558,976 TL	1,603,164 TL	604,409 TL
Munk (moderate-lo)	1,103,707 TL	2,559,391 TL	1,603,580 TL	604,826 TL
Munk (flat-hi)	336,116 TL	850,375 TL	501,534 TL	182,719 TL
Munk (flat-mod)	336,212 TL	850,471 TL	501,630 TL	182,816 TL
Munk (flat-lo)	336,308 TL	850,568 TL	501,726 TL	182,912 TL
		Individualized (housing)		
Munk (steep-hi)	2,516,069 TL	1,850,423 TL	2,170,033 TL	3,666,651 TL
Munk (steep-mod)	2,619,683 TL	1,885,502 TL	2,244,519 TL	4,546,216 TL
Munk (steep-lo)	2,713,638 TL	1,919,640 TL	2,314,174 TL	4,050,822 TL
Munk (moderate-hi)	1,061,251 TL	777,423 TL	913,347 TL	1,501,775 TL
Munk (moderate-mod)	1,105,444 TL	792,331 TL	945,023 TL	1,881,273 TL
Munk (moderate-lo)	1,145,103 TL	806,841 TL	974,647 TL	1,686,710 TL
Munk (flat-hi)	328,450 TL	256,110 TL	297,080 TL	426,696 TL
Munk (flat-mod)	340,351 TL	261,520 TL	307,038 TL	540,733 TL
Munk (flat-lo)	355,176 TL	265,574 TL	316,592 TL	479,246 TL

5.2.3 Consumption during retirement

We model that the individuals spend their annuity returns to consume baskets which cost 100 TL during the year, when our agent is 58 years old, and increase by inflation rate $\pi = 8.4\%$. Thus, every period $t > 57$ our agent consumes $\frac{annuity}{100 \cdot (1.084)^{t-58}}$. We do not provide the separate table with the value streams, as they will be implicitly included in utility values.

5.2.4 Expected utilities

Finally, we will plug the consumption streams, calculated in the previous section, into the constant relative risk-aversion expected utility functions to compare the welfare effects. The resulting expected utility values are presented in Table 5.3.

5.2.5 Conclusions

Now, we can observe the final resutls:

- Naive lifecycle investment portfolios, such as $(100 - age)\%$ don't overperform fixed-ratio Markowitz, because they don't take the risk aversion into consideration.
- Cocco et al.'s $(200 - 2.5 \cdot age)\%$ approximation is the best default portfolio. It is easy to interpret and captures lifecycle effect.
- All models perform better for higher risk aversion and worse for lower risk aversion.
- Higher stock-wage correlation considerably decreases the utility for moderate and flat wages, and doesn't affect much for steep wages.
- Merton's solution outperforms Munk's solution without housing for low levels of risk aversion, and performs save for high level of risk aversion ($\gamma = 10$).

Table 5.2. Annual Pensions by Investment Option

Option	$\gamma = 5$ (default)	$\gamma = 1.5$	$\gamma = 3$	$\gamma = 10$
		Defaults		
Anadolu Hayat (riskless)	2,617 TL	2,617 TL	2,617	2,617 TL
$(100 - age)\%$	5,065 TL	5,065 TL	5,065 TL	5,065 TL
Cocco et al.	10,459 TL	10,459 TL	10,459 TL	10,459 TL
Markowitz	2,009 TL	5,875 TL	2,681 TL	1585 TL
		Individualized (no housing)		
Merton (steep)	14,314 TL	29,611 TL	19,419 TL	9,859 TL
Merton (moderate)	6,052 TL	12,583 TL	8,229 TL	4,156 TL
Merton (flat)	1,849 TL	4,192 TL	2,581 TL	1,245 TL
Munk (steep-hi)	12,731 TL	29,330 TL	18,431 TL	7,031 TL
Munk (steep-mod)	12,736 TL	29,335 TL	18,436 TL	7,036 TL
Munk (steep-lo)	12,741 TL	29,340 TL	18,441 TL	7,041 TL
Munk (moderate-hi)	5,372 TL	12,463 TL	7,807 TL	2,942 TL
Munk (moderate-mod)	5,374 TL	12,465 TL	7,809 TL	2,944 TL
Munk (moderate-lo)	5,376 TL	12,467 TL	7,811 TL	2,946 TL
Munk (flat-hi)	1,637 TL	4,142 TL	2,443 TL	890 TL
Munk (flat-mod)	1,638 TL	4,143 TL	2,444 TL	891 TL
Munk (flat-lo)	1,638 TL	4,143 TL	2,444 TL	891 TL
		Individualized (housing)		
Munk (steep-hi)	12,256 TL	9,014 TL	10,571 TL	17,861 TL
Munk (steep-mod)	12,761 TL	9,185 TL	10,933 TL	22,146 TL
Munk (steep-lo)	13,219 TL	9,351 TL	11,273 TL	19,732 TL
Munk (moderate-hi)	5,170 TL	3,787 TL	4,449 TL	7,315 TL
Munk (moderate-mod)	5,385 TL	3,860 TL	4,603 TL	9,164 TL
Munk (moderate-lo)	5,578 TL	3,930 TL	4,748 TL	8,216 TL
Munk (flat-hi)	1,600 TL	1,248 TL	1,447 TL	2,079 TL
Munk (flat-mod)	1,658 TL	1,274 TL	1,496 TL	2,634 TL
Munk (flat-lo)	1,730 TL	1,294 TL	1,542 TL	2,335 TL

- Munk's solution with housing outperforms every other solution for high levels of risk aversion ($\gamma = 10$).
- Munk's solution with housing outperforms Munk's solution without housing for $\gamma = 5, 10$.
- Markowitz's solution outperforms both Merton's and Munk's solutions for flat wages.
- Individualizing lifecycles by wage growth rate and stock-wage correlation increases welfare for steep wagers and decreases welfare for flat wagers.

We did not provide numerical conclusions, since they are trivial — they can be obtained by calculating percentage differences in Table 5.3.

Table 5.3. Expected Utilities by Investment Option

Option	$\gamma = 5$ (default)	$\gamma = 1.5$	$\gamma = 3$	$\gamma = 10$
		Defaults		
Anadolu Hayat (riskless)	-0.0014323	-3.8493850	-0.0323960	-0.0001696
$(100 - age)\%$	-0.0001021	-2.7671080	-0.0086503	-0.0000004
Cocco et al.	-0.0000056	-1.9256860	-0.0020289	0.0000000
Markowitz	-0.0000564	-2.5692320	-0.0064289	-0.0000001
		Individualized (no housing)		
Merton (steep)	-0.0000016	-1.1444420	-0.0005885	0.0000000
Merton (moderate)	-0.0000501	-1.7555950	-0.0032775	-0.0000026
Merton (flat)	-0.0057545	-3.0415100	-0.0333239	-0.1360504
Munk (steep-hi)	-0.0000026	-1.1499060	-0.0006533	0.0000000
Munk (steep-mod)	-0.0000026	-1.1498110	-0.0006530	0.0000000
Munk (steep-lo)	-0.0000026	-1.1497160	-0.0006526	0.0000000
Munk (moderate-hi)	-0.0000807	-1.7640290	-0.0036409	-0.0000592
Munk (moderate-mod)	-0.0000806	-1.7638860	-0.0036390	-0.0000588
Munk (moderate-lo)	-0.0000804	-1.7637420	-0.0036371	-0.0000585
Munk (flat-hi)	-0.0093535	-3.0598360	-0.0371825	-2.7892210
Munk (flat-mod)	-0.0093428	-3.0596630	-0.0371682	-2.7759950
Munk (flat-lo)	-0.0093321	-3.0594900	-0.0371540	-2.7628600
		Individualized (housing)		
Munk (steep-hi)	-0.0000030	-2.0742830	-0.0019861	0.0000000
Munk (steep-mod)	-0.0000025	-2.0548970	-0.0018565	0.0000000
Munk (steep-lo)	-0.0000022	-2.0365430	-0.0017464	0.0000000
Munk (moderate-hi)	-0.0000941	-3.2001820	-0.0112116	0.0000000
Munk (moderate-mod)	-0.0000799	-3.1699320	-0.0104726	0.0000000
Munk (moderate-lo)	-0.0000694	-3.1412990	-0.0098457	0.0000000
Munk (flat-hi)	-0.0102577	-5.5755810	-0.1059719	-0.0013504
Munk (flat-mod)	-0.0088965	-5.5176110	-0.0992097	-0.0001602
Munk (flat-lo)	-0.0075016	-5.4753340	-0.0933124	-0.0004748

CHAPTER 6

CONCLUSION

In this thesis we have reviewed the current state of pension investments in Turkey and the history and recent developments of a field of financial economics. We have reviewed the concept of "lifecycle investment" and summarized the common models and heuristics.

We have presented our model as an application of Munk's (2016) recent findings and Olear's (2016) simulation techniques into Turkish retirement market. We have collected historical data to calibrate and estimate the best parameters to be used in our simulation.

Using these parameters, we have constructed heterogeneous agents, who worked and invested throughout their lifetime. We considered different investment models that our hypothetical agents would use and calculated the resulting investment capitals.

Finally, we have calculated and compared the welfare effects of all popular models to the individualized Munk's solutions. We have concluded that for rich-to-middle class citizens, the individiualized strategies considerably increase their welfare. Moreover, the solutions we proposed can be solved analytically without complex dynamic optimizations, and therefore are easy to interpret to households. We also found that even naive lifecycle investments perform better than fixed lifetime investment.

We propose these models to Turkish pension providers and to Turkish working-age households belonging to middle-to-upper class, as these options will increase their retirement welfare considerably.

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APPENDIX A

CHOLESKY DECOMPOSED ERRORS

In order to create K random series which are correlated exactly like K deterministic series we have in mind, we can multiply independent random variables with the Cholesky decomposed part of the deterministic series. To illustrate this, let Σ be a correlation matrix of matrix X consisting of variables $x_1, x_2, ..., x_K$. Obviously the matrix is symmetric and the diagonal consists of 1s. Let $\Sigma = LL'$ be a Cholesky decomposition of this matrix. Now, let Ω be a vector of K independent random variables $\epsilon_1, \epsilon_2, ..., \epsilon_K$ with variance 1. Consequently, the variance-covariance matrix of Ω is an identity matrix. Then we claim that the product $L\Omega$ has the same correlation structure as X. The proof is below:

$$cov(L\Omega) = E[(L\Omega)(L\Omega)'] = E[L\Omega\Omega'L'],$$
$$cov(L\Omega) = L \cdot E[\Omega\Omega'] \cdot L' = L \cdot var(\Omega) \cdot L',$$
$$cov(L\Omega) = L \cdot I \cdot L' = LL' = \Sigma$$

Now consider the correlation matrix:

$$R = \begin{bmatrix} 1 & \rho_{sh} & \rho_{sy} \\ \rho_{hs} & 1 & \rho_{hy} \\ \rho_{ys} & \rho_{yh} & 1 \end{bmatrix}$$

Basic algebra gives its Cholesky decomposition as:

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ \rho_{hs} & \sqrt{1-\rho_{hs}^2} & 0 \\ \rho_{ys} & \frac{\rho_{yh}-\rho_{sh}\rho_{sy}}{\sqrt{1-\rho_{sh}^2}} & \sqrt{1-\rho_{ys}^2-(\frac{\rho_{yh}-\rho_{sh}\rho_{sy}}{\sqrt{1-\rho_{sh}^2}})^2} \end{bmatrix}$$

APPENDIX B

LOGARITHMS AS PERCENTAGE CHANGE

The first order Taylor series approximation of $\log(x)$ around 1 gives $\log(x) \approx x - 1$. Thus, for any x_t, x_{t+1} such that $\frac{x_{t+1}}{x_t}$ is close to 1, we have:

$$\log(x_{t+1}) - \log(x_t) = \log(\tfrac{x_{t+1}}{x_t}) \approx \tfrac{x_{t+1}}{x_t} - 1,$$

which is a percentage change in x_t between time periods t and $t + 1$.

APPENDIX C

DERIVATION OF CEC

Certainty equivalent consumption is a level of consumption which corresponds to deterministic utility which is exactly equal to the expected utility. We have:

$$U(CEC) = E[U(c)]$$

Substituting the formulas gives:

$$\sum_{t=1}^{T} \delta^{t-1} \prod_{j=0}^{t-1} p_j \cdot \frac{CEC^{1-\gamma}}{1-\gamma} = E[U(c)]$$

Leaving CEC alone gives the desired outcome:

$$CEC = \left(\frac{E[U(c)] \cdot (1-\gamma)}{\sum_{t=1}^{T} \delta^{t-1} \prod_{j=0}^{t-1} p_j} \right)^{1/(1-\gamma)}$$

APPENDIX D

ANNUITIZATION OF WEALTH

Let W_{57} be a total wealth and let x be a constant amount to be repaid annually. Then, at each age k, a firm will pay x with probability p_k and pay nothing with probability $1-p_k$, where p_k is the survival probability at age k (assuming $p_{57}=1$). To calculate the present value of the annuity, every payment will be discounted by riskless bond return rate r_f, resulting in the following discounted sum:

$$PV = x + p_{58} \cdot \frac{x}{1+r_f} + p_{59} \cdot \frac{x}{(1+r_f)^2} + ... + p_{100} \cdot \frac{x}{(1+r_f)^{100-57}}$$

Since the present value of annuity is equal to its price, and all of the total wealth will be used to buy such annuity, we set $W_{57} = PV$. Factoring out x from the right-hand side of the above equation gives the desired formula for annual payment amount:

$$x = \frac{W_{57}}{1 + \sum_{t=58}^{100} \frac{p_t}{1+r_f}}$$

APPENDIX E

DEFAULT AND DERIVED SOLUTIONS

In this appendix you can access the risky asset shares derived by solving all models mentioned in the thesis. Most of the solutions are present in the main text as figures, here you can access the tables with actual values.

Table E.1 shows the default values used. They are taken from the standard heuristics and real life pension fund investment menus. Table E.2 shows the derived values from Merton's and Munk's formulas for flat, moderate and steep labor curves times high, moderate and low stock-wage correlations. Table E.3 shows the derived values from Merton's model with housing.

Table E.1. Default Solutions

age	(100-age)%	Bodie et al.	Anadolu Hayat Riskless	Markowitz
27	0.73	1	0.3	0.63
28	0.72	1	0.3	0.63
29	0.71	1	0.3	0.63
30	0.7	1	0.3	0.63
31	0.69	1	0.3	0.63
32	0.68	1	0.3	0.63
33	0.67	1	0.3	0.63
34	0.66	1	0.3	0.63
35	0.65	1	0.3	0.63
36	0.64	1	0.3	0.63
37	0.63	1	0.3	0.63
38	0.62	1	0.3	0.63
39	0.61	1	0.3	0.63
40	0.6	0.975	0.3	0.63
41	0.59	0.95	0.3	0.63
42	0.58	0.925	0.3	0.63
43	0.57	0.9	0.3	0.63
44	0.56	0.875	0.3	0.63
45	0.55	0.85	0.3	0.63
46	0.54	0.825	0.3	0.63
47	0.53	0.8	0.3	0.63
48	0.52	0.775	0.3	0.63
49	0.51	0.75	0.3	0.63
50	0.5	0.725	0.3	0.63
51	0.49	0.7	0.3	0.63
52	0.48	0.675	0.3	0.63
53	0.47	0.65	0.3	0.63
54	0.46	0.625	0.3	0.63
55	0.45	0.6	0.3	0.63
56	0.44	0.575	0.3	0.63

Table E.2. Derived Solutions Without Housing

	Merton			Munk								
age	steep	mod	flat	flat-hi	flat-lo	flat-mod	mod-hi	mod-lo	mod-mod	steep-hi	steep-lo	steep-mod
27	1	1	1	1	1	1	1	1	1	1	1	1
28	1	1	1	1	1	1	1	1	1	1	1	1
29	1	1	1	1	1	1	1	1	1	1	1	1
30	1	1	1	1	1	1	1	1	1	1	1	1
31	1	1	1	1	1	1	1	1	1	1	1	1
32	1	1	1	1	1	1	1	1	1	1	1	1
33	1	1	1	1	1	1	1	1	1	1	1	1
34	1	1	1	1	1	1	1	1	1	1	1	1
35	1	1	1	1	1	1	1	1	1	1	1	1
36	1	1	1	1	1	1	1	1	1	1	1	1
37	1	1	1	1	1	1	1	1	1	1	1	1
38	1	1	1	0.852	1	1	1	1	1	1	1	1
39	1	1	0.958	0.717	0.958	0.834	0.961	1	1	0.991	1	1
40	1	1	0.795	0.615	0.805	0.711	0.802	1	0.946	0.821	1	0.972
41	0.924	0.903	0.675	0.537	0.694	0.616	0.684	0.906	0.793	0.695	0.925	0.807
42	0.771	0.758	0.583	0.475	0.605	0.541	0.592	0.771	0.683	0.599	0.782	0.692
43	0.656	0.647	0.511	0.426	0.534	0.48	0.519	0.668	0.594	0.524	0.675	0.6
44	0.566	0.56	0.453	0.385	0.475	0.43	0.46	0.583	0.522	0.463	0.588	0.526
45	0.495	0.49	0.407	0.352	0.426	0.389	0.411	0.513	0.462	0.414	0.517	0.466
46	0.438	0.434	0.368	0.324	0.386	0.355	0.371	0.455	0.413	0.373	0.458	0.416
47	0.391	0.388	0.336	0.3	0.351	0.326	0.338	0.407	0.373	0.34	0.41	0.375
48	0.353	0.351	0.309	0.28	0.322	0.301	0.311	0.367	0.339	0.312	0.369	0.341
49	0.321	0.319	0.287	0.263	0.298	0.28	0.288	0.333	0.311	0.289	0.335	0.312
50	0.294	0.293	0.267	0.249	0.276	0.263	0.268	0.305	0.286	0.269	0.306	0.287
51	0.272	0.271	0.251	0.237	0.258	0.247	0.251	0.28	0.266	0.252	0.281	0.267
52	0.253	0.252	0.237	0.226	0.243	0.234	0.237	0.259	0.248	0.238	0.26	0.249
53	0.236	0.236	0.225	0.217	0.229	0.223	0.225	0.241	0.233	0.226	0.242	0.234
54	0.223	0.222	0.215	0.209	0.218	0.213	0.215	0.226	0.22	0.215	0.226	0.221
55	0.211	0.21	0.206	0.202	0.207	0.205	0.206	0.213	0.209	0.206	0.213	0.209

Table E.3. Derived Solutions With Housing

	Munk stock			Munk housing		
age	flat-hi	flat-lo	flat-mod	flat-hi	flat-lo	flat-mod
28	0.912	0.913	0.913	0.088	0.087	0.075
29	0.908	0.91	0.909	0.092	0.09	0.079
30	0.903	0.906	0.905	0.097	0.094	0.083
31	0.897	0.902	0.9	0.103	0.098	0.088
32	0.89	0.896	0.894	0.11	0.104	0.095
33	0.881	0.89	0.886	0.119	0.11	0.102
34	0.871	0.882	0.878	0.129	0.118	0.111
35	0.86	0.873	0.867	0.14	0.127	0.122
36	0.847	0.862	0.856	0.153	0.138	0.134
37	0.832	0.85	0.842	0.168	0.15	0.147
38	0.815	0.836	0.827	0.185	0.164	0.163
39	0.797	0.82	0.81	0.203	0.18	0.18
40	0.778	0.803	0.792	0.222	0.197	0.199
41	0.757	0.784	0.772	0.243	0.216	0.22
42	0.735	0.763	0.751	0.265	0.237	0.241
43	0.705	0.742	0.73	0.283	0.258	0.264
44	0.617	0.72	0.679	0.275	0.28	0.281
45	0.547	0.638	0.595	0.269	0.277	0.273
46	0.491	0.562	0.529	0.264	0.27	0.267
47	0.445	0.502	0.475	0.259	0.264	0.262
48	0.407	0.452	0.431	0.256	0.26	0.258
49	0.376	0.412	0.395	0.253	0.256	0.255
50	0.35	0.378	0.365	0.25	0.253	0.252
51	0.327	0.35	0.339	0.248	0.25	0.249
52	0.309	0.326	0.318	0.247	0.248	0.247
53	0.292	0.306	0.299	0.245	0.246	0.246
54	0.279	0.288	0.284	0.244	0.245	0.244
55	0.267	0.273	0.27	0.243	0.243	0.243
56	0.257	0.261	0.259	0.242	0.242	0.242

	Munk stock			Munk housing		
age	mod-hi	mod-lo	mod-mod	mod-hi	mod-lo	mod-mod
28	0.913	0.914	0.913	0.087	0.086	0.087
29	0.91	0.912	0.911	0.09	0.088	0.089
30	0.906	0.909	0.908	0.094	0.091	0.092
31	0.901	0.905	0.904	0.099	0.095	0.096
32	0.896	0.901	0.899	0.104	0.099	0.101
33	0.89	0.897	0.894	0.11	0.103	0.106
34	0.883	0.891	0.887	0.117	0.109	0.113
35	0.874	0.884	0.88	0.126	0.116	0.12
36	0.864	0.877	0.871	0.136	0.123	0.129
37	0.853	0.868	0.861	0.147	0.132	0.139
38	0.84	0.857	0.85	0.16	0.143	0.15
39	0.826	0.845	0.836	0.174	0.155	0.164
40	0.81	0.831	0.822	0.19	0.169	0.178
41	0.792	0.816	0.805	0.208	0.184	0.195
42	0.773	0.798	0.787	0.227	0.202	0.213
43	0.752	0.779	0.767	0.248	0.221	0.233
44	0.731	0.759	0.746	0.269	0.241	0.254
45	0.683	0.737	0.724	0.281	0.263	0.276
46	0.596	0.706	0.651	0.273	0.283	0.278
47	0.527	0.612	0.57	0.267	0.275	0.271
48	0.472	0.538	0.505	0.262	0.268	0.265
49	0.427	0.478	0.453	0.258	0.262	0.26
50	0.39	0.43	0.41	0.254	0.258	0.256
51	0.359	0.39	0.375	0.251	0.254	0.253
52	0.333	0.357	0.345	0.249	0.251	0.25
53	0.311	0.329	0.32	0.247	0.248	0.248
54	0.292	0.305	0.299	0.245	0.246	0.246
55	0.276	0.285	0.28	0.244	0.244	0.244
56	0.262	0.268	0.265	0.242	0.243	0.243

	Munk stock			Munk housing		
age	steep-hi	steep-lo	steep-mod	steep-hi	steep-lo	steep-mod
28	0.913	0.914	0.914	0.087	0.086	0.086
29	0.91	0.912	0.911	0.09	0.088	0.089
30	0.907	0.91	0.908	0.093	0.09	0.092
31	0.903	0.906	0.905	0.097	0.094	0.095
32	0.898	0.903	0.901	0.102	0.097	0.099
33	0.892	0.898	0.896	0.108	0.102	0.104
34	0.886	0.893	0.89	0.114	0.107	0.11
35	0.877	0.887	0.883	0.123	0.113	0.117
36	0.867	0.879	0.874	0.133	0.121	0.126
37	0.856	0.87	0.864	0.144	0.13	0.136
38	0.843	0.859	0.852	0.157	0.141	0.148
39	0.829	0.847	0.839	0.171	0.153	0.161
40	0.812	0.833	0.824	0.188	0.167	0.176
41	0.794	0.818	0.807	0.206	0.182	0.193
42	0.775	0.8	0.789	0.225	0.2	0.211
43	0.754	0.781	0.769	0.246	0.219	0.231
44	0.732	0.76	0.748	0.268	0.24	0.252
45	0.689	0.738	0.725	0.282	0.262	0.275
46	0.601	0.712	0.657	0.274	0.284	0.279
47	0.531	0.617	0.574	0.267	0.275	0.271
48	0.475	0.542	0.508	0.262	0.268	0.265
49	0.429	0.481	0.456	0.258	0.263	0.26
50	0.392	0.432	0.412	0.254	0.258	0.256
51	0.36	0.392	0.376	0.251	0.254	0.253
52	0.334	0.358	0.346	0.249	0.251	0.25
53	0.311	0.33	0.321	0.247	0.249	0.248
54	0.292	0.306	0.299	0.245	0.246	0.246
55	0.276	0.285	0.281	0.244	0.244	0.244
56	0.262	0.268	0.265	0.242	0.243	0.243