

*Fabio Bagliano, Carolina Fugazza and Giovanna Nicodano*

## **Optimal Life-Cycle Portfolios for Heterogeneous Workers**

# Optimal life-cycle portfolios for heterogeneous workers\*

Fabio C. Bagliano<sup>^</sup>      Carolina Fugazza<sup>^^</sup>  
Giovanna Nicodano<sup>^^^</sup>

<sup>^</sup>Università di Torino and CeRP (Collegio Carlo Alberto)

<sup>^^</sup>Università di Milano-Bicocca and CeRP (Collegio Carlo Alberto)

<sup>^^^</sup>Università di Torino and CeRP (Collegio Carlo Alberto)

**Revised draft: June 2013**

## Abstract

Household portfolios include risky bonds, beyond stocks, and respond to permanent labour income shocks. This paper brings these features into a life-cycle setting, and shows that optimal stock investment is constant or increasing in age before retirement for realistic parameter *combinations*. The driver of such inversion in the life-cycle profile is the resolution of uncertainty regarding social security pension, which increases the investor's risk appetite. This occurs if a small positive contemporaneous correlation between permanent labour income shocks and stock returns is matched by a realistically high variance of such shocks and/or risk aversion. Absent this combination, the typical downward sloping profile obtains. Overlooking differences in optimal investment profiles across heterogeneous workers results in large welfare losses, in the order of 15-30% of lifetime consumption.

*Keywords:* Life-cycle portfolio choice, background risk, age rule, investor heterogeneity, stock market participation

---

\*We thank Cristopher Carroll, Mario Padula, Loriana Pellizon, Joshua Pollet, Dori-ana Ruffino, Carsten Sprenger and participants to the CareFin-CoNSoB-Baffi Conference 2012, Workshop on Quantitative Finance 2012, Financial Management Association Meeting 2012 and Second International Moscow Finance Conference for comments, and a referee for precise and thoughtful advice. A much earlier version was presented at the conferences "Netspar-SIFR Conference on Pension Plan & Product Design" and "CeRP X Anniversary". We are grateful to Collegio Carlo Alberto and MIUR (PRIN projects) for funding, and to Raffaele Corvino and Songhua Zhang for research assistance.

# 1 Introduction

Empirical studies point to differences in labour income risk borne by investors in order to account for the observed distribution of asset holdings. The volatility in the growth rate of proprietary income, as well as its correlation with common stock returns, affect portfolio composition in the early study of Heaton and Lucas (2000). More recent work emphasizes that it is persistent, rather than temporary, income shocks that matter (Angerer and Lam, 2009). Betermier et al. (2012) find that changes in wage volatility across industries explain changes in the portfolio share invested by households in risky assets. Another indicator of the sensitivity of equity investments to labour income risk rests on asset pricing models, whose ability to explain the cross sectional distribution of equity returns improves when human capital is considered. Importantly, it is heterogeneous industry-related human capital, rather than aggregate human capital, that appears to matter (Eiling 2013).

Against this background, this paper investigates the effect of heterogeneity in permanent shocks to labour income for optimal portfolio holdings over the life cycle. The consensus is that investors should reduce their stock investments as they approach retirement age under normal circumstances (Bodie, Merton and Samuelson 1992; Viceira 2001; Cocco, Gomes and Maenhout 2005). The motive is that human capital relative to financial wealth is decreasing over the life cycle, and labour income provides a hedge against shocks to stock returns. We point out, though, that uncertainty concerning social security income falls as retirement approaches since labour income shocks are persistent. This makes financial risk bearing more attractive.

Our paper argues that the optimal portfolio share invested in stocks increases, or is constant, in age before retirement for reasonable parameter configurations. This result obtains in a standard life-cycle framework where the first pillar offers an exogenous replacement ratio and available assets include one riskless and two risky assets ("stocks" and "bonds"). The driver of this inversion of the standard life-cycle asset allocation profile is a positive contemporaneous correlation between permanent labour income shocks and innovations to stock returns, *when matched* by a relatively high variance of such shocks and/or a relatively high risk aversion. Importantly, this pattern obtains for realistic parameter values. Such parametric interactions are also able to generate non-participation in the stock market by the young - a robust empirical regularity that so far has been dealt with by resorting to various kinds of participation costs.<sup>1</sup>

---

<sup>1</sup>Our paper extends to the life-cycle framework the analysis carried out by Boyle and Guthrie (2005), who use the mean-variance model with two risky assets augmented to include human capital to show the role of the correlation between risky assets and labour

More precisely, when we simply introduce bonds as a second risky asset into the Cocco, Gomes and Maenhout (2005) model, we obtain minor variations with respect to known results. Early in the worker's life, the average asset allocation is tilted towards stocks, as labour income provides a hedge against financial risks, while it gradually shifts to bonds in the two decades before retirement because income profiles peak around age 45. Changing one parameter at a time also involves minor modifications *in profiles*, although portfolio *shares* are affected in known ways. On the contrary, a clearcut departure from previous results emerges when a moderately positive correlation between stock returns and permanent labour income innovations interacts with a slightly higher degree of risk aversion. The worker starts investing in risky stocks only around the age of 25, after accumulating a sufficient amount of financial wealth. Afterwards, the stock share increases over time to reach about 20% for the median investor at the age of 40, and remains virtually constant until retirement. The portfolio bond share is correspondingly decreased up to the age of 40, with no investment in the riskless asset at any age. Therefore, the interaction of a positive stock return-labour income correlation with a relatively high degree of risk aversion produces an opposite age pattern of stock investment with respect to standard calibrations of life-cycle models and popular target-date products. If we add to this picture a higher variance of permanent labour income shocks, a gradual decrease over time of the risky asset share applies to bonds instead of stocks and is accompanied by accumulation of the safe asset.

These results owe in part to the (small) positive correlation of income shocks with equity returns, implying that labour income becomes an imperfect substitute for stock investments inducing the investor to reduce the equity allocation (Viceira 2001). This explains higher bond investment at the beginning of the life cycle, when human capital is relative large, and possible non participation in the stock market by the young. At the same time, uncertainty over future pension income falls as retirement approaches, thereby increasing the investor's risk appetite. The interplay of these two effects determines the life-cycle investment profile.

Also Benzoni, Collin-Dufresne and Goldstein (BCG) (2007) argue for an optimal inverted life cycle stock investment profile, with no participation when young, as we do. The driver of the inversion in BCG lies in the changing tightness of the dynamic relationship between labour income and stock markets in the short and the long run. The high long-term labour income-stock return correlation faced by young investors implies a relatively low optimal investment in stocks. As retirement approaches, the lower short-term correlation in solving the asset allocation puzzle.

relation motivates an optimal increase in stock investment. Thus, increased risk taking as retirement approaches in BCG is not due to the resolution of uncertainty concerning the level of future pension income, as suggested in this paper. Dramatic investments in stocks when young may also be sub-optimal if there is housing wealth (Cocco 2004) or if the expected labour income growth rate is sensitive to the real short-term interest rate (Munk and Sorensen 2010). Here we resort to two observed features of household portfolios, namely their responsiveness to permanent income shocks and the presence of risky bonds in an otherwise benchmark model. A simple interaction between risk aversion (or background risk) and the correlation of permanent income shocks and stock returns, may even explain upward sloping age profiles and non-participation by the young. Importantly, these combination effects are specific to the three parameters we stress above, at least for realistic calibrations. For instance, when the replacement ratio falls,<sup>2</sup> simulations reveal that agents save more during their working life in anticipation of lower pension incomes, thus accumulating a higher level of financial wealth. This determines a lower optimal share of stocks at all ages and for all values of the labour income-stock return correlation, holding risk aversion fixed. However, it does not impact on the shape of life cycle profiles because income shocks, and therefore the resolution of uncertainty, are less relevant to pension income.

An implication of our analysis is that investors' heterogeneity in risk aversion and labour income risk may explain the observed patterns in portfolio age profiles. When we allow for differences in such parameters across investors, the simple life-cycle model is able to replicate both the observed average participation rate in the stock market during working years and the average equity portfolio shares conditional on participation. We also analyze how combinations of parameters characterizing an investor's type (ex-ante) affect the ex-post dispersion in investment profiles around the median one. Such ex post dispersion is generated by the realization of individual-specific labour income shocks, and is remarkable for benchmark calibrations of the model. Dispersion in optimal profiles shrinks considerably when the investor type is characterized ex ante by higher risk aversion as well as higher labour income risk and correlation with stock returns. These investors save more and accumulate larger financial wealth early in life. Accordingly, their labour (mis)fortunes impact less on their portfolio composition generating reduced ex-post heterogeneity.

Our paper also implies that multiple investment strategies ought to be

---

<sup>2</sup>Observed replacement ratios vary widely both within and across countries, ranging from 34.4% in UK to 95.7% in Greece (OECD 2007).

offered to plan participants depending on their type, i.e. their risk aversion and their specific labour income characteristics. We measure the welfare losses associated to offering a single "target-date fund" (*TDF*), mimicking those adopted by pension funds, with an initially high stock share which gradually falls in age while the bond share increases.<sup>3</sup> Such investment rule is very close to optimal for the benchmark parameters which were the focus of previous research. It generates very large welfare costs, in the order of 15-30% of lifetime consumption, when higher risk aversion is accompanied by realistically large income risk. This is because investing as much as 90% in stocks in the early working years is suboptimal for a large part of the investor population, that would not participate in the equity market, and weighs on lifetime welfare. We also consider two alternative rules of thumb. The first is an age rule, where the portfolio share allocated to risky assets decays deterministically with the worker's age, while the second one is an equally weighted portfolio of three financial assets. This echoes the "1/ $N$  rule" of DeMiguel, Garlappi and Uppal (2008) that outperforms several investment strategies in ex post portfolio experiments. The latter strategy performs consistently better than the age rule in our ex ante experiment, and appears to be preferable to the *TDF* alternative in case the pension fund ignores workers' labour income profiles.

Bodie, Merton and Samuelson (1992) already specify exceptions to 'normal' circumstances, inducing workers to choose greater risk-taking with age. These are a very risky wage or a reduction in wage risk over the life cycle. Here, we analyse such cases using the realistic stochastic process for labour income proposed by Cocco et al. (2005) and argue that these can be quite 'normal' in practice: the variance of wage shocks need not be so high for the inversion to obtain, as long as such shocks are permanent as opposed to temporary and the asset menu includes bonds. Bodie and Treussard (2008) also suggest that the standard age rule may be far from optimal when wages are perfectly correlated with stock returns and risk aversion is relatively high, in which case a duration-matched portfolio of inflation-protected bonds may lead to higher welfare. We focus on the case when the correlation of permanent wage shocks with stock return innovations is low (0.2), broadly consistent with estimates obtained by a large part of the empirical

---

<sup>3</sup>Target Date Retirement Funds (TDF) are a "safe harbor" investment default in defined-contribution (DC) plans in the US since 2006. Vanguard life cycle fund with retirement date 2015 and 2045 respectively had stock allocations of 57% and 90% as of January 2012. Sweden's AP7 introduced in 2010 a new default arrangement that allocates 100 percent in equities until age 55 and then gradually moves into fixed income investments. Several developing countries adopt decreasing age-dependent default investment options (Giacomel and Rinaldi, 2008).

literature.<sup>4</sup>

The rest of the paper is organized as follows. Section 2 presents the benchmark life-cycle model and briefly outlines the numerical solution procedure adopted. Simulation results are discussed in Section 3, which reports about the inversion in life cycle profiles (3.3) and its robustness (3.3.1). It then analyzes ex post heterogeneity in portfolio shares over the life cycle (3.3.2) and the ability of ex ante heterogeneity to account for observed patterns in investment profiles. Section 4 addresses the welfare costs of simple investment rules for heterogeneous investors. Section 5 concludes the paper.

## 2 The life-cycle model

We model an investor who maximizes the expected discounted utility of consumption over her entire life and wishes to leave a bequest as well. The effective length of her life, which lasts at most  $T$  periods, is governed by age-dependent life expectancy. At each date  $t$ , the survival probability of being alive at date  $t + 1$  is  $p_t$ , the conditional survival probability at  $t$ . The investor starts working at age  $t_0$  and retires with certainty at age  $t_0 + K$ . Investor's  $i$  preferences at date  $t$  are described by a time-separable power utility function:

$$\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0} \left[ \sum_{j=1}^T \beta^j \left( \prod_{k=0}^{j-1} p_{t_0+k} \right) \left( p_{t_0+j} \frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + (1 - p_{t_0+j}) b \frac{(X_{it_0+j}/b)^{1-\gamma}}{1-\gamma} \right) \right]$$

where  $C_{it}$  is the level of consumption at time  $t$ ,  $X_{it}$  is the amount of wealth the investor leaves as a bequest to her heirs in case of death,  $b \geq 0$  is a parameter capturing the strength of the bequest motive,  $\beta < 1$  is a utility discount factor, and  $\gamma$  is the constant relative risk aversion parameter. Following Cocco, Gomes and Maenhout (2005), we do not model labour supply decisions, whereby ignoring the insurance property of flexible work effort allowing investors to compensate for bad financial returns with higher labour income, as in Gomes, Kotlikoff and Viceira (2008), and the opportunity to switch jobs as in Ruffino (2008).

---

<sup>4</sup>For example, although this correlation is not significantly different from zero in Cocco, Gomes and Michaelides (2005) for households with any level of educational attainment, it ranges from 0.33 for households with no high-school education to 0.52 for college graduates in Campbell, Cocco, Gomes and Maenhout (2001) and Campbell and Viceira (2002). More detailed discussion of this point is provided in section 3.3 below.

## 2.1 Labour and retirement income

Available resources to finance consumption over the agent's life cycle derive from accumulated financial wealth and from the stream of labour income. At each date  $t$  during *working life*, the exogenous labour income  $Y_{it}$  is assumed to be governed by a deterministic age-dependent growth process  $f(t, \mathbf{Z}_{it})$ , and is hit by both a permanent shock  $u_{it}$  and a transitory disturbance  $n_{it}$ , the latter being uncorrelated across investors. Formally, the logarithm of  $Y_{it}$  is represented by

$$\log Y_{it} = f(t, \mathbf{Z}_{it}) + u_{it} + n_{it} \quad t_0 \leq t \leq t_0 + K \quad (1)$$

More specifically,  $f(t, \mathbf{Z}_{it})$  denotes the deterministic trend component of permanent income, which depends on age  $t$  and on a vector of individual characteristics  $\mathbf{Z}_{it}$ , such as gender, marital status, household composition and education. As in Cocco, Gomes and Maenhout (2005) and Gomes and Michaelides (2005), uncertainty of labour income is captured by the two stochastic processes,  $u_{it}$  and  $n_{it}$ , driving the permanent and the transitory component respectively. Consistent with the available empirical evidence, the permanent disturbance is assumed to follow a random walk process:

$$u_{it} = u_{it-1} + \varepsilon_{it} \quad (2)$$

where  $\varepsilon_{it}$  is distributed as  $N(0, \sigma_\varepsilon^2)$  and is uncorrelated with the idiosyncratic temporary shock  $n_{it}$ , distributed as  $N(0, \sigma_n^2)$ . Finally, the permanent disturbance  $\varepsilon_{it}$  is made up of an aggregate component, common to all investors,  $\xi_t \sim N(0, \sigma_\xi^2)$ , and an idiosyncratic component  $\omega_{it} \sim N(0, \sigma_\omega^2)$  uncorrelated across investors:

$$\varepsilon_{it} = \xi_t + \omega_{it} \quad (3)$$

As specified below, we allow for correlation between the aggregate permanent shock to labour income  $\xi_t$  and innovations to the risky asset returns.

During *retirement*, income is certain and equal to a fixed proportion  $\lambda$  of the permanent component of income in the last working year:

$$\log Y_{it} = \log \lambda + f(t_{0+K}, \mathbf{Z}_{it_{0+K}}) + u_{it_{0+K}} \quad t_0 + K < t \leq T \quad (4)$$

where the level of the replacement rate  $\lambda$  is meant to capture at least some of the features of Social Security systems. Other, less restrictive, modelling strategies are possible. For example, Campbell, Cocco, Gomes and Maenhout (2001) model a system of mandatory saving for retirement as a given fraction of the (stochastic) labour income that the investor must save for retirement and invest in the riskless asset, with no possibility of consuming it or



borrowing against it;<sup>5</sup> at retirement, the value of the wealth so accumulated is transformed into a riskless annuity until death.

## 2.2 Investment opportunities

We allow savings to be invested in a short-term riskless asset, yielding each period a constant gross real return  $R^f$ , and in two risky assets, characterized as “*stocks*” and “*bonds*”. The risky assets yield stochastic gross real returns  $R_t^s$  and  $R_t^b$  respectively. We maintain that the investment opportunities in the risky assets do not vary over time and model excess returns of stocks and bonds over the riskless asset as

$$R_t^s - R^f = \mu^s + \nu_t^s \quad (5)$$

$$R_t^b - R^f = \mu^b + \nu_t^b \quad (6)$$

where  $\mu^s$  and  $\mu^b$  are the expected stock and bond premia, and  $\nu_t^s$  and  $\nu_t^b$  are normally distributed innovations, with mean zero and variances  $\sigma_s^2$  and  $\sigma_b^2$  respectively. We allow for the two disturbances being correlated, with correlation  $\rho_{sb}$ . Moreover, we let the innovation on the stock return be potentially correlated with the aggregate permanent disturbance to the labour income, and denote this correlation by  $\rho_{sY}$ . We do not allow for excess return predictability and other forms of changing investment opportunities over time, as in Michaelides (2002) and Kojien, Nijman and Werker (2010). While both papers document market timing effects on asset allocations when parameters of the return distributions are known with certainty, there is still considerable debate as to the *ex-post* value of market timing both at short and longer horizons (De Miguel, Garlappi and Uppal 2008; Fugazza, Guidolin and Nicodano 2010) and return predictability in general (Goyal and Welch 2008) when such parameters are estimated by an asset manager.

At the beginning of each period, financial resources available for consumption and saving are given by the sum of accumulated financial wealth  $W_{it}$  plus current labour income  $Y_{it}$ , that we call *cash on hand*  $X_{it} = W_{it} + Y_{it}$ . Given the chosen level of current consumption,  $C_{it}$ , next period cash on hand is given by:

$$X_{it+1} = (X_{it} - C_{it})R_{it}^P + Y_{it+1} \quad (7)$$

where  $R_{it}^P$  is the portfolio return

$$R_{it}^P = \alpha_{it}^s R_t^s + \alpha_{it}^b R_t^b + (1 - \alpha_{it}^s - \alpha_{it}^b) R^f \quad (8)$$

---

<sup>5</sup>Kojien, Nijman and Werker (2011) argue that these mechanisms are suboptimal relative to alternative annuity designs, despite their diffusion across pension systems.

with  $\alpha_{it}^s$ ,  $\alpha_{it}^b$  and  $(1 - \alpha_{it}^s - \alpha_{it}^b)$  denoting the shares of the investor's portfolio invested in stocks, bonds and in the riskless asset respectively. We do not allow for short sales and assume that the investor is liquidity constrained, so that the nominal amount invested in each of then three financial assets are  $F_{it} \geq 0$ ,  $S_{it} \geq 0$  and  $B_{it} \geq 0$  respectively for the riskless asset, stocks and bonds, and the portfolio shares are non negative in each period.

All simulation results presented below are derived under the assumption that the investor's asset menu is the same during working life and retirement. However, the results concerning asset allocation are qualitatively similar in unreported simulations based on the alternative assumption that retirees invest in the riskless asset only.

## 2.3 Solving the life-cycle problem

In this standard intertemporal optimization framework, the investor maximizes the expected discounted utility over life time, by choosing the consumption and the portfolio rules given uncertain labour income and asset returns. Formally, the optimization problem is written as:

$$\max_{\{C_{it}\}_{t_0}^{T-1}, \{\alpha_{it}^s, \alpha_{it}^b\}_{t_0}^{T-1}} \left( \frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0} \left[ \sum_{j=1}^T \beta^j \left( \prod_{k=0}^{j-2} p_{t_0+k} \right) \left( p_{t_0+j} \frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + (1 - p_{t_0+j}) b \frac{(X_{it_0+j}/b)^{1-\gamma}}{1-\gamma} \right) \right] \right) \quad (9)$$

$$s.t. \quad X_{it+1} = (X_{it} - C_{it}) (\alpha_{it}^s R_t^s + \alpha_{it}^b R_t^b + (1 - \alpha_{it}^s - \alpha_{it}^b) R^f) + Y_{it+1}$$

with the labour income and retirement processes specified above and short sales and borrowing constraints imposed.

Given its intertemporal nature, the problem can be restated in a recursive form, rewriting the value of the optimization problem at the beginning of period  $t$  as a function of the maximized current utility and of the value of the problem at  $t + 1$  (Bellman equation):

$$V_{it}(X_{it}, u_{it}) = \max_{\{C_{it}\}_{t_0}^{T-1}, \{\alpha_{it}^s, \alpha_{it}^b\}_{t_0}^{T-1}} \left( \frac{C_{it}^{1-\gamma}}{1-\gamma} + \beta E_t [p_t V_{it+1}(X_{it+1}, u_{it+1}) + (1 - p_t) b \frac{(X_{it+1}/b)^{1-\gamma}}{1-\gamma}] \right) \quad (10)$$

At each time  $t$  the value function  $V_{it}$  describes the maximized value of the problem as a function of the two state variables, the level of cash on hand at

the beginning of time  $t$ ,  $X_{it}$ , and the level of the stochastic permanent component of income at beginning of  $t$ ,  $u_{it}$ . In order to reduce the dimensionality of the original problem to one state variable we exploit the homogeneity of degree  $(1 - \gamma)$  of the utility function, and normalize the entire problem by the permanent component of income  $u_{it}$ . Thus, we can rewrite (10) as

$$V_{it}(X_{it}) = \max_{\{C_{it}\}_{t_0}^{T-1}, \{\alpha_{it}^s, \alpha_{it}^b\}_{t_0}^{T-1}} \left( \frac{C_{it}^{1-\gamma}}{1-\gamma} + \beta E_t [p_t V_{it+1}(X_{it+1}) + (1-p_t) b \frac{(X_{it+1}/b)^{1-\gamma}}{1-\gamma}] \right) \quad (11)$$

This problem has no closed form solution: hence the optimal values for consumption and portfolio shares at each point in time are obtained by means of numerical techniques. To this aim, we apply a backward induction procedure and obtain optimal consumption and portfolio rules in terms of the state variable starting from the last possible period of life  $T$ . In particular, in the presence of bequest, the terminal condition is:

$$V_{iT+1}(X_{iT+1}) = b \frac{(X_{iT+1}/b)^{1-\gamma}}{1-\gamma}$$

and the Bellman equation (10) at  $T$  becomes

$$V_{iT}(X_{iT}) = \frac{C_{iT}^{1-\gamma}}{1-\gamma} + \beta E_t \left( b \frac{(X_{iT+1}/b)^{1-\gamma}}{1-\gamma} \right) \quad (12)$$

from which the optimal consumption and portfolio share policy rules are obtained for each possible value of the state variable (the initial level of cash on hand at  $T$ ) using the standard grid search method.<sup>6</sup> Going backwards, for every period  $t = T-1, T-2, \dots, t_0$ , the Bellman equation (10) is used to obtain the optimal rules for consumption and the portfolio shares. For each level of the state variable  $X_{it}$ , the value function at the beginning of time  $t$ ,  $V_{it}(X_{it})$ , is obtained by picking the levels of consumption and portfolio shares that maximize the sum of the utility from current consumption  $U(C_{it})$  and the discounted expected value from continuation  $\beta E_t [\cdot]$ , computed using  $V_{it+1}(X_{it+1})$  obtained from the previous iteration. In particular, given  $V_{it+1}(X_{it+1})$ , the expectation term is evaluated in two steps. We use

---

<sup>6</sup>According to this method, the problem is solved over a grid of values covering the space of the state variables and the controls, to ensure that the solution found is a global optimum.

numerical integration performed by means of the standard Gaussian Hermite quadrature method to approximate the distribution of shocks to labour income and asset returns. Then, cubic spline interpolation is employed to evaluate the value function at points that do not lie on the state space grid.

### 3 Simulation results

The numerical solution method briefly outlined above yields, for each set of parameters chosen, the optimal policy functions for the level of consumption and the shares of the financial portfolio invested in the riskless and risky assets as functions of the level of cash on hand. Using those optimal rules, it is then possible to simulate the life-cycle consumption and asset allocation choices of a large number of agents. In this section, we describe results obtained from this procedure, focusing first on a benchmark case and then presenting extensions along various dimensions.

#### 3.1 Calibration

Parameter calibration concerns the investor's preferences, the features of the labour income process during working life and retirement, and the moments of the risky asset returns. To obtain results for a benchmark case, we chose plausible sets of parameters referred to the US and based mainly on Cocco, Gomes and Maenhout (2005) and Gomes and Michaelides (2005).

The investor begins her working life at the age of 20 and works for (a maximum of) 45 periods ( $K$ ) before retiring at the age of 65. After retirement, she can live for a maximum of 35 periods until the age of 100. In each period, we take the conditional probability of being alive in the next period  $p_t$  from the life expectancy tables of the US *National Center for Health Statistics*. As regards to preferences, we set the utility discount factor  $\beta = 0.96$ , and the parameter capturing the strength of the bequest motive  $b = 2.5$  (which bears the interpretation of the number of years of her descendants' consumption that the investor intends to save for). Finally, the benchmark value for the coefficient of relative risk aversion is  $\gamma = 5$ . The latter choice is relatively standard in the literature (Gomes and Michaelides 2005; Gomes, Kotlikoff and Viceira 2008), capturing an intermediate degree of risk aversion, though Cocco, Gomes and Maenhout (2005) choose a value as high as 10 in their benchmark setting.

The labour income process is calibrated using the estimated parameters for US households with high-school education (but not a college degree) in Cocco, Gomes and Maenhout (2005). The age-dependent trend is captured

by a third-order polynomial in age, delivering the typical hump-shaped profile until retirement depicted as the dash-dotted line in Figure 1. After retirement, income is a constant proportion  $\lambda$  of the final (permanent) labour income, with  $\lambda = 0.68$ . The continuous line in the figure portrays the whole deterministic trend  $f(t, \mathbf{Z}_{it})$ , used in the simulations below, that allows also for other personal characteristics such as family size and marital status. In the benchmark case, the variances of the permanent and transitory shocks ( $\varepsilon_{it}$  and  $n_{it}$  respectively) are  $\sigma_\varepsilon^2 = 0.0106$  and  $\sigma_n^2 = 0.0738$ ; in some of the extensions below we let those parameters vary (to explore the effects of increasing labour income uncertainty) but keep the permanent-transitory ratio roughly constant at the 0.14 level. This choice is supported by the evidence in Cocco, Gomes and Maenhout (2005), showing that empirically the ratio is remarkably stable across occupational sectors despite widely different values for the labour income shock variances.

The riskless (constant) interest rate is set at 0.02, with expected stock and bond premia  $\mu^s$  and  $\mu^b$  fixed at 0.04 and 0.02 respectively. The standard deviations of the returns innovations are set at  $\sigma_s = 0.157$  and  $\sigma_b = 0.08$ ; in the benchmark case, we fix their correlation at a positive but relatively small value:  $\rho_{sb} = 0.2$ , calibrated on the historical annual correlation in the US and close to the choice of Gomes and Michaelides (2004). Finally, we initially impose a zero correlation between stock return innovations and aggregate permanent labour income disturbances ( $\rho_{sY} = 0$ ); we will assess below the impact on wealth accumulation and portfolio allocation of allowing for a moderately positive stock return-labour income shock correlation.

### 3.2 Benchmark results

In all simulations we look at the cross-sectional distribution of 10,000 agents' optimal choices over their life cycle. In the benchmark case, the typical life-cycle profiles for consumption, labour income and accumulated financial wealth are obtained over the working life and the retirement period, as in Cocco, Gomes and Maenhout (2005). Binding liquidity constraints make consumption closely track labour income until the 35-40 age range, when the consumption path becomes less steep and financial wealth is accumulated at a faster rate. After retirement at 65, wealth is gradually decumulated and consumption decreases towards retirement income.

Before presenting the age profile of optimal portfolio shares, Figure 2 displays the optimal policy rules for the risky asset shares  $\alpha_{it}^s$  and  $\alpha_{it}^b$  as functions of the level of cash on hand (the problem's state variable): in each panel, the optimal fraction of the portfolio invested in stocks and bonds is plotted against cash on hand for investors of four different ages (20, 30, 55 and

75). The basic intuition guiding the interpretation of the optimal policies, on which the following simulation results are based, is that labour income is viewed by the investor as an implicit holding of an asset (Bodie, Merton and Samuelson 1992). Although in our setting labour income is uncertain (its process being hit by both permanent and transitory shocks), as long as the correlation of asset returns' innovations and labour income disturbances is zero or sufficiently small, labour income is more similar to the risk-free than to the risky assets; therefore, when the present discounted value of the expected future labour income stream (i.e. human wealth) accounts for a sizeable portion of overall wealth, the investor is induced to tilt her portfolio towards the risky assets. The proportion of human out of total wealth is widely different across investors of different age and is one of the main determinants of their chosen portfolio composition.

Looking at Figure 2, in the case of an investor of age 75, the certain retirement income acts as a holding of the riskless asset and only relatively poor investors (with a small amount of accumulated wealth and current income) hold a financial portfolio heavily invested in stocks. Wealthier investors choose a lower portfolio share in stocks and increase their holdings of bonds, since for them the proportion of the overall wealth implicitly invested in the riskless asset (i.e. human wealth) is lower. At age 55, the investor still has a decade of relatively high expected labour income before retirement, and she will tend to balance this implicit holding of a low-risk asset with a financial portfolio more heavily invested in risky stocks (and less in bonds) than older investors: her optimal policies in Figure 2 are shifted outwards with respect to the 75-year-old agent for all levels of cash on hand.<sup>7</sup> The same intuition applies to earlier ages, for which the optimal stock and bond policies shift gradually outwards as younger investors are considered.

On the basis of such optimal investment policies, the portfolio shares of stocks, bonds and the riskless asset for 10,000 agents have been obtained by simulation over the whole investors' life cycle. Figure 3 shows the median portfolio shares for stocks (upper panels) and bonds (lower panels) from the cross-sectional distribution, plotted against age. In order to assess the amount of heterogeneity in investors' portfolio choices, also the 5th and the 95th percentiles of the cross-sectional distributions of optimal shares are shown. Two assumptions on the amount of background risk faced by agents are considered: a "normal" variance scenario (left column), in which the variances of the permanent and transitory labour income shocks are set

---

<sup>7</sup>The step-wise appearance of the policy rules is due to the choice of the grid in the numerical solution procedure. The use of a finer grid would deliver smoother policies, at the cost of additional computing time.

at the already mentioned benchmark levels ( $\sigma_\varepsilon^2 = 0.0106$  and  $\sigma_n^2 = 0.0738$ ), and a "high" variance scenario (right column), in which, while keeping the permanent to total variance ratio constant (0.14), the labour income shock variances are set at the larger values  $\sigma_\varepsilon^2 = 0.042$  and  $\sigma_n^2 = 0.30$ , consistently with the evidence presented by Cocco, Gomes and Maenhout (2005) for US workers in the agricultural sector.

Figure 3 well summarizes a relatively standard set of results on the age profiles of stock and bond portfolio shares, mainly determined by the fact that over the life cycle the proportion of overall wealth implicitly invested in the riskless asset through expected labour incomes varies, being large for young investors and declining as retirement approaches. In fact, in the "normal" labour income risk scenario, younger agents invest heavily in stocks until approximately the age of 40. Middle-age investors (between 40 and the retirement age of 65) gradually shift the composition of their portfolio away from stocks and into bonds, to reach median shares of around 55% and 45% respectively at the retirement date. After retirement, two main factors determine optimal portfolio shares. On the one hand, income becomes certain and the proportion of implicit holdings of the safe asset increases again; consequently, the share of stocks tends to increase, at the expense of bonds, to compensate for it. On the other hand, the presence of an operative bequest motive induces the investor to run down her previously accumulated financial wealth relatively slowly, whereby offsetting the incentive to increase the portfolio share of the riskiest asset. Overall, the optimal share of stocks remains nearly flat at the bottom level attained at retirement age. Throughout both working life and retirement, holdings of the riskless asset are kept at a minimum, very often zero.

The effects of increasing labour income risk on optimal asset allocation are portrayed in the right column panels of Figure 3. A larger amount of background risk induces agents to increase precautionary savings, accumulating more financial wealth over time. Therefore, there is less need for investors to tilt their asset allocation towards the riskiest asset available: the optimal share of stocks in the portfolio is reduced at any age, and the bond share is correspondingly increased. The age profiles show that investors start decreasing the stock share very early in life, to reach a bottom level (slightly larger than 40% for the median of the distribution) around the age of 50; then, the share remains remarkably flat for the remaining part of the working life and during retirement. This age profile is mirrored exactly by the bond share, with no room for investment in the safe asset at any age.

Overall, the popular financial advice of holding a portfolio share of risky stocks equal to 100 minus the investor's age (so that  $\alpha_{age}^s = (100 - \text{age})/100$ ), implying a gradual shift toward bonds over life, is not completely at variance

with the optimally designed investment policies displayed in Figure 3, at least over investors' working life. However, in our benchmark cases the decumulation of stocks is not linear, as the simple age-dependent rule would predict (with the stock share going down from 80% at the age of 20 to 35% at retirement), but depends on the relative dynamics of the investor's human and financial wealth. This helps explaining also the behavior of portfolio shares when the agent's incentives to save and accumulate financial wealth are changed in various ways.

For example, investors with lower replacement ratio,  $\lambda$ , anticipate relatively lower incomes during retirement and choose to save more during their working life, thereby accumulating a higher level of financial wealth. This determines a lower optimal share of stocks (and a correspondingly larger bond share) at all ages (and in both labour income risk scenarios) and a declining time profile over the working life, as human capital decreases relative to financial wealth, confirming the patterns displayed in Figure 3.

### 3.3 Inverted Life-Cycle Profiles

To evaluate the robustness of the life-cycle asset share profiles obtained above, we modify the benchmark setting in various ways. In this subsection, we focus on two additional important dimensions, i.e. the correlation between stock return innovations and the aggregate permanent shock to labour income ( $\rho_{sY}$ ) and the degree of investors' risk aversion ( $\gamma$ ), and their interactions with the amount of background risk faced by agents.

First, we let the stock return innovations be positively correlated with the innovations in permanent labour income. The available empirical estimates of this correlation for the US differ widely. Cocco, Gomes and Michaelides (2005) report estimated values not significantly different from zero for households with any level of educational attainment, whereas Campbell, Cocco, Gomes and Maenhout (2001) and Campbell and Viceira (2002) find higher values, ranging from 0.33 for households with no high-school education to 0.52 for college graduates.<sup>8</sup> Moreover, Cocco, Gomes, and Maenhout (2005) provide estimates between  $-0.01$  and  $0.02$ , while Heaton and Lucas (2000) between  $-0.07$  and  $0.14$ , and Munk and Sørensen (2005) report a correlation of  $0.17$ . Furthermore, as to the correlation between labour income risk and industry-specific equity risk, Davis and Willen (2000) report correlations ranging between  $-0.10$  and  $0.40$ . Since our calibration of the labour income process reflects the features of households with high-school education, we adopt an intermediate positive value of  $\rho_{sY} = 0.2$ . This choice results in a

---

<sup>8</sup>In Campbell and Viceira (2002) the correlation is estimated with a one-year lag.



modest correlation between the growth rate of individual labour income and stock return innovations, in accordance with the empirical evidence.<sup>9</sup>

Figure 4 displays the optimal portfolio shares of stocks and bonds when  $\rho_{sY} = 0.2$  and the degree of risk aversion is kept at the benchmark value  $\gamma = 5$ . The positive correlation between labour income shocks and stock returns makes labour income closer to an implicit holding of stocks rather than of the other assets, reducing the incentive to invest in stocks at all ages.<sup>10</sup> The age profile of optimal portfolio shares now depends crucially on the assumed magnitude of background risk. In the "normal" labour income variance scenario (left column), younger investors, for whom human capital is a substantial fraction of overall wealth, are therefore heavily exposed to stock market risk and will find it optimal to offset such risk by holding a considerably lower fraction of their financial portfolio in stocks if compared with the benchmark case in Figure 3. This effect decreases as workers move along the steepest part of their labour income path, determining a gradual increase in the portfolio share of stocks until around the age of 25. From that age on, the size of human capital decreases and the investor shifts her portfolio composition again towards safer bonds: this yields a hump-shaped profile for the optimal share of stocks during working life. The stock share reaches a bottom level of about 40% for the median investor at around the age of 45, and remains substantially flat until retirement. At 65 labour income becomes certain (and therefore uncorrelated with stock return innovations), and the investor rebalances her portfolio towards stocks: during retirement, the level and time profile of the stock share are very close to the benchmark

---

<sup>9</sup>In fact, using (1), (2) and (3) we can express the correlation between the growth rate of individual labour income ( $\Delta \log Y_{it}$ ) and the stock return innovation ( $\nu_t^s$ ) in terms of  $\rho_{sY}$  and the variances of the aggregate and idiosyncratic labour income shocks as:

$$\text{corr}(\Delta \log Y_{it}, \nu_t^s) = \frac{1}{\sqrt{1 + \frac{\sigma_\omega^2 + 2\sigma_n^2}{\sigma_\xi^2}}} \cdot \rho_{sY} < \rho_{sY}$$

Using our benchmark ("normal" labour income variance) value for  $\sigma_n^2 = 0.0738$  and attributing all permanent disturbances to the aggregate component, so that  $\sigma_\epsilon^2 = \sigma_\xi^2 = 0.0106$  ( $\sigma_\omega^2$  being 0), we derive an upper bound for  $\text{corr}(\Delta \log Y_{it}, \nu_t^s)$ :

$$\text{corr}(\Delta \log Y_{it}, \nu_t^s) \leq 0.26 \cdot \rho_{sY}$$

Therefore, the value for  $\rho_{sY}$  used in our simulations (0.2) implies a modest value for  $\text{corr}(\Delta \log Y_{it}, \nu_t^s)$  of (at most) 0.052. This value is only slightly changed in the "high" labour income variance scenario (0.054).

<sup>10</sup>Note that the positive correlation between stock and bond return innovations (0.2) makes also bond return positively correlated with permanent labour income innovations, but with a much smaller coefficient (0.04).

case shown in Figure 3. Again, throughout working life and retirement, the age profile of the bond share mirrors that of stocks, with no investment in the riskless asset.

Sharp differences in optimal asset allocation over the life cycle emerge when the amount of background risk is larger. In the “high” labour income variance scenario (Figure 4, right column) financial wealth is accumulated more rapidly and the portfolio share of stocks is decreased further at any age. In fact, the agent starts investing in risky stocks a positive fraction of her financial portfolio only around the age of 25, after accumulating a sufficient amount of financial wealth. Afterwards, the stock share increases over time to reach about 20% for the median investor at the age of 40, and remains virtually constant until retirement, when the portfolio is rebalanced in favour of stocks to compensate for the now riskless nature of income streams. The portfolio bond share is correspondingly decreased up to the age of 40, and then kept constant by the median investor until rebalancing occurs at the retirement date. No room for investment in the riskless asset is detected at any age. Therefore, conditional on an intermediate degree of risk aversion, the interaction of a moderately positive correlation between stock returns and permanent labour income innovations with a relatively large amount of background risk produces an opposite age pattern of stock investment with respect to standard calibrations of life-cycle models and popular financial advice. Now it is optimal for the investor not even enter the stock market when very young, and build up the stock share later during working life, when the ratio of human to financial wealth decreases due to larger savings and the hump-shaped labour income dynamics. This gradual rebalancing process towards less risky bonds stops quite early (around the age of 40), when the investor attains an optimal asset allocation which is kept constant for the rest of her working life.

The case for “inverted” life-cycle stock share profiles is further enhanced when even a moderately positive correlation between innovations to stock returns and permanent labour income interacts with a relatively higher degree of risk aversion. Figure 5 portrays the age profile of the portfolio shares of stock and bonds for investors with a risk aversion parameter  $\gamma = 8$ , keeping the stock return-labour income correlation at  $\rho_{sY} = 0.2$ . To focus on the relevance of the interaction between those two parameters in shaping optimal life-cycle asset allocation choices, we choose a value for  $\gamma$  that, though higher than in the benchmark case ( $\gamma = 5$ ), is not extreme; for example, Cocco, Gomes and Michaelides (2005) set  $\gamma = 10$  in their baseline calibration exercise, considering this value as the upper bound of the range of reasonable values. As discussed below, setting  $\gamma$  to values larger than 8 would even strengthen the results. Now, even in the “normal” background risk scenario

(Figure 5, left column), the more risk-averse (and prudent) investor saves more for precautionary reasons, accumulating financial wealth, and starts investing in risky stocks only around the age of 25, gradually increasing the stock share until about 20% at the age of 45. Then, the stock share stays flat over the remaining working life and during retirement, after a modest rebalancing when labour income becomes riskless at the retirement age. The time profile of the portfolio share invested in bonds mirrors that of stocks until the age of 50, when it starts decreasing gradually from 80% to about 60% at retirement age. Correspondingly, the investor accumulates a buffer stock of the riskless asset, which reaches a portfolio share of around 20% at retirement age and is kept constant thereafter.

Those age patterns are even more pronounced when a larger amount of labour income risk is considered (Figure 5, right column). With more background risk, precautionary savings are larger, financial wealth is accumulated more rapidly, and the optimal stock share is lower at any age. The (median) investor enters the stock market in her late 20s and slowly increases her stock portfolio share up to a modest 15% at age 45, that she keeps constant over the remaining working life; even after rebalancing the portfolio at the retirement age, the stock share reaches only 20%. The bond share decreases more smoothly than in the “normal” labour income risk scenario throughout the entire investor’s working life, as an increasingly larger portfolio share is invested into the riskless asset: at retirement, riskless asset holdings amount to about 30% of the financial portfolio for the median investor. Therefore, with high background risk, a standard age-dependent rule implying a gradual reduction over time of the risky asset share does apply to bonds instead of stocks and is accompanied by a sizeable accumulation of the safe asset.

Overall, our results show that, contrary to standard calibrations of the life-cycle model, the optimal portfolio share invested in stocks may well be increasing or constant in age before retirement when a positive correlation between permanent labour income shocks and stock return innovations is associated with a relatively high degree of investors’ risk aversion or a substantial amount of background risk.

To further explore the age profile of optimal stock investments, Figure 6 shows median optimal stock shares over the life-cycle for a broader set of values of the relevant parameters, namely four values of the risk aversion coefficient ( $\gamma = 2, 5, 8$  and  $10$ ) and a wider range of stock return-labour income correlation coefficients, from  $\rho_{sY} = 0$  to  $\rho_{sY} = 1$  (by incremental steps of 0.1). Panel (a) displays the results of those parameter combinations in the “normal” labour income shock variance scenario, whereas Panel (b) refers to

the “high” variance scenario.<sup>11</sup> Our simulations extend and generalize the findings presented in Figures 4 and 5. Now the inversion of the age profile of stock investment occurs for all levels of risk aversion: the higher the risk aversion coefficient  $\gamma$  (and the larger the magnitude of background risk), the lower is the minimum value of  $\rho_{sY}$  necessary for the inversion in the life-cycle stock investment pattern to occur. When risk aversion is low ( $\gamma = 2$ ), the optimal stock portfolio share starts to be increasing in age over most of the investor’s working life only for values of  $\rho_{sY}$  larger than 0.7 in the “normal” labour income shock variance scenario and 0.5 in the “high” variance case. As the degree of risk aversion is increased, those threshold values for  $\rho_{sY}$  are gradually reduced to around 0.1 for both background risk scenarios when  $\gamma = 10$ , the largest value of our chosen range. Indeed, high risk aversion and/or large labour income risk induce investors to save more for precautionary reasons, whereby accumulating financial wealth more rapidly, and to decrease their optimal portfolio stock share. A positive correlation between stock returns and labour income shocks makes stocks a less desirable hedge for labour income risk, further reducing the optimal stock share. The resulting age profile of the stock share may well feature a very low level (or even zero, implying non-participation in the stock market) in the earlier period of the investor’s working life, followed by a moderately increasing pattern until middle-age, even for modest values of the stock return-labour income correlation coefficient. At retirement age, income becomes uncorrelated with stock returns for any value of  $\rho_{sY}$  prevailing over working life: this feature of retirement income explains why, for all degrees of risk aversion, the magnitude of the portfolio rebalancing toward stocks at retirement age is larger the higher is  $\rho_{sY}$  during working life.

### 3.3.1 Robustness

The life-cycle investment patterns discussed above are robust to several changes in the properties of financial asset returns and investors’ preferences, and to the introduction of a greater degree of uncertainty about retirement income.<sup>12</sup>

**Correlation between bond and stock returns.** In all simulations so far, the correlation between stock and bond returns is equal to  $\rho_{sb} = 0.2$ . However, it is possible to broadly interpret “bonds” as a second risky asset

---

<sup>11</sup>All results are conditional on a replacement ratio of  $\lambda = 0.68$ , and a correlation between stock and bond returns equal to  $\rho_{sb} = 0.2$ . The next subsection examines robustness of our results to changes in these parameters.

<sup>12</sup>Detailed results are not reported for reasons of space but are available upon request from the authors.

(or a basket of other risky securities). Thus, we also explored optimal asset allocation profiles for stock-bond correlations ranging from  $\rho_{sb} = 0$  to  $\rho_{sb} = 0.6$ , the latter value being the observed correlation between returns on equity REITs and stocks in the US (Fugazza, Guidolin and Nicodano 2009). This exercise confirms our main conclusion that what drives the shape of life-cycle portfolio shares is the interaction between risk aversion, background risk, and the stock return-labour income correlation, with the stock-bond returns correlation playing only a minor role. Specifically, conditional on the levels of risk aversion and labour income risk, an increase in the stock-bond returns correlation causes only a small increase in the threshold value of  $\rho_{sY}$  that generates an inverted age profile in the portfolio stock share, for given positive risk premium on bonds. Indeed, the positive correlation between stock returns and labour income makes the latter closer to an implicit holding of stocks. This induces a young worker to invest in bonds as a hedge of labour income shocks, when their returns are uncorrelated to returns on stocks; clearly, this hedge is more attractive than the risk-free asset because of its positive risk premium. When the stock-bond correlation  $\rho_{sb}$  increases, bonds become more similar to stocks and therefore a worse hedge against labour income shocks: this may keep a young worker invested in stocks or may enlarge the portfolio share allocated to the risk-free asset earlier in life.<sup>13</sup>

**Risk aversion and intertemporal substitution.** Power utility constrains the degree of risk aversion to be inversely related to the elasticity of intertemporal substitution, both being governed by the same parameter  $\gamma$ . The recursive intertemporal utility function of Epstein and Zin (1989) makes the elasticity of intertemporal substitution independent of risk aversion, allowing us to investigate the separate role of those preference parameters in determining the life-cycle profile of optimal portfolio shares. To this aim, we extended the model of section 2 to Epstein-Zin preferences with elasticity of intertemporal substitution ranging from 0.1 to 0.5 for each value of risk aversion  $\gamma$ , and both zero or positive values for the stock returns-labour income correlation parameter  $\rho_{sY}$ . Our results show that, although both risk aversion and the elasticity of intertemporal substitution influence consumption and wealth accumulation decisions, it is the degree of risk aversion which affects the sign of the hedging component of optimal stock share, while the elasticity of intertemporal substitution impacts only on its magnitude. The patterns of age profiles for optimal portfolio shares in stocks and bonds displayed in Figure 6 are substantially confirmed, with a relatively higher share of stocks

---

<sup>13</sup>Our results are also robust to a different modification of the asset return structure, namely a reduction of the return on the safe asset with unchanged risk premia on bonds and stocks.

during retirement, consistent with Cocco, Gomes and Maenhout (2005).<sup>14</sup>

**Stochastic replacement rate.** In our setup, with pensions being a fixed proportion of permanent labour income in the final working period, uncertainty over income in retirement is gradually resolved during working life. At the retirement age the fixed replacement rate of  $\lambda = 0.68$  makes retirement income certain. This assumption may underestimate the degree of uncertainty over pensions both during working life and after retirement. In fact, as suggested by the recent experience of a number of countries, important changes in Social Security systems may occur and this very possibility can affect investors' consumption and portfolio allocation decisions throughout their lifetime. We then recognize that Social Security reforms represent an additional source of (retirement) income risk and model it as a stochastic replacement rate.<sup>15</sup> To allow for a possible permanent large fall in pension levels, we model the evolution over time of the replacement rate  $\lambda_t$  as a two-state first-order Markov process: at each date  $t$  during both working life and retirement,  $\lambda_t$  may take either a "high" value  $\lambda_t^H = 0.68$  (i.e. the *status quo ante* pension reforms) or a "low" value  $\lambda_t^L = 0.40$  (i.e. a substantially lower level implied by the pension reform) with the following transition probability matrix

$$P \equiv \begin{bmatrix} p_{LL} & p_{LH} \\ p_{HL} & p_{HH} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.04 & 0.96 \end{bmatrix}$$

where  $p_{ij}$  denotes the probability of being in state  $j$  starting from state  $i$  in the previous period, with  $i, j = L, H$ . We assume that, once implemented, the reform results in a permanent cut of pension levels: hence the lower replacement rate is an absorbing state and  $p_{LL} = 1$ . Moreover, we calibrate the process to allow, in each period, for a small probability that a reform is enacted and set the transition probability  $p_{HL} = 0.04$ . When the investor starts working life at age 20 her replacement rate is at the higher level  $\lambda_t^H = 0.68$ ; then, in each following year, with probability 0.96 no reforms in the Social Security system occur and the replacement rate is unchanged, while with probability 0.04 a reform cuts irreversibly her replacement rate to  $\lambda_t^L = 0.40$ . Figure 7 displays optimal portfolio shares for stocks and bonds (directly comparable to those of Figure 5) with a stochastic replacement rate in the case of risk aversion  $\gamma = 8$  and stock return-labour income correlation  $\rho_{sY} = 0.2$ . Our new simulations reveal that the optimal equity share falls, leaving the inverted age profile during working life unaffected. Savings increase due to higher uncertainty and financial wealth grows at all working ages: thus

---

<sup>14</sup>We also analyzed results based on two other values of the bequest intensity parameter ( $b = 0$  and  $b = 5$ ). This marginally affects investments during working life, thus leaving intact our insights concerning inverted life-cycle profiles.

<sup>15</sup>We owe this observation to the referee.

the ratio of human to financial wealth falls, prompting the reduction in the equity share. Moreover, the possibility of a fall in the replacement rate even past retirement age implies that pension income is no longer riskless for retirees, inducing a smaller portfolio rebalancing towards stocks at the retirement date. Finally, the bond share decreases throughout the entire investor’s working life, as the individual invests a larger portfolio share into the riskless asset, confirming the age-dependent pattern obtained in the fixed replacement rate case.

### 3.3.2 Ex-post optimal portfolio shares heterogeneity

So far, we discussed simulation results in terms of the median optimal portfolio shares across the investors’ population for heterogeneous types of investors. However, in our framework the presence of idiosyncratic labour income shocks may generate substantial ex-post heterogeneity in the pattern of financial wealth accumulation over time, and consequently a potentially wide dispersion of the optimal portfolio shares across individuals of the same type and age but with different levels of accumulated wealth. The degree of heterogeneity in portfolio choices is an important feature of life-cycle asset allocation models for several reasons. First, it can help rationalize observed investors’ behavior, which is characterized by a high degree of heterogeneity both in stock market participation and in the distribution of portfolio shares conditional on age (Gomes and Michaelides 2005; Benzoni, Collin-Dufresne and Goldstein 2007). Second, the amount of heterogeneity of optimal asset allocation may be relevant to the design of pension funds’ default investment options, to be offered to different classes of investors. Therefore, we now focus on the features of the distribution of optimal portfolio shares across the investors’ population, looking at the 5th and 95th percentiles of the cross-sectional distributions conditional on age and type.

In the benchmark case displayed in Figure 3, the investor type is characterized by moderate risk aversion ( $\gamma = 5$ ), labour income innovations uncorrelated with stock returns ( $\rho_{sY} = 0$ ) and “normal” background risk. The distribution of optimal stock and bond shares is highly heterogeneous for both workers and retirees, with the exception of young workers who invest the entire portfolio in stocks to compensate for the relatively riskless nature of their human capital. Heterogeneity of portfolio shares depends on the shape and movements through age of the policy functions, relating portfolio shares to the amount of resources available for investment (cash on hand), portrayed in Figure 2 for our benchmark case. Given the investor’s age, a relatively steep policy function implies that even small differences in the level of accumulated wealth (the increasingly more important component of

cash on hand) result in widely different asset allocation choices: this happens typically to young investors, who are in their initial stage of wealth accumulation. When the amount of background risk is increased (Figure 3, right column) larger savings and wealth accumulation push investors on the flatter portion of their policy functions, determining a gradually decreasing heterogeneity in optimal portfolio shares of stocks and bonds during their working life. After retirement, investors start decumulating financial wealth relatively slowly, due to the operative bequest motive, and move somewhat toward the steeper portion of their relevant policy functions: therefore the dispersion of optimal shares tends to widen again.

The pattern of decreasing ex-post heterogeneity in portfolio shares during working life is enhanced when we shift to an investor type with positive correlation between labour income shocks and stock returns ( $\rho_{sY} = 0.2$ ), keeping risk aversion at the intermediate level  $\gamma = 5$ . Figure 8 shows the policy rules for selected investors' ages (20, 30, 55 and 75) and the dispersion of optimal stock and bond portfolio shares along the life cycle. In both labour income risk scenarios, the distribution of portfolio shares shrinks rapidly around the median value. In the "normal" variance scenario portrayed in panel (a), the shape of the policy functions for 20-year old investors, who start working life with relatively small cash on hand, determines the already mentioned hump-shaped behavior of optimal portfolio shares. From the age of 30 onwards, the policy functions are very close and flat, delivering more similar asset allocation choices throughout the remaining part of working life. After retirement, the different position and shape of the policy rules (as shown for the 75-year old investor) determine an increase in the dispersion of portfolio shares for both stocks and bonds around their median values. When the background risk is larger, the shape of the policy functions for investors in their working life changes dramatically, as shown in panel (b). As regards to stocks, the policy rules for workers of any age are quite close and display a positive slope only for relatively low values of cash on hand; thereafter, they take a very flat shape. As a consequence, the optimal stock share - conditional on wealth - increases for all investors in the early part of their working life to remain constant from the age of 40 until retirement, with a rather limited dispersion of stock allocation choices across the investors' population. During retirement, policy rules become downward sloping and steeper, determining a wider dispersion of optimal portfolio shares when wealth is decumulated. Those patterns are mirrored by the policy rules and portfolio shares for bonds.

The ex-post dispersion of optimal portfolio shares is further reduced when the positive labour shock-stock return correlation interacts with a higher degree of risk aversion ( $\gamma = 8$ ), as displayed in Figure 9, where the policy



rules and the quantiles of the share distributions are shown for stocks, bonds and the riskless asset. Already in the "normal" background risk scenario, policy rules for stocks do not vary through working age and become flat from very low values of cash on hand: therefore, apart from very young investors, almost no dispersion in optimal shares is obtained throughout working life. This behavior is mirrored by the policy rules and portfolio shares for bonds with one remarkable difference: during working life, the policy functions become more steeply downward sloping from relatively high levels of cash on hands, as wealthier investors sharply decrease their optimal bond portfolio share over time in favour of the riskless asset. The resulting distribution of bond shares until retirement is therefore strongly skewed towards lower values. During retirement, the policy rules for both risky assets are relatively steep only for small values of cash on hand, determining a limited dispersion of optimal shares around median values.

The features of the policy rules just discussed are preserved when the background risk is larger, as shown in panel (b) of Figure 9. The policy functions for stocks do not vary through working age and become flat from very low values of cash on hand, determining non-participation in the stock market for the least wealthy (and youngest) investors and a narrow distribution of optimal stock shares throughout working life. Also the policy functions for bonds almost coincide for workers of different age, but their downward-sloping shape now starts from relatively low values of cash on hand. As a consequence, the reduction of the optimal bond portfolio share starts earlier and continues smoothly throughout the entire investors' working life, with a broadly constant (and roughly symmetric) dispersion of bond shares across the population. Such degree of heterogeneity is mirrored by the dispersion of the portfolio shares of the riskless asset, which is now accumulated over time also by young workers and even at relatively low levels of cash on hand.

### **3.3.3 Household portfolios and labour income risk: matching the empirical regularities**

The key implication of our model is that optimal investment profiles are sensitive to parametric *combinations*, giving rise to heterogeneity in optimal asset holdings within age groups. Heterogeneity in observed portfolio shares could thus be explained by combinations of age, volatility of permanent labour income shocks, their correlation with assets returns, and the degree of risk aversion. In particular, relatively low (high) risk aversion and zero (positive) correlations should lead to high (zero or low) equity portfolio shares when young that decrease (increase or stay constant) as retirement approaches. To the best of our knowledge, no empirical research addresses this possibility

by interacting volatility, correlation and - where possible - measures of risk aversion. This may explain why there is little consensus as to the sign of this relationship, on top of the identification problem documented in Ameriks and Zeldes (2004).<sup>16</sup>

As regards to *non-participation* in the equity market, Haliassos and Michaelides (2003) already pointed out the relevance of permanent rather than transitory income shocks. They also realized that a positive correlation was essential, but dismissed it as a plausible explanation on two grounds. First, early estimates attributed higher correlation to more educated groups and entrepreneurs, that are not the typical non-participants in the equity market. Recent investigations by Angerer and Lam (2009) find instead positive correlation between stock returns and labour income in a wide range of occupations such as craftsman, operatives, managers and administrators, farm labourers, private household workers and armed forces. As far as educational attainment is considered, correlation is positive for certificates below a college degree. The second reason for dismissing income shocks as a source of non-participation was the absence of an alternative risky asset with positive risk premium: this pushed up to 0.5 the correlation needed to achieve non-participation. In our model it is sufficient to have a relatively small positive correlation between permanent shocks to income and stock returns (0.2), which translates in an even smaller correlation between total labour income and equity returns (around 0.05). This is because risky bonds are better substitutes for equities than cash.

Also Benzoni, Collin-Dufresne and Goldstein (BCG) (2007) argue for an optimal inverted life cycle stock investment profile, with no participation when young, as we do. The driver of the inversion in BCG lies in the long-run relationship between labour income and stock markets. In particular, BCG model labour income and stock dividends as cointegrated which implies that, over the long term, their dynamics is tightly connected even though over the short term their correlation may be weak due to temporaneous deviations

---

<sup>16</sup>A downward sloping age profile for equities, both in raw data and in regression analysis, appears in Bodie and Crane (1997) who investigate the asset allocation behavior across stocks, cash and fixed income. The cross-sectional survey is restricted to TIAA-CREF participants, who are predominantly employees of colleges and universities. On the contrary, the regression coefficient of equity holdings as a share of liquid wealth on age is not statistically different from zero in the large Survey of Consumer Finance (Heaton and Lucas 2000). Ameriks and Zeldes (2004) find that equity ownership of TIAA-CREF participants has a hump-shape pattern with age, while equity shares conditional on participation are nearly constant across age groups. The inclusion of age and cohort effects leads to equity portfolio shares that increase strongly with age. In Guiso, Jappelli and Terlizzese (1996), age has again a hump-shaped effect on risky asset holdings.

from their equilibrium relationship.<sup>17</sup> The consequent high long-term labour income-stock returns correlation faced by young investors makes their human capital effectively more "stock-like", implying a relatively low optimal investment in stocks. As retirement approaches, the less tight short-term dynamics in labour and stock markets prevails, resulting in a low correlation between labour income and stocks, which makes human capital more "bond-like" and motivates an optimal increase in stock investment. Thus, increased risk taking as retirement approaches in BCG is not due to the resolution of uncertainty concerning the level of future pension income, as suggested in this paper.

The results of the sensitivity analysis summarized in Figure 6 showed that both inverted age profiles of stock investment and non participation at all in the stock market can be optimal for a wide range of parameter combinations. In order to assess to what extent our model can contribute to explain observed patterns of stock market participation rates and asset allocation conditional on participation, we depart here from the representative agent framework and allow for *ex-ante* differences, across the population of 10,000 simulated investors, in risk aversion, background risk and labour income-stock return correlation. In particular, we consider a population composed of three groups of agents and evaluate how much heterogeneity among groups is needed to match as closely as possible empirical life-cycle participation rates and conditional stock holdings, as reported in the 2001 sample of the US Survey of Consumer Finances (Gomes and Michaelides 2005). The first group, accounting for 25% of the overall population, is made up of investors with low risk aversion ( $\gamma = 2$ ) and zero labour income-stock returns correlation, whereas the second group, collecting another 25% of the population, features an intermediate degree of risk aversion ( $\gamma = 5$ ) and a moderately positive labour income-stock returns correlation ( $\rho_{sY} = 0.2$ ); both groups face a "normal" level of labour income shock variance. Finally, the remaining 50% of the population is composed of more risk-averse investors ( $\gamma = 8$ ) with a relatively higher labour income-stock returns correlation ( $\rho_{sY} = 0.4$ ) facing a larger amount of background risk (the "high" variance scenario).

Figure 10 compares the average stock market participation rates (panel (a)) and stock portfolio shares for stock market participants (panel (b)) for different age classes obtained from our model with the corresponding US SCF data. The model is able to closely match the observed life-cycle pattern of equity portfolio shares conditional on participation, yielding an average

---

<sup>17</sup>However, cointegration between labour income and stock dividends in BCD implies a counterfactual one-to-one relationship between the volatilities of dividends and stock returns.

value over the whole life cycle of 50%, to be compared with 54.8% in the data.<sup>18</sup> Also the relatively modest participation rates during working years in the data are well replicated by the model in the absence of any participation costs, yielding an average participation rate over the working life of 57%, in line with the observed value of around 50%. Only for retirees the model counterfactually predicts full participation, whereas the observed rate is about 40%.

## 4 Welfare costs of suboptimal asset allocations

Optimal asset allocation strategies tailor portfolio shares over the investor's life cycle to the characteristics of her labour income. In several instances, the optimal strategies differ substantially from simple investment rules suggested by pension funds and from popular financial advice, broadly sharing the common feature of a decreasing age profile of investment in the riskier assets. In order to provide a quantitative assessment of the welfare loss associated with adopting such simpler rules instead of optimal life-cycle strategies, we consider three alternative asset allocation patterns. The first is a typical "age rule", whereby the risky portfolio share is set at 100 minus the investor's age and equally allocated between stocks and bonds.<sup>19</sup> The second alternative (denoted as "target-date fund (TDF) rule") is designed to come closer to actual strategic asset allocation patterns adopted by Target-Date Funds. As shown in Figure 11, the stock portfolio share is set at 90% until the age of 40, is gradually decreased over the remaining working life up to 50% at the retirement age (65), and is further reduced in the early retirement period to reach a bottom of 30% at the age of 72. Over the same life span, the share of bonds increases from 10% to 40% at 65 and further up to 45% at 72; finally, the riskless asset is accumulated only in the final stage of the working life, to

---

<sup>18</sup>We focus on average values over the whole life cycle. As highlighted by Gomes and Michaelides (2005), participation and conditional stock holding age profiles obtained from the data are not robust to the cohort and time effects assumptions.

<sup>19</sup>In a two-asset framework, including only a riskless asset and stocks, several variants of the above "age rule" are adopted in the literature. For instance, Cocco, Gomes and Maenhout (2005) consider a rule whereby the portfolio share of stocks is 100% until the age of 40 and decreases linearly thereafter, to reach 50% at the age of 60. In Bodie and Treussard (2007), the investor starts saving for retirement 40 years before the target retirement date, setting the initial share of stocks at 80% and letting it fall to 40% at the target date.

reach a share of 10% at 65 and 25% at the age of 72.<sup>20</sup> The third alternative strategy fixes portfolio shares at 1/3 for each financial asset in our model: this mirrors the “1/ $N$ ” rule of DeMiguel, Garlappi and Uppal (2008), that systematically outperforms several optimal asset allocation strategies in *ex post* portfolio experiments.

The metric used to perform welfare comparisons is the standard consumption-equivalent variation as in Cocco, Gomes and Maenhout (2005) and Winter, Schlafmann and Rodepeter (2012): for each suboptimal asset allocation rule we compute the percentage increase in consumption required by the investor to obtain the same level of expected utility warranted by the optimal life-cycle strategy.<sup>21</sup> Table 1 shows the welfare losses associated with the three suboptimal asset allocation rules for several combinations of investors’ risk aversion, background risk, and correlation between innovations to labour income and stock returns.

In the benchmark case of moderate risk aversion ( $\gamma = 5$ ), uncorrelated labour income-stock return innovations ( $\rho_{sy} = 0$ ) and a “normal” level of labour income shock variance, the “age rule” and the “1/3” strategies entail losses in the range of 1-2% of life-time consumption, whereas investors following the *TDF* rule lose only 0.1%. In this case, individuals adopting the optimal investment strategy (as shown in Figure 3, left column), after a first period of liquidity-constrained working life, start wealth accumulation with a high share of stocks until the age of 40, and turn gradually to bonds as their retirement date approaches. Among the alternative investment strategies, the *TDF* rule imposes an age profile of investment in stocks and bonds which is closer to the optimal pattern than the other two alternatives, resulting in a more limited welfare loss. Very similar results emerge when the labour income-stock return correlation is set to the slightly higher value of 0.2.

When the “high” labour income shock variance scenario is considered, larger welfare losses for all strategies are obtained and the relative performance of the *TDF* rule depends on the magnitude of the labour income-stock

---

<sup>20</sup>Vanguard (2010) describes a broadly similar age profile for the strategic asset allocation of target-date funds, but with a richer asset class menu including US and international stocks, US nominal investment-grade bonds, Treasury inflation-protected securities and cash.

<sup>21</sup>The consumption-equivalent variation is obtained by simulating consumption and wealth accumulation choices of 10,000 agents following the optimal asset allocation strategy and each of the alternative (suboptimal) investment rules, and deriving the associated expected discounted life-time utility levels. From the average expected discounted utility across individuals, the constant consumption stream needed to compensate investors (in each period and state) for the adoption of suboptimal strategies is computed using the *CRRRA* utility function.

return correlation: if  $\rho_{sy} = 0$  the *TDF* rule still outperforms alternative sub-optimal strategies, whereas if  $\rho_{sy} = 0.2$  it yields the worst result with a welfare loss of about 4%. In the latter case, as shown in the right column of Figure 4, the optimal investment strategy displays an inverted age profile for stocks, with investors entering the stock market only around the age of 25 and gradually increasing the stock share over time to reach 20% at the age of 40 for the median investor. Instead, according to the *TDF* rule, the stock share is kept at 90% until 40, and only slowly reduced thereafter. The difference between the optimal strategy and the *TDF* rule is enhanced when a large amount of background risk and a moderately positive labour income-stock return correlation are combined with a relatively higher degree of risk aversion ( $\gamma = 8$ ). In this case, according to the optimal life-cycle profile (Figure 5, right column), the investor enters the stock market only in her late 20s, increases the stock share up only to 15% at the age of 45, and keeps it constant at that level over her remaining working life. Under the alternative *TDF* rule, the high risk and expected return on her financial portfolio (with a 90% stock share over the first two decades of working life) induce the investor to increase savings and wealth accumulation: consumption is therefore substantially lower than optimal over the first half of the working life, determining a sizeable decrease in expected utility that is not compensated by higher than optimal consumption levels over the remaining years of work and during retirement. Such excessive saving and wealth accumulation under the *TDF* rule yield a remarkably large welfare loss, in the range of 15-30% of lifetime consumption, whereas the other suboptimal investment strategies determine more limited welfare losses (1-3%).

Overall, the results of our welfare analysis show that investment strategies that overlook labour income characteristics of pension plan participants may entail substantial losses. In particular, the equally weighted (“1/3”) portfolio rule performs better than the “age rule”, showing lower welfare losses for most parameter combinations.<sup>22</sup> Importantly, the magnitude of welfare losses for both strategies amounts at most to 3% of life-time consumption. In this respect, a “1/*N*” strategy, entailing portfolio shares invariant to age, challenges the choice of *TDF* as default investment rule.

---

<sup>22</sup>For both alternatives, welfare losses generally fall as risk aversion increases, since high risk aversion implies reduced optimal exposure to the stock market, and risky assets in general.

## 5 Conclusions

The persistence of labour income shocks implies that a young person faces large uncertainty concerning future income and social security pension, especially in the presence of a high variance of permanent income shocks. As retirement age approaches such uncertainty resolves, making the worker more willing to take on equity market risk. Permanent shocks to labour income risk are thus able to generate, in conjunction with minor changes in other parameters, optimal equity portfolio shares that increase as retirement approaches and non-participation by young workers in the equity market. They also imply high heterogeneity in portfolio shares conditional on age, as a function of past work histories. Thus the simple life-cycle model with risky bonds is potentially able to account for several empirical regularities that so far appeared at odds with it.

Our analysis also questions the use of a one-size-fits-all default investment strategy for pension funds. A Target Date Fund investment rule, that is close to optimal when labour income risk and risk aversion are relatively low, determines deviations from the optimal life-cycle consumption resulting in large welfare losses for investors with relatively high risk aversion and background risk.

Our model considers workers as being able to know with certainty the parameters characterizing the labour income process, even at the beginning of their career, as well as the process generating financial returns, even forty years in advance. Accounting for parameter uncertainty would reduce the attractiveness of equities relative to other assets as in Barberis (2000), the more so the further away is retirement age. We leave this important extension for future work.

## References

- [1] Ameriks J., and S.P. Zeldes (2004), How do Portfolio Shares vary with Age?, GSB Columbia University and the NBER
- [2] Angerer X., and P.S. Lam (2009), Income Risk and Portfolio Choice: An Empirical Study, *Journal of Finance*, 64(2), 1037–1055
- [3] Barberis, N. (2000), Investing for the Long Run When Returns Are Predictable, *Journal of Finance*, 55, 225–64.
- [4] Benzoni L. (2008), Investing over the Life Cycle with Long-Run labour Income Risk, mimeo

- [5] Benzoni L., P. Collin-Dufresne and R.G. Goldstein (2007), Portfolio Choice over the Life-Cycle when the Stock and labour Markets are Coin-  
tegrated, *Journal of Finance*, 62, 5, 2123-2167
- [6] Betermier S., Jansson T., Parlour C.A. and J. Walden (2012), Hedging  
Labour Income Risk, *Journal of Financial Economics*, 105, 622-639
- [7] Blake D. (2008), It is all back to front: critical issues in the design of  
defined contribution pension plans, in Broeders D., S. Eijffinger and A.  
Houben (eds.), *Frontiers in Pension Finance*, Edward Elgar
- [8] Bodie Z., and D.B. Crane (1997), Personal Investing: Advice, Theory,  
and Evidence, *Financial Analysts Journal*, November/December, 1997
- [9] Bodie Z., R.C. Merton and W. Samuelson (1992), Labour Supply Flex-  
ibility and Portfolio Choice in a Life Cycle Model, *Journal of Economic  
Dynamics and Control*, 16, 427-449
- [10] Bodie, Zvi and J. Treussard (2007), Making Investment Choices as Sim-  
ple as Possible, but Not Simpler, *Financial Analyst Journal*, 63, 3, 42-47
- [11] Boyle G. W. and G. A. Guthrie (2005), Human Capital and Popular  
Investment Advice, *Review of Finance*, 9, 134-164
- [12] Cairns A.J.G., D. Blake and K. Dowd (2006), Stochastic Lifestyling:  
Optimal Dynamic Asset Allocation for Defined Contribution Pension  
Plans, *Journal of Economic Dynamics and Control*, 30, 843-877
- [13] Campbell J. Y., J. Cocco, F. Gomes and P. Maenhout (2001), Investing  
Retirement Wealth: a Life-Cycle Model, in Campbell J.Y. and M. Feld-  
stein (eds.), *Risk Aspects of Investment-Based Social Security Reform*,  
University of Chicago Press
- [14] Campbell J. Y. and L. Viceira (2002) *Strategic Asset Allocation: Port-  
folio Choice for Long-Term Investors*, Oxford University Press
- [15] Cocco J.F. (2004), Portfolio Choice in the Presence of Housing, *Review  
of Financial Studies*, 18, 535-567
- [16] Cocco J., F. Gomes and P. Maenhout (2005), Consumption and Portfolio  
Choice over the Life Cycle, *Review of Financial Studies*, 18, 491-533.
- [17] Davis S.J. and P. Willen (2000), Occupation-Level Income Shocks and  
Asset Returns: Their Covariance and Implications for Portfolio Choice,  
Working paper, GSB University of Chicago



- [18] DeMiguel V., L. Garlappi and R. Uppal (2008), Optimal versus naive diversification: how inefficient is the 1/N portfolio strategy?, *Review of Financial Studies*, 1915-1953
- [19] Eiling E. (2013), Industry-Specific Human Capital, Idiosyncratic Risk and the Cross-Section of Expected Stock Returns, *Journal of Finance*, 68, 43-84
- [20] Epstein L. and S. Zin (1989), Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework, *Econometrica*, 57, 937-969
- [21] Fugazza C., M. Guidolin and G. Nicodano (2009), Time and Risk Diversification in Real Estate Investments: the Ex-Post Performance, *Real Estate Economics*, 37(3), 341-381
- [22] Fugazza C., M. Guidolin and G. Nicodano (2010), 1/N and long run optimal portfolios: results for mixed asset menus, St. Louis Fed WP 2010-003
- [23] Giacomel E. and A.I. Rinaldi (2008), Information to Members of DC Pension Plans. Conceptual Framework and International Trends, IOPS Working Paper, 5
- [24] Gomes F.J., L.J. Kotlikoff and L.M. Viceira (2008), Optimal Life-Cycle Investing with Flexible labour Supply: A Welfare Analysis of Life-Cycle Funds, *American Economic Review*, 98, 2, 297-303.
- [25] Gomes F.J. and A. Michaelides (2004), A Human Capital Explanation for an Asset Allocation Puzzle, mimeo.
- [26] Gomes F.J. and A. Michaelides (2005), Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence, *Journal of Finance*, 60, 869-904.
- [27] Goyal, A. and I. Welch (2008), A Comprehensive Look at the Empirical Performance of Equity Premium Prediction, *Review of Financial Studies*, 21 (4): 1453-1454.
- [28] Guiso L., T. Jappelli and D. Terlizzese (1996), Income Risk, Borrowing Constraints and Portfolio Choice, *American Economic Review*, 86(1), 158-162.
- [29] Haliassos M. and A. Michaelides (2003), Portfolio Choice and Liquidity Constraints, *International Economic Review*, (44)1, 143-177.

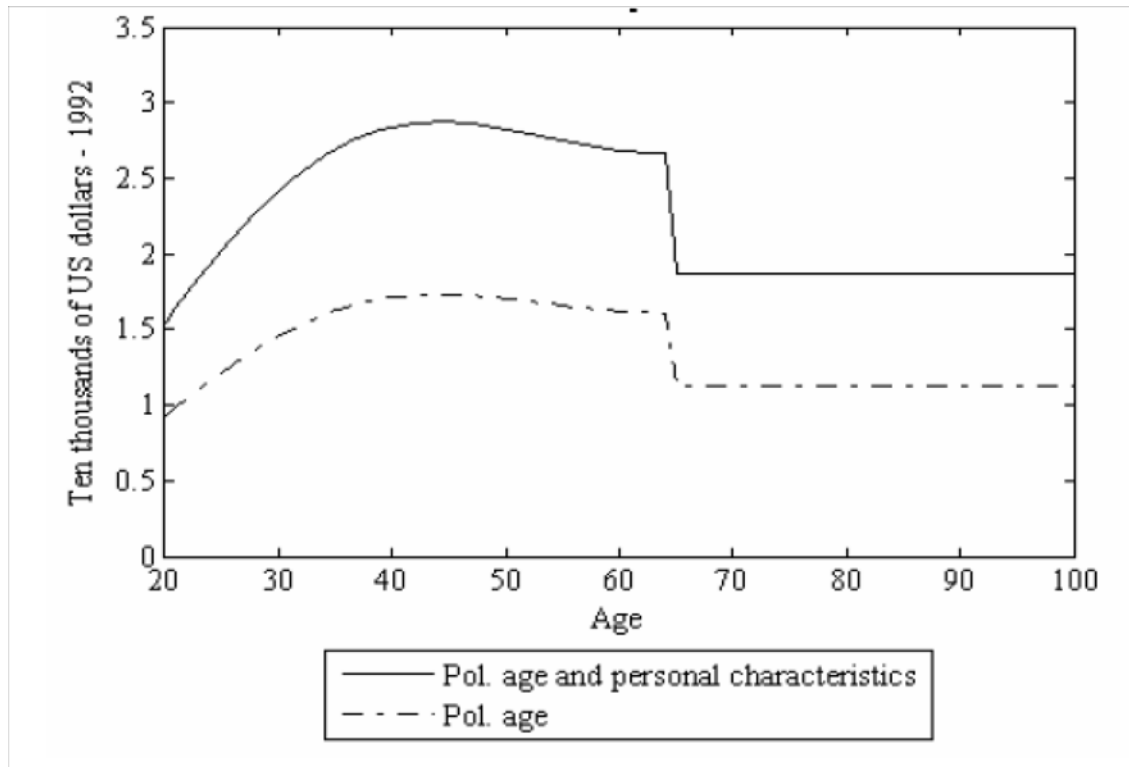
- [30] Heaton J. and D. Lucas (2000), Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk. *Journal of Finance*, 55(3), 1163-1198.
- [31] Koijen R.S.J, T. Nijman and B.J.M. Werker (2010), When Can Life Cycle Investors Benefit from Time Varying Risk Premia?, *Review of Financial Studies*, 23, 741-780.
- [32] Koijen R.S.J, T. Nijman and B.J.M. Werker (2011), Optimal annuity risk management, *Review of Finance*, 15(4), 799-833.
- [33] Michaelides A. (2002), Portfolio Choice, Liquidity Constraints and Stock Market Mean Reversion, available at papers.ssrn.com.
- [34] Munk, C. and C. Sørensen (2010), Dynamic asset allocation with stochastic income and interest rates, *Journal of Financial Economics*, 96(3), 433-462.
- [35] OECD (2007), Pensions at a Glance - Public Policies Across OECD Countries, OECD Publishing.
- [36] Poterba J. , J. Rauh, S. Venti and D. Wise (2007), Defined Contribution Plans, Defined Benefit Plans, and the Accumulation of Retirement Wealth, *Journal of Public Economics*, 91, 2062-2086
- [37] Ruffino D. (2008), Individual labour-flexibility and rational portfolio allocation: Businessman risk and employer stock ownership, mimeo.
- [38] Samuelson P.A. (1969), Lifetime Portfolio Selection by Dynamic Stochastic Programming”, *Review of Economics and Statistics*, 51, 239-246.
- [39] Vanguard (2010), Vanguard’s Approach to Target-Date Funds, Vanguard Research, September.
- [40] Viceira L.M. (2001), Optimal Portfolio Choice for Long-Horizon Investors with Nontradable labour Income, *Journal of Finance*, 56, 433-470.
- [41] Viceira, L.M. (2009), Life-Cycle Funds, in A. Lusardi ed., *Overcoming the Saving Slump: How to Increase the Effectiveness of Financial Education and Saving Programs*, University of Chicago Press.
- [42] Winter J.K., K. Schlafmann and R. Rodepeter (2012), Rules of Thumb in Life-Cycle Saving Decisions, *Economic Journal*, 122(3), 479-501.

**Table 1.** Welfare losses from suboptimal life-cycle  
asset allocation strategies  
(percentage of life-time consumption)

	"normal" labour income shock variance			"high" labour income shock variance		
	"age-rule"	"TDF rule"	1/N	"age-rule"	"TDF rule"	1/N
Risk aversion $\gamma = 5$						
$\rho_{sy} = 0$	1.6	0.1	1.4	1.8	1.3	2.0
$\rho_{sy} = 0.2$	1.1	0.1	1.0	2.1	3.9	1.6
Risk aversion $\gamma = 8$						
$\rho_{sy} = 0$	1.0	0.3	0.7	1.1	14.8	1.3
$\rho_{sy} = 0.2$	0.7	1.1	0.5	3.1	31.0	3.2

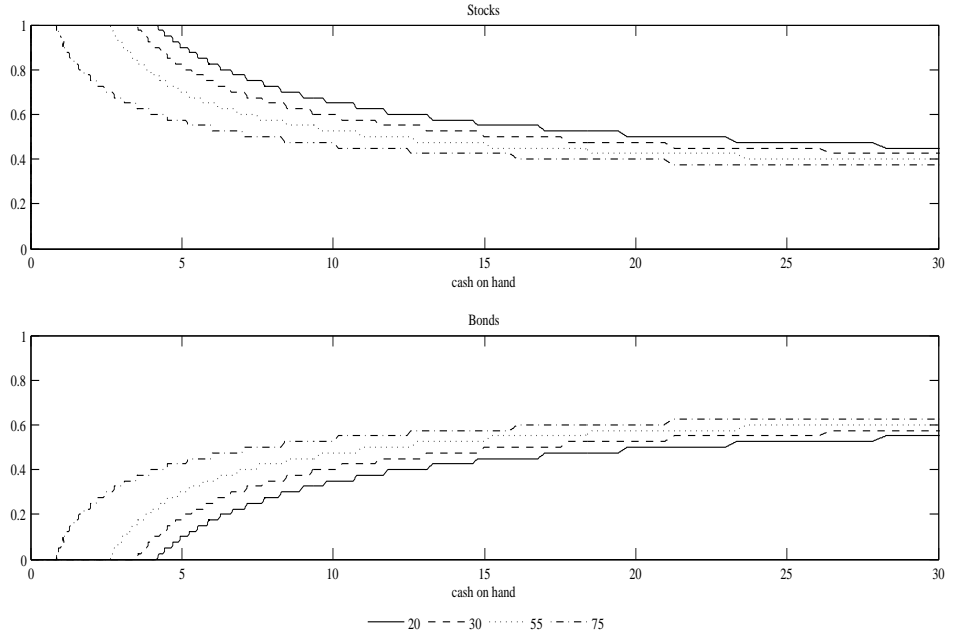
Life-time welfare losses are expressed as the percentage increases in the constant consumption level that would ensure the same expected discounted utility as with optimal asset allocation strategies. "Normal" labour income shock variance:  $\sigma_\varepsilon^2 = 0.0106$  and  $\sigma_n^2 = 0.0738$ ; "high" labour income shock variance:  $\sigma_\varepsilon^2 = 0.0418$  and  $\sigma_n^2 = 0.296$ . Other relevant parameters: replacement ratio  $\lambda = 0.68$ , correlation between stock and bond returns  $\rho_{sb} = 0.2$ .

**Figure 1.** Labour income process



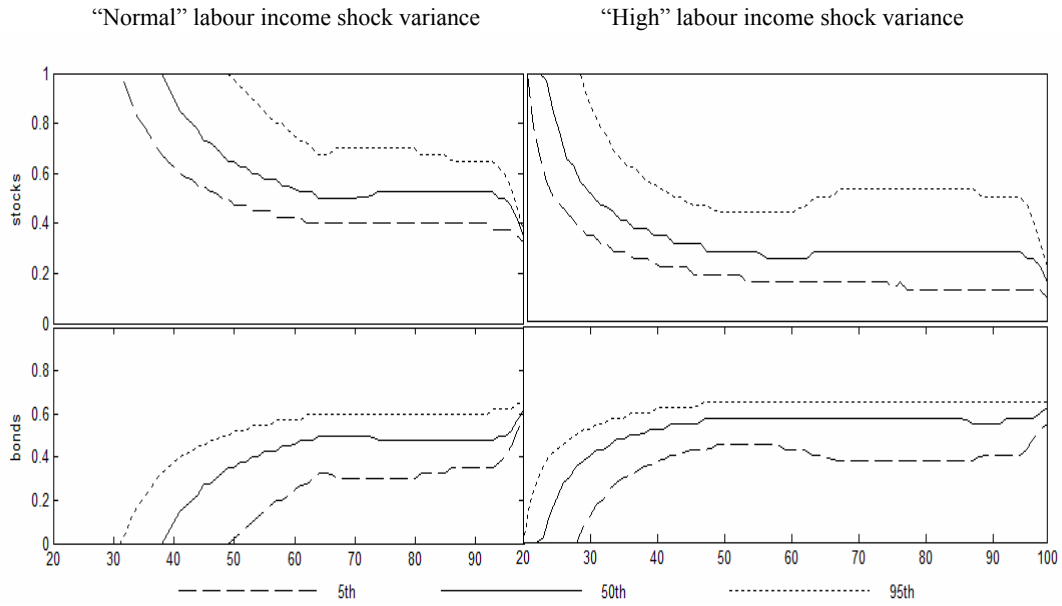
The figure reports the fitted polynomial in age (dash-dotted line) and in age and personal characteristics (continuous line) derived using the calibration in Cocco et al. (2005) for households with high school education.

**Figure 2.** Optimal policy rules for stocks and bonds in the benchmark case



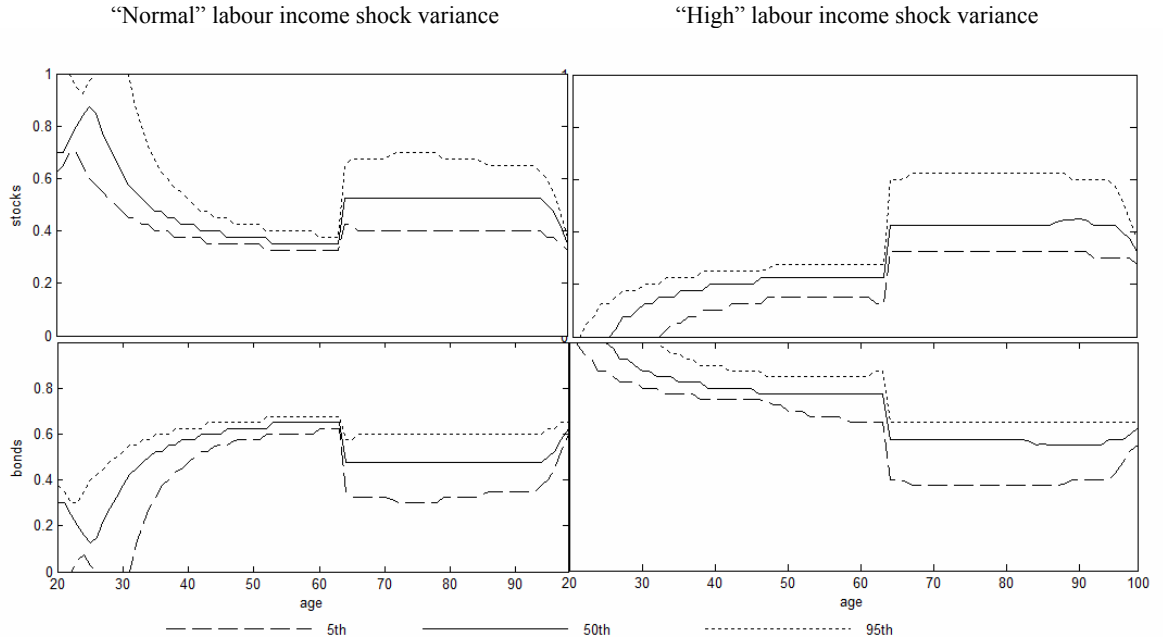
The figure shows the portfolio rules for stocks and bonds as a function of normalized cash on hand for individuals of age 20, 30, 55 and 75.

**Figure 3.** Portfolio shares of stocks and bonds in the benchmark case



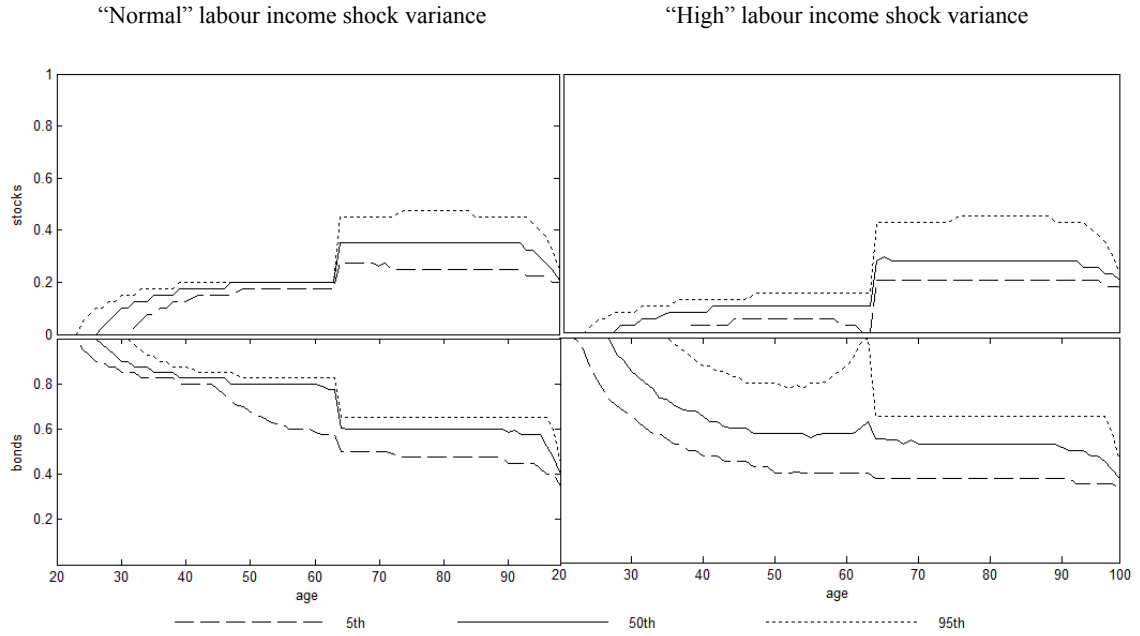
The figure displays the 5<sup>th</sup>, 50<sup>th</sup> and 95<sup>th</sup> percentiles for the simulated stock and bond profiles for individuals of age 20 to 100. Left column: “normal” labour income shock variance ( $\sigma_\varepsilon^2 = 0.0106$  and  $\sigma_n^2 = 0.0738$ ). Right column: “high” labour income shock variance ( $\sigma_\varepsilon^2 = 0.0418$  and  $\sigma_n^2 = 0.296$ ). Other relevant parameters: risk aversion  $\gamma=5$ , replacement ratio  $\lambda=0.68$ , correlation between shocks to labour income and stock returns  $\rho_{sY}=0$ , correlation between stock and bond returns  $\rho_{sb}=0.2$ .

**Figure 4.** Portfolio shares of stocks and bonds with positive labour income-stock returns correlation ( $\rho_{sY}=0.2$ )



The figure displays the 5<sup>th</sup>, 50<sup>th</sup> and 95<sup>th</sup> percentiles for the simulated stock and bond profiles for individuals of age 20 to 100. Left column: “normal” labour income shock variance ( $\sigma_{\varepsilon}^2 = 0.0106$  and  $\sigma_n^2 = 0.0738$ ). Right column: “high” labour income shock variance ( $\sigma_{\varepsilon}^2 = 0.0418$  and  $\sigma_n^2 = 0.296$ ). Other relevant parameters: risk aversion  $\gamma=5$ , replacement ratio  $\lambda=0.68$ , correlation between shocks to labour income and stock returns  $\rho_{sY}=0.2$ , correlation between stock and bond returns  $\rho_{sb}=0.2$ .

**Figure 5.** Portfolio shares of stocks and bonds with positive labour income-stock returns correlation ( $\rho_{sY}=0.2$ ) and high risk aversion ( $\gamma=8$ )

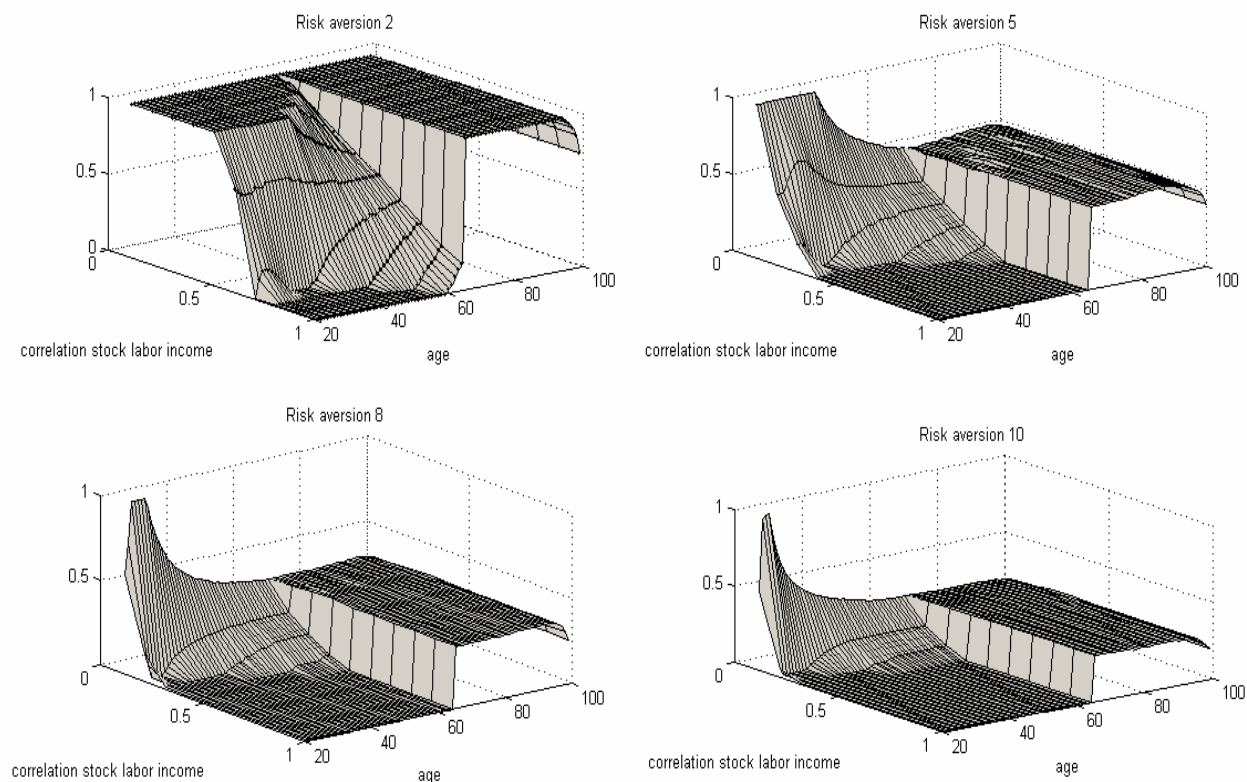


The figure reports the 5<sup>th</sup>, 50<sup>th</sup> and 95<sup>th</sup> percentiles for the simulated stock and bond profiles for individuals of age 20 to 100. Left column: “normal” labour income shock variance ( $\sigma_{\varepsilon}^2 = 0.0106$  and  $\sigma_n^2 = 0.0738$ ). Right column: “high” labour income shock variance ( $\sigma_{\varepsilon}^2 = 0.0418$  and  $\sigma_n^2 = 0.296$ ). Other relevant parameters: risk aversion  $\gamma=8$ , replacement ratio  $\lambda=0.68$ , correlation between shocks to labour income and stock returns  $\rho_{sY}=0.2$ , correlation between stock and bond returns  $\rho_{sb}=0.2$ .

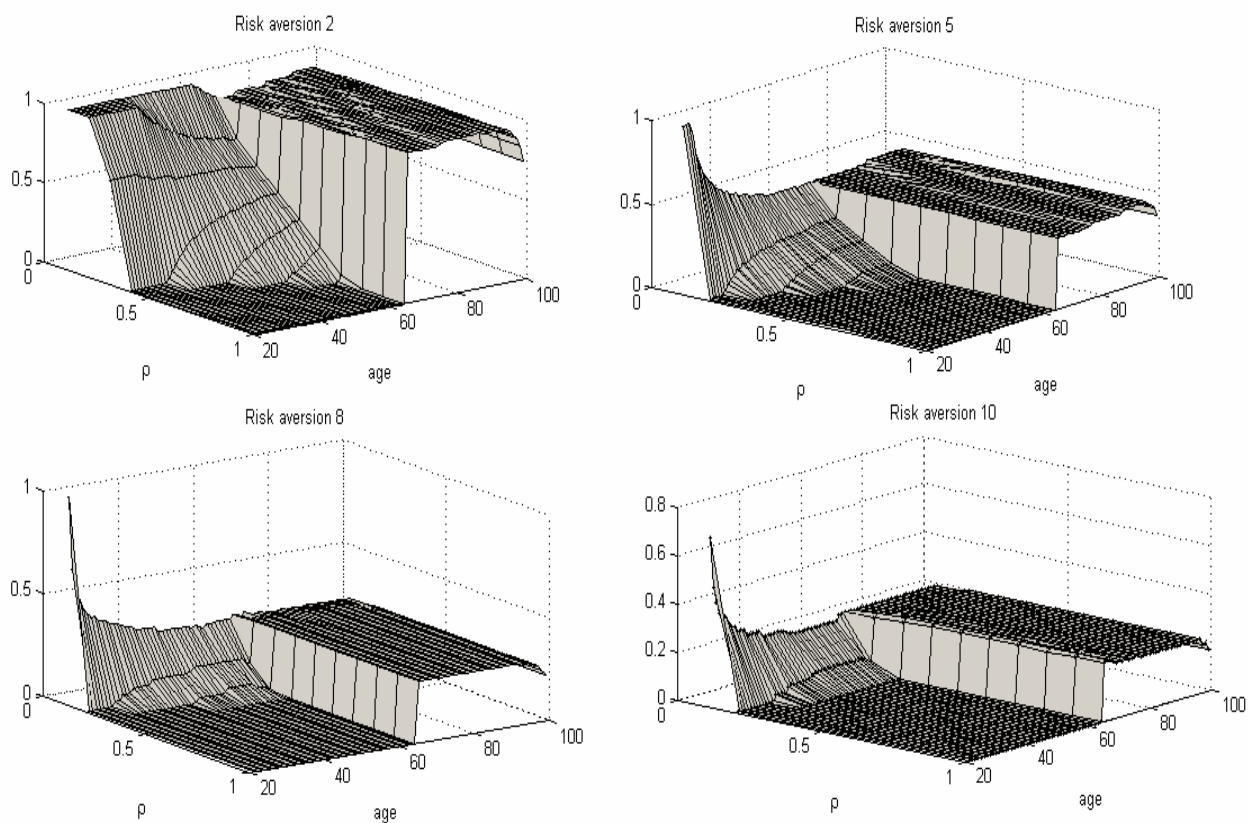


**Figure 6.** Sensitivity of median portfolio share of stocks to risk aversion, stock returns-labour income correlation and labour income shock variance

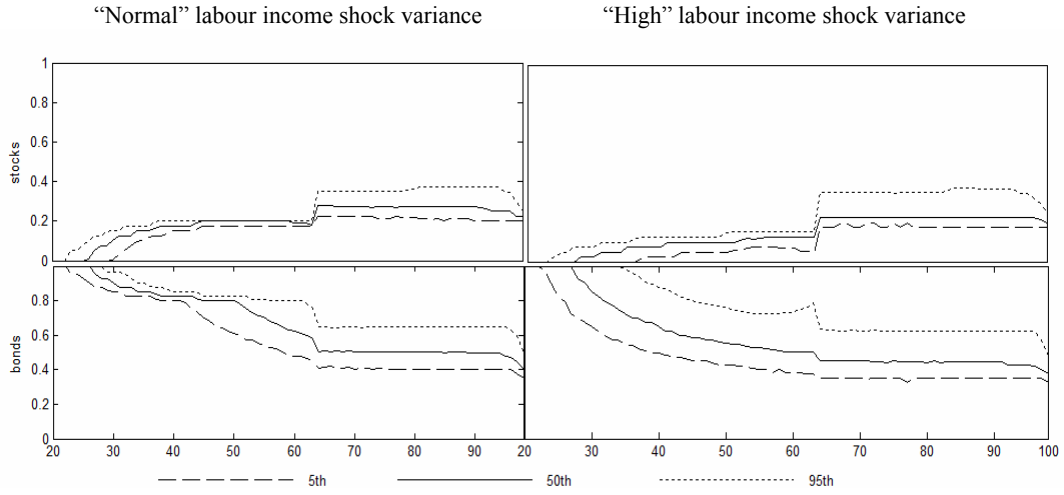
(a) “normal” labor income shock variance



(b) “high” labor income shock variance



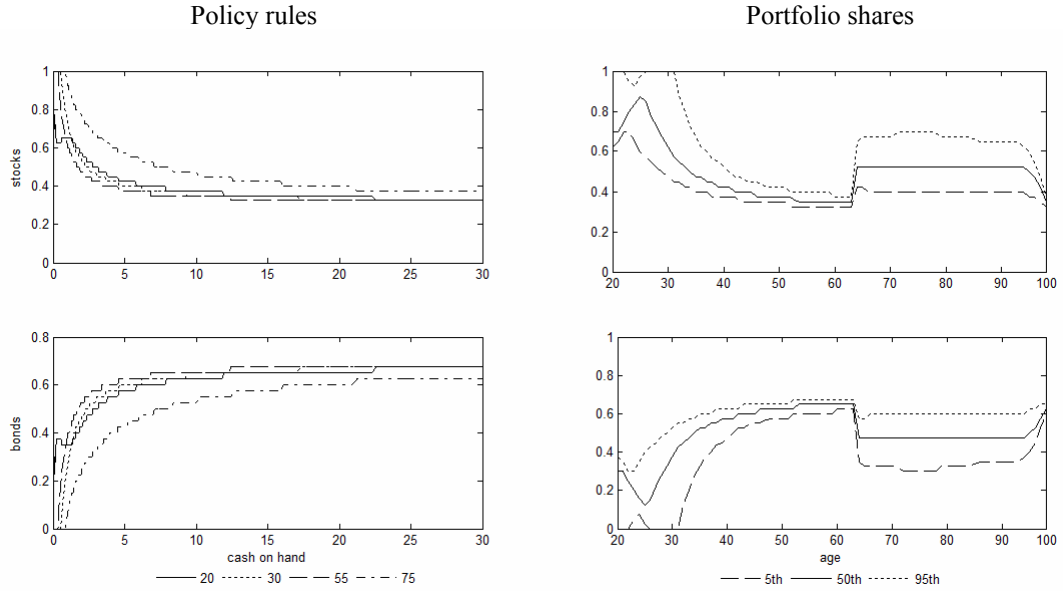
**Figure 7.** Portfolio shares of stocks and bonds with positive labour income-stock returns correlation ( $\rho_{sY}=0.2$ ), high risk aversion ( $\gamma=8$ ) and stochastic replacement ratio



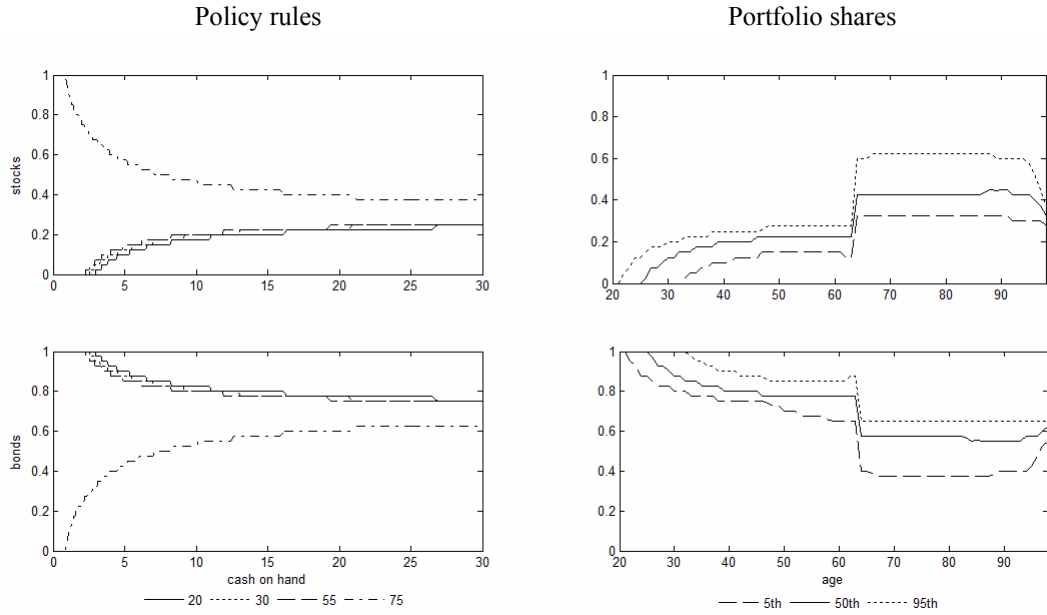
The figure reports the 5<sup>th</sup>, 50<sup>th</sup> and 95<sup>th</sup> percentiles for the simulated stock and bond profiles for individuals of age 20 to 100. Left column: “normal” labour income shock variance ( $\sigma_\varepsilon^2 = 0.0106$  and  $\sigma_n^2 = 0.0738$ ). Right column: “high” labour income shock variance ( $\sigma_\varepsilon^2 = 0.0418$  and  $\sigma_n^2 = 0.296$ ). The replacement rate  $\lambda$  is stochastic and modelled as described in Section 3.3.1. The correlation between stock and bond returns innovations is  $\rho_{sb}=0.2$ .

**Figure 8.** Optimal policy rules and portfolio shares heterogeneity with positive labor income-stock returns correlation ( $\rho_{sY}=0.2$ ) and moderate risk aversion ( $\gamma=5$ )

(a) “normal” labor income shock variance

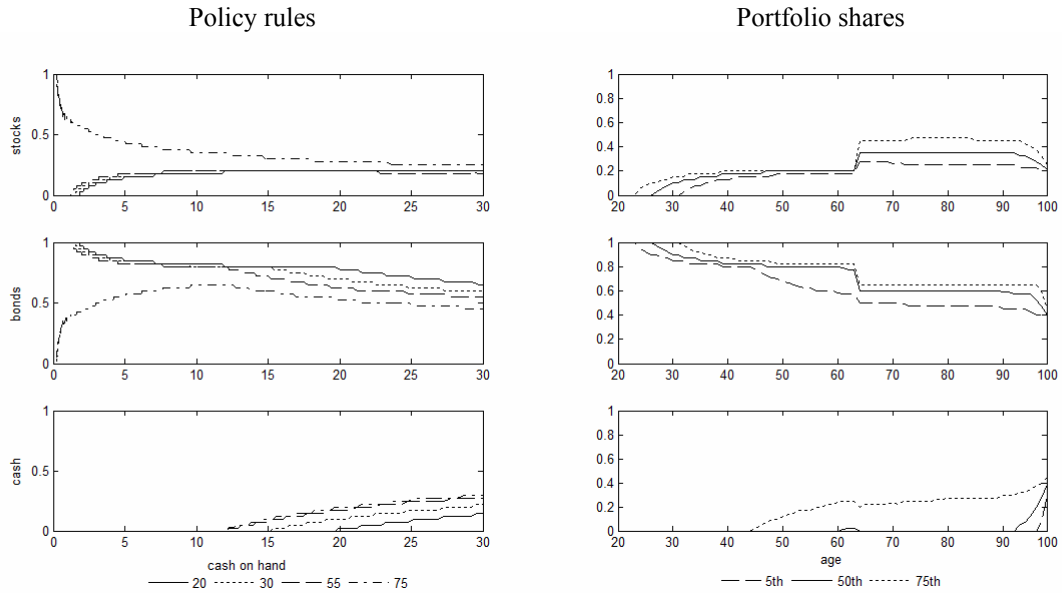


(b) “high” labor income shock variance

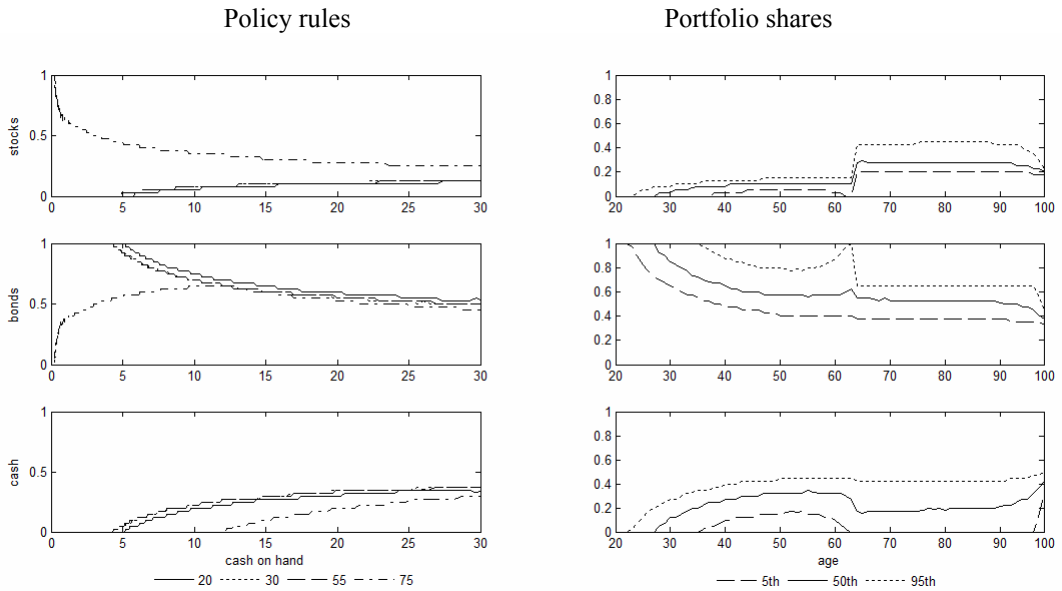


**Figure 9.** Optimal policy rules and portfolio shares heterogeneity with positive labor income-stock returns correlation ( $\rho_{sY}=0.2$ ) and high risk aversion ( $\gamma=8$ )

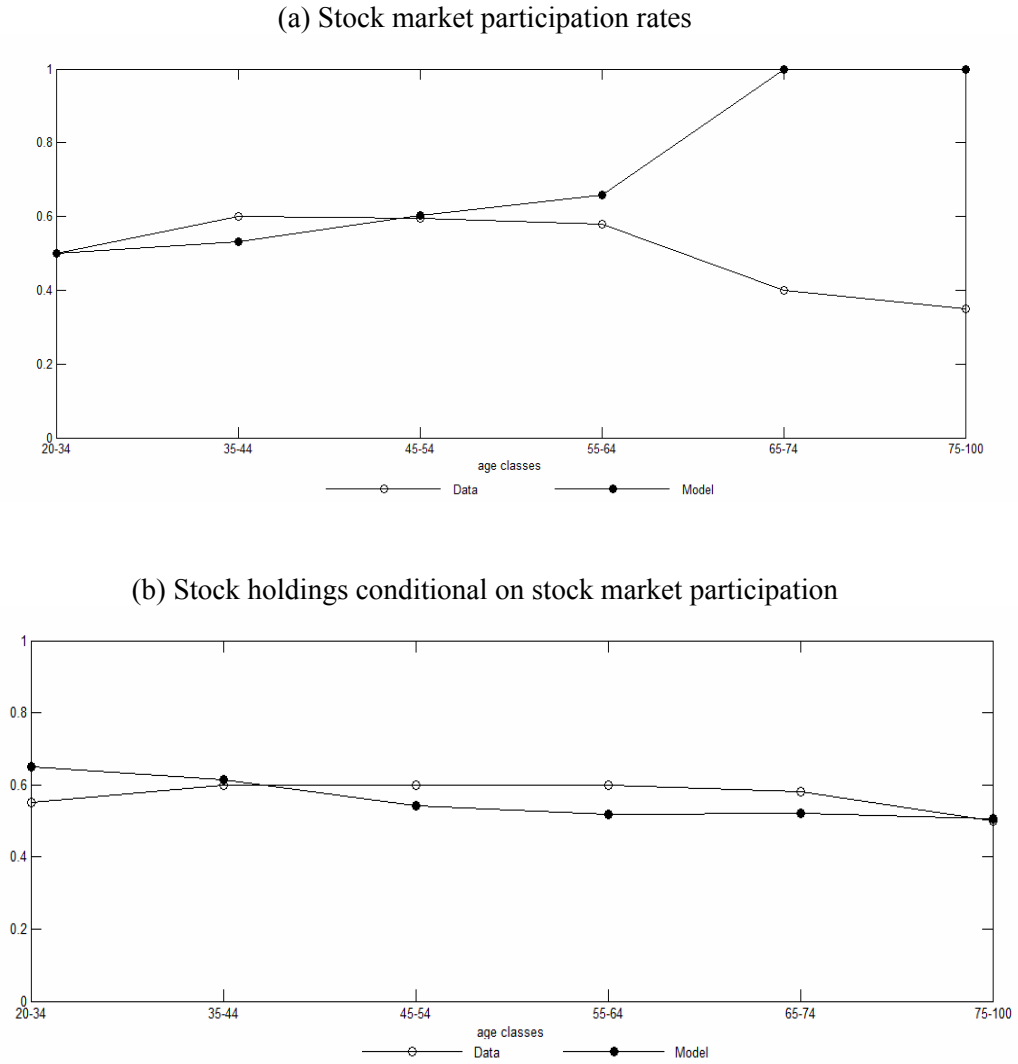
(a) “normal” labor income shock variance



(b) “high” labor income shock variance



**Figure 10.** Predicted and empirical stock market participation rates and conditional stock holdings



The figure shows stock market participation rates (panel a) and average portfolio shares invested in stocks conditional on participation in the stock market (panel b) for age classes comparing data from the 2001 sample of the US Survey of Consumer Finances (“Data”) and results obtained from simulating a population composed of three groups of investors: (i)  $\gamma = 2$ ,  $\rho_{sY} = 0$  and “normal” labour income shock variance; (ii)  $\gamma = 5$ ,  $\rho_{sY} = 0.2$  and “normal” labour income shock variance; (iii)  $\gamma = 8$ ,  $\rho_{sY} = 0.4$  and “high” labour income shock variance, with weights 25%, 25% and 50% respectively. Other relevant parameters: replacement ratio  $\lambda=0.68$ , correlation between stock and bond returns  $\rho_{sb} = 0.2$ .

**Figure 11.** Portfolio shares for stocks, bonds and the riskless asset from a typical Target-Date Fund asset allocation strategy.

