

南京大学 ACM-ICPC 集训队代码模版库



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1 General

1.1 Code library checksum

```
ab14 #!/usr/bin/python3
c502 import re, sys, hashlib
427e
f7db for line in sys.stdin.read().strip().split("\n") :
ddf5     print(hashlib.md5(re.sub(r'\s|//[.]*', '', line).encode('utf8')).hexdigest()
        [-4:], line)
```

1.2 Makefile

```
dab2 .PHONY : run
427e
207e $(t) : $(t).cpp
2d16     g++ --std=c++14 -Wall -D__LOCAL_DEBUG__ -fsanitize=undefined -fsanitize=
        address -ggdb -pipe -o $@ $<
427e
5f25 run : $(t)
bf3e     ./$<(t) < $(t).in
```

1.3 .vimrc

```
914c set nocompatible
733d syntax on
6bbc colorscheme slate
7db5 set number
b0e3 set cursorline
061b set shiftwidth=2
8011 set softtabstop=2
a66d set tabstop=2
d23a set expandtab
5245 set magic
740c set smartindent
bee8 set backspace=indent,eol,start
815d set cmdheight=1
0a40 set laststatus=2
1c67 set whichwrap=b,s,<,>,[,]
```

1.4 Stack

```
const int STK_SZ = 2000000;
char STK[STK_SZ * sizeof(void)];
void *STK_BAK;

#if defined(__i386__)
#define SP "%esp"
#elif defined(__x86_64__)
#define SP "%rsp"
#endif

int main() {
    asm volatile("movl SP, %0; movl %1, SP: =g(STK_BAK):g(STK+sizeof(STK));");
    ;

    // main program

    asm volatile("movl %0, SP::g(STK_BAK);");
    return 0;
}
```

1.5 Template

```
#include <bits/stdc++.h>
using namespace std;

#ifdef __LOCAL_DEBUG__
# define _debug(fmt, ...) fprintf(stderr, "[%s] " fmt "\n", \
    __func__, ##__VA_ARGS__)
#else
# define _debug(...) ((void) 0)
#endif

#define rep(i, n) for (int i=0; i<(n); i++)
#define Rep(i, n) for (int i=1; i<=(n); i++)
#define range(x) begin(x), end(x)
typedef long long LL;
typedef unsigned long long ULL;
```

2 Miscellaneous Algorithms

2.1 2-SAT

```

0f42 const int MAXN = 100005;
03a9 struct twoSAT{
5c83     int n;
8f72     vector<int> G[MAXN*2];
d060     bool mark[MAXN*2];
b42d     int S[MAXN*2], c;
427e
d34f     void init(int n){
b985         this->n = n;
f9ec         for (int i=0; i<n*2; i++) G[i].clear();
0609         memset(mark, 0, sizeof(mark));
95cf     }
427e
3bd5     bool dfs(int x){
bd70         if (mark[x^1]) return false;
c96a         if (mark[x]) return true;
fd23         mark[x] = true;
4bea         S[c++] = x;
1ce6         for (int i=0; i<G[x].size(); i++)
d942             if (!dfs(G[x][i])) return false;
3361         return true;
95cf     }
427e
5894     void add_clause(int x, bool xval, int y, bool yval){
6afe         x = x * 2 + xval;
e680         y = y * 2 + yval;
81cc         G[x^1].push_back(y);
6835         G[y^1].push_back(x);
95cf     }
427e
d0cb     bool solve() {
7c39         for (int i=0; i<n*2; i+=2){
e63f             if (!mark[i] && !mark[i+1]){
88fb                 c = 0;
f4b9                 if (!dfs(i)){
3f03                     while (c > 0) mark[S[--c]] = false;
86c5                     if (!dfs(i+1)) return false;
95cf                 }
95cf             }

```

```

    }
    return true;
}

inline bool value(unsigned i){return mark[2*i+1];}
};

```

95cf
3361
95cf
427e
5f0a
329b

2.2 Knuth's optimization

```

int n;
int dp[256][256], dc[256][256];

template <typename T>
void compute(T cost) {
    for (int i = 0; i <= n; i++) {
        dp[i][i] = 0;
        dc[i][i] = i;
    }
    rep (i, n) {
        dp[i][i+1] = 0;
        dc[i][i+1] = i;
    }
    for (int len = 2; len <= n; len++) {
        for (int i = 0; i + len <= n; i++) {
            int j = i + len;
            int lbnd = dc[i][j-1], rbnd = dc[i+1][j];
            dp[i][j] = INT_MAX / 2;
            int c = cost(i, j);
            for (int k = lbnd; k <= rbnd; k++) {
                int res = dp[i][k] + dp[k][j] + c;
                if (res < dp[i][j]) {
                    dp[i][j] = res;
                    dc[i][j] = k;
                }
            }
        }
    }
};

```

5c83
d77c
427e
b7ec
0bc7
0423
8f5e
9488
95cf
be8e
95b5
aa0f
95cf
ec08
88b8
d3da
9824
a24a
f933
90d2
9bd0
26b5
e6af
9c88
95cf
95cf
95cf
95cf
329b

2.3 Mo's algorithm

All intervals are closed on both sides. When running functions `enter()` and `leave()`, the global `l` and `r` has not changed yet.

Usage:

```
add_query(id, l, r)    Add id-th query [l, r].
run()                 Run Mo's algorithm.
init()                TODO. Initialize the range [l, r].
yield(id)             TODO. Yield answer for id-th query.
enter(o)              TODO. Add o-th element.
leave(o)              TODO. Remove o-th element.
```

```
5194 constexpr int BLOCK_SZ = 300;
427e
3ec4 struct query { int l, r, id; };
d26a vector<query> queries;
427e
1e30 void add_query(int id, int l, int r) {
54c9     queries.push_back(query{l, r, id});
95cf }
427e
9f6b int l, r;
427e
427e // ----- functions to implement -----
62b4 inline void init();
50e1 inline void yield(int id);
b20d inline void enter(int o);
13af inline void leave(int o);
427e
37f0 void run() {
ab0b     if (queries.empty()) return;
8508     sort(range(queries), [](query lhs, query rhs) {
c7f8         int lb = lhs.l / BLOCK_SZ, rb = rhs.l / BLOCK_SZ;
03e7         if (lb != rb) return lb < rb;
0780         return lhs.r < rhs.r;
b251     });
6196     l = queries[0].l;
9644     r = queries[0].r;
07e2     init();
5bc9     for (query q : queries) {
7bc7         while (l > q.l) enter(l - 1), l--;
d646         while (r < q.r) enter(r + 1), r++;
13f0         while (l < q.l) leave(l), l++;
e1c6         while (r > q.r) leave(r), r--;
```

```
        yield(q.id);
    }
}
```

```
82f5
95cf
95cf
```

3 String

3.1 Knuth-Morris-Pratt algorithm

```
const int SIZE = 10005;

struct kmp_matcher {
    char p[SIZE];
    int fail[SIZE];
    int len;

    void construct(const char* needle) {
        len = strlen(p);
        strcpy(p, needle);
        fail[0] = fail[1] = 0;
        for (int i = 1; i < len; i++) {
            int j = fail[i];
            while (j && p[i] != p[j]) j = fail[j];
            fail[i + 1] = p[i] == p[j] ? j + 1 : 0;
        }
    }

    inline void found(int pos) {
        // ! add codes for having found at pos
    }

    void match(const char* haystack) { // must be called after construct
        const char* t = haystack;
        int n = strlen(t);
        int j = 0;
        rep(i, n) {
            while (j && p[j] != t[i]) j = fail[j];
            if (p[j] == t[i]) j++;
            if (j == len) found(i - len + 1);
        }
    }
};
```

```
2836
427e
d02b
2d81
9847
57b7
427e
60cf
aaa1
3a87
3dd4
d8a8
147f
3c79
4643
95cf
95cf
427e
c464
427e
95cf
427e
2daf
700f
8482
8fd0
be8e
4e19
b5d5
f024
95cf
95cf
329b
```

3.2 Manacher algorithm

```

81d4 struct Manacher {
cd09     int Len;
9255     vector<int> lc;
b301     string s;
427e
ec07     void work() {
c033         lc[1] = 1;
6bef         int k = 1;
427e
491f         for (int i = 2; i <= Len; i++) {
7957             int p = k + lc[k] - 1;
5e04             if (i <= p) {
24a1                 lc[i] = min(lc[2 * k - i], p - i + 1);
8e2e             } else {
e0e5                 lc[i] = 1;
95cf             }
74ff             while (s[i + lc[i]] == s[i - lc[i]]) lc[i]++;
2b9a             if (i + lc[i] > k + lc[k]) k = i;
95cf         }
95cf     }
427e
bfd5     void init(const char *tt) {
aaaf         int len = strlen(tt);
f701         s.resize(len * 2 + 10);
7045         lc.resize(len * 2 + 10);
8e13         s[0] = '*';
ae54         s[1] = '#';
1321         for (int i = 0; i < len; i++) {
e995             s[i * 2 + 2] = tt[i];
69fd             s[i * 2 + 1] = '#';
95cf         }
43fd         s[len * 2 + 1] = '#';
75d1         s[len * 2 + 2] = '\0';
61f7         Len = len * 2 + 2;
3e7a         work();
95cf     }
427e
b194     pair<int, int> maxpal(int l, int r) {
901a         int center = l + r + 1;
ffb2         int rad = lc[center] / 2;
ab54         int rmid = (l + r + 1) / 2;

```

```

int rl = rmid - rad, rr = rmid + rad - 1;
if ((r ^ 1) & 1) {
} else rr++;
return {max(l, rl), min(r, rr)};
}
};

```

```

17e4
3908
69f3
69dc
95cf
329b

```

3.3 Aho-corasick automaton

```

struct AC : Trie {
    int fail[MAXN];
    int last[MAXN];

    void construct() {
        queue<int> q;
        fail[0] = 0;
        rep(c, CHARN) {
            if (int u = tr[0][c]) {
                fail[u] = 0;
                q.push(u);
                last[u] = 0;
            }
        }
        while (!q.empty()) {
            int r = q.front();
            q.pop();
            rep(c, CHARN) {
                int u = tr[r][c];
                if (!u) {
                    tr[r][c] = tr[fail[r]][c];
                    continue;
                }
                q.push(u);
                int v = fail[r];
                while (v && !tr[v][c]) v = fail[v];
                fail[u] = tr[v][c];
                last[u] = tag[fail[u]] ? fail[u] : last[fail[u]];
            }
        }
    }

    void found(int pos, int j) {

```

```

a1ad
9143
daca
427e
8690
93d2
a7a6
ce3c
b1c6
a506
3e14
f689
95cf
95cf
cc78
31f0
15dd
ce3c
ab59
0ef5
9d58
b333
95cf
3e14
b3ff
d2ea
c275
654c
95cf
95cf
95cf
427e
7752

```

```

043e     if (j) {
427e         // ! add codes for having found word with tag[j]
4a96         found(pos, last[j]);
95cf     }
95cf }
427e
9785 void find(const char* text) { // must be called after construct()
80a4     int p = 0, c, len = strlen(text);
9c94     rep(i, len) {
b3db         c = id(text[i]);
f119         p = tr[p][c];
f08e         if (tag[p])
389b             found(i, p);
1e67         else if (last[p])
299e             found(i, last[p]);
95cf     }
95cf }
329b };

```

3.4 Trie

```

e6f1 const int MAXN = 12000;
dd87 const int CHARN = 26;
427e
8ff5 inline int id(char c) { return c - 'a'; }
427e
a281 struct Trie {
5c83     int n;
f4f5     int tr[MAXN][CHARN]; // Trie tree, 0 denotes fail
35a5     int tag[MAXN];
427e
4fee     Trie() {
3ccc         memset(tr[0], 0, sizeof(tr[0]));
4d52         tag[0] = 0;
46bf         n = 1;
95cf     }
427e
427e     // tag should not be 0
30b0 void add(const char* s, int t) {
d50a     int p = 0, c, len = strlen(s);
9c94     rep(i, len) {
3140         c = id(s[i]);

```

```

if (!tr[p][c]) {
    memset(tr[n], 0, sizeof(tr[n]));
    tag[n] = 0;
    tr[p][c] = n++;
}
p = tr[p][c];
}
tag[p] = t;
}

// returns 0 if not found
// AC automaton does not need this function
int search(const char* s) {
    int p = 0, c, len = strlen(s);
    rep(i, len) {
        c = id(s[i]);
        if (!tr[p][c]) return 0;
        p = tr[p][c];
    }
    return tag[p];
}
};

```

```

d6c8
26dd
2e5c
73bb
95cf
f119
95cf
35ef
95cf
427e
427e
427e
216c
d50a
9c94
3140
f339
f119
95cf
840e
95cf
329b

```

3.5 Suffix array

The character immediately after the end of the string **MUST** be set to the **UNIQUE SMALLEST** element.

Usage:

<code>s[]</code>	the source string
<code>sa[i]</code>	the index of starting position of i -th suffix
<code>rk[i]</code>	the number of suffixes less than the suffix starting from i
<code>h[i]</code>	the longest common prefix between the i -th and $(i-1)$ -th lexicographically smallest suffixes
<code>n</code>	size of source string
<code>m</code>	size of character set

```

void radix_sort(int x[], int y[], int sa[], int n, int m) {
    static int cnt[1000005]; // size > max(n, m)
    fill(cnt, cnt + m, 0);
    rep(i, n) cnt[x[y[i]]]++;
    partial_sum(cnt, cnt + m, cnt);
    for (int i = n - 1; i >= 0; i--) sa[--cnt[x[y[i]]]] = y[i];
}

```

```

de09
ec00
6066
93b7
9154
acac
95cf

```

```

427e void suffix_array(int s[], int sa[], int rk[], int n, int m) {
c939     static int y[1000005]; // size > n
a69a     copy(s, s + n, rk);
7306     iota(y, y + n, 0);
afb6     radix_sort(rk, y, sa, n, m);
7b42     for (int j = 1, p = 0; j <= n; j <= 1, m = p, p = 0) {
c8c2         for (int i = n - j; i < n; i++) y[p++] = i;
8c3a         rep (i, n) if (sa[i] >= j) y[p++] = sa[i] - j;
9323         radix_sort(rk, y, sa, n, m + 1);
9e9d         swap_ranges(rk, rk + n, y);
ae41         rk[sa[0]] = p = 1;
ffd2         for (int i = 1; i < n; i++)
445e             rk[sa[i]] = ((y[sa[i]] == y[sa[i-1]] and y[sa[i]+j] == y[sa[i-1]+j])
f8dc                 ? p : ++p);
02f0         if (p == n) break;
95cf     }
97d9     rep (i, n) rk[sa[i]] = i;
95cf }
427e
1715 void calc_height(int s[], int sa[], int rk[], int h[], int n) {
c41f     int k = 0;
f313     h[0] = 0;
be8e     rep (i, n) {
0883         k = max(k - 1, 0);
527d         if (rk[i]) while (s[i+k] == s[sa[rk[i]-1]+k]) ++k;
56b7         h[rk[i]] = k;
95cf     }
95cf }

```

3.6 Rolling hash

PLEASE call `init_hash()` in `int main()`!

Usage:

`build(str)` Construct the hasher with given string.
`operator()(l, r)` Get hash value of substring $[l, r)$.

```

1e42 const LL mod = 1006658951440146419, g = 967;
9f60 const int MAXN = 200005;
0291 LL pg[MAXN];
427e
dfe7 inline LL mul(LL x, LL y) { return __int128_t(x) * y % mod; }
427e

```

```

void init_hash() { // must be called in `int main()`
    pg[0] = 1;
    for (int i = 1; i < MAXN; i++) pg[i] = mul(pg[i-1], g);
}

struct hasher {
    LL val[MAXN];

    void build(const char *str) { // assume lower-case letter only
        for (int i = 0; str[i]; i++)
            val[i+1] = (mul(val[i], g) + str[i]) % mod;
    }

    LL operator() (int l, int r) { // [l, r)
        return (val[r] - mul(val[l], pg[r-l]) + mod) % mod;
    }
};

```

4 Math

4.1 Extended Euclidean algorithm and Chinese remainder theorem

```

void exgcd(LL a, LL b, LL &g, LL &x, LL &y) {
    if (!b) g = a, x = 1, y = 0;
    else {
        exgcd(b, a % b, g, y, x);
        y -= x * (a / b);
    }
}

LL crt(LL r[], LL p[], int n) {
    LL q = 1, ret = 0;
    rep (i, n) q *= p[i];
    rep (i, n) {
        LL m = q / p[i];
        LL d, x, y;
        exgcd(p[i], m, d, x, y);
        ret = (ret + y * m * r[i]) % q;
    }
    return (q + ret) % q;
}

```


4.2 Linear basis

```

8b44 const int MAXD = 30;
03a6 struct linearbasis {
3558     ULL b[MAXD] = {};
427e
1566     bool insert(LL v) {
9b2b         for (int j = MAXD - 1; j >= 0; j--) {
de3e             if (!(v & (1ll << j))) continue;
ee78             if (b[j] v ^= b[j]
037f             else {
7836                 for (int k = 0; k < j; k++)
f0b4                     if (v & (1ll << k)) v ^= b[k];
b0aa                 for (int k = j + 1; k < MAXD; k++)
46c9                     if (b[k] & (1ll << j)) b[k] ^= v;
8295                 b[j] = v;
3361                 return true;
95cf             }
95cf         }
438e         return false;
95cf     }
329b };

```

4.3 Gauss elimination over finite field

```

b784 const LL p = 1000000007;
427e
2a2c LL powmod(LL b, LL e) {
95a2     LL r = 1;
3e90     while (e) {
1783         if (e & 1) r = r * b % p;
5549         b = b * b % p;
16fc         e >>= 1;
95cf     }
547e     return r;
95cf }
427e
c130 typedef vector<LL> VLL;
42ac typedef vector<VLL> WVLL;
427e
2c62 LL gauss(WVLL &a, WVLL &b) {
561b     const int n = a.size(), m = b[0].size();

```

```

vector<int> irow(n), icol(n), ipiv(n);
LL det = 1;

rep (i, n) {
    int pj = -1, pk = -1;
    rep (j, n) if (!ipiv[j])
        rep (k, n) if (!ipiv[k])
            if (pj == -1 || a[j][k] > a[pj][pk]) {
                pj = j;
                pk = k;
            }
    if (a[pj][pk] == 0) return 0;
    ipiv[pk]++;
    swap(a[pj], a[pk]);
    swap(b[pj], b[pk]);
    if (pj != pk) det = (p - det) % p;
    irow[i] = pj;
    icol[i] = pk;
}

```

```

LL c = powmod(a[pk][pk], p - 2);
det = det * a[pk][pk] % p;
a[pk][pk] = 1;
rep (j, n) a[pk][j] = a[pk][j] * c % p;
rep (j, m) b[pk][j] = b[pk][j] * c % p;
rep (j, n) if (j != pk) {
    c = a[j][pk];
    a[j][pk] = 0;
    rep (k, n) a[j][k] = (a[j][k] + p - a[pk][k] * c % p) % p;
    rep (k, m) b[j][k] = (b[j][k] + p - b[pk][k] * c % p) % p;
}
}

for (int j = n - 1; j >= 0; j--) if (irow[j] != icol[j]) {
    for (int k = 0; k < n; k++) swap(a[k][irow[j]], a[k][icol[j]]);
}
return det;
}

```

4.4 Berlekamp-Massey algorithm

Call `berlekamp()` with input sequence $(x_0, x_1, \dots, x_{n-1})$. Return a vector of coefficients $(c_0 = 1, c_1, \dots, c_{m-1})$ with minimum m , such that $\sum_{i=0}^m c_i x_{j-i} = 0$ for all possible j .

a25e
2976
427e
be8e
d2b5
6b4a
e582
6112
a905
657b
95cf
d480
0305
8dad
aad8
be4d
d080
f156
427e
4ecd
865b
c36a
dd36
1b23
f8f3
e97f
c449
820b
f039
95cf
95cf
427e
37e1
50dc
95cf
f27f
95cf

```

6e50 LL mod = 1000000007;
97db vector<LL> berlekamp(const vector<LL>& a) {
8904     vector<LL> p = {1}, r = {1};
075b     LL dif = 1;
8bc9     rep (i, a.size()) {
1b35         LL u = 0;
bd0b         rep (j, p.size()) u = (u + p[j] * a[i-j]) % mod;
eae9         if (u == 0) {
b14c             r.insert(r.begin(), 0);
8e2e         } else {
0c78             auto op = p;
02f6             p.resize(max(p.size(), r.size() + 1));
0a2e             LL idif = powmod(dif, mod - 2);
9b57             rep (j, r.size())
dacc                 p[j+1] = (p[j+1] - r[j] * idif % mod * u % mod + mod) % mod;
bcd1             dif = u; r = op;
95cf         }
95cf     }
e149     return p;
95cf }

```

4.5 Fast Walsh-Hadamard transform

```

061e void fwt(int* a, int n){
5595     for (int d = 1; d < n; d <= 1)
05f2         for (int i = 0; i < n; i += d << 1)
b833             rep (j, d){
7796                 int x = a[i+j], y = a[i+j+d];
427e                 // a[i+j] = x+y, a[i+j+d] = x-y; // xor
427e                 // a[i+j] = x+y; // and
427e                 // a[i+j+d] = x+y; // or
95cf             }
95cf }
427e
4db1 void ifwt(int* a, int n){
5595     for (int d = 1; d < n; d <= 1)
05f2         for (int i = 0; i < n; i += d << 1)
b833             rep (j, d){
7796                 int x = a[i+j], y = a[i+j+d];
427e                 // a[i+j] = (x+y)/2, a[i+j+d] = (x-y)/2; // xor
427e                 // a[i+j] = x-y; // and
427e                 // a[i+j+d] = y-x; // or

```

```

    }
}

void conv(int* a, int* b, int n){
    fwt(a, n);
    fwt(b, n);
    rep(i, n) a[i] *= b[i];
    ifwt(a, n);
}

```

4.6 Fast fourier transform

```

const int NMAX = 1<<20;

typedef complex<double> cplx;

const double PI = 2*acos(0.0);
struct FFT{
    int rev[NMAX];
    cplx omega[NMAX], oinv[NMAX];
    int K, N;

    FFT(int k){
        K = k; N = 1 << k;
        rep (i, N){
            rev[i] = (rev[i>>1]>>1) | ((i&1)<<(K-1));
            omega[i] = polar(1.0, 2.0 * PI / N * i);
            oinv[i] = conj(omega[i]);
        }
    }

    void dft(cplx* a, cplx* w){
        rep (i, N) if (i < rev[i]) swap(a[i], a[rev[i]]);
        for (int l = 2; l <= N; l *= 2){
            int m = l/2;
            for (cplx* p = a; p != a + N; p += l)
                rep (k, m){
                    cplx t = w[N/l*k] * p[k+m];
                    p[k+m] = p[k] - t; p[k] += t;
                }
        }
    }
}

```

```

427e void fft(cplx* a){dft(a, omega);}
617b void ifft(cplx* a){
a123     dft(a, oinv);
3b2f     rep (i, N) a[i] /= N;
57fc }
95cf
427e void conv(cplx* a, cplx* b){
bdc0     fft(a); fft(b);
6497     rep (i, N) a[i] *= b[i];
12a5     ifft(a);
f84e }
95cf };
329b

```

4.7 Number theoretic transform

```

4ab9 const int NMAX = 1<<21;
427e
427e // 998244353 = 7*17*2^23+1, G = 3
fb9a const int P = 1004535809, G = 3; // = 479*2^21+1
427e
87ab struct NTT{
c47c     int rev[NMAX];
0eda     LL omega[NMAX], oinv[NMAX];
81af     int g, g_inv; // g: g_n = G^((P-1)/n)
9827     int K, N;
427e
2a2c     LL powmod(LL b, LL e){
95a2         LL r = 1;
3e90         while (e){
6624             if (e&1) r = r * b % P;
489e             b = b * b % P;
16fc             e >>= 1;
95cf         }
547e         return r;
95cf     }
427e
f420     NTT(int k){
e209         K = k; N = 1 << k;
7652         g = powmod(G, (P-1)/N);
4b3a         g_inv = powmod(g, N-1);
e04f         omega[0] = oinv[0] = 1;

```

```

rep (i, N){
    rev[i] = (rev[i>>1]>>1) | ((i&1)<<(K-1));
    if (i){
        omega[i] = omega[i-1] * g % P;
        oinv[i] = oinv[i-1] * g_inv % P;
    }
}

void _ntt(LL* a, LL* w){
    rep (i, N) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int l = 2; l <= N; l *= 2){
        int m = l/2;
        for (LL* p = a; p != a + N; p += l)
            rep (k, m){
                LL t = w[N/l*k] * p[k+m] % P;
                p[k+m] = (p[k] - t + P) % P;
                p[k] = (p[k] + t) % P;
            }
    }
}

void ntt(LL* a){_ntt(a, omega);}
void intt(LL* a){
    LL inv = powmod(N, P-2);
    _ntt(a, oinv);
    rep (i, N) a[i] = a[i] * inv % P;
}

void conv(LL* a, LL* b){
    ntt(a); ntt(b);
    rep (i, N) a[i] = a[i] * b[i] % P;
    intt(a);
}
};

```

4.8 Sieve of Euler

```

const int MAXX = 1e7+5;
bool p[MAXX];
int prime[MAXX], sz;

```

b393
7ba3
ad4f
8d8b
9e14
95cf
95cf
95cf
427e
9668
a215
ac6e
2969
7a1d
c24f
0ad3
6209
fa1b
95cf
95cf
95cf
427e
92ea
5daf
1f2a
9910
a873
95cf
427e
3a5b
ad16
e49e
5748
95cf
329b

```

9bc6 void sieve(){
9628     p[0] = p[1] = 1;
1ec8     for (int i = 2; i < MAXX; i++){
bf28         if (!p[i]) prime[sz++] = i;
e82c         for (int j = 0; j < sz && i*prime[j] < MAXX; j++){
b6a9             p[i*prime[j]] = 1;
5f51             if (i % prime[j] == 0) break;
95cf         }
95cf     }
95cf }

```

```

        } else {
            pval[x] = prime[j];
            pcnt[x] = 1;
        }
        if (x != pval[x]) {
            f[x] = f[x / pval[x]] * f[pval[x]]
        }
        if (i % prime[j] == 0) break;
    }
}
}
}
}

```

```

8e2e
cc91
6322
95cf
6191
d614
95cf
5f51
95cf
95cf
95cf
95cf
95cf

```

4.9 Sieve of Euler (General)

```

b62e namespace sieve {
6589     constexpr int MAXN = 10000007;
e982     bool p[MAXN]; // true if not prime
6ae8     int prime[MAXN], sz;
cbf7     int pval[MAXN], pcnt[MAXN];
6030     int f[MAXN];
427e
76f6     void exec(int N = MAXN) {
9628         p[0] = p[1] = 1;
427e
8a8a         pval[1] = 1;
bdda         pcnt[1] = 0;
c6b9         f[1] = 1;
427e
a643         for (int i = 2; i < N; i++) {
01d6             if (!p[i]) {
b2b2                 prime[sz++] = i;
37d9                 for (LL j = i; j < N; j *= i) {
758c                     int b = j / i;
81fd                     pval[j] = i * pval[b];
e0f3                     pcnt[j] = pcnt[b] + 1;
a96c                     f[j] = ____; // f[j] = f(i^pcnt[j])
95cf                 }
95cf             }
34c0             for (int j = 0; i * prime[j] < N; j++) {
f87a                 int x = i * prime[j]; p[x] = 1;
20cc                 if (i % prime[j] == 0) {
9985                     pval[x] = pval[i] * prime[j];
3f93                     pcnt[x] = pcnt[i] + 1;

```

4.10 Miller-Rabin primality test

The array `a[]` (excluding sentinel, i.e. `LLONG_MAX`) should be

{2}	when $n < 2,047$.
{2, 7, 61}	when $n < 4,759,123,141 (2^{32})$.
{2, 3, 5, 7, 11}	when $n < 2.1 \times 10^{12}$.
{2, 325, 9375, 28178, 450775, 9780504, 1795265022}	when $n < 2^{64}$.

```

bool test(LL n){
    if (n < 3) return n==2;
    // ! The array a[] should be modified if the range of x changes.
    const LL a[] = {2LL, 7LL, 61LL, LLONG_MAX};
    LL r = 0, d = n-1, x;
    while (~d & 1) d >>= 1, r++;
    for (int i=0; a[i] < n; i++){
        x = powmod(a[i], d, n); // ! powmod must use for 64bit mulmod
        if (x == 1 || x == n-1) goto next;
        rep (i, r) {
            x = mulmod(x, x, n);
            if (x == n-1) goto next;
        }
        return false;
    }
next:;
}
return true;
}

```

```

f16f
59f2
427e
3f11
c320
f410
2975
ece1
7f99
e257
d7ff
8d2e
95cf
438e
d490
95cf
3361
95cf

```

4.11 Integer factorization (Pollard's rho)

```

2e6b ULL gcd(ULL a, ULL b) {return b ? gcd(b, a % b) : a;}
427e
54a5 ULL PollardRho(ULL n){
45eb     ULL c, x, y, d = n;
d3e5     if (~n&1) return 2;
3c69     while (d == n){
0964         x = y = 2;
4753         d = 1;
5952         c = rand() % (n - 1) + 1;
9e5b         while (d == 1){
33d5             x = (mulmod(x, x, n) + c) % n;
e1bf             y = (mulmod(y, y, n) + c) % n;
e1bf             y = (mulmod(y, y, n) + c) % n;
a313             d = gcd(x>y ? x-y : y-x, n);
95cf         }
95cf     }
5d89     return d;
95cf }

```

5 Graph Theory

5.1 Strongly connected component

```

837c const int MAXV = 100005;
427e
2ea0 struct graph{
88e3     vector<int> adj[MAXV];
9cad     stack<int> s;
3d02     int V; // number of vertices
8b6c     int pre[MAXV], lnk[MAXV], scc[MAXV];
27ee     int time, sccn;
427e
bfab     void add_edge(int u, int v){
c71a         adj[u].push_back(v);
95cf     }
427e
d714     void dfs(int u){
7e41         pre[u] = lnk[u] = ++time;
80f6         s.push(u);

```

```

for (int v : adj[u]){
    if (!pre[v]){
        dfs(v);
        lnk[u] = min(lnk[u], lnk[v]);
    } else if (!scc[v]){
        lnk[u] = min(lnk[u], pre[v]);
    }
}
if (lnk[u] == pre[u]){
    sccn++;
    int x;
    do {
        x = s.top(); s.pop();
        scc[x] = sccn;
    } while (x != u);
}
}

void find_scc(){
    time = sccn = 0;
    memset(scc, 0, sizeof scc);
    memset(pre, 0, sizeof pre);
    Rep (i, V){
        if (!pre[i]) dfs(i);
    }
}

vector<int> adjc[MAXV];
void contract(){
    Rep (i, V)
        rep (j, adj[i].size()){
            if (scc[i] != scc[adj[i][j]])
                adjc[scc[i]].push_back(scc[adj[i][j]]);
        }
}
};

```

18f6
173e
5f3c
002c
6068
d5df
95cf
95cf
8de2
660f
3c9e
a69f
3834
b0e9
6757
95cf
95cf
427e
4c88
f4a2
8de7
8c2f
6901
56d1
95cf
95cf
427e
27ce
364d
1a1e
21a2
b730
b46e
95cf
95cf
329b

5.2 Vertex biconnected component

```

const int MAXN = 100005;
struct graph {
    int pre[MAXN], iscut[MAXN], bccno[MAXN], dfs_clock, bcc_cnt;

```

0f42
2ea0
33ae

```

848f    vector<int> adj[MAXN], bcc[MAXN];
6b06    set<pair<int, int>> bcce[MAXN];
427e
76f7    stack<pair<int, int>> s;
427e
bfab    void add_edge(int u, int v) {
c71a        adj[u].push_back(v);
a717        adj[v].push_back(u);
95cf    }
427e
7d3c    int dfs(int u, int fa) {
9fe6        int lowu = pre[u] = ++dfs_clock;
ec14        int child = 0;
18f6        for (int v : adj[u]) {
173e            if (!pre[v]) {
e7f8                s.push({u, v});
fdcf                child++;
f851                int lowv = dfs(v, u);
189c                lowu = min(lowu, lowv);
b687                if (lowv >= pre[u]) {
6323                    iscut[u] = 1;
57eb                    bcc[bcc_cnt].clear();
90b8                    bcce[bcc_cnt].clear();
a147                    while (1) {
a6a3                        int xu, xv;
a0c3                        tie(xu, xv) = s.top(); s.pop();
0ef5                        bcce[bcc_cnt].insert({min(xu, xv), max(xu, xv)});
3db2                        if (bccno[xu] != bcc_cnt) {
e0db                            bcc[bcc_cnt].push_back(xu);
d27f                            bccno[xu] = bcc_cnt;
95cf                        }
f357                        if (bccno[xv] != bcc_cnt) {
752b                            bcc[bcc_cnt].push_back(xv);
57c9                            bccno[xv] = bcc_cnt;
95cf                        }
7096                        if (xu == u && xv == v) break;
95cf                    }
03f5                    bcc_cnt++;
95cf                }
7470            } else if (pre[v] < pre[u] && v != fa) {
e7f8                s.push({u, v});
f115                lowu = min(lowu, pre[v]);
95cf            }
95cf        }

```

```

        if (fa < 0 && child == 1) iscut[u] = 0;
        return lowu;
    }

    void find_bcc(int n) {
        memset(pre, 0, sizeof pre);
        memset(iscut, 0, sizeof iscut);
        memset(bccno, -1, sizeof bccno);
        dfs_clock = bcc_cnt = 0;
        rep (i, n) if (!pre[i]) dfs(i, -1);
    }
};

```

e104
1160
95cf
427e
17be
8c2f
e2d2
40d3
fae2
5c63
95cf
329b

5.3 Cut vertices

If the graph is unconnected, the algorithm should be run on each component. One may run `Rep (i, n)if (!dfn[i])tarjan(i, i)` for unconnected graph.

Usage:

<code>add_edge(u, v)</code>	Add an undirected edge (u, v) .
<code>tarjan(u, fa)</code>	Run Tarjan's algorithm on tree rooted at fa . Please call with identical u and fa .
<code>cut[v]</code>	Whether v is a cut vertex.

```

const int MAXN = 200005;
vector<int> adj[MAXN];
int dfn[MAXN], low[MAXN], idx;
bool cut[MAXN];

void add_edge(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
}

void tarjan(int u, int fa) {
    dfn[u] = low[u] = ++idx;
    int child = 0;
    for (int v : adj[u]) {
        if (!dfn[v]) {
            tarjan(v, fa); low[u] = min(low[u], low[v]);
            if (low[v] >= dfn[u] && u != fa) cut[u] = true;
            child += u == fa;
        }
        low[u] = min(low[u], dfn[v]);
    }
}

```

9f60
0b32
18e4
d39d
427e
bfab
c71a
a717
95cf
427e
50aa
9891
ec14
18f6
3c64
9636
f368
7923
95cf
769a

```

95cf     }
7927     if (u == fa && child > 1) cut[u] = true;
95cf }

```

5.4 Minimum spanning arborescence, faster

All vertices are 1-based. Clear the fields when reuse the struct.

Usage:

add_edge(u, v, w) Add an edge from u to v with weight w .
run(n, rt) Compute the total weight of MSA rooted at rt . If not
 exist, return `LLONG_MIN`.

Time Complexity: $O((|E| + |V| \log |V|) \log |V|)$

```

5ece const int MAXN = 300005;
2fef typedef pair<LL, int> pii;
1495 struct MDST {
01b2     priority_queue<pii, vector<pii>, greater<pii>> heap[MAXN];
321d     LL shift[MAXN];
fc06     int fa[MAXN], vis[MAXN];
427e
38dd     int find(int x) { return fa[x] == x ? x : fa[x] = find(fa[x]); }
427e
29b0     void unite(int x, int y) {
0c14         x = find(x); y = find(y); fa[y] = x; if (x == y) return;
6fa0         if (heap[x].size() < heap[y].size()) {
9c26             swap(heap[x], heap[y]);
2ffc             swap(shift[x], shift[y]);
95cf         }
9959         while (heap[y].size()) {
175b             auto p = heap[y].top(); heap[y].pop();
c0c5             heap[x].emplace(p.first - shift[y] + shift[x], p.second);
95cf         }
95cf     }
427e
0bbd     void add_edge(int u, int v, LL w) { heap[v].emplace(w, u); }
427e
a526     LL run(int n, int rt) {
f7ff         LL ans = 0;
81f2         iota(fa, fa + n + 1, 0);
19b3         Rep (i, n) if (find(i) != find(rt)) {
a7b1             int u = find(i);
010e             stack<int, vector<int>> s;
eff5             while (find(u) != find(rt)) {

```

```

if (vis[u]) while (s.top() != u) {
    vis[s.top()] = 0; unite(u, s.top()); s.pop();
} else { vis[u] = 1; s.push(u); }
while (heap[u].size()) {
    ans += heap[u].top().first - shift[u];
    shift[u] = heap[u].top().first;
    if (find(heap[u].top().second) != u) break;
    heap[u].pop();
}
if (heap[u].empty()) return LLONG_MIN;
u = find(heap[u].top().second);
}
while (s.size()) { vis[s.top()] = 0; unite(rt, s.top()); s.pop(); }
}
return ans;
}
};

```

0dda
c593
83c4
c76e
b385
dde2
da47
9fbb
95cf
6961
87e6
95cf
2d46
95cf
4206
95cf
329b

5.5 Maximum flow (Dinic)

Usage:

add_edge(u, v, c) Add an edge from u to v with capacity c .
max_flow(s, t) Compute maximum flow from s to t .

Time Complexity: For general graph, $O(V^2E)$; for network with unit capacity, $O(\min\{V^{2/3}, \sqrt{E}\}E)$; for bipartite network, $O(\sqrt{VE})$.

```

struct edge{
    int from, to;
    LL cap, flow;
};

const int MAXN = 1005;
struct Dinic {
    int n, m, s, t;
    vector<edge> edges;
    vector<int> G[MAXN];
    bool vis[MAXN];
    int d[MAXN];
    int cur[MAXN];

    void add_edge(int from, int to, LL cap) {
        edges.push_back(edge{from, to, cap, 0});
        edges.push_back(edge{to, from, 0, 0});
    }
};

```

bcf8
60e2
5e6d
329b
427e
e2cd
9062
4dbf
9f0c
b891
bbb6
b40a
ddec
427e
5973
7b55
1db7

```

fe77     m = edges.size();
dff5     G[from].push_back(m-2);
8f2d     G[to].push_back(m-1);
95cf     }
427e
1836     bool bfs() {
3b73         memset(vis, 0, sizeof(vis));
93d2         queue<int> q;
5d13         q.push(s);
2cd2         vis[s] = 1;
721d         d[s] = 0;
cc78         while (!q.empty()) {
66ba             int x = q.front(); q.pop();
3b61             for (int i = 0; i < G[x].size(); i++) {
b510                 edge& e = edges[G[x][i]];
bba9                 if (!vis[e.to] && e.cap > e.flow) {
cd72                     vis[e.to] = 1;
cf26                     d[e.to] = d[x] + 1;
ca93                     q.push(e.to);
95cf                 }
95cf             }
95cf         }
b23b         return vis[t];
95cf     }
427e
9252     LL dfs(int x, LL a) {
6904         if (x == t || a == 0) return a;
8bf9         LL flow = 0, f;
f515         for (int& i = cur[x]; i < G[x].size(); i++) {
b510             edge& e = edges[G[x][i]];
2374             if(d[x] + 1 == d[e.to] && (f = dfs(e.to, min(a, e.cap-e.flow))) > 0)
                {
1cce                 e.flow += f;
e16d                 edges[G[x][i]^1].flow -= f;
a74d                 flow += f;
23e5                 a -= f;
97ed                 if(a == 0) break;
95cf             }
95cf         }
84fb         return flow;
95cf     }
427e
5bf2     LL max_flow(int s, int t) {
590d         this->s = s; this->t = t;

```

```

LL flow = 0;
while (bfs()) {
    memset(cur, 0, sizeof(cur));
    flow += dfs(s, LLONG_MAX);
}
return flow;
}

vector<int> min_cut() { // call this after maxflow
    vector<int> ans;
    for (int i = 0; i < edges.size(); i++) {
        edge& e = edges[i];
        if(vis[e.from] && !vis[e.to] && e.cap > 0) ans.push_back(i);
    }
    return ans;
}
};

```

5.6 Maximum cardinality bipartite matching (Hungarian)

```

#include <bits/stdc++.h>
using namespace std;

#define rep(i, n) for (int i = 0; i < (n); i++)
#define Rep(i, n) for (int i = 1; i <= (n); i++)
#define range(x) (x).begin(), (x).end()
typedef long long LL;

struct Hungarian{
    int nx, ny;
    vector<int> mx, my;
    vector<vector<int>> > e;
    vector<bool> mark;

    void init(int nx, int ny){
        this->nx = nx;
        this->ny = ny;
        mx.resize(nx); my.resize(ny);
        e.clear(); e.resize(nx);
        mark.resize(nx);
    }
}

```



```

4589 inline void add(int a, int b){
486c     e[a].push_back(b);
95cf }
427e
0c2b bool augment(int i){
207c     if (!mark[i]) {
dae4         mark[i] = true;
6a1e         for (int j : e[i]){
0892             if (my[j] == -1 || augment(my[j])){
9ca3                 mx[i] = j; my[j] = i;
3361                 return true;
95cf             }
95cf         }
95cf     }
438e     return false;
95cf }
427e
3fac int match(){
5b57     int ret = 0;
b0f1     fill(range(mx), -1);
b957     fill(range(my), -1);
4ed1     rep (i, nx){
13a5         fill(range(mark), false);
cc89         if (augment(i)) ret++;
95cf     }
ee0f     return ret;
95cf }
329b };

```

5.7 Maximum matching of general graph (Edmond's blossom)

Usage:

init(n)	Initialize the template with n vertices, numbered from 1.
add_edge(u, v)	Add an undirected edge uv .
solve()	Find the maximum matching. Return the number of matched edges.
mate[]	The mate of a matched vertex. If it is not matched, then the value is 0.

Time Complexity: $O(|V|^3)$, but extremely fast in practice.

```

c041 const int MAXN = 1024;
6ab1 struct Blossom {

```

```

vector<int> adj[MAXN];
queue<int> q;
int n;
int label[MAXN], mate[MAXN], save[MAXN], used[MAXN];

void init(int nv) {
    n = nv; for (auto& v : adj) v.clear();
    fill(range(label), 0); fill(range(mate), 0);
    fill(range(save), 0); fill(range(used), 0);
}

void add_edge(int u, int v) { adj[u].push_back(v); adj[v].push_back(u); }

void rematch(int x, int y) {
    int m = mate[x]; mate[x] = y;
    if (mate[m] == x) {
        if (label[x] <= n) {
            mate[m] = label[x]; rematch(label[x], m);
        } else {
            int a = 1 + (label[x] - n - 1) / n;
            int b = 1 + (label[x] - n - 1) % n;
            rematch(a, b); rematch(b, a);
        }
    }
}

void traverse(int x) {
    Rep (i, n) save[i] = mate[i];
    rematch(x, x);
    Rep (i, n) {
        if (mate[i] != save[i]) used[i] ++;
        mate[i] = save[i];
    }
}

void relabel(int x, int y) {
    Rep (i, n) used[i] = 0;
    traverse(x); traverse(y);
    Rep (i, n) {
        if (used[i] == 1 and label[i] < 0) {
            label[i] = n + x + (y - 1) * n;
            q.push(i);
        }
    }
}

```

0b32
93d2
5c83
0de2
427e
2186
3728
477d
bb35
95cf
427e
c2dd
427e
2a48
8af8
1aa4
f4ba
740a
8e2e
3341
2885
ef33
95cf
95cf
95cf
427e
8a50
43c0
2ef7
34d7
62c5
97ef
95cf
95cf
427e
8bf8
d101
c4ea
34d7
dee9
1c22
eb31
95cf
95cf

```

95cf    }
427e
a0ce    int solve() {
34d7        Rep (i, n) {
a073            if (mate[i]) continue;
1fc0            Rep (j, n) label[j] = -1;
7676            label[i] = 0; q = queue<int>(); q.push(i);
1c7d            while (q.size()) {
66ba                int x = q.front(); q.pop();
b98c                for (int y : adj[x]) {
c07f                    if (mate[y] == 0 and i != y) {
7f36                        mate[y] = x; rematch(x, y); q = queue<int>(); break;
95cf                    }
d315                    if (label[y] >= 0) { relabel(x, y); continue; }
58ec                    if (label[mate[y]] < 0) {
c9c4                        label[mate[y]] = x; q.push(mate[y]);
95cf                    }
95cf                }
95cf            }
8abb            int cnt = 0;
b52f            Rep (i, n) cnt += (mate[i] > i);
6808            return cnt;
95cf        }
329b    };

```

5.8 Minimum cost maximum flow

```

bcf8    struct edge{
60e2        int from, to;
d698        int cap, flow;
32cc        LL cost;
329b    };
427e
cc3e    const LL INF = LLONG_MAX / 2;
2aa8    const int MAXN = 5005;
c6cb    struct MCMF {
9ceb        int s, t, n, m;
9f0c        vector<edge> edges;
b891        vector<int> G[MAXN];
f74f        bool inq[MAXN]; // queue
8f67        LL d[MAXN];    // distance

```

```

int p[MAXN];    // previous
int a[MAXN];    // improvement

void add_edge(int from, int to, int cap, LL cost) {
    edges.push_back(edge{from, to, cap, 0, cost});
    edges.push_back(edge{to, from, 0, 0, -cost});
    m = edges.size();
    G[from].push_back(m-2);
    G[to].push_back(m-1);
}

bool spfa(){
    queue<int> q;
    fill(d, d + MAXN, INF); d[s] = 0;
    memset(inq, 0, sizeof(inq));
    q.push(s); inq[s] = true;
    p[s] = 0; a[s] = INT_MAX;
    while (!q.empty()){
        int u = q.front(); q.pop(); inq[u] = false;
        for (int i : G[u]) {
            edge& e = edges[i];
            if (e.cap > e.flow && d[e.to] > d[u] + e.cost){
                d[e.to] = d[u] + e.cost;
                p[e.to] = G[u][i];
                a[e.to] = min(a[u], e.cap - e.flow);
                if (!inq[e.to]) q.push(e.to), inq[e.to] = true;
            }
        }
    }
    return d[t] != INF;
}

void augment(){
    int u = t;
    while (u != s){
        edges[p[u]].flow += a[t];
        edges[p[u]^1].flow -= a[t];
        u = edges[p[u]].from;
    }
}

#ifdef GIVEN_FLOW
bool min_cost(int s, int t, int f, LL& cost) {
    this->s = s; this->t = t;

```

```

21d4     int flow = 0;
23cb     cost = 0;
22dc     while (spfa()) {
bcdcb         augment();
a671         if (flow + a[t] >= f){
b14d             cost += (f - flow) * d[t]; flow = f;
3361             return true;
8e2e         } else {
2a83             flow += a[t]; cost += a[t] * d[t];
95cf         }
95cf     }
438e     return false;
95cf }
a8cb #else
f9a9     int min_cost(int s, int t, LL& cost) {
590d         this->s = s; this->t = t;
21d4         int flow = 0;
23cb         cost = 0;
22dc         while (spfa()) {
bcdcb             augment();
2a83             flow += a[t]; cost += a[t] * d[t];
95cf         }
84fb         return flow;
95cf     }
1937 #endif
329b };

```

5.9 Fast LCA

All indices of the tree are 1-based.

Usage:

```

preprocess(root)    Initialize with tree rooted at root.
lca(u, v)           Query the lowest common ancestor of  $u$  and  $v$ .

```

```

0e34 const int MAXN = 500005;
0b32 vector<int> adj[MAXN];
fccb int id[MAXN], nid;
1356 pair<int, int> st[MAXN << 1][33 - __builtin_clz(MAXN)];
427e
e16d void dfs(int u, int p, int d) {
0df2     st[id[u] = nid++][0] = {d, u};
18f6     for (int v : adj[u]) {
bd87         if (v == p) continue;

```

```

        dfs(v, u, d + 1);
        st[nid++][0] = {d, u};
    }
}

void preprocess(int root) {
    nid = 0;
    dfs(root, 0, 1);
    int l = 31 - __builtin_clz(nid);
    rep (j, l) rep (i, 1+nid-(1<<j))
        st[i][j+1] = min(st[i][j], st[i+(1<<j)][j]);
}

int lca(int u, int v) {
    tie(u, v) = minmax(id[u], id[v]);
    int k = 31 - __builtin_clz(v-u+1);
    return min(st[u][k], st[v-(1<<k)+1][k]).second;
}

```

5.10 Heavy-light decomposition

Time Complexity: The decomposition itself takes linear time. Each query takes $O(\log n)$ operations.

```

const int MAXN = 100005;
vector<int> adj[MAXN];
int sz[MAXN], top[MAXN], fa[MAXN], son[MAXN], depth[MAXN], id[MAXN];

void dfs1(int x, int dep, int par){
    depth[x] = dep;
    sz[x] = 1;
    fa[x] = par;
    int maxn = 0, s = 0;
    for (int c: adj[x]){
        if (c == par) continue;
        dfs1(c, dep + 1, x);
        sz[x] += sz[c];
        if (sz[c] > maxn){
            maxn = sz[c];
            s = c;
        }
    }
    son[x] = s;
}

```

```

95cf }
427e
ba54 int cid = 0;
3644 void dfs2(int x, int t){
8d96     top[x] = t;
d314     id[x] = ++cid;
c4a1     if (son[x]) dfs2(son[x], t);
c861     for (int c: adj[x]){
9881         if (c == fa[x]) continue;
5518         if (c == son[x]) continue;
13f9         else dfs2(c, c);
95cf     }
95cf }
427e
0f04 void decomp(int root){
9fa4     dfs1(root, 1, 0);
1c88     dfs2(root, root);
95cf }
427e
2c98 void query(int u, int v){
03a1     while (top[u] != top[v]){
45ec         if (depth[top[u]] < depth[top[v]]) swap(u, v);
427e         // id[top[u]] to id[u]
005b         u = fa[top[u]];
95cf     }
6083     if (depth[u] > depth[v]) swap(u, v);
427e     // id[u] to id[v]
95cf }

```

5.11 Centroid decomposition

Note that the centroid here is not the exact centroid of the graph. It only guarantees that the size of each subtree does not exceed half of that of the original tree. This is enough to guarantee the correct time complexity. All vertices are numbered from 1. Call `decomp(root)` to use.

Usage:

`decomp(u, p)` Decompose the tree rooted at u with parent p .

Time Complexity: The decomposition itself takes $O(n \log n)$ time.

```

1fb6 vector<int> adj[100005];
88e0 int sz[100005], sum;
427e
f93d void getsz(int u, int p) {

```

```

    sz[u] = 1; sum++;
    for (int v : adj[u]) {
        if (v == p) continue;
        getsz(v, u);
        sz[u] += sz[v];
    }
}

int getcent(int u, int p) {
    for (int v : adj[u])
        if (v != p and sz[v] > sum / 2)
            return getcent(v, u);
    return u;
}

void decompose(int u) {
    sum = 0; getsz(u, 0);
    u = getcent(u, 0); // update u to the centroid

    for (int v : adj[u]) {
        // get answer for subtree v
    }
    // get answer for the whole tree
    // don't forget to count the centroid itself

    for (int v : adj[u]) { // divide and conquer
        adj[v].erase(find(range(adj[v]), u));
        decompose(v);
        adj[v].push_back(u); // restore deleted edge
    }
}

```

5.12 DSU on tree

This implementation avoids parallel existence of multiple data structures but requires that the data structure is invertible. To use this template, implement `merge`, `enter`, `leave` as needed; first call `decomp(root, 0)`, then call `work(root, 0, false)`. Labels of vertices start from 1.

Usage:

`decomp(u, p)` Decompose the tree u .
`work(u, p, keep)` Work for subtree u . When `keep` is set, information is not cleared.

Time Complexity: $O(n \log n)$ times the complexity for merge, enter, leave.

```

1fb6 vector<int> adj[100005];
901d int sz[100005], son[100005];
427e
5559 void decomp(int u, int p) {
50c0     sz[u] = 1;
18f6     for (int v : adj[u]) {
bd87         if (v == p) continue;
a851         decomp(v, u);
8449         sz[u] += sz[v];
d28c         if (sz[v] > sz[son[u]]) son[u] = v;
95cf     }
95cf }
427e
b7ec template <typename T>
62f5 void trav(T fn, int u, int p) {
4412     fn(u);
30b3     for (int v : adj[u]) if (v != p) trav(fn, v, u);
95cf }
427e
7467 #define for_light(v) for (int v : adj[u]) if (v != p and v != son[u])
33ff void work(int u, int p, bool keep) {
72a2     for_light(v) work(v, u, 0); // process light children
427e
427e     // process heavy child
427e     // current data structure contains info of heavy child
9866     if (son[u]) work(son[u], u, 1);
427e
18a9     auto merge = [u] (int c) { /* count contribution of c */ };
1ab0     auto enter = [] (int c) { /* add vertex c */ };
f241     auto leave = [] (int c) { /* remove vertex c */ };
427e
3d3b     for_light(v) {
74c6         trav(merge, v, u);
c13d         trav(enter, v, u);
95cf     }
427e
427e     // count answer for root and add it
427e     // Warning: special check may apply to root!
c54f     merge(u);
9dec     enter(u);
427e
427e     // Leave current tree

```

```

    if (!keep) trav(leave, u, p);
}

```

4e3e
95cf

6 Data Structures

6.1 Fenwick tree (point update range query)

```

struct bit_purq { // point update, range query
    int N;
    vector<LL> tr;

    void init(int n) { tr.resize(N = n + 5); }

    LL sum(int n) {
        LL ans = 0;
        while (n) { ans += tr[n]; n &= n - 1; }
        return ans;
    }

    void add(int n, LL x){
        while (n < N) { tr[n] += x; n += n & -n; }
    }
};

```

9976
d7af
99ff
427e
456d
427e
63d0
f7ff
6770
4206
95cf
427e
f4bd
968e
95cf
329b

6.2 Fenwick tree (range update point query)

```

struct bit_rupq{ // range update, point query
    int N;
    vector<LL> tr;

    void init(int n) { tr.resize(N = n + 5);}

    LL query(int n) {
        LL ans = 0;
        while (n < N) { ans += tr[n]; n += n & -n; }
        return ans;
    }

    void add(int n, LL x) {
        while (n) { tr[n] += x; n &= n - 1; }
    }
}

```

3d03
d7af
99ff
427e
456d
427e
38d4
f7ff
3667
4206
95cf
427e
f4bd
0a2b

```
95cf    }
329b    };
```

6.3 Segment tree

```
3942    LL p;
1ebb    const int MAXN = 4 * 100006;
451a    struct segtree {
27be        int l[MAXN], m[MAXN], r[MAXN];
4510        LL val[MAXN], tadd[MAXN], tmul[MAXN];
427e
ac35    #define lson (o<<1)
1294    #define rson (o<<1|1)
427e
1344    void pull(int o) {
bbe9        val[o] = (val[lson] + val[rson]) % p;
95cf    }
427e
e4bc    void push_add(int o, LL x) {
5dd6        val[o] = (val[o] + x * (r[o] - l[o])) % p;
6eff        tadd[o] = (tadd[o] + x) % p;
95cf    }
427e
d658    void push_mul(int o, LL x) {
b82c        val[o] = val[o] * x % p;
aa86        tadd[o] = tadd[o] * x % p;
649f        tmul[o] = tmul[o] * x % p;
95cf    }
427e
b149    void push(int o) {
3159        if (l[o] == m[o]) return;
0a90        if (tmul[o] != 1) {
0f4a            push_mul(lson, tmul[o]);
045e            push_mul(rson, tmul[o]);
ac0a            tmul[o] = 1;
95cf        }
1b82        if (tadd[o]) {
9547            push_add(lson, tadd[o]);
0e73            push_add(rson, tadd[o]);
6234            tadd[o] = 0;
95cf        }
95cf    }
```

```
void build(int o, int ll, int rr) {
    int mm = (ll + rr) / 2;
    l[o] = ll; r[o] = rr; m[o] = mm;
    tmul[o] = 1;
    if (ll == mm) {
        scanf("%lld", val + o);
        val[o] %= p;
    } else {
        build(lson, ll, mm);
        build(rson, mm, rr);
        pull(o);
    }
}

void add(int o, int ll, int rr, LL x) {
    if (ll <= l[o] && r[o] <= rr) {
        push_add(o, x);
    } else {
        push(o);
        if (m[o] > ll) add(lson, ll, rr, x);
        if (m[o] < rr) add(rson, ll, rr, x);
        pull(o);
    }
}

void mul(int o, int ll, int rr, LL x) {
    if (ll <= l[o] && r[o] <= rr) {
        push_mul(o, x);
    } else {
        push(o);
        if (ll < m[o]) mul(lson, ll, rr, x);
        if (m[o] < rr) mul(rson, ll, rr, x);
        pull(o);
    }
}

LL query(int o, int ll, int rr) {
    if (ll <= l[o] && r[o] <= rr) {
        return val[o];
    } else {
        push(o);
        if (rr <= m[o]) return query(lson, ll, rr);
        if (ll >= m[o]) return query(rson, ll, rr);
```

```
427e
471c
0e87
9d27
ac0a
5c92
001f
e5b6
8e2e
7293
5e67
ba26
95cf
95cf
427e
4406
3c16
db32
8e2e
c4b0
4305
d5a6
ba26
95cf
95cf
427e
48cd
3c16
e7d0
8e2e
c4b0
d1ba
67f3
ba26
95cf
95cf
427e
0f62
3c16
6dfe
8e2e
c4b0
462a
5cca
```

```

bbf9     return query(lson, ll, rr) + query(rson, ll, rr);
95cf     }
95cf     }
4d99     } seg;

```

6.4 Treap

Self-balanced binary search tree which supports split and merge.

Usage:

push(x)	Push lazy tags to children.
pull(x)	Update statistics of node x .
Init(x, v)	Initialize node x with value v .
Add(x, v)	Apply addition to subtree x .
Reverse(x)	Apply reversion to subtree x .
Merge(x, y)	Merge trees rooted at x and y . Return the root of new tree.
Split(t, k, x, y)	Split out the left k elements of tree t . The roots of left part and right part are stored in x and y , respectively.
init(n)	Initialize the treap with array of size n .
work(op, l, r)	Range operation over $[l, r)$.

Time Complexity: Expected $O(\log n)$ per operation.

```

9f60 const int MAXN = 200005;
a7c5 mt19937 gen(time(NULL));
9542 struct Treap {
6d61     int ch[MAXN][2];
3948     int sz[MAXN], key[MAXN], val[MAXN];
5d9a     int add[MAXN], rev[MAXN];
2b1b     LL sum[MAXN] = {0};
a773     int maxv[MAXN] = {INT_MIN}, minv[MAXN] = {INT_MAX};
427e
a629 void Init(int x, int v) {
5a00     ch[x][0] = ch[x][1] = 0;
d8cd     key[x] = gen(); val[x] = v; pull(x);
95cf }
427e
3bf9 void pull(int x) {
e1c3     sz[x] = 1 + sz[ch[x][0]] + sz[ch[x][1]];
99f8     sum[x] = val[x] + sum[ch[x][0]] + sum[ch[x][1]];
94e9     maxv[x] = max({val[x], maxv[ch[x][0]], maxv[ch[x][1]]});
6bb9     minv[x] = min({val[x], minv[ch[x][0]], minv[ch[x][1]]});
95cf }
427e

```

```

void Add(int x, int a) {
    val[x] += a; add[x] += a;
    sum[x] += LL(sz[x]) * a; maxv[x] += a; minv[x] += a;
}

```

```

void Reverse(int x) {
    rev[x] ^= 1;
    swap(ch[x][0], ch[x][1]);
}

```

```

void push(int x) {
    for (int c : ch[x]) if (c) {
        Add(c, add[x]);
        if (rev[x]) Reverse(c);
    }
    add[x] = 0; rev[x] = 0;
}

```

```

int Merge(int x, int y) {
    if (!x || !y) return x | y;
    push(x); push(y);
    if (key[x] > key[y]) {
        ch[x][1] = Merge(ch[x][1], y); pull(x); return x;
    } else {
        ch[y][0] = Merge(x, ch[y][0]); pull(y); return y;
    }
}

```

```

void Split(int t, int k, int &x, int &y) {
    if (t == 0) { x = y = 0; return; }
    push(t);
    if (sz[ch[t][0]] < k) {
        x = t; Split(ch[t][1], k - sz[ch[t][0]] - 1, ch[t][1], y);
    } else {
        y = t; Split(ch[t][0], k, x, ch[t][0]);
    }
    if (x) pull(x); if (y) pull(y);
}
} treap;

```

```
int root;
```

```

void init(int n) {
    Rep(i, n) {

```

```

8c8e
a7b1
832a
95cf
427e
aaf6
52c6
7850
95cf
427e
1a53
5fe5
fd76
7a53
95cf
49ee
95cf
427e
9d2c
1b09
cd7e
bffa
a3df
8e2e
bf9e
95cf
95cf
427e
dc7e
6303
f26b
3465
ffd8
8e2e
8a23
95cf
89e3
95cf
b1f4
427e
24b6
427e
d34f
34d7

```

```

7681     int x; scanf("%d", &x);
0ed8     treap.Init(i, x);
bcc8     root = (i == 1) ? 1 : treap.Merge(root, i);
95cf }
95cf }
427e
d030 void work(int op, int l, int r) {
6639     int tl, tm, tr;
b6c4     treap.Split(root, l, tl, tm);
8de3     treap.Split(tm, r - 1, tm, tr);
3658     if (op == 1) {
c039         int x; scanf("%d", &x); treap.Add(tm, x);
1dcb     } else if (op == 2) {
ae78         treap.Reverse(tm);
581d     } else if (op == 3) {
e092         printf("%lld,%d,%d\n",
867f             treap.sum[tm], treap.minv[tm], treap.maxv[tm]);
95cf     }
6188     root = treap.Merge(treap.Merge(tl, tm), tr);
95cf }

```

6.5 Link/cut tree

Dynamic connectivity of undirected acyclic graph. Support single-vertex update, path aggregation and relative LCA query. Vertices are numbered from 1. Zero initialization is enough except for the statistic information.

Usage:

pull(x)	Update statistics of node <i>x</i> .
Root(u)	Get the root of tree where vertex <i>u</i> is in.
Link(u, v)	Link two unconnected trees.
Cut(u, v)	Cut an existent edge.
Query(u, v)	Path aggregation.
Update(u, x)	Single point modification.
LCA(u, v, root)	Get the lowest common ancestor of <i>u</i> and <i>v</i> in tree rooted at root.

Time Complexity: $O(\log n)$ per operation

```

2e73 const int MAXN = 1000005;
ca06 struct LCT {
6a6d     int fa[MAXN], ch[MAXN][2], val[MAXN], sum[MAXN];
c6e1     bool rev[MAXN];
427e

```

```

bool isroot(int x) { return ch[fa[x]][0] == x || ch[fa[x]][1] == x; }
void pull(int x) { sum[x] = val[x] ^ sum[ch[x][0]] ^ sum[ch[x][1]]; }
void reverse(int x) { swap(ch[x][0], ch[x][1]); rev[x] ^= 1; }
void push(int x) {
    if (rev[x]) rep (i, 2) if (ch[x][i]) reverse(ch[x][i]); rev[x] = 0;
}
void rotate(int x) {
    int y = fa[x], z = fa[y], k = ch[y][1] == x, w = ch[x][!k];
    if (isroot(y)) ch[z][ch[z][1] == y] = x;
    ch[x][!k] = y; ch[y][k] = w; if (w) fa[w] = y;
    fa[y] = x; fa[x] = z; pull(y);
}
void pushall(int x) { if (isroot(x)) pushall(fa[x]); push(x); }
void splay(int x) {
    int y = x, z = 0;
    for (pushall(y); isroot(x); rotate(x)) {
        y = fa[x]; z = fa[y];
        if (isroot(y)) rotate((ch[y][0] == x) ^ (ch[z][0] == y) ? x : y);
    }
    pull(x);
}
void access(int x) {
    int z = x;
    for (int y = 0; x; x = fa[y = x]) { splay(x); ch[x][1] = y; pull(x); }
    splay(z);
}
void chroot(int x) { access(x); reverse(x); }
void split(int x, int y) { chroot(x); access(y); }

int Root(int x) {
    for (access(x); ch[x][0]; x = ch[x][0]) push(x);
    splay(x); return x;
}
void Link(int u, int v) { chroot(u); fa[u] = v; }
void Cut(int u, int v) { split(u, v); fa[u] = ch[v][0] = 0; pull(v); }
int Query(int u, int v) { split(u, v); return sum[v]; }
void Update(int u, int x) { splay(u); val[u] = x; }
int LCA(int x, int y, int root) {
    chroot(root); access(x); splay(y);
    while (fa[y]) splay(y = fa[y]);
    return y;
}
};

```

eba3
 f19f
 1c4d
 1a53
 89a0
 95cf
 425f
 51af
 e1fe
 1e6f
 6d09
 95cf
 52c6
 f69c
 d095
 c494
 ceef
 4449
 95cf
 78a0
 95cf
 6229
 1548
 8854
 7afd
 95cf
 a067
 126d
 427e
 d87a
 f4f1
 0d77
 95cf
 9e46
 7c10
 0691
 a999
 1f42
 6cb2
 02e5
 c218
 95cf
 329b

6.6 Balanced binary search tree from pb_ds

```

0475 #include <ext/pb_ds/assoc_container.hpp>
332d using namespace __gnu_pbds;
427e
43a7 tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update>
    rkt;
427e // null_tree_node_update
427e
427e // SAMPLE USAGE
190e rkt.insert(x);           // insert element
05d4 rkt.erase(x);          // erase element
add5 rkt.order_of_key(x);   // obtain the number of elements less than x
b064 rkt.find_by_order(i);   // iterator to i-th (numbered from 0) smallest element
c103 rkt.lower_bound(x);
4ff4 rkt.upper_bound(x);
b19b rkt.join(rkt2);        // merge tree (only if their ranges do not intersect)
cb47 rkt.split(x, rkt2);     // split all elements greater than x to rkt2

```

6.7 Persistent segment tree, range k-th query

```

f1a7 struct node {
2ff6     static int n, pos;
427e
7cec     int value;
70e2     node *left, *right;
427e
20b0     void* operator new(size_t size);
427e
3dc0     static node* Build(int l, int r) {
b6c5         node* a = new node;
ce96         if (r > l + 1) {
181e             int mid = (l + r) / 2;
3ba2             a->left = Build(l, mid);
8aaf             a->right = Build(mid, r);
8e2e         } else {
bfc4             a->value = 0;
95cf         }
5ffd         return a;
95cf     }
427e
5a45     static node* init(int size) {

```

```

n = size;
pos = 0;
return Build(0, n);
}

```

```

static int Query(node* lt, node *rt, int l, int r, int k) {
    if (r == l + 1) return l;
    int mid = (l + r) / 2;
    if (rt->left->value - lt->left->value < k) {
        k -= rt->left->value - lt->left->value;
        return Query(lt->right, rt->right, mid, r, k);
    } else {
        return Query(lt->left, rt->left, l, mid, k);
    }
}

```

```

static int query(node* lt, node *rt, int k) {
    return Query(lt, rt, 0, n, k);
}

```

```

node *Inc(int l, int r, int pos) const {
    node* a = new node(*this);
    if (r > l + 1) {
        int mid = (l + r) / 2;
        if (pos < mid)
            a->left = left->Inc(l, mid, pos);
        else
            a->right = right->Inc(mid, r, pos);
    }
    a->value++;
    return a;
}

```

```

node *inc(int index) {
    return Inc(0, n, index);
}
} nodes[8000000];

```

```

int node::n, node::pos;
inline void* node::operator new(size_t size) {
    return nodes + (pos++);
}

```

2c46
7ee3
be52
95cf
427e
93c0
d30c
181e
cb5a
8edb
2412
8e2e
0119
95cf
95cf
427e
c9ad
9e27
95cf
427e
b19c
5794
ce96
181e
203d
f44a
649a
1024
95cf
2b3e
5ffd
95cf
427e
e80f
c246
95cf
865a
427e
99ce
1987
bb3c
95cf

6.8 Block list

All indices are 0-based. All ranges are left-closed right-open.

Usage:

<code>block::fix()</code>	Apply tags to the current block.
<code>Init(l, r)</code>	Range initializer.
<code>Reverse(l, r)</code>	Reverse the range.
<code>Add(l, r, x)</code>	Add x to the range.
<code>Query(l, r)</code>	Range aggregation.

```
fd9e const int BLOCK = 800;
76b3 typedef vector<int> vi;
427e
a771 struct block {
8fbc     vi data;
e3b5     LL sum; int minv, maxv;
41db     int add; bool rev;
427e
d7eb     block(vi&& vec) : data(move(vec)),
1f0c         sum(accumulate(range(data), 0ll)),
8216         minv(*min_element(range(data))),
527d         maxv(*max_element(range(data))),
6437         add(0), rev(0) { }
427e
b919     void fix() {
0694         if (rev) reverse(range(data));         rev = 0;
0527         if (add) for (int& x : data) x += add;   add = 0;
95cf     }
427e
8bc4     void merge(block& another) {
b895         fix(); another.fix();
f516         vi temp(move(data));
d02c         temp.insert(temp.end(), range(another.data));
88ea         *this = block(move(temp));
95cf     }
427e
42e8     block split(int pos) {
3e79         fix();
ccab         block result(vi(data.begin() + pos, data.end()));
861a         data.resize(pos); *this = block(move(data));
56b0         return result;
95cf     }
329b };
427e
```

```
typedef list<block>::iterator lit;

struct blocklist {
    list<block> blk;

    void maintain() {
        lit it = blk.begin();
        while (it != blk.end() && next(it) != blk.end()) {
            lit it2 = it;
            while (next(it2) != blk.end() &&
                    it2->data.size() + next(it2)->data.size() <= BLOCK) {
                it2->merge(*next(it2));
                blk.erase(next(it2));
            }
            ++it;
        }
    }

    lit split(int pos) {
        for (lit it = blk.begin(); ; it++) {
            if (pos == 0) return it;
            while (it->data.size() > pos)
                blk.insert(next(it), it->split(pos));
            pos -= it->data.size();
        }
    }

    void Init(int *l, int *r) {
        for (int *cur = l; cur < r; cur += BLOCK)
            blk.emplace_back(vi(cur, min(cur + BLOCK, r)));
    }

    void Reverse(int l, int r) {
        lit it = split(l), it2 = split(r);
        reverse(it, it2);
        while (it != it2) {
            it->rev ^= 1;
            it++;
        }
        maintain();
    }

    void Add(int l, int r, int x) {
```

```
2a18
427e
ce14
5540
427e
7b8e
3131
4628
852d
188c
3600
93e1
e1fa
95cf
5771
95cf
95cf
427e
b7b3
2273
5502
8e85
2099
a5a1
427e
95cf
95cf
427e
1c7b
9919
8950
95cf
427e
a22f
997b
dfd0
8f89
6a06
5283
95cf
b204
95cf
427e
3cce
```

```

997b     lit it = split(l), it2 = split(r);
8f89     while (it != it2) {
e927         it->sum += LL(x) * it->data.size();
03d3         it->minv += x; it->maxv += x;
4511         it->add += x; it++;
95cf     }
b204     maintain();
95cf }
427e
3ad3     void Query(int l, int r) {
997b         lit it = split(l), it2 = split(r);
c33d         LL sum = 0; int minv = INT_MAX, maxv = INT_MIN;
8f89         while (it != it2) {
e472             sum += it->sum;
72c4             minv = min(minv, it->minv);
e1c4             maxv = max(maxv, it->maxv);
5283             it++;
95cf         }
b204         maintain();
8792         printf("%lld_%d_%d\n", sum, minv, maxv);
95cf     }
958e } lst;

```

6.9 Persistent block list

Block list that supports persistence. All indices are 0-based. All ranges are left-closed right-open. `std::shared_ptr` is used to ease memory management. One should modify the constructor of `block` to maintain extra information. Here we use this policy that the size of each block does not exceed `BLOCK`, while the sum of sizes of two adjacent blocks does not less than `BLOCK`.

When some operation that breaks block list property, please call `maintain` in time to restore the property.

Usage:

<code>maintain()</code>	Maintain the block list property.
<code>split(pos)</code>	Split the block list at position <code>pos</code> . Returns an iterator to a block starting at <code>pos</code> .
<code>sum(l, r)</code>	An example function of list traversal between $[l, r)$.

Time Complexity: When `BLOCK` is properly selected, the time complexity is $O(\sqrt{n})$ per operation.

```

a19e constexpr int BLOCK = 800;
76b3 typedef vector<int> vi;

```

```

typedef shared_ptr<vi> pvi;
typedef shared_ptr<const vi> pcvi;

struct block {
    pcvi data;
    LL sum;

    // add information to maintain
    block(pcvi ptr) :
        data(ptr),
        sum(accumulate(ptr->begin(), ptr->end(), 0ll))
    { }

    void merge(const block& another) {
        pvi temp = make_shared<vi>(data->begin(), data->end());
        temp->insert(temp->end(), another.data->begin(), another.data->end());
        *this = block(temp);
    }

    block split(int pos) {
        block result(make_shared<vi>(data->begin() + pos, data->end()));
        *this = block(make_shared<vi>(data->begin(), data->begin() + pos));
        return result;
    }
};

typedef list<block>::iterator lit;

struct blocklist {
    list<block> blk;

    void maintain() {
        lit it = blk.begin();
        while (it != blk.end() and next(it) != blk.end()) {
            lit it2 = it;
            while (next(it2) != blk.end() and
                it2->data->size() + next(it2)->data->size() <= BLOCK) {
                it2->merge(*next(it2));
                blk.erase(next(it2));
            }
            ++it;
        }
    }
};

```

0563
013b
427e
a771
2989
8fd0
427e
427e
a613
24b5
0cf0
e93b
427e
5c0f
0b18
ac21
6467
95cf
427e
42e8
dac1
01db
56b0
95cf
329b
427e
2a18
427e
ce14
5540
427e
7b8e
3131
5e44
852d
0b03
029f
93e1
e1fa
95cf
5771
95cf
427e

```

b7b3     lit split(int pos) {
2273         for (lit it = blk.begin(); ; it++) {
5502             if (pos == 0) return it;
d480             while (it->data->size() > pos) {
2099                 blk.insert(next(it), it->split(pos));
95cf             }
a1c8             pos -= it->data->size();
95cf         }
95cf     }
427e
fd38     LL sum(int l, int r) { // traverse
48b4         lit it1 = split(l), it2 = split(r);
ac09         LL res = 0;
9f1d         while (it1 != it2) {
8284             res += it1->sum;
61fd             it1++;
95cf         }
b204         maintain();
244d         return res;
95cf     }
329b };

```

6.10 Sparse table, range minimum query

The array is 0-based and the range is left-closed right-open.

```

db63     const int MAXN = 100007;
cefd     int a[MAXN], st[MAXN][30];
427e
d34f     void init(int n){
c73d         int l = log2(n);
cf75         rep (i, n) st[i][0] = a[i];
426b         rep (j, l) rep (i, 1+n-(1<<j))
1131             st[i][j+1] = min(st[i][j], st[i+(1<<j)][j]);
95cf     }
427e
c863     int rmq(int l, int r){
f089         int k = log2(r - l);
6117         return min(st[l][k], st[r-(1<<k)][k]);
95cf     }

```

7 Geometrics

7.1 2D geometric template

```

#include <bits/stdc++.h>
using namespace std;

typedef int T;
typedef struct pt {
    T x, y;
    T operator , (pt a) { return x*a.x + y*a.y; } // inner product
    T operator * (pt a) { return x*a.y - y*a.x; } // outer product
    pt operator + (pt a) { return {x+a.x, y+a.y}; }
    pt operator - (pt a) { return {x-a.x, y-a.y}; }

    pt operator * (T k) { return {x*k, y*k}; }
    pt operator - () { return {-x, -y}; }
} vec;

typedef pair<pt, pt> seg;

bool ptOnSeg(pt& p, seg& s){
    vec v1 = s.first - p, v2 = s.second - p;
    return (v1, v2) <= 0 && v1 * v2 == 0;
}

// 0 not on segment
// 1 on segment except vertices
// 2 on vertices
int ptOnSeg2(pt& p, seg& s){
    vec v1 = s.first - p, v2 = s.second - p;
    T ip = (v1, v2);
    if (v1 * v2 != 0 || ip > 0) return 0;
    return (v1, v2) ? 1 : 2;
}

// if two orthogonal rectangles do not touch, return true
inline bool nIntRectRect(seg a, seg b){
    return min(a.first.x, a.second.x) > max(b.first.x, b.second.x) ||
           min(a.first.y, a.second.y) > max(b.first.y, b.second.y) ||
           min(b.first.x, b.second.x) > max(a.first.x, a.second.x) ||
           min(b.first.y, b.second.y) > max(a.first.y, a.second.y);
}

```

302f
421c
427e
4553
c0ae
7a9d
ffaa
3ec7
221a
8b34
427e
368b
90f4
ba8c
427e
0ea6
427e
8d6e
ce77
de97
95cf
427e
427e
427e
427e
8421
ce77
70ca
8b14
0847
95cf
427e
427e
72bb
f9ac
f486
39ce
80c7
95cf

```

427e // >0 in order
427e // <0 out of order
427e // =0 not standard
7538 inline double rotOrder(vec a, vec b, vec c){return double(a*b)*(b*c);}
427e
31ed inline bool intersect(seg a, seg b){
427e     // ! if (nIntRectRect(a, b)) return false; // if commented, assume that a
        and b are non-collinear
cb52     return rotOrder(b.first-a.first, a.second-a.first, b.second-a.first) >= 0 &&
059e         rotOrder(a.first-b.first, b.second-b.first, a.second-b.first) >= 0;
95cf }
427e
427e // 0 not intersect
427e // 1 standard intersection
427e // 2 vertex-line intersection
427e // 3 vertex-vertex intersection
427e // 4 collinear and have common point(s)
4d19 int intersect2(seg& a, seg& b){
5dc4     if (nIntRectRect(a, b)) return 0;
42c0     vec va = a.second - a.first, vb = b.second - b.first;
2096     double j1 = rotOrder(b.first-a.first, va, b.second-a.first),
72fe         j2 = rotOrder(a.first-b.first, vb, a.second-b.first);
5ac6     if (j1 < 0 || j2 < 0) return 0;
9400     if (j1 != 0 && j2 != 0) return 1;
83db     if (j1 == 0 && j2 == 0){
6b0c         if (va * vb == 0) return 4; else return 3;
fb17     } else return 2;
95cf }
427e
2c68 template <typename Tp = T>
5894 inline pt getIntersection(pt P, vec v, pt Q, vec w){
6850     static_assert(is_same<Tp, double>::value, "must_be_double!");
7c9a     return P + v * (w*(P-Q)/(v*w));
95cf }
427e
427e // -1 outside the polygon
427e // 0 on the border of the polygon
427e // 1 inside the polygon
cbdd int ptOnPoly(pt p, pt* poly, int n){
5fb4     int wn = 0;
1294     for (int i = 0; i < n; i++) {
427e
3cae         T k, d1 = poly[i].y - p.y, d2 = poly[(i+1)%n].y - p.y;
```

```

        if (k = (poly[(i+1)%n] - poly[i])*(p - poly[i])){
            if (k > 0 && d1 <= 0 && d2 > 0) wn++;
            if (k < 0 && d2 <= 0 && d1 > 0) wn--;
        } else return 0;
    }
    return wn ? 1 : -1;
}

istream& operator >> (istream& lhs, pt& rhs){
    lhs >> rhs.x >> rhs.y;
    return lhs;
}

istream& operator >> (istream& lhs, seg& rhs){
    lhs >> rhs.first >> rhs.second;
    return lhs;
}
}
```

b957
8c40
3c4d
aad3
95cf
0a5f
95cf
427e
d4a3
fa86
331a
95cf
427e
07ae
5cab
331a
95cf

8 Appendices

8.1 Primes

8.1.1 First primes

p	$g(p)$	p	$g(p)$	p	$g(p)$	p	$g(p)$	p	$g(p)$
2	1	3	2	5	2	7	3	11	2
13	2	17	3	19	2	23	5	29	2
31	3	37	2	41	6	43	3	47	5
53	2	59	2	61	2	67	2	71	7
73	5	79	3	83	2	89	3	97	5
101	2	103	5	107	2	109	6	113	3
127	3	131	2	137	3	139	2	149	2
151	6	157	5	163	2	167	5	173	2
179	2	181	2	191	19	193	5	197	2
199	3	211	2	223	3	227	2	229	6

8.1.2 Arbitrary length primes

$\lg p$	p	$g(p)$	p	$g(p)$
3	967	5	1031	14
4	9859	2	10273	10
5	96331	10	102931	3
6	958543	6	1031137	5
7	9594539	2	10169651	2
8	96243449	3	103211039	7
9	980483981	2	1042484357	2
10	9858935453	2	10261276009	7
11	95748666809	3	101759940101	2
12	950781833849	3	1012797784423	5
13	9739822952371	7	10037217092377	7
14	96181051140397	5	104974966380359	11
15	981030138360889	13	1029038416465403	2
16	9655206098080843	3	10116299875820773	2
17	97687777921994419	3	101506415998163437	2

8.1.3 $\sim 1 \times 10^9$

p	$g(p)$	p	$g(p)$	p	$g(p)$
954854573	3	967607731	2	973215833	3
975831713	3	978949117	2	980766497	3
983879921	3	985918807	3	986608921	29
991136977	5	991752599	13	997137961	11
1003911991	3	1009775293	2	1012423549	6
1021000537	5	1023976897	7	1024153643	2
1037027287	3	1038812881	11	1044754639	3
1045125617	3	1047411427	3	1047753349	6

8.1.4 $\sim 1 \times 10^{18}$

p	$g(p)$	p	$g(p)$
951970612352230049	3	963284339889659609	3
967495386904694119	3	969751761517096213	2
983238274281901499	2	984647442475101409	23
989286107138674069	11	1002507954383424641	3
1006658951440146419	2	1020152326159075903	3
1034876265966119449	7	1042753851435034019	2
1043609016597371563	2	1045571042176595707	2
1048364250160580293	2	1049495624119026949	2

8.2 Pell's equation

$x^2 - ny^2 = 1$, where n is a positive nonsquare integer.

Let (x_0, y_0) be the smallest positive solution of the equation, then the k -th solution is:

$$\begin{pmatrix} x_k \\ y_k \end{pmatrix} = \begin{pmatrix} x_0 & ny_0 \\ y_0 & x_0 \end{pmatrix}^k \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

Some smallest solutions to Pell's equation:

n	2	3	5	6	7	8	10	11	12	13	14	15	17	18	19	20
x	3	2	9	5	8	3	19	10	7	649	15	4	33	17	170	9
y	2	1	4	2	3	1	6	3	2	180	4	1	8	4	39	2

8.3 Burnside's lemma and Polya's enumeration theorem

The Burnside's lemma says that

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

where G is a group acting on X , X^g is the set of elements in X that are fixed by g , i.e. $X^g = \{x \in X : gx = x\}$.

The unweighted version of Pólya enumeration theorem says that

$$|Y^X/G| = \frac{1}{|G|} \sum_{g \in G} m^{c_g}$$

where $m = |X|$ is the number of colors, c_g is the number of the cycles of permutation g .

8.4 Lagrange's interpolation

For sample points $(x_0, y_0), \dots, (x_k, y_k)$, define

$$l_j(x) = \prod_{0 \leq m \leq k, m \neq j} \frac{x - x_m}{x_j - x_m}$$

then the Lagrange polynomial is

$$L(x) = \sum_{j=0}^k y_j l_j(x).$$

To use the script below, type two lines

```
x0 x1 x2 ... xn
y0 y1 y2 ... yn
```

the script will print the fractional coefficient of the polynomial in ascending exponent order.

```
#!/usr/bin/python2
from fractions import *

def polymul(a, b) :
    p = [0] * (len(a)+len(b)-1)
    for e1, c1 in enumerate(a) :
        for e2, c2 in enumerate(b) :
            p[e1+e2] += c1*c2
    return p

x, y = [map(Fraction, raw_input().split()) for _ in 0,0]
n = len(x)
lj = [reduce(polymul, [[-x[m]/(x[j]-x[m]), 1/(x[j]-x[m])]
    for m in range(n) if m != j]]) for j in range(n)]
print '\n'.join(map(str, map(sum, zip(*map(
    lambda a, b : [x*a for x in b], y, lj)))))
```

6dc9
4b2b
427e
796b
83e4
f697
156c
dfce
5849
427e
f06d
e80a
a649
9dfa
3cae
7c0d