

南京大学 ACM-ICPC 集训队代码模版库



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1 General

1.1 Code library checksum

```
ab14 #!/usr/bin/python3
c502 import re, sys, hashlib
427e
f7db for line in sys.stdin.read().strip().split("\n") :
ddf5     print(hashlib.md5(re.sub(r'\s|//[.]*', '', line).encode('utf8')).hexdigest()
        [-4:], line)
```

1.2 .vimrc

```
914c set nocompatible
733d syntax on
6bbc colorscheme slate
7db5 set number
b0e3 set cursorline
061b set shiftwidth=2
8011 set softtabstop=2
a66d set tabstop=2
d23a set expandtab
5245 set magic
740c set smartindent
bee8 set backspace=indent,eol,start
815d set cmdheight=1
0a40 set laststatus=2
e458 set statusline=\ %<F[%1%M%*%n%R%H]%=\ %y\ %0(%{&fileformat}\ %{&encoding}\ %c
    :%l/%L%\
1c67 set whichwrap=b,s,<,>,[,]
```

1.3 Template

```
302f #include <bits/stdc++.h>
421c using namespace std;
427e
426f #ifdef __LOCAL_DEBUG__
3341 # define _debug(fmt, ...) fprintf(stderr, "[%s]_" fmt "\n", \
611f     __func__, ##__VA_ARGS__)
a8cb #else
```

```
# define _debug(...) ((void) 0)
#endif
#define rep(i, n) for (int i=0; i<(n); i++)
#define Rep(i, n) for (int i=1; i<=(n); i++)
#define range(x) (x).begin(), (x).end()
typedef long long LL;
typedef unsigned long long ULL;

template <unsigned p>
struct Zp{
    unsigned x;
    Zp(unsigned x):x(x){}
    operator unsigned(){return x;}
    Zp operator ^ (ULL e) {
        Zp b=x, r=1;
        while (e) {
            if (e&1) r=r*b;
            b=b*b;
            e>>=1;
        }
        return r;
    }
    Zp operator + (Zp rhs) {return (x+rhs)%p;}
    Zp operator - (Zp rhs) {return (x+p-rhs)%p;}
    Zp operator * (Zp rhs) {return x*rhs%p;}
    Zp operator / (Zp rhs) {return Zp(x)*(rhs^(p-2));}
};

typedef Zp<1000000007> zp;

zp operator"" _ (ULL n){return n;}
```

```
e6b5
1937
0d6c
cfe3
8843
5cad
b773
427e
5120
87b8
7797
ff67
22e3
fecc
4fce
3e90
5421
2059
16fc
95cf
547e
95cf
a2f5
664b
3ec4
7cfd
329b
427e
370f
427e
0795
```

2 Miscellaneous Algorithms

2.1 2-SAT

```
const int MAXN = 100005;
struct twoSAT{
    int n;
    vector<int> G[MAXN*2];
    bool mark[MAXN*2];
    int S[MAXN*2], c;
```

```
0f42
03a9
5c83
8f72
d060
b42d
```

```

427e void init(int n){
d34f     this->n = n;
b985     for (int i=0; i<n*2; i++) G[i].clear();
f9ec     memset(mark, 0, sizeof(mark));
0609 }
95cf
427e bool dfs(int x){
3bd5     if (mark[x^1]) return false;
bd70     if (mark[x]) return true;
c96a     mark[x] = true;
fd23     S[++] = x;
4bea     for (int i=0; i<G[x].size(); i++)
1ce6         if (!dfs(G[x][i])) return false;
d942     return true;
3361 }
95cf
427e void add_clause(int x, bool xval, int y, bool yval){
5894     x = x * 2 + xval;
6afe     y = y * 2 + yval;
e680     G[x^1].push_back(y);
81cc     G[y^1].push_back(x);
6835 }
95cf
427e bool solve() {
d0cb     for (int i=0; i<n*2; i+=2){
7c39         if (!mark[i] && !mark[i+1]){
e63f             c = 0;
88fb             if (!dfs(i)){
f4b9                 while (c > 0) mark[S[--c]] = false;
3f03                 if (!dfs(i+1)) return false;
86c5             }
95cf         }
95cf     }
3361     return true;
95cf }
427e
5f0a inline bool value(unsigned i){return mark[2*i+1];}
329b };

```

2.2 Knuth's optimization

```

int n;
int dp[256][256], dc[256][256];

template <typename T>
void compute(T cost) {
    for (int i = 0; i <= n; i++) {
        dp[i][i] = 0;
        dc[i][i] = i;
    }
    rep (i, n) {
        dp[i][i+1] = 0;
        dc[i][i+1] = i;
    }
    for (int len = 2; len <= n; len++) {
        for (int i = 0; i + len <= n; i++) {
            int j = i + len;
            int lbnd = dc[i][j-1], rbnd = dc[i+1][j];
            dp[i][j] = INT_MAX / 2;
            int c = cost(i, j);
            for (int k = lbnd; k <= rbnd; k++) {
                int res = dp[i][k] + dp[k][j] + c;
                if (res < dp[i][j]) {
                    dp[i][j] = res;
                    dc[i][j] = k;
                }
            }
        }
    }
};

```

3 String

3.1 Knuth-Morris-Pratt algorithm

```

const int SIZE = 10005;
int fail[SIZE];
int len;

void construct(const char* p) {
    len = strlen(p);
    fail[0] = fail[1] = 0;
    for (int i = 1; i < len; i++) {

```

```

5c83
d77c
427e
b7ec
0bc7
0423
8f5e
9488
95cf
be8e
95b5
aa0f
95cf
ec08
88b8
d3da
9824
a24a
f933
90d2
9bd0
26b5
e6af
9c88
95cf
95cf
95cf
329b

```

```

2836
9847
57b7
427e
182f
aaa1
3dd4
d8a8

```

```

147f      int j = fail[i];
3c79      while (j && p[i] != p[j]) j = fail[j];
4643      fail[i + 1] = p[i] == p[j] ? j + 1 : 0;
95cf    }
95cf  }
427e
c464  inline void found(int pos) {
427e      // ! add codes for having found at pos
95cf  }
427e
1932  void match(const char* t, const char* p) { // must be called after construct
8482      int n = strlen(t);
8fd0      int j = 0;
be8e      rep(i, n) {
4e19          while (j && p[j] != t[i]) j = fail[j];
b5d5          if (p[j] == t[i]) j++;
f024          if (j == len) found(i - len + 1);
95cf      }
95cf  }

```

3.2 Manacher algorithm

```

81d4  struct Manacher {
cd09      int Len;
9255      vector<int> lc;
b301      string s;
427e
ec07      void work() {
c033          lc[1] = 1;
6bef          int k = 1;
427e
491f          for (int i = 2; i <= Len; i++) {
7957              int p = k + lc[k] - 1;
5e04              if (i <= p) {
24a1                  lc[i] = min(lc[2 * k - i], p - i + 1);
8e2e              } else {
e0e5                  lc[i] = 1;
95cf              }
74ff              while (s[i + lc[i]] == s[i - lc[i]]) lc[i]++;
2b9a              if (i + lc[i] > k + lc[k]) k = i;
95cf          }
95cf  }

```

```

void init(const char *tt) {
    int len = strlen(tt);
    s.resize(len * 2 + 10);
    lc.resize(len * 2 + 10);
    s[0] = '*';
    s[1] = '#';
    for (int i = 0; i < len; i++) {
        s[i * 2 + 2] = tt[i];
        s[i * 2 + 1] = '#';
    }
    s[len * 2 + 1] = '#';
    s[len * 2 + 2] = '\0';
    Len = len * 2 + 2;
    work();
}

```

```

pair<int, int> maxpal(int l, int r) {
    int center = l + r + 1;
    int rad = lc[center] / 2;
    int rmid = (l + r + 1) / 2;
    int rl = rmid - rad, rr = rmid + rad - 1;
    if ((r ^ 1) & 1) {
        } else rr++;
    return {max(1, rl), min(r, rr)};
}
};

```

3.3 Aho-corasick automaton

```

struct AC : Trie {
    int fail[MAXN];
    int last[MAXN];

    void construct() {
        queue<int> q;
        fail[0] = 0;
        rep(c, CHARN) {
            if (int u = tr[0][c]) {
                fail[u] = 0;
                q.push(u);
                last[u] = 0;
            }
        }
    }
}

```

```

95cf     }
95cf     }
cc78     while (!q.empty()) {
31f0         int r = q.front();
15dd         q.pop();
ce3c         rep(c, CHARN) {
ab59             int u = tr[r][c];
0ef5             if (!u) {
9d58                 tr[r][c] = tr[fail[r]][c];
b333                 continue;
95cf             }
3e14         q.push(u);
b3ff         int v = fail[r];
d2ea         while (v && !tr[v][c]) v = fail[v];
c275         fail[u] = tr[v][c];
654c         last[u] = tag[fail[u]] ? fail[u] : last[fail[u]];
95cf     }
95cf     }
95cf     }
427e
7752     void found(int pos, int j) {
043e         if (j) {
427e             // ! add codes for having found word with tag[j]
4a96             found(pos, last[j]);
95cf         }
95cf     }
427e
9785     void find(const char* text) { // must be called after construct()
80a4         int p = 0, c, len = strlen(text);
9c94         rep(i, len) {
b3db             c = id(text[i]);
f119             p = tr[p][c];
f08e             if (tag[p])
389b                 found(i, p);
1e67             else if (last[p])
299e                 found(i, last[p]);
95cf         }
95cf     }
329b };

```

3.4 Trie

```

const int MAXN = 12000;
const int CHARN = 26;

inline int id(char c) { return c - 'a'; }

struct Trie {
    int n;
    int tr[MAXN][CHARN]; // Trie tree, 0 denotes fail
    int tag[MAXN];

    Trie() {
        memset(tr[0], 0, sizeof(tr[0]));
        tag[0] = 0;
        n = 1;
    }

    // tag should not be 0
    void add(const char* s, int t) {
        int p = 0, c, len = strlen(s);
        rep(i, len) {
            c = id(s[i]);
            if (!tr[p][c]) {
                memset(tr[n], 0, sizeof(tr[n]));
                tag[n] = 0;
                tr[p][c] = n++;
            }
            p = tr[p][c];
        }
        tag[p] = t;
    }

    // returns 0 if not found
    // AC automaton does not need this function
    int search(const char* s) {
        int p = 0, c, len = strlen(s);
        rep(i, len) {
            c = id(s[i]);
            if (!tr[p][c]) return 0;
            p = tr[p][c];
        }
        return tag[p];
    }
};

```

```

e6f1
dd87
427e
8ff5
427e
a281
5c83
f4f5
35a5
427e
4fee
3ccc
4d52
46bf
95cf
427e
427e
30b0
d50a
9c94
3140
d6c8
26dd
2e5c
73bb
95cf
f119
95cf
35ef
95cf
427e
427e
216c
d50a
9c94
3140
f339
f119
95cf
840e
95cf
329b

```

4 Linear Algebra

4.1 Matrix powermod

```

44b4 const int MAXN = 105;
92df const LL modular = 1000000007;
5c83 int n; // order of matrices
427e
8864 struct matrix{
3180     LL m[MAXN][MAXN];
427e
43c5     void operator *=(matrix& a){
e735         static LL t[MAXN][MAXN];
34d7         Rep (i, n){
4c11             Rep (j, n){
ee1e                 t[i][j] = 0;
c4a7                 Rep (k, n){
fc9f                     t[i][j] += (m[i][k] * a.m[k][j]) % modular;
199e                     t[i][j] %= modular;
95cf                 }
95cf             }
95cf         }
dad4         memcpy(m, t, sizeof(t));
95cf     }
329b };
427e
63d8 matrix r;
3ec2 void m_powmod(matrix& b, LL e){
83f0     memset(r.m, 0, sizeof(r.m));
a7c3     Rep(i, n)
de64         r.m[i][i] = 1;
3e90     while (e){
5a0e         if (e & 1) r *= b;
35c5         b *= b;
16fc         e >>= 1;
95cf     }
95cf }

```

4.2 Linear basis

```

8b44 const int MAXD = 30;
03a6 struct linearbasis {

```

```

ULL b[MAXD] = {};

bool insert(LL v) {
    for (int j = MAXD - 1; j >= 0; j--) {
        if (!(v & (1LL << j))) continue;
        if (b[j]) v ^= b[j]
        else {
            for (int k = 0; k < j; k++)
                if (v & (1LL << k)) v ^= b[k];
            for (int k = j + 1; k < MAXD; k++)
                if (b[k] & (1LL << j)) b[k] ^= v;
            b[j] = v;
            return true;
        }
    }
    return false;
}
};

```

4.3 Gauss elimination over finite field

```

const LL p = 1000000007;

LL powmod(LL b, LL e) {
    LL r = 1;
    while (e) {
        if (e & 1) r = r * b % p;
        b = b * b % p;
        e >>= 1;
    }
    return r;
}

```

```

typedef vector<LL> VLL;
typedef vector<VLL> WLL;

```

```

LL gauss(WLL &a, WLL &b) {
    const int n = a.size(), m = b[0].size();
    vector<int> irow(n), icol(n), ipiv(n);
    LL det = 1;

    rep (i, n) {

```

3558
427e
842f
9b2b
de36
ee78
037f
7836
f0b4
b0aa
46c9
8295
3361
95cf
95cf
438e
95cf
329b

b784
427e
2a2c
95a2
3e90
1783
5549
16fc
95cf
547e
95cf
427e
c130
42ac
427e
2c62
561b
a25e
2976
427e
be8e

```

d2b5     int pj = -1, pk = -1;
6b4a     rep (j, n) if (!ipiv[j])
e582         rep (k, n) if (!ipiv[k])
6112             if (pj == -1 || a[j][k] > a[pj][pk]) {
a905                 pj = j;
657b                 pk = k;
95cf             }
d480     if (a[pj][pk] == 0) return 0;
0305     ipiv[pk]++;
8dad     swap(a[pj], a[pk]);
aad8     swap(b[pj], b[pk]);
be4d     if (pj != pk) det = (p - det) % p;
d080     irow[i] = pj;
f156     icol[i] = pk;
427e
4ecd     LL c = powmod(a[pk][pk], p - 2);
865b     det = det * a[pk][pk] % p;
c36a     a[pk][pk] = 1;
dd36     rep (j, n) a[pk][j] = a[pk][j] * c % p;
1b23     rep (j, m) b[pk][j] = b[pk][j] * c % p;
f8f3     rep (j, n) if (j != pk) {
e97f         c = a[j][pk];
c449         a[j][pk] = 0;
820b         rep (k, n) a[j][k] = (a[j][k] + p - a[pk][k] * c % p) % p;
f039         rep (k, m) b[j][k] = (b[j][k] + p - b[pk][k] * c % p) % p;
95cf     }
95cf }
427e
37e1     for (int j = n - 1; j >= 0; j--) if (irow[j] != icol[j]) {
50dc         for (int k = 0; k < n; k++) swap(a[k][irow[j]], a[k][icol[j]]);
95cf     }
f27f     return det;
95cf }

```

4.4 Berlekamp-Massey algorithm

```

2b86     const LL MOD = 1000000007;
427e
391d     LL inverse(LL b) {
32d3         LL e = MOD - 2, r = 1;
3e90         while (e) {
9a62             if (e & 1) r = r * b % MOD;

```

```

        b = b * b % MOD;
        e >>= 1;
    }
    return r;
}

struct Poly {
    vector<int> a;

    Poly() { a.clear(); }

    Poly(vector<int> &a) : a(a) {}

    int length() const { return a.size(); }

    Poly move(int d) {
        vector<int> na(d, 0);
        na.insert(na.end(), a.begin(), a.end());
        return Poly(na);
    }

    int calc(vector<int> &d, int pos) {
        int ret = 0;
        for (int i = 0; i < (int)a.size(); ++i) {
            if ((ret += (long long)d[pos - i] * a[i] % MOD) >= MOD) {
                ret -= MOD;
            }
        }
        return ret;
    }

    Poly operator - (const Poly &b) {
        vector<int> na(max(this->length(), b.length()));
        for (int i = 0; i < (int)na.size(); ++i) {
            int aa = i < this->length() ? this->a[i] : 0,
                bb = i < b.length() ? b.a[i] : 0;
            na[i] = (aa + MOD - bb) % MOD;
        }
        return Poly(na);
    }
};

Poly operator * (const int &c, const Poly &p) {
    vector<int> na(p.length());

```

29ea
16fc
95cf
547e
95cf
427e
32a6
afe0
427e
9794
427e
de81
427e
8087
427e
16de
b31d
f915
cecf
95cf
427e
fa1a
5b57
501c
5de5
3041
95cf
95cf
ee0f
95cf
427e
c856
bd55
d1a7
3507
2bee
9526
95cf
cecf
95cf
329b
427e
5473
72de


```

d1a7  for (int i = 0; i < (int)na.size(); ++i) {
bf0c      na[i] = (long long)c * p.a[i] % MOD;
95cf  }
aaab  return na;
95cf  }
427e
afff  vector<int> solve(vector<int> a) {
9f23      int n = a.size();
58d0      Poly s, b;
4e8f      s.a.push_back(1), b.a.push_back(1);
c2aa      for (int i = 1, j = 0, ld = a[0]; i < n; ++i) {
4158          int d = s.calc(a, i);
d503          if (d) {
c29d              if ((s.length() - 1) * 2 <= i) {
db9d                  Poly ob = b;
6bce                  b = s;
1d0e                  s = s - (long long)d * inverse(ld) % MOD * ob.move(i - j);
0889                  j = i;
64f1                  ld = d;
8e2e              } else {
714e                  s = s - (long long)d * inverse(ld) % MOD * b.move(i - j);
95cf              }
95cf          }
95cf      }
427e      // Caution: s.a might be shorter than expected
e235      return s.a;
95cf  }

```

4.5 Fast Walsh-Hadamard transform

```

061e  void fwt(int* a, int n){
5595      for (int d = 1; d < n; d <= 1)
05f2          for (int i = 0; i < n; i += d < 1)
b833              rep (j, d){
7796                  int x = a[i+j], y = a[i+j+d];
427e                  // a[i+j] = x+y, a[i+j+d] = x-y;    // xor
427e                  // a[i+j] = x+y;                    // and
427e                  // a[i+j+d] = x+y;                    // or
95cf              }
95cf      }
427e
4db1  void ifwt(int* a, int n){

```

```

for (int d = 1; d < n; d <= 1)
    for (int i = 0; i < n; i += d < 1)
        rep (j, d){
            int x = a[i+j], y = a[i+j+d];
            // a[i+j] = (x+y)/2, a[i+j+d] = (x-y)/2;    // xor
            // a[i+j] = x-y;                            // and
            // a[i+j+d] = y-x;                            // or
        }
}

void conv(int* a, int* b, int n){
    fwt(a, n);
    fwt(b, n);
    rep(i, n) a[i] *= b[i];
    ifwt(a, n);
}

```

4.6 Fast fourier transform

```

const int NMAX = 1<<20;

typedef complex<double> cplx;

const double PI = 2*acos(0.0);
struct FFT{
    int rev[NMAX];
    cplx omega[NMAX], oinv[NMAX];
    int K, N;

    FFT(int k){
        K = k; N = 1 << k;
        rep (i, N){
            rev[i] = (rev[i>>1]>>1) | ((i&1)<<(K-1));
            omega[i] = polar(1.0, 2.0 * PI / N * i);
            oinv[i] = conj(omega[i]);
        }
    }

    void dft(cplx* a, cplx* w){
        rep (i, N) if (i < rev[i]) swap(a[i], a[rev[i]]);
        for (int l = 2; l <= N; l *= 2){
            int m = l/2;

```

```

b3cf      for (cplx* p = a; p != a + N; p += 1)
c24f          rep (k, m){
fe06              cplx t = w[N/l*k] * p[k+m];
ecbf              p[k+m] = p[k] - t; p[k] += t;
95cf          }
95cf      }
95cf  }
427e

617b  void fft(cplx* a){dft(a, omega);}
a123  void ifft(cplx* a){
3b2f      dft(a, oinv);
57fc      rep (i, N) a[i] /= N;
95cf  }
427e

bdc0  void conv(cplx* a, cplx* b){
6497      fft(a); fft(b);
12a5      rep (i, N) a[i] *= b[i];
f84e      ifft(a);
95cf  }
329b };

```

4.7 Number theoretic transform

```

4ab9  const int NMAX = 1<<21;
427e
427e  // 998244353 = 7*17*2^23+1, G = 3
fb9a  const int P = 1004535809, G = 3; // = 479*2^21+1
427e
87ab  struct NTT{
c47c      int rev[NMAX];
0eda      LL omega[NMAX], oinv[NMAX];
81af      int g, g_inv; // g: g_n = G^((P-1)/n)
9827      int K, N;
427e
2a2c      LL powmod(LL b, LL e){
95a2          LL r = 1;
3e90          while (e){
6624              if (e&1) r = r * b % P;
489e              b = b * b % P;
16fc              e >>= 1;
95cf          }
547e      return r;

```

```

}

NTT(int k){
    K = k; N = 1 << k;
    g = powmod(G, (P-1)/N);
    g_inv = powmod(g, N-1);
    omega[0] = oinv[0] = 1;
    rep (i, N){
        rev[i] = (rev[i>>1]>>1) | ((i&1)<<(K-1));
        if (i){
            omega[i] = omega[i-1] * g % P;
            oinv[i] = oinv[i-1] * g_inv % P;
        }
    }
}

void _ntt(LL* a, LL* w){
    rep (i, N) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int l = 2; l <= N; l *= 2){
        int m = l/2;
        for (LL* p = a; p != a + N; p += l)
            rep (k, m){
                LL t = w[N/l*k] * p[k+m] % P;
                p[k+m] = (p[k] - t + P) % P;
                p[k] = (p[k] + t) % P;
            }
    }
}

void ntt(LL* a){_ntt(a, omega);}
void intt(LL* a){
    LL inv = powmod(N, P-2);
    _ntt(a, oinv);
    rep (i, N) a[i] = a[i] * inv % P;
}

void conv(LL* a, LL* b){
    ntt(a); ntt(b);
    rep (i, N) a[i] = a[i] * b[i] % P;
    intt(a);
}
};

```

```

95cf
427e
f420
e209
7652
4b3a
e04f
b393
7ba3
ad4f
8d8b
9e14
95cf
95cf
95cf
427e
9668
a215
ac6e
2969
7a1d
c24f
0ad3
6209
fa1b
95cf
95cf
95cf
427e
92ea
5daf
1f2a
9910
a873
95cf
427e
3a5b
ad16
e49e
5748
95cf
329b

```

5 Number Theory

5.1 Sieve of Euler

```

b62e namespace sieve {
6589     constexpr int MAXN = 10000007;
e982     bool p[MAXN]; // true if not prime
6ae8     int prime[MAXN], sz;
cbf7     int pval[MAXN], pcnt[MAXN];
6030     int f[MAXN];

427e     void exec(int N = MAXN) {
76f6         p[0] = p[1] = 1;

8a8a         pval[1] = 1;
bdda         pcnt[1] = 0;
c6b9         f[1] = 1;

427e         for (int i = 2; i < N; i++) {
a643             if (!p[i]) {
01d6                 prime[sz++] = i;
b2b2                 for (LL j = i; j < N; j *= i) {
37d9                     int b = j / i;
758c                     pval[j] = i * pval[b];
81fd                     pcnt[j] = pcnt[b] + 1;
e0f3                     f[j] = ____; // f[j] = f(i^pcnt[j])
a96c                 }
95cf             }
95cf         }
34c0         for (int j = 0; i * prime[j] < N; j++) {
f87a             int x = i * prime[j]; p[x] = 1;
20cc             if (i % prime[j] == 0) {
9985                 pval[x] = pval[i] * prime[j];
3f93                 pcnt[x] = pcnt[i] + 1;
8e2e             } else {
cc91                 pval[x] = prime[j];
6322                 pcnt[x] = 1;
95cf             }
6191             if (x != pval[x]) {
d614                 f[x] = f[x / pval[x]] * f[pval[x]]
95cf             }
5f51             if (i % prime[j] == 0) break;
95cf         }
95cf     }

```

```

}
}

```

95cf
95cf

5.2 Miller-Rabin primality test

The array `a[]` (excluding sentinel, i.e. `LLONG_MAX`) should be

<code>{2}</code>	when $n < 2,047$.
<code>{2, 7, 61}</code>	when $n < 4,759,123,141$ (2^{32}).
<code>{2, 3, 5, 7, 11}</code>	when $n < 2.1 \times 10^{12}$.
<code>{2, 325, 9375, 28178, 450775, 9780504, 1795265022}</code>	when $n < 2^{64}$.

```

bool test(LL n){
    if (n < 3) return n==2;
    // ! The array a[] should be modified if the range of x changes.
    const LL a[] = {2LL, 7LL, 61LL, LLONG_MAX};
    LL r = 0, d = n-1, x;
    while (~d & 1) d >>= 1, r++;
    for (int i=0; a[i] < n; i++){
        x = powmod(a[i], d, n); // ! powmod must use for 64bit mulmod
        if (x == 1 || x == n-1) goto next;
        rep (i, r) {
            x = mulmod(x, x, n);
            if (x == n-1) goto next;
        }
        return false;
next:;
    }
    return true;
}

```

f16f
59f2
427e
3f11
c320
f410
2975
ece1
7f99
e257
d7ff
8d2e
95cf
438e
d490
95cf
3361
95cf

5.3 Pollard's rho algorithm

```

ULL gcd(ULL a, ULL b) {return b ? gcd(b, a % b) : a;}

ULL PollardRho(ULL n){
    ULL c, x, y, d = n;
    if (~n&1) return 2;
    while (d == n){
        x = y = 2;

```

2e6b
427e
54a5
45eb
d3e5
3c69
0964

```

4753     d = 1;
5952     c = rand() % (n - 1) + 1;
9e5b     while (d == 1){
33d5         x = (mulmod(x, x, n) + c) % n;
e1bf         y = (mulmod(y, y, n) + c) % n;
e1bf         y = (mulmod(y, y, n) + c) % n;
a313         d = gcd(x>y ? x-y : y-x, n);
95cf     }
95cf }
5d89 return d;
95cf }

```

6 Graph Theory

6.1 Strongly connected component

```

837c const int MAXV = 100005;
427e
2ea0 struct graph{
88e3     vector<int> adj[MAXV];
9cad     stack<int> s;
3d02     int V; // number of vertices
8b6c     int pre[MAXV], lnk[MAXV], scc[MAXV];
27ee     int time, sccn;
427e
bfab     void add_edge(int u, int v){
c71a         adj[u].push_back(v);
95cf     }
427e
d714     void dfs(int u){
7e41         pre[u] = lnk[u] = ++time;
80f6         s.push(u);
18f6         for (int v : adj[u]){
173e             if (!pre[v]){
5f3c                 dfs(v);
002c                 lnk[u] = min(lnk[u], lnk[v]);
6068             } else if (!scc[v]){
d5df                 lnk[u] = min(lnk[u], pre[v]);
95cf             }
95cf         }
8de2         if (lnk[u] == pre[u]){
660f             sccn++;

```

```

        int x;
        do {
            x = s.top(); s.pop();
            scc[x] = sccn;
        } while (x != u);
    }

void find_scc(){
    time = sccn = 0;
    memset(scc, 0, sizeof scc);
    memset(pre, 0, sizeof pre);
    Rep (i, V){
        if (!pre[i]) dfs(i);
    }
}

vector<int> adjc[MAXV];
void contract(){
    Rep (i, V)
        rep (j, adj[i].size()){
            if (scc[i] != scc[adj[i][j]])
                adjc[scc[i]].push_back(scc[adj[i][j]]);
        }
}

};

```

3c9e
a69f
3834
b0e9
6757
95cf
95cf
427e
4c88
f4a2
8de7
8c2f
6901
56d1
95cf
95cf
427e
27ce
364d
1a1e
21a2
b730
b46e
95cf
95cf
329b

6.2 Vertex biconnected component

```

const int MAXN = 100005;
struct graph {
    int pre[MAXN], iscut[MAXN], bccno[MAXN], dfs_clock, bcc_cnt;
    vector<int> adj[MAXN], bcc[MAXN];
    set<pair<int, int>> bcce[MAXN];

    stack<pair<int, int>> s;

    void add_edge(int u, int v) {
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
}

```

0f42
2ea0
33ae
848f
6b06
427e
76f7
427e
bfab
c71a
a717
95cf
427e

```

7d3c  int dfs(int u, int fa) {
9fe6      int lowu = pre[u] = ++dfs_clock;
ec14      int child = 0;
18f6      for (int v : adj[u]) {
173e          if (!pre[v]) {
e7f8              s.push({u, v});
fdcf              child++;
f851              int lowv = dfs(v, u);
189c              lowu = min(lowu, lowv);
b687              if (lowv >= pre[u]) {
6323                  iscut[u] = 1;
57eb                  bcc[bcc_cnt].clear();
90b8                  bcce[bcc_cnt].clear();
a147                  while (1) {
a6a3                      int xu, xv;
a0c3                      tie(xu, xv) = s.top(); s.pop();
0ef5                      bcce[bcc_cnt].insert({min(xu, xv), max(xu, xv)});
3db2                      if (bccno[xu] != bcc_cnt) {
e0db                          bcc[bcc_cnt].push_back(xu);
d27f                          bccno[xu] = bcc_cnt;
95cf                      }
f357                      if (bccno[xv] != bcc_cnt) {
752b                          bcc[bcc_cnt].push_back(xv);
57c9                          bccno[xv] = bcc_cnt;
95cf                      }
7096                      if (xu == u && xv == v) break;
95cf                      }
03f5                      bcc_cnt++;
95cf                      }
7470                  } else if (pre[v] < pre[u] && v != fa) {
e7f8                      s.push({u, v});
f115                      lowu = min(lowu, pre[v]);
95cf                  }
95cf          }
e104      if (fa < 0 && child == 1) iscut[u] = 0;
1160      return lowu;
95cf  }
427e
17be  void find_bcc(int n) {
8c2f      memset(pre, 0, sizeof pre);
e2d2      memset(iscut, 0, sizeof iscut);
40d3      memset(bccno, -1, sizeof bccno);
fae2      dfs_clock = bcc_cnt = 0;
5c63      rep (i, n) if (!pre[i]) dfs(i, -1);

```

```

    }
};

```

```

95cf
329b

```

6.3 Minimum spanning arborescence (Chu-Liu)

Usage: `getans(n, root, edges)`

All vertices are 1-based.

```

struct edge {
    int u, v;
    LL w;
};

const int MAXN = 10005;
LL in[MAXN];
int pre[MAXN], vis[MAXN], id[MAXN];

LL getans(int n, int rt, vector<edge>& edges) {
    LL ans = 0;
    int cnt = 0;
    while (1) {
        Rep (i, n) in[i] = LLONG_MAX, id[i] = vis[i] = 0;
        for (auto e : edges) {
            if (e.u != e.v and e.w < in[e.v]) {
                pre[e.v] = e.u;
                in[e.v] = e.w;
            }
        }
        in[rt] = 0;
        Rep (i, n) {
            if (in[i] == LLONG_MAX) return -1;
            ans += in[i];
            int u;
            for (u = i; u != rt && vis[u] != i && !id[u]; u = pre[u])
                vis[u] = i;
            if (u != rt && !id[u]) {
                id[u] = ++cnt;
                for (int v = pre[u]; v != u; v = pre[v])
                    id[v] = cnt;
            }
        }
        if (!cnt) return ans;
        Rep (i, n) if (!id[i]) id[i] = ++cnt;
    }
}

```

```

bcf8
54f1
309c
329b
427e
f5a4
7124
1c1d
427e
5a43
f7ff
8abb
a147
641a
0705
073a
c1df
5fbc
95cf
95cf
3fdb
34d7
3c97
cf57
a763
4b0e
88a2
4b22
b66e
0443
5c22
95cf
95cf
91e9
5e22

```

```

7400     for (auto& e : edges) {
7750         LL laz = in[e.v];
97ae         e.u = id[e.u];
fae6         e.v = id[e.v];
bdd2         if (e.u != e.v) e.w -= laz;
95cf     }
6cc4     n = cnt; rt = id[rt]; cnt = 0;
95cf }
95cf }

```

6.4 Maximum flow (Dinic)

```

bcf8 struct edge{
60e2     int from, to;
5e6d     LL cap, flow;
329b };
427e
e2cd const int MAXN = 1005;
9062 struct Dinic {
4dbf     int n, m, s, t;
9f0c     vector<edge> edges;
b891     vector<int> G[MAXN];
bbb6     bool vis[MAXN];
b40a     int d[MAXN];
ddec     int cur[MAXN];
427e
5973     void add_edge(int from, int to, LL cap) {
7b55         edges.push_back(edge{from, to, cap, 0});
1db7         edges.push_back(edge{to, from, 0, 0});
fe77         m = edges.size();
dff5         G[from].push_back(m-2);
8f2d         G[to].push_back(m-1);
95cf     }
427e
1836     bool bfs() {
3b73         memset(vis, 0, sizeof(vis));
93d2         queue<int> q;
5d13         q.push(s);
2cd2         vis[s] = 1;
721d         d[s] = 0;
cc78         while (!q.empty()) {
66ba             int x = q.front(); q.pop();

```

```

        for (int i = 0; i < G[x].size(); i++) {
            edge& e = edges[G[x][i]];
            if (!vis[e.to] && e.cap > e.flow) {
                vis[e.to] = 1;
                d[e.to] = d[x] + 1;
                q.push(e.to);
            }
        }
    }
    return vis[t];
}

LL dfs(int x, LL a) {
    if (x == t || a == 0) return a;
    LL flow = 0, f;
    for (int& i = cur[x]; i < G[x].size(); i++) {
        edge& e = edges[G[x][i]];
        if (d[x] + 1 == d[e.to] && (f = dfs(e.to, min(a, e.cap-e.flow))) > 0)
        {
            e.flow += f;
            edges[G[x][i]^1].flow -= f;
            flow += f;
            a -= f;
            if (a == 0) break;
        }
    }
    return flow;
}

LL max_flow(int s, int t) {
    this->s = s; this->t = t;
    LL flow = 0;
    while (bfs()) {
        memset(cur, 0, sizeof(cur));
        flow += dfs(s, LLONG_MAX);
    }
    return flow;
}

vector<int> min_cut() { // call this after maxflow
    vector<int> ans;
    for (int i = 0; i < edges.size(); i++) {
        edge& e = edges[i];
        if (vis[e.from] && !vis[e.to] && e.cap > 0) ans.push_back(i);
    }
}

```

```

95cf    }
4206    return ans;
95cf    }
329b    };

```

6.5 Maximum cardinality bipartite matching (Hungarian)

```

302f    #include <bits/stdc++.h>
421c    using namespace std;
427e
0d6c    #define rep(i, n) for (int i = 0; i < (n); i++)
cfe3    #define Rep(i, n) for (int i = 1; i <= (n); i++)
8843    #define range(x) (x).begin(), (x).end()
5cad    typedef long long LL;
427e
84ee    struct Hungarian{
fbf6        int nx, ny;
9ec6        vector<int> mx, my;
9d4c        vector<vector<int>> > e;
edec        vector<bool> mark;
427e
8324        void init(int nx, int ny){
c1d1            this->nx = nx;
f9c1            this->ny = ny;
ac92            mx.resize(nx); my.resize(ny);
3f11            e.clear(); e.resize(nx);
1023            mark.resize(nx);
95cf        }
427e
4589        inline void add(int a, int b){
486c            e[a].push_back(b);
95cf        }
427e
0c2b        bool augment(int i){
207c            if (!mark[i]) {
dae4                mark[i] = true;
6a1e                for (int j : e[i]){
0892                    if (my[j] == -1 || augment(my[j])){
9ca3                        mx[i] = j; my[j] = i;
3361                        return true;
95cf                    }
95cf                }

```

```

    }
    return false;
}

int match(){
    int ret = 0;
    fill(range(mx), -1);
    fill(range(my), -1);
    rep (i, nx){
        fill(range(mark), false);
        if (augment(i)) ret++;
    }
    return ret;
}
};

```

```

95cf
438e
95cf
427e
3fac
5b57
b0f1
b957
4ed1
13a5
cc89
95cf
ee0f
95cf
329b

```

6.6 Minimum cost maximum flow

```

struct edge{
    int from, to;
    int cap, flow;
    LL cost;
};

const LL INF = LLONG_MAX / 2;
const int MAXN = 5005;
struct MCMF {
    int s, t, n, m;
    vector<edge> edges;
    vector<int> G[MAXN];
    bool inq[MAXN]; // queue
    LL d[MAXN];     // distance
    int p[MAXN];    // previous
    int a[MAXN];    // improvement

    void add_edge(int from, int to, int cap, LL cost) {
        edges.push_back(edge{from, to, cap, 0, cost});
        edges.push_back(edge{to, from, 0, 0, -cost});
        m = edges.size();
        G[from].push_back(m-2);
        G[to].push_back(m-1);
    }
}

```

```

bcf8
60e2
d698
32cc
329b
427e
cc3e
2aa8
c6cb
9ceb
9f0c
b891
f74f
8f67
9524
b330
427e
f7f2
24f0
95f0
fe77
dff5
8f2d
95cf

```

```

427e
3c52  bool spfa(){
93d2     queue<int> q;
8494     fill(d, d + MAXN, INF); d[s] = 0;
fd48     memset(inq, 0, sizeof(inq));
5e7c     q.push(s); inq[s] = true;
2dae     p[s] = 0; a[s] = INT_MAX;
cc78     while (!q.empty()){
b0aa         int u = q.front(); q.pop(); inq[u] = false;
ddff         rep (i, G[u].size()){
c234             edge& e = edges[G[u][i]];
3601             if (e.cap > e.flow && d[e.to] > d[u] + e.cost){
55bc                 d[e.to] = d[u] + e.cost;
0bea                 p[e.to] = G[u][i];
8249                 a[e.to] = min(a[u], e.cap - e.flow);
e5d3                 if (!inq[e.to]) q.push(e.to), inq[e.to] = true;
95cf             }
95cf         }
95cf     }
6d7c     return d[t] != INF;
95cf }
427e
71a4  void augment(){
06f1     int u = t;
b19d     while (u != s){
db09         edges[p[u]].flow += a[t];
25a9         edges[p[u]^1].flow -= a[t];
e6c9         u = edges[p[u]].from;
95cf     }
95cf }
427e
6e20  #ifndef GIVEN_FLOW
5972  bool min_cost(int s, int t, int f, LL& cost) {
590d      this->s = s; this->t = t;
21d4      int flow = 0;
23cb      cost = 0;
22dc      while (spfa()) {
bcd8          augment();
a671          if (flow + a[t] >= f){
9c87              cost += (f - flow) * a[t]; flow = f;
3361              return true;
8e2e          } else {
2a83              flow += a[t]; cost += a[t] * d[t];
95cf          }

```

```

    }
    return false;
}
#else
int min_cost(int s, int t, LL& cost) {
    this->s = s; this->t = t;
    int flow = 0;
    cost = 0;
    while (spfa()) {
        augment();
        flow += a[t]; cost += a[t] * d[t];
    }
    return flow;
}
#endif
};

```

95cf
438e
95cf
a8cb
f9a9
590d
21d4
23cb
22dc
bcd8
2a83
95cf
84fb
95cf
1937
329b

6.7 Global minimum cut (Stoer-Wagner)

```

typedef vector<LL> VI;
typedef vector<VI> WVI;

pair<LL, VI> stoer(WVI &w) {
    int n = w.size();
    VI used(n), c, bestc;
    LL bestw = -1;

    for (int ph = n - 1; ph >= 0; ph--) {
        VI wt = w[0], added = used;
        int prev, last = 0;
        rep (i, ph) {
            prev = last;
            last = -1;
            for (int j = 1; j < n; j++)
                if (!added[j] && (last == -1 || wt[j] > wt[last]))
                    last = j;
            if (i == ph - 1) {
                rep (j, n) w[prev][j] += w[last][j];
                rep (j, n) w[j][prev] = w[prev][j];
                used[last] = true;
                c.push_back(last);
                if (bestw == -1 || wt[last] < bestw) {

```

f9d7
045e
427e
f012
66f7
4d98
329d
427e
cd21
ec6e
f20e
4b32
8bfc
0706
4942
c4b9
887d
71bc
9cfa
1f25
5613
8e11
bb8e


```

bab6         bestc = c;
372e         bestw = wt[last];
95cf     }
8e2e     } else {
caeb         rep (j, n) wt[j] += w[last][j];
8b92         added[last] = true;
95cf     }
95cf     }
95cf     }
038c     return {bestw, bestc};
95cf }

```

6.8 Heavy-light decomposition

```

0f42 const int MAXN = 100005;
0b32 vector<int> adj[MAXN];
42f2 int sz[MAXN], top[MAXN], fa[MAXN], son[MAXN], depth[MAXN], id[MAXN];
427e
be5c void dfs1(int x, int dep, int par){
7489     depth[x] = dep;
2ee7     sz[x] = 1;
adb4     fa[x] = par;
b79d     int maxn = 0, s = 0;
c861     for (int c: adj[x]){
fe45         if (c == par) continue;
fd2f         dfs1(c, dep + 1, x);
b790         sz[x] += sz[c];
f0f1         if (sz[c] > maxn){
c749             maxn = sz[c];
fe19             s = c;
95cf         }
95cf     }
0e08     son[x] = s;
95cf }
427e
ba54 int cid = 0;
3644 void dfs2(int x, int t){
8d96     top[x] = t;
d314     id[x] = ++cid;
c4a1     if (son[x]) dfs2(son[x], t);
c861     for (int c: adj[x]){
9881         if (c == fa[x]) continue;

```

```

         if (c == son[x]) continue;
         else dfs2(c, c);
    }
}

void decomp(int root){
    dfs1(root, 1, 0);
    dfs2(root, root);
}

void query(int u, int v){
    while (top[u] != top[v]){
        if (depth[top[u]] < depth[top[v]]) swap(u, v);
        // id[top[u]] to id[u]
        u = fa[top[u]];
    }
    if (depth[u] > depth[v]) swap(u, v);
    // id[u] to id[v]
}

```

7 Data Structures

7.1 Segment tree

```

LL p;
const int MAXN = 4 * 100006;
struct segtree {
    int l[MAXN], m[MAXN], r[MAXN];
    LL val[MAXN], tadd[MAXN], tmul[MAXN];

#define lson (o<<1)
#define rson (o<<1|1)

    void pull(int o) {
        val[o] = (val[lson] + val[rson]) % p;
    }

    void push_add(int o, LL x) {
        val[o] = (val[o] + x * (r[o] - l[o])) % p;
        tadd[o] = (tadd[o] + x) % p;
    }
}

```

```

d658 void push_mul(int o, LL x) {
b82c     val[o] = val[o] * x % p;
aa86     tadd[o] = tadd[o] * x % p;
649f     tmul[o] = tmul[o] * x % p;
95cf }
427e
b149 void push(int o) {
3159     if (l[o] == m[o]) return;
0a90     if (tmul[o] != 1) {
0f4a         push_mul(lson, tmul[o]);
045e         push_mul(rson, tmul[o]);
ac0a         tmul[o] = 1;
95cf     }
1b82     if (tadd[o]) {
9547         push_add(lson, tadd[o]);
0e73         push_add(rson, tadd[o]);
6234         tadd[o] = 0;
95cf     }
95cf }
427e
471c void build(int o, int ll, int rr) {
0e87     int mm = (ll + rr) / 2;
9d27     l[o] = ll; r[o] = rr; m[o] = mm;
ac0a     tmul[o] = 1;
5c92     if (ll == mm) {
001f         scanf("%lld", val + o);
e5b6         val[o] %= p;
8e2e     } else {
7293         build(lson, ll, mm);
5e67         build(rson, mm, rr);
ba26         pull(o);
95cf     }
95cf }
427e
4406 void add(int o, int ll, int rr, LL x) {
3c16     if (ll <= l[o] && r[o] <= rr) {
db32         push_add(o, x);
8e2e     } else {
c4b0         push(o);
4305         if (m[o] > ll) add(lson, ll, rr, x);
d5a6         if (m[o] < rr) add(rson, ll, rr, x);
ba26         pull(o);
95cf     }
95cf }

```

```

void mul(int o, int ll, int rr, LL x) {
    if (ll <= l[o] && r[o] <= rr) {
        push_mul(o, x);
    } else {
        push(o);
        if (ll < m[o]) mul(lson, ll, rr, x);
        if (m[o] < rr) mul(rson, ll, rr, x);
        pull(o);
    }
}

LL query(int o, int ll, int rr) {
    if (ll <= l[o] && r[o] <= rr) {
        return val[o];
    } else {
        LL ans = 0;
        push(o);
        if (m[o] > ll) ans += query(lson, ll, rr);
        if (m[o] < rr) ans += query(rson, ll, rr);
        return ans % p;
    }
}
} seg;

```

```

427e
48cd
3c16
e7d0
8e2e
c4b0
d1ba
67f3
ba26
95cf
95cf
427e
0f62
3c16
6dfe
8e2e
f7ff
c4b0
c5f8
ef81
a420
95cf
95cf
4d99

```

7.2 Link/cut tree

```

// about 0.13s per 100k ops @Luogu.org

namespace LCT {
    const int MAXN = 300005;
    int fa[MAXN], ch[MAXN][2], val[MAXN], sum[MAXN];
    bool rev[MAXN];

    bool isroot(int x) {
        return ch[fa[x]][0] == x || ch[fa[x]][1] == x;
    }

    void pull(int x) {
        sum[x] = val[x] ^ sum[ch[x][0]] ^ sum[ch[x][1]];
    }
}

```

```

427e
427e
ed4d
5ece
6a6d
c6e1
427e
7839
45a9
95cf
427e
3bf9
6664
95cf
427e

```

```

3698 void reverse(int x) {
7850     swap(ch[x][0], ch[x][1]);
52c6     rev[x] ^= 1;
95cf }
427e
1a53 void push(int x) {
8f1f     if (rev[x]) {
ebf3         if (ch[x][0]) reverse(ch[x][0]);
6eb0         if (ch[x][1]) reverse(ch[x][1]);
8fc1         rev[x] = 0;
95cf     }
95cf }
427e
425f void rotate(int x) {
51af     int y = fa[x], z = fa[y], k = ch[y][1] == x, w = ch[x][!k];
e1fe     if (isroot(y)) ch[z][ch[z][1] == y] = x;
af46     ch[x][!k] = y; ch[y][k] = w;
fa6f     if (w) fa[w] = y;
3540     fa[y] = x; fa[x] = z;
72ef     pull(y);
95cf }
427e
bc1b void pushall(int x) {
a316     if (isroot(x)) pushall(fa[x]);
a97b     push(x);
95cf }
427e
f69c void splay(int x) {
d095     int y = x, z = 0;
8ab3     pushall(y);
f244     while (isroot(x)) {
ceef         y = fa[x]; z = fa[y];
4449         if (isroot(y)) rotate((ch[y][0] == x) ^ (ch[z][0] == y) ? x : y);
cf90         rotate(x);
95cf     }
78a0     pull(x);
95cf }
427e
6229 void access(int x) {
1548     int z = x;
ba78     for (int y = 0; x; x = fa[y = x]) {
8fec         splay(x);
b05d         ch[x][1] = y;
78a0         pull(x);

```

```

    }
    splay(z);
}

void chroot(int x) {
    access(x);
    reverse(x);
}

void split(int x, int y) {
    chroot(x);
    access(y);
}

int Root(int x) {
    access(x);
    while (ch[x][0]) {
        push(x);
        x = ch[x][0];
    }
    splay(x);
    return x;
}

void Link(int u, int v) { // assume unconnected before
    chroot(u);
    fa[u] = v;
}

void Cut(int u, int v) { // assume connected before
    split(u, v);
    fa[u] = ch[v][0] = 0;
    pull(v);
}

int Query(int u, int v) {
    split(u, v);
    return sum[v];
}

void Update(int u, int x) {
    splay(u);
    val[u] = x;
}

```

```

95cf
7afd
95cf
427e
502e
766a
cb0d
95cf
427e
471a
3015
29b5
95cf
427e
d87a
766a
874d
a97b
b83a
95cf
8fec
d074
95cf
427e
70d3
b8a5
2448
95cf
427e
c2f4
e8ce
fd95
743b
95cf
427e
6ca2
e8ce
a5ba
95cf
427e
eaba
46ce
1d62
95cf

```

329b };

7.3 Balanced binary search tree from pb_ds

```

0475 #include <ext/pb_ds/assoc_container.hpp>
332d using namespace __gnu_pbds;
427e
43a7 tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update>
    rkt;
427e // null_tree_node_update
427e
427e // SAMPLE USAGE
190e rkt.insert(x);           // insert element
05d4 rkt.erase(x);          // erase element
add5 rkt.order_of_key(x);   // obtain the number of elements less than x
b064 rkt.find_by_order(i);  // iterator to i-th (numbered from 0) smallest element
c103 rkt.lower_bound(x);
4ff4 rkt.upper_bound(x);
b19b rkt.join(rkt2);        // merge tree (only if their ranges do not intersect)
cb47 rkt.split(x, rkt2);     // split all elements greater than x to rkt2

```

7.4 Persistent segment tree, range k-th query

```

f1a7 struct node {
2ff6     static int n, pos;
427e
7cec     int value;
70e2     node *left, *right;
427e
20b0     void* operator new(size_t size);
427e
3dc0     static node* Build(int l, int r) {
b6c5         node* a = new node;
ce96         if (r > l + 1) {
181e             int mid = (l + r) / 2;
3ba2             a->left = Build(l, mid);
8aaf             a->right = Build(mid, r);
8e2e         } else {
bfc4             a->value = 0;
95cf         }
5ffd         return a;

```

}

```

static node* init(int size) {
    n = size;
    pos = 0;
    return Build(0, n);
}

```

```

static int Query(node* lt, node *rt, int l, int r, int k) {
    if (r == l + 1) return l;
    int mid = (l + r) / 2;
    if (rt->left->value - lt->left->value < k) {
        k -= rt->left->value - lt->left->value;
        return Query(lt->right, rt->right, mid, r, k);
    } else {
        return Query(lt->left, rt->left, l, mid, k);
    }
}

```

```

static int query(node* lt, node *rt, int k) {
    return Query(lt, rt, 0, n, k);
}

```

```

node *Inc(int l, int r, int pos) const {
    node* a = new node(*this);
    if (r > l + 1) {
        int mid = (l + r) / 2;
        if (pos < mid)
            a->left = left->Inc(l, mid, pos);
        else
            a->right = right->Inc(mid, r, pos);
    }
    a->value++;
    return a;
}

```

```

node *inc(int index) {
    return Inc(0, n, index);
}
} nodes[8000000];

```

```

int node::n, node::pos;
inline void* node::operator new(size_t size) {
    return nodes + (pos++);
}

```

95cf
427e
5a45
2c46
7ee3
be52
95cf
427e
93c0
d30c
181e
cb5a
8edb
2412
8e2e
0119
95cf
95cf
427e
c9ad
9e27
95cf
427e
b19c
5794
ce96
181e
203d
f44a
649a
1024
95cf
2b3e
5ffd
95cf
427e
e80f
c246
95cf
865a
427e
99ce
1987
bb3c

95cf }

8 Appendices

8.1 Primes

8.1.1 First primes

p	$g(p)$	p	$g(p)$	p	$g(p)$	p	$g(p)$	p	$g(p)$
2	1	3	2	5	2	7	3	11	2
13	2	17	3	19	2	23	5	29	2
31	3	37	2	41	6	43	3	47	5
53	2	59	2	61	2	67	2	71	7
73	5	79	3	83	2	89	3	97	5
101	2	103	5	107	2	109	6	113	3
127	3	131	2	137	3	139	2	149	2
151	6	157	5	163	2	167	5	173	2
179	2	181	2	191	19	193	5	197	2
199	3	211	2	223	3	227	2	229	6

7.5 Sparse table, range extremum query

The array is 0-based and the range is closed.

```

db63 const int MAXN = 100007;
b330 int a[MAXN];
69ae int st[MAXN][32 - __builtin_clz(MAXN)];
427e
8041 inline int ext(int x, int y){return x>y?x:y;} // ! max
427e
d34f void init(int n){
ce01     int l = 31 - __builtin_clz(n);
cf75     rep (i, n) st[i][0] = a[i];
b811     rep (j, l)
6937         rep (i, 1+n-(1<<j))
082a         st[i][j+1] = ext(st[i][j], st[i+(1<<j)][j]);
95cf }
427e
c863 int rmq(int l, int r){
92f5     int k = 31 - __builtin_clz(r-l+1);
baa2     return ext(st[l][k], st[r-(1<<k)+1][k]);
95cf }
```

8.1.2 Arbitrary length primes

$\lg p$	p	$g(p)$	p	$g(p)$
3	967	5	1031	14
4	9859	2	10273	10
5	96331	10	102931	3
6	958543	6	1031137	5
7	9594539	2	10169651	2
8	96243449	3	103211039	7
9	980483981	2	1042484357	2
10	9858935453	2	10261276009	7
11	95748666809	3	101759940101	2
12	950781833849	3	1012797784423	5
13	9739822952371	7	10037217092377	7
14	96181051140397	5	104974966380359	11
15	981030138360889	13	1029038416465403	2
16	9655206098080843	3	10116299875820773	2
17	97687777921994419	3	101506415998163437	2

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$\lg p$	p	$g(p)$	p	$g(p)$
3	967	5	1031	14
4	9859	2	10273	10
5	96331	10	102931	3
6	958543	6	1031137	5
7	9594539	2	10169651	2
8	96243449	3	103211039	7
9	980483981	2	1042484357	2
10	9858935453	2	10261276009	7
11	95748666809	3	101759940101	2
12	950781833849	3	1012797784423	5
13	9739822952371	7	10037217092377	7
14	96181051140397	5	104974966380359	11
15	981030138360889	13	1029038416465403	2
16	9655206098080843	3	10116299875820773	2
17	97687777921994419	3	101506415998163437	2

8.1.4 $\sim 1 \times 10^9$

p	$g(p)$	p	$g(p)$	p	$g(p)$
954854573	3	967607731	2	973215833	3
975831713	3	978949117	2	980766497	3
983879921	3	985918807	3	986608921	29
991136977	5	991752599	13	997137961	11
1003911991	3	1009775293	2	1012423549	6
1021000537	5	1023976897	7	1024153643	2
1037027287	3	1038812881	11	1044754639	3
1045125617	3	1047411427	3	1047753349	6

8.1.5 $\sim 1 \times 10^{18}$

p	$g(p)$	p	$g(p)$
951970612352230049	3	963284339889659609	3
967495386904694119	3	969751761517096213	2
983238274281901499	2	984647442475101409	23
989286107138674069	11	1002507954383424641	3
1006658951440146419	2	1020152326159075903	3
1034876265966119449	7	1042753851435034019	2
1043609016597371563	2	1045571042176595707	2
1048364250160580293	2	1049495624119026949	2

8.2 Pell's equation

$x^2 - ny^2 = 1$, where n is a positive nonsquare integer.

Let (x_0, y_0) be the smallest positive solution of the equation, then the k -th solution is:

$$\begin{pmatrix} x_k \\ y_k \end{pmatrix} = \begin{pmatrix} x_0 & ny_0 \\ y_0 & x_0 \end{pmatrix}^k \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

Some smallest solutions to Pell's equation:

n	2	3	5	6	7	8	10	11	12	13	14	15	17	18	19	20
x	3	2	9	5	8	3	19	10	7	649	15	4	33	17	170	9
y	2	1	4	2	3	1	6	3	2	180	4	1	8	4	39	2

8.3 Burnside's lemma and Polya's enumeration theorem

The Burnside's lemma says that

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

where G is a group acting on X , X^g is the set of elements in X that are fixed by g , i.e. $X^g = \{x \in X : gx = x\}$.

The unweighted version of Pólya enumeration theorem says that

$$|Y^X/G| = \frac{1}{|G|} \sum_{g \in G} m^{c_g}$$

where $m = |X|$ is the number of colors, c_g is the number of the cycles of permutation g .