1. gyakorlat

$$\left[\frac{5}{\sqrt{x}} - \frac{x^4}{4}\right]' = 5 \cdot \left[\frac{1}{\sqrt{x}}\right]' - \frac{1}{4}\left[x^4\right]' =$$

$$\frac{1}{\sqrt{x}} = x^{-1/2}$$

$$= 5 \cdot (-1/2) \cdot x^{-3/2} - \frac{1}{4} \cdot 4 \cdot x^3 = -\frac{5}{2} \cdot x^{-3/2} - x^3$$

$$\left[3 \ln x + \frac{e^x}{2} - \sin 2x\right] = 3 \cdot \frac{1}{x} + \frac{1}{2} \cdot e^x - \cos(2x) \cdot 2$$

$$\left[\operatorname{arctg}(e^x)\right]' = \frac{1}{1 + (e^x)^2} \cdot e^x = \frac{e^x}{1 + e^{2x}}$$

$$f(y) = \operatorname{arctg} y \qquad f'(y) = \frac{1}{1 + y^2}$$

$$g(x) = e^x \qquad g'(x) = e^x$$

$$\left[\sqrt{1 - x^2}\right]' = 1/2 \cdot (1 - x^2)' \cdot (-2x) = -\frac{x}{\sqrt{1 - x^2}}$$

$$f(y) = \sqrt{y} \qquad f'(y) = 1/2 \cdot y^{-1/2}$$

$$g(x) = 1 - x^2 \qquad g'(x) = -2x$$

$$\int -\frac{x}{\sqrt{1-x^2}}\,dx = \sqrt{1-x^2}' + C$$

$$[x^2]' = 2x \qquad \int 2x\,dx = x^2 + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \text{ mert } \left[\frac{x^{n+1}}{n+1} + C\right]' = \frac{1}{n+1}(n+1)\cdot x^n = x^n$$

$$\int \sin x\,dx = -\cos x + C, \text{ mert } [\cos x + C]' =$$
$$= -(-\sin x) + 0$$

$$\int \frac{e^x}{5} - \frac{x^6}{6} + 2^x - \frac{4}{1+x^2} + \frac{3}{\sqrt[3]{x}}\,dx =$$

$$= \frac{1}{5}\int e^x - \frac{1}{6}\int x^6 + \int 2^x - 4\int \frac{1}{1+x^2} + 3\int \frac{1}{\sqrt[3]{x}} =$$

$$= \frac{1}{5}\cdot e^x - \frac{1}{6}\cdot\frac{x^7}{7} + \frac{2^x}{\ln 2} - 4\cdot \operatorname{arctg} x + 3\cdot\frac{x^{2/3}}{2/3} + C$$

$$\int \frac{4}{3x^2} - \frac{3^x}{2} + \frac{\sqrt{x}}{6} - \frac{2}{x}\,dx =$$

$$= \frac{4}{3}\int x^{-2}dx - \frac{1}{2}\int 3^x dx + \frac{1}{6}\int x^{1/2}dx - 2\int \frac{1}{x}dx =$$

$$= \frac{4}{3}\cdot\frac{x^{-1}}{-1} - \frac{1}{2}\cdot\frac{3^x}{\ln 3} + \frac{1}{6}\cdot\frac{x^{3/2}}{3/2} - 2\ln x + C$$

$$\int \frac{2}{4x^3} - \frac{\cos x}{3} - \frac{2}{\sqrt{1-x^2}}\,dx =$$

$$= \frac{2}{4}\int x^{-3}\,dx - \frac{1}{3}\int \cos x\,dx - 2\int \frac{1}{\sqrt{1-x^2}}\,dx =$$

$$= \frac{1}{2}\cdot\frac{x^{-2}}{-2} - \frac{1}{3}(+\sin x) - 2\cdot \arcsin x + C$$

Tipusok

$$\int f(ax+b)\,dx = \frac{F(ax+b)}{a} + C$$

$$F(g(x)) + C = \int f(g(x))\,g'(x)$$

$$\int (2x+27)^{2023}\,dx = \frac{(2x+27)^{2024}}{2024} + C$$

$$2$$

$a = 2 \quad b = 27$

$f(y) = y^{2023}$ $\qquad F(y) = \frac{y^{2024}}{2024}$

$g(x) = 2x + 27$

$$\int \frac{6}{(5x-1)^7}\,dx = 6\int \frac{1}{(5x-1)^7} = \frac{6\cdot\frac{(5x-1)^{-6}}{-6}}{5} + C$$

$f(y) = \frac{1}{y^7}$ $\qquad F(y) = \frac{y^{-6}}{-6}$

$g(x) = (5x-1)$

$a = 5 \quad b = -1$

$$\int \frac{\sqrt{2-3x}}{5}\,dx = \frac{1}{5}\int \sqrt{2-3x}\,dx =$$

$$f(y) = \sqrt{y} \qquad F(y) = \frac{y^{3/2}}{3/2}$$

$$g(x) = -3x+2$$

$$a = -3 \qquad b = 2$$

$$= \frac{1/5\,(2-3x)^{3/2}}{3/2} + C$$

$$-3$$