1. gyakorlat

$$\left[(x^2-1)^8\right]' = 8\cdot(x^2-1)^7\cdot 2x$$

$f(y) = y^8 \qquad g(x) = x^2-1$

$f'(y) = 8\cdot y^7 \qquad g'(x) = 2x$

$$\left[\sqrt{2x+1}\right]' = 1/2\cdot(2x+1)^{-1/2}\cdot 2 = (2x+1)^{-1/2}$$

$f(y) = \sqrt{y} \qquad g(y) = 2x+1$

$f'(y) = 1/2\cdot y^{-1/2} \qquad g'(y) = 2$

$$\int \frac{\sqrt[7]{x}}{7} - \frac{4}{\cos^2 x} + \frac{3}{4x^4} + \frac{2}{\sqrt{1-x^2}}\,dx =$$

$$= \frac{1}{7}\int \sqrt[7]{x}\,dx - 4\cdot\int\frac{1}{\cos^2 x}\,dx + \frac{3}{4}\cdot\int\frac{1}{x^4}\,dx +$$

$$+ 2\cdot\int\frac{1}{\sqrt{1-x^2}}\,dx = \frac{1}{7}\cdot\frac{x^{\frac{8}{7}}}{8/7} - 4\cdot \operatorname{tg} x + \frac{3}{4}\cdot\frac{x^{-3}}{-3} +$$

$$+ 2\cdot\arcsin x + C$$

$$\int \frac{2}{3x} - \frac{4^x}{5} + \frac{2\cdot \sin x}{5} - \frac{6}{1+x^2}\,dx =$$

$$= \frac{2}{3}\int \frac{1}{x}\,dx - \frac{1}{5}\cdot\int 4^x\,dx + \frac{2}{5}\cdot\int \sin x\,dx - 6\int \frac{1}{1+x^2}\,dx =$$

$$= \frac{2}{3}\cdot \ln|x| - \frac{1}{5}\,\frac{4^x}{\ln 4} + \frac{2}{5}\cdot(-\cos x) - 6\,\mathrm{arctg}\,x + C$$

$$\int f(ax+b)\,dx = \frac{F(ax+b)}{a} + C$$

$$\int \frac{6}{7x-8}\,dx = 6\cdot\int \frac{1}{7x-8}\,dx = 6\cdot\frac{\ln|7x-8|}{7} + C$$

$$f(y) = \frac{1}{y} \qquad F(y) = \ln|y|$$

$$ax+b = 7x-8$$

$$\int \sin(1-2x)\,dx = \frac{-\cos(-2x+1)}{-2} + C$$

$$f(y) = \sin y \qquad F(y) = -\cos y$$

$$ax+b = -2x+1 \rightarrow a = -2$$

$$2\cdot\frac{\frac{(4x+5)^{2/3}}{2/3}}{4} + C$$

$$\int \frac{2}{\sqrt[3]{4x+5}}\,dx = 2\cdot\int \frac{1}{\sqrt[3]{4x+5}}\,dx$$

$$f(y) = \frac{1}{\sqrt[3]{y}}$$

$$F(y) = \frac{y^{2/3}}{2/3}$$

$$\int \frac{4}{\sqrt{1-4x^2}}\,dx = 4 \cdot \int \frac{1}{\sqrt{1-(2x)^2}}\,dx$$

$$f(y) = \frac{1}{\sqrt{1-y^2}} \qquad F(y) = \arcsin y$$

$$ax+b = 2x+0 \Rightarrow a = -2$$

$$= 4 \cdot \frac{\arcsin(+2x)}{+2} + C$$

$$\int \frac{g'(x)}{g(x)}\,dx = \ln|g(x)| + C$$

$$\int \frac{e^x}{3+e^x}\,dx = \ln|3+e^x| + C$$

$$\int \frac{\cos x}{1+2\sin x}\,dx = \frac{1}{2}\int \frac{2\cdot\cos x}{1+2\cdot\sin x}\,dx = \ln|1+2\sin x| + C$$

$$g(x) = 1 + 2\cdot\sin x$$

$$g'(x) = 2\cdot\cos x$$

$$\int \frac{6}{7x-8}\,dx = \frac{6}{7}\int \frac{7}{7x-8}\,dx = \ln|7x-8|+C$$

$$g(x) = 7x-8$$

$$g'(x) = 7$$

$$\int \operatorname{tg}(4x)\,dx = \int \frac{\sin 4x}{\cos 4x}\,dx =$$

$$= -\frac{1}{4}\int \frac{-4\cdot\sin(4x)}{\cos 4x}\,dx =$$

$$= \ln|\cos 4x|+C$$

$$[\cos 4x]' = -\sin 4x \cdot 4$$

$$f(y) = \cos y \qquad f'(y) = -\sin y$$

$$g(x) = 4x \qquad g'(x) = 4$$

$$\int \frac{1}{x\cdot\ln x}\,dx = \int \frac{1/x}{\ln x}\,dx = \ln|\ln x|+C$$

Hiperbolikus függvények

$$\operatorname{sh}x = \frac{e^{x}-e^{-x}}{2}\,; \quad \operatorname{ch}x = \frac{e^{x}+e^{-x}}{2}$$

$$\operatorname{th}x = \frac{\operatorname{sh}x}{\operatorname{ch}x} \qquad \operatorname{cth}x = \frac{\operatorname{ch}x}{\operatorname{sh}x}$$

$$\int th x \, dx = \int \frac{sh x}{ch x} dx = \int \frac{\frac{e^x - e^{-x}}{2}}{\frac{e^x + e^{-x}}{2}} dx =$$

$$= \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx = \ln |e^x + e^{-x}| + C$$

$$\left[e^x + e^{-x}\right]' = e^x + e^{-x} \cdot (-1) = e^x - e^{-x}$$