3. óra

$$\int \cos x \cdot \sin x \, dx = -\int \cos x \cdot (-\sin x)\, dx =$$

$$\boxed{[\cos x]' = -\sin x}$$

$$= -\left(\frac{\cos^2 x}{2}\right) + C$$

$$\boxed{\int g^n(x) \cdot g'(x)\, dx}$$

$$\int x^2 \cdot (2x^3+4)^2 \, dx = \frac{1}{6} \int \underbrace{6x^2}_{g'} \underbrace{(2x^3+4)^2}_{g^n} dx =$$

$$= \frac{1}{6} \cdot \frac{(2x^3+4)^3}{3} + C$$

$$\int \frac{x}{\sqrt{x^2+1}}\, dx = \int x \cdot (x^2+1)^{-1/2} = \frac{1}{2} \int \underbrace{2x}_{g'} \underbrace{(x^2+1)^{-1/2}}_{g^n} dy$$

$$= \cancel{\frac{1}{2}} \cdot \frac{(x^2+1)^{1/2}}{\cancel{1/2}} + C = \sqrt{x^2+1} + C$$

$$\int \frac{2}{(3+4x)^5}\, dy = \left| \begin{array}{l} t = 4y+3 \\ x = \frac{t-3}{4} \\ y' = 1/4 \end{array} \right| = 2 \int t^{-5}\, dt \cdot 1/4 =$$

$$= 1/2 \cdot \frac{t^{-4}}{-4} + C =$$

$$= \frac{(3+4x)^{-4}}{-8} + C$$

$$\int x \cdot \sin(x^2)\,dx = \left| \begin{array}{l} t = x^2 \\ x = \sqrt{t} \\ x' = \frac{1}{2} \cdot t^{-1/2} \end{array} \right| =$$

$$= \int \cancel{\sqrt{t}} \cdot \sin t \cdot \frac{1}{2} \cdot \frac{1}{\cancel{\sqrt{t}}} = \frac{1}{2} \int \sin t \, dt =$$

$$= \frac{1}{2} \cdot (-\cos t) + c = \frac{1}{2} \cdot (-\cos(x^2)) + C$$

$$\int \frac{e^x}{\sqrt[3]{1+e^x}}\,dx = \left| \begin{array}{l} t = e^x \\ x = \ln|t| \\ x' = \frac{1}{t} \end{array} \right| =$$

$$= \int \frac{\cancel{t}}{\sqrt[3]{1+t}} \cdot \frac{1}{\cancel{t}}\,dt = \int (1+t)^{-\frac{1}{3}}\,dt =$$

$$= \left| \begin{array}{l} s = 1+t \\ t = s-1 \\ t' = 1 \end{array} \right| = \int s^{-\frac{1}{3}} \cdot 1 \, ds = \frac{s^{2/3}}{2/3} + C =$$

$$= \frac{(1+t)^{2/3}}{2/3} + C = \frac{(1+e^x)^{2/3}}{2/3} + C$$

$$\int \frac{\sqrt{\ln x}}{x}\, dx = \left| \begin{array}{l} t = \ln x \\ x = e^{t} \\ x' = e^{t} \end{array} \right| = \int \frac{\sqrt{t}}{e^{t}} \cdot e^{t} dt$$

$$= \frac{t^{3/2}}{3/2} + C = \frac{(\ln x)^{3/2}}{3/2} + C$$

$$\int \frac{e^{x}}{\sqrt{1 - e^{2x}}}\, dx = \left| \begin{array}{l} t = e^{x} \\ x = \ln t \\ x' = \frac{1}{t} \end{array} \right| = \int \frac{\cancel{t}}{\sqrt{1 - t^{2}}} \cdot \frac{1}{\cancel{t}}\, dt$$

$$= \arcsin t + C = \arcsin(e^{x}) + C$$

$$\int \frac{1}{x + \sqrt{x}}\, dx = \left| \begin{array}{l} t = \sqrt{x} \\ x = t^{2} \\ x' = 2t \end{array} \right| =$$

$$= \int \frac{1}{t^{2} + t} \cdot 2t = \int \frac{1}{\cancel{t}(t+1)} \cdot 2\cancel{t} =$$

$$= 2 \ln|t+1| + C = 2 \ln|\sqrt{x} + 1| + C$$