2. előadás

$$\int f(ax+b)\,dx = \frac{F(ax+b)}{a} + C$$

$$\int \frac{g'(x)}{g(x)}\,dx = \ln|g(x)| + C$$

$$\int f(g(x)) \cdot g'(x)\,dx = F(g(x)) + C$$

$$\int g^n(x) \cdot g'(x) = \frac{g^{n+1}(x)}{n+1} + C$$

P!
$$\int x \cdot \sqrt{\underbrace{x^2+1}_{g(x)}}\,dx = \frac{1}{2}\int (x^2+1)^{1/2} \cdot 2x\,dx = 1/2 \cdot \frac{(x^2+1)^{3/2}}{3/2} + C$$

$g(x) = x^2+1 \quad g'(x) = 2x$

$n = 1/2$

$$\int \cos^3 x\,dx =$$

$$= \int \cos^2 x \cdot \cos x\,dx = \int (1-\sin^2 x) \cdot \cos x\,dx =$$

$$= \int \cos x - \sin^2 x \cdot \cos x = \sin x - \frac{\sin^3 x}{3} + C$$

$g(x) = \sin x \quad g'(x) = \cos x$

$n = 2$

Helyettesítéses integrálás

Összetett fgv. deriváltja $\left[F(x(t))\right]' =$

$$= F'(x(t)) \cdot x'(t) = f(x(t)) \cdot x'(t)$$

$$\int [F(x(t))]' \, dt = \int f(x(t)) \cdot x'(t) \, dt$$

$$F(x(t)) + C = \int f(x(t)) \cdot x'(t) \, dt$$

$$\int f(x(t)) \cdot x'(t) \, dt = F(x(t)) + C$$

$$f \xrightarrow[\int]{} F$$

$$\int f(x) \, dx = F(x) + C$$

Pl. $\int \underbrace{x \cdot \sqrt{x+2}}_{f(x)} \, dx =$

$t = \sqrt{x+2}$

új változó

$t^2 = x + 2$

$x(t) = t^2 - 2$

$x'(t) = 2t$

$$= \int \underbrace{(t^2 - 2)}_{f(x(t))} \cdot t \cdot \underbrace{2t}_{x'(t)} \, dt =$$

$$= 2\int t^4 - 2t^2 \, dt = 2\left(\frac{t^5}{5} - 2\frac{t^3}{3}\right) + C$$

$$= 2\left(\frac{\sqrt{x+2}^{\,5}}{5} - 2\frac{\sqrt{x+2}^{\,3}}{3}\right) + C$$

$$\int 2x \, dx = \left| \begin{array}{l} t = 2x \\ \frac{t}{2} = x \\ \frac{1}{2} = x' \end{array} \right| = \int t \cdot \frac{1}{2} \, dt = \frac{t^2}{2} \cdot \frac{1}{2} + C =$$

$$= \frac{(2x)^2}{4} + C = x^2 + C$$

$$\int x \cdot \sqrt{x^2+1}\, dx = \left| \begin{array}{l} t = \sqrt{x^2+1} \\ t^2 = x^2+1 \\ x = \sqrt{t^2-1} \\ x'(t) = \underbrace{\tfrac{1}{2} \cdot (t^2-1)^{-1/2} \cdot 2t}_{\frac{t}{\sqrt{t^2-1}}} \end{array} \right\} \Rightarrow$$

$$= \int \cancel{\sqrt{t^2-1}} \cdot t \cdot \frac{t}{\cancel{\sqrt{t^2-1}}}\, dt = \frac{t^3}{3} + C = \frac{\sqrt{x^2+1}^{\,3}}{3} + C$$

$$\int x \cdot e^{-x^2}\, dx = \left| \begin{array}{l} -x^2 = t \\ -t = x^2 \\ x = \sqrt{-t} \\ x' = \frac{-1}{\sqrt{-t}} \cdot \frac{1}{2} \end{array} \right| = \int \cancel{\sqrt{-t}} \cdot e^t \cdot -\frac{1}{2} \cdot \frac{1}{\cancel{\sqrt{-t}}}\, dt$$

$$= \int -\tfrac{1}{2} e^t\, dt = -\frac{1}{2} e^{-x^2} + C$$

$$\int \frac{1}{x \cdot \ln x}\, dx = \left| \begin{array}{l} t = \ln x \\ e^t = x \\ x' = e^t \end{array} \right| = \int \frac{1}{\cancel{e^t} \cdot t} \cdot \cancel{e^t}\, dt =$$

$$= \int \frac{1}{t}\, dt = \ln t + C = \ln |\ln x| + C$$

$$\int (3-8x)^7\, dx = \left| \begin{array}{l} t = 3-8x \\ x = \frac{3-t}{8} \\ x' = \tfrac{1}{8} \cdot -1 = -\tfrac{1}{8} \end{array} \right| \begin{array}{l} = \int t^7 \cdot -\tfrac{1}{8} = \\ = -\tfrac{1}{8} \cdot \frac{t^8}{8} + C \end{array}$$

$$\int \sin^2 y \cdot \cos x \, dx =$$

$$\cos^2 x = 1 - \sin^2 x$$
$$\cos x = \sqrt{1-\sin^2 y}$$

$$= \left| \begin{array}{l} t = \sin y \\ x = \arcsin t \\ x' = \dfrac{1}{\sqrt{1-t^2}} \end{array} \right| = \int t^2 \cdot \cancel{\sqrt{1-t^2}} \cdot \frac{1}{\cancel{\sqrt{1-t^2}}} \, dt =$$

$$\cos x = \sqrt{1-t^2}$$

$$= \frac{t^3}{3} + C = \frac{\sin^3 x}{3} + C$$

Pl. $$\int \sqrt{1-x^2} \, dx = \left| \begin{array}{l} x(t) = \sin t \\ x'(t) = \cos t \\ t = \arcsin(x) \end{array} \right| = \int \sqrt{1-\sin^2 t} \cdot \cos t =$$

$$= \int \cos t \cdot \cos t \, dt = \int \cos^2 t \, dt = \int \frac{1+\cos 2t}{2} \, dt =$$

$$= 1/1 \cdot \left(t + \frac{\sin 2t}{2} \right) + C = 1/2 \left(t + \frac{\cancel{2} \cdot \sin t \cos t}{\cancel{2}} \right) + C =$$

$$= \frac{1}{2} \left(\arcsin x + x \cdot \sqrt{1-x^2} \right) + C$$

Parciális integrálás

Pl. $$\int x \cdot \sin x \, dx$$

$$[u \cdot v]' = u' v + u \cdot v' \qquad \int (u \cdot v)' = \int u' v + \int u \cdot v'$$

$$u \cdot v = \int u' \cdot v + \int u \cdot v' \quad / - \int u' \cdot v$$

$$\int u \cdot v' = uv - \int u' \cdot v$$

$$\int x \cdot \sin x \, dx = \left| \begin{array}{ll} u(x) = x & u'(x) = 1 \\ v'(x) = \sin x & \\ v(x) = -\cos x & \end{array} \right| =$$

$$= x \cdot -\cos x - \int 1 \cdot -\cos x = -x \cdot \cos x + \sin x + C$$

$$\int \ln x \, dx = \int \underset{v'}{1} \cdot \underset{u}{\ln x} \, dx = \left| \begin{array}{l} u = \ln x \\ u' = 1/x \\ v' = 1 \\ v = x \end{array} \right| = x \cdot \ln x \int \frac{1}{x} \cdot x \, dx$$

$$= x \cdot \ln x - x + C$$