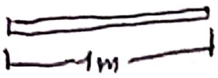
Homework #5.

1. At what speed does a meter stick move if its length is observed to shrink to 0.5m?

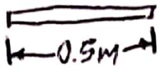


In normal conditions the stick it is measured as 1m. However, if we consider the relativistic effects we consider that,

$$l = l_0\sqrt{1-\frac{u^2}{c^2}} \Rightarrow l = l_0\sqrt{1-\beta^2} \Rightarrow l = \frac{l_0}{\gamma}$$

This is the length contraction, so,

$$l = l_0\sqrt{1-\frac{u^2}{c^2}}$$



l and l_0 are the proper lengths for each observer.

$$\left[\frac{l}{l_0}\right]^2 = 1-\frac{u^2}{c^2}$$

$$u^2 = \left(1-\left[\frac{l}{l_0}\right]^2\right)c^2$$

$$u = \left(1-\left[\frac{l}{l_0}\right]^2\right)^{1/2} c$$

$\boxed{u \approx 0.86c} \Rightarrow$ with $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = 299.80\times10^6 m/s$

$$\boxed{u = 259.63\times10^6\ m/s}$$

2. An atomic clock is placed in a jet airplane. The clock measures a time interval of 3600 s when the jet moves with speed 400m/s. How much larger a time interval does an identical clock held by an observer at rest on the ground measure?

For time dilatation we have that

Δt and Δt_0 are the proper times for each observer

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}} \Rightarrow \Delta t = \frac{\Delta t_0}{\sqrt{1-\beta^2}} \Rightarrow \Delta t = \gamma\Delta t_0$$

So, if we consider that the time interval measured in the ground for an stationary observer is Δt we can assume that,

$$\Delta t = \gamma \Delta t_0 \longrightarrow \Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Delta t = \frac{3600\,s}{\sqrt{1-\beta^2}}$$

, in this example we can consider that the factor β is very small so, for facility we can consider that $c = 3\times10^8\,m/s$ and solve the problem with a binomial expansion.

Hence,

$$\Delta t = \frac{3600\,s}{\sqrt{1-\frac{400\,m/s^2}{3\times10^8\,m/s^2}}}$$

$$\beta^2 = \frac{v^2}{c^2} = \frac{(400\,m/s)^2}{(3\times10^8\,m/s)^2} = \frac{160\times10^3\,\cancel{m/s}}{9\times10^{16}\,\cancel{m/s}}$$

$$\beta^2 = 1.78\times10^{-12}$$

Now, we got the expression, and,

$$(1-1.76\times10^{-12})^{1/2}$$

using the binomial expansion we have,

$$(1-1.76\times10^{-12})^{1/2} = 1 + \left(\tfrac{1}{2}\right)(1.76\times10^{-12}) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)(1.76\times10^{-12})^2}{2!} + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)(1.76\times10^{-12})^3}{3!}$$

$$\sqrt{1-\beta^2} \approx 0.\overline{9}1^{[13]} \approx 0.9999\ldots$$

$$\Delta t = \frac{3600\,s}{\sqrt{1-\beta^2}} = 3600.00000000326$$

The time measured on Earth is slightly major than that measured on the airplane.

3. How fast and in what direction must galaxy A be moving if an absorption line found at wavelength 5500 Å for a stationary galaxy is shifted to 7000 Å (a redshift) for galaxy A?

This is considering that the form v/c is less than 1, so $\Delta f/f$ is very small in the Doppler effect given by $f = \sqrt{\frac{c+v}{c-v}}\, f_0$,

If the shift measured is redshift we infer that the object is moving radially apart from the observer because the wavelengths are getting wider, so, in the consideration that,

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$$

where v is the radial velocity and $\Delta\lambda$ the change in wavelength, thus

$$5500\,Å \times \frac{0.1\,nm}{1\,Å} = 550\,nm$$

$$7000\,Å \times \frac{0.1\,nm}{Å} = 700\,nm$$

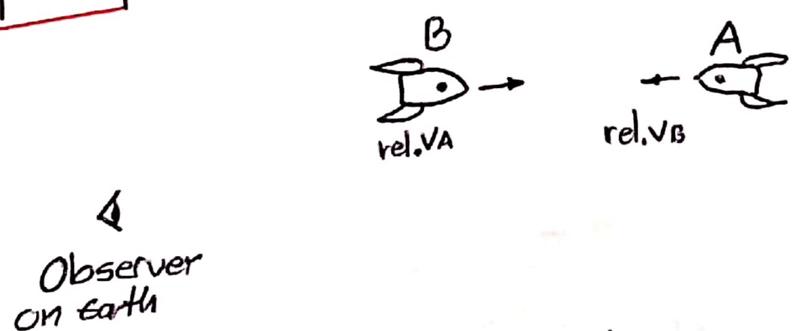
$$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{v}{c}$$

The positive value indicates that the object is moving apart from the observer.

$$\frac{700\,nm - 550\,nm}{550\,nm} = \frac{v}{c} \Rightarrow \frac{150\,nm}{550\,nm} \times c = v$$

$$v = \frac{150\times10^{-9}\,m}{550\times10^{-9}\,m}\, c \Rightarrow 0.273c \quad \text{or} \quad 81.843\times10^{6}\,m/s \quad \text{with } c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

4. Two spaceships approach each other, each moving with the same speed as measured by a stationary observer on Earth. Their relative speed is 0.70c. Determine the velocities of each spaceship as measured by the stationary observer on Earth.



Here we have to apply the Lorentz transformations for different frames of reference, so, knowing that,

$$v' = \frac{u+v}{1+\frac{uv}{c^2}}$$

- Now, for consideration we have that the velocity of A relative to B is equal to $u_{AB} = 0.70c$.
- The velocity of the spaceship B is $u_B = -u$ because it is moving away.
- The velocity of the spaceship A is $u_A = u$ because it is approaching Earth.
- The velocity of the observer on Earth relative to B is $u_B = u$
- The velocity u_{AB} is the relative speed between the spaceships so $u_{AB} = v'$
- The velocities u_A and u_B are reciprocally u and v

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Now, arranging the previous statements we have that,

$$V' = \frac{u+v}{1+\frac{uv}{c^2}}$$

$$u_{AB} = \frac{u_A + u_B}{1+\frac{u_A u_B}{c^2}}$$ and given that $u_A = u_B = u$, we have that

$$u_{AB} = \frac{u+u}{1+\frac{uu}{c^2}}$$

$$0.70c = \frac{2u}{1+\frac{u^2}{c^2}}$$

$$0.70c\left(1+\frac{u^2}{c^2}\right) = 2u$$

$$0.70c + \frac{0.70\cancel{c}u^2}{c^{\cancel{2}}} - 2u = 0$$

$$\frac{0.70u^2}{c} - 2u + 0.70c = 0$$

$$\frac{0.70u^2}{c^2} - \frac{2u}{c} + 0.70 = 0$$

$$0.70x^2 - 2x + 0.70 = 0 \quad \text{where } x = \frac{u}{c}$$

Solving for $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ we obtain

This value is not possible. so the choice is x_2

$\cancel{x_1 = 2.44}$ $x_2 = 0.41$

Thus, $x = \frac{u}{c} \Rightarrow \boxed{0.41c = u}$