Ejercicio #2.

Con el elemento de línea,

$$ds^2 = dv^2 - v^2 du^2$$

tendremos la métrica,

$$g_{\alpha\beta} = \begin{bmatrix} 1 & 0 \\ 0 & -v^2 \end{bmatrix} \text{ con inversa, } g^{\alpha\beta} = \begin{bmatrix} 1 & 0 \\ 0 & -v^{-2} \end{bmatrix}$$

Calculamos símbolos de Christoffel

$$\Gamma^i_{kl} = \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m} \right)$$

$$\Gamma^0_{kl} = \frac{1}{2} g^{0m} \frac{\partial g_{mk}}{\partial x^l} + \frac{1}{2} g^{0m} \frac{\partial g_{ml}}{\partial x^k} - \frac{1}{2} g^{0m} \frac{\partial g_{kl}}{\partial x^m}$$

$$= \frac{1}{2} g^{00} \frac{\partial g_{0k}}{x^l} + \frac{1}{2} g^{01} \frac{\partial g_{1k}}{x^l} + \frac{1}{2} g^{00} \frac{\partial g_{0l}}{x^k} + \frac{1}{2} g^{01} \frac{\partial g_{1l}}{x^k}$$

$$- \left(\frac{1}{2} g^{00} \frac{\partial g_{kl}}{x^0} + \frac{1}{2} g^{01} \frac{\partial g_{kl}}{x^1} \right)$$

$$\Gamma^0_{0l} = \frac{1}{2} g^{00} \frac{\partial g_{0k}}{x^0} + \frac{1}{2} g^{01} \frac{\partial g_{1k}}{x^0} + \frac{1}{2} g^{00} \frac{\partial g_{00}}{x^k} + \frac{1}{2} g^{01} \frac{\partial g_{10}}{x^k}$$

$$- \left(\frac{1}{2} g^{00} \frac{\partial g_{k0}}{x^0} + \frac{1}{2} g^{01} \frac{\partial g_{k0}}{x^1} \right)$$

$$\Gamma^0_{00} = \frac{1}{2} g^{00} \frac{\partial g_{00}}{x^0} + \frac{1}{2} g^{01} \frac{\partial g_{10}}{x^0} + \frac{1}{2} g^{00} \frac{\partial g_{00}}{x^0} + \frac{1}{2} g^{01} \frac{\partial g_{10}}{x^0}$$

$$- \left(\frac{1}{2} g^{00} \frac{\partial g_{00}}{x^0} + \frac{1}{2} g^{01} \frac{\partial g_{00}}{x^1} \right)$$

$$\Gamma^{0}_{00} = \frac{1}{2} g^{00} \frac{\partial g_{00}}{x^0} + \cancel{\frac{1}{2} g^{01} \frac{\partial g_{10}}{x^0}}^{\,0} + \frac{1}{2} g^{00} \frac{\partial g_{00}}{x^0} + \cancel{\frac{1}{2} g^{01} \frac{\partial g_{10}}{x^0}}^{\,0}$$

$$- \left(\frac{1}{2} g^{00} \frac{\partial g_{00}}{x^0} + \cancel{\frac{1}{2} g^{01} \frac{\partial g_{00}}{x^1}}^{\,0} \right)$$

$$\Gamma^{0}_{00} = \frac{1}{2} g^{00} \frac{\partial g_{00}}{x^0} + \frac{1}{2} g^{00} \frac{\partial g_{00}}{x^0} - \frac{1}{2} g^{00} \frac{\partial g_{00}}{x^0}$$

$$\Gamma^{10}_{00} = \cancel{\frac{1}{2} \cdot (1) \cdot \frac{\partial(1)}{\partial v}}^{\,0} + \cancel{\frac{1}{2} (1) g^{00} \frac{\partial(1)}{\partial v}}^{\,0} - \cancel{\frac{1}{2} (1) \frac{\partial(1)}{\partial v}}^{\,0}$$

$$\Gamma^{10}_{00} = 0$$

$$\Gamma^{0}_{11} = \frac{1}{2} g^{00} \frac{\partial g_{01}}{x^1} + \frac{1}{2} g^{01} \frac{\partial g_{11}}{x^1} + \frac{1}{2} g^{00} \frac{\partial g_{01}}{x^1} + \frac{1}{2} g^{01} \frac{\partial g_{11}}{x^1}$$

$$- \left(\frac{1}{2} g^{00} \frac{\partial g_{11}}{x^0} + \frac{1}{2} g^{01} \frac{\partial g_{11}}{x^1} \right)$$

$$\Gamma^{0}_{11} = \frac{1}{2} g^{00} \frac{\partial g_{01}}{x^1} + \cancel{\frac{1}{2} g^{01} \frac{\partial g_{11}}{x^1}}^{\,0} + \frac{1}{2} g^{00} \frac{\partial g_{01}}{x^1} + \cancel{\frac{1}{2} g^{01} \frac{\partial g_{11}}{x^1}}^{\,0}$$

$$- \left(\frac{1}{2} g^{00} \frac{\partial g_{11}}{x^0} + \cancel{\frac{1}{2} g^{01} \frac{\partial g_{11}}{x^1}}^{\,0} \right)$$

$$\Gamma^{10}_{11} = \frac{1}{2} g^{00} \frac{\partial g_{01}}{x^1} + \frac{1}{2} g^{00} \frac{\partial g_{01}}{x^1} - \frac{1}{2} g^{00} \frac{\partial g_{11}}{x^0}$$

$$\Gamma^{10}_{11} = \cancel{\frac{1}{2} \cdot 1 \cdot \frac{\partial(0)}{\partial u}}^{\,0} + \cancel{\frac{1}{2} \cdot 1 \cdot \frac{\partial(0)}{\partial u}}^{\,0} - \frac{1}{2} \cdot (1) \cdot \frac{\partial(-v^2)}{\partial v}$$

$$\Gamma^{0}_{11} = -\frac{1}{2} \cdot -2v \quad = \Gamma^{10}_{11} = v = \Gamma^{v}_{uu}$$

Realizando el cómputo para los demás índices tendremos que,

$$\Gamma^{v}_{uu} = v \quad , \quad \Gamma^{u}_{vu} = \frac{1}{v} = \Gamma^{u}_{uv}$$

Con base en este resultado intentaremos calcular $R^{u}{}_{vuv}$.

$$R^{\alpha}{}_{\beta\mu\nu} = \partial_\mu \Gamma^{\alpha}_{\beta\nu} - \partial_\nu \Gamma^{\alpha}_{\beta\mu} + \Gamma^{\alpha}_{\sigma\mu}\Gamma^{\sigma}_{\beta\nu} - \Gamma^{\alpha}_{\sigma\nu}\Gamma^{\sigma}_{\beta\mu}$$

$$R^{0}{}_{010} = \underbrace{\partial_1 \Gamma^{0}_{00}}_{0} - \underbrace{\partial_0 \Gamma^{0}_{01}}_{0} + \underbrace{\Gamma^{0}_{01}\Gamma^{0}_{00}}_{0} + \Gamma^{0}_{11}\underbrace{\Gamma^{1}_{00}}_{0} - \left(\underbrace{\Gamma^{0}_{00}\Gamma^{0}_{01}}_{0} + \underbrace{\Gamma^{0}_{10}\Gamma^{1}_{01}}_{0}\right)$$

$$R^{0}{}_{010} = 0$$

$$R^{1}{}_{010} = \underbrace{\partial_1 \Gamma^{1}_{00}}_{0} - \partial_0 \Gamma^{1}_{01} + \overbrace{\Gamma^{1}_{01}\underbrace{\Gamma^{0}_{00}}_{0}}^{0} + \overbrace{\underbrace{\Gamma^{1}_{11}}_{0}\Gamma^{1}_{00}}^{0} - \left(\overbrace{\Gamma^{1}_{00}\underbrace{\Gamma^{0}_{01}}_{0}}^{0} + \Gamma^{1}_{10}\Gamma^{1}_{01}\right)$$

$$R^{1}{}_{010} = -\partial_0 \Gamma^{1}_{01} - \Gamma^{1}_{10}\Gamma^{1}_{01}$$

$$R^{1}{}_{010} = -\frac{\partial(1/v)}{\partial v} - \frac{1}{v}\cdot\frac{1}{v} = -\left(-\frac{1}{v^2}\right) - \left(\frac{1}{v^2}\right) = \frac{1}{v^2} - \frac{1}{v^2} = 0$$

Ahora bien,

$$R_{1010} = R_{uvuv} = g_{u\sigma}R^{\sigma}{}_{vuv} = g_{1\sigma}R^{\sigma}{}_{010}$$

$$= g_{10}R^{0}{}_{010} + g_{11}R^{1}{}_{010} = 0\cdot 0 + -v^2\cdot 0 = 0$$

$$R_{uvuv} = 0$$

Ejercicio #3.

Con el elemento de línea $ds^2 = (b + a\,\text{sen}\varphi)^2 d\theta^2 + a^2 d\varphi^2$ con a y b constantes tendremos que,

$$g_{\alpha\beta} = \begin{bmatrix} (b + a\,\text{sen}\varphi)^2 & 0 \\ 0 & a^2 \end{bmatrix}$$

cuya inversa será,

$$g^{\alpha\beta} = \begin{bmatrix} \frac{1}{(b + a\,\text{sen}\varphi)^2} & 0 \\ 0 & a^{-2} \end{bmatrix}$$

Usando la relación,

$$\Gamma^{i}_{kl} = \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m} \right)$$

tendremos que los símbolos de Christoffel distintos a cero serán,

$$\Gamma^{\theta}_{\theta\varphi} = \Gamma^{\theta}_{\varphi\theta} = \frac{a(a\,\text{sen}\varphi + b)\cos\varphi}{(a\,\text{sen}\varphi + b)^2}$$

$$\Gamma^{\varphi}_{\theta\theta} = -\frac{(a\,\text{sen}\varphi + b)\cos\varphi}{a}$$

* Para este inciso me apoyé en el snippet de Python que escribí para la tarea 6. Por favor, revisar la nota en dicha tarea.

A partir de esto, y sabiendo que el tensor de Riemann sigue la relación,

$$R^{\alpha}_{\beta\mu\nu} = \partial_\mu \Gamma^{\alpha}_{\beta\nu} - \partial_\nu \Gamma^{\alpha}_{\beta\mu} + \Gamma^{\alpha}_{\sigma\mu}\Gamma^{\sigma}_{\beta\nu} - \Gamma^{\alpha}_{\sigma\nu}\Gamma^{\sigma}_{\beta\mu}$$

Veremos que, aquellos componentes del tensor de Riemann distintos a cero serán,

$$R^{\theta}_{\varphi\theta\varphi} = \frac{a\,\mathrm{sen}\,\varphi}{a\,\mathrm{sen}\,\varphi + b} \quad , \quad R^{\theta}_{\varphi\varphi\theta} = -\frac{a\,\mathrm{sen}\,\varphi}{a\,\mathrm{sen}\,\varphi + b}$$

$$R^{\varphi}_{\theta\theta\varphi} = -\frac{(a\,\mathrm{sen}\,\varphi + b)\,\mathrm{sen}\,\varphi}{a} , \quad R^{\varphi}_{\theta\varphi\theta} = \frac{(a\,\mathrm{sen}\,\varphi + b)\,\mathrm{sen}\,\varphi}{a}$$

Ahora bien, calculando la forma totalmente covariante del tensor de Riemann tendremos que,

$$R_{\alpha\beta\mu\nu} = g_{\alpha\sigma} R^{\sigma}_{\beta\mu\nu}$$

$$R_{\theta\varphi\theta\varphi} = a(a\,\mathrm{sen}\,\varphi + b)\,\mathrm{sen}\,\varphi$$

$$R_{\theta\varphi\varphi\theta} = -a(a\,\mathrm{sen}\,\varphi + b)\,\mathrm{sen}\,\varphi$$

$$R_{\varphi\theta\theta\varphi} = -a(a\,\mathrm{sen}\,\varphi + b)\,\mathrm{sen}\,\varphi$$

$$R_{\varphi\theta\varphi\theta} = a(a\,\mathrm{sen}\,\varphi + b)\,\mathrm{sen}\,\varphi$$

Usando condiciones de antisimetría y pares veremos que,

$$R_{\theta\varphi\theta\varphi} = a(a\,\mathrm{sen}\,\varphi + b)\,\mathrm{sen}\,\varphi \;//$$

Ejercicio #4.

Teniendo que

$$T^{\mu\nu} = \frac{1}{4\pi}\left(F^{\mu\alpha}F^{\nu}{}_{\alpha} - \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right)$$

Vamos a ver que,

$$T_{\mu}^{\mu} = g_{\mu\nu}T^{\mu\nu}$$

$$T_{\mu}^{\mu} = \frac{1}{4\pi}\left[g_{\mu\nu}F^{\mu\alpha}F^{\nu}{}_{\alpha} - g_{\mu\nu}g^{\mu\nu}\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}\right]$$

Con la métrica de Schwarzchild tendremos que,

$$g_{\mu\nu} = \begin{bmatrix} -\left(1-\frac{2GM}{r}\right) & 0 & 0 & 0 \\ 0 & \left(1-\frac{2GM}{r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & \\ 0 & 0 & 0 & r^2 \operatorname{sen}^2\theta \end{bmatrix}$$

cuya inversa será,

$$g^{\mu\nu} = \begin{bmatrix} -\left(1-\frac{2GM}{r}\right)^{-1} & 0 & 0 & 0 \\ 0 & 1-\frac{2GM}{r} & 0 & 0 \\ 0 & 0 & r^{-2} & \\ 0 & 0 & 0 & r^{-2} \operatorname{sen}^{-2}\theta \end{bmatrix}$$

Dado que $g^{\mu\nu} g_{\mu\nu} = \delta^{\mu}_{\mu}$

$$\delta^{\mu}_{\mu} = g^{\mu\nu} g_{\mu\nu} = g^{00} g_{00} + \sum_{i=1}^{3} g^{ii} g_{ii}$$

$$= -\left(1 - \frac{2GM}{r}\right) \cdot -\left(1 - \frac{2GM}{r}\right)^{-1} + \left(1 - \frac{2GM}{r}\right)\left(1 - \frac{2GM}{r}\right)^{-1} + r^2 \cdot \frac{1}{r^2}$$

$$+ r^2 \operatorname{sen}^2\theta \cdot \frac{1}{r^2 \operatorname{sen}^2\theta} =$$

$$= 1 + 1 + 1 + 1 = 4$$

Propiedad del tensor metrico para bajar índices.

$$T^{\mu}_{\mu} = \frac{1}{4\pi}\left[F^{\mu\alpha} g_{\mu\nu} F^{\nu}{}_{\alpha} - g_{\mu\nu} g^{\mu\nu} \frac{1}{4} F^{\alpha\beta} F_{\alpha\beta}\right]$$

$$T^{\mu}_{\mu} = \frac{1}{4\pi}\left[F^{\mu\alpha} F_{\mu\alpha} - \cancel{\frac{4}{4}}^{\,1} F^{\alpha\beta} F_{\alpha\beta}\right]$$

$$T^{\mu}_{\mu} = \frac{1}{4\pi}\left[F^{\mu\alpha} F_{\mu\alpha} - F^{\mu\alpha} F_{\mu\alpha}\right] \rightarrow$$ Cambio de nombre a índices mudos

$$T^{\mu}_{\mu} = \frac{1}{4\pi}\left[0\right] \Rightarrow T^{\mu}_{\mu} = 0$$

Ejercicio #5

Teniendo que,

$$G^{\mu\nu} = 8\pi G T^{\mu\nu}$$

$$R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = 8\pi G T^{\mu\nu}$$

Aplicamos $g_{\mu\nu}$ para obtener el escalar,

$$g_{\mu\nu}\left(R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu}\right) = 8\pi G g_{\mu\nu} T^{\mu\nu}$$

$$g_{\mu\nu} R^{\mu\nu} - \frac{1}{2} R g_{\mu\nu} g^{\mu\nu} = 8\pi G g_{\mu\nu} T^{\mu\nu}$$

$$R - \frac{4}{2} R = 8\pi G T$$

$$R - 2R = 8\pi G T$$

$$R(1-2) = 8\pi G T$$

$$-R = 8\pi G T$$

$$R = -8\pi G T$$

Vemos que, dado que $T = g_{\mu\nu} T^{\mu\nu}$ y del ejercicio anterior encontramos que $T^{\mu}_{\mu} = g_{\mu\nu} T^{\mu\nu}$, concluimos que $T^{\mu}_{\mu} = T$, por lo tanto,

$$R = -8\pi G T^{\mu}_{\mu}$$

$$R = -8\pi G (0)$$

$$R = 0 \;//$$

Ejercicio #6

Si tenemos el elemento de línea $ds^2 = d\theta^2 + \text{sen}^2\theta\, d\varphi^2$ cuya métrica asociada es

$$g_{\alpha\beta} = \begin{bmatrix} 1 & 0 \\ 0 & \text{sen}^2\theta \end{bmatrix}$$

, y, además, tenemos que, $\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$

Operando obtendremos que,

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$$

Dado que, $\nabla_\mu \xi = \partial_\mu \xi_\nu - \Gamma^\alpha_{\mu\nu} \xi_\alpha$

$$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu - \Gamma^\alpha_{\mu\nu} \xi_\alpha = 0$$

Tendremos entonces que, con $\Gamma^\alpha_{\mu\nu} = \Gamma^\alpha_{\nu\mu}$

$$\partial_\theta \xi_\theta + \partial_\theta \xi_\theta - 2\Gamma^\alpha_{\theta\theta} \xi_\alpha = 0$$

$$\partial_\theta \xi_\varphi + \partial_\varphi \xi_\theta - 2\Gamma^\alpha_{\theta\varphi} \xi_\alpha = 0$$

$$\partial_\varphi \xi_\varphi + \partial_\varphi \xi_\varphi - 2\Gamma^\alpha_{\varphi\varphi} \xi_\alpha = 0$$

Del ejercicio #7 de este examen sabemos que,

$$\Gamma^\theta_{\varphi\varphi} = -\text{sen}\theta\cos\theta \quad , \Gamma^\varphi_{\theta\varphi} = \Gamma^\varphi_{\varphi\theta} = \frac{\cos\theta}{\text{sen}\theta}$$

Así pues, tenemos que,

$\partial_\theta \xi_\theta = 0$ Dado que Γ^θ_{ij} son cero tal como se muestra en el siguiente ejercicio.

$$\partial_\varphi \xi_\varphi + 2\Gamma^\varphi_{\theta\varphi} \xi_\varphi = \partial_\varphi \xi_\varphi + 2\cot\theta\, \xi_\varphi = 0$$

$$\partial_\theta \xi_\varphi + \partial_\varphi \xi_\theta - 2\Gamma^\theta_{\varphi\varphi} \xi^\varphi = \partial_\theta \xi_\varphi + \partial_\varphi \xi_\theta + 2\,\mathrm{sen}\,\theta\cos\theta\, \xi_\varphi = 0$$

Ejercicio #7

Con el elemento de línea $ds^2 = d\theta^2 + \text{sen}^2\theta\, d\varphi^2$ tendremos la métrica

$$g_{\alpha\beta} = \begin{bmatrix} 1 & 0 \\ 0 & \text{sen}^2\theta \end{bmatrix}, \text{ teniendo que } g^{\alpha\beta} = \begin{bmatrix} 1 & 0 \\ 0 & \text{sen}^{-2}\theta \end{bmatrix}$$

$$\Gamma^0_{\kappa\iota} = \frac{1}{2} g^{00} \frac{\partial g_{0\kappa}}{x^\iota} + \cancel{\frac{1}{2} g^{01} \frac{\partial g_{1\kappa}}{x^\iota}}^{\,0} + \frac{1}{2} g^{00} \frac{\partial g_{0\iota}}{x^\kappa} + \cancel{\frac{1}{2} g^{01} \frac{\partial g_{1\iota}}{x^\kappa}}^{\,0}$$
$$- \left(\frac{1}{2} g^{00} \frac{\partial g_{\kappa\iota}}{x^0} + \cancel{\frac{1}{2} g^{01} \frac{\partial g_{\kappa\iota}}{x^1}}^{\,0} \right)$$

$$\Gamma^0_{\kappa\iota} = \frac{1}{2} g^{00} \frac{\partial g_{0\kappa}}{x^\iota} + \frac{1}{2} g^{00} \frac{\partial g_{0\iota}}{x^\kappa} - \frac{1}{2} g^{00} \frac{\partial g_{\kappa\iota}}{x^0}$$

$$\Gamma^0_{00} = \frac{1}{2} g^{00} \frac{\partial g_{00}}{x^0} + \frac{1}{2} g^{00} \frac{\partial g_{00}}{x^0} - \frac{1}{2} g^{00} \frac{\partial g_{00}}{x^0} = 0$$

$$\Gamma^0_{01} = \frac{1}{2} g^{00} \frac{\partial g_{00}}{x^1} + \frac{1}{2} g^{00} \frac{\partial g_{01}}{x^0} - \frac{1}{2} g^{00} \frac{\partial g_{01}}{x^0} = 0$$

$$\Gamma^0_{10} = \frac{1}{2} g^{00} \frac{\partial g_{01}}{x^0} + \frac{1}{2} g^{00} \frac{\partial g_{00}}{x^1} - \frac{1}{2} g^{00} \frac{\partial g_{10}}{x^0} = 0$$

$$\Gamma^0_{11} = \cancel{\frac{1}{2} g^{00} \frac{\partial g_{01}}{x^1}}^{\,0} + \cancel{\frac{1}{2} g^{00} \frac{\partial g_{01}}{x^1}}^{\,0} - \frac{1}{2} g^{00} \frac{\partial g_{11}}{x^0} \neq 0$$

$$\Gamma^0_{11} = -\frac{1}{2} \cdot 1 \cdot 2\,\text{sen}\theta\cos\theta = -\text{sen}\theta\cos\theta$$

$$\Gamma^{1}_{\kappa\iota} = \cancelto{0}{\frac{1}{2} g^{10} \frac{\partial g_{0\kappa}}{x^{\iota}}} + \frac{1}{2} g^{11} \frac{\partial g_{1\kappa}}{x^{\iota}} + \cancelto{0}{\frac{1}{2} g^{10} \frac{\partial g_{0\iota}}{x^{\kappa}}} + \frac{1}{2} g^{11} \frac{\partial g_{1\iota}}{x^{\kappa}}$$

$$- \left(\cancelto{0}{\frac{1}{2} g^{10} \frac{\partial g_{\kappa\iota}}{x^{0}}} + \frac{1}{2} g^{11} \frac{\partial g_{\kappa\iota}}{x^{1}} \right)$$

$$\Gamma^{1}_{\kappa\iota} = \frac{1}{2} g^{11} \frac{\partial g_{1\kappa}}{x^{\iota}} + \frac{1}{2} g^{11} \frac{\partial g_{1\iota}}{x^{\kappa}} - \frac{1}{2} g^{11} \frac{\partial g_{\kappa\iota}}{x^{1}}$$

$$\Gamma^{1}_{00} = \frac{1}{2} g^{11} \frac{\partial g_{10}}{x^{0}} + \frac{1}{2} g^{11} \frac{\partial g_{10}}{x^{0}} - \frac{1}{2} g^{11} \frac{\partial g_{00}}{x^{1}} = 0$$

$$\Gamma^{1}_{01} = \frac{1}{2} g^{11} \frac{\partial g_{10}}{x^{1}} + \frac{1}{2} g^{11} \frac{\partial g_{11}}{x^{0}} - \frac{1}{2} g^{11} \frac{\partial g_{01}}{x^{1}} = \cos\theta / \operatorname{sen}\theta$$

$$\Gamma^{1}_{10} = \frac{1}{2} g^{11} \frac{\partial g_{11}}{x^{0}} + \frac{1}{2} g^{11} \frac{\partial g_{10}}{x^{1}} - \frac{1}{2} g^{11} \frac{\partial g_{10}}{x^{1}} = \cos\theta / \operatorname{sen}\theta$$

$$\Gamma^{1}_{11} = \frac{1}{2} g^{11} \frac{\partial g_{11}}{x^{1}} + \frac{1}{2} g^{11} \frac{\partial g_{11}}{x^{1}} - \frac{1}{2} g^{11} \frac{\partial g_{11}}{x^{1}} = 0$$

Ahora bien, teniendo que,

$$R^{\alpha}_{\beta\mu\nu} = \partial_{\mu} \Gamma^{\alpha}_{\beta\nu} - \partial_{\nu} \Gamma^{\alpha}_{\beta\mu} + \Gamma^{\alpha}_{\sigma\mu} \Gamma^{\sigma}_{\beta\nu} - \Gamma^{\alpha}_{\sigma\nu} \Gamma^{\sigma}_{\beta\mu}$$

Obtendríamos,

$$R^{\alpha}_{\beta\mu\nu} = \partial_{\mu} \Gamma^{\alpha}_{\beta\nu} - \partial_{\nu} \Gamma^{\alpha}_{\beta\mu} + \Gamma^{\alpha}_{0\mu} \Gamma^{0}_{\beta\nu} + \Gamma^{\alpha}_{1\mu} \Gamma^{1}_{\beta\nu} - \left(\Gamma^{\alpha}_{0\nu} \Gamma^{0}_{\beta\mu} + \Gamma^{\alpha}_{1\nu} \Gamma^{1}_{\beta\mu} \right)$$

$R^{\theta}{}_{\theta\theta\theta} = 0 = R^{0}{}_{000}$

$R^{\theta}{}_{\theta\theta\varphi} = 0 = R^{0}{}_{001}$

$R^{\theta}{}_{\theta\varphi\theta} = 0 = R^{0}{}_{010}$

$R^{\theta}{}_{\theta\varphi\varphi} = 0 = R^{0}{}_{011}$

$R^{\theta}{}_{\varphi\theta\theta} = 0 = R^{0}{}_{100}$

$R^{\theta}{}_{\varphi\theta\varphi} = \mathrm{sen}^2\theta = R^{0}{}_{101}$

$R^{\theta}{}_{\varphi\varphi\theta} = -\mathrm{sen}^2\theta = R^{0}{}_{110}$

$R^{\theta}{}_{\varphi\varphi\varphi} = 0 = R^{0}{}_{111}$

$R^{\varphi}{}_{\theta\theta\theta} = 0 = R^{1}{}_{000}$

$R^{\varphi}{}_{\theta\theta\varphi} = -1 = R^{1}{}_{001}$

$R^{\varphi}{}_{\theta\varphi\theta} = 1 = R^{1}{}_{010}$

$R^{\varphi}{}_{\theta\varphi\varphi} = 0 = R^{1}{}_{011}$

$R^{\varphi}{}_{\varphi\theta\theta} = 0 = R^{1}{}_{100}$

$R^{\varphi}{}_{\varphi\theta\varphi} = 0 = R^{1}{}_{101}$

$R^{\varphi}{}_{\varphi\varphi\theta} = 0 = R^{1}{}_{110}$

$R^{\varphi}{}_{\varphi\varphi\varphi} = 0 = R^{1}{}_{111}$

$R^{\theta}{}_{\varphi\theta\varphi} = \mathrm{sen}^2\theta = R^{0}{}_{101}$ $\qquad$ $R^{\varphi}{}_{\theta\theta\varphi} = -1 = R^{1}{}_{001}$

$R^{\theta}{}_{\varphi\varphi\theta} = -\mathrm{sen}^2\theta = R^{0}{}_{110}$ $\qquad$ $R^{\varphi}{}_{\theta\varphi\theta} = 1 = R^{1}{}_{010}$

Para calcular el tensor y el escalar de Ricci vamos a contraer el tensor de Riemann de la forma,

$$R_{\alpha\beta} = R^{\mu}{}_{\alpha\mu\beta}$$

$$R_{00} = R^{0}{}_{000} + R^{1}{}_{010} = 1$$

$$R_{01} = R^{0}{}_{001} + R^{1}{}_{011} = 0$$

$$R_{10} = R^{0}{}_{100} + R^{1}{}_{110} = 0$$

$$R_{11} = R^{0}{}_{101} + R^{1}{}_{111} = \mathrm{sen}^2\theta$$

$$\Rightarrow R_{\alpha\beta} = \begin{bmatrix} 1 & 0 \\ 0 & \mathrm{sen}^2\theta \end{bmatrix}$$

Contrayendo una vez más tenemos que,

$$R = g^{\mu\nu} R_{\mu\nu}$$

$$R = g^{0\nu} R_{0\nu} + g^{1\nu} R_{1\nu}$$

$$R = g^{00} R_{00} + g^{01} R_{01} + g^{10} R_{10} + g^{11} R_{11}$$

$$R = 1 \cdot 1 + 0 \cdot 0 + 0 \cdot 0 + \frac{1}{\operatorname{sen}^2\theta} \cdot \operatorname{sen}^2\theta$$

$$R = 1 + 1 = 2$$