TAREA 4 DE RG

20 de Noviembre/2020. Resuelva todos los ejercicios posibles. **Fecha de entrega: 2 de Diciembre/2020**.

1. Resuelva los siguientes ejercicios del capítulo 5 en el libro de Schutz. Los paréntesis cuentan 1 punto.

7 Calculate all elements of the transformation matrices $\Lambda^{\alpha'}{}_{\beta}$ and $\Lambda^{\mu}{}_{\nu'}$ for the transformation from Cartesian (x, y) – the unprimed indices – to polar (r, θ) – the primed indices.

8 (a) (Uses the result of Exer. 7.) Let $f = x^2 + y^2 + 2xy$, and in Cartesian coordinates $\vec{V} \to (x^2 + 3y, y^2 + 3x)$, $\vec{W} \to (1, 1)$. Compute f as a function of r and θ, and find the components of $\vec{V}$ and $\vec{W}$ on the polar basis, expressing them as functions of r and θ.

(b) Find the components of $\tilde{d}f$ in Cartesian coordinates and obtain them in polars (i) by direct calculation in polars, and (ii) by transforming components from Cartesian.

(c) (i) Use the metric tensor in polar coordinates to find the polar components of the one-forms $\tilde{V}$ and $\tilde{W}$ associated with $\vec{V}$ and $\vec{W}$. (ii) Obtain the polar components of $\tilde{V}$ and $\tilde{W}$ by transformation of their Cartesian components.

11 (Uses the result of Exers. 7 and 8.) For the vector field $\vec{V}$ whose Cartesian components are $(x^2 + 3y, y^2 + 3x)$, compute: (a) $V^{\alpha}{}_{,\beta}$ in Cartesian; (b) the transformation $\Lambda^{\mu'}{}_{\alpha}\Lambda^{\beta}{}_{\nu'}V^{\alpha}{}_{,\beta}$ to polars; (c) the components $V^{\mu'}{}_{;\nu'}$ directly in polars using the Christoffel symbols, Eq. (5.45), in Eq. (5.50); (d) the divergence $V^{\alpha}{}_{,\alpha}$ using your results in (a); (e) the divergence $V^{\mu'}{}_{;\mu'}$ using your results in either (b) or (c); (f) the divergence $V^{\mu'}{}_{;\mu'}$ using Eq. (5.56) directly.

12 For the one-form field $\tilde{p}$ whose Cartesian components are $(x^2 + 3y, y^2 + 3x)$, compute: (a) $p_{\alpha,\beta}$ in Cartesian; (b) the transformation $\Lambda^{\alpha}{}_{\mu'}\Lambda^{\beta}{}_{\nu'}\, p_{\alpha,\beta}$ to polars; (c) the components $p_{\mu';\nu'}$ directly in polars using the Christoffel symbols, Eq. (5.45), in Eq. (5.63).

14 For the tensor whose polar components are $(A^{rr} = r^2, A^{r\theta} = r \sin\theta, A^{\theta r} = r\cos\theta, A^{\theta\theta} = \tan\theta)$, compute Eq. (5.65) in polars for all possible indices.

15 For the vector whose polar components are $(V^r = 1, V^{\theta} = 0)$, compute in polars all components of the second covariant derivative $V^{\alpha}{}_{;\mu;\nu}$. (Hint: to find the second derivative, treat the first derivative $V^{\alpha}{}_{;\mu}$ as any $\binom{1}{1}$ tensor: Eq. (5.66).)