TAREA 6 DE RELATIVIDAD GENERAL
Fecha límite de entrega: POR DETERMINAR. Tarea INDIVIDUAL.

1. Nombre

29 In polar coordinates, calculate the Riemann curvature tensor of the sphere of unit radius, whose metric is given in Exer. 28. (Note that in two dimensions there is only *one* independent component, by the same arguments as in Exer. 18(b). So calculate $R_{\theta\phi\theta\phi}$ and obtain all other components in terms of it.)

32 A four-dimensional manifold has coordinates $(u,\ v,\ w,\ p)$ in which the metric has components $g_{uv} = g_{ww} = g_{pp} = 1$, all other independent components vanishing.

(a) Show that the manifold is flat and the signature is $+2$.

(b) The result in (a) implies the manifold must be Minkowski spacetime. Find a coordinate transformation to the usual coordinates $(t,\ x,\ y,\ z)$. (You may find it a useful hint to calculate $\vec{e}_v \cdot \vec{e}_v$ and $\vec{e}_u \cdot \vec{e}_u$.)

35 Compute 20 independent components of $R_{\alpha\beta\mu\nu}$ for a manifold with line element $\mathrm{d}s^2 = -e^{2\Phi}\,\mathrm{d}t^2 + e^{2\Lambda}\,\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$, where Φ and Λ are arbitrary functions of the coordinate r alone. (First, identify the coordinates and the components $g_{\alpha\beta}$; then compute $g^{\alpha\beta}$ and the Christoffel symbols. Then decide on the indices of the 20 components of $R_{\alpha\beta\mu\nu}$ you wish to calculate, and compute them. Remember that you can deduce the remaining 236 components from those 20.)

3 Calculate all the Christoffel symbols for the metric given by Eq. (7.8), to first order in ϕ. Assume ϕ is a general function of *t, x, y,* and *z*.

3 (a) Calculate in geometrized units:

(i) the Newtonian potential ϕ of the Sun at the Sun's surface, radius 6.960×10^8 m;

(ii) the Newtonian potential ϕ of the Sun at the radius of Earth's orbit, $r = 1\ \mathrm{AU} = 1.496 \times 10^{11}$ m;

(iii) the Newtonian potential ϕ of Earth at its surface, radius $= 6.371 \times 10^6$ m;

(iv) the velocity of Earth in its orbit around the Sun.

(b) You should have found that your answer to (ii) was larger than to (iii). Why, then, do we on Earth feel Earth's gravitational pull much more than the Sun's?

(c) Show that a circular orbit around a body of mass M has an orbital velocity, in Newtonian theory, of $\boldsymbol{v}^2 = -\phi$, where ϕ is the Newtonian potential.