18.787. Topics in Number Theory (Fall 2012)

• Instructor: Sug Woo Shin (swshin@mit.edu)

• Class hours: TR 1-2:30

Classroom: 2-151Office: 2-167

• Office hours: Mon 10-11, Thu 2:30-3:30, or by appointment

Homework: None Exams: None

• Grade: Undergrads will be asked to write term papers to receive grades.

What to expect The following are sample topics to be covered. Certainly it's too much - I will select some of them as we go along.

- Basics on group schemes, abelian varieties, p-divisible groups
- CM abelian varieties
- Dieudonné theory (classification of p-div. groups over $\overline{\mathbb{F}}_p$ up to isogeny)
- Honda-Tate theory (classification of abel. var. over $\overline{\mathbb{F}}_p$ up to isogeny)
- Serre-Tate theorem/theory on deformations of p-div. groups and abel. var.
- Grothendieck-Messing theory on deformations of p-div. groups
- Galois representations associated with abelian varieties
- Moduli spaces of abelian varieties

Some useful references

- Mumford, Abelian Varieties, TIFR (reasonably priced in the online AMS bookstore)
- Bosch, Lütkebohmert and Raynaud, Néron Models, Ergebnisse der Math. 21, Springer-Verlag (Find at http://math.arizona.edu/~cais/scans/.)
- Messing, The Crystals Associated to Barsotti-Tate Groups: with Applications to Abelian Schemes, LNM 264, Springer-Verlag
- Shatz, Group Schemes, Formal Groups, and p-Divisible Groups (in "Arithmetic Geometry", Springer)
- Tate, Finite Flat Group Schemes (in "Modular Forms and Fermat's Last Theorem", Springer)
- Tate, p-divisible Groups (in "Local Fields" proceeding, Springer)
- Grothendieck, Groupe de Barsotti-Tate et Cristaux de Dieudonné (Find at http://www.math.jussieu.fr/~leila/grothendieckcircle/barsottitate.pdf.)

Prerequisite

• Reasonable knowledge of general algebra, algebraic number theory, and scheme theory would be needed. It would help if you are familiar with elliptic curves.

Other remarks

• This course is designed as a graduate course. An undergraduate with proper background knowledge is welcome to attend my class but keep in mind that you may be required to invest a lot to follow the course.