LU Decomposition, Partial pivoting

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1.0001 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

$$k = 1 \qquad\qquad p(1) = 1$$

$$P_{\text{new}}A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P_{\text{new}} \begin{pmatrix} \boxed{1} & 1 & 1 \\ 1 & 1.0001 & 2 \\ 1 & 2 & 2 \end{pmatrix} \qquad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1.0001 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

$$p = \begin{Bmatrix} 1 & 2 & 3 \end{Bmatrix} \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$p_{\text{new}} = \text{swap}_{1,1} \circ \begin{Bmatrix} 1 & 2 & 3 \end{Bmatrix} \qquad P_{\text{new}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{Bmatrix} 1 & 2 & 3 \end{Bmatrix} \qquad = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$k = 1 \qquad i = 2 \qquad p(1) = 1 \qquad p(2) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} \boxed{1} & 1 & 1 \\ \mathbf{1} & \mathbf{1.0001} & \mathbf{2} \\ 1 & 2 & 2 \end{pmatrix}} \qquad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0.0001 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

$$p = \begin{Bmatrix} 1 & 2 & 3 \end{Bmatrix} \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{p(2)} \leftarrow\!\shortmid R_{p(2)} - \frac{1}{1} R_{p(1)}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{1} & \mathbf{1.0001} & \mathbf{2} \\ 1 & 2 & 2 \end{pmatrix}} = P \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0.0001} & \mathbf{1} \\ 1 & 2 & 2 \end{pmatrix}$$

$$k = 1 \qquad i = 2 \qquad p(1) = 1 \qquad p(2) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} \boxed{1} & 1 & 1 \\ \mathbf{1} & \mathbf{1.0001} & \mathbf{2} \\ 1 & 2 & 2 \end{pmatrix}} \qquad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0.0001 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

$$p = \begin{Bmatrix} 1 & 2 & 3 \end{Bmatrix} \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{p(2)} \mapsto R_{p(2)} - \frac{1}{1} R_{p(1)}$$

$$P \boxed{\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{1} & \mathbf{1.0001} & \mathbf{2} \\ 1 & 2 & 2 \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0.0001} & \mathbf{1} \\ 1 & 2 & 2 \end{pmatrix}$$

$$k = 1 \qquad i = 2 \qquad p(1) = 1 \qquad p(2) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} \boxed{1} & 1 & 1 \\ \mathbf{0} & \mathbf{0.0001} & \mathbf{1} \\ 1 & 2 & 2 \end{pmatrix} \qquad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0.0001 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

$$p = \begin{Bmatrix} 1 & 2 & 3 \end{Bmatrix} \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$k = 1 \qquad i = 3 \qquad p(1) = 1 \qquad p(3) = 3$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} \boxed{1} & 1 & 1 \\ 0 & 0.0001 & 1 \\ \mathbf{1} & \mathbf{2} & \mathbf{2} \end{pmatrix}} \qquad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0.0001 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$p = \begin{Bmatrix} 1 & 2 & 3 \end{Bmatrix} \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{p(3)} \leftarrow\!\shortmid R_{p(3)} - \frac{1}{1} R_{p(1)}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0.0001 & 1 \\ \mathbf{1} & \mathbf{2} & \mathbf{2} \end{pmatrix}} = P \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0.0001 & 1 \\ \mathbf{0} & \mathbf{1} & \mathbf{1} \end{pmatrix}$$

$$k = 1 \qquad i = 3 \qquad p(1) = 1 \qquad p(3) = 3$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} \boxed{1} & 1 & 1 \\ 0 & 0.0001 & 1 \\ \mathbf{1} & \mathbf{2} & \mathbf{2} \end{pmatrix}} \qquad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0.0001 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$p = \begin{Bmatrix} 1 & 2 & 3 \end{Bmatrix} \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{p(3)} \leftarrow\!\shortmid\, R_{p(3)} - \frac{1}{1} R_{p(1)}$$

$$P \boxed{\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0.0001 & 1 \\ \mathbf{1} & \mathbf{2} & \mathbf{2} \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0.0001 & 1 \\ \mathbf{0} & \mathbf{1} & \mathbf{1} \end{pmatrix}$$

$$k = 1 \qquad i = 3 \qquad p(1) = 1 \qquad p(3) = 3$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} P \begin{pmatrix} \boxed{1} & 1 & 1 \\ 0 & 0.0001 & 1 \\ \mathbf{0} & \mathbf{1} & \mathbf{1} \end{pmatrix} \qquad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0.0001 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$p = \begin{Bmatrix} 1 & 2 & 3 \end{Bmatrix} \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$k = 2 \qquad\qquad p(3) = 3$$

$$P_{\text{new}} A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} P_{\text{new}} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0.0001 & 1 \\ 0 & \boxed{1} & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0.0001 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$p = \begin{Bmatrix} 1 & 2 & 3 \end{Bmatrix} \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$p_{\text{new}} = \text{swap}_{2,3} \circ \begin{Bmatrix} 1 & 2 & 3 \end{Bmatrix} \qquad P_{\text{new}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{Bmatrix} 1 & 3 & 2 \end{Bmatrix} \qquad = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$k = 2 \qquad i = 3 \qquad p(2) = 3 \qquad p(3) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0.0001} & \mathbf{1} \\ 0 & \boxed{1} & 1 \end{pmatrix}} \qquad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0.0001 & 0.9999 \\ 1 & 1 & 1 \end{pmatrix}$$

$$p = \begin{Bmatrix} 1 & 3 & 2 \end{Bmatrix} \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$R_{p(3)} \leftarrow\!\shortmid\; R_{p(3)} - \frac{0.0001}{1} R_{p(2)}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -0.0001 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0.0001} & \mathbf{1} \\ 0 & 1 & 1 \end{pmatrix}} = P \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0.9999} \\ 0 & 1 & 1 \end{pmatrix}$$

$$k = 2 \qquad i = 3 \qquad p(2) = 3 \qquad p(3) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0.0001} & \mathbf{1} \\ 0 & \boxed{1} & 1 \end{pmatrix}} \qquad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0.0001 & 0.9999 \\ 1 & 1 & 1 \end{pmatrix}$$

$$p = \begin{Bmatrix} 1 & 3 & 2 \end{Bmatrix} \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$R_{p(3)} \mapsto R_{p(3)} - \frac{0.0001}{1} R_{p(2)}$$

$$P \boxed{\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0.0001} & \mathbf{1} \\ 0 & 1 & 1 \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.0001 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0.9999} \\ 0 & 1 & 1 \end{pmatrix}$$

$$k = 2 \qquad i = 3 \qquad p(2) = 3 \qquad p(3) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0.0001 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0.9999} \\ 0 & \boxed{1} & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0.0001 & 0.9999 \\ 1 & 1 & 1 \end{pmatrix}$$

$$p = \begin{Bmatrix} 1 & 3 & 2 \end{Bmatrix} \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0.0001 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0.9999 \\ 0 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0.0001 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0.9999 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$