LU Decomposition, Partial pivoting

$$A = \begin{pmatrix} 1 & 5 & 0 \\ 4 & 8 & 12 \\ 2 & 8 & 10 \end{pmatrix}$$

$$k = 1 \qquad\qquad p(2) = 2$$

$$P_{\text{new}} A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P_{\text{new}} \begin{pmatrix} 1 & 5 & 0 \\ \boxed{4} & 8 & 12 \\ 2 & 8 & 10 \end{pmatrix} \qquad \begin{pmatrix} 1 & 5 & 0 \\ 4 & 8 & 12 \\ 2 & 8 & 10 \end{pmatrix}$$

$$p = \begin{Bmatrix} 1 & 2 & 3 \end{Bmatrix} \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$p_{\text{new}} = \text{swap}_{1,2} \circ \begin{Bmatrix} 1 & 2 & 3 \end{Bmatrix} \qquad P_{\text{new}} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{Bmatrix} 2 & 1 & 3 \end{Bmatrix} \qquad = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$k = 1 \qquad i = 2 \qquad p(1) = 2 \qquad p(2) = 1$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} \mathbf{1} & \mathbf{5} & \mathbf{0} \\ \boxed{4} & 8 & 12 \\ 2 & 8 & 10 \end{pmatrix}} \qquad \begin{pmatrix} 0.25 & 3 & -3 \\ 4 & 8 & 12 \\ 2 & 8 & 10 \end{pmatrix}$$

$$p = \begin{Bmatrix} 2 & 1 & 3 \end{Bmatrix} \qquad P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{p(2)} \leftarrow R_{p(2)} - \frac{1}{4} R_{p(1)}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ -0.25 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} \mathbf{1} & \mathbf{5} & \mathbf{0} \\ 4 & 8 & 12 \\ 2 & 8 & 10 \end{pmatrix}} = P \begin{pmatrix} \mathbf{0} & \mathbf{3} & \mathbf{-3} \\ 4 & 8 & 12 \\ 2 & 8 & 10 \end{pmatrix}$$

$$k = 1 \qquad i = 2 \qquad p(1) = 2 \qquad p(2) = 1$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} \mathbf{1} & \mathbf{5} & \mathbf{0} \\ \boxed{4} & 8 & 12 \\ 2 & 8 & 10 \end{pmatrix}} \qquad \begin{pmatrix} 0.25 & 3 & -3 \\ 4 & 8 & 12 \\ 2 & 8 & 10 \end{pmatrix}$$

$$p = \begin{Bmatrix} 2 & 1 & 3 \end{Bmatrix} \qquad P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{p(2)} \leftarrow R_{p(2)} - \frac{1}{4} R_{p(1)}$$

$$P \boxed{\begin{pmatrix} \mathbf{1} & \mathbf{5} & \mathbf{0} \\ 4 & 8 & 12 \\ 2 & 8 & 10 \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} \mathbf{0} & \mathbf{3} & \mathbf{-3} \\ 4 & 8 & 12 \\ 2 & 8 & 10 \end{pmatrix}$$

$$k = 1 \qquad i = 2 \qquad p(1) = 2 \qquad p(2) = 1$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} \mathbf{0} & \mathbf{3} & \mathbf{-3} \\ \boxed{4} & 8 & 12 \\ 2 & 8 & 10 \end{pmatrix} \qquad \begin{pmatrix} 0.25 & 3 & -3 \\ 4 & 8 & 12 \\ 2 & 8 & 10 \end{pmatrix}$$

$$p = \begin{Bmatrix} 2 & 1 & 3 \end{Bmatrix} \qquad P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$k = 1 \qquad i = 3 \qquad p(1) = 2 \qquad p(3) = 3$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} 0 & 3 & -3 \\ \boxed{4} & 8 & 12 \\ \mathbf{2} & \mathbf{8} & \mathbf{10} \end{pmatrix}} \qquad \begin{pmatrix} 0.25 & 3 & -3 \\ 4 & 8 & 12 \\ 0.5 & 4 & 4 \end{pmatrix}$$

$$p = \begin{Bmatrix} 2 & 1 & 3 \end{Bmatrix} \qquad P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{p(3)} \leftarrow\!\shortmid R_{p(3)} - \frac{2}{4} R_{p(1)}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -0.5 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} 0 & 3 & -3 \\ 4 & 8 & 12 \\ \mathbf{2} & \mathbf{8} & \mathbf{10} \end{pmatrix}} = P \begin{pmatrix} 0 & 3 & -3 \\ 4 & 8 & 12 \\ \mathbf{0} & \mathbf{4} & \mathbf{4} \end{pmatrix}$$

$$k = 1 \qquad i = 3 \qquad p(1) = 2 \qquad p(3) = 3$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} 0 & 3 & -3 \\ \boxed{4} & 8 & 12 \\ \mathbf{2} & \mathbf{8} & \mathbf{10} \end{pmatrix}} \qquad \begin{pmatrix} 0.25 & 3 & -3 \\ 4 & 8 & 12 \\ 0.5 & 4 & 4 \end{pmatrix}$$

$$p = \begin{Bmatrix} 2 & 1 & 3 \end{Bmatrix} \qquad P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{p(3)} \leftarrow\!\shortmid R_{p(3)} - \frac{2}{4} R_{p(1)}$$

$$P \boxed{\begin{pmatrix} 0 & 3 & -3 \\ 4 & 8 & 12 \\ \mathbf{2} & \mathbf{8} & \mathbf{10} \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.5 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 0 & 3 & -3 \\ 4 & 8 & 12 \\ \mathbf{0} & \mathbf{4} & \mathbf{4} \end{pmatrix}$$

$$k = 1 \qquad i = 3 \qquad p(1) = 2 \qquad p(3) = 3$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0.25 & 1 & 0 \\ 0.5 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 0 & 3 & -3 \\ \boxed{4} & 8 & 12 \\ \mathbf{0} & \mathbf{4} & \mathbf{4} \end{pmatrix} \qquad \begin{pmatrix} 0.25 & 3 & -3 \\ 4 & 8 & 12 \\ 0.5 & 4 & 4 \end{pmatrix}$$

$$p = \begin{Bmatrix} 2 & 1 & 3 \end{Bmatrix} \qquad P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$k = 2 \qquad\qquad p(3) = 3$$

$$P_{\text{new}}A = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.25 & 0 & 1 \end{pmatrix} P_{\text{new}} \begin{pmatrix} 0 & 3 & -3 \\ 4 & 8 & 12 \\ 0 & \boxed{4} & 4 \end{pmatrix} \qquad \begin{pmatrix} 0.25 & 3 & -3 \\ 4 & 8 & 12 \\ 0.5 & 4 & 4 \end{pmatrix}$$

$$p = \begin{Bmatrix} 2 & 1 & 3 \end{Bmatrix} \qquad P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$p_{\text{new}} = \text{swap}_{2,3} \circ \begin{Bmatrix} 2 & 1 & 3 \end{Bmatrix} \qquad P_{\text{new}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{Bmatrix} 2 & 3 & 1 \end{Bmatrix} \qquad = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$k = 2 \qquad i = 3 \qquad p(2) = 3 \qquad p(3) = 1$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.25 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} \mathbf{0} & \mathbf{3} & \mathbf{-3} \\ 4 & 8 & 12 \\ 0 & \boxed{4} & 4 \end{pmatrix}} \qquad \begin{pmatrix} 0.25 & 0.75 & -6 \\ 4 & 8 & 12 \\ 0.5 & 4 & 4 \end{pmatrix}$$

$$p = \begin{Bmatrix} 2 & 3 & 1 \end{Bmatrix} \qquad P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$R_{p(3)} \leftarrow\!\shortmid R_{p(3)} - \frac{3}{4} R_{p(2)}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -0.75 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} \mathbf{0} & \mathbf{3} & \mathbf{-3} \\ 4 & 8 & 12 \\ 0 & 4 & 4 \end{pmatrix}} = P \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{-6} \\ 4 & 8 & 12 \\ 0 & 4 & 4 \end{pmatrix}$$

$$k = 2 \qquad i = 3 \qquad p(2) = 3 \qquad p(3) = 1$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.25 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} \mathbf{0} & \mathbf{3} & \mathbf{-3} \\ 4 & 8 & 12 \\ 0 & \boxed{4} & 4 \end{pmatrix}} \qquad \begin{pmatrix} 0.25 & 0.75 & -6 \\ 4 & 8 & 12 \\ 0.5 & 4 & 4 \end{pmatrix}$$

$$p = \begin{Bmatrix} 2 & 3 & 1 \end{Bmatrix} \qquad P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$R_{p(3)} \leftarrow\!\shortmid R_{p(3)} - \frac{3}{4} R_{p(2)}$$

$$P \boxed{\begin{pmatrix} \mathbf{0} & \mathbf{3} & \mathbf{-3} \\ 4 & 8 & 12 \\ 0 & 4 & 4 \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.75 & 1 \end{pmatrix} P \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{-6} \\ 4 & 8 & 12 \\ 0 & 4 & 4 \end{pmatrix}$$

$$k = 2 \qquad i = 3 \qquad p(2) = 3 \qquad p(3) = 1$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.25 & 0.75 & 1 \end{pmatrix} P \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{-6} \\ 4 & 8 & 12 \\ 0 & \boxed{4} & 4 \end{pmatrix} \qquad \begin{pmatrix} 0.25 & 0.75 & -6 \\ 4 & 8 & 12 \\ 0.5 & 4 & 4 \end{pmatrix}$$

$$p = \begin{Bmatrix} 2 & 3 & 1 \end{Bmatrix} \qquad P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.25 & 0.75 & 1 \end{pmatrix} P \begin{pmatrix} 0 & 0 & -6 \\ 4 & 8 & 12 \\ 0 & 4 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.25 & 0.75 & 1 \end{pmatrix} \begin{pmatrix} 4 & 8 & 12 \\ 0 & 4 & 4 \\ 0 & 0 & -6 \end{pmatrix}$$

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$