LU Decomposition, Partial pivoting

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 1 & 0 & 1 \\ -2 & 2 & 4 \end{pmatrix}$$

$$k=1 p(3)=1$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 2 & 4 \\ 1 & 0 & 1 \\ \hline -2 & 2 & 4 \end{pmatrix}$$

$$p = \begin{cases} 1 & 2 & 3 \end{cases} \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$p_{\text{new}} = \text{swap}_{1,3} \circ \begin{cases} 1 & 2 & 3 \end{cases} \qquad P_{\text{new}} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{cases} 3 & 2 & 1 \end{cases} \qquad = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$k = 1$$
 $i = 2$ $p(1) = 3$ $p(2) = 2$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} 1 & 2 & 4 \\ \mathbf{1} & \mathbf{0} & \mathbf{1} \\ -\mathbf{2} & 2 & 4 \end{pmatrix}}$$

$$p = \left\{ 3 \quad 2 \quad 1 \right\} \qquad \qquad P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$R_{p(2)} \leftarrow R_{p(2)} - \frac{1}{-2} R_{p(1)}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{vmatrix} 1 & 2 & 4 \\ 1 & 0 & 1 \\ -2 & 2 & 4 \end{vmatrix} = P \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 3 \\ -2 & 2 & 4 \end{pmatrix}$$

$$k = 1$$
 $i = 2$ $p(1) = 3$ $p(2) = 2$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{bmatrix} 1 & 2 & 4 \\ \mathbf{1} & \mathbf{0} & \mathbf{1} \\ -\mathbf{2} & 2 & 4 \end{bmatrix}$$

$$p = \left\{ 3 \quad 2 \quad 1 \right\} \qquad \qquad P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$R_{p(2)} \leftarrow R_{p(2)} - \frac{1}{-2} R_{p(1)}$$

$$P \left| \begin{pmatrix} 1 & 2 & 4 \\ \mathbf{1} & \mathbf{0} & \mathbf{1} \\ -2 & 2 & 4 \end{pmatrix} \right| = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 2 & 4 \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \\ -2 & 2 & 4 \end{pmatrix}$$

A = LU

$$k = 1 i = 2 p(1) = 3 p(2) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 2 & 4 \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \\ -2 & 2 & 4 \end{pmatrix}$$

$$p = \begin{cases} 3 & 2 & 1 \end{cases} P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

A = LU

$$k = 1$$
 $i = 3$ $p(1) = 3$ $p(3) = 1$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{4} \\ 0 & 1 & 3 \\ \hline -\mathbf{2} & 2 & 4 \end{pmatrix}$$

$$p = \left\{ 3 \quad 2 \quad 1 \right\} \qquad \qquad P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$R_{p(3)} \leftarrow R_{p(3)} - \frac{1}{-2} R_{p(1)}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.5 & 0 & 1 \end{pmatrix} P \overline{\begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{4} \\ 0 & 1 & 3 \\ -2 & 2 & 4 \end{pmatrix}} = P \begin{pmatrix} \mathbf{0} & \mathbf{3} & \mathbf{6} \\ 0 & 1 & 3 \\ -2 & 2 & 4 \end{pmatrix}$$

$$k = 1$$
 $i = 3$ $p(1) = 3$ $p(3) = 1$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{4} \\ 0 & 1 & 3 \\ \hline -\mathbf{2} & 2 & 4 \end{pmatrix}$$

$$p = \left\{ 3 \quad 2 \quad 1 \right\} \qquad P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$R_{p(3)} \leftarrow R_{p(3)} - \frac{1}{-2} R_{p(1)}$$

$$k = 1 i = 3 p(1) = 3 p(3) = 1$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -0.5 & 0 & 1 \end{pmatrix} P \begin{pmatrix} \mathbf{0} & \mathbf{3} & \mathbf{6} \\ 0 & 1 & 3 \\ \hline -2 & 2 & 4 \end{pmatrix}$$

$$p = \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$k = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -0.5 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 0 & \boxed{3} & 6 \\ 0 & 1 & 3 \\ -2 & 2 & 4 \end{pmatrix}$$

$$p = \begin{cases} 3 & 2 & 1 \end{cases}$$

$$p_{\text{new}} = \text{swap}_{2,3} \circ \begin{cases} 3 & 2 & 1 \end{cases}$$

$$= \begin{cases} 3 & 1 & 2 \end{cases}$$

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$P_{\text{new}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

p(3) = 2

$$k = 2$$
 $i = 3$ $p(2) = 1$ $p(3) = 2$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -0.5 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} 0 & \boxed{3} & 6 \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \\ -2 & 2 & 4 \end{pmatrix}}$$

$$p = \begin{cases} 3 & 1 & 2 \end{cases} \qquad P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$R_{p(3)} \leftarrow R_{p(3)} - \frac{1}{3}R_{p(2)}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -0.333333 & 1 \end{pmatrix} P \begin{vmatrix} 0 & 3 & 6 \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \\ -2 & 2 & 4 \end{vmatrix} = P \begin{pmatrix} 0 & 3 & 6 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ -2 & 2 & 4 \end{vmatrix}$$

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$$k = 2$$
 $i = 3$ $p(2) = 1$ $p(3) = 2$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -0.5 & 0 & 1 \end{pmatrix} P \boxed{\begin{pmatrix} 0 & \boxed{3} & 6 \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \\ -2 & 2 & 4 \end{pmatrix}}$$

$$p = \left\{ 3 \ 1 \ 2 \right\} \qquad P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$R_{p(3)} \leftarrow R_{p(3)} - \frac{1}{3} R_{p(2)}$$

$$P \left[\begin{pmatrix} 0 & 3 & 6 \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \\ -2 & 2 & 4 \end{pmatrix} \right] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \mathbf{0.3333333} & 1 \end{pmatrix} P \begin{pmatrix} 0 & 3 & 6 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ -2 & 2 & 4 \end{pmatrix}$$

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$$k = 2 i = 3 p(2) = 1 p(3) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -0.5 & 0.333333 & 1 \end{pmatrix} P \begin{pmatrix} 0 & \boxed{3} & 6 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ -2 & 2 & 4 \end{pmatrix}$$

$$p = \begin{pmatrix} 3 & 1 & 2 \end{pmatrix} P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -0.5 & 0.333333 & 1 \end{pmatrix} P \begin{pmatrix} 0 & 3 & 6 \\ 0 & 0 & 1 \\ -2 & 2 & 4 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -0.5 & 0.333333 & 1 \end{pmatrix} \begin{pmatrix} -2 & 2 & 4 \\ 0 & 3 & 6 \\ 0 & 0 & 1 \end{pmatrix}$$
$$P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$