

LU Decomposition, No pivoting

$$A = \begin{pmatrix} 2 & -1 & -1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{pmatrix}$$

$$k = 1$$

$$i = 2$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \left[\begin{pmatrix} \boxed{2} & -1 & -1 \\ \color{blue}{3} & \mathbf{3} & \mathbf{9} \\ 3 & 3 & 5 \end{pmatrix} \right] \quad \begin{pmatrix} 2 & -1 & -1 \\ \color{red}{1.5} & 4.5 & 10.5 \\ 3 & 3 & 5 \end{pmatrix}$$

$$R_2 \leftarrow R_2 - \frac{\color{blue}{3}}{\color{red}{2}} R_1$$

$$\begin{array}{r} \begin{pmatrix} \mathbf{3} & \mathbf{3} & \mathbf{9} \end{pmatrix} \\ -\frac{\color{blue}{3}}{\color{red}{2}} \begin{pmatrix} 2 & -1 & -1 \end{pmatrix} \\ \hline \begin{pmatrix} \mathbf{0} & \mathbf{4.5} & \mathbf{10.5} \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ \color{red}{-1.5} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \left[\begin{pmatrix} 2 & -1 & -1 \\ \mathbf{3} & \mathbf{3} & \mathbf{9} \\ 3 & 3 & 5 \end{pmatrix} \right] = \begin{pmatrix} 2 & -1 & -1 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{10.5} \\ 3 & 3 & 5 \end{pmatrix}$$

$$k = 1$$

$$i = 2$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \left[\begin{pmatrix} \boxed{2} & -1 & -1 \\ \textcolor{blue}{3} & \mathbf{3} & \mathbf{9} \\ 3 & 3 & 5 \end{pmatrix} \right] \begin{pmatrix} 2 & -1 & -1 \\ \textcolor{red}{1.5} & 4.5 & 10.5 \\ 3 & 3 & 5 \end{pmatrix}$$

$$R_2 \leftarrow R_2 - \frac{\textcolor{blue}{3}}{\textcolor{red}{2}} R_1$$

$$\begin{array}{r} \begin{pmatrix} \mathbf{3} & \mathbf{3} & \mathbf{9} \end{pmatrix} \\ -\frac{\textcolor{blue}{3}}{\textcolor{red}{2}} \begin{pmatrix} 2 & -1 & -1 \end{pmatrix} \\ \hline \begin{pmatrix} \mathbf{0} & \mathbf{4.5} & \mathbf{10.5} \end{pmatrix} \end{array}$$

$$\boxed{\begin{pmatrix} 2 & -1 & -1 \\ \mathbf{3} & \mathbf{3} & \mathbf{9} \\ 3 & 3 & 5 \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ \textcolor{red}{1.5} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{10.5} \\ 3 & 3 & 5 \end{pmatrix}$$

$$k = 1$$

$$i = 2$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ \color{red}{1.5} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \boxed{2} & -1 & -1 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{10.5} \\ 3 & 3 & 5 \end{pmatrix} \quad \begin{pmatrix} 2 & -1 & -1 \\ \color{red}{1.5} & 4.5 & 10.5 \\ 3 & 3 & 5 \end{pmatrix}$$

$$k = 1$$

$$i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \color{red}{2} & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \color{blue}{3} & \mathbf{3} & \mathbf{5} \end{pmatrix}} \quad \begin{pmatrix} 2 & -1 & -1 \\ \color{red}{1.5} & 4.5 & 10.5 \\ \color{red}{1.5} & 4.5 & 6.5 \end{pmatrix}$$

$$R_3 \leftarrow R_3 - \frac{\color{blue}{3}}{\color{red}{2}} R_1$$

$$\begin{array}{r} \begin{pmatrix} \mathbf{3} & \mathbf{3} & \mathbf{5} \end{pmatrix} \\ -\frac{\color{blue}{3}}{\color{red}{2}} \begin{pmatrix} 2 & -1 & -1 \end{pmatrix} \\ \hline \begin{pmatrix} \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \color{red}{-1.5} & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{3} & \mathbf{3} & \mathbf{5} \end{pmatrix}} = \begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix}$$

$$k = 1$$

$$i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \color{red}{2} & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \color{blue}{3} & \mathbf{3} & \mathbf{5} \end{pmatrix}} \quad \begin{pmatrix} 2 & -1 & -1 \\ \color{red}{1.5} & 4.5 & 10.5 \\ \color{red}{1.5} & 4.5 & 6.5 \end{pmatrix}$$

$$R_3 \leftarrow R_3 - \frac{\color{blue}{3}}{\color{red}{2}} R_1$$

$$\begin{array}{r} \begin{pmatrix} \mathbf{3} & \mathbf{3} & \mathbf{5} \end{pmatrix} \\ -\frac{\color{blue}{3}}{\color{red}{2}} \begin{pmatrix} 2 & -1 & -1 \end{pmatrix} \\ \hline \begin{pmatrix} \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix} \end{array}$$

$$\boxed{\begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{3} & \mathbf{3} & \mathbf{5} \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \color{red}{1.5} & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix}$$

$$k = 1$$

$$i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ \textcolor{red}{1.5} & 0 & 1 \end{pmatrix} \begin{pmatrix} \boxed{2} & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix} \quad \begin{pmatrix} 2 & -1 & -1 \\ \textcolor{red}{1.5} & 4.5 & 10.5 \\ \textcolor{red}{1.5} & 4.5 & 6.5 \end{pmatrix}$$

$$k = 2$$

$$i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 1.5 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 2 & -1 & -1 \\ 0 & \textcolor{red}{4.5} & 10.5 \\ \mathbf{0} & \textcolor{blue}{4.5} & \mathbf{6.5} \end{pmatrix}} \quad \begin{pmatrix} 2 & -1 & -1 \\ \textcolor{red}{1.5} & 4.5 & 10.5 \\ \textcolor{red}{1.5} & \textcolor{red}{1} & -4 \end{pmatrix}$$

$$R_3 \leftarrow R_3 - \frac{\textcolor{blue}{4.5}}{\textcolor{red}{4.5}} R_2$$

$$\begin{array}{r} \begin{pmatrix} \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix} \\ -\frac{\textcolor{blue}{4.5}}{\textcolor{red}{4.5}} \begin{pmatrix} 0 & 4.5 & 10.5 \end{pmatrix} \\ \hline \begin{pmatrix} \mathbf{0} & \mathbf{0} & -4 \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \textcolor{red}{-1} & 1 \end{pmatrix} \boxed{\begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix}} = \begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{0} & \mathbf{0} & -4 \end{pmatrix}$$

$$k = 2$$

$$i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 1.5 & 0 & 1 \end{pmatrix} \left[\begin{pmatrix} 2 & -1 & -1 \\ 0 & \boxed{4.5} & 10.5 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix} \right] \begin{pmatrix} 2 & -1 & -1 \\ \mathbf{1.5} & 4.5 & 10.5 \\ \mathbf{1.5} & \mathbf{1} & -4 \end{pmatrix}$$

$$R_3 \leftarrow R_3 - \frac{4.5}{4.5} R_2$$

$$\begin{array}{r} \begin{pmatrix} \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix} \\ - \frac{4.5}{4.5} \begin{pmatrix} 0 & 4.5 & 10.5 \end{pmatrix} \\ \hline \begin{pmatrix} \mathbf{0} & \mathbf{0} & -4 \end{pmatrix} \end{array}$$

$$\left[\begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix} \right] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \mathbf{1} & \mathbf{1} \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{0} & \mathbf{0} & -4 \end{pmatrix}$$

$$k = 2$$

$$i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 1.5 & \mathbf{1} & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ 0 & \boxed{4.5} & 10.5 \\ \mathbf{0} & \mathbf{0} & -\mathbf{4} \end{pmatrix} \quad \begin{pmatrix} 2 & -1 & -1 \\ \mathbf{1.5} & 4.5 & 10.5 \\ \mathbf{1.5} & \mathbf{1} & -4 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 1.5 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ 0 & 0 & -4 \end{pmatrix}$$