

LU Decomposition, Partial pivoting

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1.0001 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

$$k = 1$$

$$p(1) = 1$$

$$P_{\text{new}}A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P_{\text{new}} \begin{pmatrix} \boxed{1} & 1 & 1 \\ 1 & 1.0001 & 2 \\ 1 & 2 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1.0001 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

$$p = \{1 \ 2 \ 3\}$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$p_{\text{new}} = \text{swap}_{1,1} \circ \{1 \ 2 \ 3\}$$

$$P_{\text{new}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \{1 \ 2 \ 3\}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$k = 1$$

$$i = 2$$

$$p(1) = 1$$

$$p(2) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} \boxed{1} & 1 & 1 \\ \textcolor{blue}{1} & \mathbf{1.0001} & \mathbf{2} \\ 1 & 2 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 1 \\ \textcolor{red}{1} & 0.0001 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

$$p = \{1 \quad 2 \quad 3\}$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{p(2)} \leftarrow R_{p(2)} - \frac{1}{\textcolor{red}{1}} R_{p(1)}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ \textcolor{red}{-1} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{1} & \mathbf{1.0001} & \mathbf{2} \\ 1 & 2 & 2 \end{pmatrix} = P \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0.0001} & \mathbf{1} \\ 1 & 2 & 2 \end{pmatrix}$$

$$k = 1$$

$$i = 2$$

$$p(1) = 1$$

$$p(2) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} \boxed{1} & 1 & 1 \\ \textcolor{blue}{1} & \mathbf{1.0001} & \mathbf{2} \\ 1 & 2 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 1 \\ \textcolor{red}{1} & 0.0001 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

$$p = \{1 \ 2 \ 3\}$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{p(2)} \leftarrow R_{p(2)} - \frac{1}{\textcolor{red}{1}} R_{p(1)}$$

$$P \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{1} & \mathbf{1.0001} & \mathbf{2} \\ 1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \textcolor{red}{1} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0.0001} & \mathbf{1} \\ 1 & 2 & 2 \end{pmatrix}$$

$$k = 1$$

$$i = 2$$

$$p(1) = 1$$

$$p(2) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ \color{red}{1} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P = \begin{pmatrix} \boxed{1} & 1 & 1 \\ \mathbf{0} & \mathbf{0.0001} & \mathbf{1} \\ 1 & 2 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 1 \\ \color{red}{1} & 0.0001 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

$$p = \{1 \quad 2 \quad 3\}$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$k = 1$$

$$i = 3$$

$$p(1) = 1$$

$$p(3) = 3$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \left(\begin{pmatrix} \boxed{1} & 1 & 1 \\ 0 & 0.0001 & 1 \\ \textcolor{blue}{1} & \mathbf{2} & \mathbf{2} \end{pmatrix} \right) \quad \begin{pmatrix} 1 & 1 & 1 \\ \textcolor{red}{1} & 0.0001 & 1 \\ \textcolor{red}{1} & 1 & 1 \end{pmatrix}$$

$$p = \{1 \quad 2 \quad 3\}$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{p(3)} \leftarrow R_{p(3)} - \frac{\textcolor{blue}{1}}{\textcolor{red}{1}} R_{p(1)}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \textcolor{red}{-1} & 0 & 1 \end{pmatrix} P \left(\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0.0001 & 1 \\ \mathbf{1} & \mathbf{2} & \mathbf{2} \end{pmatrix} \right) = P \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0.0001 & 1 \\ \mathbf{0} & \mathbf{1} & \mathbf{1} \end{pmatrix}$$

$$k = 1$$

$$i = 3$$

$$p(1) = 1$$

$$p(3) = 3$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \left[\begin{pmatrix} \boxed{1} & 1 & 1 \\ 0 & 0.0001 & 1 \\ \textcolor{blue}{1} & \mathbf{2} & \mathbf{2} \end{pmatrix} \right] \begin{pmatrix} 1 & 1 & 1 \\ \textcolor{red}{1} & 0.0001 & 1 \\ \textcolor{red}{1} & 1 & 1 \end{pmatrix}$$

$$p = \{1 \ 2 \ 3\}$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{p(3)} \leftarrow R_{p(3)} - \frac{\textcolor{blue}{1}}{\textcolor{red}{1}} R_{p(1)}$$

$$P \left[\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0.0001 & 1 \\ \mathbf{1} & \mathbf{2} & \mathbf{2} \end{pmatrix} \right] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \textcolor{red}{1} & 0 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0.0001 & 1 \\ \mathbf{0} & \mathbf{1} & \mathbf{1} \end{pmatrix}$$

$$k = 1$$

$$i = 3$$

$$p(1) = 1$$

$$p(3) = 3$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ \textcolor{red}{1} & 0 & 1 \end{pmatrix} P = \begin{pmatrix} \boxed{1} & 1 & 1 \\ 0 & 0.0001 & 1 \\ \mathbf{0} & \mathbf{1} & \mathbf{1} \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 1 \\ \textcolor{red}{1} & 0.0001 & 1 \\ \textcolor{red}{1} & 1 & 1 \end{pmatrix}$$

$$p = \{1 \quad 2 \quad 3\}$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$k = 2$$

$$p(3) = 3$$

$$P_{\text{new}}A = \begin{pmatrix} 1 & 0 & 0 \\ \color{red}{1} & 1 & 0 \\ \color{red}{1} & 0 & 1 \end{pmatrix} P_{\text{new}} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0.0001 & 1 \\ 0 & \boxed{1} & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 1 \\ \color{red}{1} & 0.0001 & 1 \\ \color{red}{1} & 1 & 1 \end{pmatrix}$$

$$p = \{1 \ 2 \ 3\}$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$p_{\text{new}} = \text{swap}_{\color{red}{2},\color{blue}{3}} \circ \{1 \ 2 \ 3\}$$

$$P_{\text{new}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & \color{red}{1} \\ 0 & \color{red}{1} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \{1 \ 3 \ 2\}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$k = 2$$

$$i = 3$$

$$p(2) = 3$$

$$p(3) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0.0001} & \mathbf{1} \\ 0 & \boxed{1} & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{1} & \mathbf{0.0001} & 0.9999 \\ \mathbf{1} & 1 & 1 \end{pmatrix}$$

$$p = \{1 \ 3 \ 2\}$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$R_{p(3)} \leftarrow R_{p(3)} - \frac{0.0001}{\mathbf{1}} R_{p(2)}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \mathbf{-0.0001} & 1 \end{pmatrix} P \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0.0001} & \mathbf{1} \\ 0 & 1 & 1 \end{pmatrix} = P \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0.9999} \\ 0 & 1 & 1 \end{pmatrix}$$

$$k = 2$$

$$i = 3$$

$$p(2) = 3$$

$$p(3) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0.0001} & \mathbf{1} \\ 0 & \boxed{1} & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{1} & \mathbf{0.0001} & 0.9999 \\ \mathbf{1} & 1 & 1 \end{pmatrix}$$

$$p = \{1 \ 3 \ 2\}$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$R_{p(3)} \leftarrow R_{p(3)} - \frac{0.0001}{\mathbf{1}} R_{p(2)}$$

$$P \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0.0001} & \mathbf{1} \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \mathbf{0.0001} & 1 \end{pmatrix} P \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0.9999} \\ 0 & 1 & 1 \end{pmatrix}$$

$$k = 2$$

$$i = 3$$

$$p(2) = 3$$

$$p(3) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & \textcolor{red}{0.0001} & 1 \end{pmatrix} P = \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0.9999} \\ 0 & \boxed{1} & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 1 \\ \textcolor{red}{1} & \textcolor{red}{0.0001} & 0.9999 \\ \textcolor{red}{1} & 1 & 1 \end{pmatrix}$$

$$p = \{1 \quad 3 \quad 2\}$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{aligned}
 PA &= \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0.0001 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0.9999 \\ 0 & 1 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0.0001 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0.9999 \end{pmatrix} \\
 P &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}
 \end{aligned}$$