

LU Decomposition, No pivoting

$$A = \begin{pmatrix} 2 & -1 & -1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{pmatrix}$$

$$k = 1$$

$$i = 2$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \color{red}{2} & -1 & -1 \\ \color{blue}{3} & \mathbf{3} & \mathbf{9} \\ 3 & 3 & 5 \end{pmatrix}}$$

$$R_2 \leftarrow R_2 - \frac{\color{blue}{3}}{\color{red}{2}} R_1$$

$$\begin{array}{r} \begin{pmatrix} \mathbf{3} & \mathbf{3} & \mathbf{9} \end{pmatrix} \\ -\frac{\color{blue}{3}}{\color{red}{2}} \begin{pmatrix} 2 & -1 & -1 \end{pmatrix} \\ \hline \begin{pmatrix} \mathbf{0} & \mathbf{4.5} & \mathbf{10.5} \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ \color{red}{-1.5} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 2 & -1 & -1 \\ \mathbf{3} & \mathbf{3} & \mathbf{9} \\ 3 & 3 & 5 \end{pmatrix}} = \begin{pmatrix} 2 & -1 & -1 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{10.5} \\ 3 & 3 & 5 \end{pmatrix}$$

$$k = 1$$

$$i = 2$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \color{red}{2} & -1 & -1 \\ \color{blue}{3} & \mathbf{3} & \mathbf{9} \\ 3 & 3 & 5 \end{pmatrix}}$$

$$R_2 \leftarrow R_2 - \frac{\color{blue}{3}}{\color{red}{2}} R_1$$

$$\begin{array}{r} \begin{pmatrix} \mathbf{3} & \mathbf{3} & \mathbf{9} \end{pmatrix} \\ -\frac{\color{blue}{3}}{\color{red}{2}} \begin{pmatrix} 2 & -1 & -1 \end{pmatrix} \\ \hline \begin{pmatrix} \mathbf{0} & \mathbf{4.5} & \mathbf{10.5} \end{pmatrix} \end{array}$$

$$\boxed{\begin{pmatrix} 2 & -1 & -1 \\ \mathbf{3} & \mathbf{3} & \mathbf{9} \\ 3 & 3 & 5 \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ \color{red}{1.5} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{10.5} \\ 3 & 3 & 5 \end{pmatrix}$$

$$k = 1$$

$$i = 2$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ \textcolor{red}{1.5} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \boxed{2} & -1 & -1 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{10.5} \\ 3 & 3 & 5 \end{pmatrix}$$

$$k = 1$$

$$i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \color{red}{2} & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \color{blue}{3} & \mathbf{3} & \mathbf{5} \end{pmatrix}}$$

$$R_3 \leftarrow R_3 - \frac{\color{blue}{3}}{\color{red}{2}} R_1$$

$$\begin{array}{r} \begin{pmatrix} \mathbf{3} & \mathbf{3} & \mathbf{5} \end{pmatrix} \\ -\frac{\color{blue}{3}}{\color{red}{2}} \begin{pmatrix} 2 & -1 & -1 \end{pmatrix} \\ \hline \begin{pmatrix} \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \color{red}{-1.5} & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{3} & \mathbf{3} & \mathbf{5} \end{pmatrix}} = \begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix}$$

$$k = 1$$

$$i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \color{red}{2} & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \color{blue}{3} & \mathbf{3} & \mathbf{5} \end{pmatrix}}$$

$$R_3 \leftarrow R_3 - \frac{\color{blue}{3}}{\color{red}{2}} R_1$$

$$\begin{array}{r} \begin{pmatrix} \mathbf{3} & \mathbf{3} & \mathbf{5} \end{pmatrix} \\ -\frac{\color{blue}{3}}{\color{red}{2}} \begin{pmatrix} 2 & -1 & -1 \end{pmatrix} \\ \hline \begin{pmatrix} \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix} \end{array}$$

$$\boxed{\begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{3} & \mathbf{3} & \mathbf{5} \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \color{red}{1.5} & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix}$$

$$k = 1$$

$$i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ \textcolor{red}{1.5} & 0 & 1 \end{pmatrix} \begin{pmatrix} \boxed{2} & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix}$$

$$k = 2$$

$$i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 1.5 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ 0 & 4.5 & 6.5 \end{pmatrix}}$$

$$R_3 \leftarrow R_3 - \frac{4.5}{4.5} R_2$$

$$\begin{array}{r} \begin{pmatrix} 0 & 4.5 & 6.5 \end{pmatrix} \\ -\frac{4.5}{4.5} \begin{pmatrix} 0 & 4.5 & 10.5 \end{pmatrix} \\ \hline \begin{pmatrix} 0 & 0 & -4 \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ 0 & 4.5 & 6.5 \end{pmatrix}} = \begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ 0 & 0 & -4 \end{pmatrix}$$

$$k = 2$$

$$i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 1.5 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ 0 & 4.5 & 6.5 \end{pmatrix}}$$

$$R_3 \leftarrow R_3 - \frac{4.5}{4.5} R_2$$

$$\begin{array}{r} \begin{pmatrix} 0 & 4.5 & 6.5 \\ 0 & 4.5 & 10.5 \end{pmatrix} \\ - \frac{4.5}{4.5} \begin{pmatrix} 0 & 4.5 & 10.5 \end{pmatrix} \\ \hline \begin{pmatrix} 0 & 0 & -4 \end{pmatrix} \end{array}$$

$$\boxed{\begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ 0 & 4.5 & 6.5 \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ 0 & 0 & -4 \end{pmatrix}$$

$$k = 2$$

$$i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 1.5 & \color{red}{1} & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ 0 & \boxed{4.5} & 10.5 \\ \mathbf{0} & \mathbf{0} & -4 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 1.5 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ 0 & 0 & -4 \end{pmatrix}$$