LU Decomposition, No pivoting

$$A = \begin{pmatrix} 1 & 5 & 0 \\ 4 & 8 & 12 \\ 2 & 8 & 10 \end{pmatrix}$$

$$k = 1 \qquad\qquad i = 2$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \boxed{1} & 5 & 0 \\ \mathbf{4} & \mathbf{8} & \mathbf{12} \\ 2 & 8 & 10 \end{pmatrix}} \qquad \begin{pmatrix} 1 & 5 & 0 \\ 4 & -12 & 12 \\ 2 & 8 & 10 \end{pmatrix}$$

$$R_2 \leftarrow\!\shortmid\; R_2 - \frac{4}{1} R_1$$

$$\begin{array}{rc} & \begin{pmatrix} \mathbf{4} & \mathbf{8} & \mathbf{12} \end{pmatrix} \\ -\frac{4}{1} & \begin{pmatrix} 1 & 5 & 0 \end{pmatrix} \\ \hline & \begin{pmatrix} \mathbf{0} & \mathbf{-12} & \mathbf{12} \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 1 & 5 & 0 \\ \mathbf{4} & \mathbf{8} & \mathbf{12} \\ 2 & 8 & 10 \end{pmatrix}} = \begin{pmatrix} 1 & 5 & 0 \\ \mathbf{0} & \mathbf{-12} & \mathbf{12} \\ 2 & 8 & 10 \end{pmatrix}$$

$$k = 1 \qquad\qquad i = 2$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \boxed{1} & 5 & 0 \\ \mathbf{4} & \mathbf{8} & \mathbf{12} \\ 2 & 8 & 10 \end{pmatrix}} \qquad \begin{pmatrix} 1 & 5 & 0 \\ 4 & -12 & 12 \\ 2 & 8 & 10 \end{pmatrix}$$

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$$\boxed{\begin{pmatrix} 1 & 5 & 0 \\ \mathbf{4} & \mathbf{8} & \mathbf{12} \\ 2 & 8 & 10 \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 & 0 \\ \mathbf{0} & \mathbf{-12} & \mathbf{12} \\ 2 & 8 & 10 \end{pmatrix}$$

$$k = 1 \qquad\qquad i = 2$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \boxed{1} & 5 & 0 \\ \mathbf{0} & \mathbf{-12} & \mathbf{12} \\ 2 & 8 & 10 \end{pmatrix} \qquad \begin{pmatrix} 1 & 5 & 0 \\ 4 & -12 & 12 \\ 2 & 8 & 10 \end{pmatrix}$$

$$k = 1 \qquad\qquad i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \boxed{1} & 5 & 0 \\ 0 & -12 & 12 \\ \mathbf{2} & \mathbf{8} & \mathbf{10} \end{pmatrix}} \qquad \begin{pmatrix} 1 & 5 & 0 \\ 4 & -12 & 12 \\ 2 & -2 & 10 \end{pmatrix}$$

$$R_3 \mapsto R_3 - \frac{2}{1} R_1$$

$$\begin{array}{rc} & \begin{pmatrix} \mathbf{2} & \mathbf{8} & \mathbf{10} \end{pmatrix} \\ -\frac{2}{1} & \begin{pmatrix} 1 & 5 & 0 \end{pmatrix} \\ \hline & \begin{pmatrix} \mathbf{0} & \mathbf{-2} & \mathbf{10} \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ \mathbf{2} & \mathbf{8} & \mathbf{10} \end{pmatrix}} = \begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ \mathbf{0} & \mathbf{-2} & \mathbf{10} \end{pmatrix}$$

$$k = 1 \qquad\qquad i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \boxed{1} & 5 & 0 \\ 0 & -12 & 12 \\ \mathbf{2} & \mathbf{8} & \mathbf{10} \end{pmatrix}} \qquad \begin{pmatrix} 1 & 5 & 0 \\ 4 & -12 & 12 \\ 2 & -2 & 10 \end{pmatrix}$$

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$$\boxed{\begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ \mathbf{2} & \mathbf{8} & \mathbf{10} \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ \mathbf{0} & \mathbf{-2} & \mathbf{10} \end{pmatrix}$$

$$k = 1 \qquad\qquad i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} \boxed{1} & 5 & 0 \\ 0 & -12 & 12 \\ \mathbf{0} & \mathbf{-2} & \mathbf{10} \end{pmatrix} \qquad \begin{pmatrix} 1 & 5 & 0 \\ 4 & -12 & 12 \\ 2 & -2 & 10 \end{pmatrix}$$

$$k = 2 \qquad\qquad i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 1 & 5 & 0 \\ 0 & \boxed{-12} & 12 \\ \mathbf{0} & \mathbf{-2} & \mathbf{10} \end{pmatrix}} \qquad \begin{pmatrix} 1 & 5 & 0 \\ 4 & -12 & 12 \\ 2 & 0.166667 & 8 \end{pmatrix}$$

$$R_3 \mapsto R_3 - \frac{-2}{-12} R_2$$

$$\begin{array}{rc} & \begin{pmatrix} \mathbf{0} & \mathbf{-2} & \mathbf{10} \end{pmatrix} \\ -\frac{-2}{-12} & \begin{pmatrix} 0 & -12 & 12 \end{pmatrix} \\ \hline & \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{8} \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -0.166667 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ \mathbf{0} & \mathbf{-2} & \mathbf{10} \end{pmatrix}} = \begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ \mathbf{0} & \mathbf{0} & \mathbf{8} \end{pmatrix}$$

$$k = 2 \qquad\qquad i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 1 & 5 & 0 \\ 0 & \boxed{-12} & 12 \\ \mathbf{0} & \mathbf{-2} & \mathbf{10} \end{pmatrix}} \qquad \begin{pmatrix} 1 & 5 & 0 \\ 4 & -12 & 12 \\ 2 & 0.166667 & 8 \end{pmatrix}$$

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$$\begin{array}{rl} & \begin{pmatrix} \mathbf{0} & \mathbf{-2} & \mathbf{10} \end{pmatrix} \\ -\frac{-2}{-12} & \begin{pmatrix} 0 & -12 & 12 \end{pmatrix} \\ \hline & \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{8} \end{pmatrix} \end{array}$$

$$\boxed{\begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ \mathbf{0} & \mathbf{-2} & \mathbf{10} \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.166667 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ \mathbf{0} & \mathbf{0} & \mathbf{8} \end{pmatrix}$$

$k = 2$ $\qquad\qquad$ $i = 3$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & 0.166667 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 & 0 \\ 0 & \boxed{-12} & 12 \\ \mathbf{0} & \mathbf{0} & \mathbf{8} \end{pmatrix} \qquad \begin{pmatrix} 1 & 5 & 0 \\ 4 & -12 & 12 \\ 2 & 0.166667 & 8 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & 0.166667 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ 0 & 0 & 8 \end{pmatrix}$$