

LU Decomposition, No pivoting

$$A = \begin{pmatrix} 1 & 5 & 0 \\ 4 & 8 & 12 \\ 2 & 8 & 10 \end{pmatrix}$$

$$k = 1$$

$$i = 2$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \color{red}{1} & 5 & 0 \\ \color{blue}{4} & \mathbf{8} & \mathbf{12} \\ 2 & 8 & 10 \end{pmatrix}} \quad \begin{pmatrix} 1 & 5 & 0 \\ \color{red}{4} & -12 & 12 \\ 2 & 8 & 10 \end{pmatrix}$$

$$R_2 \leftarrow R_2 - \frac{\color{blue}{4}}{\color{red}{1}} R_1$$

$$\begin{array}{r} \begin{pmatrix} \mathbf{4} & \mathbf{8} & \mathbf{12} \end{pmatrix} \\ -\frac{\color{blue}{4}}{\color{red}{1}} \begin{pmatrix} 1 & 5 & 0 \end{pmatrix} \\ \hline \begin{pmatrix} \mathbf{0} & \mathbf{-12} & \mathbf{12} \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ \color{red}{-4} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 1 & 5 & 0 \\ \mathbf{4} & \mathbf{8} & \mathbf{12} \\ 2 & 8 & 10 \end{pmatrix}} = \begin{pmatrix} 1 & 5 & 0 \\ \mathbf{0} & \mathbf{-12} & \mathbf{12} \\ 2 & 8 & 10 \end{pmatrix}$$

$$k = 1$$

$$i = 2$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \color{red}{1} & 5 & 0 \\ \color{blue}{4} & \mathbf{8} & \mathbf{12} \\ 2 & 8 & 10 \end{pmatrix}} \quad \begin{pmatrix} 1 & 5 & 0 \\ \color{red}{4} & -12 & 12 \\ 2 & 8 & 10 \end{pmatrix}$$

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$$\boxed{\begin{pmatrix} 1 & 5 & 0 \\ \mathbf{4} & \mathbf{8} & \mathbf{12} \\ 2 & 8 & 10 \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ \color{red}{4} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 & 0 \\ \mathbf{0} & \mathbf{-12} & \mathbf{12} \\ 2 & 8 & 10 \end{pmatrix}$$

$$k = 1$$

$$i = 2$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \boxed{1} & 5 & 0 \\ \mathbf{0} & -\mathbf{12} & \mathbf{12} \\ 2 & 8 & 10 \end{pmatrix} \quad \begin{pmatrix} 1 & 5 & 0 \\ 4 & -12 & 12 \\ 2 & 8 & 10 \end{pmatrix}$$

$$k = 1$$

$$i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \color{red}{1} & 5 & 0 \\ 0 & -12 & 12 \\ \color{blue}{2} & \mathbf{8} & \mathbf{10} \end{pmatrix}} \quad \begin{pmatrix} 1 & 5 & 0 \\ \color{red}{4} & -12 & 12 \\ \color{red}{2} & -2 & 10 \end{pmatrix}$$

$$R_3 \leftarrow R_3 - \frac{\color{blue}{2}}{\color{red}{1}} R_1$$

$$\begin{array}{r} \begin{pmatrix} \mathbf{2} & \mathbf{8} & \mathbf{10} \end{pmatrix} \\ -\frac{\color{blue}{2}}{\color{red}{1}} \begin{pmatrix} 1 & 5 & 0 \end{pmatrix} \\ \hline \begin{pmatrix} \mathbf{0} & \mathbf{-2} & \mathbf{10} \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \color{red}{-2} & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ \mathbf{2} & \mathbf{8} & \mathbf{10} \end{pmatrix}} = \begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ \mathbf{0} & \mathbf{-2} & \mathbf{10} \end{pmatrix}$$

$$k = 1$$

$$i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \color{red}{1} & 5 & 0 \\ 0 & -12 & 12 \\ \color{blue}{2} & \mathbf{8} & \mathbf{10} \end{pmatrix}} \quad \begin{pmatrix} 1 & 5 & 0 \\ \color{red}{4} & -12 & 12 \\ \color{red}{2} & -2 & 10 \end{pmatrix}$$

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$$\boxed{\begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ \mathbf{2} & \mathbf{8} & \mathbf{10} \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \color{red}{2} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ \mathbf{0} & \mathbf{-2} & \mathbf{10} \end{pmatrix}$$

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$$i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ \color{red}{2} & 0 & 1 \end{pmatrix} \begin{pmatrix} \boxed{1} & 5 & 0 \\ 0 & -12 & 12 \\ \mathbf{0} & -\mathbf{2} & \mathbf{10} \end{pmatrix} \quad \begin{pmatrix} 1 & 5 & 0 \\ \color{red}{4} & -12 & 12 \\ \color{red}{2} & -2 & 10 \end{pmatrix}$$

$$k = 2$$

$$i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \begin{bmatrix} \begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ 0 & -2 & 10 \end{pmatrix} \end{bmatrix} \begin{pmatrix} 1 & 5 & 0 \\ 4 & -12 & 12 \\ 2 & 0.166667 & 8 \end{pmatrix}$$

$$R_3 \leftarrow R_3 - \frac{-2}{-12} R_2$$

$$\begin{array}{r} \begin{pmatrix} 0 & -2 & 10 \\ 0 & -12 & 12 \end{pmatrix} \\ -\frac{-2}{-12} \begin{pmatrix} 0 & -12 & 12 \end{pmatrix} \\ \hline \begin{pmatrix} 0 & 0 & 8 \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -0.166667 & 1 \end{pmatrix} \begin{bmatrix} \begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ 0 & -2 & 10 \end{pmatrix} \end{bmatrix} = \begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ 0 & 0 & 8 \end{pmatrix}$$

$$k = 2$$

$$i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \left[\begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ 0 & -2 & 10 \end{pmatrix} \right] \begin{pmatrix} 1 & 5 & 0 \\ 4 & -12 & 12 \\ 2 & 0.166667 & 8 \end{pmatrix}$$

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$$\begin{array}{r} \begin{pmatrix} 0 & -2 & 10 \\ 0 & -12 & 12 \end{pmatrix} \\ -\frac{-2}{-12} \begin{pmatrix} 0 & -12 & 12 \end{pmatrix} \\ \hline \begin{pmatrix} 0 & 0 & 8 \end{pmatrix} \end{array}$$

$$\left[\begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ 0 & -2 & 10 \end{pmatrix} \right] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.166667 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ 0 & 0 & 8 \end{pmatrix}$$

$$k = 2$$

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$$A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & 0.166667 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 & 0 \\ 0 & \boxed{-12} & 12 \\ 0 & 0 & 8 \end{pmatrix} \quad \begin{pmatrix} 1 & 5 & 0 \\ 4 & -12 & 12 \\ 2 & 0.166667 & 8 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & 0.166667 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 & 0 \\ 0 & -12 & 12 \\ 0 & 0 & 8 \end{pmatrix}$$