LU Decomposition, No pivoting

$$A = \begin{pmatrix} 2 & -1 & -1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{pmatrix}$$

$$k = 1 \qquad\qquad i = 2$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \boxed{2} & -1 & -1 \\ \mathbf{3} & \mathbf{3} & \mathbf{9} \\ 3 & 3 & 5 \end{pmatrix}} \qquad \begin{pmatrix} 2 & -1 & -1 \\ 1.5 & 4.5 & 10.5 \\ 3 & 3 & 5 \end{pmatrix}$$

$$R_2 \leftarrow\!\shortmid\; R_2 - \frac{3}{2} R_1$$

$$\begin{array}{rc} & \begin{pmatrix} \mathbf{3} & \mathbf{3} & \mathbf{9} \end{pmatrix} \\ -\frac{3}{2} & \begin{pmatrix} 2 & -1 & -1 \end{pmatrix} \\ \hline & \begin{pmatrix} \mathbf{0} & \mathbf{4.5} & \mathbf{10.5} \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ -1.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 2 & -1 & -1 \\ \mathbf{3} & \mathbf{3} & \mathbf{9} \\ 3 & 3 & 5 \end{pmatrix}} = \begin{pmatrix} 2 & -1 & -1 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{10.5} \\ 3 & 3 & 5 \end{pmatrix}$$

$$k = 1 \qquad\qquad i = 2$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \boxed{2} & -1 & -1 \\ \mathbf{3} & \mathbf{3} & \mathbf{9} \\ 3 & 3 & 5 \end{pmatrix}} \qquad \begin{pmatrix} 2 & -1 & -1 \\ 1.5 & 4.5 & 10.5 \\ 3 & 3 & 5 \end{pmatrix}$$

$$R_2 \leftarrow\!\shortmid\; R_2 - \frac{3}{2} R_1$$

$$\begin{array}{rc} & (\ \mathbf{3} \quad \mathbf{3} \quad \mathbf{9}\ ) \\ -\frac{3}{2} & (\ 2 \quad -1 \quad -1\ ) \\ \hline & (\ \mathbf{0} \quad \mathbf{4.5} \quad \mathbf{10.5}\ ) \end{array}$$

$$\boxed{\begin{pmatrix} 2 & -1 & -1 \\ \mathbf{3} & \mathbf{3} & \mathbf{9} \\ 3 & 3 & 5 \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{10.5} \\ 3 & 3 & 5 \end{pmatrix}$$

$k = 1$ $\qquad\qquad$ $i = 2$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \boxed{2} & -1 & -1 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{10.5} \\ 3 & 3 & 5 \end{pmatrix} \qquad \begin{pmatrix} 2 & -1 & -1 \\ 1.5 & 4.5 & 10.5 \\ 3 & 3 & 5 \end{pmatrix}$$

$$k = 1 \qquad\qquad i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \boxed{2} & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{3} & \mathbf{3} & \mathbf{5} \end{pmatrix}} \qquad \begin{pmatrix} 2 & -1 & -1 \\ 1.5 & 4.5 & 10.5 \\ 1.5 & 4.5 & 6.5 \end{pmatrix}$$

$$R_3 \mapsto R_3 - \frac{3}{2} R_1$$

$$\begin{array}{rc} & \begin{pmatrix} \mathbf{3} & \mathbf{3} & \mathbf{5} \end{pmatrix} \\ -\frac{3}{2} & \begin{pmatrix} 2 & -1 & -1 \end{pmatrix} \\ \hline & \begin{pmatrix} \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1.5 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{3} & \mathbf{3} & \mathbf{5} \end{pmatrix}} = \begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix}$$

$$k = 1 \qquad\qquad i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} \boxed{2} & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{3} & \mathbf{3} & \mathbf{5} \end{pmatrix}} \qquad \begin{pmatrix} 2 & -1 & -1 \\ 1.5 & 4.5 & 10.5 \\ 1.5 & 4.5 & 6.5 \end{pmatrix}$$

$$R_3 \mapsto R_3 - \frac{3}{2} R_1$$

$$\begin{array}{rc} & \begin{pmatrix} \mathbf{3} & \mathbf{3} & \mathbf{5} \end{pmatrix} \\ -\frac{3}{2} & \begin{pmatrix} 2 & -1 & -1 \end{pmatrix} \\ \hline & \begin{pmatrix} \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix} \end{array}$$

$$\boxed{\begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{3} & \mathbf{3} & \mathbf{5} \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1.5 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix}$$

$$k = 1 \qquad\qquad i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 1.5 & 0 & 1 \end{pmatrix} \begin{pmatrix} \boxed{2} & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix} \qquad \begin{pmatrix} 2 & -1 & -1 \\ 1.5 & 4.5 & 10.5 \\ 1.5 & 4.5 & 6.5 \end{pmatrix}$$

$$k = 2 \qquad\qquad i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 1.5 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 2 & -1 & -1 \\ 0 & \boxed{4.5} & 10.5 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix}} \qquad \begin{pmatrix} 2 & -1 & -1 \\ 1.5 & 4.5 & 10.5 \\ 1.5 & 1 & -4 \end{pmatrix}$$

$$R_3 \mapsto R_3 - \frac{4.5}{4.5} R_2$$

$$\begin{array}{rc} & \begin{pmatrix} \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix} \\ -\frac{4.5}{4.5} & \begin{pmatrix} 0 & 4.5 & 10.5 \end{pmatrix} \\ \hline & \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{-4} \end{pmatrix} \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix}} = \begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{0} & \mathbf{0} & \mathbf{-4} \end{pmatrix}$$

$$k = 2 \qquad\qquad i = 3$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 1.5 & 0 & 1 \end{pmatrix} \boxed{\begin{pmatrix} 2 & -1 & -1 \\ 0 & \boxed{4.5} & 10.5 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix}} \qquad \begin{pmatrix} 2 & -1 & -1 \\ 1.5 & 4.5 & 10.5 \\ 1.5 & 1 & -4 \end{pmatrix}$$

$$R_3 \mapsto R_3 - \frac{4.5}{4.5} R_2$$

$$\begin{array}{rc} & \begin{pmatrix} \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix} \\ -\frac{4.5}{4.5} & \begin{pmatrix} 0 & 4.5 & 10.5 \end{pmatrix} \\ \hline & \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{-4} \end{pmatrix} \end{array}$$

$$\boxed{\begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{0} & \mathbf{4.5} & \mathbf{6.5} \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ \mathbf{0} & \mathbf{0} & \mathbf{-4} \end{pmatrix}$$

$k = 2$ $\qquad\qquad$ $i = 3$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 1.5 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ 0 & \boxed{4.5} & 10.5 \\ \mathbf{0} & \mathbf{0} & \mathbf{-4} \end{pmatrix} \qquad \begin{pmatrix} 2 & -1 & -1 \\ 1.5 & 4.5 & 10.5 \\ 1.5 & 1 & -4 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 1.5 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ 0 & 4.5 & 10.5 \\ 0 & 0 & -4 \end{pmatrix}$$