

LU Decomposition, Partial pivoting

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 1 & 0 & 1 \\ -2 & 2 & 4 \end{pmatrix}$$

$$k = 1$$

$$p(3) = 3$$

$$P_{\text{new}}A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P_{\text{new}} \begin{pmatrix} 1 & 2 & 4 \\ 1 & 0 & 1 \\ \boxed{-2} & 2 & 4 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 4 \\ 1 & 0 & 1 \\ -2 & 2 & 4 \end{pmatrix}$$

$$p = \{1 \ 2 \ 3\}$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$p_{\text{new}} = \text{swap}_{1,3} \circ \{1 \ 2 \ 3\}$$

$$P_{\text{new}} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \{3 \ 2 \ 1\}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$k = 1$$

$$i = 2$$

$$p(1) = 3$$

$$p(2) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 2 & 4 \\ \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \boxed{-2} & 2 & 4 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 4 \\ -0.5 & 1 & 3 \\ -2 & 2 & 4 \end{pmatrix}$$

$$p = \{3 \quad 2 \quad 1\}$$

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$R_{p(2)} \leftarrow R_{p(2)} - \frac{1}{-2} R_{p(1)}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ \mathbf{0.5} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 2 & 4 \\ \mathbf{1} & \mathbf{0} & \mathbf{1} \\ -2 & 2 & 4 \end{pmatrix} = P \begin{pmatrix} 1 & 2 & 4 \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \\ -2 & 2 & 4 \end{pmatrix}$$

$$k = 1$$

$$i = 2$$

$$p(1) = 3$$

$$p(2) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 2 & 4 \\ \mathbf{1} & \mathbf{0} & \mathbf{1} \\ -2 & 2 & 4 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 4 \\ -0.5 & 1 & 3 \\ -2 & 2 & 4 \end{pmatrix}$$

$$p = \{3 \quad 2 \quad 1\}$$

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$R_{p(2)} \leftarrow R_{p(2)} - \frac{1}{-2} R_{p(1)}$$

$$P \begin{pmatrix} 1 & 2 & 4 \\ \mathbf{1} & \mathbf{0} & \mathbf{1} \\ -2 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 2 & 4 \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \\ -2 & 2 & 4 \end{pmatrix}$$

$$k = 1$$

$$i = 2$$

$$p(1) = 3$$

$$p(2) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 1 & 2 & 4 \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \\ \boxed{-2} & 2 & 4 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 4 \\ -0.5 & 1 & 3 \\ -2 & 2 & 4 \end{pmatrix}$$

$$p = \{3 \quad 2 \quad 1\}$$

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$k = 1$$

$$i = 3$$

$$p(1) = 3$$

$$p(3) = 1$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{4} \\ 0 & 1 & 3 \\ \mathbf{-2} & 2 & 4 \end{pmatrix} \begin{pmatrix} \mathbf{-0.5} & 3 & 6 \\ \mathbf{-0.5} & 1 & 3 \\ -2 & 2 & 4 \end{pmatrix}$$

$$p = \{3 \quad 2 \quad 1\}$$

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$R_{p(3)} \leftarrow R_{p(3)} - \frac{1}{-2} R_{p(1)}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \mathbf{0.5} & 0 & 1 \end{pmatrix} P \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{4} \\ 0 & 1 & 3 \\ -2 & 2 & 4 \end{pmatrix} = P \begin{pmatrix} \mathbf{0} & \mathbf{3} & \mathbf{6} \\ 0 & 1 & 3 \\ -2 & 2 & 4 \end{pmatrix}$$

$$k = 1$$

$$i = 3$$

$$p(1) = 3$$

$$p(3) = 1$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} P \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{4} \\ 0 & 1 & 3 \\ \mathbf{-2} & 2 & 4 \end{pmatrix} \begin{pmatrix} \mathbf{-0.5} & 3 & 6 \\ \mathbf{-0.5} & 1 & 3 \\ -2 & 2 & 4 \end{pmatrix}$$

$$p = \{3 \quad 2 \quad 1\}$$

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$R_{p(3)} \leftarrow R_{p(3)} - \frac{1}{-2} R_{p(1)}$$

$$P \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{4} \\ 0 & 1 & 3 \\ -2 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \mathbf{-0.5} & 0 & 1 \end{pmatrix} P \begin{pmatrix} \mathbf{0} & \mathbf{3} & \mathbf{6} \\ 0 & 1 & 3 \\ -2 & 2 & 4 \end{pmatrix}$$

$$k = 1$$

$$i = 3$$

$$p(1) = 3$$

$$p(3) = 1$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -0.5 & 0 & 1 \end{pmatrix} P \begin{pmatrix} \mathbf{0} & \mathbf{3} & \mathbf{6} \\ 0 & 1 & 3 \\ \boxed{-2} & 2 & 4 \end{pmatrix} \quad \begin{pmatrix} -0.5 & 3 & 6 \\ -0.5 & 1 & 3 \\ -2 & 2 & 4 \end{pmatrix}$$

$$p = \{3 \quad 2 \quad 1\}$$

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$k = 2$$

$$p(3) = 1$$

$$P_{\text{new}}A = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -0.5 & 0 & 1 \end{pmatrix} P_{\text{new}} \begin{pmatrix} 0 & \boxed{3} & 6 \\ 0 & 1 & 3 \\ -2 & 2 & 4 \end{pmatrix} \quad \begin{pmatrix} -0.5 & 3 & 6 \\ -0.5 & 1 & 3 \\ -2 & 2 & 4 \end{pmatrix}$$

$$p = \{3 \ 2 \ 1\}$$

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$p_{\text{new}} = \text{swap}_{2,3} \circ \{3 \ 2 \ 1\}$$

$$P_{\text{new}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \{3 \ 1 \ 2\}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$k = 2$$

$$i = 3$$

$$p(2) = 1$$

$$p(3) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -0.5 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 0 & \boxed{3} & 6 \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \\ -2 & 2 & 4 \end{pmatrix} \begin{pmatrix} -0.5 & 3 & 6 \\ -0.5 & 0.333333 & 1 \\ -2 & 2 & 4 \end{pmatrix}$$

$$p = \{3 \quad 1 \quad 2\}$$

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$R_{p(3)} \leftarrow R_{p(3)} - \frac{1}{3} R_{p(2)}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -0.333333 & 1 \end{pmatrix} P \begin{pmatrix} 0 & 3 & 6 \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \\ -2 & 2 & 4 \end{pmatrix} = P \begin{pmatrix} 0 & 3 & 6 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ -2 & 2 & 4 \end{pmatrix}$$

$$k = 2$$

$$i = 3$$

$$p(2) = 1$$

$$p(3) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -0.5 & 0 & 1 \end{pmatrix} P \begin{pmatrix} 0 & \boxed{3} & 6 \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \\ -2 & 2 & 4 \end{pmatrix} \begin{pmatrix} -0.5 & 3 & 6 \\ -0.5 & 0.333333 & 1 \\ -2 & 2 & 4 \end{pmatrix}$$

$$p = \{3 \quad 1 \quad 2\}$$

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$R_{p(3)} \leftarrow R_{p(3)} - \frac{1}{3} R_{p(2)}$$

$$P \begin{pmatrix} 0 & 3 & 6 \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \\ -2 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.333333 & 1 \end{pmatrix} P \begin{pmatrix} 0 & 3 & 6 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ -2 & 2 & 4 \end{pmatrix}$$

$$k = 2$$

$$i = 3$$

$$p(2) = 1$$

$$p(3) = 2$$

$$PA = \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -0.5 & 0.333333 & 1 \end{pmatrix} P = \begin{pmatrix} 0 & \boxed{3} & 6 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ -2 & 2 & 4 \end{pmatrix} \quad \begin{pmatrix} -0.5 & 3 & 6 \\ -0.5 & 0.333333 & 1 \\ -2 & 2 & 4 \end{pmatrix}$$

$$p = \{3 \quad 1 \quad 2\}$$

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{aligned}
 PA &= \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -0.5 & 0.333333 & 1 \end{pmatrix} P \begin{pmatrix} 0 & 3 & 6 \\ 0 & 0 & 1 \\ -2 & 2 & 4 \end{pmatrix} \\
 &= \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -0.5 & 0.333333 & 1 \end{pmatrix} \begin{pmatrix} -2 & 2 & 4 \\ 0 & 3 & 6 \\ 0 & 0 & 1 \end{pmatrix} \\
 P &= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}
 \end{aligned}$$