6-11

A car manufacturer wants to estimate the average MPG highway rating for a new model. From experience with similar models, the manufacture believes the MPG standard deviation is 4.6. From a random sample of 100 highway runs, the new model yields a sample mean of 32. Give a 95% confidence interval for the population mean

$$n = 100, \qquad \sigma = 4.6, \qquad \bar{X} = 32$$

$$95\%\ Conf\ Interval = \bar{X} \pm Z_{0.025}\frac{\sigma}{\sqrt{n}}$$

$$= 32 \pm 1.96\frac{4.6}{\sqrt{100}}$$

$$= [32 - 0.9016, 32 + 0.9016]$$

$$95\%\ Confidence\ Interval = [31.098\ , 32.902]$$

6-39

Data of daily consumption of fuel by a delivery truck, in gallons, recorded during 25 randomly selected working days are as follows: 9.7, 8.9, 9.7, 10.9, 10.3, 10.1, 10.7, 10.6, 10.4, 10.6, 11.6, 11.7, 9.7, 9.7, 9.7, 9.8, 12, 10.4, 8.8, 8.9, 8.4, 9.7, 10.3, 10, 9.2. Give a 90% confidence interval for the population mean given that the population mean and standard deviation are unknown.

$$n = 25, d.f. = 24$$

$$\bar{x} = \frac{\sum x_i}{n} \quad and \quad s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$

$$90\ \%\ Conf\ Interval = \bar{x} \pm t_{0.05,d.f.}\frac{s}{\sqrt{n}}$$

$$\bar{x} = \frac{9.7 + 8.9 + 9.7 + \cdots + 9.2}{25} \Rightarrow \frac{251.8}{25} \Rightarrow 10.07$$

$$s = \sqrt{\frac{(9.7 - 10.072)^2 + (8.9 - 10.072)^2 + (9.7 - 10.072)^2 + \cdots + (9.2 - 10.072)^2}{24}}$$

$$s = \sqrt{0.8121} \Rightarrow 0.9012$$

$$t_{0.05,df=24} = 1.711$$

$$90\%\ Conf\ Interval = 10.07 \pm 1.711\frac{0.9012}{\sqrt{25}}$$

$$90\%\ CI = [10.07 - 0.3084, 10.07 + 0.3084] \Rightarrow [9.76, 10.38]$$

6-51

According to *Forbes*, 78% of patients of physicians who offer online appointments take advantage of the option. If this statistic is based on a random sample of 1,000 patients, 780 of whom do their appointments online, give a 90% confidence interval for the proportion of patients who do their appointments online.

$$n = 1000, \hat{p} = 0.78, \hat{q} = 0.22$$

$$90\%\ Conf\ Interval = \hat{p} \pm Z_{0.05}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$90\%\ CI = 0.78 \pm 1.645\sqrt{\frac{0.1716}{1000}} \Rightarrow [0.78 - 0.0215, 0.78 + 0.0215] \Rightarrow [75.85\%, 80.15\%]$$

6-58

The service time in queues should not have a large variance. Otherwise, the queue tends to build up. A bank regularly checks service time by its tellers to determine its variance. A sample of 22 (n = 22, d.f. = 21) service times (in minutes) gives $s^2 = 8$. Give a 95% confidence interval for the variance of service time at the bank.

$$95\%\ Conf\ Interval = \left[\frac{(n-1)s^2}{\chi^2_{0.025}}, \frac{(n-1)s^2}{\chi^2_{0.975}}\right]$$

$$95\%\ CI = \left[\frac{(21 \times 8)}{35.5}, \frac{(21 \times 8)}{10.8}\right] = [4.732, 16.311]$$

6-63

Find the minimum required sample size of accounts of United Airlines in 2011 if the proportion of accounts in error is to be estimated within 0.02 with 95% Confidence. A rough guess of accounts in error is equal to 0.10.

$$p = 0.10, q = 0.90, Z_{\alpha/2} = 1.96, B = 0.02$$

$$n = \frac{(Z^2)(p)(q)}{B^2} \Rightarrow \frac{1.96^2 \times 0.90 \times 0.10}{0.02^2} \Rightarrow 864.36 \Rightarrow 865\ accounts\ should\ be\ sampled$$