Aerodynamic Shape Optimisation of Axisymmetric Bodies in Supersonic Flow

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by

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<u>CERTIFICATE</u>

*This is to certify that the work contained in this thesis entitled "**Aerodynamic Shape Optimisation of Axisymmetric Bodies in Supersonic Flow**" is a bonafide work of **Ashwin K Raghu** (**Roll No. 131601008**), carried out in the Department of Mechanical Engineering, Indian Institute of Technology Palakkad under my supervision and that it has not been submitted elsewhere for a degree.*

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Chapter 1

Introduction

Computer simulations play an essential role in research. They are capable of producing results that are as accurate as those obtained by experiments. Their reliability and celerity in producing results have entitled them as the "third method of discovery," after theory and experimentation. When it comes to the design of an optimal aerodynamic shape, CFD, along with optimisation techniques, are coupled together in an Aerodynamic Shape Optimisation (ASO) framework. Such frameworks are widely used in Rocket and Missile industries to determine minimum-drag geometries, which helps in increasing the range of flight. Some commercial solvers perform ASO, but their computational cost is very high as they are built to solve the fundamental governing equations, which are general and applicable to any given geometry. But specific geometries do not pose the need for evaluating all the governing equations, and in such cases, the equations can be simplified to obtain the solution quickly at a lower cost. Advanced research in computational mechanics involves attempts to devise and modify numerical solvers that use simplified governing equations, unlike traditional CFD solvers. In this work, a low fidelity ASO framework is devised to reduce the wave drag on an axisymmetric forebody in supersonic flow by slightly modifying its shape while adhering to the geometrical constraints. It is targeted to be used in the design of optimal shapes for bullets, long-range missiles, or rockets. The solver utilizes simplified governing equations called the Taylor Maccoll equations for flow field calculations, which are derived based on certain assumptions. The solver represents the geometry using a NURBS Curve and performs optimisation using the steepest descent algorithm. The celerity of the solver helps to optimise numerous variations of axisymmetric forebodies in shorter periods, thus making the design procedure easy. The approximate estimate of the optimal shape obtained from the numerical solver can be fed into a traditional optimisation framework to derive the accurate optimal shape. The optimised geometry is convex and has a nearly 15% lower drag coefficient than the initial geometry. The optimal shape derived is found to have a weak Mach number dependence at high supersonic or hypersonic flows.

1.1 Organization of The Report

- **Chapter 1, Introduction** - Gives an introduction to the thesis, explain the motivation behind it and gives a brief description of its implementation and application.
- **Chapter 2, Review of Prior Works** - Describes the theory and previous research which is included or utilized in this work.
- **Chapter 3, Design Methodology** - Explains the implementation of this work, gives a detailed explanation about the algorithm created in this work.
- **Chapter 4, Results and Discussion** - Discusses the outputs of this work and its validation.
- **Chapter 5, Conclusion and Future Work** - Summarises the results, provides a brief conclusion to the thesis, and discusses the scope of adopting newer methods to improvise this work.
- **References**

Chapter 2

Review of Prior Works

2.1 Conical Shock

When a body moves faster than the speed of sound, shock waves are formed. These shock waves create regions where abrupt changes in flow properties are observed. In the case of a wedge placed in supersonic flow, an oblique shock formation immediately deflects the flow and makes it parallel to the wedge. This phenomenon is bolstered by spectrographic evidence of shocks on wedges. But in the case of a cone, a conical shock is formed, which deflects the flow continuously and curls it gradually until it becomes parallel to the surface [1]. The static pressure and the Mach number varies continuously after the shock to the surface of the cone.

2.2 Taylor Maccoll Equations

The flow is assumed to be isentropic before and after the conical shock. As the body is axisymmetric and conical, properties are independent of r and ϕ (in terms of spherical coordinates [r, ϕ, θ]). Hence, the continuity equation in spherical coordinates can be simplified using the aforementioned assumptions of independence. Further assumptions to simplify the equation include straight shocks, calorically perfect gas, and steady and adiabatic flow - which makes stagnation enthalpy constant everywhere inside the domain. Thus, according to Crocco's equation, the flow becomes irrotational, and the flow properties along a ray originating from vertex remain constant. The Crocco's equations, along with Euler and ideal gas equations are substituted back into the simplified continuity equations to generate a new set of equations, called the Taylor-Maccoll Equations [2]. These equations can be solved using numerical methods (e.g., Runge-Kutta method) to obtain the flow Mach number. Isentropic relations remain valid according to our assumptions and can be utilized to determine surface pressure, drag, and drag coefficient. The Taylor-Maccoll equations are given below:

$$V_\theta = \frac{\partial V_r'}{\partial \theta} \tag{2.1}$$

$$\frac{\gamma-1}{2}\left[1 - V_r^2 - (\frac{dV_r'}{d\theta})^2\right]\left[2V_r' + \frac{dV_r'}{d\theta}cot\theta + \frac{d^2V_r'}{d\theta^2}\right] - \frac{dV_r'}{d\theta}\left[V_r'\frac{dV_r'}{d\theta} + \frac{dV_r'}{d\theta}\frac{d^2V_r'}{d\theta^2}\right]$$

2.3 Tangent-Cone Method

Since the Taylor Maccoll Equations are valid only to cones with an attached shock, they can only be applied to other axisymmetric forebodies using the tangent cone method. For utilizing this method, the body should be parameterized using a mathematical model. While using the tangent cone method, the parameterized geometry is sliced along the length into frustums, and the elemental flow fields are calculated. These are later combined to get the flow field around the whole body.

2.4 Geometry Parameterizaton

A NURBS (Non-Uniform Rational B-Spline) curve is an ideal choice for geometry parameterization. It is widely used in CAD/CAM softwares. The curve is very flexible and can represent free form shapes. A NURBS curve can be defined if its control points, order, knot vector, and the set of weights are given [3] . Its representation is as follows:

$$C(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u) w_i P_i}{\sum_{i=0}^{n} N_{i,p}(u) w_i} \tag{2.2}$$

where n denotes the number of control points, P_i denotes the control points, w_i denotes the weights, and p denotes the degree of the curve. The coefficient $N_{i,p}$ is determined the Cox-de Boor recursion formula,

$$N_{i,0} = \begin{cases} 1 & if\ u \in [u_i, u_{i+1}) \\ 0 & otherwise \end{cases} \tag{2.3}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p+1}(u)$$

2.5 Steepest Descent Method

For optimisation, techniques like Genetic Algorithm is unavailing as it requires an initial population. Instead, a simple and reliable procedure called the method of Steepest Descent can be employed. It is an iterative process that is aimed at finding the global minimum of a particular cost function, if it exists. The drag on a body is associated with geometry parameters, which can be tweaked using methods like the steepest descent to derive the parametric values corresponding to minimum-drag geometry. In this method, the derivative of the cost function with respect to the geometry parameter determines the direction of the steepest descent. It is multiplied with a learning rate, and their product indicates the magnitude of descent. In the ASO framework, an initial value for the parameter is assumed, and in each iteration, a step is taken from the previous value in the direction of steepest descent until the local or global minima of the cost function are obtained, or a prescribed tolerance condition is satisfied. The value of the learning rate (λ) can be adaptively controlled based on the value of the cost function (Drag), which helps in faster convergence [4].

$$\lambda = \begin{cases} \alpha\lambda^{k-1} & if\ C_d^k > C_d^{k-1} \\ \beta\lambda^{k-1} & otherwise \end{cases} \tag{2.4}$$

Previous optimisation studies [5] have shown that the solution is obtained quickly with the least number of optimisation cycles when the value of α and β are chosen to be 0.5 and 1.4, respectively.

Chapter 3

Design Methodology

The proposed Aerodynamic Shape Optimisation framework is divided into three sections: Optimisation algorithm, Flow solver, and Geometric parameterization.

- Geometry parameterization is a mathematical representation of the surface of the body, which is modified during the optimisation. In this project, a NURBS curve is utilized for geometry representation.
- The Flow solver computes the flow properties such as Mach number, pressure, drag, drag coefficient on the surface of an input conical element using Taylor-Maccoll equations.
- The Optimisation algorithm is iterative and determines the geometry parameters of minimum-drag geometry. In this project, the steepest descent algorithm is employed for optimisation. It minimizes the cost function by altering the design (or geometry) parameters.

3.1 Optimisation Algorithm

In this work, the steepest descent method is chosen for optimisation. It is a simple algorithm that achieves global convergence, if it exists. The optimisation starts with an initial trial solution and iterates towards the minimal solution. The control points of the NURBS curve serve as the design variables for optimisation. Out of the four control points, two are at the ends and are fixed (geometry constraints) as they are characteristic of the input geometry. The coordinates of the remaining two control points are represented as vector X = $[x_2, y_2, x_3, y_3]$. This vector is updated in each optimisation cycle, where the overall shape changes and the drag reduces. The optimisation cycle breaks when a tolerance condition is satisfied, or a prescribed number of cycles is reached. It then outputs the minimum-drag geometry and displays the value of the minimum drag coefficient. Before implementing optimisation using NURBS Curve, a power law profile of the formula $y = ax^n$ is utilised to make debugging easier. Previous research [5] have shown that it can provide approximate results. Once the framework is complete, the power-law profile is replaced with a NURBS representation.

The optimisation algorithm is the central part of the framework that drives both the flow solver and geometry parameterization algorithm.

3.1.1 Implementation

- Accepts freestream flow properties, length and radius of the input cone.
- Sets the initial values of lambda and control points such that the initial geometry is a straight line representing the input two dimensional cone .
- Starts the optimisation cycle by passing the control point vector to the geometry parameterization function to generate the coordinate matrix of the fit curve.
- Passes the coordinates to the flow solver for calculating the drag coefficient.
- Perturbs each coordinate of the control point vector by a small amount ϵ and updates the coordinate using the formula:

$$X_i^{k+1} = X_i^k - \lambda \frac{\partial C_d}{\partial X_i} \tag{3.1}$$

$$\frac{\partial C_d}{\partial X_i} = \frac{C_d^k(X_i + \epsilon) - C_d^k(X_i)}{\epsilon}$$

where $C_d^k(X_i + \epsilon)$ is the coefficient of drag calculated after the perturbing the i^{th} coordinate.

Note: If a power law profile is used, there is only one parameter for optimisation. Hence X_i is replaced by n in Equation 3.1

- Updates the learning rate according to Equation 2.4, using $\alpha = 0.5$ and $\beta = 1.4$.

- Continues the optimisation cycle until one of the following criteria is met:

  - Maximum number of iterations reached
  - $|\frac{C_d^{k+1} - C_d^k}{C_d^k}| \leq \tau$ *(tolerance value)*

3.2 Geometry Parameterization

This function is responsible for fitting the mathematical model using the inputs given by the optimisation algorithm. A NURBS Curve is chosen to represent the geometry because it is very flexible and can easily represent free form shapes. Before implementing the NURBS Curve, the geometry is modeled using a power-law profile ($y = ax^n$). Once the framework is complete, the power-law profile is replaced with a NURBS representation.

3.2.1 Implementation

- Accepts the control points and the total no. of points (or slices) required on the curve.

  Note: If a power law profile is to be fit, the value of n is accepted instead of the control points

- Fits a cubic NURBS curve using unit weights and a clamped knot vector [0,0,0,0,1,1,1,1].

Note: If a power law profile is to be fit, the algorithm sets $y = \frac{R}{L^n}x^n$, where R and L are the radius and length of the input cone

- Slices the curve into elements (frustums) and stores the coordinates of the elements in a matrix having separate columns for x and y.
- Returns the coordinate matrix to the optimisation algorithm.

3.3 Flow Solver

The flow solver is called by the optimisation algorithm to calculate the drag on a given element. It accepts the endpoints of the element (frustum) and calculates the drag after solving Taylor-Maccoll equations on an infinite imaginary cone passing through the element.

3.3.1 Implementation

- Accesses the freestream flow properties
- Accepts the endpoints of the element from the optimisation algorithm and considers an infinite cone with an attached shock passing through these points.
- Derives the shock angle using the cone angle and Mach number through an iterative trial and error method, as given below:
  - Assumes the initial shock angle to be $sin^{-1}(\frac{1}{M})$ (minimum angle of shock possible for the given Mach number)
  - Calculates the velocity after the shock, splits them into its r and θ components and normalizes them to V_{max} $(or \sqrt{2C_pT_o})$.
  - Uses the normalized velocity components as initial conditions and solves the Taylor-Maccoll equations iteratively using the Runge Kutta Method.

  - Stops the iterations when the normal velocity component becomes zero or crosses zero to negative, and records the cone angle.
  - Compares the calculated cone angle with the given cone angle. If they are very close, the assumed shock angle is accepted. If not, a new assumption is made by slightly incrementing the current shock angle and the Taylor-Maccoll equations are solved again.

- Calculates the Mach Number at the surface using the relation:

$$M_s = \sqrt{\frac{2}{(\gamma - 1)(V_s^{-2} - 1)}} \tag{3.2}$$

$$M_s = Surface\ Mach\ number,\ \ V_s = Surface\ velocity$$

- Derives the surface pressure using the surface Mach number and isentropic relations.

- Calculates the drag force on the element according to the equation given below and stores it.

$$Drag = Pcos(\theta)\pi(y_2^2 - y_1^2) \tag{3.3}$$

  where y_2 and y_1 are y coordinates of the endpoints of the element.

- Repeats the process for every element and outputs the total drag force on the body.

3.4 Conclusion

The optimisation algorithm is the backbone of this framework. It calls the functions Geometry Parameterization and Flow Solver during the execution. The algorithm has handles whose values can be varied to improve accuracy, but they come at the cost of an increase in execution time. The flow solver developed in this work assumes inviscid flow; hence only an approximate answer can be determined from this framework.

Chapter 4

Results and Discussion

A cone having a 240 mm length and a 63.5 mm base radius is fed into the ASO framework for optimisation. The supersonic flow is assumed to have a freestream pressure of 97kPa and a temperature of 300K. The fluid is considered to be a perfect gas with a heat capacity ratio, $\gamma = 1.4$ and the input geometry is discretized into 200 points. The value of ϵ is set to 0.001 and the tolerance value is set to 10^{-6}. The framework is run for Mach numbers 2,3,5,6 and 8 using both Power law and NURBS representation.

4.1 Results obtained using Power Law representation

The optimisation process for M = 2 completed in 1473.95 s with an average cycle time of 46.06 s. The results obtained for different Mach Numbers are tabulated in Table 4.1.

M	$C_{d,initial}$	$C_{d,optimised}$	n	Drag Reduction, %
2	0.1420	0.1245	0.734	11.99
3	0.0842	0.07502	0.750	13.60
5	0.0532	0.0450	0.767	15.15
6	0.0477	0.0402	0.768	15.76
8	0.0422	0.0353	0.776	16.52

Table 4.1 Results from ASO Framework using power law representation

- An average drag reduction of 14.6 % is found for the cone of the same fineness ratio (Length to Radius ratio).

- In all cases, the optimal body shape is convex (n < 1) and the value of n varies between 0.73 to 0.78.

- As Mach number increases, the value of drag coefficient decreases and the drag reduction increases. This result is in good agreement with the theoretical studies related to power law based configurations [5].

The value of n oscillated between the optimal value during the optimisation process and slowly converged to the optimal value of 0.767. It took 36 cycles to complete the optimisation for freestream Mach number 5. The values of n calculated in each cycle during optimisation for M = 5 are plotted in Fig. 4.1.

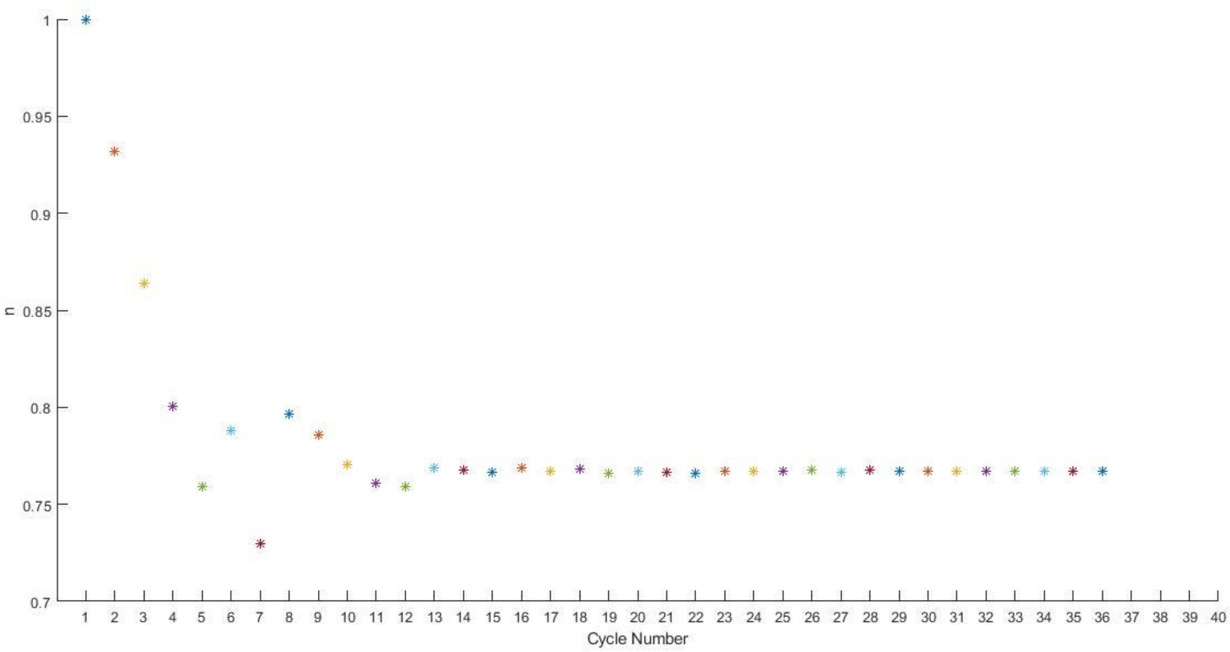


Fig. 4.1 Value of n vs cycle number(M=5)

The values of drag coefficient of power law profiles at M = 5 having n close to that of the optimised profile are tabulated in Table 4.2. These results exhibit a minimum drag coefficient(C_d) at n = 0.767.

n	C_d
0.765	0.45052
0.766	0.45051
0.767	0.45049
0.768	0.45050
0.769	0.45052

Table 4.2 Value of n vs C_d

4.1.1 Verification of results using ANSYS

The optimal shape is verified by performing traditional CFD simulations using ANSYS Fluent 18.2 for freestream Mach number 5, and power-law profiles having n = 0.765, 0.766 and 0.767. The simulations are performed on a 1000 mm X 400 mm grid with a maximum cell size of 10 mm and a minimum cell size of 0.3125 mm. The drag values of power-law profiles obtained from ANSYS are tabulated in Table 4.3(a). These values validate the minimum of drag coefficient obtained earlier at n = 0.767. Table 4.3(b) compares the C_d of the optimised profile obtained using tangent cone method with that obtained using ANSYS Fluent for M = 2. The results indicate that the drag reduction obtained from ANSYS is sufficiently close to that obtained using the Tangent cone method.

(a) n vs Drag (M = 5)

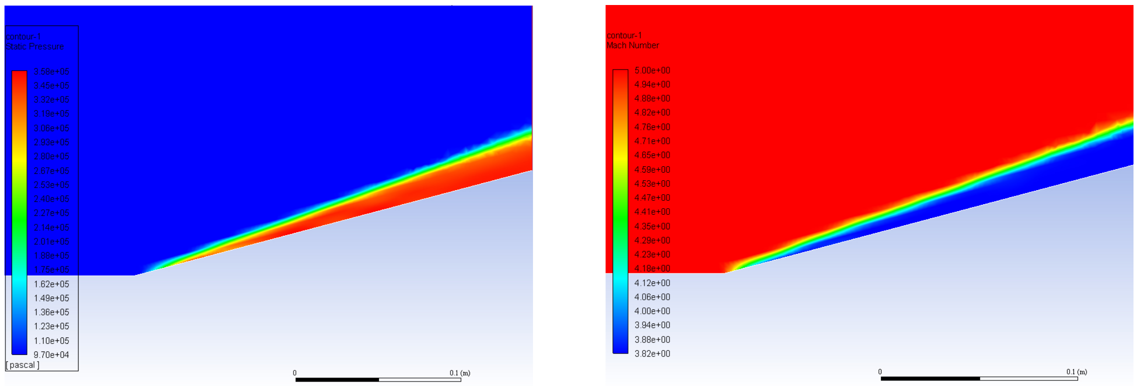
n	Drag (N)
0.765	2615.4889
0.766	2615.1860
0.767	2618.7691

(b) Drag Coefficient Comparison, (M = 2)

	$C_{d,Initial}$	$C_{d,optimised}$	Drag Reduction (%)
Tangent-cone	0.1420	0.1245	12.32
Fluent	0.196	0.157	19.89

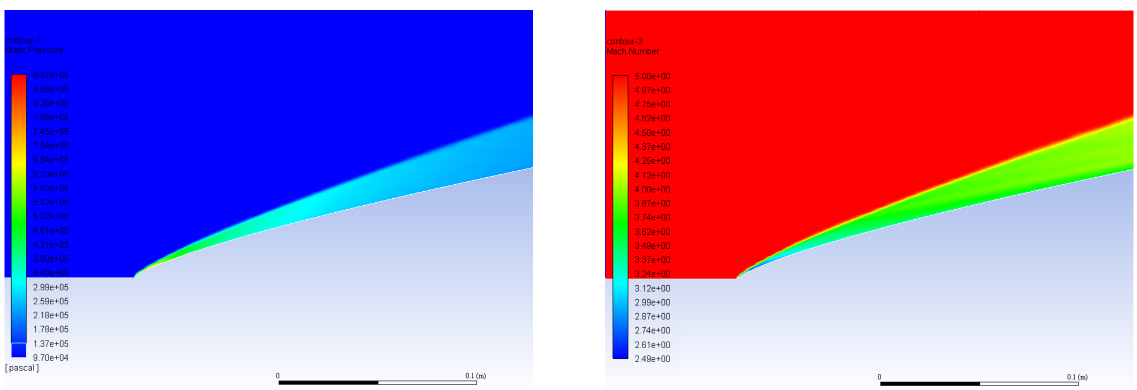
Table 4.3 Results from ANSYS Fluent 18.2

Figures 4.2(a) and 4.2(b) display the static pressure and Mach number distribution obtained from the ANSYS simulation of initial geometry and Figures 4.3(a) and 4.3(b) display the same for the optimal geometry. The reduction in drag on the optimised geometry is due to the decrease in area having high surface pressure. It is clearly observed from Fig. 4.2(a) that the static pressure is uniform on the surface of the initial cone and its magnitude is high, whereas in the optimised shape (Fig. 4.3(a)), the area bearing high pressure is lower and closer towards the nose. Further, the flow also expands in the optimised shape following the curvature of the body and results in lower surface pressure.



(a) Static Pressure (b) Mach Number

Fig. 4.2 Results from initial geometry (M = 5)



(a) Static Pressure (b) Mach Number

Fig. 4.3 Results from optimised geometry (M = 5)

4.2 Results obtained using NURBS representation

The optimisation process for M = 2 completed in 5212.44 seconds with an average cycle time of 32.57 seconds. The variation of each control point during the optimisation process is plotted in Fig. 4.4. An average drag reduction of 13.46% is obtained from the optimisation performed using Mach numbers 2,3,4,6,and 8 and the results are tabulated in Table 4.4. The optimal geometric profile is found to vary with Mach number but this variation reduces at higher Mach numbers (Fig 4.5). At high supersonic or hypersonic velocities, the profile becomes independent of the Mach number.

Fig. 4.6 shows the surface pressure distribution on both initial and final geometries analyzed at Mach number 6. It is clear that on the optimised geometry, the high surface pressure is distributed over a smaller area, whereas in the initial geometry it is evenly distributed. The optimised profile allows the flow to curves over its surface thus leading to flow expansion which results in lower surface pressure.

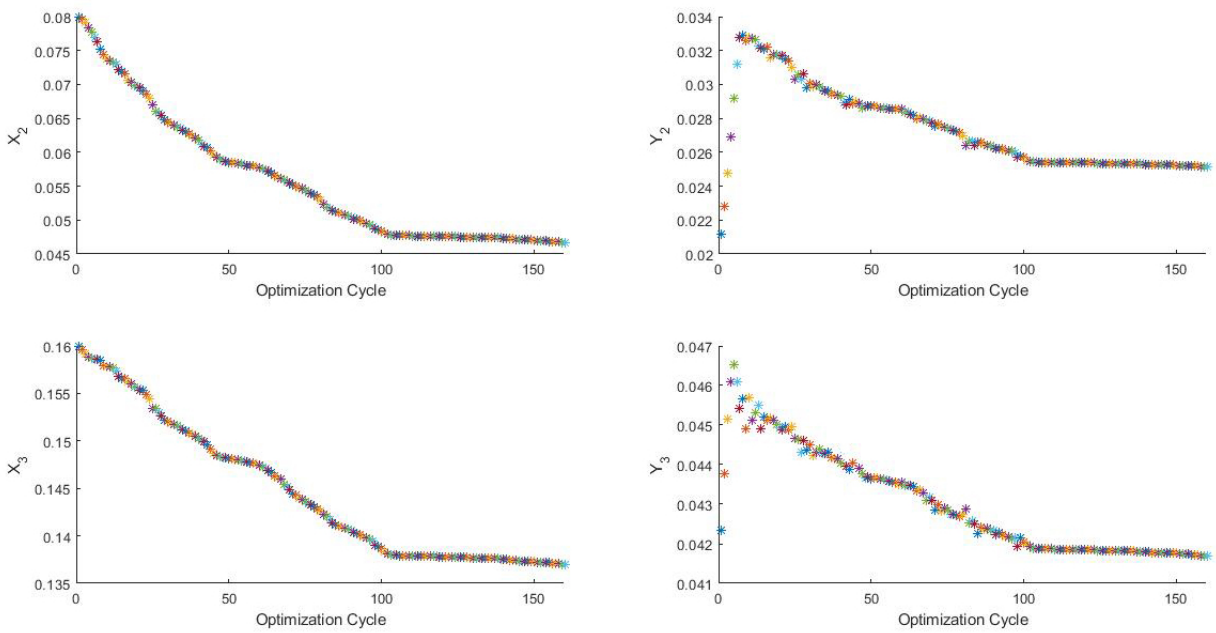


Fig. 4.4 Variation of control points during optimisation(M = 2)

M	C_d Initial	C_d optimised	X_2	Y_2	X_3	Y_3	Drag Reduction, %
2	0.1420	0.1254	0.02833	0.01884	0.12162	0.03932	11.69
3	0.0842	0.0733	0.02782	0.01830	0.13450	0.04159	12.94
5	0.0532	0.0459	0.04719	0.02300	0.13504	0.04051	13.72
6	0.0477	0.0410	0.05266	0.02444	0.13925	0.04100	14.04
8	0.0422	0.0359	0.04951	0.02346	0.14240	0.04204	14.93

Table 4.4 Results from ASO Framework using NURBS representation

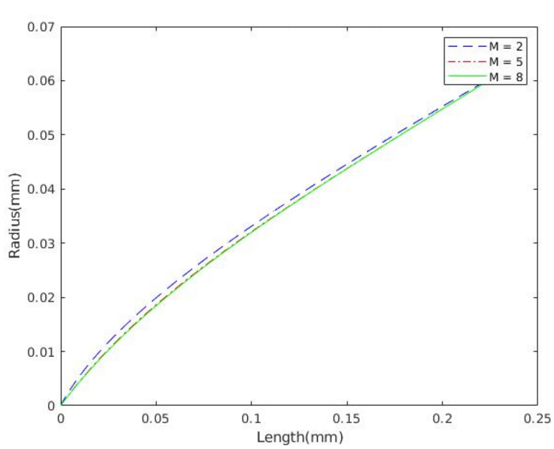


Fig. 4.5 Variation of optimal profile with Mach number

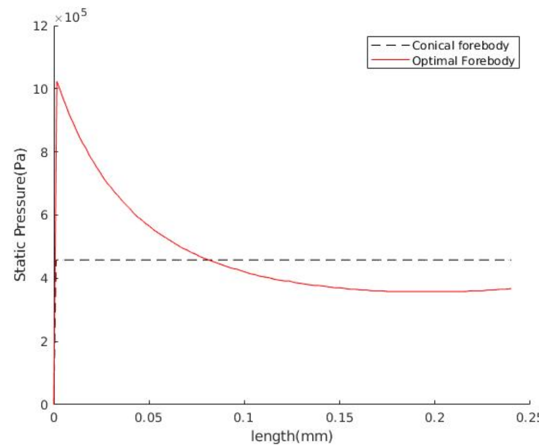
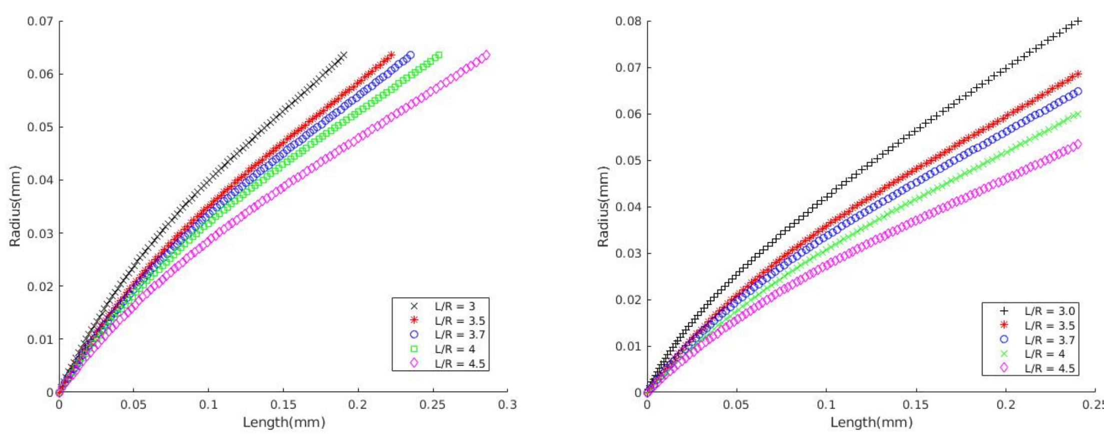


Fig. 4.6 Surface pressure distribution (M = 6)

The variation of optimal profile with fineness ratio is also analyzed at constant Mach number, M = 2. The results indicate a lower slope at the nose for a geometry with high fineness ratio (Fig. 4.7). It is also found that the volume of optimal shape reduces when the radius is varied and the length is fixed, but not necessarily so when the length is varied and the radius is fixed.



(a) a

(b) b

Fig. 4.7 Optimal shapes for different fineness ratio at M = 2 with variations in length(a) and radius (b).

Chapter 5

Conclusion and Future Work

An ASO framework is developed to reduce the wave drag on an axisymmetric forebody by modifying its shape. Assuming an initial geometry in inviscid supersonic flow with an attached shock constraint, the framework uses the tangent cone method and solves Taylor Maccoll equations to calculate the flow field around the body. It uses both Power law and NURBS curves for geometry parameterization and optimises the geometrical parameters using the steepest descent method. The NURBS representation has more degrees of freedom, hence it produced more reduction in drag. The optimisation process is quick and incurs low cost when compared to traditional methods. The optimal geometry derived is convex in nature and varies with Mach number, but this Mach number dependency reduces considerably at higher Mach numbers. Using the developed framework, under 15% drag reduction is achieved for an input cone of length 240mm and base radius 63.5mm. The framework is simple, efficient, and reliable and obtains a good estimate of the accurate optimal geometry. It has applications in bullets, missiles, and rocket industries and can be used in the design of long-range vehicles. In the future, the inviscid regime can be extended to the viscous regime for a more accurate and realistic solution by incorporating the boundary layer equations. If extended, it can provide solutions that are very close to that produced by traditional CFD solvers.

References

[1] G. I. Taylor and J. W. Maccoll, "The air pressure on a cone moving at high speeds —i," *Proc. R. Soc. Lond*, vol. A, no. 139, p. 278–297, 1997.

[2] J. D. Anderson, *Fundamentals of Aerodynamics*. McGraw-Hill, 2010.

[3] L. Piegl and W. Tiller, *The NURBS Book*. Springer, 1996.

[4] G. Natrajan, N. Sahoo, and V. Kulkarni, "Optimal forebody shape for minimum drag in supersonic flow," *J. Inst. Eng. India Ser. C*, vol. 96, no. 1, pp. 5–11, 2015.

[5] A. Eggers, M. Resnikoff, and D. Dennis, "Bodies of revolution having minimum drag at high supersonic airspeeds," NACA TR-1306, Tech. Rep., 1958.

[6] S. Brahmachary, G. Natarajan, and N. Sahoo, "On maximum ballistic coefficient axisymmetric geometries in hypersonic flows," *Journal of Spacecraft and Rockets*, vol. 55, no. 2, pp. 518–521, 2018.