In [1]:
```
import numpy as np
import math
import matplotlib.pyplot as plt
import random
from scipy.stats import binom
import pandas as pd
```

Assignment Requirement

A popular TCP protocol is the TCP Reno, which uses the Additive-IncreaseMultiplicative-Decrease (AIMD) algorithm to control the amount of data sent. The AIMD algorithm controls the congestion window with two parameters. For source s, let the additive and the multiplicative parameters be set as, respectively, alpha and 0 < beta < 1. Typically, these two additive and the multiplicative parameters are set as 1 and 0.5, and this leads to a trajectory as shown below for two senders over a common switching link. Discuss how you might set the AIMD parameters in a data center environment with its integrated tiered architecture. Will fairness still be important in a data center?

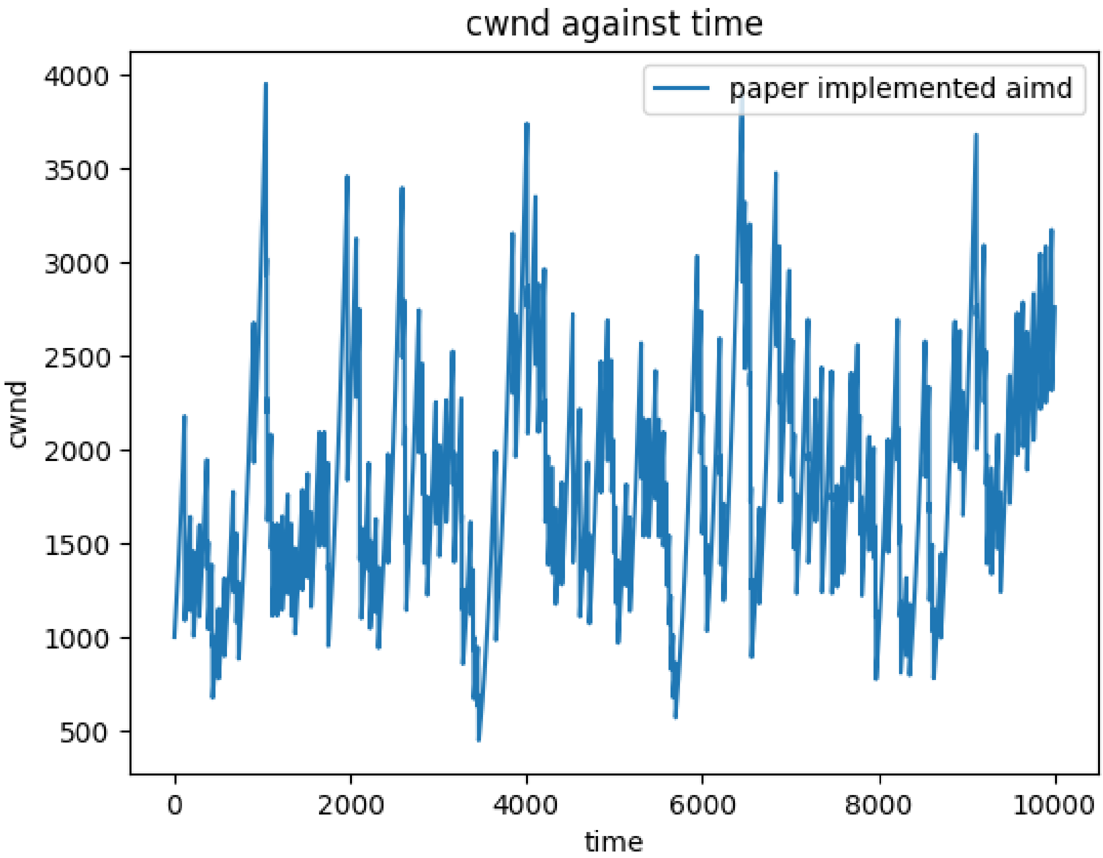
Assignment 1 is Question 3 of Tutorial 1 on the AIMD mechanism of TCP and is Individual Submission by each student (not group work). Please submit a report (not more than five pages) written in Microsoft Word or Latex (https://www.overleaf.com/ (https://www.overleaf.com/)). Your report should consist of a section where you explore tuning AIMD parameters. For example, I demonstrated a code with the alpha parameter bring the logarithmic function of window size in the tutorial class. For reference, see https://www.evl.uic.edu/eric/atp/HighSpeedTCP.pdf (https://www.evl.uic.edu/eric/atp/HighSpeedTCP.pdf) and consider different functions to design alpha and beta parameters. Your resport should have a section of numerical examples and experiments with different number of TCP users/flows sharing a single bottleneck. Show that the TCP dynamics converge numerically to some solution (e.g., the right eigenvector of the matrix A in the Lecture slides). Lastly, your report should have discussions based on your numerical experiments and a conclusion (attaching your code as Appendix). You may write your code in any programming language (we have demonstrated a Matlab/Python code in tutorial). You can use Python Jupyther Notebook and attach it as Code in Appendix or convert into a readable PDF or Word document for upload.

In [2]:
```
# simulate congestion probability: 90% of not dropping
def next_packet(cwnd, p=0.1):
    binomial_dist = binom(cwnd, p)
    packets_dropped = binomial_dist.ppf(random.random() )
    if packets_dropped == 0:
        return (packets_dropped, "Transfer")
    else:
        return (packets_dropped, "Dropped")
```

In [3]:

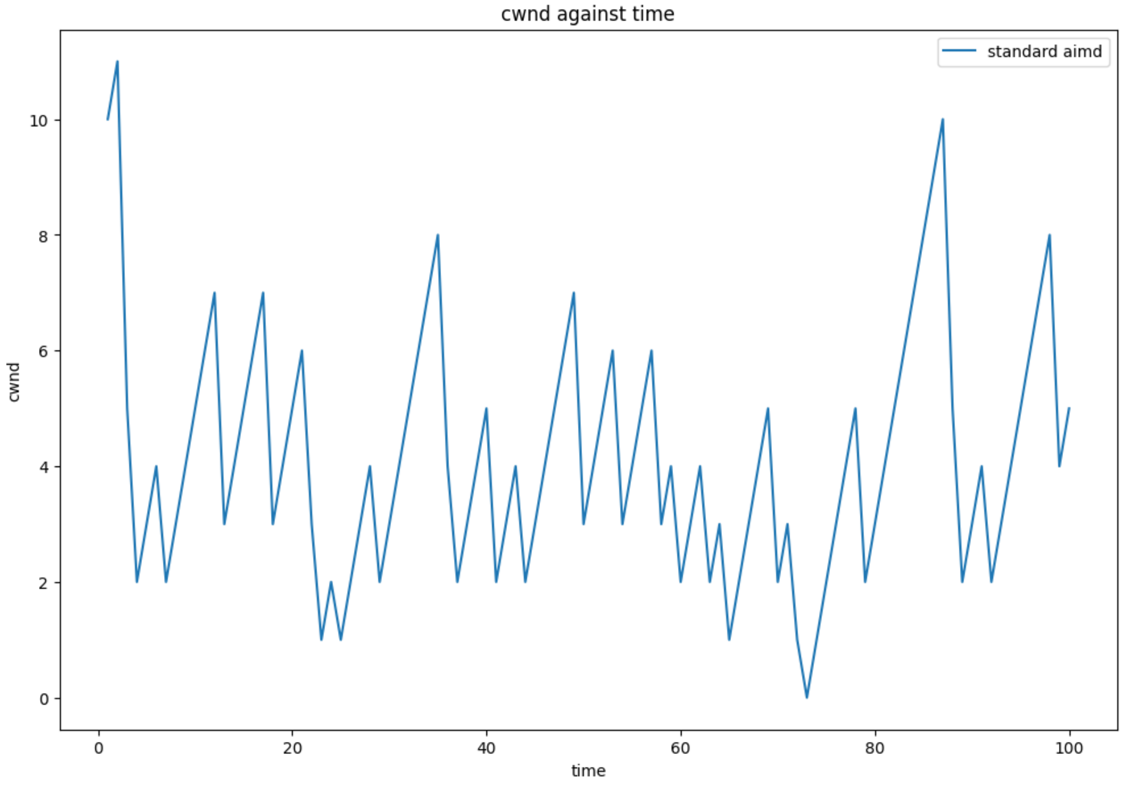
```python
def paper_aimd(rtt_tracker, data_packet, cwnd=10):
    def b(cwnd):
        return (0.1 - 0.5) * (math.log10(cwnd) - math.log10(38))

    _, status = data_packet
    if status == 'Transfer':
        if (2 - b(cwnd)) > 0:
            a = cwnd**2 * (0.078 / cwnd**1.2) * 2 * b(cwnd) / (2
        cwnd += round(a)

    else:
        cwnd = round(cwnd * (1-b(cwnd)))
        if cwnd<1: cwnd=1
    rtt_tracker.append(cwnd)
    return cwnd


ITERATIONS = 10000
cwnd = 1000
pdr = [0, 0]
rtt_tracker = [cwnd]

# iterate
for i in range(ITERATIONS):
    data_packet = next_packet(cwnd, p=0.00001)
    pdr[0] += cwnd
    pdr[1] += data_packet[0]
    cwnd = paper_aimd(rtt_tracker, data_packet, cwnd=cwnd)

print("packet drop rate for highspeed aimd is", pdr[1]/pdr[0])

# visualise
x, y = [], []
for i in range(ITERATIONS):
    x.append(i + 1)
    y.append(rtt_tracker[i])

plt.plot(x, y, label="paper implemented aimd")
plt.xlabel("time")
plt.ylabel("cwnd")
plt.legend()
plt.title(f"cwnd against time")
plt.show()
```

```
packet drop rate for highspeed aimd is 9.573768705297681e-06
```


cwnd against time
paper implemented aimd
cwnd
time
4000
3500
3000
2500
2000
1500
1000
500
0
2000
4000
6000
8000
10000

In [4]:

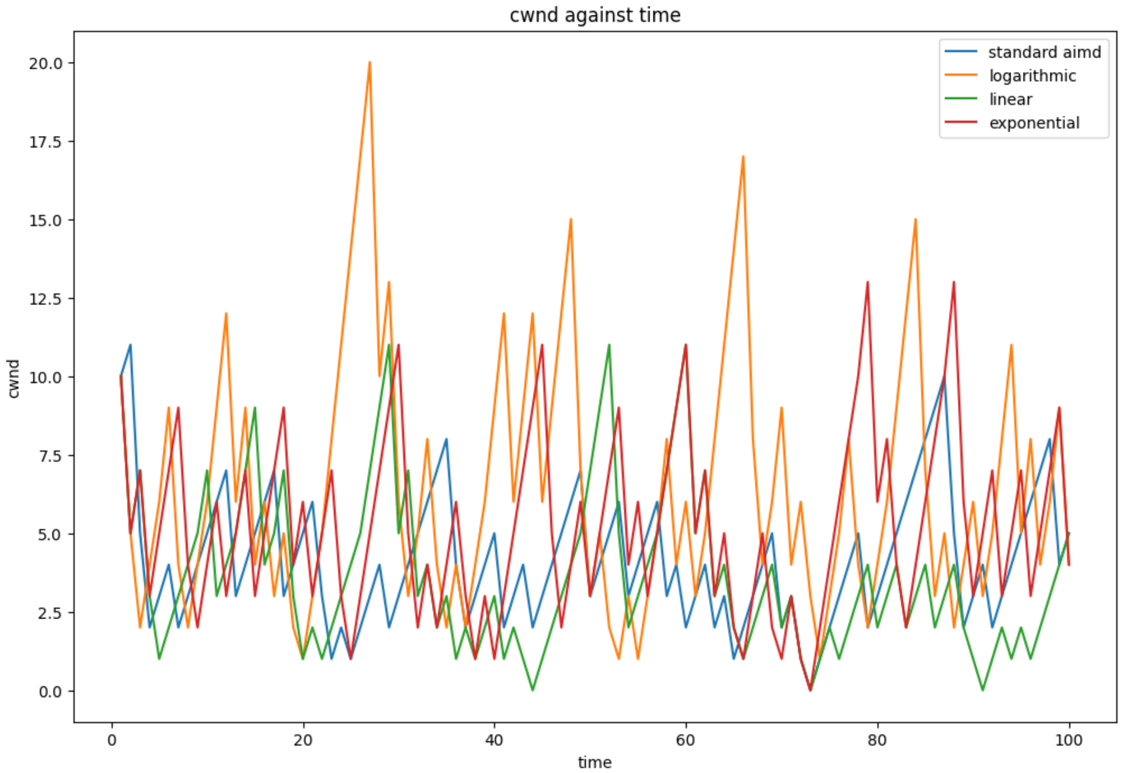
```
# standard tcp congestion control
def standard_aimd(rtt_tracker, data_packet, cwnd=10, a=1, b=0.5)
    _, status = data_packet
    if status == 'Transfer':
        cwnd += a
    else:
        cwnd = math.floor(cwnd * b)
    rtt_tracker.append(cwnd)
    return cwnd


ITERATIONS = 10000
pdr = []
cwnd_1 = 10
pdr_1 = [0, 0]

rtt_tracker_1 = [cwnd_1]

# iterate
for i in range(ITERATIONS):
    data_packet = next_packet(cwnd_1)
    pdr_1[0] += cwnd_1
    pdr_1[1] += data_packet[0]
    cwnd_1 = standard_aimd(rtt_tracker_1, data_packet, cwnd=cwnd

pdr.append(pdr_1)

# visualise
x, y1 = [], []
for i in range(ITERATIONS):
    x.append(i + 1)
    y1.append(rtt_tracker_1[i])

plt.figure(figsize=(12,8))
plt.plot(x[:100], y1[:100], label="standard aimd")
plt.xlabel("time")
plt.ylabel("cwnd")
plt.legend()
plt.title(f"cwnd against time")
plt.show()
```


cwnd against time
standard aimd
cwnd
time
0
2
4
6
8
10
0
20
40
60
80
100

In [5]:

```
# highspeed tcp as per https://www.evl.uic.edu/eric/atp/HighSpee
# w += a(w), a(w) increases with w; (1-b(w))*w, b(w) decreases w
def ai_logarithmic(rtt_tracker, data_packet, cwnd=10, b=0.5):
    _, status = data_packet
    if status == 'Transfer':
        cwnd += math.ceil(math.log(cwnd+math.e)) # a(w) is a log
    else:
        cwnd = math.floor(cwnd * b)
    rtt_tracker.append(cwnd)
    return cwnd


def ai_linear(rtt_tracker, data_packet, cwnd=10, b=0.5, denom=5)
    _, status = data_packet
    if status == 'Transfer': # a(w) is a linear function, mentic
        cwnd += cwnd//denom + 1
    else:
        cwnd = math.floor(cwnd * b)
    rtt_tracker.append(cwnd)
    return cwnd


def ai_exponential(rtt_tracker, data_packet, cwnd=10, b=0.5, lam
    _, status = data_packet
    if status == 'Transfer': # a(w) is an exponential function
        cwnd += math.floor(math.exp(lambda_*cwnd+1))
    else:
        cwnd = math.floor(cwnd * b)
    rtt_tracker.append(cwnd)
    return cwnd



cwnd_2 = cwnd_3 = cwnd_4 = 10
pdr_2, pdr_3, pdr_4 = [0, 0], [0, 0], [0, 0]
rtt_tracker_2, rtt_tracker_3, rtt_tracker_4 = [cwnd_2], [cwnd_3]


# iterate
for i in range(ITERATIONS):
    data_packet = next_packet(cwnd_2)
    pdr_2[0] += cwnd_2
    pdr_2[1] += data_packet[0]
    cwnd_2 = ai_logarithmic(rtt_tracker_2, data_packet, cwnd=cwn

    data_packet = next_packet(cwnd_3)
    pdr_3[0] += cwnd_3
    pdr_3[1] += data_packet[0]
    cwnd_3 = ai_linear(rtt_tracker_3, data_packet, cwnd=cwnd_3)

    data_packet = next_packet(cwnd_4)
    pdr_4[0] += cwnd_4
    pdr_4[1] += data_packet[0]
    cwnd_4 = ai_exponential(rtt_tracker_4, data_packet, cwnd=cwn

pdr.extend([pdr_2, pdr_3, pdr_4])

# visualise
y2, y3, y4 = [], [], []
for i in range(ITERATIONS):
    y2.append(rtt_tracker_2[i])
```

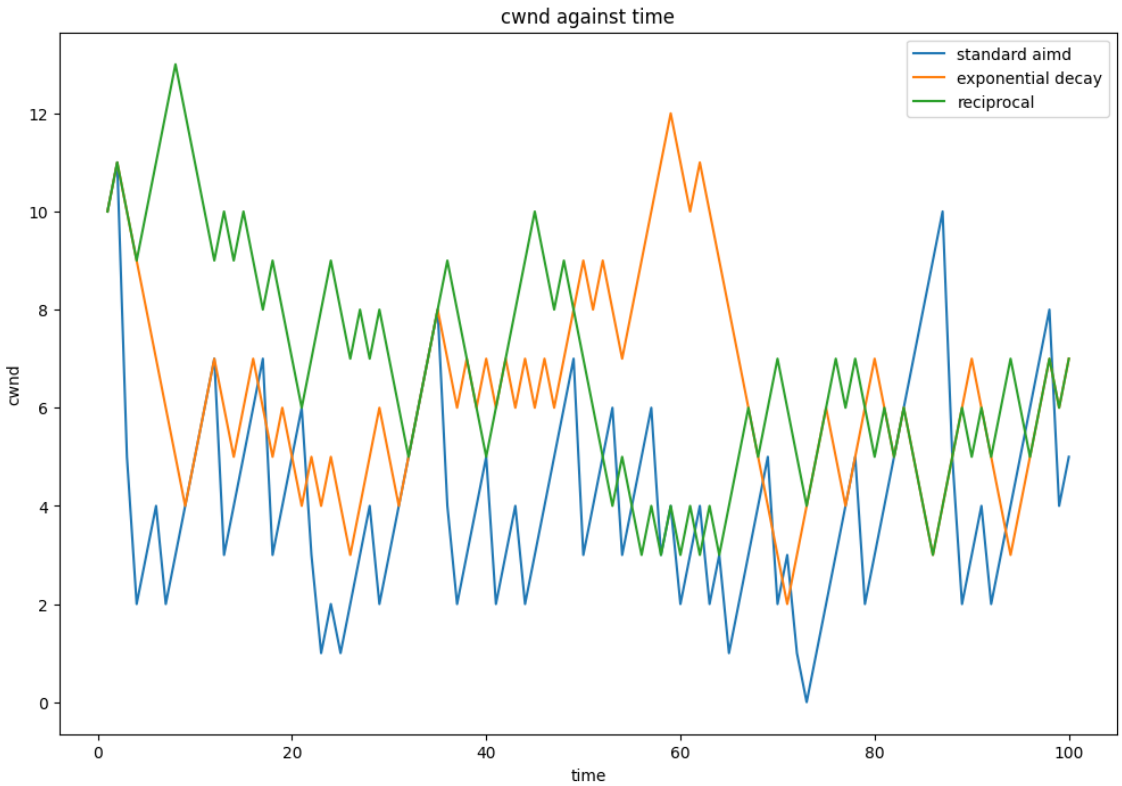
```python
    y3.append(rtt_tracker_3[i])
    y4.append(rtt_tracker_4[i])

plt.figure(figsize=(12,8))
plt.plot(x[:100], y1[:100], label="standard aimd")
plt.plot(x[:100], y2[:100], label="logarithmic")
plt.plot(x[:100], y3[:100], label="linear")
plt.plot(x[:100], y4[:100], label="exponential")
plt.xlabel("time")
plt.ylabel("cwnd")
plt.legend()
plt.title(f"cwnd against time")
plt.show()
```



In [6]:

```
# highspeed tcp as per https://www.evl.uic.edu/eric/atp/HighSpee
# w += a(w), a(w) increases with w; (1-b(w))*w, b(w) decreases w
def md_exponential_decay(rtt_tracker, data_packet, cwnd=10, a=1,
    _, status = data_packet
    if status == 'Transfer':
        cwnd += a
    else:
        cwnd = math.floor(cwnd * (1-math.exp(-lambda_ * cwnd)))
        if cwnd<1: cwnd=1
    rtt_tracker.append(cwnd)
    return cwnd


def md_reciprocal(rtt_tracker, data_packet, cwnd=10, a=1):
    _, status = data_packet
    if status == 'Transfer':
        cwnd += a
    else:
        cwnd = math.ceil(cwnd * (1-1/cwnd)) #b(w) is a reciproca
    rtt_tracker.append(cwnd)
    return cwnd




cwnd_5 = cwnd_6 = 10
pdr_5, pdr_6 = [0, 0], [0, 0]
rtt_tracker_5, rtt_tracker_6 = [cwnd_5], [cwnd_6]


# iterate
for i in range(ITERATIONS):
    data_packet = next_packet(cwnd_5)
    pdr_5[0] += cwnd_5
    pdr_5[1] += data_packet[0]
    cwnd_5 = md_exponential_decay(rtt_tracker_5, data_packet, cw

    data_packet = next_packet(cwnd_6)
    pdr_6[0] += cwnd_6
    pdr_6[1] += data_packet[0]
    cwnd_6 = md_reciprocal(rtt_tracker_6, data_packet, cwnd=cwnd

pdr.extend([pdr_5, pdr_6])
# visualise
y5, y6 = [], []
for i in range(ITERATIONS):
    y5.append(rtt_tracker_5[i])
    y6.append(rtt_tracker_6[i])

plt.figure(figsize=(12,8))
plt.plot(x[:100], y1[:100], label="standard aimd")
plt.plot(x[:100], y5[:100], label="exponential decay")
plt.plot(x[:100], y6[:100], label="reciprocal")
plt.xlabel("time")
plt.ylabel("cwnd")
plt.legend()
plt.title(f"cwnd against time")
```

```
plt.show()
```



In [7]:

```
result = {
    "aimd": [
        "standard",
        "ai_logarithmic",
        "ai_linear",
        "ai_exponential",
        "md_exponential_decay",
        "md_reciprocal",
    ],

    "total_packets_dropped": [x[1] for x in pdr],
    "total_cwnd": [x[0] for x in pdr]
}
result = pd.DataFrame(result)
result["packet_drop_rate"] = result["total_packets_dropped"]/res
result["avg_cwnd"] = result["total_cwnd"]/ITERATIONS
result
```

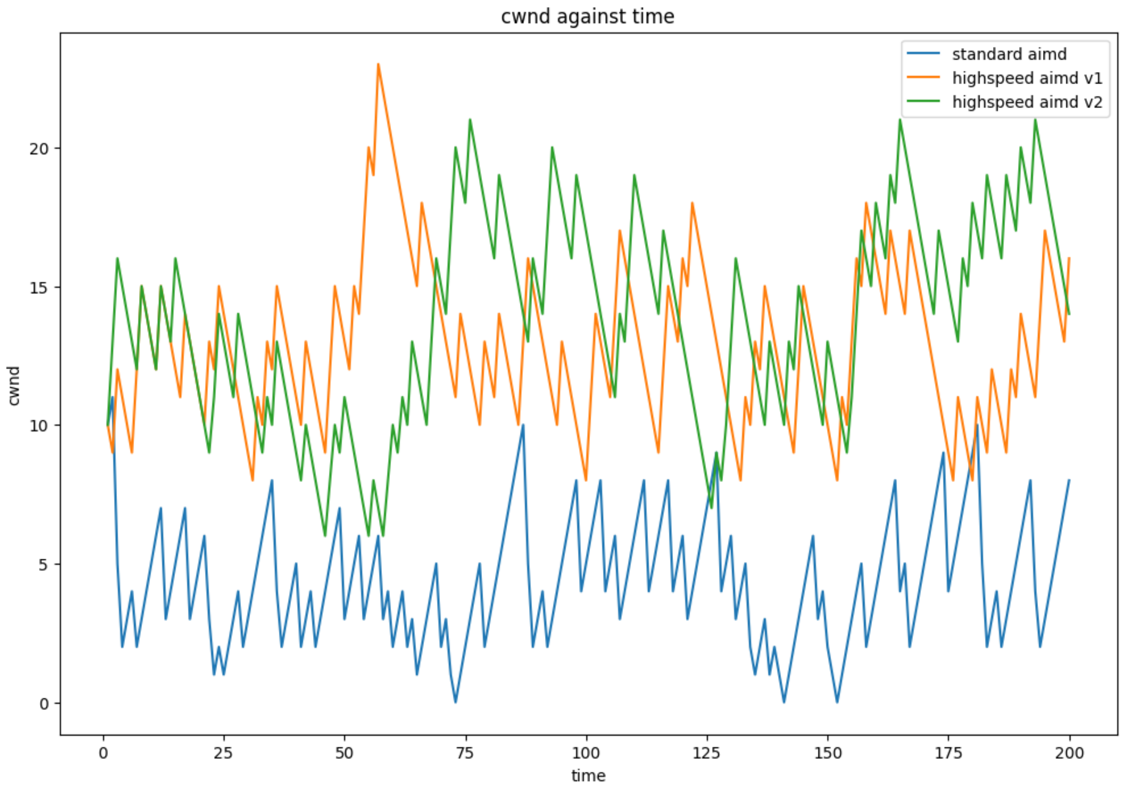
Out[7]:

	aimd	total_packets_dropped	total_cwnd	packet_drop_rate	avg_cwnd
0	standard	3499.0	35057	0.099809	3.5057
1	ai_logarithmic	5308.0	52705	0.100712	5.2705
2	ai_linear	3721.0	37051	0.100429	3.7051
3	ai_exponential	4947.0	49185	0.100579	4.9185
4	md_exponential_decay	6836.0	68606	0.099641	6.8606
5	md_reciprocal	6844.0	68416	0.100035	6.8416

In [8]:

```python
# highspeed tcp as per https://www.evl.uic.edu/eric/atp/HighSpee
# w += a(w), a(w) increases with w; (1-b(w))*w, b(w) decreases w
def highspeed_aimd_v1(rtt_tracker, data_packet, cwnd=10, lambda_
    """
    a(w): logarithmic
    b(w): exponential decay
    """
    _, status = data_packet
    if status == 'Transfer':
            cwnd += math.ceil(math.log(cwnd+math.e)) # a(w) is a
    else:
        cwnd = math.floor(cwnd * (1-math.exp(-lambda_ * cwnd)))
        if cwnd<1: cwnd=1
    rtt_tracker.append(cwnd)
    return cwnd


def highspeed_aimd_v2(rtt_tracker, data_packet, cwnd=10, lambda_
    """
    a(w): exponential
    b(w): reciprocal
    """
    _, status = data_packet
    if status == 'Transfer':
        cwnd += math.floor(math.exp(lambda_*cwnd+1))
    else:
        cwnd = math.ceil(cwnd * (1-1/cwnd))
    rtt_tracker.append(cwnd)
    return cwnd



cwnd_7 = cwnd_8 = 10
pdr_7, pdr_8 = [0, 0], [0, 0]
rtt_tracker_7, rtt_tracker_8 = [cwnd_7], [cwnd_8]

# iterate
for i in range(ITERATIONS):
    data_packet = next_packet(cwnd_7)
    pdr_7[0] += cwnd_7
    pdr_7[1] += data_packet[0]
    cwnd_7 = highspeed_aimd_v1(rtt_tracker_7, data_packet, cwnd=

    data_packet = next_packet(cwnd_8)
    pdr_8[0] += cwnd_8
    pdr_8[1] += data_packet[0]
    cwnd_8 = highspeed_aimd_v2(rtt_tracker_8, data_packet, cwnd=


pdr.extend([pdr_7, pdr_8])

# visualise
y7, y8 = [], []
for i in range(ITERATIONS):
    y7.append(rtt_tracker_7[i])
    y8.append(rtt_tracker_8[i])

plt.figure(figsize=(12,8))
plt.plot(x[:200], y1[:200], label="standard aimd")
plt.plot(x[:200], y7[:200], label="highspeed aimd v1")
plt.plot(x[:200], y8[:200], label="highspeed aimd v2")
```

```
plt.xlabel("time")
plt.ylabel("cwnd")
plt.legend()
plt.title(f"cwnd against time")
plt.show()
```



In [9]:

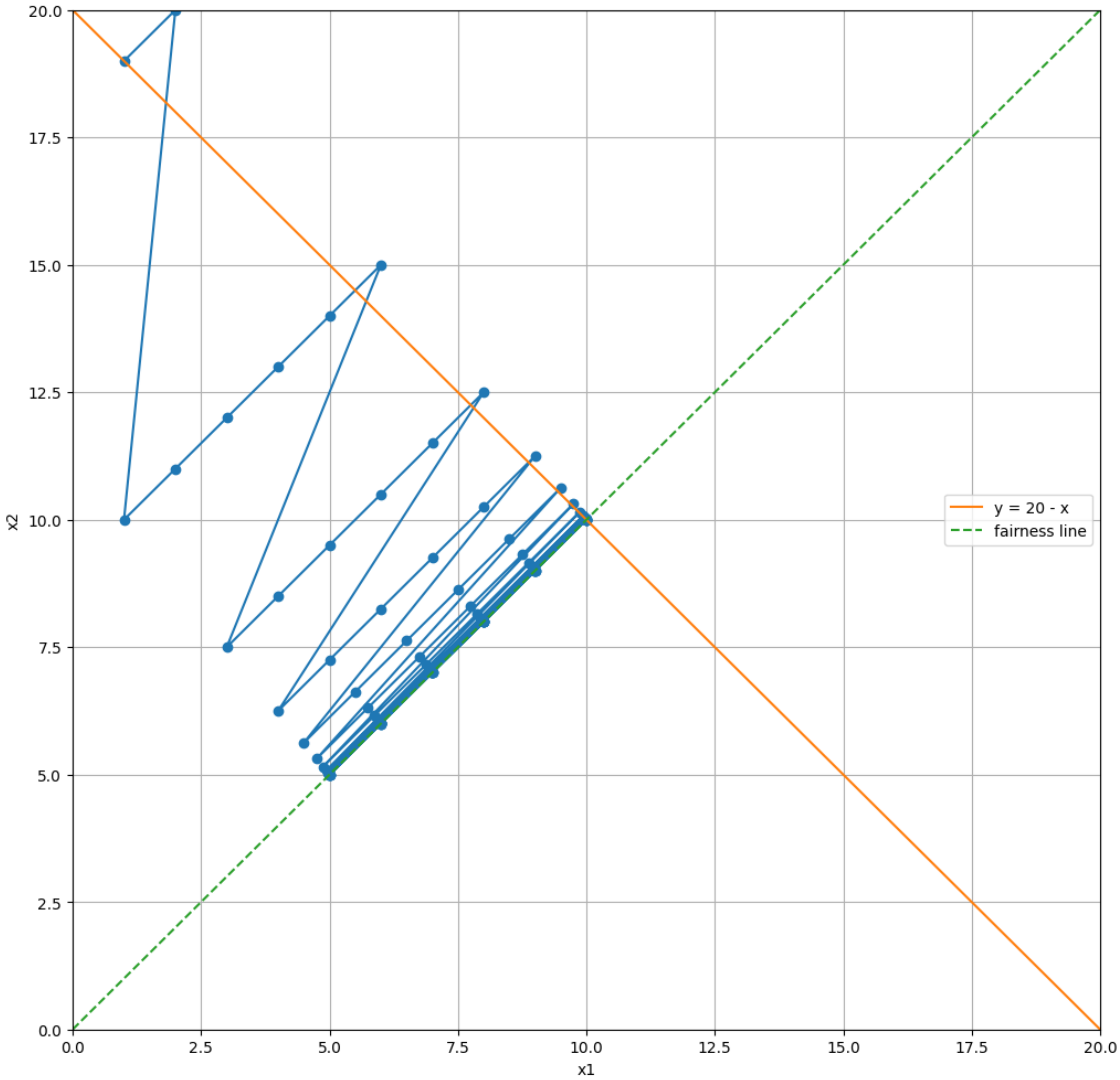
```
result = {
    "aimd": [
        "standard",
        "ai_logarithmic",
        "ai_linear",
        "ai_exponential",
        "md_exponential_decay",
        "md_reciprocal",
        "highspeed_aimd_v1",
        "highspeed_aimd_v2"
    ],

    "total_packets_dropped": [x[1] for x in pdr],
    "total_cwnd": [x[0] for x in pdr]
}
result = pd.DataFrame(result)
result["packet_drop_rate"] = result["total_packets_dropped"]/res
result["avg_cwnd"] = result["total_cwnd"]/ITERATIONS
result
```

Out[9]:

	aimd	total_packets_dropped	total_cwnd	packet_drop_rate	avg_cwnd
0	standard	3499.0	35057	0.099809	3.5057
1	ai_logarithmic	5308.0	52705	0.100712	5.2705
2	ai_linear	3721.0	37051	0.100429	3.7051
3	ai_exponential	4947.0	49185	0.100579	4.9185
4	md_exponential_decay	6836.0	68606	0.099641	6.8606
5	md_reciprocal	6844.0	68416	0.100035	6.8416
6	highspeed_aimd_v1	13982.0	140007	0.099866	14.0007
7	highspeed_aimd_v2	13175.0	131454	0.100225	13.1454

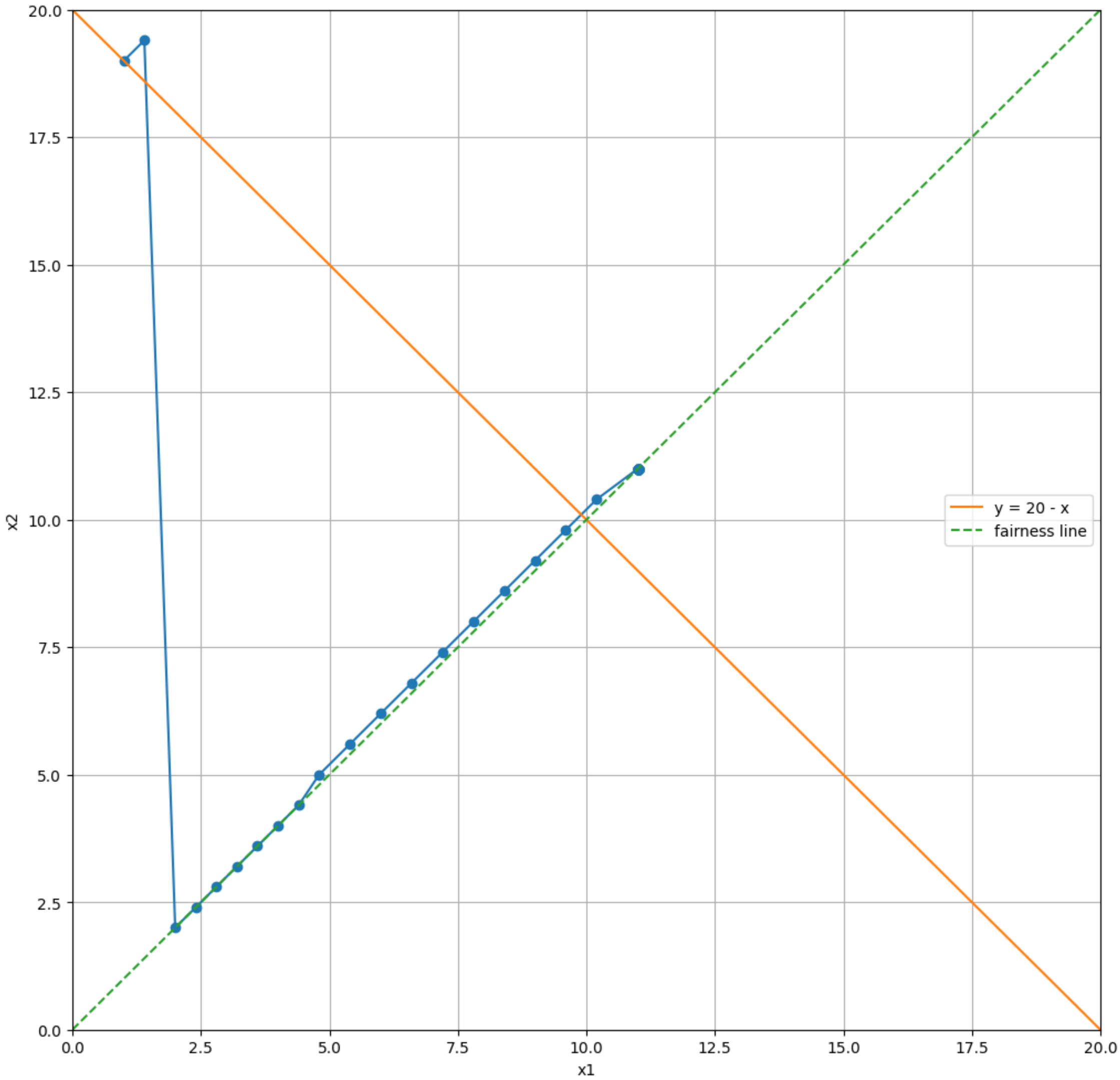
In [10]:

```
ITERATESMAX = 100
C = 20
alpha = 1
beta = 0.5
x1 = 1
x2 = 19
x1_values = np.zeros(ITERATESMAX)
x2_values = np.zeros(ITERATESMAX)
for i in range(ITERATESMAX):
    x1_values[i] = x1
    x2_values[i] = x2
    if (x1 + x2 <= C):
        x1 += alpha
        x2 += alpha
    else:
        x1 *= beta
        x2 *= beta

plt.figure(figsize=(12,12))
plt.plot(x1_values, x2_values, marker='o', linestyle='-')
plt.xlabel('x1')
plt.ylabel('x2')
x = np.linspace(0, C, 100)
y = C - x # capacity
y2 = x # fairness line
plt.plot(x, y, label=f'y = {C} - x')
plt.plot(x, y2, linestyle="--", label='fairness line')
plt.grid(True)
plt.xlim(0, 20)
plt.ylim(0, 20)
plt.legend()
plt.show()
```



20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
x2
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
x1
y = 20 - x
fairness line

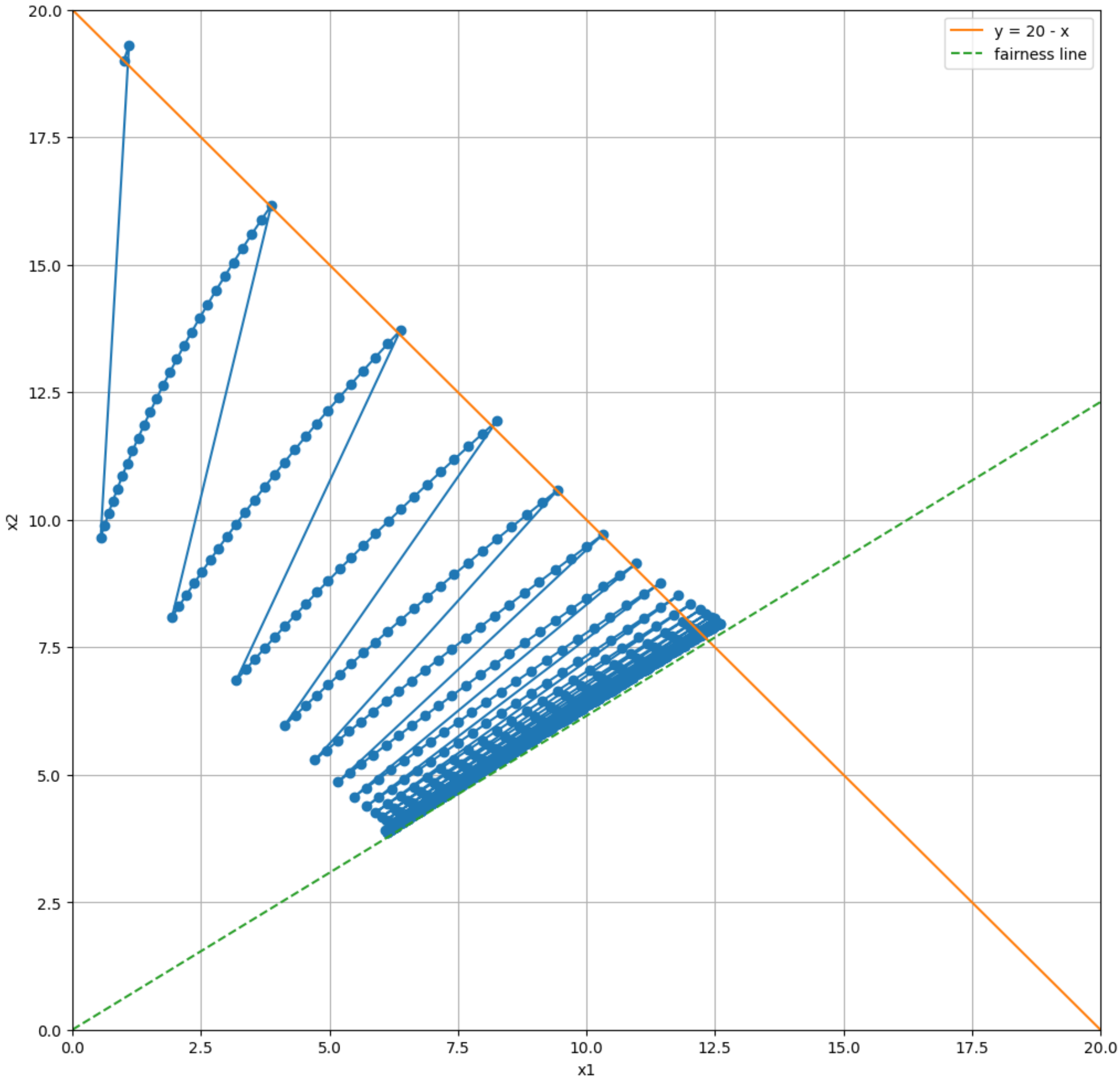
In [11]:
```
ITERATESMAX = 100
C = 20
alpha = 0.2
lambda_ = 0.01
beta = 2
x1 = 1
x2 = 19
x1_values = np.zeros(ITERATESMAX)
x2_values = np.zeros(ITERATESMAX)
for i in range(ITERATESMAX):
    x1_values[i] = x1
    x2_values[i] = x2
    if (x1 + x2 <= C):
        x1 += alpha*math.ceil(math.log(x1+math.e))
        x2 += alpha*math.ceil(math.log(x1+math.e))
    else:
        x1 = math.ceil(x1 * (1-math.exp(-beta * x1)))
        x2 = math.ceil(x1 * (1-math.exp(-beta * x1)))

plt.figure(figsize=(12,12))
plt.plot(x1_values, x2_values, marker='o', linestyle='-')
plt.xlabel('x1')
plt.ylabel('x2')
x = np.linspace(0, C, 100)
y = C - x # capacity
y2 = x # fairness line
plt.plot(x, y, label=f'y = {C} - x')
plt.plot(x, y2, linestyle="--", label='fairness line')
plt.grid(True)
plt.xlim(0, 20)
plt.ylim(0, 20)
plt.legend()
plt.show()
```



20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
x2
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
x1
y = 20 - x
fairness line

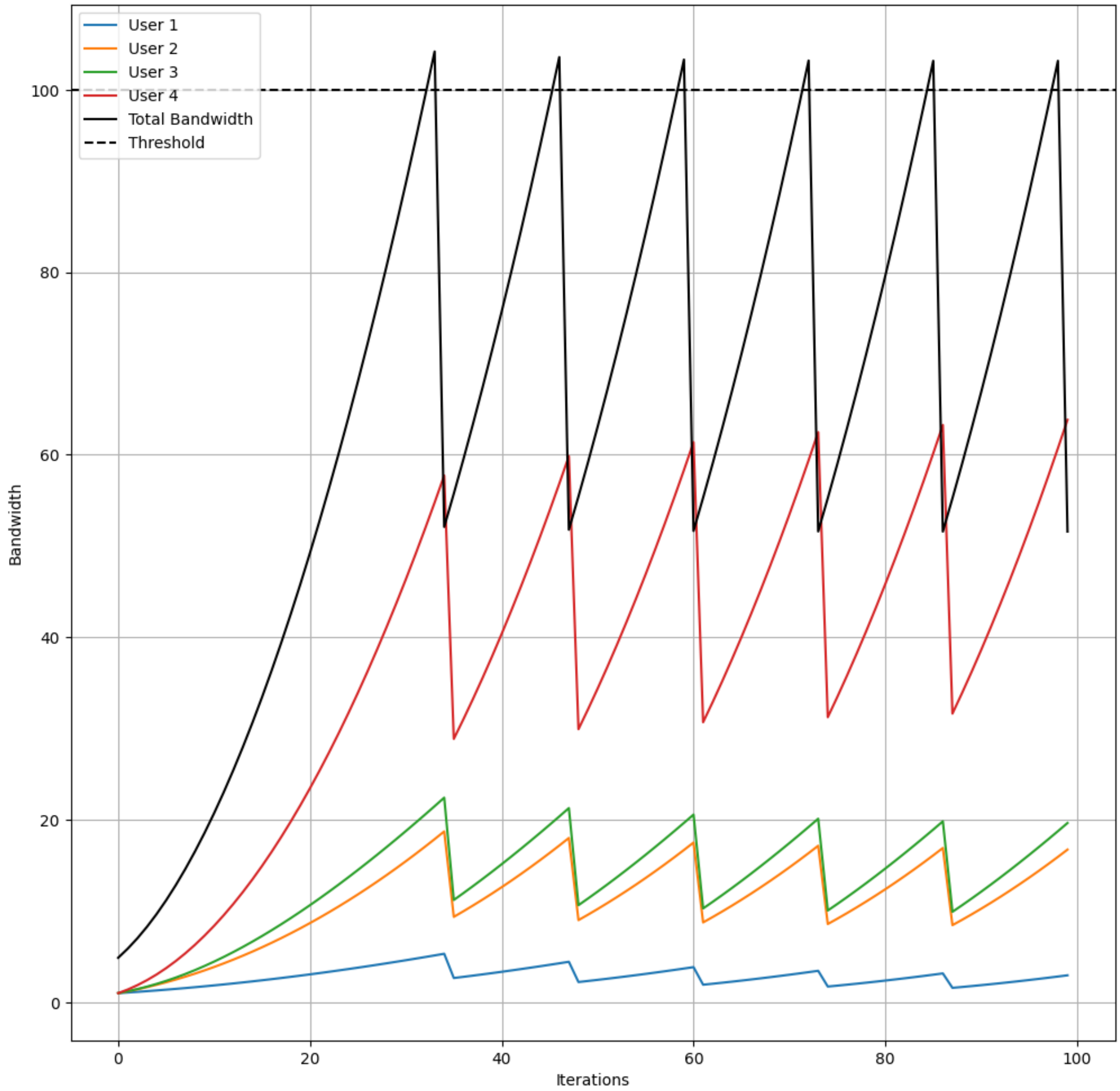
In [12]:

```
ITERATESMAX = 1000
C = 20
alpha = 0.1
beta = 0.5
x1 = 1
x2 = 19
alpha1 = 0
alpha2 = 0
x1_values = np.zeros(ITERATESMAX)
x2_values = np.zeros(ITERATESMAX)
for i in range(ITERATESMAX):
    x1_values[i] = x1
    x2_values[i] = x2
    if (x1 + x2 <= C):
        x1 += alpha * np.power(x1, beta)
        x2 += alpha * np.log(x2 + 1)
    else:
        x1 = x1 * beta
        x2 = x2 * beta


plt.figure(figsize=(12,12))
plt.plot(x1_values, x2_values, marker='o', linestyle='-')
plt.xlabel('x1')
plt.ylabel('x2')
x = np.linspace(0, C, 100)
y = C - x # capacity
y2 = 8/13*x # fairness line
plt.plot(x, y, label=f'y = {C} - x')
plt.plot(x, y2, linestyle="--", label='fairness line')
plt.grid(True)
plt.xlim(0, C)
plt.ylim(0, C)
plt.legend()
plt.show()
```


y = 20 - x
fairness line
x1
x2
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

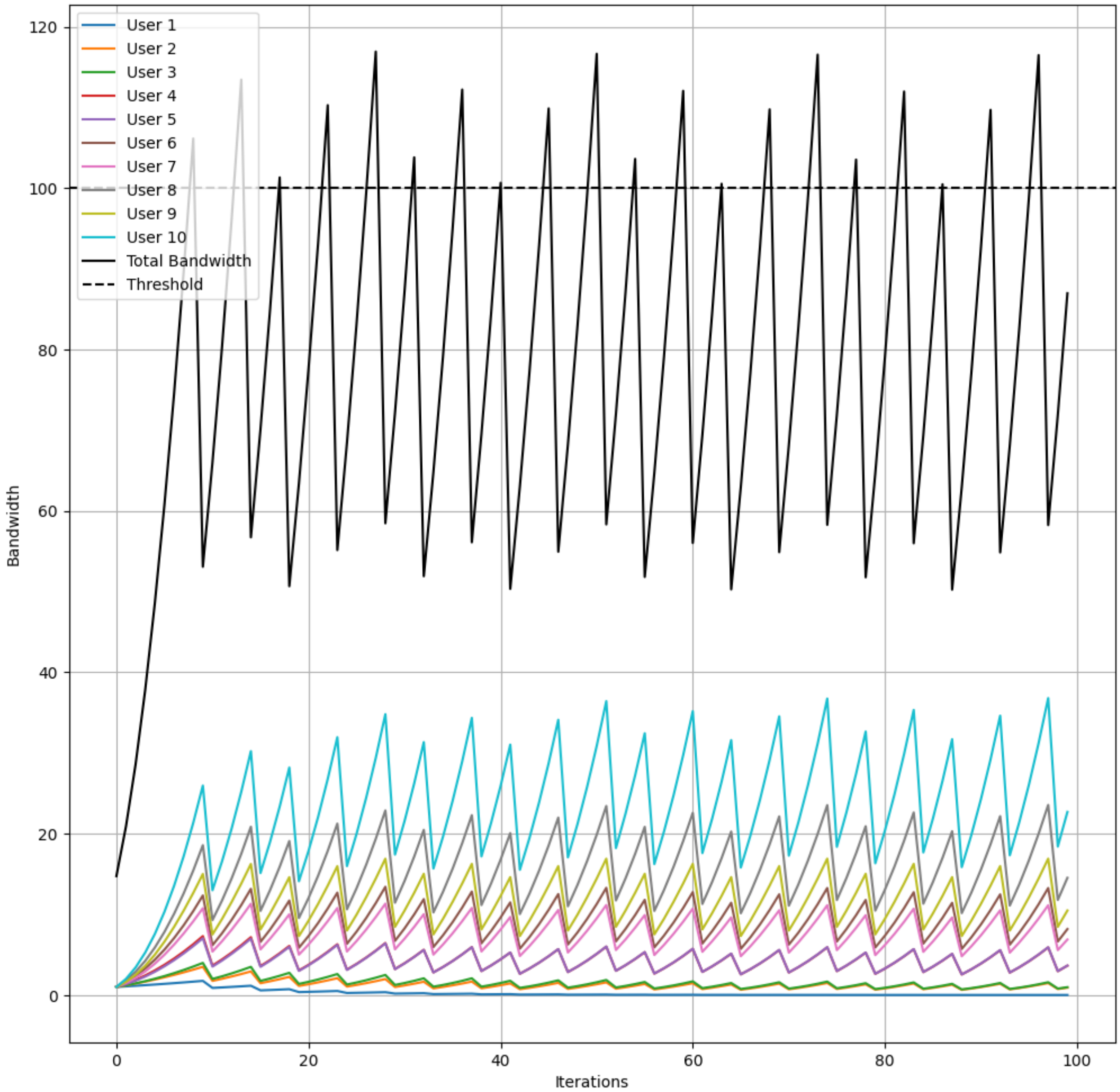
In [13]:

```
import numpy as np
import matplotlib.pyplot as plt

ITERATESMAX = 100
C = 100
alpha = 0.1
beta = 0.5
n_user = 4  # n_users
exponent1 = 0.5

x_values = np.zeros((ITERATESMAX, n_user))
sum_bandwidth = np.zeros(ITERATESMAX)
x = np.ones(n_user) # initialisation
for i in range(ITERATESMAX):
    x_values[i] = x

    if np.sum(x) <= C:
        for j in range(n_user):
            if j % 2 == 0:  # Even-numbered users
                x[j] += alpha * (j+1) * np.log(x[j] + 1)
            else:  # Odd-numbered users
                x[j] += alpha * (j+1) * np.power(x[j], exponent1
    else:
        x *= beta

    sum_bandwidth[i] = np.sum(x)

plt.figure(figsize=(12, 12))
for i in range(n_user):
    plt.plot(range(ITERATESMAX), x_values[:, i], linestyle='-',

plt.plot(range(ITERATESMAX), sum_bandwidth, linestyle='-', color
plt.axhline(y=100, color='black', linestyle='--', label='Thresho
plt.xlabel('Iterations')
plt.ylabel('Bandwidth')
plt.grid(True)
plt.legend(loc='upper left')
plt.show()
```


User 1
User 2
User 3
User 4
Total Bandwidth
Threshold
Bandwidth
Iterations
0
20
40
60
80
100

In [14]:

```
import numpy as np
import matplotlib.pyplot as plt

ITERATESMAX = 100
C = 100
alpha = 0.1
beta = 0.5
n_user = 10  # n_users
exponent1 = 0.5

x_values = np.zeros((ITERATESMAX, n_user))
sum_bandwidth = np.zeros(ITERATESMAX)
x = np.ones(n_user) # initialisation
for i in range(ITERATESMAX):
    x_values[i] = x

    if np.sum(x) <= C:
        for j in range(n_user):
            if j % 2 == 0:  # Even-numbered users
                x[j] += alpha * (j+1) * np.log(x[j] + 1)
            else:  # Odd-numbered users
                x[j] += alpha * (j+1) * np.power(x[j], exponent1
    else:
        x *= beta

    sum_bandwidth[i] = np.sum(x)

plt.figure(figsize=(12, 12))
for i in range(n_user):
    plt.plot(range(ITERATESMAX), x_values[:, i], linestyle='-',

plt.plot(range(ITERATESMAX), sum_bandwidth, linestyle='-', color
plt.axhline(y=100, color='black', linestyle='--', label='Thresho
plt.xlabel('Iterations')
plt.ylabel('Bandwidth')
plt.grid(True)
plt.legend(loc='upper left')
plt.show()

```


User 1
User 2
User 3
User 4
User 5
User 6
User 7
User 8
User 9
User 10
Total Bandwidth
Threshold
Bandwidth
Iterations
0
20
40
60
80
100
120

In [ ]:
```
1
```