Report on Proximity, Stability, and Leverage as Leading Indicators of Systemic Market Downturns

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Problem

The stock market is an integral part of the U.S. economy that allows for the exchange of capital. Savings, retirement funds, company success, and consumer prices are staked on the success or failure of the stock market. When the stock market takes a downturn, it can signal an economic recession characterized by increased unemployment rates and lower consumer spending. If someone is unprepared when the stock market crashes, they risk losing a large portion of money they have invested. Given how important it is to be prepared when the stock market crashes, economists and investors are always searching for ways to predict when the stock market is about to crash. Such predictors are called leading indicators.

Metric Classes

Proximity is a measure of how close an asset is to the efficient frontier. The proximity function is a ratio of the volatility of the Markowitz frontier and the asset. When an asset is on the Markowitz frontier, the proximity is 0 and the proximity increases proportionally to the distance between the asset and the Markowitz frontier. The proximity of the market index fund (MI, VFINX), total bond index fund (BI, VBMFX), and efficient long tangent allocation will be examined.

$$\omega^{\sigma}(\mathbf{f}) = \sqrt{1 - \frac{\sigma_{\mathrm{mf}}(\mu(\mathbf{f}))^2}{\sigma(\mathbf{f})^2}}. \qquad \text{From Appendix K}$$

Stability is a measure of the Tobin frontier tangency point and asymptote intersection. A decrease in the stability metric correlates to the increase in stability. The tangency point and asymptote intersection metrics will both be examined for the safe investment portfolio and the credit line portfolio.

$$\omega_{\mathrm{rf}}^{\mathrm{tg}} = \frac{\nu_{\mathrm{mv}}^2}{\nu_{\mathrm{rf}}^2}, \qquad \omega_{\mathrm{rf}}^{\mathrm{as}} = \frac{\nu_{\mathrm{mv}}}{\nu_{\mathrm{rf}}}. \qquad \text{From Appendix K}$$

Leverage is a measure of the leverage ratio which is the proportion of debt to equity in a portfolio. As the leverage ratio increases, the leverage metric approaches its upper bound of 1. The leverage metric will be examined for the minimum volatility allocation, safe tangent allocation, and the credit tangent allocation.

$$\omega^{\lambda}(\mathbf{f}) = \frac{\lambda(\mathbf{f})}{1 + \lambda(\mathbf{f})}$$ From Appendix K

Economic Downturns

Over the past 16 years (2006-2021), there have been a significant number of systemic economic downturns. The first is the real estate market crash in 2008 caused by the housing bubble bursting (Amadeo, 2022). Signs of the 2008 crash began in 2007 when the housing prices fell triggering defaults on mortgages. The following downturn was the European debt crisis which began in 2010 and peaked again in 2011 (Kenny, 2021). The European debt crisis began when the 2008 crash exposed unsustainable financial deficits in countries like Greece, Portugal, Ireland, Italy, and Spain. Smaller countries like Greece and Ireland were bailed out by the E.U. while larger countries like Italy and Spain were unable to be saved. The European debt crisis lasted for years, but the most notable downturns in the U.S. were in 2010 and 2011. The next downturn was the Chinese stock crash in 2015 (Investopedia, 2021). This crisis was caused by a campaign in China in 2014 to promote investment that led to a huge boom in the economy followed by a swift crash when the bubble burst. While this crash was devastating in China, it had a smaller effect on the U.S. economy than the European debt crisis. The Trade War with China during Trump's presidency in 2018 was the next significant economic downturn (Bown & Kolb, 2022). This resulted from a sequence of raised tariffs between the U.S. and China that began with virtually no warning. There will likely be no signs leading up to this crisis since it occurred almost on a whim of President Trump's. The last downturn was a product of the COVID-19 pandemic which came in 2020 with no economic basis and swept the globe (Broady, Edelberg, & O'Donnell, 2022). Once again, this economic crisis will likely have no leading indicators in the market since it is not systemically based.

Model

When examining leading indicators to economic downturns, a market index must be used to track the success or failure of the market. In this model, VFINX is used as the representation of the market. The success of VFINX is modeled both by being mapped on the sigma-mu plane and using a timeline from 2006-2020 with yearly rolling return averages. Yearly rolling averages were used to reduce noise and incorporate more past data compared to another method such as quarterly rolling return averages.

Sigma-Mu Plane

In this model, the Markowitz frontier and VFINX were graphed on the sigma-mu plane for every year within the 16-year timeframe. The sigma-mu plane has volatility on the horizontal axis and return mean on the vertical axis. Ideally, assets have high return mean and low volatility indicating low-risk, high-reward. The Markowitz frontier is the positive volatility portion of the hyperbola mapped in the sigma-mu plane, separating efficient portfolios from inefficient portfolios. While portfolios above and to the left of the frontier are considered optimal, realistically portfolios are contained in the area to the right of the Markowitz frontier. While the focus of this simulation is on the changes in the metrics, the Markowitz frontier and the movement of VFINX around the sigma-mu plane can lend clarity to the activity of the metrics.

Assets

The model involved examining three groups of assets. Group A consisted of VFINX, VBMFX, and VGSLX. These three assets are all Vanguard index funds. VFINX aims to track the S&P 500 which measures the U.S. stock market (Yahoo! Finance). The fund mostly contains stocks of large U.S. companies since those dominate the U.S. stock market. VBMFX aims to track the performance of the Bloomberg Barclays U.S. Aggregate Float Adjusted Index which measures public securities in the U.S. by holding 80% U.S. bonds (Yahoo! Finance). VGSLX is a non-diversified index fund that follows the performance of the real-estate market by holding stocks in the industry (Yahoo! Finance). The second group of assets, group AB, is made up of Group A as well as three additional assets:VIMAX, VGTSX, and VGHCX. These three assets are once again Vanguard index funds. VIMAX aims to track the performance of the CRSP US Mid Cap (Yahoo! Finance). To do so, the fund holds stocks of mid-size U.S. companies in

diversified sectors. VGTSX tracks the returns from foreign developed and emerging markets by holding companies in such markets (Yahoo! Finance). VGHCX is a non-diversified fund that holds its stocks in companies that are involved in the healthcare industry (Yahoo! Finance). The last group, group ABC, contains the previously mentioned assets as well as GS, JPM, and WFC. These assets are stocks from companies in the financial services industry: Goldman Sachs, JP Morgan, and Wells Fargo & Company respectively. All three of these companies are large, worldwide businesses involved in banking, investing, and other financial services. These assets were observed on the sigma-mu plane as well as through different metrics throughout the 16-year timeframe. The metrics were also used when calculating different allocations used in the metric models.

Metric Models

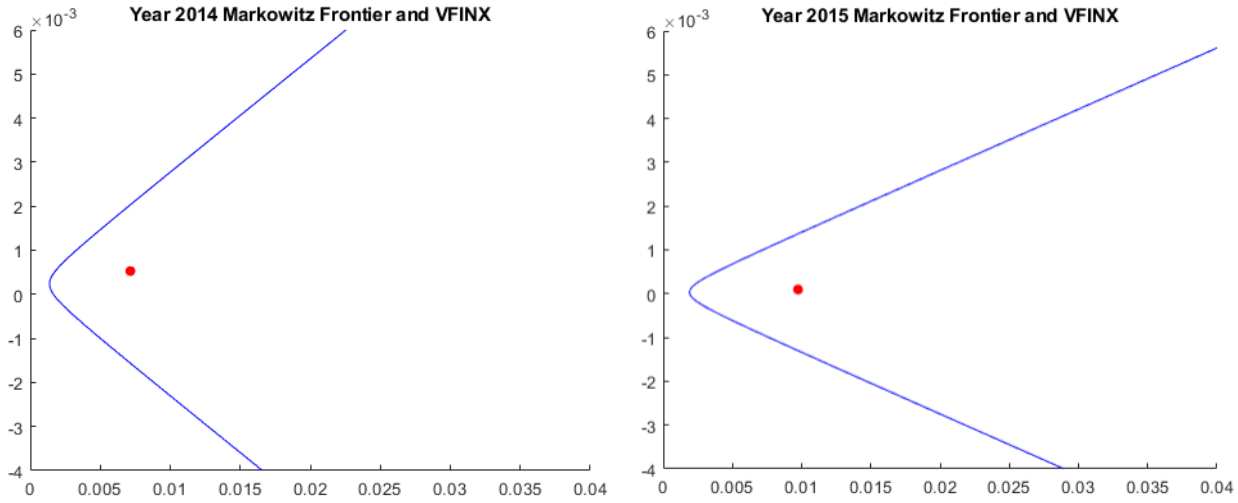
For each metric class, the individual metrics are graphed with their quarterly changes over the 16 years being examined. Quarter intervals are used to increase the frequency of data without also increasing the noise that would come with monthly data points. The graphs are compared by placing them above and below each other on a matching timeline. The placement of these graphs allows them to be seen separately with ease while still allowing for comparison to one another. The VFINX returns graph is likewise placed above the metric graphs running on the same timeline. Vertical lines are placed across all of the graphs for that metric class showing where each economic downturn begins. The markers at the beginning of an economic decline help pinpoint changes in the metrics preceding the decline. While markers are placed at each significant economic decline, both the trade war and the COVID-19 pandemic downturns in 2018 and 2020 respectively were not caused by preceding systemic issues and are therefore not examined for leading indicators.

Results

Below is the graph of VFINX with markers indicating the start of each of the financial crashes from appendix J. The different economic downturns can be seen and their effect on the market. While the 2008 crash was the largest downturn, the economy recovered relatively quickly. The European debt crisis in 2010 and the subsequent crash in 2011 had the longest recovery time. The Chinese stock crash in 2015 seemed to have the least impact on the U.S. economy. The 2018 and 2020 crashes are obvious, but once again had little to no systemic warning and as such will not be examined for leading indicators.

Sigma-Mu Plane

During economic declines, the Markowitz frontier narrows. This means the most optimal allocations have high volatility and low return mean, indicating high-risk, low-reward. During economic downturns, VFINX responded by increasing in volatility and decreasing in return, which once again corresponds with high-risk, low-reward. These trends can be noted in the graphs below as the Chinese stock crash hits in 2015.



From Appendix I: Sigma-Mu plane with Markowitz frontier and VFINX for 2014 and 2015

Assets

During years of market crashes, the assets seem to collectively move lower on the plane. The assets may not necessarily have a return mean below zero as a result of market crashes, but they seem to consistently move lower relative to the previous year. This corresponds to lower return. VBMFX is the least responsive asset to economic downturns which is expected since the bond market is less affected by crashes and generally considered a safer, less volatile investment. It can be noted in Appendices E, F, and G which show the allocations of the safe tangent, credit tangent, and efficient long tangent portfolios, that VBMFX tends to gain a large portion of the allocation during years with economic downturns.

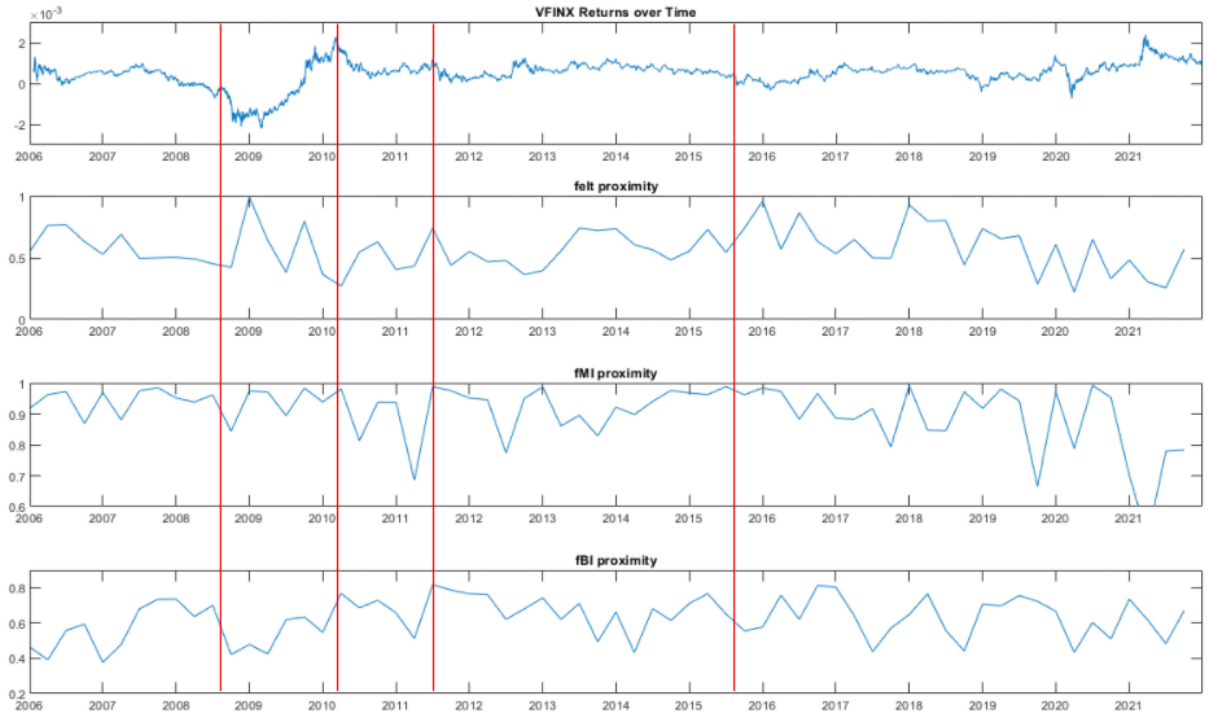
The volatility and mean returns of the safe tangent and credit tangent portfolios tend to move further away from the vertex of the Markowitz frontier during years of economic downturns. Particularly the safe tangent portfolio moves drastically away in a few cases. While the portfolios may increase in return during these changes, usually the increase in volatility is so drastic that it makes the potential increase in return undesirable.

Metrics

When analyzing the pattern of these metrics, we will only consider the first 3 downturn events, since the Trade War and COVID pandemic were spontaneous and not caused by rooted issues in the market.

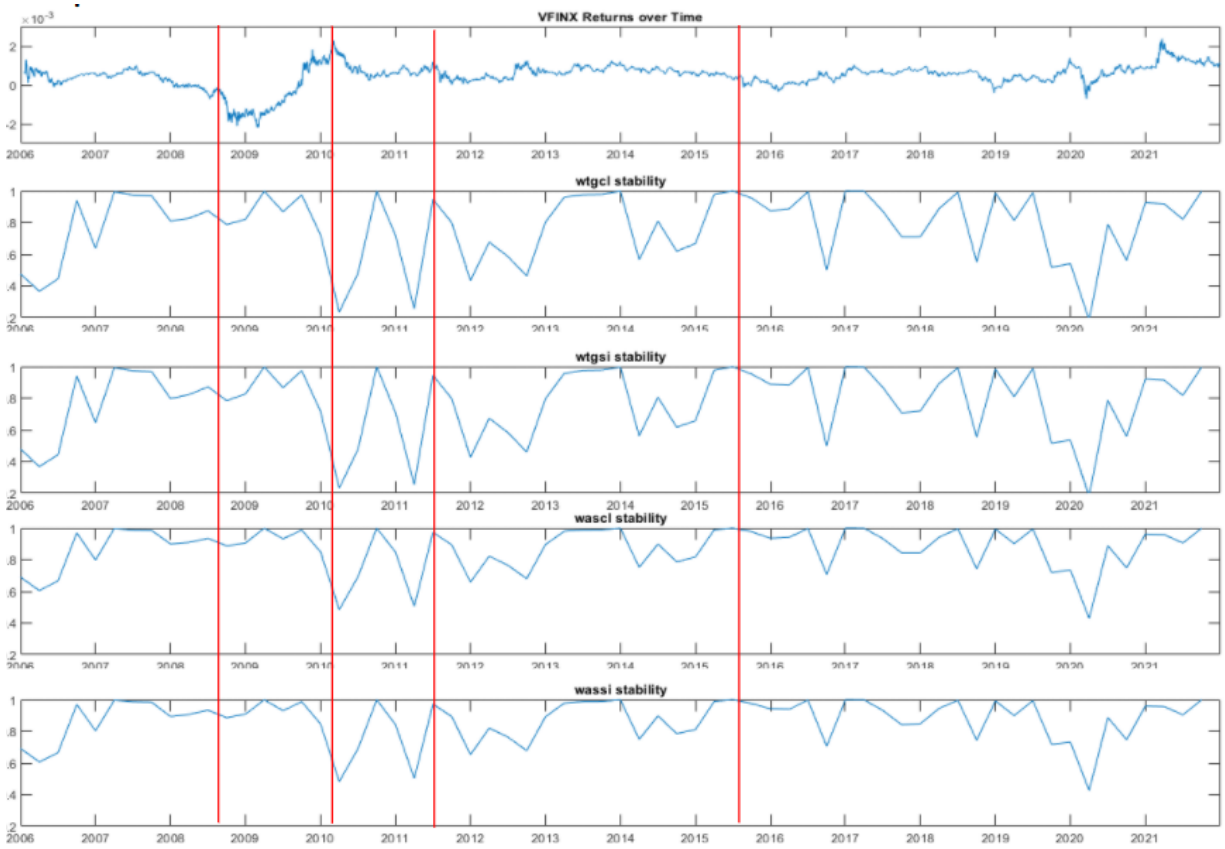
We will first consider proximity (its graph shown below), which is applied to the efficient long tangent allocation, market index fund, and the bond index fund. When looking at the graphs, it can be seen that the metric used for the market index fund has a pattern of peaking right before or at the start of a downturn, a pattern not seen as consistently with the other two allocations. The market index fund having the most clear pattern is unsurprising due to the fact that VFINX is such a major portion of the market. When proximity peaks, it implies that the asset reaches its locally maximum difference in volatility from the efficient frontier, which corroborates the event of a crash. It's worth noting that high volatility doesn't necessarily imply a crash is looming, however a crash of a large magnitude due to rooted issues would require high volatility

preceding it, making tracking this metric very useful to keep an eye on the risk of coming downturns.

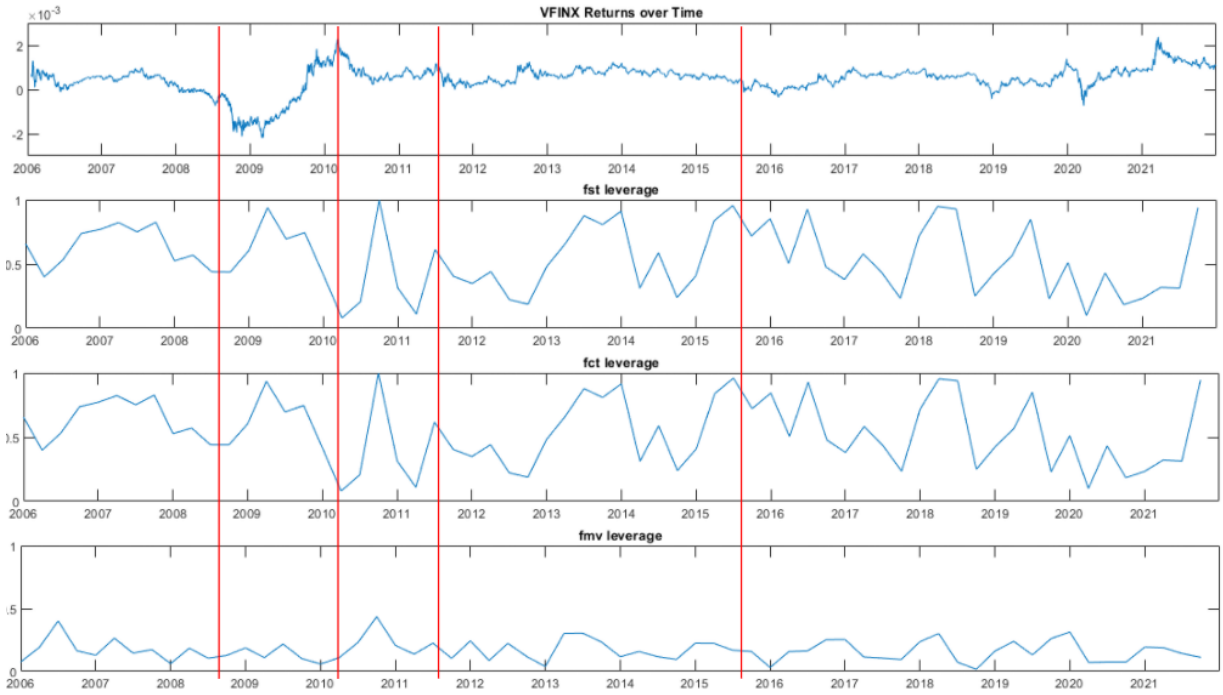


Next, we consider stability applied to the tangency point and asymptote intersection metric for both the safe investment and credit line risk-free rates, resulting in 4 metrics total. In the graph shown below, the difference visually between the safe investment and credit line metrics are negligible, so we are able to only examine one of each. When looking at the tangency point and asymptote intersection graphs, we see that they too are very similar, the only major difference being that the fluctuations in the tangency point graph are more pronounced. We notice that except for the 2008 crash, the other 2 downturn events we are examining have fairly major troughs/spikes beforehand indicating either high or low stability respectively. These major fluctuations correlate to an increase in volatility, which we just discussed also result in high proximity. For example, we can see two major fluctuations during 2010 and 2011 which correlate to the two waves of the European debt crisis. Stability prior to the 2008 crash did display a dip, but it was not as pronounced as the fluctuations in the other downturns that followed. This is not unexpected as a crash doesn't necessarily need extremely high volatility beforehand, albeit being

a good indicator, but also can be due to more long-term causes. As we can see in the 2008 case, we have a fairly long period of high instability with moderate volatility, starting in early 2007. This combination, when put into context, makes the crash more predictable.



Finally, we consider leverage as a potential leading indicator. Once again, leverage ultimately measures the debt compared to equity in a portfolio, therefore long portfolios always have a leverage of 0. On the other hand, short portfolios have their leverage metric increase the more debt they accumulate on the allocation. On the graph shown below, we notice that just like in the stability graphs, we see no major differences in the safe and credit tangent graphs. For the minimum volatility graph, we unsurprisingly see that the range of leverage on the graph is much lower when compared to the tangent graphs. When it comes to indications of downturns, we notice that debt spikes usually occur once the crash has occurred, notably by increases seen after 2008, late 2010, and just when the Chinese market crash occurred. From the data we see, we will classify leverage as a lagging metric.



Conclusion

Reiterating our objective, we were interested in exploring metrics that may be able to display specific behaviors preceding downturn events, coining them leading indicators. We explored the context of certain downturns that were caused by internal issues in the market, and plotted the behaviors of our metrics during these periods. After analyzing these metrics with the appropriate context, we concluded that Proximity is effective due to its behavior to spike right before downturns, indicating high volatility. We also classified Stability to be a leading indicator, whose fluctuations similarly indicated an increase in volatility and therefore the potential for a downturn. When it comes to Leverage however, the measure of debt tends to be a lagging indicator since most of it follows the crash, displaying a behavior only once a downturn has already occurred.

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Appendix A

With μ_{mv} as umv, σ_{mv} as omv, and ν_{mv} as vmv

tableA =

3×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
umvA	0.00020094	0.00029579	6.3793e-05	0.00029261	0.00027788	0.00025586	0.00023581	0.00014629
omvA	0.0020397	0.0021348	0.0034749	0.0025687	0.0019826	0.0018258	0.0013042	0.0017567
vmvA	0.1089	0.084256	0.079649	0.046816	0.055816	0.040786	0.057609	0.20054

2014	2015	2016	2017	2018	2019	2020	2021
0.00023447	1.6771e-05	0.0001464	0.00030867	4.3919e-06	0.00042396	0.00033807	3.9666e-05
0.0013966	0.0019737	0.0017893	0.0013834	0.0016068	0.00167	0.0028349	0.002234
0.12178	0.012667	0.04128	0.12422	0.020461	0.10254	0.061624	0.15326

tableAB =

3×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
umvAB	0.00020681	0.00025501	6.8477e-05	0.0002887	0.00024535	0.00030623	0.0002394	0.00020191
omvAB	0.0020161	0.0020718	0.003438	0.0025532	0.001913	0.0017848	0.0012956	0.0017145
vmvAB	0.1639	0.12675	0.1084	0.10851	0.12615	0.16828	0.071657	0.27969

2014	2015	2016	2017	2018	2019	2020	2021
0.00022085	2.1763e-06	0.00014509	0.00027903	-2.3688e-05	0.00042436	0.000333	-5.7426e-06
0.0013942	0.0019673	0.0017812	0.0013454	0.0015851	0.0016529	0.0028199	0.0021557
0.24468	0.10617	0.14625	0.18477	0.1578	0.11334	0.087642	0.18538

tableABC =

3×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
umvABC	0.00019049	0.00025354	9.0275e-05	0.00025107	0.000244	0.00026916	0.00024609	0.00019989
omvABC	0.0019795	0.0020453	0.0033887	0.0024908	0.0018898	0.0017675	0.0012827	0.0017092
vmvABC	0.19064	0.13603	0.14756	0.15446	0.13083	0.23297	0.093851	0.28054

2014	2015	2016	2017	2018	2019	2020	2021
0.00023489	3.3409e-05	0.00018064	0.00025035	-3.5519e-05	0.00045633	0.0002743	2.3634e-05
0.0013798	0.0019045	0.0016831	0.0012893	0.0014819	0.0015312	0.0027216	0.001853
0.25619	0.13961	0.19378	0.22613	0.22089	0.13905	0.14935	0.2155

Appendix B

tablefmvA =

5×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
VFINX	0.106	0.135	0.076	0.118	0.165	0.202	0.135	0.217
VBMFX	0.92	0.882	0.93	0.907	0.867	0.874	0.872	0.916
VGSLX	-0.026	-0.017	-0.006	-0.025	-0.033	-0.076	-0.007	-0.133
Leverage	0.025	0.016	0.006	0.025	0.032	0.07	0.007	0.117
Liquidity	0.005	0.008	0.012	0.007	0.007	0.005	0.005	0.007

2014	2015	2016	2017	2018	2019	2020	2021
0.157	0.197	0.185	0.264	0.094	0.188	0.086	0.085
0.887	0.907	0.901	0.799	0.955	0.888	0.946	0.906
-0.044	-0.103	-0.086	-0.063	-0.049	-0.076	-0.032	0.01
0.042	0.094	0.079	0.06	0.046	0.071	0.031	0
0.004	0.007	0.006	0.004	0.004	0.005	0.017	0.009

tablefmvAB =

8×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
VFINX	0.219	0.068	0.153	0.137	0.267	0.299	0.136	0.312
VBMFX	0.905	0.825	0.938	0.915	0.827	0.852	0.857	0.906
VGSLX	-0.017	-0.022	-0.012	-0.03	-0.022	-0.065	-0.004	-0.121
VIMAX	-0.085	0.071	-0.005	0.03	-0.084	-0.094	-0.018	-0.08
VGTSX	-0.025	-0.092	-0.043	-0.037	-0.071	-0.036	-0.014	-0.079
VGHCX	0.003	0.149	-0.03	-0.015	0.084	0.044	0.043	0.062
Leverage	0.113	0.102	0.083	0.076	0.151	0.164	0.034	0.219
Liquidity	0.005	0.008	0.012	0.007	0.006	0.004	0.005	0.007

2014	2015	2016	2017	2018	2019	2020	2021
0.16	0.148	0.186	0.288	0.001	0.12	0.14	-0.011
0.888	0.903	0.901	0.832	0.933	0.894	0.953	0.868
-0.045	-0.103	-0.085	-0.07	-0.044	-0.084	-0.019	0.01
0.01	0.048	0.017	0.061	0.047	0.092	-0.043	-0.014
0.006	0.018	-0.028	-0.053	0.043	0.011	-0.027	0.097
-0.018	-0.014	0.01	-0.058	0.019	-0.033	-0.005	0.05
0.06	0.105	0.102	0.153	0.042	0.105	0.085	0.025
0.005	0.007	0.006	0.004	0.004	0.005	0.016	0.008

tablefmvABC =

11×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
VFINX	0.317	0.164	0.149	0.254	0.335	0.259	0.094	0.289
VBMFX	0.886	0.818	0.93	0.903	0.822	0.853	0.864	0.907
VGSLX	-0.012	-0.011	-0.021	-0.017	-0.02	-0.068	-0.006	-0.121
VIMAX	-0.111	0.033	0.023	-0.03	-0.104	-0.078	-0.005	-0.073
VGTSX	-0.025	-0.1	-0.049	-0.042	-0.071	-0.038	-0.013	-0.077
VGHCX	0.005	0.134	-0.023	-0.046	0.058	0.056	0.045	0.057
GS	-0.007	-0.014	-0.019	-0.01	0.007	0.021	-0.004	-0.007
JPM	-0.049	0.001	0.006	-0.001	-0.005	-0.017	0.008	0.009
WFC	-0.004	-0.025	0.004	-0.012	-0.022	0.01	0.017	0.015
Leverage	0.172	0.131	0.1	0.136	0.182	0.167	0.028	0.217
Liquidity	0.005	0.008	0.012	0.006	0.006	0.004	0.005	0.006

2014	2015	2016	2017	2018	2019	2020	2021
0.112	0.02	0.094	0.188	-0.105	0.052	0.123	-0.047
0.887	0.91	0.924	0.856	0.93	0.892	0.958	0.868
-0.043	-0.092	-0.06	-0.055	-0.033	-0.052	-0.038	0.012
0.014	0.056	-0.005	0.05	0.081	0.046	-0.058	-0.064
0.007	0.023	-0.035	-0.056	0.026	-0.016	-0.048	0.07
-0.014	0.01	0.014	-0.039	0.032	-0.009	0.03	0.068
0	-0.024	0.019	0.013	-0.001	0.009	-0.02	-0.011
0.006	0.045	0.04	0.018	0.053	0.062	0.033	0.087
0.033	0.053	0.009	0.024	0.016	0.016	0.018	0.016
0.055	0.104	0.091	0.131	0.121	0.071	0.14	0.109
0.005	0.007	0.005	0.004	0.004	0.004	0.015	0.007

Appendix C

With μ_{si} and μ_{cl} represented as usi and ucl, w_{si}^{tg} and w_{si}^{as} as wtgsi and wassi, and v_{si} and v_{cl} as vsi and vcl

Note: All values are rounded to the nearest thousandth. As such, all the μ values were rounded to zero.

tableratesA =

8×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
usi	0	0	0	0	0	0	0	0
vsi	0.109	0.113	0.081	0.117	0.146	0.144	0.185	0.216
wtgsi	1	0.559	0.969	0.161	0.146	0.081	0.097	0.861
wassi	1	0.748	0.984	0.401	0.383	0.284	0.311	0.928
ucl	0	0	0	0	0	0	0	0
vcl	0.109	0.112	0.081	0.116	0.145	0.143	0.185	0.216
wtgcl	1	0.563	0.97	0.162	0.148	0.081	0.097	0.863
wascl	1	0.75	0.985	0.402	0.384	0.285	0.312	0.929

2014	2015	2016	2017	2018	2019	2020	2021
0	0	0	0	0	0	0	0
0.202	0.013	0.076	0.213	0.065	0.239	0.133	0.154
0.362	0.912	0.297	0.339	0.1	0.183	0.215	0.995
0.602	0.955	0.545	0.582	0.316	0.428	0.463	0.998
0	0	0	0	0	0	0	0
0.202	0.013	0.075	0.213	0.065	0.239	0.133	0.154
0.365	0.887	0.302	0.341	0.098	0.184	0.216	0.995
0.604	0.942	0.549	0.584	0.313	0.429	0.465	0.998

tableratesAB =

8×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
usi	0	0	0	0	0	0	0	0
vsi	0.164	0.139	0.11	0.152	0.176	0.239	0.194	0.302
wtgsi	0.999	0.83	0.979	0.511	0.514	0.497	0.137	0.855
wassi	1	0.911	0.99	0.715	0.717	0.705	0.37	0.925
ucl	0	0	0	0	0	0	0	0
vcl	0.164	0.139	0.109	0.151	0.176	0.238	0.193	0.302
wtgcl	0.999	0.833	0.98	0.513	0.517	0.499	0.138	0.857
wascl	1	0.913	0.99	0.717	0.719	0.707	0.371	0.926

2014	2015	2016	2017	2018	2019	2020	2021
0	0	0	0	0	0	0	0
0.288	0.107	0.159	0.242	0.177	0.246	0.146	0.186
0.721	0.989	0.843	0.582	0.796	0.211	0.361	0.997
0.849	0.994	0.918	0.763	0.892	0.46	0.601	0.999
0	0	0	0	0	0	0	0
0.288	0.107	0.159	0.242	0.177	0.246	0.146	0.186
0.723	0.987	0.846	0.585	0.793	0.213	0.362	0.997
0.851	0.994	0.92	0.765	0.89	0.461	0.602	0.998

tableratesABC =

8×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
usi	0	0	0	0	0	0	0	0
vsi	0.191	0.148	0.149	0.181	0.18	0.277	0.209	0.303
wtgsi	1	0.849	0.977	0.731	0.529	0.707	0.201	0.858
wassi	1	0.921	0.989	0.855	0.727	0.841	0.448	0.926
ucl	0	0	0	0	0	0	0	0
vcl	0.191	0.147	0.149	0.18	0.179	0.277	0.209	0.303
wtgcl	1	0.851	0.978	0.733	0.532	0.709	0.202	0.859
wascl	1	0.923	0.989	0.856	0.729	0.842	0.45	0.927

2014	2015	2016	2017	2018	2019	2020	2021
0	0	0	0	0	0	0	0
0.304	0.14	0.213	0.267	0.24	0.292	0.179	0.216
0.71	0.999	0.83	0.72	0.848	0.226	0.693	1
0.842	0.999	0.911	0.848	0.921	0.476	0.833	1
0	0	0	0	0	0	0	0
0.304	0.14	0.212	0.266	0.24	0.292	0.179	0.216
0.712	0.999	0.832	0.722	0.846	0.227	0.695	1
0.844	1	0.912	0.85	0.92	0.477	0.834	1

Appendix D

With asset 1 as VFINX, asset 2 as VBMFX, and asset 3 as VGSLX

tablemetricsA =

12×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
wliquidity1	0.019	0.035	0.089	0.054	0.04	0.067	0.025	0.026
wliquidity2	0.005	0.006	0.01	0.007	0.008	0.01	0.006	0.01
wliquidity3	0.027	0.054	0.193	0.117	0.059	0.09	0.029	0.042
wsharpe1	0.447	0.907	1	0.469	0.634	0.914	0.573	0.251
wsharpe2	1	0.463	0.475	0.363	0.366	0.257	0.476	1
wsharpe3	0.022	1	1	0.627	0.557	0.816	0.562	0.935
wefficiency1	0.331	0.505	0.703	0.342	0.373	0.513	0.234	0.254
wefficiency2	0.519	0.414	0.346	0.538	0.495	0.474	0.807	0.903
wefficiency3	0.212	0.705	0.563	0.377	0.302	0.473	0.213	0.511
wproximity1	0.893	0.979	0.906	0.938	0.954	0.992	0.836	0.844
wproximity2	0.358	0.562	0.484	0.511	0.67	0.758	0.539	0.354
wproximity3	0.798	0.905	0.989	0.967	0.913	0.994	0.81	0.984

2014	2015	2016	2017	2018	2019	2020	2021
0.023	0.04	0.036	0.019	0.041	0.031	0.121	0.027
0.005	0.007	0.01	0.006	0.005	0.007	0.017	0.007
0.033	0.048	0.039	0.022	0.037	0.023	0.18	0.035
0.641	0.446	0.286	0.22	1	0.44	0.701	0.203
0.4	1	0.6	0.79	1	0.497	0.314	1
0.269	0.115	0.572	0.861	1	0.469	0.936	0.044
0.418	0.477	0.405	0.213	0.524	0.193	0.408	0.211
0.519	0.54	0.64	0.66	0.405	0.779	0.473	0.642
0.268	0.46	0.451	0.497	0.536	0.222	0.506	0.154
0.968	0.98	0.961	0.78	0.989	0.774	0.975	0.795
0.628	0.653	0.633	0.686	0.574	0.604	0.474	0.601
0.87	0.981	0.983	0.979	0.987	0.815	0.995	0.708

Appendix E

tablefstA =

6×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
VFINX	-1.277	0.404	-1.164	0.188	0.098	0.124	0.15	1.005
VBMFX	-11.065	0.846	1.685	0.841	0.856	0.888	0.823	0.491
VGSLX	13.341	-0.25	0.479	-0.029	0.046	-0.012	0.026	-0.496
Leverage	0.925	0.2	0.538	0.028	0	0.012	0	0.331
Liquidity	0.046	0.034	0.035	0.034	0.034	0.034	0.034	0.035
Sharpe	0	0	0	0	0	0	0	0

2014	2015	2016	2017	2018	2019	2020	2021
0.099	0.314	0.309	0.472	0.1	0.233	0.233	1.019
0.774	1.449	0.772	0.638	0.9	0.778	0.909	-2.644
0.127	-0.762	-0.081	-0.11	0	-0.011	-0.142	2.626
0	0.433	0.075	0.099	0	0.011	0.125	0.726
0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.038
0	1	0	0	1	0	0	0

tablefstAB =

9×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
VFINX	14.742	-0.026	0.047	-0.546	-0.203	0.366	0.122	0.377
VBMFX	-3.799	1.042	1.032	0.861	0.937	0.761	0.784	0.534
VGSLX	6.726	-0.345	0.524	-0.044	-0.022	-0.002	0.037	-0.352
VIMAX	-15.864	0.366	-1.346	0.531	0.495	-0.22	-0.08	0.036
VGTSX	7.812	0.508	-0.845	0.087	-0.102	-0.288	0.018	-0.474
VGHCX	-8.617	-0.545	1.588	0.11	-0.104	0.384	0.12	0.88
Leverage	0.966	0.478	0.687	0.371	0.301	0.338	0.074	0.452
Liquidity	0.045	0.034	0.036	0.034	0.034	0.034	0.034	0.035
Sharpe	0	0	0	0	0	0	0	0

2014	2015	2016	2017	2018	2019	2020	2021
0.304	-3.025	1.245	0.578	-0.718	0.328	0.154	-7.085
0.813	0.982	0.773	0.532	0.723	0.806	0.959	4.304
0.177	-0.264	-0.125	-0.096	0.035	-0.013	-0.206	-3.123
-0.159	5.242	0.01	-0.276	0.745	0.011	0.304	3.415
-0.392	0.865	-0.305	0.292	0.57	-0.057	-0.159	3.429
0.257	-2.801	-0.598	-0.031	-0.355	-0.076	-0.053	0.06
0.355	0.859	0.507	0.287	0.518	0.127	0.295	0.911
0.034	0.032	0.034	0.034	0.033	0.034	0.034	0.026
0	1	0	0	1	0	0	1

tablefstABC =

12×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
VFINX	-12.253	0.154	-1.617	-0.955	-0.051	0.596	-0.007	0.432
VBMFX	4.897	1.06	0.889	0.881	0.926	0.749	0.816	0.523
VGSLX	-9.04	-0.319	0.029	-0.067	-0.02	0.023	0.021	-0.357
VIMAX	24.171	0.199	-0.04	0.702	0.434	-0.331	-0.038	0.032
VGTSX	-10.689	0.461	-0.222	0.107	-0.101	-0.288	0.002	-0.475
VGHCX	10.744	-0.505	1.611	0.218	-0.142	0.339	0.137	0.875
GS	-5.68	0.113	-0.253	0.117	0.004	-0.15	0.019	-0.01
JPM	-0.228	-0.063	0.1	0.009	-0.055	0.065	0.017	-0.027
WFC	-0.921	-0.1	0.504	-0.012	0.004	-0.002	0.033	0.007
Leverage	0.975	0.497	0.681	0.508	0.269	0.435	0.042	0.465
Liquidity	0.012	0.035	0.037	0.034	0.034	0.035	0.034	0.035
Sharpe	1	0	0	0	0	0	0	0

2014	2015	2016	2017	2018	2019	2020	2021
0.112	3.185	0.663	0.668	-0.79	0.18	0.118	15.05
0.812	0.979	0.921	0.448	0.808	0.828	0.951	-5.239
0.171	0.993	-0.022	-0.07	0.036	0.014	-0.191	6.753
-0.148	-12.008	-0.093	-0.318	0.613	-0.031	0.394	-9.938
-0.352	-1.299	-0.261	0.331	0.369	-0.095	-0.164	-8.45
0.243	7.376	-0.388	-0.036	-0.255	-0.019	-0.097	1.193
0.027	-4.36	0.13	-0.128	0.239	0.009	0.075	4.426
-0.013	6.893	0.29	0.155	-0.185	0.13	0.085	-4.009
0.149	-0.759	-0.241	-0.051	0.164	-0.015	-0.17	1.214
0.339	0.949	0.501	0.376	0.552	0.138	0.384	0.965
0.034	0.041	0.035	0.034	0.033	0.034	0.034	0.094
0	0	0	0	1	0	0	0

Appendix F

tablefctA =

6×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
VFINX	-1.84	0.406	-1.195	0.188	0.098	0.124	0.15	1.011
VBMFX	-15.952	0.846	1.704	0.841	0.856	0.888	0.823	0.487
VGSLX	18.793	-0.252	0.491	-0.029	0.046	-0.012	0.027	-0.499
Leverage	0.947	0.201	0.544	0.028	0	0.012	0	0.333
Liquidity	0.051	0.034	0.035	0.034	0.034	0.034	0.034	0.035
Sharpe	0	0	0	0	0	0	0	0

2014	2015	2016	2017	2018	2019	2020	2021
0.098	0.298	0.31	0.473	0.1	0.233	0.234	1.067
0.774	1.377	0.771	0.638	0.901	0.778	0.909	-2.827
0.128	-0.675	-0.081	-0.11	0	-0.011	-0.143	2.76
0	0.403	0.075	0.099	0	0.011	0.125	0.739
0.034	0.034	0.034	0.034	0.034	0.034	0.034	0.039
0	1	0	0	1	0	0	0

tablefctAB =

9×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
VFINX	16.645	-0.027	0.044	-0.549	-0.205	0.366	0.122	0.377
VBMFX	-4.415	1.044	1.034	0.861	0.937	0.76	0.783	0.532
VGSLX	7.609	-0.348	0.536	-0.044	-0.022	-0.002	0.037	-0.354
VIMAX	-17.932	0.369	-1.376	0.534	0.498	-0.221	-0.08	0.036
VGTSX	8.839	0.513	-0.863	0.088	-0.102	-0.289	0.018	-0.476
VGHCX	-9.747	-0.551	1.624	0.11	-0.105	0.385	0.12	0.884
Leverage	0.97	0.481	0.691	0.372	0.303	0.338	0.074	0.453
Liquidity	0.046	0.034	0.036	0.034	0.034	0.034	0.034	0.035
Sharpe	0	0	0	0	0	0	0	0

2014	2015	2016	2017	2018	2019	2020	2021
0.305	-2.865	1.256	0.58	-0.712	0.329	0.154	-6.71
0.813	0.978	0.771	0.53	0.725	0.806	0.959	4.122
0.178	-0.255	-0.125	-0.096	0.034	-0.012	-0.206	-2.957
-0.16	4.981	0.01	-0.278	0.738	0.011	0.306	3.233
-0.394	0.822	-0.307	0.294	0.565	-0.057	-0.16	3.252
0.259	-2.66	-0.605	-0.03	-0.351	-0.076	-0.053	0.059
0.357	0.853	0.509	0.288	0.515	0.127	0.295	0.906
0.034	0.032	0.034	0.034	0.033	0.034	0.034	0.027
0	1	0	0	1	0	0	1

tablefctABC =

12×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
VFINX	-10.398	0.154	-1.645	-0.961	-0.053	0.597	-0.007	0.433
VBMFX	4.305	1.062	0.888	0.881	0.927	0.748	0.815	0.521
VGSLX	-7.708	-0.322	0.03	-0.068	-0.02	0.023	0.021	-0.359
VIMAX	20.587	0.2	-0.041	0.706	0.437	-0.333	-0.038	0.032
VGTSX	-9.115	0.466	-0.225	0.107	-0.101	-0.289	0.002	-0.477
VGHCX	9.159	-0.511	1.637	0.219	-0.143	0.34	0.137	0.88
GS	-4.843	0.114	-0.257	0.118	0.004	-0.151	0.019	-0.01
JPM	-0.202	-0.064	0.102	0.009	-0.055	0.066	0.017	-0.027
WFC	-0.786	-0.1	0.512	-0.012	0.004	-0.002	0.033	0.007
Leverage	0.971	0.499	0.684	0.51	0.271	0.436	0.043	0.466
Liquidity	0.016	0.035	0.037	0.035	0.034	0.035	0.034	0.035
Sharpe	1	0	0	0	0	0	0	0

2014	2015	2016	2017	2018	2019	2020	2021
0.112	3.671	0.667	0.671	-0.784	0.18	0.118	17.607
0.811	0.99	0.921	0.446	0.809	0.828	0.951	-6.273
0.172	1.16	-0.021	-0.07	0.036	0.014	-0.192	7.894
-0.149	-13.862	-0.094	-0.32	0.608	-0.031	0.396	-11.61
-0.354	-1.503	-0.263	0.333	0.367	-0.095	-0.165	-9.893
0.245	8.508	-0.391	-0.036	-0.253	-0.019	-0.097	1.384
0.027	-5.026	0.131	-0.129	0.237	0.009	0.076	5.177
-0.014	7.946	0.292	0.156	-0.183	0.13	0.085	-4.702
0.15	-0.884	-0.243	-0.051	0.163	-0.015	-0.171	1.416
0.341	0.955	0.503	0.377	0.55	0.139	0.385	0.97
0.034	0.042	0.035	0.034	0.033	0.034	0.035	0.11
0	0	0	0	1	0	0	0

Appendix G

With μ_{si} as usi, μ_{mn} as umn, μ_{mx} as umx, σ_{mx} as omx, σ_{elf} as oelf, and μ_{elf} as uelf

Note: Since the numbers were all rounded to the nearest thousandth, many of the numbers were rounded to zero.

tablefeltA =

13×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
VFINX	0	0.11	0	0.135	0.105	0.111	0.151	1
VBMFX	0	0.885	1	0.865	0.857	0.888	0.821	0
VGSLX	1	0.005	0	0	0.038	0.001	0.029	0
Liquidity	0.027	0.008	0.01	0.011	0.007	0.007	0.005	0.026
Efficiency	0	1	0.101	0.065	0	0.001	0	0.135
Proximity	0	0.001	0.283	0.231	0	0.02	0.002	0.662
Sharpe	0.022	0.416	0.475	0.027	0.001	0.002	0	0.251
usi	0	0	0	0	0	0	0	0
umn	0	-0.001	-0.001	0	0	0	0	0
umx	0.001	0	0	0.002	0.001	0.001	0.001	0.001
omx	0.009	0.002	0.004	0.042	0.018	0.019	0.009	0.007
oelf	0.009	0.002	0.004	0.003	0.002	0.002	0.001	0.007
uelf	0.001	0	0	0	0	0	0	0.002

2014	2015	2016	2017	2018	2019	2020	2021
0.099	0	0.272	0.444	0	0.227	0.082	0.284
0.775	0	0.727	0.555	0	0.767	0.918	0.001
0.126	1	0	0	1	0.006	0	0.715
0.007	0.048	0.01	0.006	0.037	0.006	0.019	0.03
0	0	0.051	0.087	0.992	0.003	0.548	0
0.001	0.001	0.286	0.361	0.179	0.047	0.265	0.032
0	0.115	0.045	0.067	1	0.002	0.158	0.02
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0.001	0	0	0.001	0	0.001	0.001	0.001
0.007	0.011	0.008	0.004	0.002	0.008	0.022	0.009
0.002	0.011	0.002	0.002	0.01	0.002	0.003	0.008
0	0	0	0.001	0	0.001	0	0.001

tablefeltAB =

16×16 table

	2006	2007	2008	2009	2010	2011	2012	2013
VFINX	0	0	0	0	0.001	0	0.036	0
VBMFX	0.001	0.85	1	0.864	0.853	0.837	0.79	0
VGSLX	0.643	0	0	0	0.001	0	0.027	0
VIMAX	0	0	0	0.091	0.144	0	0.003	0
VGTSX	0.356	0.149	0	0.03	0	0	0.015	0
VGHCX	0	0	0	0.015	0.001	0.162	0.13	1
Liquidity	0.022	0.008	0.01	0.012	0.007	0.006	0.005	0.028
Efficiency	0.145	0.229	0.228	0.188	0.088	0.429	0.036	0.15
Proximity	0.68	0.423	0.409	0.417	0.276	0.425	0.138	0.689
Sharpe	0.306	0.395	0.612	0.171	0.104	0.317	0.01	0.276
usi	0	0	0	0	0	0	0	0
umn	0	-0.001	-0.002	0	0	0	0	0
umx	0.001	0.001	0	0.002	0.001	0.001	0.001	0.001
omx	0.009	0.012	0.004	0.042	0.018	0.019	0.01	0.007
oelf	0.008	0.002	0.004	0.003	0.002	0.002	0.001	0.007
uelf	0.002	0.001	0	0	0	0	0	0.002

2014	2015	2016	2017	2018	2019	2020	2021
0.001	0	0.273	0.232	0	0.229	0.075	0.286
0.76	0.403	0.727	0.455	0	0.767	0.916	0
0.1	0	0	0	0	0	0	0.714
0.001	0	0	0	0	0.003	0.006	0
0	0	0	0.309	0	0	0.001	0
0.139	0.597	0	0.004	1	0	0.002	0
0.006	0.023	0.01	0.008	0.041	0.005	0.02	0.03
0.154	0.242	0.371	0.073	0.468	0.067	0.493	0.073
0.467	0.812	0.634	0.41	0.985	0.238	0.298	0.503
0.217	0.548	0.546	0.089	1	0.03	0.234	0.189
0	0	0	0	0	0	0	0
0	0	0	0	-0.001	0	0	0
0.001	0.001	0	0.001	0	0.001	0.001	0.001
0.007	0.011	0.008	0.004	0.01	0.008	0.023	0.009
0.002	0.006	0.002	0.002	0.01	0.002	0.003	0.008
0.001	0.001	0	0.001	0.002	0.001	0	0.002

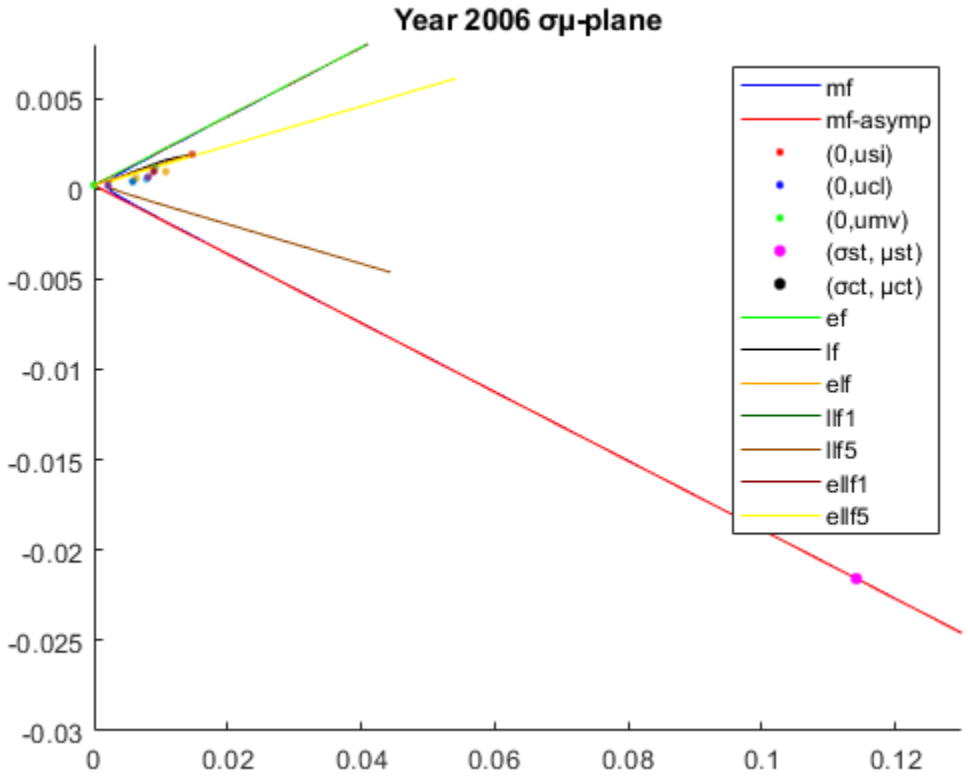
tablefeltABC =

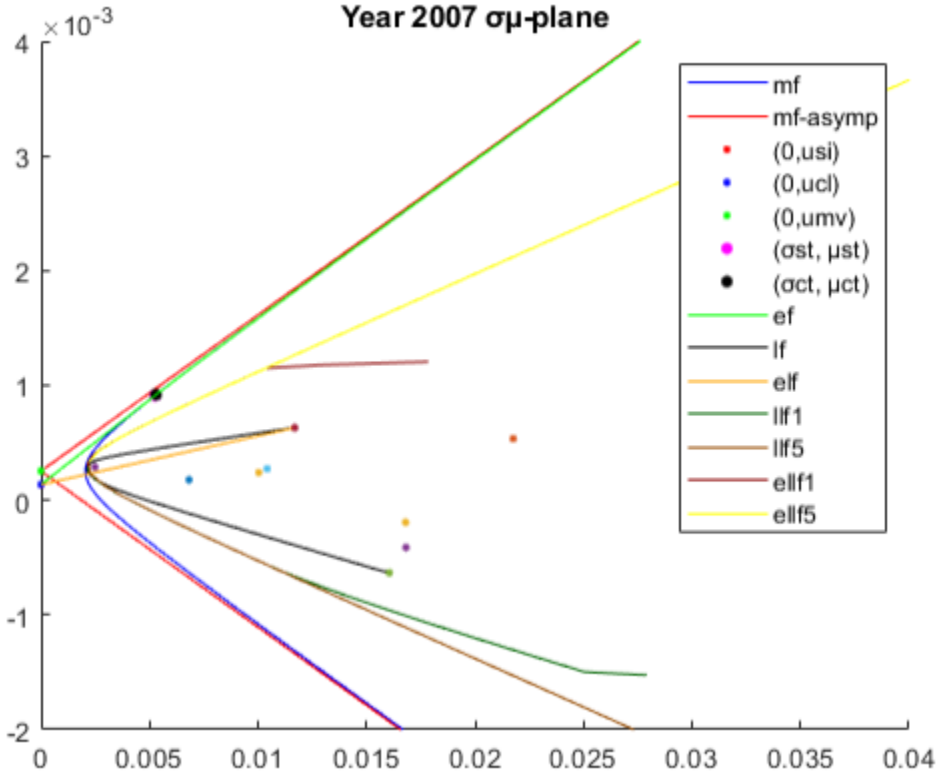
19×16 table

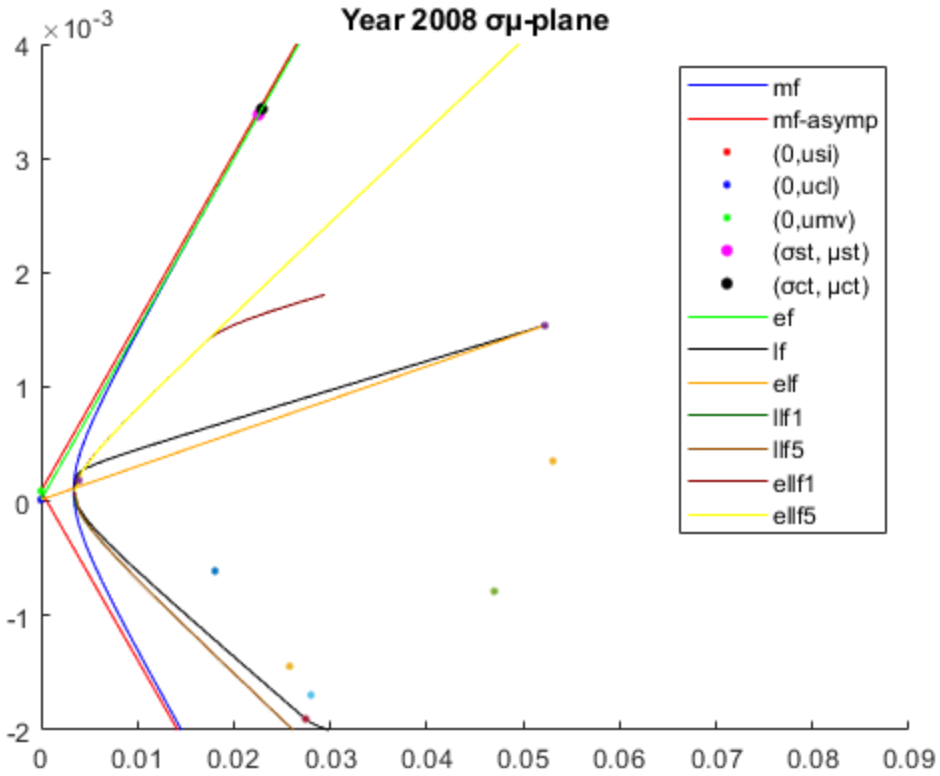
	2006	2007	2008	2009	2010	2011	2012	2013
VFINX	0	0	0	0	0.001	0.003	0.003	0
VBMFX	0	0.849	0.947	0.872	0.855	0.851	0.83	0.008
VGSLX	0.509	0	0	0	0	0	0.009	0
VIMAX	0	0.001	0	0.035	0.142	0.001	0.002	0
VGTSX	0.096	0.148	0	0.017	0	0	0.002	0
VGHCX	0	0	0	0.03	0.001	0.143	0.098	0.992
GS	0.391	0.001	0	0.045	0	0	0.012	0
JPM	0	0	0	0	0	0	0.017	0
WFC	0.004	0	0.052	0	0	0.001	0.027	0
Liquidity	0.032	0.008	0.015	0.016	0.007	0.006	0.005	0.028
Efficiency	0.151	0.254	0.226	0.187	0.112	0.369	0.014	0.151
Proximity	0.7	0.458	0.425	0.521	0.314	0.415	0.086	0.691
Sharpe	0.321	0.431	0.595	0.257	0.124	0.417	0.009	0.277
usi	0	0	0	0	0	0	0	0
umn	0	-0.001	-0.002	0	0	-0.002	0	0
umx	0.002	0.001	0.002	0.004	0.001	0.001	0.002	0.001
omx	0.015	0.012	0.052	0.038	0.018	0.019	0.018	0.014
oelf	0.01	0.002	0.004	0.003	0.002	0.002	0.001	0.007
uelf	0.002	0.001	0	0.001	0	0	0	0.002

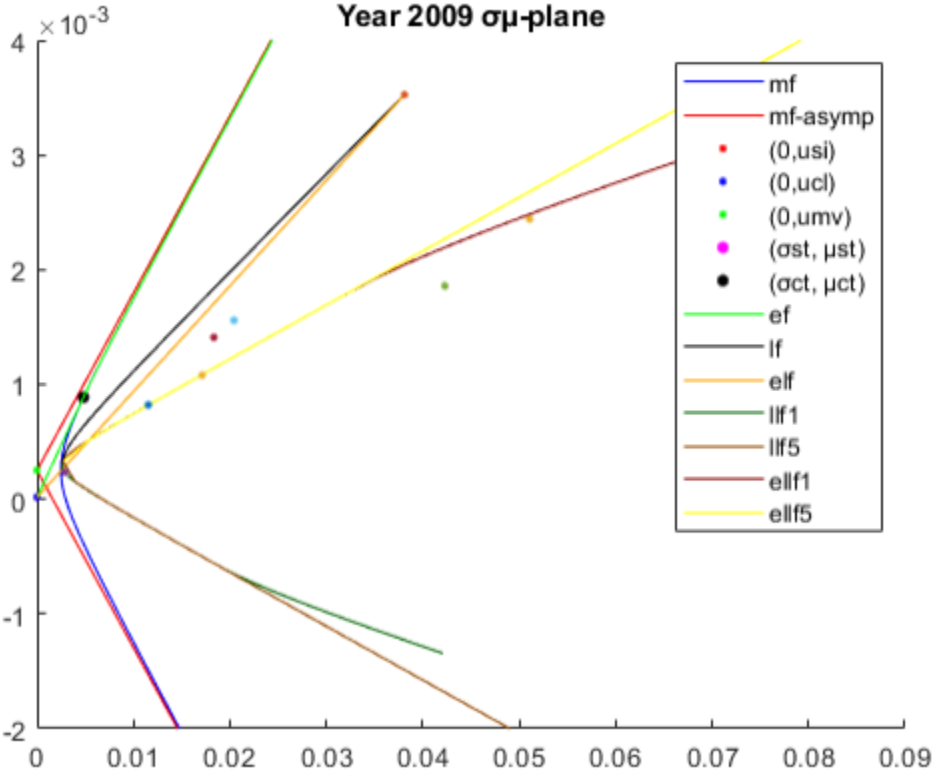
2014	2015	2016	2017	2018	2019	2020	2021
0.001	0	0.001	0.166	0	0.056	0.055	0.037
0.775	0.406	0.835	0.508	0	0.798	0.935	0.007
0.067	0	0.001	0	0	0.019	0	0.722
0.001	0	0.001	0	0	0	0	0
0	0	0	0.281	0	0	0	0
0.084	0.591	0	0.001	1	0	0.001	0
0.002	0	0.053	0	0	0.004	0.006	0.134
0.001	0.003	0.11	0.044	0	0.123	0.001	0
0.069	0	0	0	0	0	0	0.1
0.006	0.023	0.007	0.007	0.041	0.004	0.019	0.032
0.157	0.324	0.296	0.116	0.475	0.049	0.324	0.116
0.403	0.889	0.602	0.497	0.987	0.188	0.305	0.627
0.231	0.655	0.445	0.161	1	0.021	0.381	0.245
0	0	0	0	0	0	0	0
0	0	0	0	-0.002	0	-0.001	0
0.001	0.001	0.001	0.001	0	0.002	0.001	0.002
0.007	0.011	0.017	0.01	0.01	0.012	0.033	0.021
0.002	0.006	0.002	0.002	0.01	0.002	0.003	0.009
0	0.001	0	0.001	0.002	0.001	0	0.002

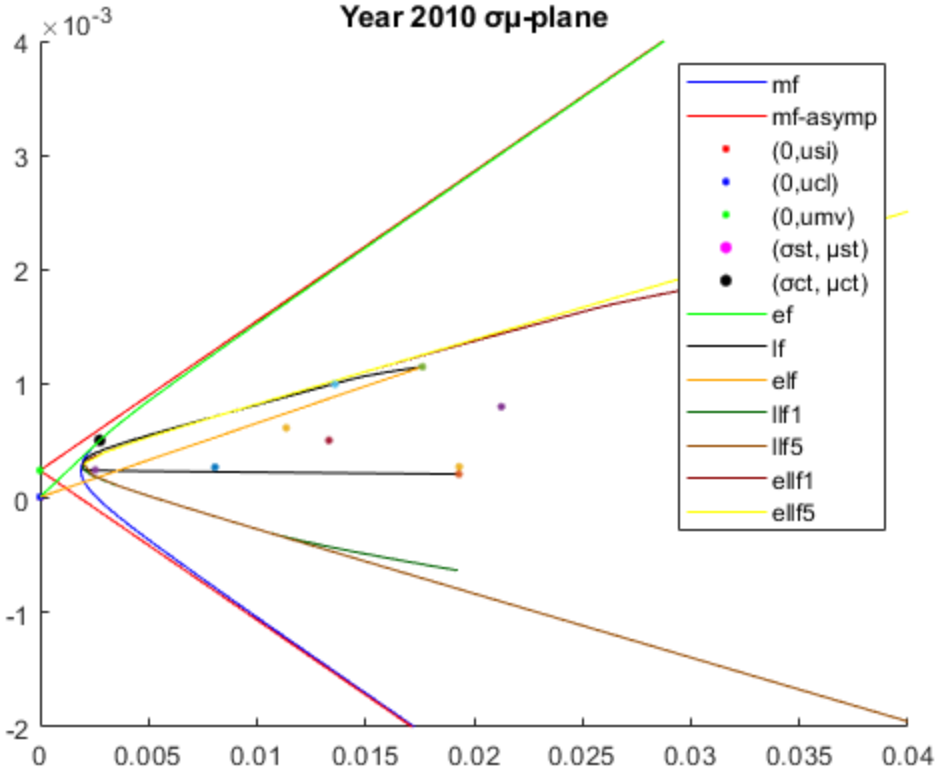
Appendix H

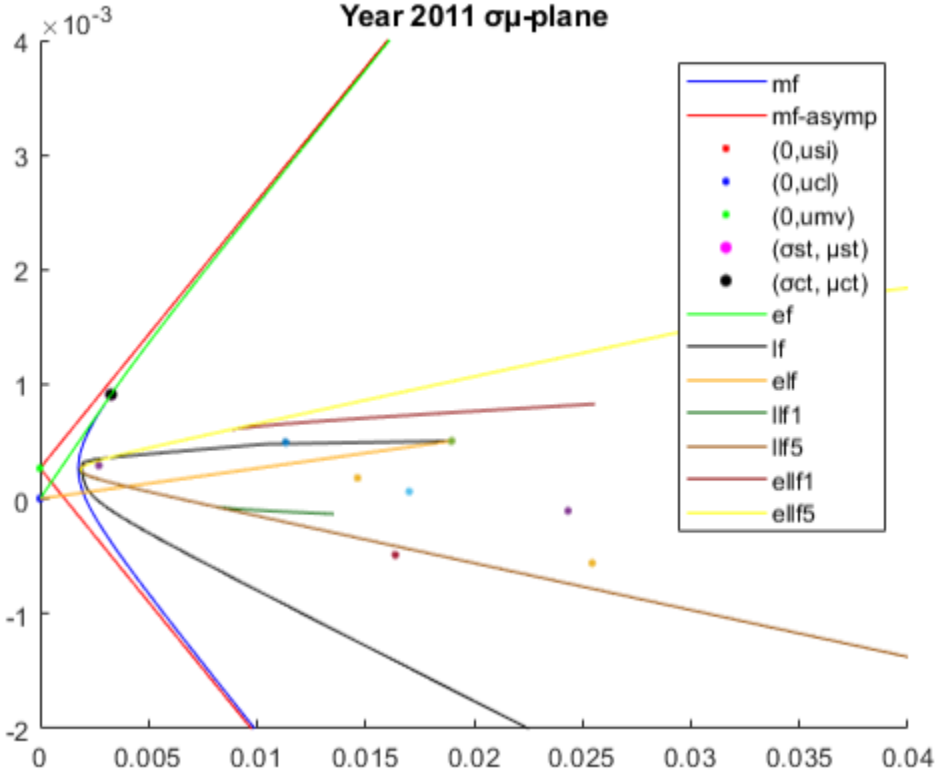

Year 2006 σμ-plane
mf
mf-asymp
(0,usi)
(0,ucl)
(0,umv)
(σst, μst)
(σct, μct)
ef
lf
elf
llf1
llf5
ellf1
ellf5
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
-0.03
0
0.02
0.04
0.06
0.08
0.1
0.12

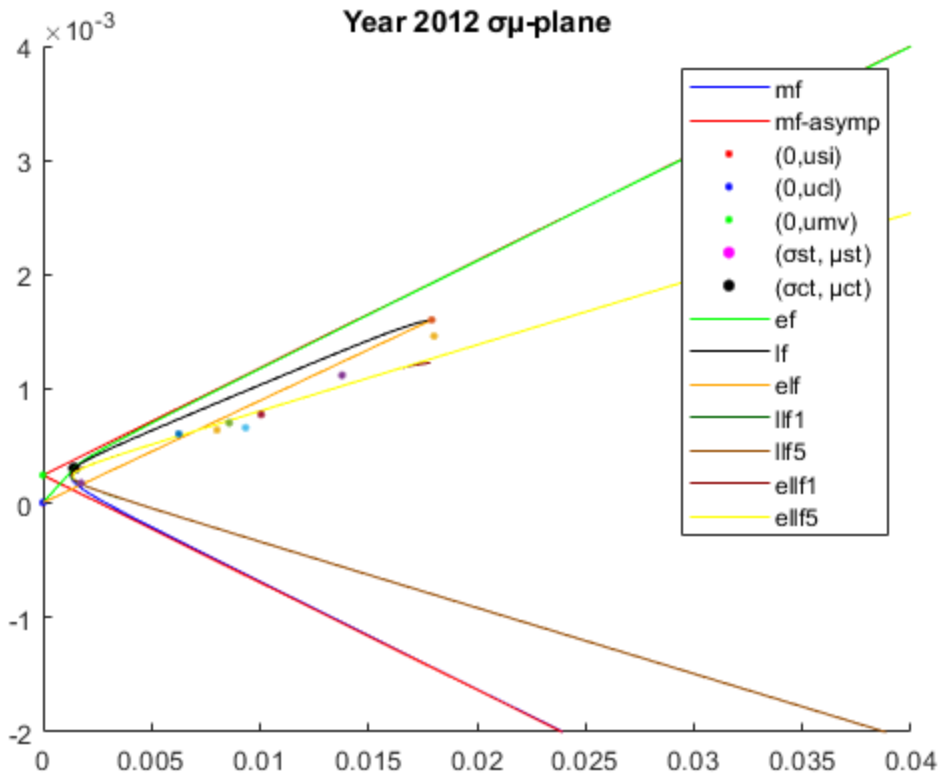

Year 2007 σμ-plane
×10^-3
mf
mf-asymp
(0,usi)
(0,ucl)
(0,umv)
(σst, μst)
(σct, μct)
ef
lf
elf
llf1
llf5
ellf1
ellf5
4
3
2
1
0
-1
-2
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04

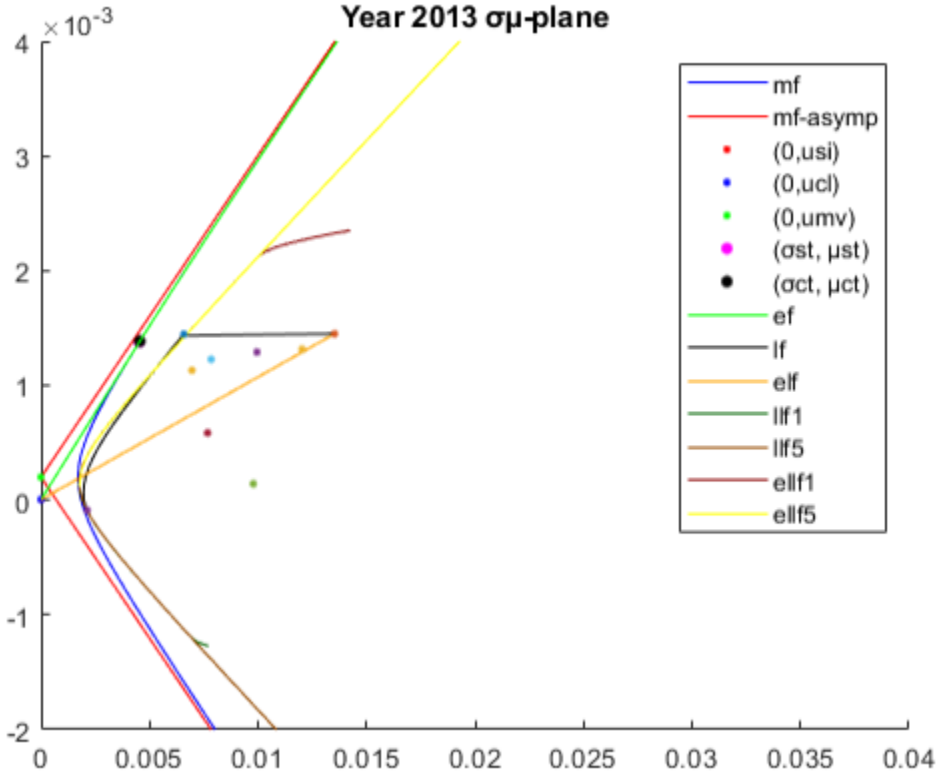

Year 2008 σμ-plane
×10^-3
4
3
2
1
0
-1
-2
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
mf
mf-asymp
(0,usi)
(0,ucl)
(0,umv)
(σst, μst)
(σct, μct)
ef
lf
elf
llf1
llf5
ellf1
ellf5

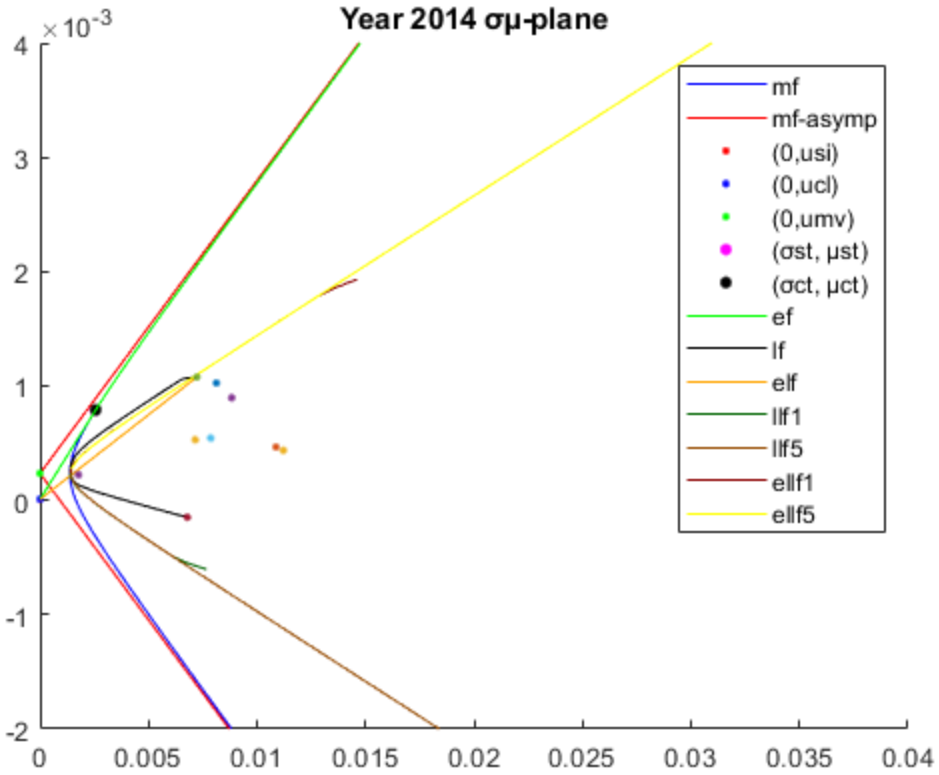

Year 2009 σμ-plane
×10^-3
4
3
2
1
0
-1
-2
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
mf
mf-asymp
(0,usi)
(0,ucl)
(0,umv)
(σst, μst)
(σct, μct)
ef
lf
elf
llf1
llf5
ellf1
ellf5

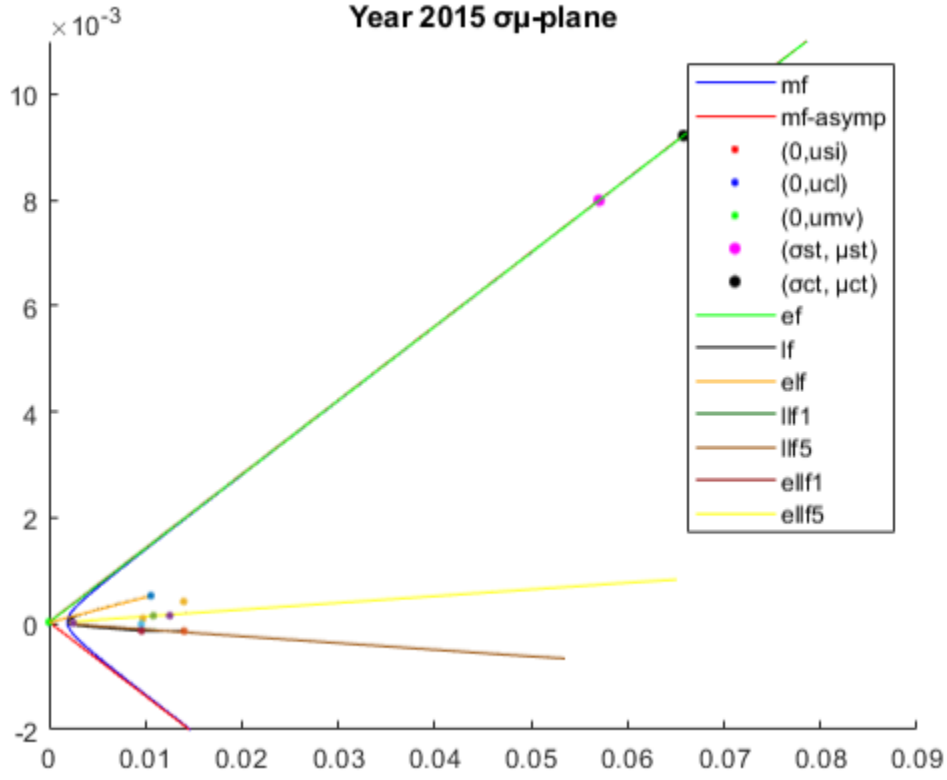

Year 2010 σμ-plane
×10-3
4
3
2
1
0
-1
-2
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
mf
mf-asymp
(0,usi)
(0,ucl)
(0,umv)
(σst, μst)
(σct, μct)
ef
lf
elf
llf1
llf5
ellf1
ellf5

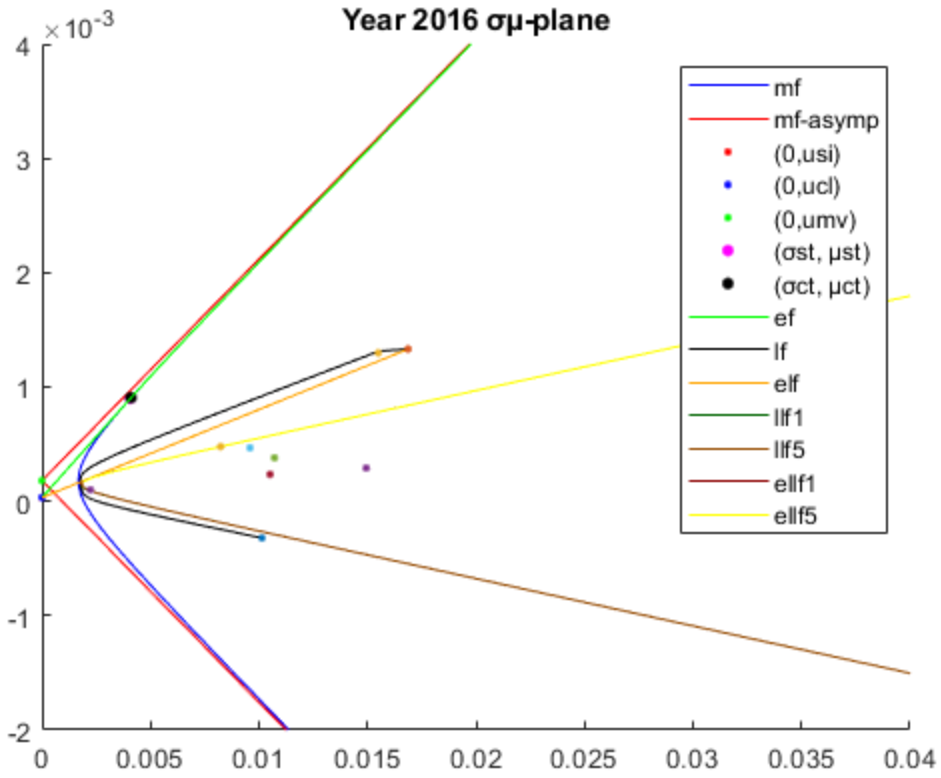

Year 2011 σμ-plane
×10-3
4
3
2
1
0
-1
-2
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
mf
mf-asymp
(0,usi)
(0,ucl)
(0,umv)
(σst, μst)
(σct, μct)
ef
lf
elf
llf1
llf5
ellf1
ellf5


Year 2012 σμ-plane
×10^-3
4
3
2
1
0
-1
-2
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
mf
mf-asymp
(0,usi)
(0,ucl)
(0,umv)
(σst, μst)
(σct, μct)
ef
lf
elf
llf1
llf5
ellf1
ellf5


Year 2013 σμ-plane
×10^-3
4
3
2
1
0
-1
-2
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
mf
mf-asymp
(0,usi)
(0,ucl)
(0,umv)
(σst, μst)
(σct, μct)
ef
lf
elf
llf1
llf5
ellf1
ellf5


Year 2014 σμ-plane
×10^-3
4
3
2
1
0
-1
-2
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
mf
mf-asymp
(0,usi)
(0,ucl)
(0,umv)
(σst, μst)
(σct, μct)
ef
lf
elf
llf1
llf5
ellf1
ellf5


Year 2015 σμ-plane
×10^-3
10
8
6
4
2
0
-2
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
mf
mf-asymp
(0,usi)
(0,ucl)
(0,umv)
(σst, μst)
(σct, μct)
ef
lf
elf
llf1
llf5
ellf1
ellf5


Year 2016 σμ-plane
×10^-3
4
3
2
1
0
-1
-2
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
mf
mf-asymp
(0,usi)
(0,ucl)
(0,umv)
(σst, μst)
(σct, μct)
ef
lf
elf
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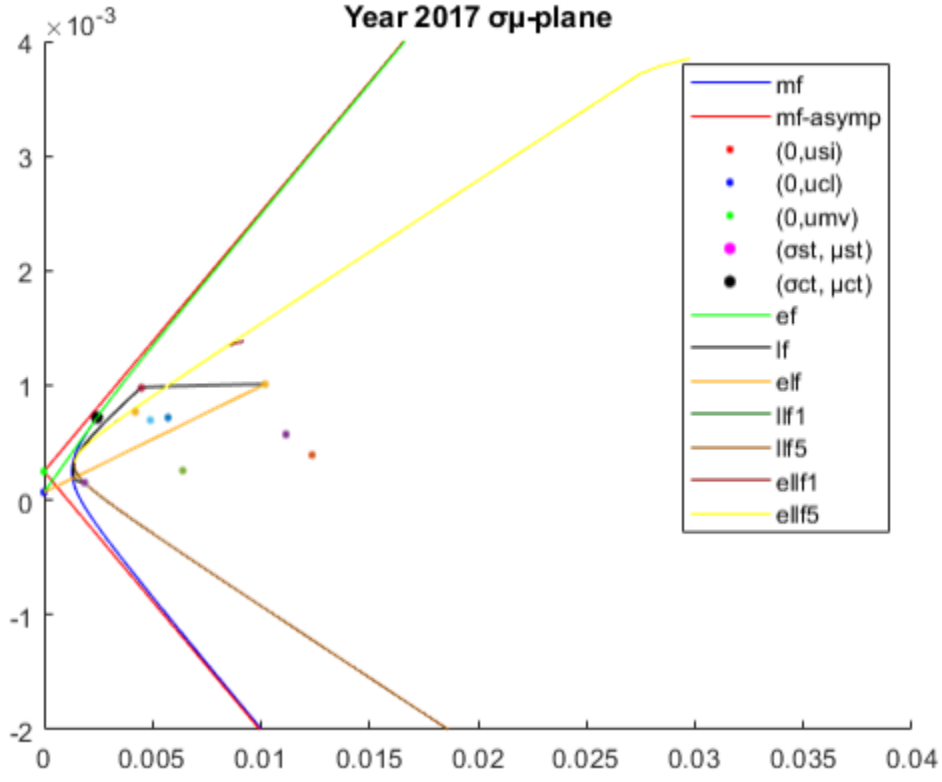
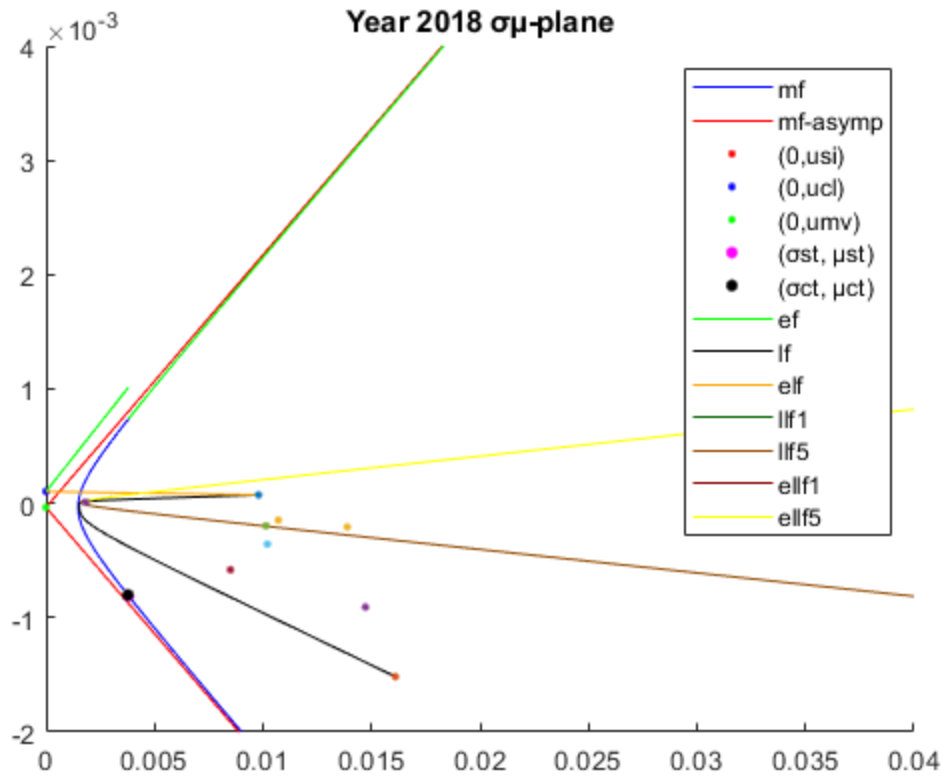
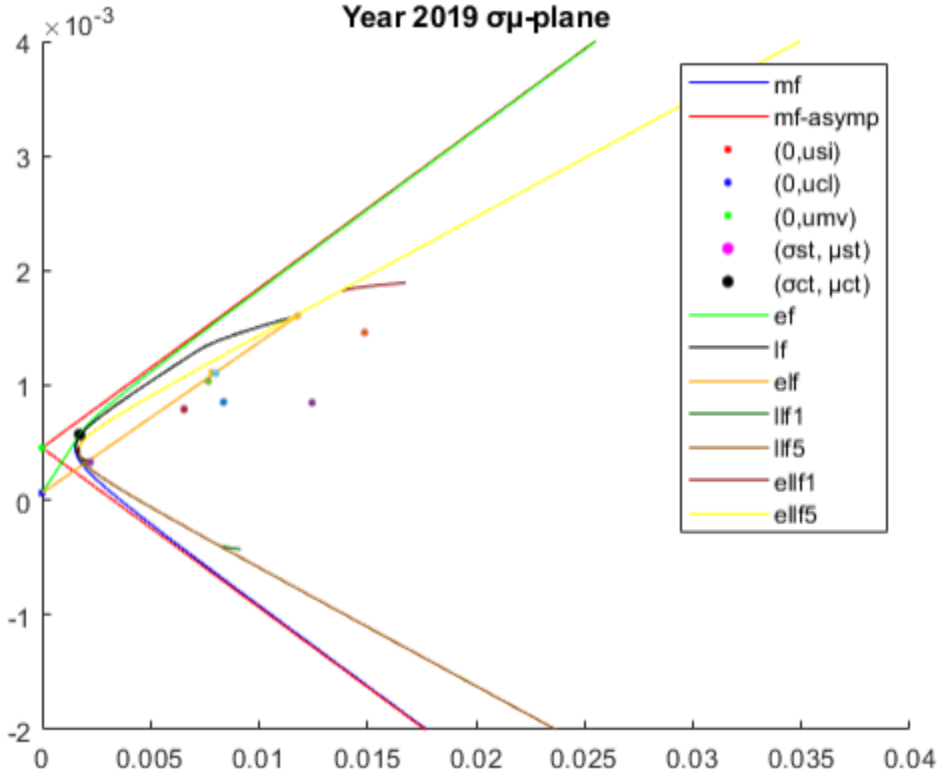
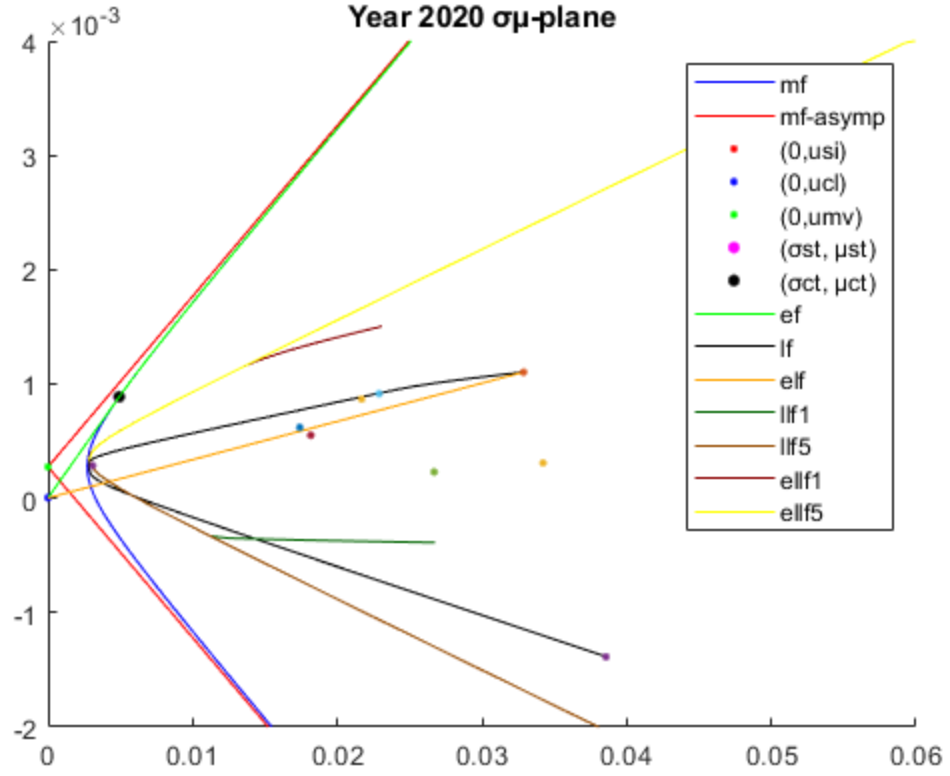
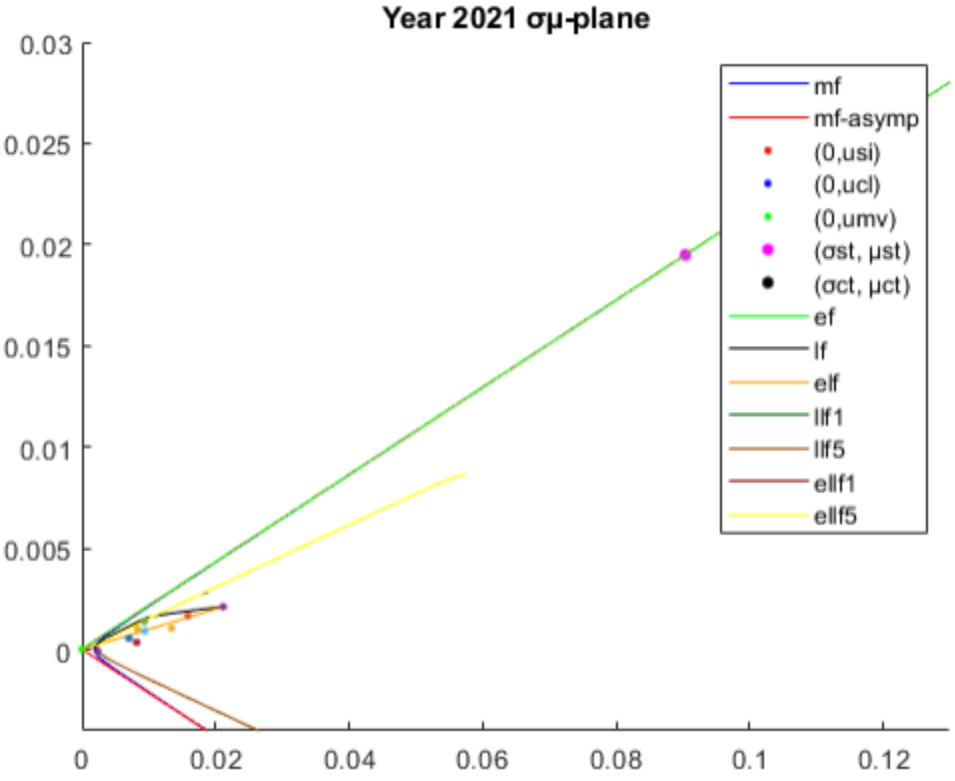
llf5
ellf1
ellf5


Year 2017 σμ-plane
×10^-3
4
3
2
1
0
-1
-2
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
mf
mf-asymp
(0,usi)
(0,ucl)
(0,umv)
(σst, μst)
(σct, μct)
ef
lf
elf
llf1
llf5
ellf1
ellf5

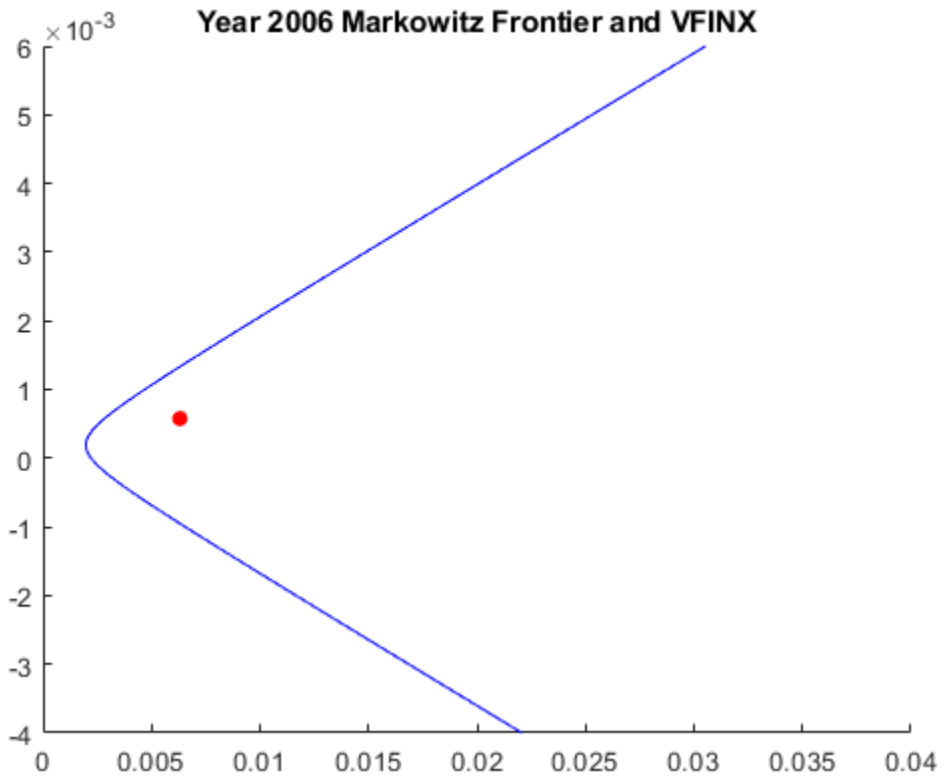

Year 2018 σμ-plane
×10^-3
4
3
2
1
0
-1
-2
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
mf
mf-asymp
(0,usi)
(0,ucl)
(0,umv)
(σst, μst)
(σct, μct)
ef
lf
elf
llf1
llf5
ellf1
ellf5


Year 2019 σμ-plane
×10^-3
4
3
2
1
0
-1
-2
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
mf
mf-asymp
(0,usi)
(0,ucl)
(0,umv)
(σst, μst)
(σct, μct)
ef
lf
elf
llf1
llf5
ellf1
ellf5


Year 2020 σμ-plane
×10^-3
4
3
2
1
0
-1
-2
0
0.01
0.02
0.03
0.04
0.05
0.06
mf
mf-asymp
(0,usi)
(0,ucl)
(0,umv)
(σst, μst)
(σct, μct)
ef
lf
elf
llf1
llf5
ellf1
ellf5


Year 2021 σμ-plane
0.03
0.025
0.02
0.015
0.01
0.005
0
0
0.02
0.04
0.06
0.08
0.1
0.12
mf
mf-asymp
(0,usi)
(0,ucl)
(0,umv)
(σst, μst)
(σct, μct)
ef
lf
elf
llf1
llf5
ellf1
ellf5

Appendix I


Year 2006 Markowitz Frontier and VFINX
×10^-3
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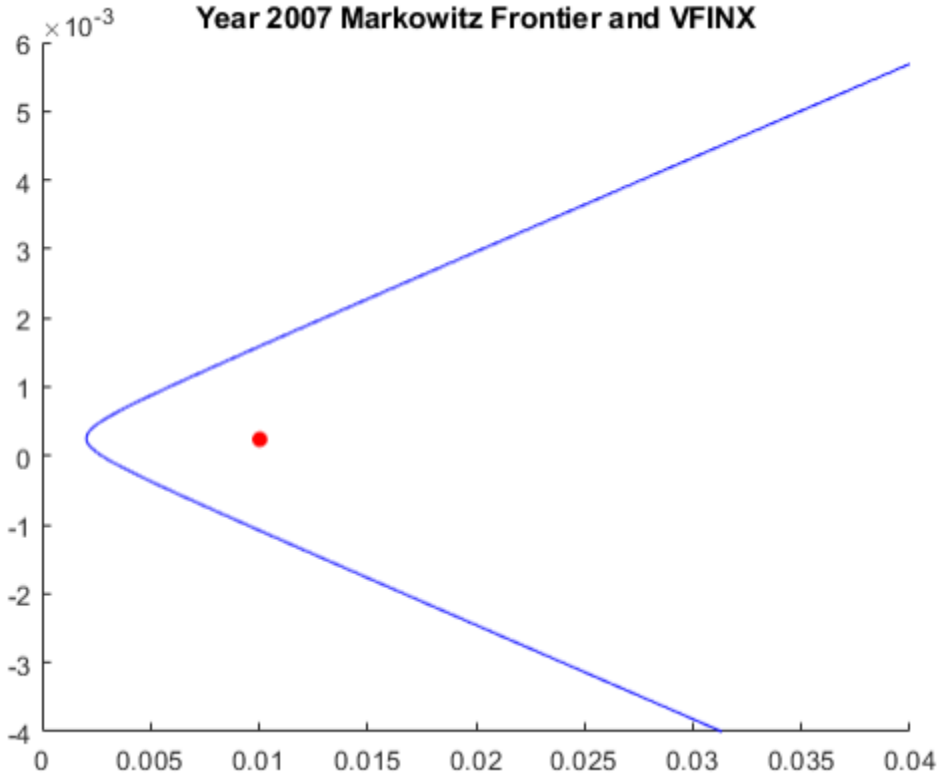
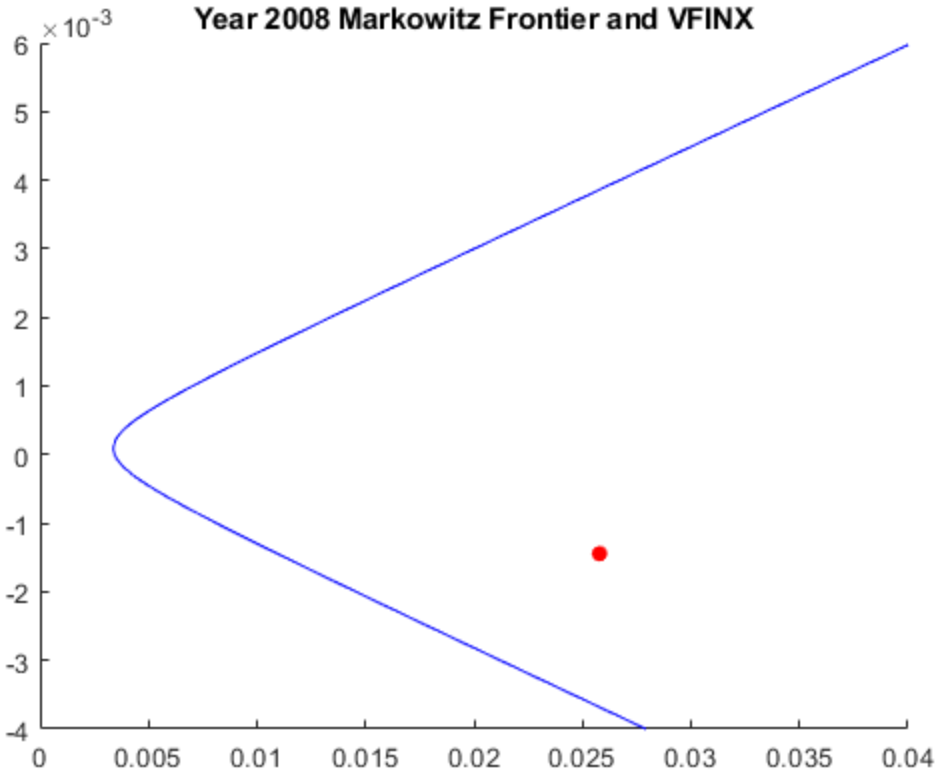
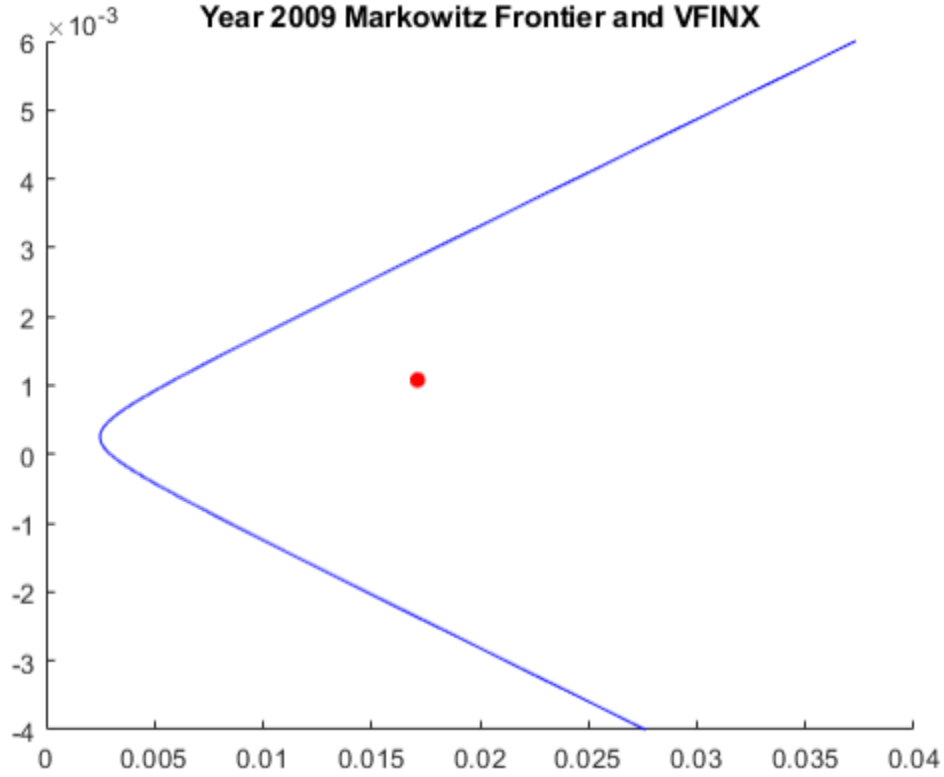
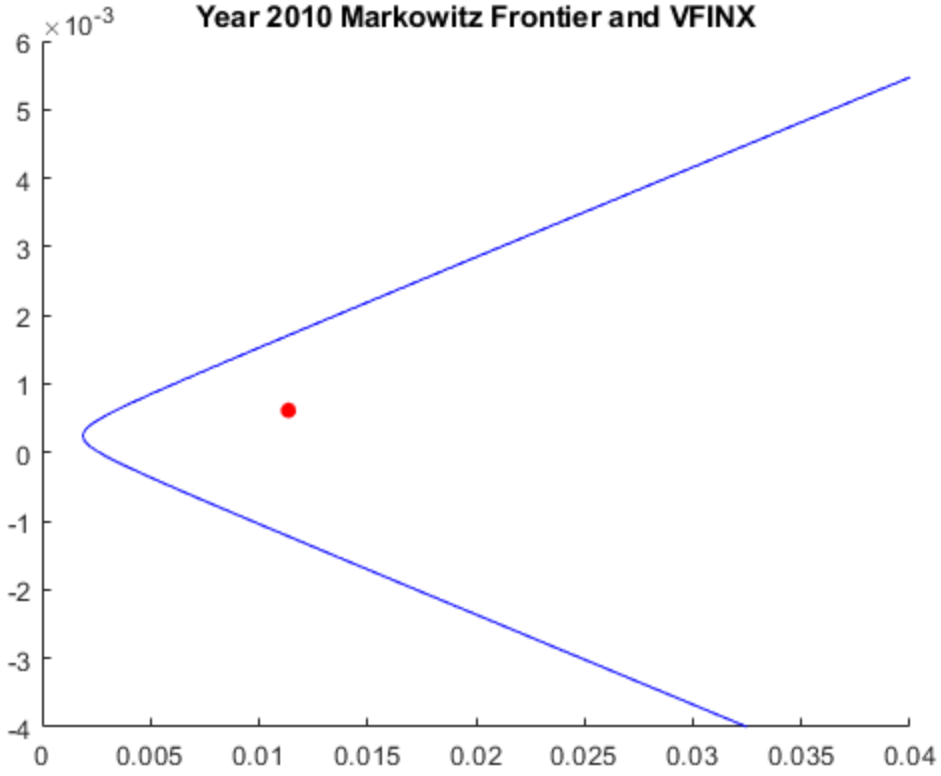
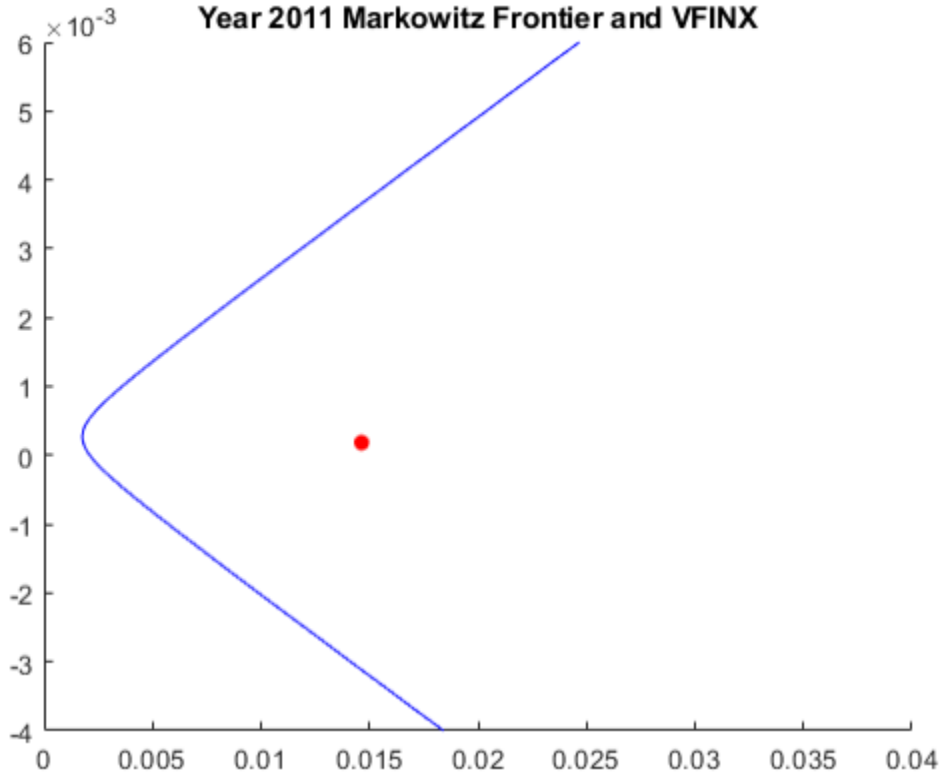
5
4
3
2
1
0
-1
-2
-3
-4
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04


Year 2007 Markowitz Frontier and VFINX
×10^-3
6
5
4
3
2
1
0
-1
-2
-3
-4
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04


Year 2008 Markowitz Frontier and VFINX
×10^-3
6
5
4
3
2
1
0
-1
-2
-3
-4
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04


Year 2009 Markowitz Frontier and VFINX
×10^-3
6
5
4
3
2
1
0
-1
-2
-3
-4
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04


Year 2010 Markowitz Frontier and VFINX
×10^-3
6
5
4
3
2
1
0
-1
-2
-3
-4
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04


Year 2011 Markowitz Frontier and VFINX
×10^-3
6
5
4
3
2
1
0
-1
-2
-3
-4
0
0.005
0.01
0.015
0.02
0.025
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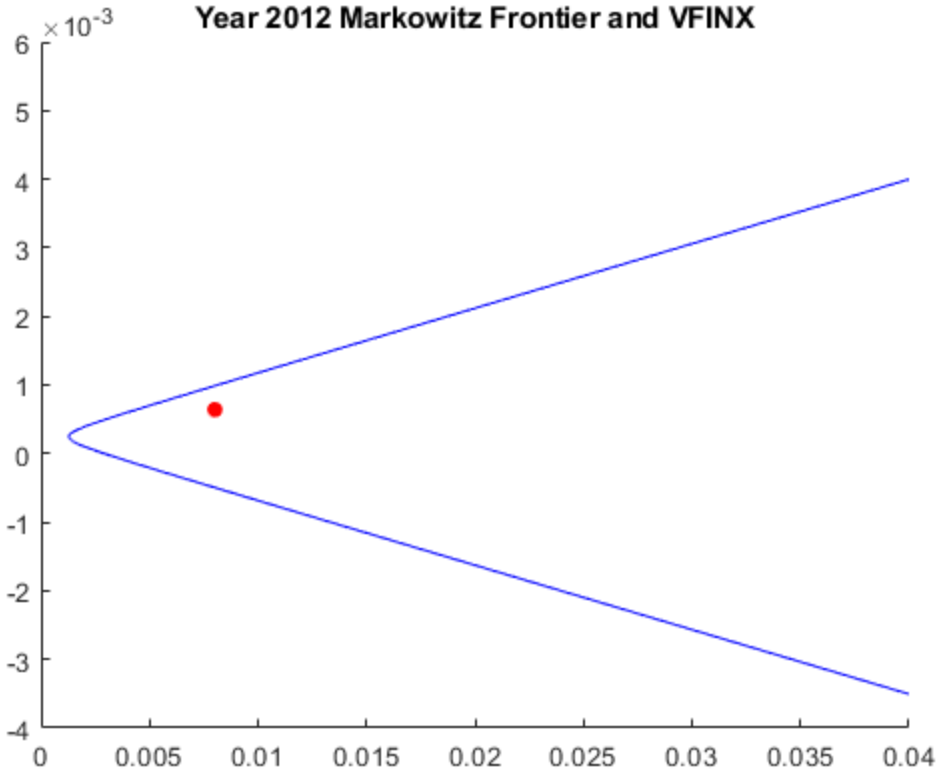
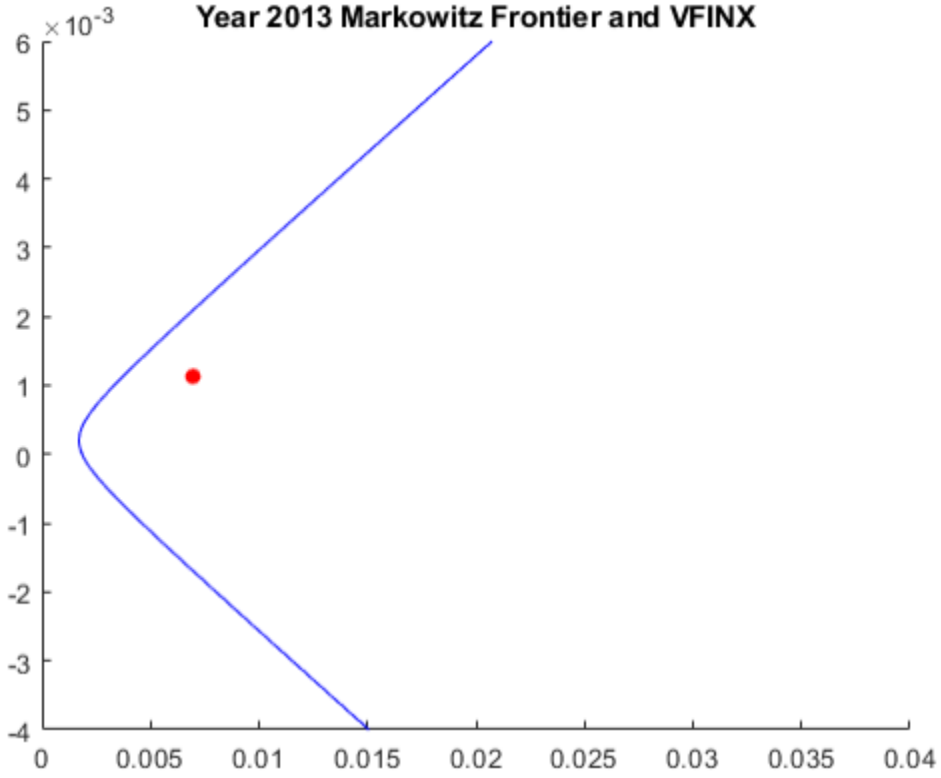
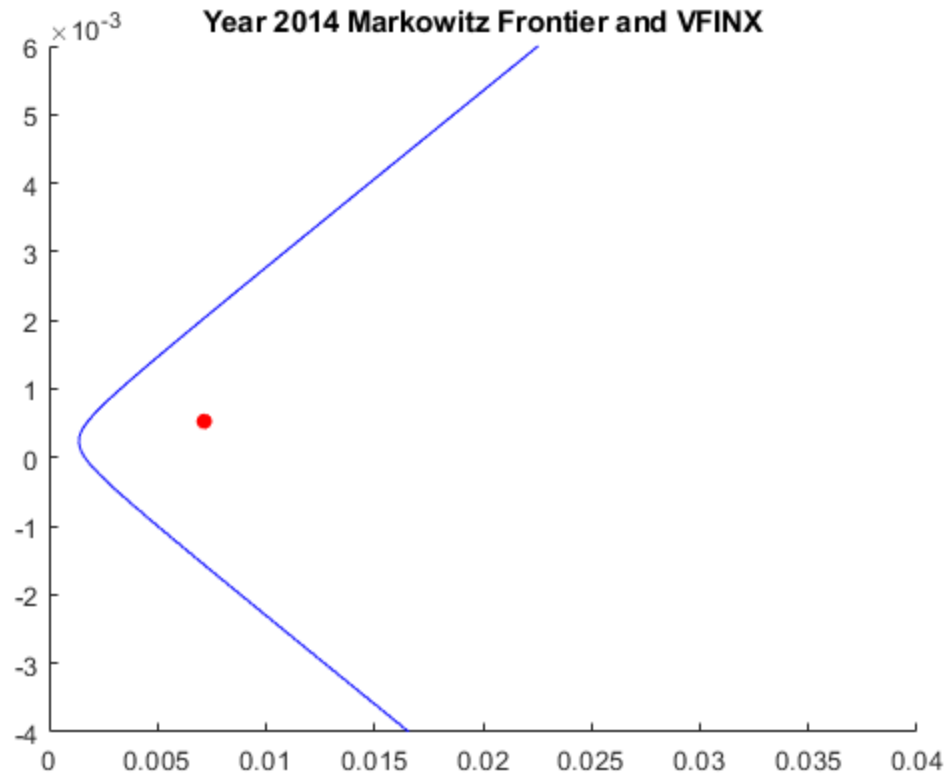
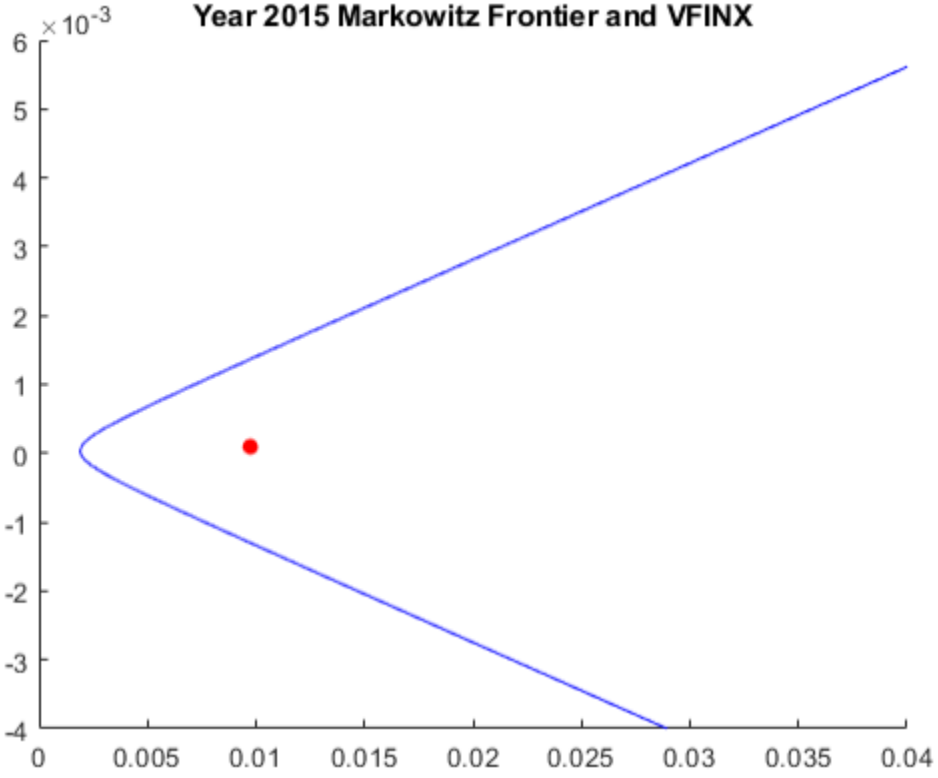
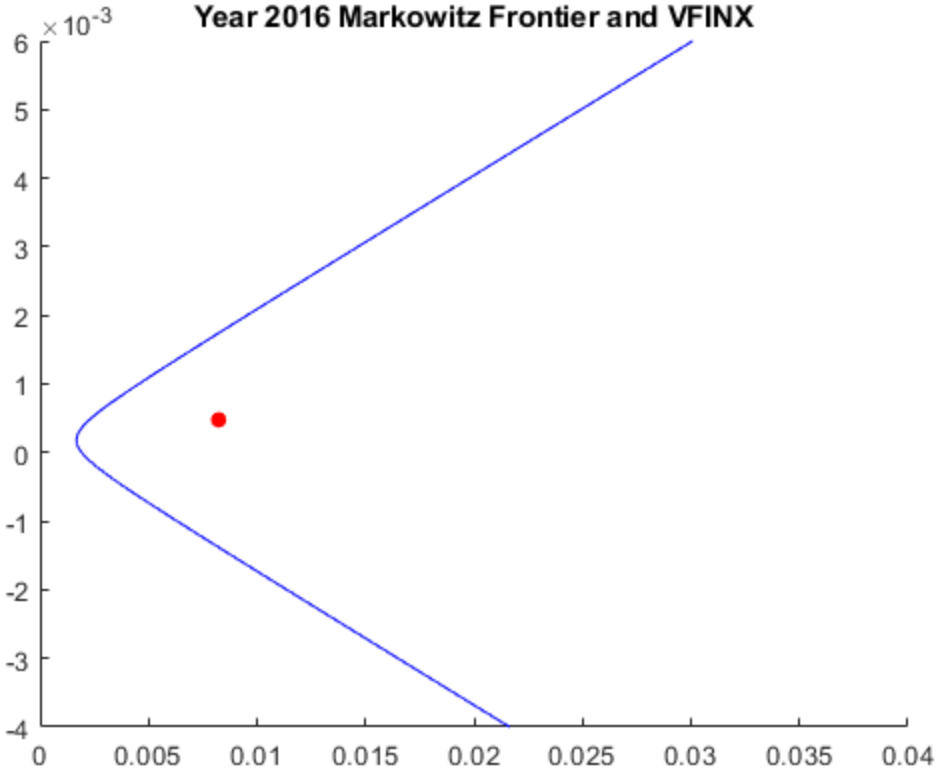
0.035
0.04

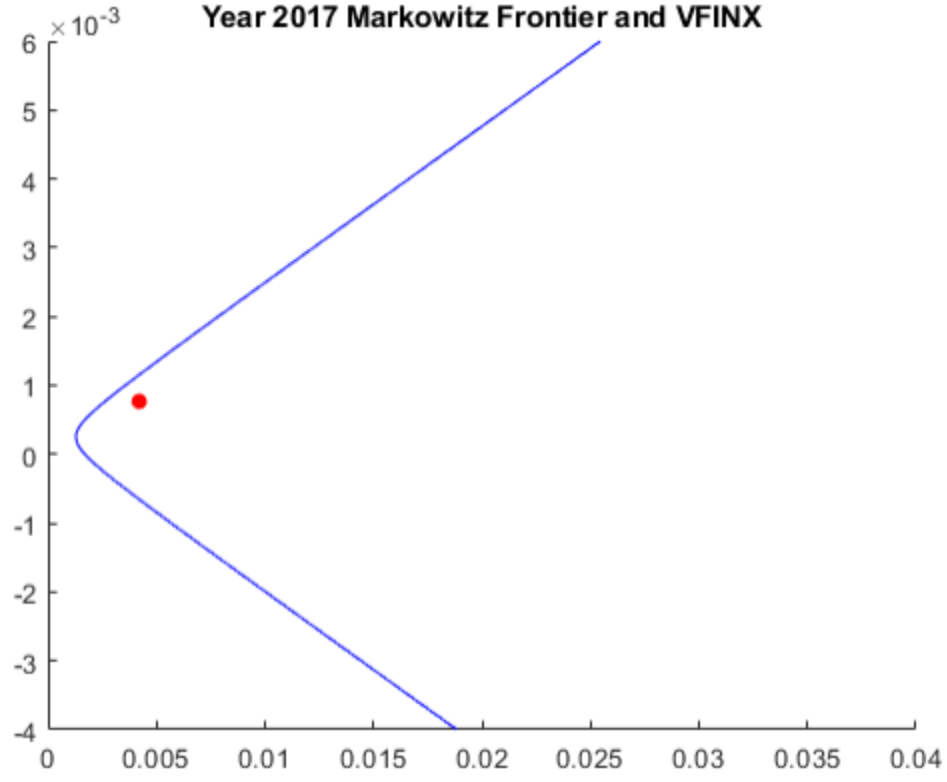

Year 2012 Markowitz Frontier and VFINX
×10^-3
6
5
4
3
2
1
0
-1
-2
-3
-4
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04

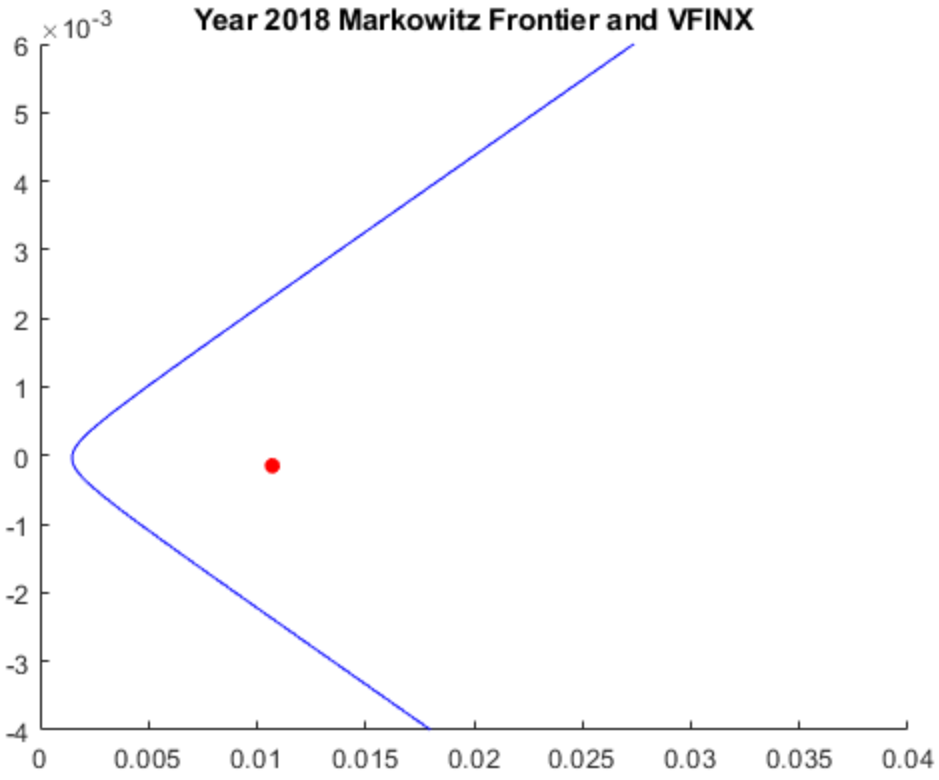

Year 2013 Markowitz Frontier and VFINX
×10^-3
6
5
4
3
2
1
0
-1
-2
-3
-4
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04

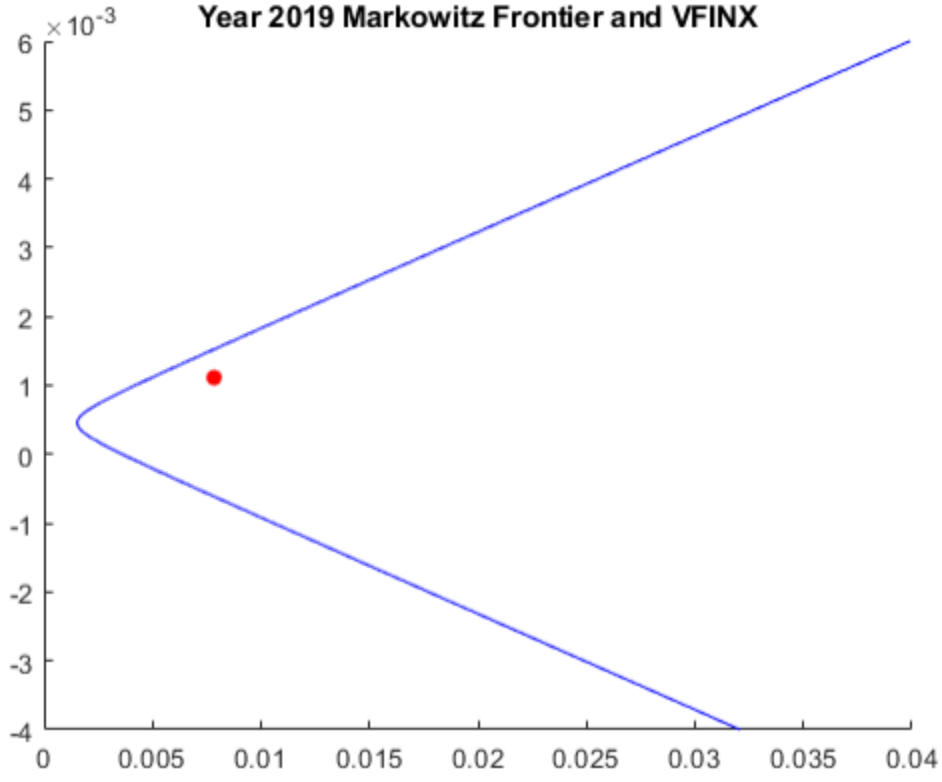

Year 2014 Markowitz Frontier and VFINX
×10^-3
6
5
4
3
2
1
0
-1
-2
-3
-4
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04

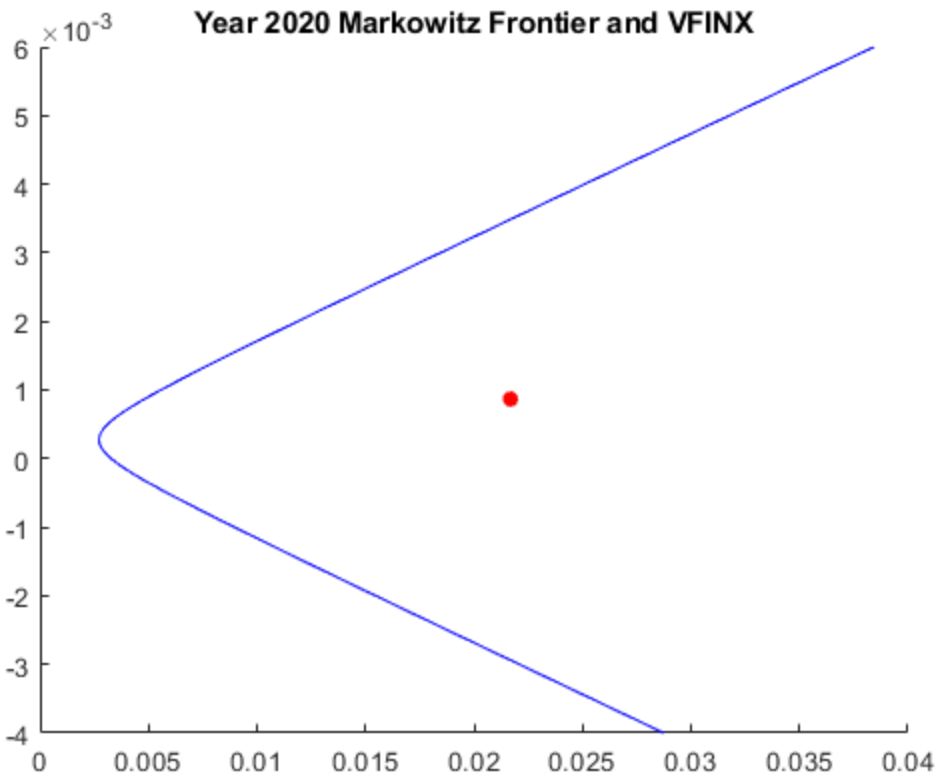

Year 2015 Markowitz Frontier and VFINX
×10^-3
6
5
4
3
2
1
0
-1
-2
-3
-4
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04

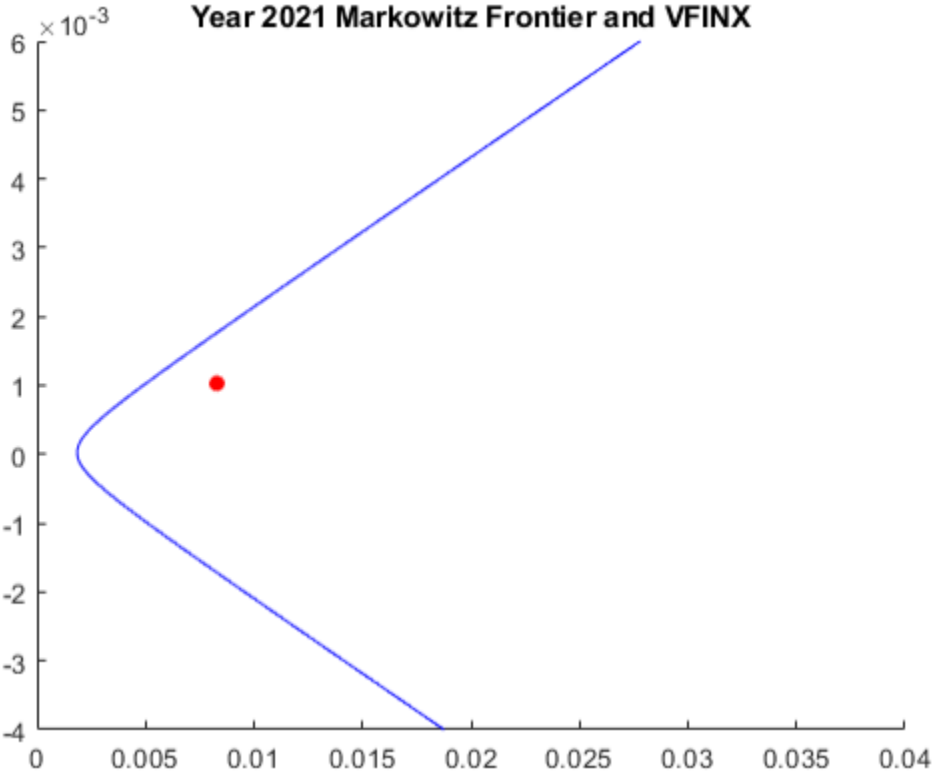

Year 2016 Markowitz Frontier and VFINX
×10^-3
6
5
4
3
2
1
0
-1
-2
-3
-4
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04


Year 2017 Markowitz Frontier and VFINX
×10^-3
6
5
4
3
2
1
0
-1
-2
-3
-4
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04


Year 2018 Markowitz Frontier and VFINX
×10^-3
6
5
4
3
2
1
0
-1
-2
-3
-4
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04

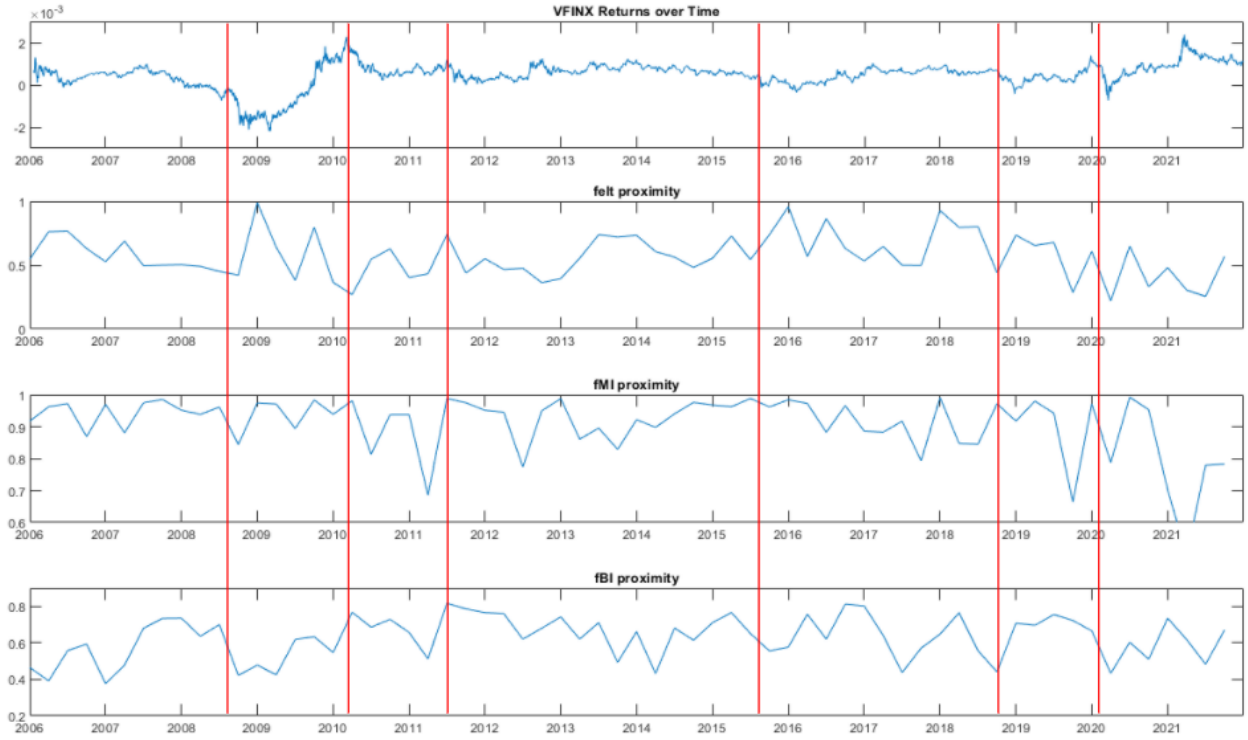

Year 2019 Markowitz Frontier and VFINX
×10^-3
6
5
4
3
2
1
0
-1
-2
-3
-4
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04


Year 2020 Markowitz Frontier and VFINX
×10^-3
6
5
4
3
2
1
0
-1
-2
-3
-4
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04

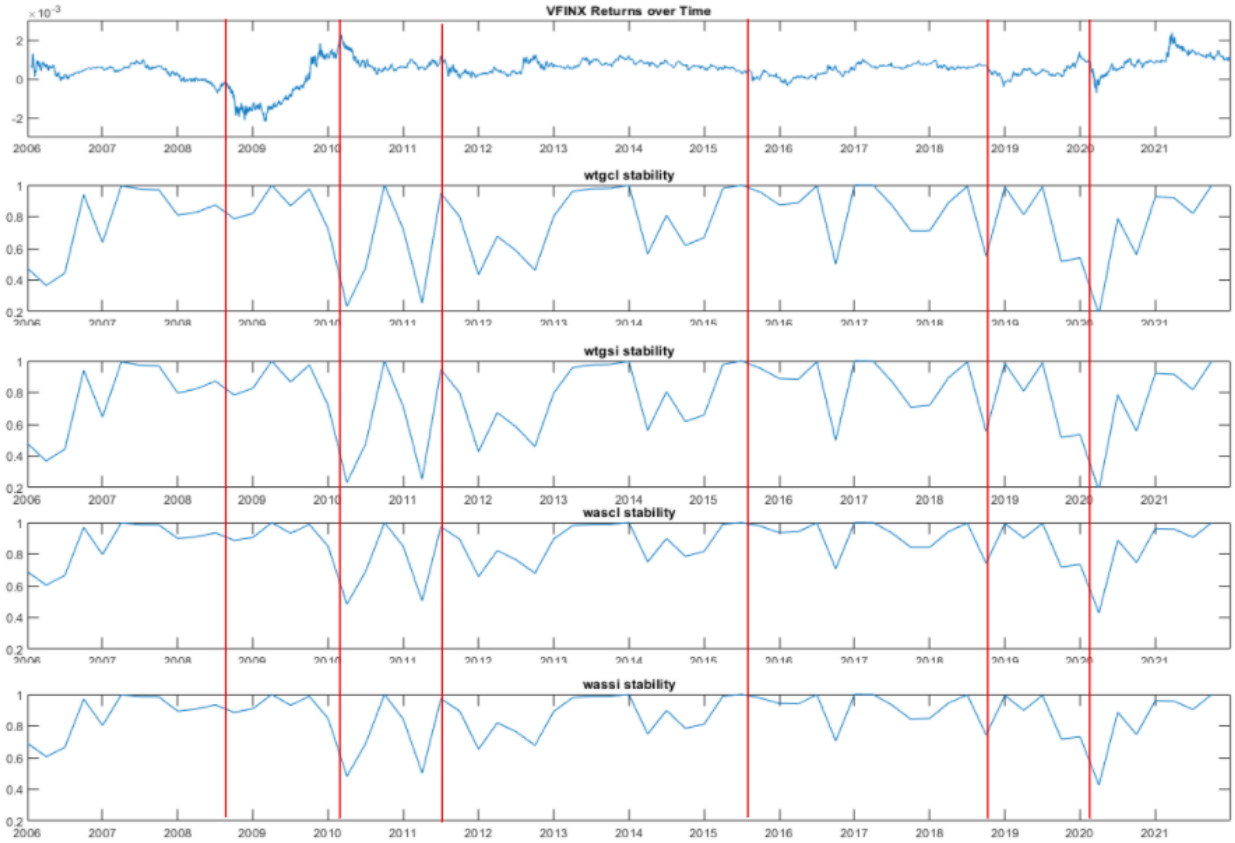

Year 2021 Markowitz Frontier and VFINX
×10^-3
6
5
4
3
2
1
0
-1
-2
-3
-4
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04

Appendix J: Metric Models

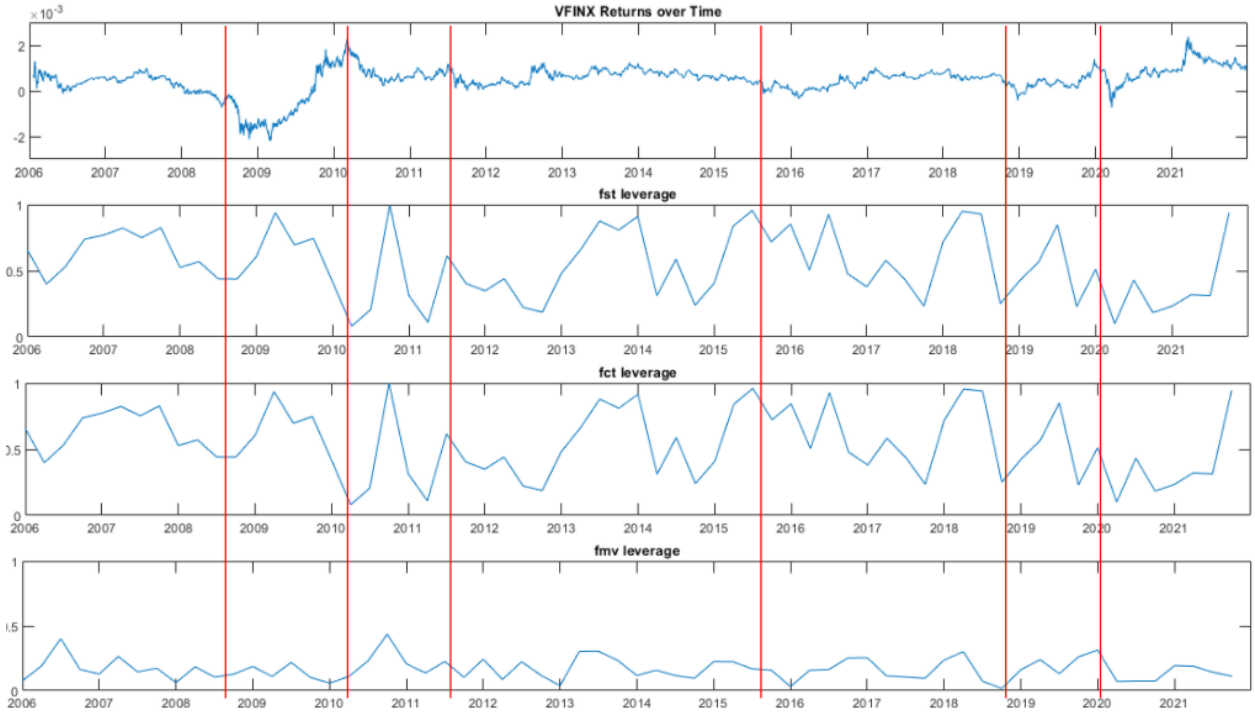
Proximity



Stability


VFINX Returns over Time
wtgcl stability
wtgsi stability
wascl stability
wassi stability

Leverage


VFINX Returns over Time
fst leverage
fct leverage
fmv leverage
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019
2020
2021

Appendix K: Metric Formulas

Proximity:

$$\omega^{\sigma}(\mathbf{f}) = \sqrt{1 - \frac{\sigma_{\text{mf}}(\mu(\mathbf{f}))^2}{\sigma(\mathbf{f})^2}}.$$

Stability:

$$\omega_{\text{rf}}^{\text{tg}} = \frac{\nu_{\text{mv}}^2}{\nu_{\text{rf}}^2}, \qquad \omega_{\text{rf}}^{\text{as}} = \frac{\nu_{\text{mv}}}{\nu_{\text{rf}}}.$$

Leverage:

$$\omega^{\lambda}(\mathbf{f}) = \frac{\lambda(\mathbf{f})}{1 + \lambda(\mathbf{f})}$$