

2. (1)

当 $\frac{m}{V_1} = 4 \text{ kg/m}^3$ 时开始液化, 有

$$V_1 = \frac{0.027}{4} = 6.75 \times 10^{-3} \text{ m}^3$$

(2)

$$V_2 = \frac{0.027}{1.3 \times 10^3} = 2.0769 \times 10^{-5} \text{ m}^3$$

(3) 设气体体积为 V_1 , 液体体积为 V_2

$$\begin{cases} V_1 + V_2 = 1 \times 10^{-3} \\ 4V_1 + 1.3 \times 10^3 V_2 = 0.027 \end{cases}$$

$$\text{得} \begin{cases} V_1 = 9.8225 \times 10^{-4} \text{ m}^3 \\ V_2 = 1.775 \times 10^{-5} \text{ m}^3 \end{cases}$$

7. 由克拉珀龙方程得 $(PV = RT)$

$$\frac{dp}{dT} = \frac{\lambda}{T(V_2 - V_1)} \approx \frac{\lambda P}{RT^2}$$

$$\text{得} \ln P + C = -\frac{\lambda}{RT}$$

$$\text{故而} \ln \frac{P_1}{P_2} = \frac{\lambda}{R} \left(\frac{1}{T_2} - \frac{1}{T_1} \right)$$

$$\text{得} \lambda \approx 92.606 \text{ J/mol}$$

18. 在恒温条件下, 亥姆赫兹自由能为

$$F(T, x) = F(T, 0) - \int_0^x F' dx$$

$$\text{而 } F' = -kx$$

$$\text{有 } F(T, x) = F(T, 0) + \frac{1}{2} k(T) x^2$$

$$\text{而 } \left(\frac{\partial S}{\partial x} \right)_T = \left(\frac{\partial F'}{\partial T} \right)_x = - \frac{dk(T)}{dT} x$$

$$\text{故而 } S(T, x) = S(T, 0) - \frac{1}{2} \frac{dk(T)}{dT} x^2$$

$$\begin{aligned} \left(\frac{\partial U}{\partial x} \right)_T &= T \left(\frac{\partial S}{\partial x} \right)_T + kx \\ &= -T \frac{dk(T)}{dT} x + kx \end{aligned}$$

$$\text{故而 } U(T, x) = U(T, 0) + \frac{1}{2} \left(k - T \frac{dk(T)}{dT} \right) x^2$$

24. (1) 系统在可逆等温过程中, 有

$$Q = T \Delta S$$

$$\text{而 } \left(\frac{\partial S}{\partial H} \right)_T = V M_0 \left(\frac{\partial M}{\partial T} \right)_H$$

$$\left(\frac{\partial M}{\partial T}\right)_H = -\frac{C}{T^2} H$$

$$\text{故得} \left(\frac{\partial S}{\partial H}\right)_T = V\mu_0 \left(-\frac{C}{T^2} H\right) = -\frac{VC\mu_0}{T^2} H$$

$$\Delta S = \int_0^H \left(\frac{\partial S}{\partial H}\right)_T dH = -\frac{CV\mu_0 H^2}{T^2}$$

$$Q = T\Delta S = -\frac{CV\mu_0 H^2}{2T}$$

$$(2) F(T, M) = F(T, 0) + \int \mu_0 H dM = F(T, 0) + \frac{\mu_0 T M^2}{2C}$$

$$S(T, M) = -\frac{dF}{dT} = S(T, 0) - \frac{\mu_0 M^2}{2C}$$

$$U = F + TS = F(T, 0) + TS(T, 0)$$

$$U = U(T, 0)$$

$$2b. \quad C_p = \left(\frac{\partial H}{\partial T}\right)_p \quad C_v = \left(\frac{\partial U}{\partial T}\right)_v$$

$$C_v = T \left(\frac{\partial S}{\partial T}\right)_v$$

$$C_p = T \left(\frac{\partial S}{\partial T}\right)_p$$

$$C_p - C_v = T \left(\frac{\partial S}{\partial T}\right)_v + T \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p - T \left(\frac{\partial S}{\partial T}\right)_v$$

$$= T \left(\frac{\partial S}{\partial V} \right)_T \left(\frac{\partial V}{\partial T} \right)_P$$

$$= T \left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial V}{\partial T} \right)_P$$

$$\text{若 } \left(\frac{\partial P}{\partial V} \right)_T < 0$$

$$\text{则由 } \left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial V}{\partial P} \right)_T \left(\frac{\partial T}{\partial V} \right)_P = -1$$

$$\text{得 } \left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial V}{\partial T} \right)_P > 0$$

故而 $C_P > 0$

$$\text{而 } \left(\frac{\partial P}{\partial V} \right)_S = \left(\frac{\partial P}{\partial V} \right)_T + \left(\frac{\partial P}{\partial S} \right)_V \left(\frac{\partial S}{\partial V} \right)_T$$

$$= \left(\frac{\partial P}{\partial V} \right)_T + \left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial S}{\partial T} \right)_V$$

$$\text{而 } \frac{1}{\left(\frac{\partial P}{\partial S} \right)_V} = \left(\frac{\partial S}{\partial P} \right)_V$$

$$= \frac{C_P}{T} \left(\frac{\partial T}{\partial P} \right)_V - \left(\frac{\partial V}{\partial T} \right)_P$$

$$\left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial P}{\partial S} \right)_V = \frac{1}{\frac{C_P}{T} \left(\frac{\partial T}{\partial P} \right)_V - \left(\frac{\partial V}{\partial T} \right)_P \left(\frac{\partial T}{\partial P} \right)_V}$$

$$\text{有 } \frac{C_P}{T} \left(\frac{\partial T}{\partial P} \right)_V - \left(\frac{\partial V}{\partial T} \right)_P \left(\frac{\partial T}{\partial P} \right)_V < 0$$

故有 $\left(\frac{\partial P}{\partial V}\right)_T < 0$

32. 相变潜热 $\lambda = h^\beta - h^\alpha$

$$\begin{aligned}\text{而 } \frac{dh}{dT} &= \left(\frac{\partial h}{\partial T}\right)_P + \left(\frac{\partial h}{\partial P}\right)_T \frac{dP}{dT} \\ &= C_P + \left[V - T \left(\frac{\partial V}{\partial T}\right)_P \right] \frac{dP}{dT}\end{aligned}$$

由克拉珀龙方程, 有

$$\frac{dP}{dT} = \frac{\lambda}{T(V_m^\beta - V_m^\alpha)}$$

$$\text{故有 } \frac{dh}{dT} = C_P + \left[V - T \left(\frac{\partial V}{\partial T}\right)_P \right] \frac{\lambda}{T(V_m^\beta - V_m^\alpha)}$$

$$\text{进而 } \frac{d\lambda}{dT} = C_P^\beta - C_P^\alpha + \frac{\lambda}{T} + \left[\left(\frac{\partial V_m^\alpha}{\partial T}\right)_P - \left(\frac{\partial V_m^\beta}{\partial T}\right)_P \right] \frac{\lambda}{(V_m^\beta - V_m^\alpha)}$$

当 β 相为气相, α 是液相时,

$$V_m^\beta \gg V_m^\alpha$$

$$\text{且 } V_m^\beta \propto T$$

$$\text{故有 } \frac{\partial V_m^\beta}{\partial T} = \frac{V_m^\beta}{T}$$

$$\text{有 } \frac{d\lambda}{dT} = C_P^\beta - C_P^\alpha + \frac{\lambda}{T} - \frac{\lambda}{T} = C_P^\beta - C_P^\alpha$$

33.

$$d(pV_m) = d(RT)$$

$$V_m dp + p dV_m = R dT$$

$$V_m \frac{dp}{dT} + p \frac{dV_m}{dT} = R$$

$$p \frac{dV_m}{dT} = R - V_m \frac{dp}{dT} \approx R - V_m \frac{\lambda}{TV_m}$$

$$\frac{dV_m}{dT} = \frac{1}{p} \left(R - \frac{\lambda}{T} \right)$$

$$\frac{1}{V_m} \frac{dV_m}{dT} = \frac{1}{RT} \left(R - \frac{\lambda}{T} \right)$$

$$\frac{1}{V_m} \frac{dV_m}{dT} = \frac{1}{T} \left(1 - \frac{\lambda}{RT} \right)$$