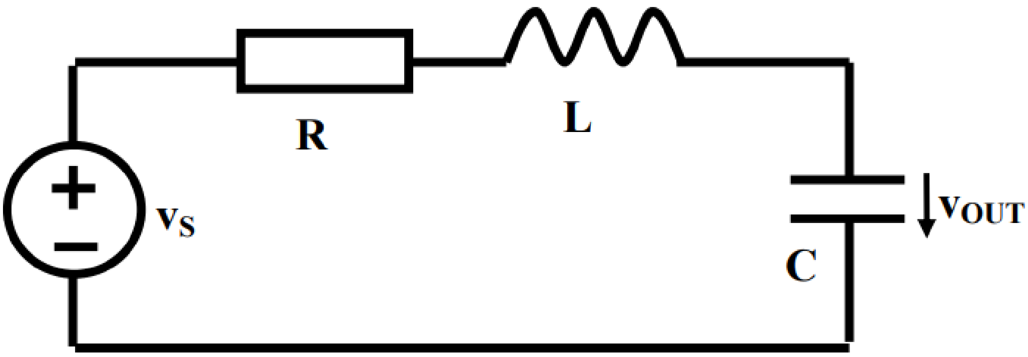
RLC串联电路仿真

无04 2019012137 张鸿琳

对下面电路进行分析：



可知传递函数为：

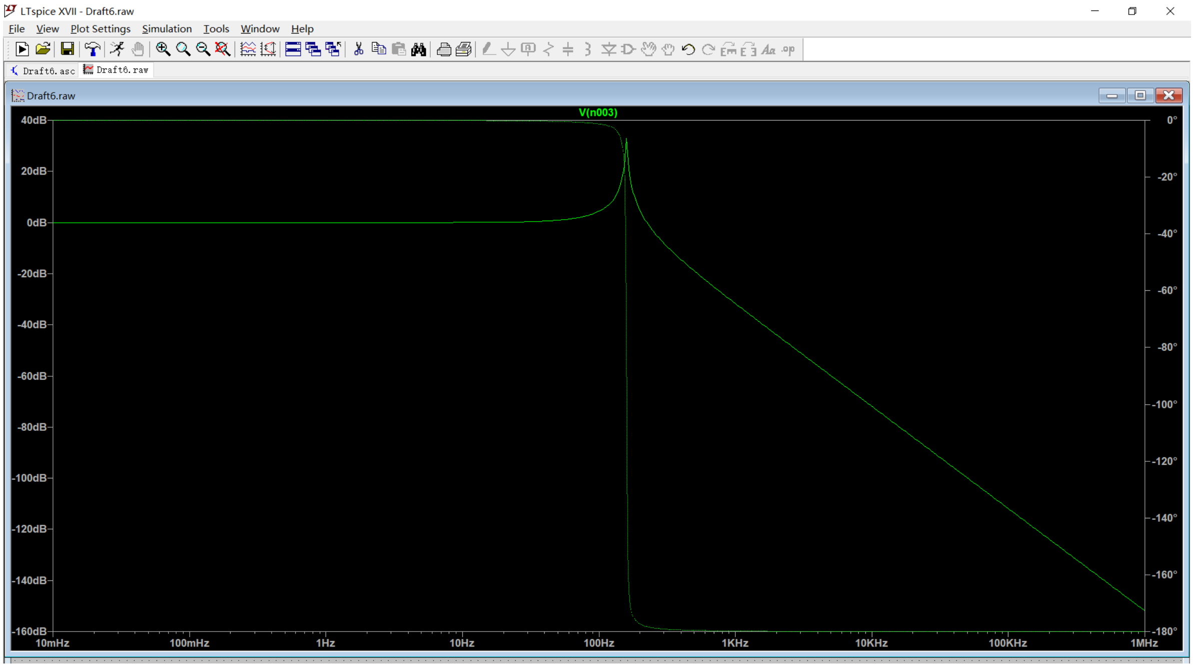
$$H(s)=\frac{1}{s^2LC+sRC+1}=\frac{\omega_0^2}{s^2+2\xi\omega_0 s+\omega_0^2} \tag{2}$$

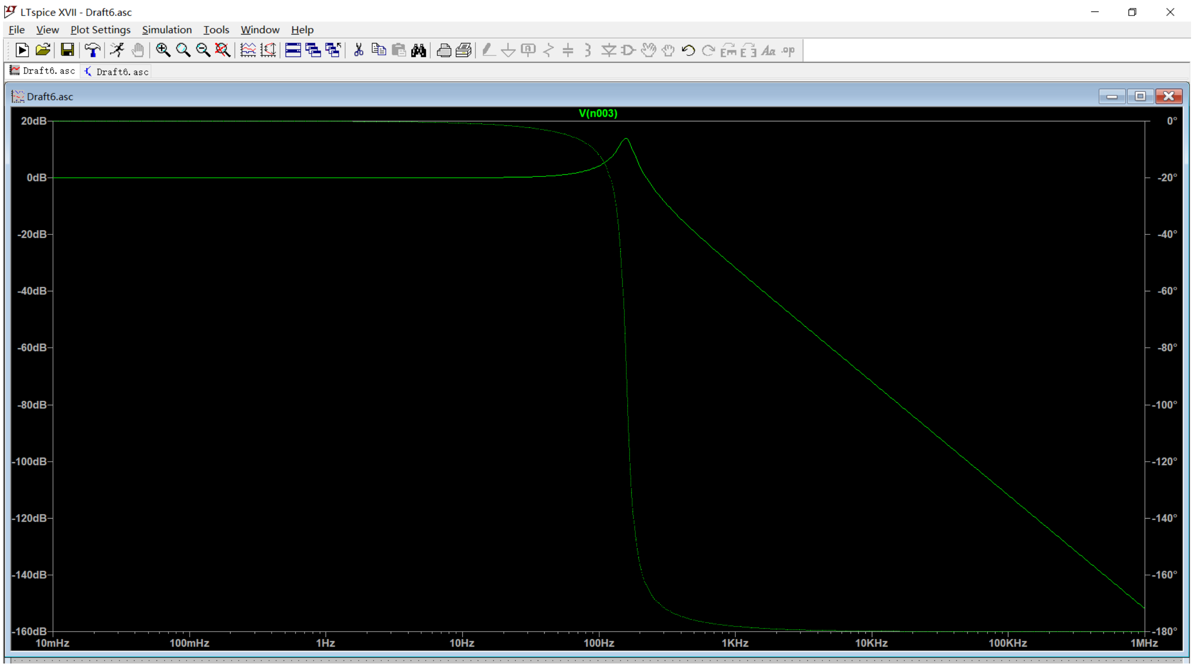
自由震荡频率为$\omega_0=\frac{1}{\sqrt{LC}}$，阻尼系数为$\xi=\frac{1}{2}R\sqrt{\frac{C}{L}}$。

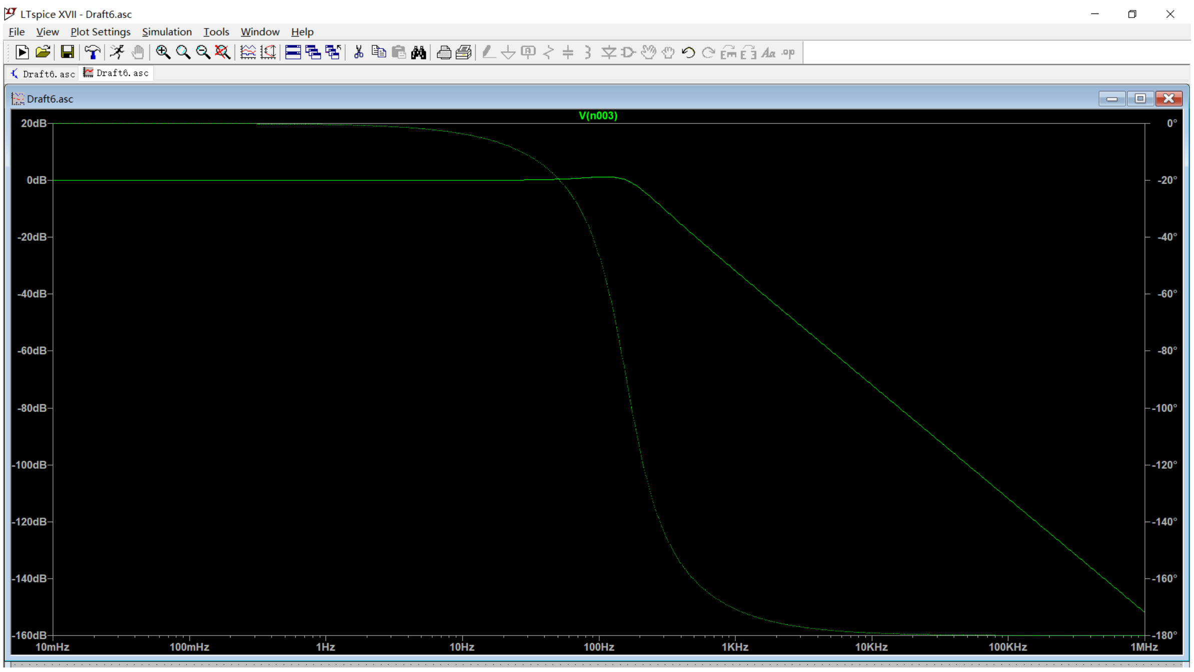
首先研究其幅频渐进特性，可以看出$|H(j\omega)|=\frac{1}{\sqrt{1+(4\xi^2-2)(\omega/\omega_0)^2+(\omega/\omega_0)^4}}$，当$\omega$较小时，$|H(j\omega)|\approx 1$，当$\xi<\frac{1}{\sqrt{2}}$时，$|H(j\omega)|$存在最大值，这样在$\omega=\sqrt{(1-2\xi^2)}\omega_0$存在一个极大值点，极大值为$|H(j\omega)|=\frac{1}{2\xi\sqrt{1-\xi^2}}$，而后当频率进一步增大，$|H(j\omega)|\approx\frac{1}{(\omega/\omega_0)^2}$，在伯德图上近似表现为斜率为$-40$的直线。当$\xi\geq\frac{1}{\sqrt{2}}$时，将不会再出现极大值点，幅频特性可以近似用两条相交直线表示。而当ξ进一步增大，阻尼系数的影响进一步扩大，会使转折点明显左移，同时也会导致ω较大时的负斜率直线不再平直，会出现一段$|H(j\omega)|\approx\frac{1}{(\omega/\omega_0)\sqrt{4\xi^2-2}}$的区域，在伯德图上表现为斜率近似为$-20$的直线，同时阻尼系数越大，该段直线的跨度会越大。而更精准的近似可以通过观察$H(j\omega)$的零点和极点分布得到。

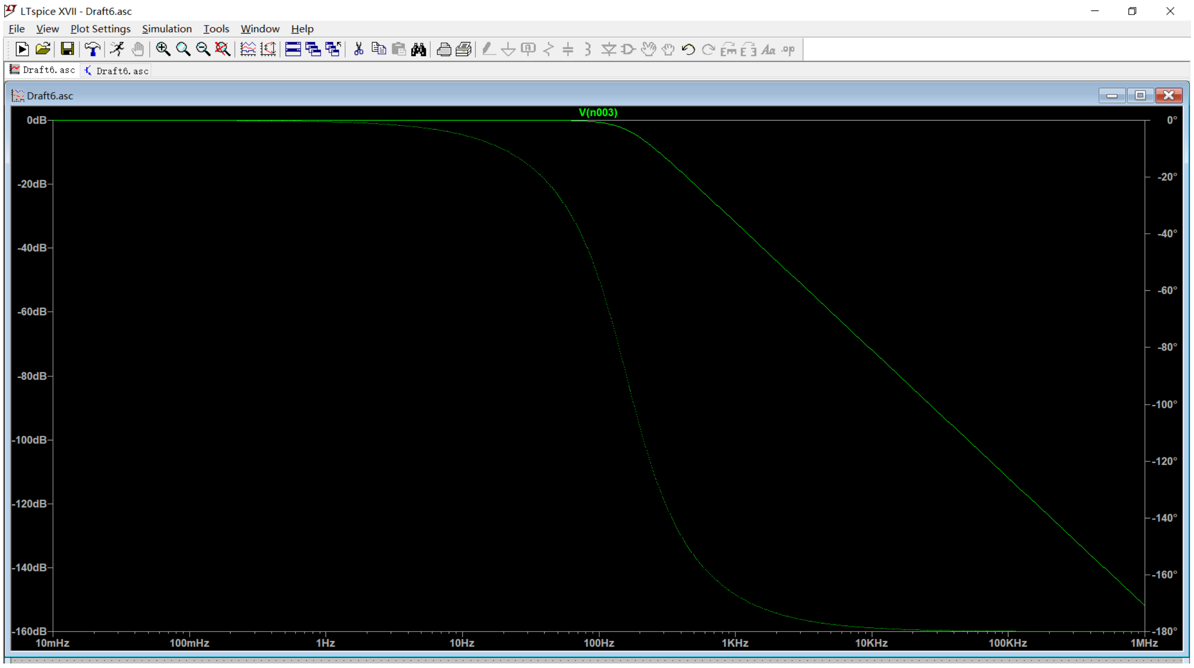
再分析相频特性有$\angle H(j\omega)=-\arctan\left(\frac{2\xi\omega_0\omega}{\omega_0^2-\omega^2}\right)$，当$\omega$较小时，显然$\angle H(j\omega)\approx 0$。而根据$H(jw)$式子，利用其极点和零点分布进行近似，可以得到，存在两个极点$\omega_1=2\xi\omega_0+2\omega_0\sqrt{\xi^2-1}$，$\omega_2=2\xi\omega_0-2\omega_0\sqrt{\xi^2-1}$，每遇到一个极点，相位下降$90^\circ$。当$\xi<1$时，相频曲线在$\omega_0$附近直接下降$180^\circ$，而当$\xi>1$时，存在两个实极点，那么会出现一段近似平台，范围是$4\omega_0\sqrt{\xi^2-1}$，随着ξ增大而扩大，但是$\angle H(j\omega_0)=-90^\circ$始终成立。同时随着$\xi$的增大，$\omega_2$不断减小，$\omega_1$不断增大，也就是相变范围也不断扩大。

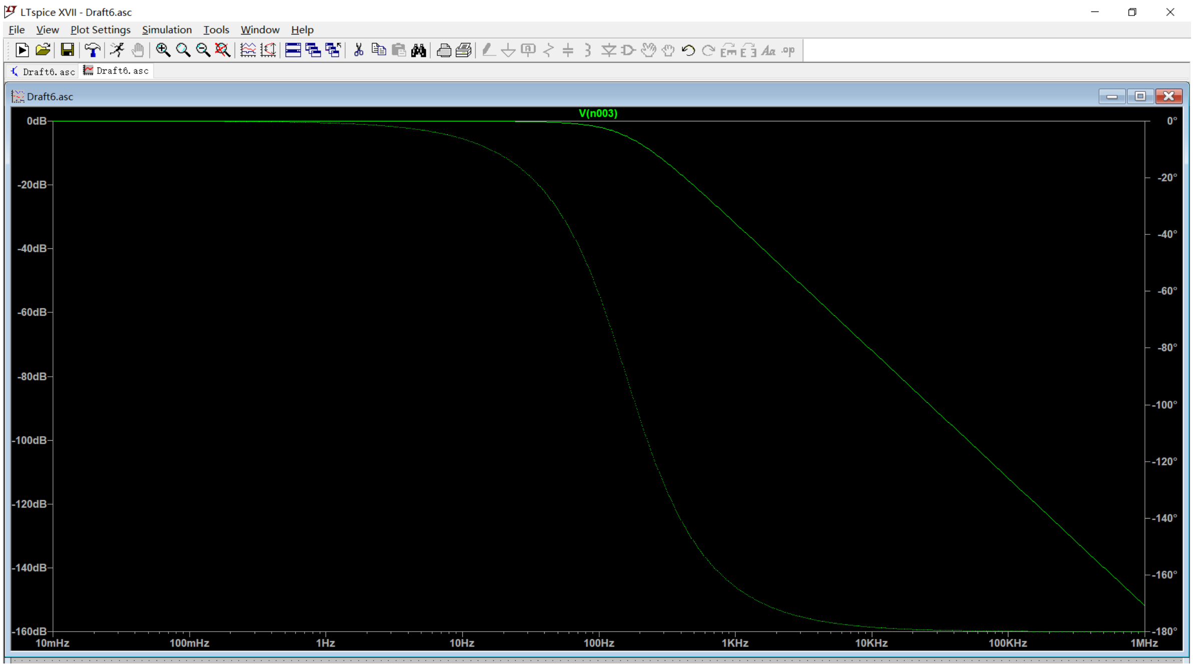
假设固定L和C，可以通过调节R的大小确定阻尼系数。本次仿真取$L=1mH$，$C=1mF$，变化电阻的大小来调节阻尼系数，当阻尼系数ξ分别为$0.01,0.1,0.5,0.707,0.866,1,2,10,50,100$时的伯德图如下（虚线为相频特性，实线为幅频特性）：

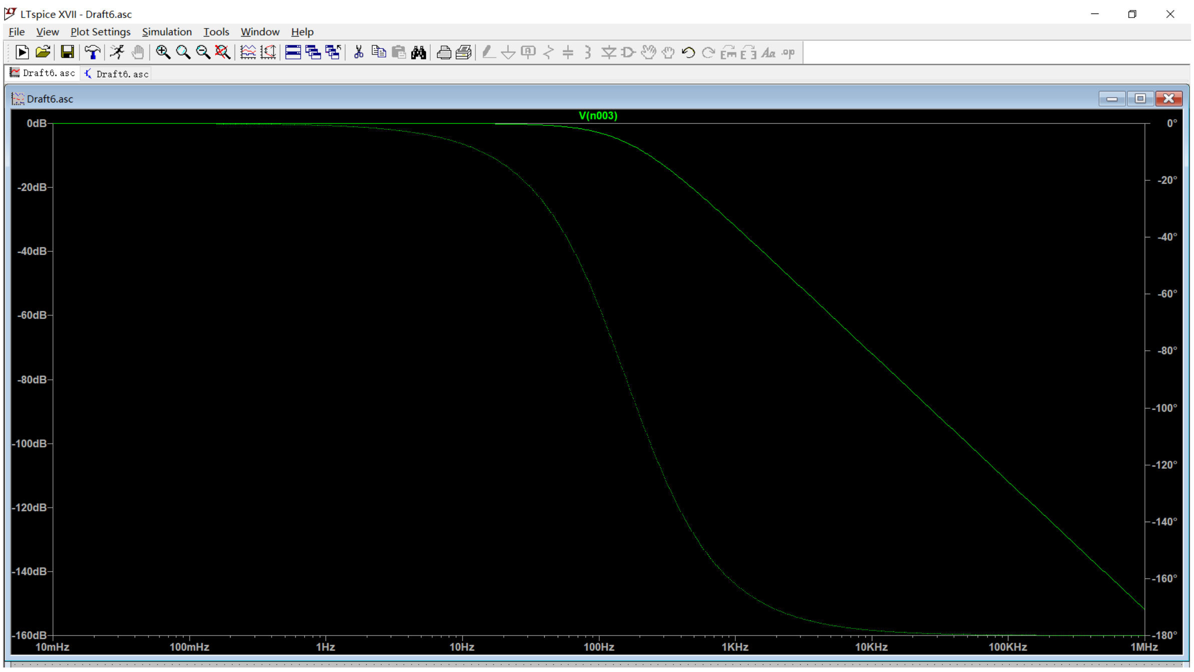

LTspice XVII - Draft6.raw
File View Plot Settings Simulation Tools Window Help
Draft6.asc
Draft6.raw
Draft6.raw
V(n003)
40dB
20dB
0dB
-20dB
-40dB
-60dB
-80dB
-100dB
-120dB
-140dB
-160dB
0°
-20°
-40°
-60°
-80°
-100°
-120°
-140°
-160°
-180°
10mHz
100mHz
1Hz
10Hz
100Hz
1KHz
10KHz
100KHz
1MHz

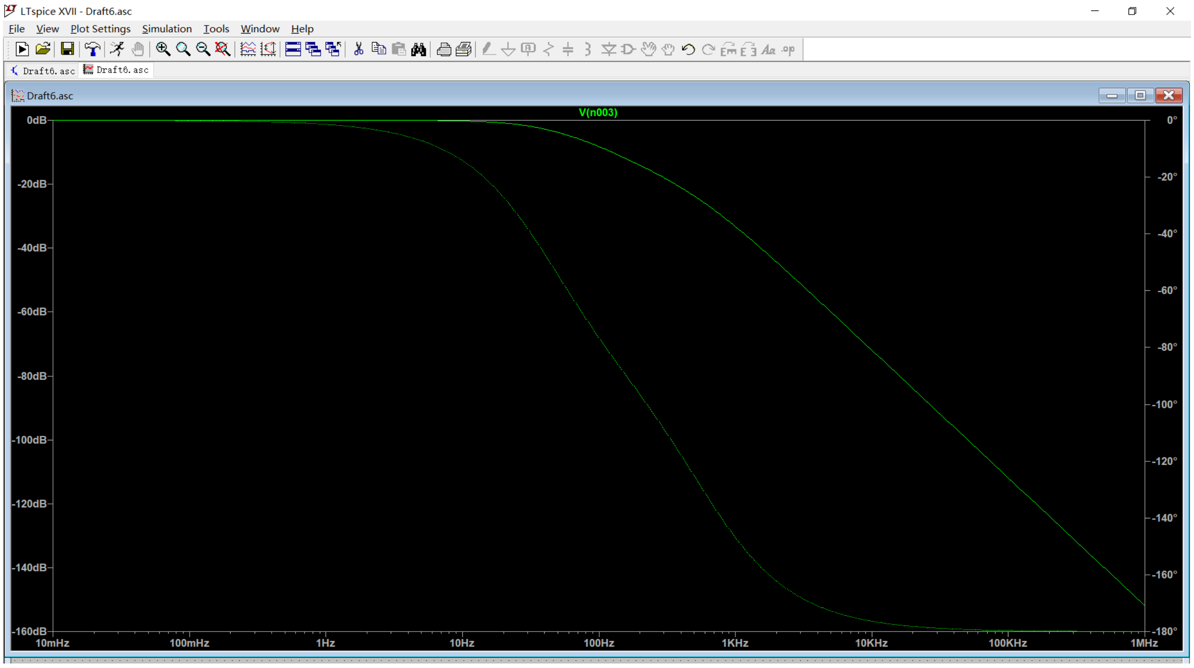

LTspice XVII - Draft6.asc
File View Plot Settings Simulation Tools Window Help
Draft6.asc
Draft6.asc
Draft6.asc
V(n003)
20dB
0dB
-20dB
-40dB
-60dB
-80dB
-100dB
-120dB
-140dB
-160dB
0°
-20°
-40°
-60°
-80°
-100°
-120°
-140°
-160°
-180°
10mHz
100mHz
1Hz
10Hz
100Hz
1KHz
10KHz
100KHz
1MHz

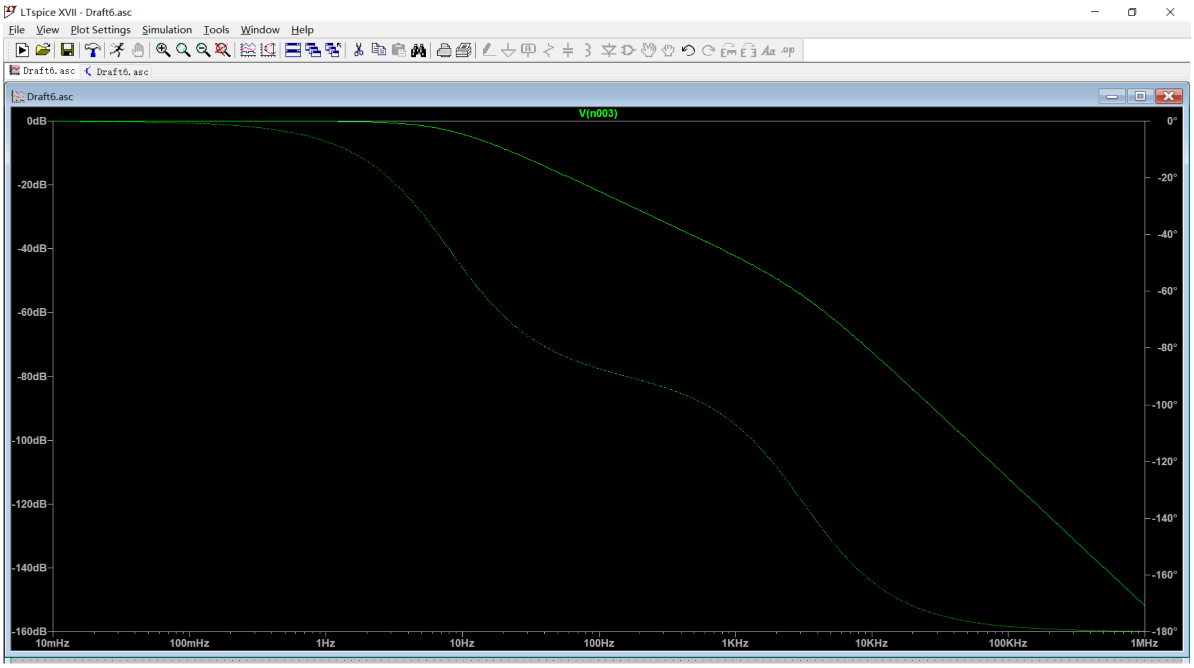

LTspice XVII - Draft6.asc
File View Plot Settings Simulation Tools Window Help
Draft6.asc
Draft6.asc
Draft6.asc
V(n003)
20dB
0dB
-20dB
-40dB
-60dB
-80dB
-100dB
-120dB
-140dB
-160dB
0°
-20°
-40°
-60°
-80°
-100°
-120°
-140°
-160°
-180°
10mHz
100mHz
1Hz
10Hz
100Hz
1KHz
10KHz
100KHz
1MHz

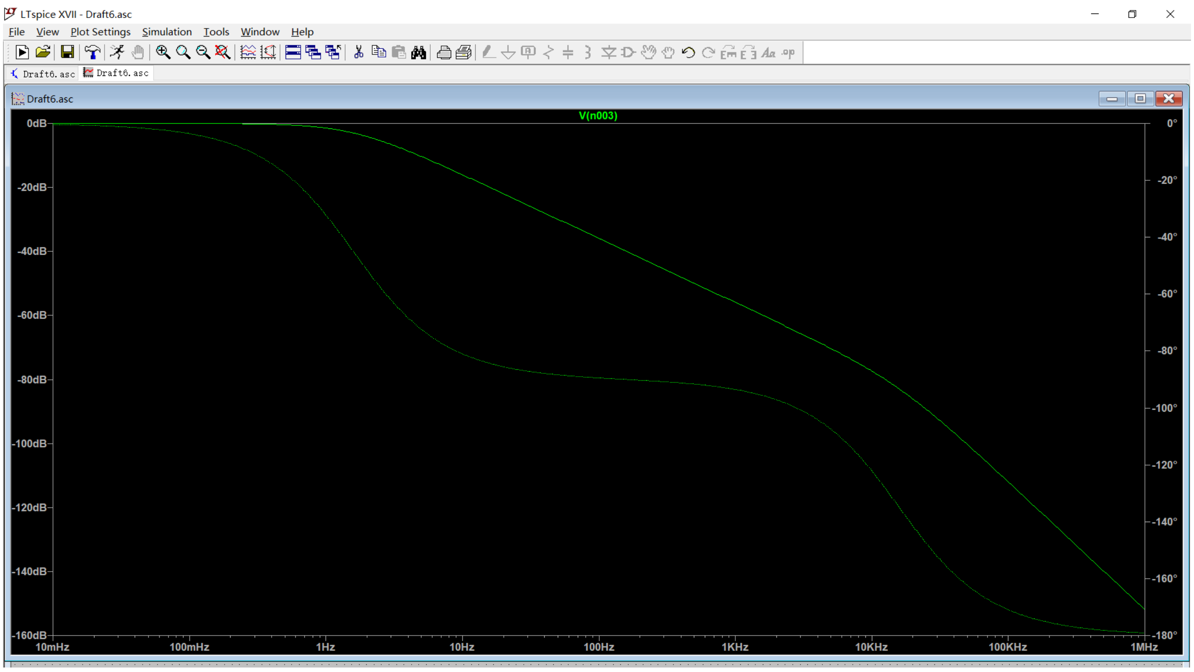

LTspice XVII - Draft6.asc
File View Plot Settings Simulation Tools Window Help
Draft6.asc
V(n003)
0dB -20dB -40dB -60dB -80dB -100dB -120dB -140dB -160dB
0° -20° -40° -60° -80° -100° -120° -140° -160° -180°
10mHz 100mHz 1Hz 10Hz 100Hz 1KHz 10KHz 100KHz 1MHz

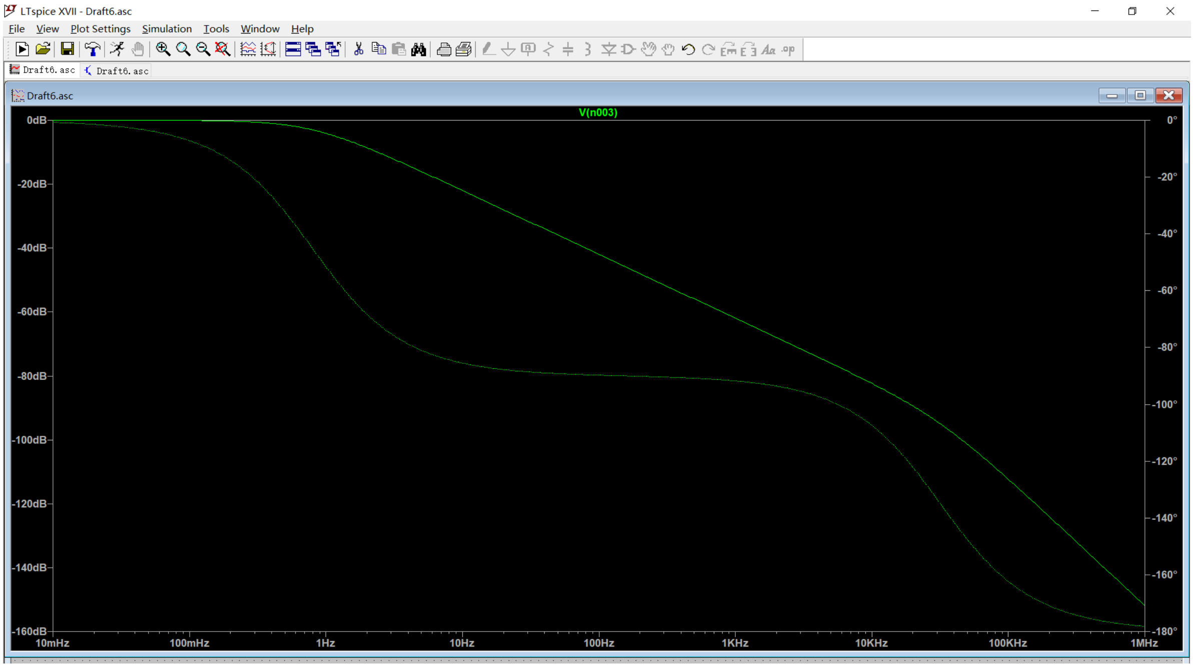

LTspice XVII - Draft6.asc
File View Plot Settings Simulation Tools Window Help
Draft6.asc
V(n003)
0dB -20dB -40dB -60dB -80dB -100dB -120dB -140dB -160dB
0° -20° -40° -60° -80° -100° -120° -140° -160° -180°
10mHz 100mHz 1Hz 10Hz 100Hz 1KHz 10KHz 100KHz 1MHz


LTspice XVII - Draft6.asc
File View Plot Settings Simulation Tools Window Help
Draft6.asc
V(n003)
0dB -20dB -40dB -60dB -80dB -100dB -120dB -140dB -160dB
0° -20° -40° -60° -80° -100° -120° -140° -160° -180°
10mHz 100mHz 1Hz 10Hz 100Hz 1KHz 10KHz 100KHz 1MHz


LTspice XVII - Draft6.asc
File View Plot Settings Simulation Tools Window Help
Draft6.asc
V(n003)
0dB
-20dB
-40dB
-60dB
-80dB
-100dB
-120dB
-140dB
-160dB
0°
-20°
-40°
-60°
-80°
-100°
-120°
-140°
-160°
-180°
10mHz 100mHz 1Hz 10Hz 100Hz 1KHz 10KHz 100KHz 1MHz

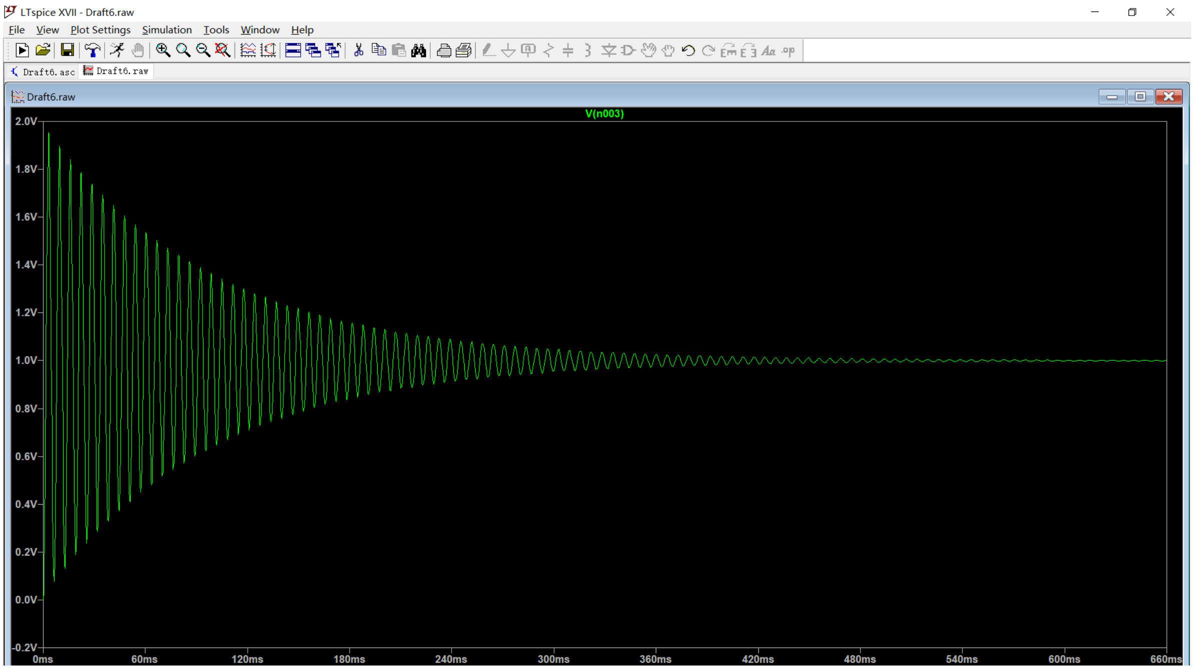

LTspice XVII - Draft6.asc
File View Plot Settings Simulation Tools Window Help
Draft6.asc
V(n003)
0dB
-20dB
-40dB
-60dB
-80dB
-100dB
-120dB
-140dB
-160dB
0°
-20°
-40°
-60°
-80°
-100°
-120°
-140°
-160°
-180°
10mHz 100mHz 1Hz 10Hz 100Hz 1KHz 10KHz 100KHz 1MHz

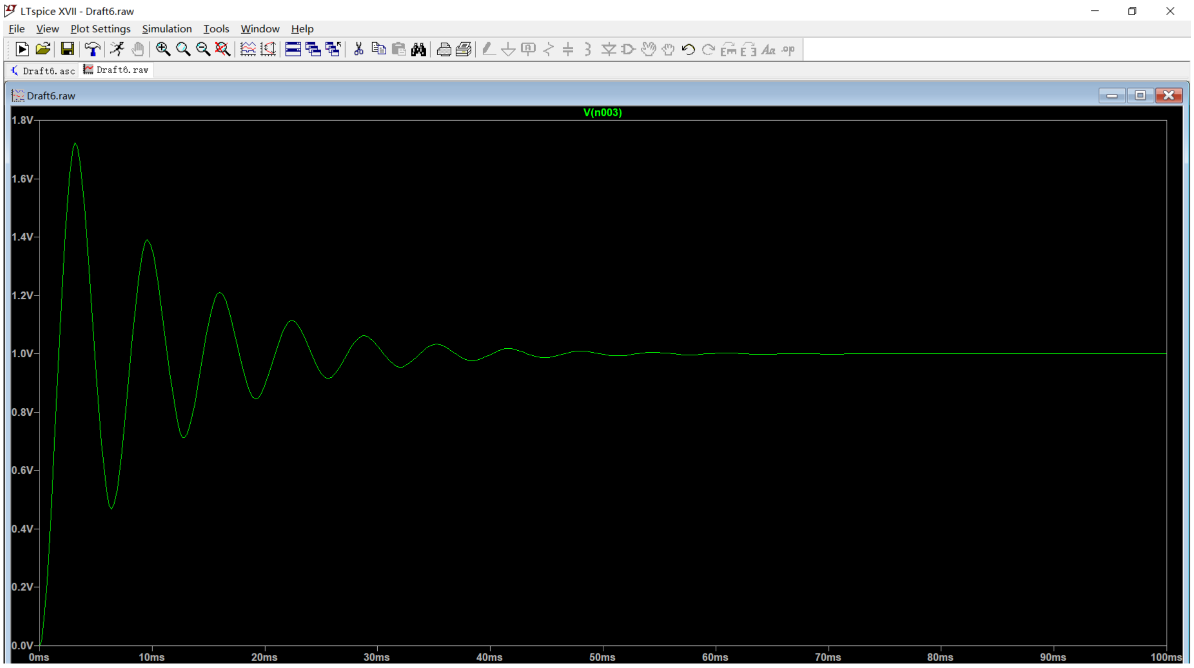

LTspice XVII - Draft6.asc
File View Plot Settings Simulation Tools Window Help
Draft6.asc
V(n003)
0dB
-20dB
-40dB
-60dB
-80dB
-100dB
-120dB
-140dB
-160dB
0°
-20°
-40°
-60°
-80°
-100°
-120°
-140°
-160°
-180°
10mHz 100mHz 1Hz 10Hz 100Hz 1KHz 10KHz 100KHz 1MHz

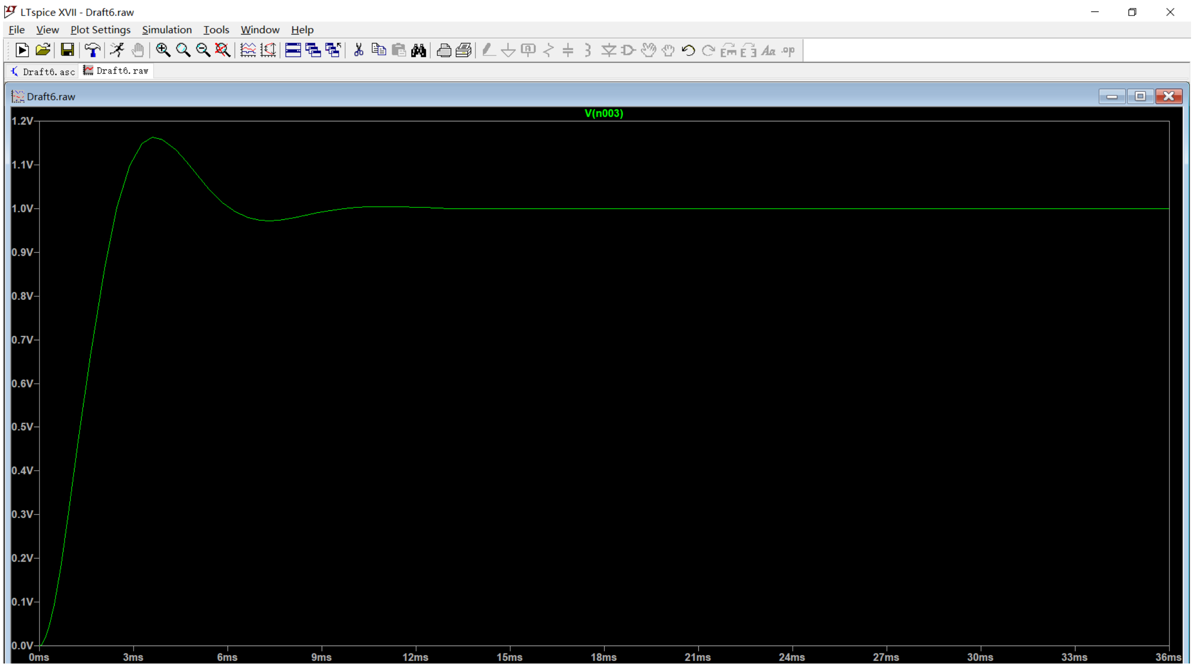


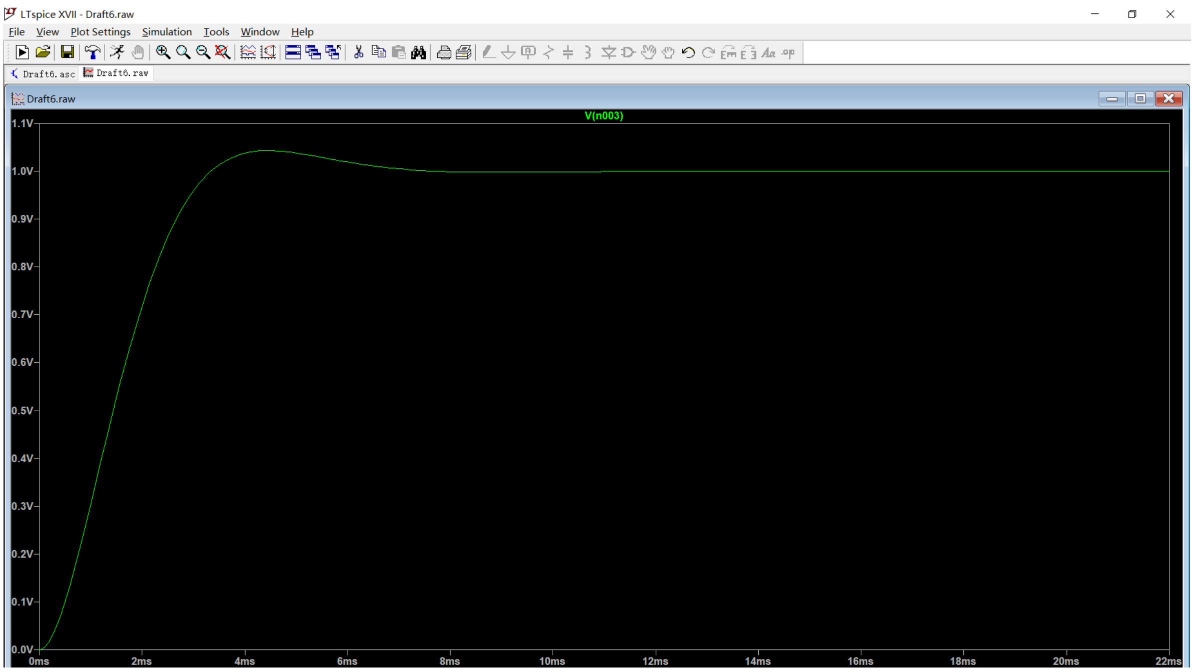
可以看出伯德图与前面的分析基本一致。

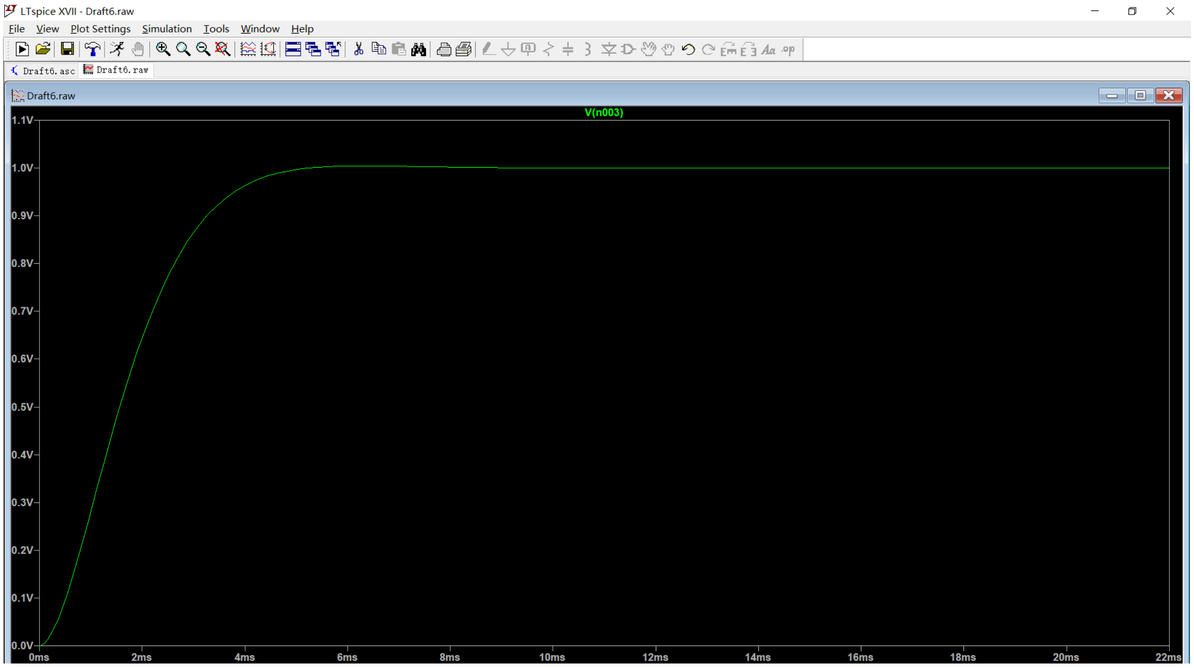
下面再同样的条件下，测试输出电压的时域特性，输入电压设定为单位阶跃电压，当调节电阻阻值，使得阻尼系数分别为0.01, 0.1, 0.5, 0.707, 0.866, 1, 2, 10, 50, 100时，得到的输出电压时域波形如下：

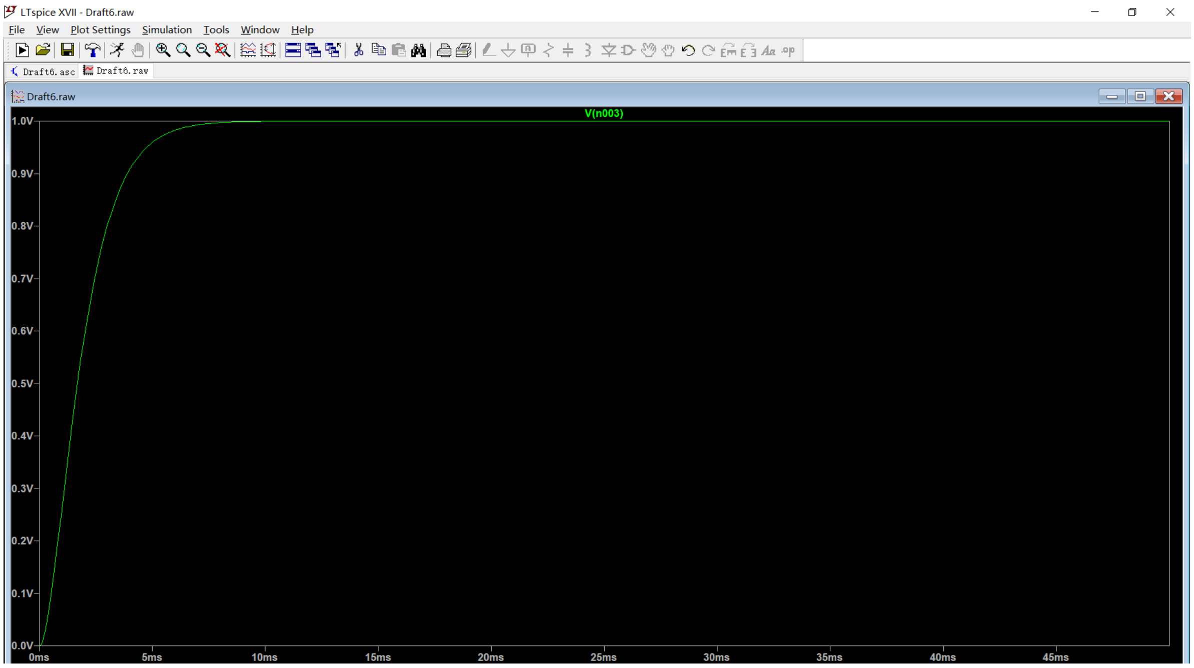


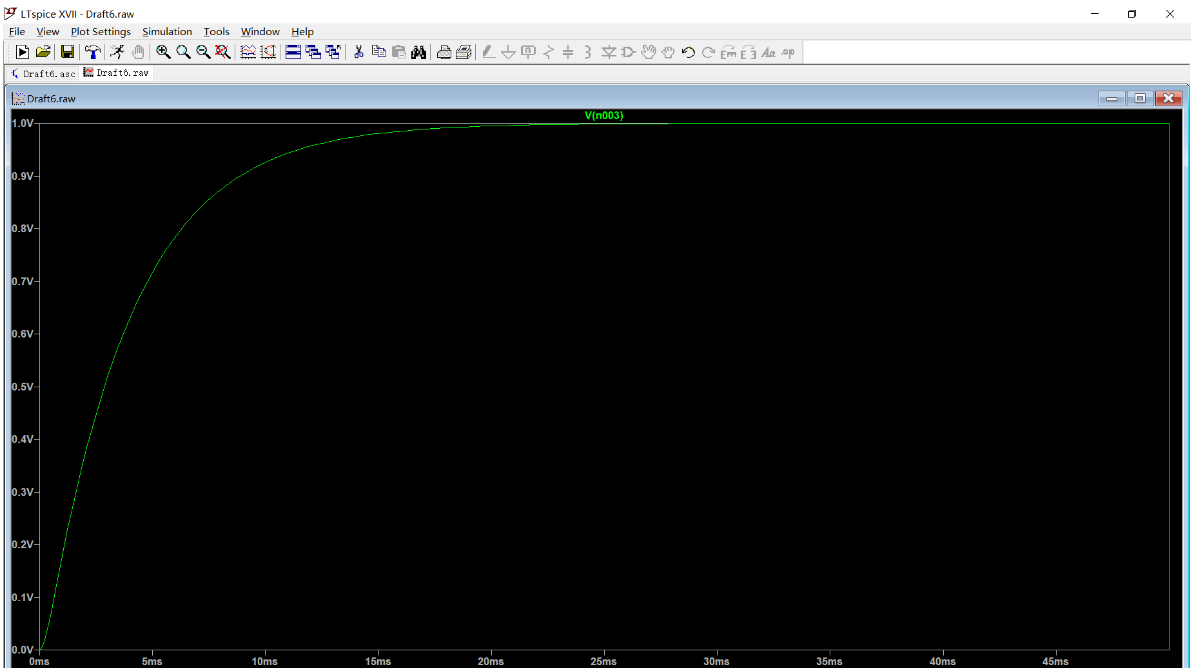

LTspice XVII - Draft6.raw
File View Plot Settings Simulation Tools Window Help
Draft6.asc
Draft6.raw
Draft6.raw
V(n003)
1.8V
1.6V
1.4V
1.2V
1.0V
0.8V
0.6V
0.4V
0.2V
0.0V
0ms
10ms
20ms
30ms
40ms
50ms
60ms
70ms
80ms
90ms
100ms

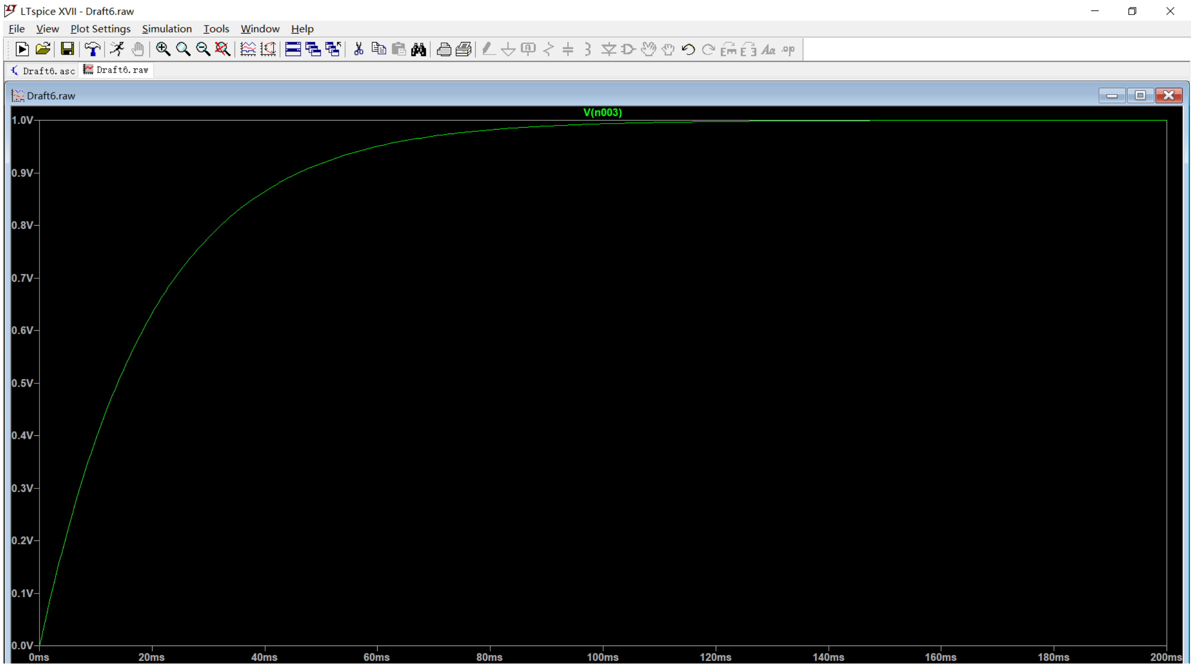

LTspice XVII - Draft6.raw
File View Plot Settings Simulation Tools Window Help
Draft6.asc
Draft6.raw
Draft6.raw
V(n003)
1.2V
1.1V
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
0ms
3ms
6ms
9ms
12ms
15ms
18ms
21ms
24ms
27ms
30ms
33ms
36ms


LTspice XVII - Draft6.raw
File View Plot Settings Simulation Tools Window Help
Draft6.asc
Draft6.raw
Draft6.raw
V(n003)
1.1V
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
0ms
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
20ms
22ms


LTspice XVII - Draft6.raw
File View Plot Settings Simulation Tools Window Help
Draft6.asc
Draft6.raw
Draft6.raw
V(n003)
1.1V
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
0ms
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
20ms
22ms


LTspice XVII - Draft6.raw
File View Plot Settings Simulation Tools Window Help
Draft6.asc
Draft6.raw
Draft6.raw
V(n003)
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
0ms
5ms
10ms
15ms
20ms
25ms
30ms
35ms
40ms
45ms


LTspice XVII - Draft6.raw
File View Plot Settings Simulation Tools Window Help
Draft6.asc
Draft6.raw
Draft6.raw
V(n003)
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
0ms
5ms
10ms
15ms
20ms
25ms
30ms
35ms
40ms
45ms



联系课上所学，可知当$\xi < 1$时，为弱阻尼，会出现震荡逐渐消失的现象，前5幅图像可以看到该现象，且随着阻尼系数不断增大，达到平衡的时间不断减小，耗时近似与ξ成反比。而当$\xi = 1$时，达到临界阻尼，不再出现震荡现象，且会以最快的速度达到平衡，而后随着ξ进一步增大，进入过阻尼状态，达到平衡的时间随着阻尼系数的增大而不断增加，达到平衡耗时近似为$\frac{1}{(\xi-\sqrt{\xi^2-1})\omega_0}$。