1.

$$\frac{dp}{p}=\frac{dN}{N}=-\frac{\bar{v}}{4V}dt \qquad \frac{p_0 V N_A}{RT}e^{-\frac{\bar{v}}{4V}t}$$

有 $p'=Ce^{-\frac{\bar{v}}{4V}t'}$

$$e^{-\frac{\bar{v}}{4V}t'}=\frac{p}{p_0}$$

得 $t'=-\frac{4V\ln\left(\frac{p}{p_0}\right)}{\bar{v}}=-4V\sqrt{\frac{\pi m}{8kT}}\ln\left(\frac{p}{p_0}\right)$

总能量为

$$E=N\cdot\frac{1}{2}m\overline{v^2}=N\cdot\frac{3}{2}kT=\frac{3}{2}NkT$$

$$=\frac{3p_0VN_ANk}{2R}e^{-\frac{\bar{v}}{4V}t}$$

$$\left(\frac{\partial E}{\partial t}\right)_{t=t'}=-\frac{3p_0VN_ANk\bar{v}}{8VR}\frac{p}{p_0}=-\frac{3pN_ANk\bar{v}}{8R}$$

$$=-\frac{3pN_ANk}{8R}\sqrt{\frac{8kT}{\pi m}}$$

2. (1) $pV=\nu RT$

$$N=\nu N_A=\frac{pV}{RT}N_A=7.24\times10^{22}$$

(2) $\bar{v}=\sqrt{\frac{8kT}{\pi m}}=\sqrt{\frac{8kTN_A}{\pi M}}=398.5\ m/s$

(3) $N'=\frac{1}{4}n\bar{v}\Delta S=\frac{1}{4}\frac{N}{V}\bar{v}\Delta S=7.2\times10^{20}$

(4) $$\Delta t = \frac{4V}{\bar{v}\Delta S} = 100.5S$$

(5) 若绝热，则随着分子出射，

$$dN = V dn = -\Gamma \Delta S\, dt = -n(t)\sqrt{\frac{kT(t)}{2\pi m}}\,\Delta S\, dt$$

而出射粒子的平均方根速率为

$$\overline{v^2} = \int_0^\infty F(v)\, v^2 dv = \frac{1}{2}\left(\frac{m}{kT}\right)^2 \int_0^\infty v^5 e^{-\frac{mv^2}{2kT}} dv$$

$$= \frac{4kT}{m}$$

故而 $\bar{\varepsilon}_{出} = 2kT$

dt内耗散能量为

$$d\left(\frac{3}{2}kTVn\right) = \frac{3}{2}k(dT)Vn + \frac{3}{2}kTV dn = V dn \cdot 2kT$$

得 $$3\cancel{k}Vn\, dT = \cancel{k}VT dn$$

$$\frac{3dT}{T} = \frac{dn}{n}$$

即 $$T^3 = An$$

那么 $$V dn = -n(t)\sqrt{\frac{kA^{\frac{1}{3}}n^{\frac{1}{3}}}{2\pi m}}\,\Delta S\, dt$$

$$n^{-\frac{7}{6}} dn = -\sqrt{\frac{kA^{\frac{1}{3}}}{2\pi m}} \cdot \frac{\Delta S}{V} dt$$

$$-6 n^{-\frac{1}{6}} = C - \sqrt{\frac{kA}{2\pi m}}\frac{\Delta S}{V} t$$

$$\Delta t \cdot \frac{\Delta S}{V}\sqrt{\frac{kA}{2\pi m}} = 6\left(\left(\frac{n_0}{e}\right)^{-\frac{1}{6}} - n_0^{-\frac{1}{6}}\right)$$

得$\Delta t \approx 3454.1\,s$

4. 解：(1) $\frac{1}{3}A{V_0}^3 = 1 \qquad A = \frac{3}{{V_0}^3}$

(2) $$\bar{V} = \int_0^{V_0} AV^3\,dV = \frac{3}{4}V_0$$

$$\overline{V^2} = \int_0^{V_0} AV^4\,dV = \frac{3}{5}{V_0}^2$$

$$\sqrt{\overline{V^2}} = \frac{\sqrt{15}}{5}V_0$$