Homework for General physics II Set4

1. For a TTL periodic pulse with duty cycle of 50%, it is a pulse with period of λ, fundamental frequency $K_0 = \dfrac{2\pi}{\lambda}$, within one period: $f(x) = \begin{cases} 1 & x \subset (0, \lambda/2) \\ 0 & x \subset (\lambda/2, \lambda) \end{cases}$

It repeats this pattern over periods. Find the Fourier Expansion series for this TTL pulse.

2. Hecht's 7.36

7.36* Show that the Fourier series representation of the function $f(\theta) = |\sin\theta|$ is

$$f(\theta) = \frac{2}{\pi} - \frac{4}{\pi}\sum_{m=1}^{\infty} \frac{\cos 2m\theta}{4m^2 - 1}$$

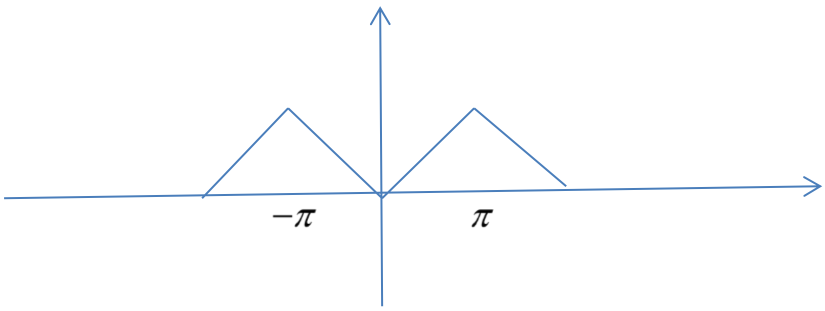
3. For periodic function, within a period of $-\pi$ to $+\pi$, its form is:

$$f(x) = x^2 \quad -\pi \le x \le \pi$$

It repeats itself over other region. Find its Fourier Expansion and prove a famous math relation: $\dfrac{\pi^2}{6} = \sum_{n=1}^{\infty} \dfrac{1}{n^2}$ (which is a part of Riemann zeta function)

4. For a triangular periodic wave represented by (within one period):

$$f(x) = \begin{cases} x & 0 < x < \pi \\ -x & -\pi < x < 0 \end{cases}$$



Find its Fourier Expansion.

5. In my note and Zhao's book, we use the convention that for function f(x) and its Fourier Transform F(K), their relations are:

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} F(K) e^{iKx} dK$$

$$F(K) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-iKx} dx$$

However, this is not the universal convention, you will find in Hecht's book, he defines:

$$f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}F'(K)e^{-iKx}dK$$

$$F'(K)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}f(x)e^{+iKx}dx$$

Of course there is a relation between the Fourier Transforms defined by above conventions (I chose F' as function form of Fourier Transform in Hecht's convention, while he just uses F), prove that:

$F'(K)=F(-K)$, i.e. the Fourier Transform used by Hecht is just our Transform with –K (the function flipped over K axis)

6. 1) For a square function with height A width a (A, *a* are constants), and zero elsewhere:

$$f(x)=\begin{cases}A & |x|<\dfrac{a}{2}\\ 0 & \text{elsewhere}\end{cases}$$

Find its Fourier transform (spectrum) F(K)

2) For a truncated cosine function with duration of *a*, i.e.

$$f(x)=\begin{cases}A\cos\omega_0 t & |t|<\dfrac{a}{2}\\ 0 & \text{t elsewhere}\end{cases}$$

It is basically a cosine function times the above square function. (Of course here instead of x, we use t; instead of K we will use ω)

Find the Fourier transform for the above truncated cosine function $F(\omega)$

7. Starting from our basic formula for Fourier Transform, we study transforms for even and odd f(x).

1) If f(x) is an even function f(x)=f(-x), prove that:

$$F(K)=\sqrt{\frac{2}{\pi}}\int_0^{\infty}f(x)\cos(Kx)dx$$

$$f(x)=\sqrt{\frac{2}{\pi}}\int_0^{\infty}F(K)\cos(Kx)dK$$

This is called Fourier Cosine transform for even functions (analogy to Fourier expansion for even periodic functions only contains cosine terms)

2) If f(x) is an odd function, f(x)=-f(-x), prove that:

$$F(K) = -iG(K) \text{ where } G(K) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x)\sin(Kx)dx$$

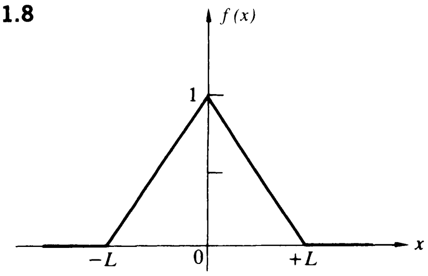
$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty G(K)\sin(Kx)dK$$

This is called Fourier sine transform for odd functions.

8. Hecht's 11.8:

11.8 Compute the Fourier transform of the triangular pulse shown in Fig. P.11.8. Make a sketch of your answer, labeling all the pertinent values on the curve.

Figure P.11.8



The function is: $f(x) = 1 - x/L$ for x between (0, L); and $f(x) = 1 + x/L$ for x between (-L,0); f(x)=0 elsewhere.

(Note: the answer is at Hecht's solution section, Zhao's result on pg. 94 has a mistake: a^2 should be a; both answers differ from our definition of F(K) by a constant factor $\sqrt{\frac{1}{2\pi}}$. Needless to say this problem is asking you to do the dirty work of calculation instead of copying answers from books. You may check the integration table for integrals)

9. (a) Show that $g(-K) = g^*(K)$ is a necessary and sufficient condition for $f(x)$ to be real.

(b) Show that $g(-K) = -g^*(K)$ is a necessary and sufficient condition for $f(x)$ to be pure imaginary.

g(K) is the Fourier transform of f(x)