Homework 4

Dec. 03, 2021

NOTE: Homework 4 is due Dec. 10, 2021.

1. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample of size $n = 15$ from a normal distribution $N(\mu, \sigma^2)$, and the observed values are:

$$3.0, 2.7, 2.9, 2.8, 3.1, 2.6, 2.5, 2.8, 2.4, 2.9, 2.7, 2.6, 3.2, 3.0, 2.8$$

   (i) Construct a 95% confidence interval for μ when $\sigma = 0.23$.

   (ii) Construct a 95% confidence interval for μ when σ is unknown.

   (iii) Construct a 95% confidence interval for σ^2 when $\mu = 2.85$.

   (iv) Construct a 95% confidence interval for σ^2 when μ is unknown.

   (v) Suppose that both μ and σ are unknown. Let $\theta = \mu^2$, derive the moment estimate and MLE for θ.

2. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from a distribution with unknown (finite) mean μ and known (finite) variance σ^2, and suppose that n is large. Then:

   (i) Construct a confidence interval for μ with confidence level approximately $1 - \alpha$.

   (ii) Provide the form of the interval in part (i) for $n = 100$, $\sigma = 1$, and $\alpha = 0.05$.

   (iii) Refer to part (i) and suppose that $\sigma = 1$ and $\alpha = 0.05$. Then determine the sample size n, so that the length of the confidence interval is less than 0.1.

   (iv) Show that the length of the interval in part (i) tends to 0 in probability as $n \to \infty$, for any σ and α.

3. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from a distribution with unknown (finite) mean μ and unknown (finite) variance σ^2, and suppose that n is large. Then:

   (i) Construct a confidence interval for μ with confidence level approximately $1 - \alpha$.

   (ii) Provide the form of the interval in part (i) for $n = 100$, $\alpha = 0.05$.

   (iii) Show that the length of the interval in part (i) tends to 0 in probability as $n \to \infty$.

4. Consider the data set "data.csv". There are 1475 houses which have central air-conditioner (Central_Air = Y) and 123 houses which don't have air-conditioner (Central_Air = N). Compare the mean SalePrice and the variance of SalePrice of the houses with central air-conditioner (Central_Air = Y) and those without central air-conditioner (Central_Air = N). In particular, let $X_1, \cdots, X_{1475}$ denote the SalePrice of the houses with central air-conditioner and suppose that they are i.i.d. r.v.'s. Let $Y_1, \cdots, Y_{123}$ denote the SalePrice of the houses without central air-conditioner and suppose that they are i.i.d. r.v.'s too. Suppose that X_i's and Y_j's are independent.

   (i) Suppose that $X_i \ i.i.d. \sim N(\mu_1, \sigma^2)$, $Y_j \ i.i.d. \sim N(\mu_2, \sigma^2)$, $\sigma = 79400$. Construct a 95% confidence interval for $\mu_1 - \mu_2$.

(ii) Suppose that X_i *i.i.d.* $\sim N(\mu_1, \sigma^2)$, Y_j *i.i.d.* $\sim N(\mu_2, \sigma^2)$, σ is unknown. Construct a 95% confidence interval for $\mu_1 - \mu_2$.

(iii) Suppose that X_i *i.i.d.* $\sim N(\mu_1, \sigma_1^2)$, Y_j *i.i.d.* $\sim N(\mu_2, \sigma_2^2)$, $\sigma_1^2 \neq \sigma_2^2$ are unknown. Construct a 95% confidence interval for σ_1/σ_2.

(iv) Refer to part (iii), construct approximately 95% confidence intervals for $\mu_1 - \mu_2$ by using central limit theory.