Homework 11 for GP II

Following problems 1 and 2 are from Griffiths' book, section 2.4 (the solutions are in his book and his solution manual book)

1. For a free particle with mass m, its initial wavefunction is in form of:

$\psi(x,t=0)=\begin{cases} A & \text{if } -a<x<a \\ 0 & \text{elsewhere} \end{cases}$ (a square function with width of 2a), A, a are real, positive constants. Find form of $\psi(x,t)$ (express it in integral form would be ok, it does not have analytical formula as Gaussian wave packet)

(problem 1 is example 2.6 in Griffiths')

2. (Griffiths' problem 2.21) A free particle has initial wavefunction:

$\psi(x,t=0)=Ae^{-a|x|}$, A, a are real positive constants.

(a) Normalize $\psi(x,0)$

(b)Find $\phi(k)$ (I used g(k) in my lecture)

(c)Construct $\psi(x,t)$, in the form of the integral.

(d)Discuss the limiting case (*a* very small and *a* very large)

You may also read his problem 2.22 (which is the Gaussian wave packet I solved in lecture)

3. Griffiths P2.5.

A particle in the infinite square well has its initial wave function in form of mixture of first two stationary states: $\psi(x,0)=A(\psi_1(x)+\psi_2(x))$

(a) Normalize the $\psi(x,0)$.

(b) Find the $\psi(x,t)$ and $|\psi(x,t)|^2$. Express the latter as a sinusoidal function of time. To simplify the result, let $\omega \equiv \pi^2\hbar / 2ma^2$

(c) Compute <x>. Notice that it oscillates in time. What is the angular frequency and amplitude of the oscillation?

(d) Compute <p> and better do it the quick way.

(e) If you measure the energy of this state, what values you might get and what is the probability of getting each of them? Find the expectation value of H.

4. Griffiths P2.4.

Calculate <x>, <x^2>, <p>, <p^2>, σ_x and σ_p ($\sigma_x^2 = <x^2> - <x>^2$), for the n^{th} stationary state of the infinite well. Check that the uncertainty principle is satisfied and what state comes closest to the uncertainty limit?

5. A particle of mass m is in a 1-D infinite square well where V(x)=0 when $0 \le x \le a$, and infinity elsewhere. At t=0 its normalized wave function is:

$$\psi(x,t=0) = \sqrt{8/5a}[1+\cos(\frac{\pi x}{a})]\sin(\frac{\pi x}{a})$$

(a) What is the wave function at a later time $t=t_0$?

(b) What is the average energy of the system at t=0 and $t= t_0$?

(c) What is the probability of finding the particle at the left half of the box (i.e. [0, *a*/2]) at $t= t_0$?

6. For a general 3-D problem, let's consider one particular general as well as simple case, that is the potential V(r) can be written as:

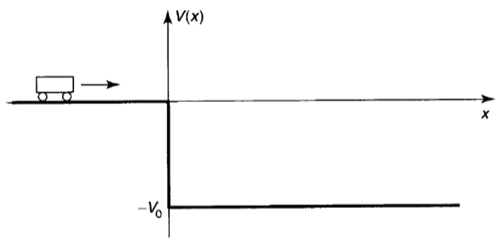
$V(x,y,z)=V_x(x)+V_y(y)+V_z(z)$, then prove that the energy would be in form of: Solving the 3-D problem will become solving three 1-D equations, with $E=E_x+E_y+E_z$, and the stationary state wave function is in form of: $\psi(r)=\psi(x,y,z)=\psi_x(x)\psi_y(y)\psi_z(z)$

7. We already learned for the 1-D free (V(x)=0) particle, the wave function with particular momentum is (expressed with k_x instead of p_x):

$\psi(x)=\frac{1}{\sqrt{2\pi}}e^{ik_x x}$, with momentum $p_x=k_x\hbar, E=\frac{\hbar^2 k_x^2}{2m}$. Now consider a 3-D free particle, what is the general expression of wave function with particular momentum and what is the expression for energy? (You will see the function is really like a 3-D plane wave)

8. Griffiths P2.35

A particle of mass m and kinetic energy E>0 approaches an abrupt potential drop V_0:



(a) What is the probability that it will "reflect" back, if E=V_0/3?

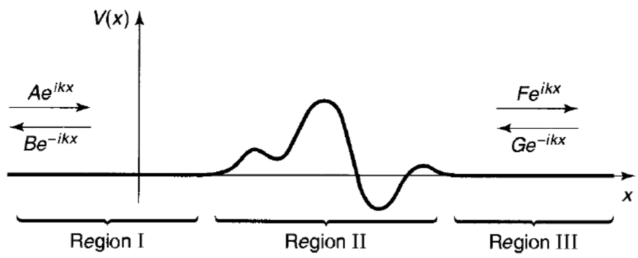
(b) The figure is drawn as a approaching a cliff, obviously the probability of bouncing back from the edge of cliff is far smaller than what you got in (a). Explain why this potential does not correctly represent a cliff.

(c) When a free neutron enters a nucleus, it experiences a sudden drop in potential energy, from V=0 outside to around V=-12MeV inside.

Suppose a neutron emitted with kinetic energy 4 MeV by a fission event, strikes such nucleus. What is the probability it will be absorbed? Hint: You may use T=1-R to get the transmission ratio.

9. (adapted from Griffiths P2.52,2.53)

We have talked about scattering S-matrix and transfer M-matrix:



For S-matrix, it relates:

$$\begin{pmatrix} B \\ F \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} A \\ G \end{pmatrix}$$ (Knowing the incoming, we know the outgoing)

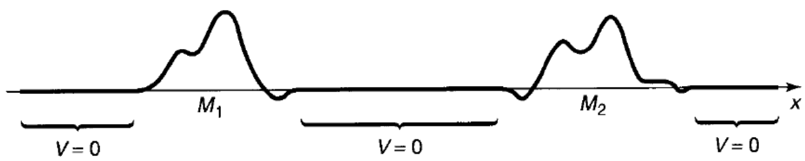
The M-matrix relates:

$$\begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$$ (Knowing one side we know the other side)

(a) Find the four elements of the M-matrix, in terms of the elements of the S-matrix, and vice versa.

(b) If we have scattering from left, i.e. G=0, express the reflection and transmission coefficients in terms of S and M elements.

(c) For a potential consisting of separated pieces:



Show that the overall M-matrix is the product of the two M-matrices for the each section: $M=M_2M_1$ (note the order).

10. An electron with energy E=1eV is incident on a rectangle barrier with height V_0=2eV and width d. What is the value of d so that the transmission probability is 10^{-3}.

11. (OPTIONAL, you may skip this if you like and it won't relate to exam. The goal for this problem is to let you know the delta function potential which is covered in Griffiths' book)

Griffiths P2.27

Consider the double delta-function potential:

$V(x) = -\alpha[\delta(x+a) + \delta(x-a)]$

Where α, a are positive constants.

(a) Sketch the potential.

(b) How many bound states does it possess? Find the allowed energies , for $\alpha = \hbar^2 / ma$ and for $\alpha = \hbar^2 / 4ma$, and sketch the wave functions. (Hint: Solve the problem by separate regions and consider the boundary conditions. The details are quite long and please refer to the solution manual book)