# Statistical Inference - Homework 3

Nov. 25, 2021

**NOTE: Homework 3 is due Dec. 03, 2021.**

1. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from uniform distribution $U(\theta, 2\theta)$, $0 < \theta < +\infty$. Derive the MLE of $\theta$. Is the MLE unbiased? If not, find an unbiased estimate based on the MLE.

2. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from the distribution with p.d.f.

$$f(x;\theta) = \frac{1}{2\sigma} \exp\{-|x-a|/\sigma\},$$

where $\sigma > 0$, $-\infty < a < +\infty$. Find the MLE of $a$ and $\sigma$.

3. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from the Weibull distribution with p.d.f.

$$f(x) = \alpha\beta x^{\beta-1} e^{-\alpha x^{\beta}}, \quad x > 0 \quad (\alpha, \beta > 0).$$

Suppose $\beta$ is known, determine the MLE of $\alpha$.

4. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from the p.d.f

$$f(x) = \theta x^{\theta-1}, \quad 0 \le x \le 1, 0 < \theta < \infty.$$

Find the MLE of $\theta$ , and show that is variance goes to 0 when $n$ goes to $\infty$.

5. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from normal distribution $N(\mu, \sigma^2)$ (both $\mu$ and $\sigma^2$ are unknown), derive the UMVUE of (1) $3\mu + 4\sigma^2$, and (2) $\mu^2/(4\sigma^2)$.

6. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from normal distribution $N(0, \sigma^2)$, show that

$$\hat{\sigma} = \frac{\Gamma(\frac{n}{2})}{\sqrt{2}\Gamma(\frac{n+1}{2})} \left( \sum_{i=1}^{n} X_i^2 \right)^{1/2}$$

is the UMVUE of $\sigma$ and determine its efficiency.

7. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from Geometric distribution:

$$P(X_1 = i) = \theta(1-\theta)^{i-1},\ i = 1, 2, \cdots,\ 0 < \theta < 1.$$

Derive the UMVUE of $\theta^{-1}$ and $\theta$.

8. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from Bernoulli(p). For $n \ge 4$, show that the product $X_1X_2X_3X_4$ is an unbiased estimator of $p^4$, and use this factor to find the best unbiased estimator for $p^4$.