10.

$$dU = C_{V,m} dT - p dV$$

$$U = \int_{V_0}^{U} dU = C_{V,m}(T - T_0) + RT \ln \frac{V_m}{V_0} + a\left(\frac{1}{V_m} - \frac{1}{V_0}\right)$$

11. $$U = c(T - T_0) + \frac{V_m^2 - V_0^2}{2b} + \frac{(V_0 + aT)(V_0 - V_m)}{b}$$

12. (1) ① $$\left(\frac{\partial U}{\partial T}\right)_p = \left(\frac{\partial H}{\partial T}\right)_p - P\left(\frac{\partial V}{\partial T}\right)_p$$

$$= C_p - \alpha PV$$

② $$\left(\frac{\partial U}{\partial P}\right)_T = T\left(\frac{\partial S}{\partial P}\right)_T + kPV$$

$$= -T\left(\frac{\partial V}{\partial T}\right)_p + kPV$$

$$= -\frac{(C_p - C_v)}{T} \cdot T\left(\frac{\partial P}{\partial T}\right)_V + kPV$$

$$= -P(C_p - C_v)\beta + kPV$$

$$= kPV - (C_p - C_v)\frac{k}{\alpha}$$

(2) ① $\left(\frac{\partial U}{\partial p}\right)_V = C_V \left(\frac{\partial T}{\partial p}\right)_V = C_V \frac{1}{\left(\frac{\partial p}{\partial T}\right)_V} = \frac{C_V}{\beta p}$

$$= \frac{k C_V}{\alpha}$$

② $\left(\frac{\partial U}{\partial V}\right)_p = \left(\frac{\partial H}{\partial V}\right)_p - p = \left(\frac{\partial H}{\partial T}\right)_p \left(\frac{\partial T}{\partial V}\right)_p - p$

$$= C_p \frac{1}{\left(\frac{\partial V}{\partial T}\right)_p} - p$$

$$= \frac{C_p}{\alpha V} - p$$