Homework 6

Dec. 15, 2021

1. A coin, with probability θ of falling heads, is tossed independently 100 times and 60 heads are observed. At level of significance $\alpha = 0.1$:
   (i) Use the LR test in order to test the hypothesis $H_0 : \theta = 1/2 \ \leftrightarrow \ H_1 : \theta \neq 1/2$.
   (ii) Employ the appropriate approximation to determine the critical value.

2. A medical researcher wishes to determine whether a pill has the undesirable side effect of reducing the blood pressure of the user. The study requires recording the initial blood pressure of n college-age women. After the use of the pill regularly for 6 months, their blood pressures are again recorded. With μ denoting the difference of blood pressure after the usage of the pill and before it, the claim is that $\mu < 0$.
   (i) Check this claim by testing the hypothesis $H_0 : \mu \geq 0 \ \leftrightarrow \ H_1 : \mu < 0$ at level of significance α, by using the likelihood ratio test.
   (ii) Carry out the test if $n = 90$ and $\alpha = 0.05$.

3. The diameters of certain cylindrical items produced by a machine are r.v.'s distributed as $N(\mu, 0.01)$. A sample of size 16 is taken and it is found that $\bar{x} = 2.48$ inches.
   (i) If the desired value for μ is 2.5 inches, formulate the appropriate testing hypothesis problem and carry out the likelihood ratio test at level of significance $\alpha = 0.05$.
   (ii) Determine the power of the test.

4. Let $X_i, i = 1, \cdots, 9$ and $Y_j, j = 1, \cdots, 10$ be independent r.v.'s from the distributions $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, respectively. Suppose that the observed values of the sample variance are $s_x^2 = 4$, $s_y^2 = 9$.
   (i) At level of significance $\alpha = 0.05$, test the hypothesis $H_0 : \sigma_1 = \sigma_2, \leftrightarrow \ H_1 : \sigma_1 \neq \sigma_2$ by using likelihood ratio test.
   (ii) Find an expression for the computation of the power of the test for $\sigma_1 = 2$ and $\sigma_2 = 3$.

5. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from uniform distribution $U(\theta, 1)$ where $\theta < 1$ is unknown.
   (i) At level of significance α, carry out the likelihood ratio test of the hypothesis
   $$H_0 : \theta \geq \theta_0 \ \longleftrightarrow \ H_1 : \theta < \theta_0,$$
   where $\theta_0 < 1$ is given.
   (ii) Determine the power function of the test.

6. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from Normal distribution $N(\mu, \sigma^2)$, where μ is unknown and σ is known.
   (i) Derive the UMP test for testing the hypothesis $H_0 : \mu = \mu_0 \ \longleftrightarrow \ H_1 : \mu > \mu_0$ at level of significance α.
   (ii) Carry out the testing hypothesis for $n = 100, \sigma^2 = 4, \mu_0 = 3, \bar{x} = 3.2, \alpha = 0.01$ and compute the power for $\mu = 3.5$.

7. Let $\boldsymbol{X} = (X_1, \cdots, X_n)$ be a random sample from Gamma distribution Gamma(α_0, β) with α_0 known and β unknown.

   (i) Construct the MP test for testing the hypothesis $H_0 : \beta = \beta_1 \longleftrightarrow H_1 : \beta = \beta_2$ $(\beta_2 > \beta_1)$ at level of significance α.

   (ii) Show that $X_1 + \cdots + X_n \sim$ Gamma$(n\alpha_0, \beta)$.

   (iii) Use the CLT to carry out the test when $n = 30, \alpha_0 = 10, \beta_1 = 2.5, \beta_2 = 3, \alpha = 0.05$ and compute the power.

8. Let X be a r.v. distributed as $B(n, \theta)$, $\theta \in \Theta = (0, 1)$.

   (i) Derive the UMP test for testing the hypothesis $H_0 : \theta \leq \theta_0 \longleftrightarrow H_1 : \theta > \theta_0$ at level of significance α.

   (ii) Specify the test in part (i) for $n = 10, \theta_0 = 0.25$, and $\alpha = 0.05$.

   (iii) Compute the power of the test for $\theta = 0.375, 0.500$.

   (iv) Use the CLT in order to determine the sample size n if $\theta_0 = 0.125, \alpha = 0.1$ and $\pi(0.25) = 0.9$.

9. Two testers (A and B) analyzed the same product samples and obtained the following results:

tester	sample 1	sample 2	sample 3	sample 4	sample 5	sample 6	sample 7	sample 8
A	4.3	3.2	3.8	3.5	3.5	4.8	3.3	3.9
B	3.7	4.1	3.8	3.8	4.6	3.9	2.8	4.4

   If we could not make normality assumption about the data, use sign test and Wilcoxon signed rank test to test whether there is a significant difference (in means) of the analysis results of testers A and B. Take $\alpha = 0.05$.

10. On the basis of the following scores, appropriately taken, test whether there are differences in mathematical ability of boys and girls (as is often claimed!). Take $\alpha = 0.05$ and use Wilcoxon two-sample rank-sum test.

Boys:	80	96	98	87	75	83	70	92	97	82
Girls:	82	90	84	70	80	97	87	88	88	

11. The following record shows a classification of 41,208 births in Wisconsin (courtesy of Professor Jerome Klotz). Set up a suitable probability model and check whether or not the births are Uniformly distributed over all 12 months of the year ($\alpha = 0.05$).

Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total
3478	3333	3771	3542	3479	3304	3476	3495	3490	3331	3188	3321	41208