

**[Proof]**

1. [15 pts] Consider  $P(a, b, c)$  modeled using an MRF with an undirected graph  $H$  shown below:

Where:  $P(a, b, c) = \frac{1}{Z} \psi_{A,B}(a, b) \psi_{B,C}(b, c)$ .

Prove  $A \perp\!\!\!\perp C \mid B$  by showing  $P(A, C \mid B = b) = P(A \mid B = b)P(C \mid B = b)$  using the above expression.

$A \perp\!\!\!\perp C \mid B$ 를 증명하려면  $P(A, C \mid B) = P(A \mid B)P(C \mid B) \dots$  ①임을 보여야 한다.  
 베이즈 정리를 이용해 ①의 좌변을 풀어서,  $P(A, C \mid B) = P(A \mid B, C) \cdot P(C \mid B)$  ②이다.  
 즉, ①과 ②를 비교하면  $P(A \mid B) \cdot P(C \mid B) = P(A \mid B, C) \cdot P(C \mid B)$ 를 보여야 한다.

given expression:  $P(a, b, c) = \frac{1}{Z} \cdot \psi_{A,B}(a, b) \cdot \psi_{B,C}(b, c)$ 에서,  

$$\left. \begin{aligned} P(a \mid b) &= \frac{1}{Z} \cdot \psi_{A,B}(a, b) \\ P(c \mid b) &= \frac{1}{Z} \cdot \psi_{B,C}(b, c) \end{aligned} \right\} \dots \text{③을 도출할 수 있다.}$$

③을 이용하면,  $P(A, C \mid B) = \left(\frac{1}{Z}\right)^2 \cdot \psi_{A,B}(a, b) \cdot \psi_{B,C}(b, c)$ 이다.  
 이를  $P(C \mid B)$ 로 나누면,  $P(A \mid B, C) = \frac{1}{Z} \psi_{A,B}(a, b)$ 이다.  
 이를 ③에 대입하면,  

$$P(A \mid B, C) = \frac{1}{Z} \cdot \psi_{A,B}(a, b) \cdot \psi_{B,C}(b, c) \text{이다. (③ 이용)}$$

따라서,  $P(A \mid B)P(C \mid B) = P(A \mid B, C) \cdot P(C \mid B)$   

$$\Rightarrow \frac{1}{Z} \cdot \psi_{A,B}(a, b) \cdot \frac{1}{Z} \cdot \psi_{B,C}(b, c) = \frac{1}{Z} \cdot \psi_{A,B}(a, b) \cdot \frac{1}{Z} \cdot \psi_{B,C}(b, c) \text{이다.}$$

즉,  $P(A, C \mid B) = P(A \mid B) \cdot P(C \mid B)$ 이다.

또한,  $P(A, C \mid B)$ 는 결국 "A와 B", "B와 C"의 pair만 고려하므로,  
 결론적으로  $A \perp\!\!\!\perp C \mid B$ 이다.

**[Reading homeworks]**

2. [5 pts] Read XGBoost paper <https://arxiv.org/abs/1603.02754> up to Section 3.1. Please indicate (just write yes) that you will read it by Midterm exam.

→ Yes

3. [5 pts] (Reading homework) Read GPML Book <http://gaussianprocess.org/gpml/chapters/RW.pdf> about Mauna Loa Atmospheric Carbon Dioxide (p118–122). Please indicate that you will read it by Midterm exam.

→ Yes

4. [5 pts] (Reading homework) Read 'How to Use t-SNE Effectively' <https://distill.pub/2016/misread-tsne/>. Please indicate that you will read it by Midterm exam.

→ Yes