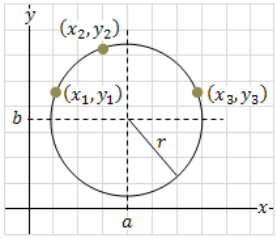
Equation of a circle passing through 3 points (x_1, y_1) (x_2, y_2) ***and*** (x_3, y_3)

The equation of the circle after solving the determinant is of the form $Ax^2 + Ay^2 + Bx + Cy + D = 0$



$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ {x_1}^2+{y_1}^2 & x_1 & y_1 & 1 \\ {x_2}^2+{y_2}^2 & x_2 & y_2 & 1 \\ {x_3}^2+{y_3}^2 & x_3 & y_3 & 1 \end{vmatrix} = 0$$

And the circle equation coefficients are:

$$A = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \quad B = -\begin{vmatrix} {x_1}^2+{y_1}^2 & y_1 & 1 \\ {x_2}^2+{y_2}^2 & y_2 & 1 \\ {x_3}^2+{y_3}^2 & y_3 & 1 \end{vmatrix} \quad C = \begin{vmatrix} {x_1}^2+{y_1}^2 & x_1 & 1 \\ {x_2}^2+{y_2}^2 & x_2 & 1 \\ {x_3}^2+{y_3}^2 & x_3 & 1 \end{vmatrix} \quad D = -\begin{vmatrix} {x_1}^2+{y_1}^2 & x_1 & y_1 \\ {x_2}^2+{y_2}^2 & x_2 & y_2 \\ {x_3}^2+{y_3}^2 & x_3 & y_3 \end{vmatrix}$$

$$A = x_1(y_2 - y_3) - y_1(x_2 - x_3) + x_2y_3 - x_3y_2$$
$$B = ({x_1}^2+{y_1}^2)(y_3 - y_2) + ({x_2}^2+{y_2}^2)(y_1 - y_3) + ({x_3}^2+{y_3}^2)(y_2 - y_1)$$
$$C = ({x_1}^2+{y_1}^2)(x_2 - x_3) + ({x_2}^2+{y_2}^2)(x_3 - x_1) + ({x_3}^2+{y_3}^2)(x_1 - x_2)$$
$$D = ({x_1}^2+{y_1}^2)(x_3y_2 - x_2y_3) + ({x_2}^2+{y_2}^2)(x_1y_3 - x_3y_1) + ({x_3}^2+{y_3}^2)(x_2y_1 - x_1y_2)$$

Center point (x, y) ***and the radius*** (r) ***of a circle passing through 3 points*** (x_1, y_1) (x_2, y_2) ***and*** (x_3, y_3)

$$x = \frac{({x_1}^2+{y_1}^2)(y_2 - y_3) + ({x_2}^2+{y_2}^2)(y_3 - y_1) + ({x_3}^2+{y_3}^2)(y_1 - y_2)}{2(x_1(y_2 - y_3) - y_1(x_2 - x_3) + x_2y_3 - x_3y_2)} = -\frac{B}{2A}$$

$$y = \frac{({x_1}^2+{y_1}^2)(x_3 - x_2) + ({x_2}^2+{y_2}^2)(x_1 - x_3) + ({x_3}^2+{y_3}^2)(x_2 - x_1)}{2(x_1(y_2 - y_3) - y_1(x_2 - x_3) + x_2y_3 - x_3y_2)} = -\frac{C}{2A}$$

$$r = \sqrt{(x - x_1)^2 + (y - y_1)^2} = \sqrt{\frac{B^2 + C^2 - 4AD}{4A^2}}$$

Example: *Find the equation of a circle passing through the points* $(-3,4)$ $(4,5)$ $(1,-4)$.

$A = -3(5 + 4) - 4(4 - 1) + 4 \cdot (-4) - 1 \cdot 5 = -60$
$B = (9 + 16)(-4 - 5) + (16 + 25)(4 + 4) + (1 + 16)(5 - 4) = 120$
$C = (9 + 16)(4 - 1) + (16 + 25)(1 + 3) + (1 + 16)(-3 - 4) = 120$
$D = (9 + 16)\big(1 \cdot 5 - 4 \cdot (-4)\big) + (16 + 25)(-3 \cdot (-4) - 1 \cdot 4) + (1 + 16)(4 \cdot 4 - (-3)5) = 1380$

Divide all terms by -60 *to obtain:* $\boldsymbol{x^2 + y^2 - 2x - 2y - 23 = 0}$

The center of the circle by solving x and y is at point $(1,1)$

The radius of the circle is: $r = \sqrt{(1 + 3)^2 + (1 - 4)^2} = 5$

The equation of the circle by standard form is: $(x - 1)^2 + (y - 1)^2 = 25$