

RECENT DEVELOPMENTS IN QUANTITATIVE MODELS OF SOVEREIGN DEFAULT

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Abstract. The current crisis and discussions, in the euro area in particular, show that sovereign debt crises/defaults are no longer confined to developing economies. Following crises in many Latin American countries, the literature on quantitative dynamic macro models of sovereign default has been advancing rapidly. Current debate should take note of the findings of this literature – an extensive overview of which has been provided in this paper. This paper also discusses the inherent difficulties as well as possibilities of integrating this type of model into standard business cycle models (RBC and DSGE models). This is likely to be particularly helpful when using models to analyse upcoming issues in the euro area, such as a suitable sovereign insolvency law or the assumption of joint liability.

Keywords. Default risk; Endogenous borrowing constraints; Small open economy; Sovereign debt

1. Introduction

The current crisis has reawakened interest in sovereign defaults in general as well as in stabilization measures aimed in particular at preventing the occurrence of such events. Debate in the euro area shows that such issues are no longer confined to developing countries but are also becoming increasingly more relevant for developed economies. Topics, such as contagion, suitable insolvency legislation or a workable – at least partial – assumption of joint liability in the context of the European Stability Mechanism (ESM), the European Financial Stability Facility (EFSF) and even Eurobonds, for example, have made it onto the agenda (see Arghyrou and Kontonikas, 2011). Business cycle models (i.e. RBC or DSGE models) are generally a suitable theory-based analytical tool for evaluating/simulating various (fiscal) policies. Many major institutions are also currently using business cycle models in this way to analyse a wide range of crisis-related issues.¹ However, for understandable technical reasons, they are rather reticent when it comes to issues related to sovereign defaults. This paper provides an overview of recent developments in the literature on quantitative dynamic macro models of sovereign default and a brief explanation of how this model class could be more closely integrated into standard RBC/DSGE models. Integrating sovereign default into such economic models is likely to prove quite helpful for a micro-based discussion of the issues mentioned above. Nevertheless, the findings of quantitative models of sovereign default gained to date should also be considered in the current debate. To the author's knowledge, no extensive overview of this literature exists yet. The main section of this paper has deliberately been written intuitively to make the analysis of this model class accessible to non-theoreticians too. A simple base model has been included, however.

The literature on quantitative dynamic macro models of sovereign default primarily evolved in the wake of government crises in many Latin American countries over the past two decades. In the literature, a sovereign's decision on *whether* to service its debt *or* default is endogenized. This therefore

means an analysis of the aspects of being ‘willing to repay debts’ rather than those of being ‘able to repay debts’. However, this should not be seen as a limitation. Many sovereign defaults were or are brought about by a loss of public support for measures to restructure the government budget that would probably enable debts to be repaid. Even in the current European context, this point is likely to be important.²

The quantitative models of sovereign debt that are primarily examined in this paper endogenize the sovereign’s default decision by comparing present value functions of whether to meet its payment obligations. Governments borrow from abroad. International investors fix interest rates based on their expectations of the government’s repayment behaviour. The models are able to explain interest rate developments and sovereign default using a country’s output fluctuations. Nevertheless, in the majority of cases, the output process is assumed to be exogenous. Repercussions of interest rate developments on the productivity of the country in question are usually not shown endogenously. Another branch of the literature that endogenizes the output process in the sense of RBC models, but assumes interest rate developments to be exogenous, shows that there is a negative correlation between the two quantities.³ However, combining these two approaches may be a suitable way of addressing the shortcoming in each individual branch of the literature (see also, Mendoza and Yue, 2011). We will revisit this point at the end of the paper once we have integrated the models into the general literature on sovereign defaults and summarized the present findings gained from the literature.

In brief, the literature on quantitative dynamic macro models of sovereign default finds that (1) despite the absence of international insolvency legislation, sovereigns have an interest in repaying debt owing to reputational damage and negative output effects, both aspects, however, included by assumption; (2) in the event of transitory shocks, debt level is the key factor determining the probability of default; in the event of trend shocks, the output level also plays a decisive role; (3) the political stability of a country reduces its probability of default; (4) bailouts or the possibility of borrowing from international financial institutions, such as the IMF, can increase the probability of default; (5) fiscal policy acts procyclically if it takes the probability of default and the impact on interest rates into account; (6) taking private sector debt into account does not change the above-mentioned qualitative statements, but can increase a government’s propensity to borrow in relative terms; (7) endogenizing negotiations on debt recovery rates tends to increase the interest rates on sovereign debt, but a government’s propensity to borrow can still rise; (8) it can be efficient for borrowers and creditors to delay negotiations on restructuring sovereign debt, although this is heavily dependent on the assumed bargaining game; (9) long-term bonds usually have a higher rate of interest than short-term ones; (10) contagion effects owing to risk-averse investors as well as information shocks affecting investors that impact on the interest rate can even cause fundamentally sound governments to default; and (11) the formal method of solving these models also plays a role in quantitative terms. All of these aspects and the relevant effects that lead to these conclusions are explained in detail below. It should also be emphasized here that these findings are results of quantitative models. So far, not all findings have resilient empirical support.

The rest of the paper is structured as follows. Section 2 integrates the model class we focus on here into the general literature on government debt. Section 3 presents the underlying articles that led to the development of newer quantitative models of sovereign default. Furthermore, this section contains a stylized (and simplified) base model and describes the solution method. Section 4 describes recent developments in this literature. Section 5 outlines the approaches to improving the integration of this literature with RBC models. A summary can be found in Section 6.

2. Integrating Quantitative Models of Sovereign Default into the General Literature on Government Debt

In order to be able to improve the understanding of the model class discussed below, it appears appropriate, first of all, to integrate it into the literature on government debt. It is particularly important

to examine why governments borrow, what motivates them to repay their debt, and which problems arise should they default.

2.1 *Why Governments Borrow*

In the political economy literature there is, in principle, little doubt that governments have a propensity to borrow (for more details, see Eslava, 2011). This is triggered by a certain degree of fiscal illusion or by ‘common pool’ problems. In both cases there are clear points in favour of debt-financed spending for a certain group, where the financing costs are distributed over a larger base (the common pool) or put off until a future date (see, e.g. von Hagen, 1992; Velasco, 2000). Furthermore, a government uses debt financing *inter alia* to increase its chances of being reelected or – if it has reason to believe it will be voted out of office – to limit the new government’s discretionary leeway (Alesina and Tabellini, 1990). In conventional politico-economic (usually one country) models, long-term equilibrium is established by corresponding changes on the capital market (e.g. risk premiums and a corresponding decline in private investment) and in fiscal policy, in some cases, rule bound, which thus bring spending into line with potential revenue (see Woo, 2005; Stähler, 2009). The exclusion of Ponzi games generally results in a mechanism that prevents bankruptcy.

The (New) Keynesian theory can, at least, if there is a prolonged sequence of unfavourable shocks, also be used to justify government borrowing as countercyclical fiscal policy, particularly when aided by automatic stabilizers, smoothes the consumption path, which is usually a welcome development from a welfare perspective (see, e.g. Schmitt-Grohe and Uribe, 2007). However, in this theory, debt should be repaid and replaced by a formation of assets in periods of upswing, as is shown, for example, by Fatás and Mihov (2002), Galí and Perotti (2003) and Wyplosz (2006).

Quantitative models of sovereign default rationalize the reason why sovereigns default in such an environment – that is if, in terms of discounted welfare, the financing costs of government debt are higher than the costs for a defaulting sovereign. Thus these models examine the ‘willingness’ rather than the ‘ability’ to repay debts. It should be noted in this context that the model class discussed below is confined to real foreign debt. Aspects of domestic debt, and hence the potential monetization of government debt or issues related to currency mismatch, are not examined.⁴ Nevertheless, these are certainly important areas of future work.

2.2 *Why Governments Service their Debt*

In contrast to insolvencies in the private sector, it is impossible – or at least extremely difficult – to force independent sovereigns to repay outstanding debts, especially to foreign creditors. Politico-economic arguments suggest that sovereigns are nevertheless keen to repay debts if no distinction can be drawn between domestic and foreign creditors and if the incumbent government can maximize the utility for domestic households or if it is concerned that there may be domestic unrest and/or it may be voted out of office (see Broner *et al.*, 2006; Bornsztein and Panizza, 2009). However, the models discussed below focus on foreign debt, thus leaving almost no place for these arguments.

The potential partial exclusion from the capital market as a major incentive to repay debts was identified at an early stage in the literature. It is assumed that foreign debt presents an opportunity for governments to insure households against fluctuations in output (and the resulting fluctuations in utility or production). Eaton and Gersovitz (1981) assume that debtor countries are excluded from the capital market if they do not service their debt. This is sufficient grounds to always service outstanding debt – at least up to a certain limit. This paper is discussed in greater detail in Section 3.1. Even if the extreme assumption of permanent exclusion and approval of other insurance options for sovereigns is relaxed, situations may arise in which a partial exclusion from the capital market represents a serious

threat (see, e.g. Kletzer and Wright, 2000; Wright, 2002). However, from an empirical perspective, this threat does not appear to be the main incentive to service debts as any exclusion from the capital market (especially in the recent past) is of a rather limited duration and cyclical fluctuations on credit markets themselves seem to have a much larger impact on individual sovereigns' access to the capital market (see Richmond and Dias, 2009; Gelos *et al.*, 2011). The increased price of borrowing options following a loss of reputation also seems to be rather short lived as corresponding risk premiums no longer show up in the statistics 3 years later (see Borensztein and Panizza, 2009).⁵

It seems plausible that a country's refusal to repay its debts could have negative growth consequences within that country, too. For example, domestic and foreign economic agents may see sovereign default as a sign of a country's (structural) situation being worse than anticipated, which could result in a correction of expectations about future fiscal policy, capital outflows, lower investment and potential banking crises (see, e.g. Catão and Kapur, 2006; Catão *et al.*, 2007; Sandleris, 2008). For instance, Sturzenegger and Zettelmeyer (2007) and Borensztein and Panizza (2009) find relatively clear evidence of a statistical link between sovereign default and banking crises. Sturzenegger (2004), de Paoli *et al.* (2006) and Panizza *et al.* (2009) find a negative correlation between bankruptcy and growth, while Levy-Yeyati and Panizza (2011) cannot confirm such a finding. Also, causality does not appear to be 100% clear (for a discussion, see Panizza *et al.*, 2009).

In summary it can be said that, from an empirical perspective, a sovereign's incentives to repay its debt may primarily be explained by the domestic cost of sovereign default, while uncertainty about the form and the extent of debt restructuring can intensify the effects mentioned above (Panizza *et al.*, 2009). However, the empirical evidence for the existence of such costs is still weak. The literature discussed below nevertheless assumes a mix of domestic unrest and a temporary exclusion from the capital market following sovereign default which, to some extent, can be interpreted as a disconnect between the assumptions used in the theoretical models and the empirical evidence on the costs of default.

2.3 Problems in the Event of Sovereign Default

The fact that there are no legal provisions to force a country to repay outstanding government debt also means that there are no provisions on what to do should such a situation actually arise. In this case, an unequal distribution of rights between creditors and debtors can result in considerable inefficiencies. For the most part, current literature reduces this issue to collective action problems which can be summarized into three categories (for what follows, see Rogoff and Zettelmeyer, 2002; Roubini, 2002; Sturzenegger and Zettelmeyer, 2007; Bernhard and Kellermann, 2008; Berensmann and Herzberg, 2009 for more details).

The first category is 'holding out' for original claims (free-rider effects). It is very difficult to establish an evolved restructuring procedure that could be advantageous for a majority of creditors if a minority of creditors can circumvent the procedure and may thus not have to participate in the burden of restructuring. Instead, the minority counts on full repayment of the claims once restructuring has been completed. This problem is intensified in situations where there is no perfect information and thus uncertainty about the behaviour of other creditors.

The second category is 'rush to the exit'. As soon as there is the risk of a debt crisis in a debtor country (even in the case of a short-term reconcilable liquidity crisis), there is a tendency for creditors to engage in disorderly and immediate ('panic') sales or redemptions of their claims. This can put a stop to new bond issues and inflate risk premiums and interest rates. Ultimately, it may even lead to a self-fulfilling liquidity and debt crisis.

The third category is 'rush to the courthouse'. If a debtor offers to restructure the debt burden, the creditor can refuse and instead try to sue to enforce its claims. The debt is then repaid from any assets

that the debtor still has. This can result in a collective action problem if the debtor's assets are limited. The first creditor, whose legal claim is met in full, then leaves a smaller insolvency estate for any other remaining creditors. As soon as there is an indication of a debtor's insolvency, the courts may be stormed with every creditor insisting on repayment in full.

Authors such as Roubini (2002) and Rogoff and Zettelmeyer (2002) have identified another key problem area for sovereign default on the part of the debtor, namely a potential 'rush to default'. This could be caused by a debtor country's potential immunity, especially in the case of claims on physical assets by creditors. If costs of default for a debtor country are low from the start, then the moral hazard on the part of the debtor increases the incentive to default opportunistically. Merely observing a 'rush to default' in a debtor country often automatically leads to higher opportunity costs in the form of rising interest rates. However, there does not yet seem to be resilient empirical evidence for the 'rush to default' theory (see Borensztein and Panizza, 2009, 2010).

Berensmann and Herzberg (2009), Häselser (2009) and Panizza *et al.* (2009), to name but a few, describe and compare various existing proposals to solve these problems culminating in an international insolvency law for states. To date, with two exceptions, the literature outlined below refrains from analysing such rules. The author believes that there is great potential for extending the literature on this subject.

3. Base Model of Sovereign Default

The literature on quantitative models of sovereign default discussed below analyses default in a micro-based model following Eaton and Gersovitz (1981) as well as Grossman and van Huyck (1988). The first part of this section gives a brief account of these two articles. In the second part, we present a stylized quantitative base model and provide a brief overview of the solution method used. Such an overview would seem helpful in gaining a better understanding of the findings of the literature described in Section 4.

3.1 *The Foundations*

The analysis by Eaton and Gersovitz (1981) and by Grossman and van Huyck (1988) can, in principle, be seen as a theoretical basis for the dynamic (stochastic) macro models that endeavour to portray government crises using numerical solutions. Sovereigns cannot be forced to repay outstanding government debt and there is no international insolvency law governing debt restructuring. Sovereign default is thus interpreted as a contingent claim. Both Eaton and Gersovitz (1981) and Grossman and van Huyck (1988) formalize, from a theoretical point of view, the reasons why sovereigns usually choose to service their debt. The models assume a sovereign that maximizes households' intertemporal utility resulting from (the path of) consumption. Owing to budget constraints, the government has to finance public spending and interest on outstanding government debt with revenue and, possibly, borrowing (debt rollover). Eaton and Gersovitz (1981) assume that debtor countries which default once are permanently excluded from the capital markets and thus cannot rollover debt in the future. As households have a preference for smoothing future consumption owing to their utility function, this provides a strong incentive to service debts. By not servicing present government debts, the potential for current consumption can be increased. However, following a default, future consumption can no longer be 'brought forward' and the effects of income fluctuations on consumption can no longer be smoothed.

When deciding whether to default, the sovereign compares the present value of the utility with and without the option of smoothing consumption. If the latter is higher, the outstanding debt is no longer serviced. Due to this (theoretical) possibility of lenders being left holding their claims, these lenders

will endogenously set an upper credit ceiling depending on how high they expect the government's utility loss resulting from an exclusion from the credit market to be. The model's structure implies that debts are serviced in a state of equilibrium. Grossman and Huyck (1988) relax the assumption of a permanent exclusion from the capital markets and assume that the exclusion is only temporary which, as we explained in Section 2.2, is a more realistic scenario. They show that a sovereign may default in a state of equilibrium. Because, in this model, lenders form expectations about the trustworthiness of a sovereign and also differentiate between excusable default and unjustifiable repudiation, the sovereign in question is keen to maintain a good reputation and to restrict its non-servicing of debts to 'bad times' only. Lenders consider a sovereign to be trustworthy if it borrows only what it can pay back in 'normal times'. The debt level can be calculated in the model. If a sovereign still defaults in normal or even in good times, then it loses its trustworthy reputation and the costs of default rise relatively sharply or lenders are no longer willing to provide loans. This also means that in this model (1) defaults in a state of equilibrium occur only in 'bad times' and (2) debt is not built up in excess of that which can be paid back in normal times. Furthermore, default is usually only partial. From a lender's perspective, only the worst state of the world can produce total default.

3.2 A Base Model: Setup and Solution Method

Model setup and equilibrium: the literature generally considers a small open economy that receives a stochastic stream of income, y . The sovereign trades bonds, d , with risk neutral competitive foreign investors at price $q(d, y)$, but debt contracts are not enforceable. Every period, the sovereign is, therefore, in one of two states: default or non-default, D and N , respectively. Its default decision can be represented by

$$V(d, y) = \max_D \{V^D(y), V^N(d, y)\} \quad (1)$$

where D is the binary choice to default ($D = 1$) or not ($D = 0$), output y follows a stochastic process explained below and d denotes the asset stock of the sovereign. Hence, for an indebted country it holds that $d < 0$. The sovereign maximizes the economy's utility by choosing whether to default or not. Default, $D = 1$ is chosen only if $V^D > V^N$, where

$$V^D(y) = u(y^{def}) + \beta \int_{y'} [\theta V(0, y') + (1 - \theta)V^D(y')] f(y', y) dy \quad (2)$$

is the present value function of being in the default state, while

$$V^N(d, y) = \max_{d'} \left\{ u(y - q(d', y)d' + d) + \beta \int_{y'} V(d', y') f(y', y) dy \right\} \quad (3)$$

is the present value function of being in the non-default state. A prime indicates expected next period's values of the corresponding variable. $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, with $\sigma > 0$, is a CRRA per period utility function of consumption, c , β is the discount factor and θ the exogenously determined probability of re-entry to the credit market when in the default state.

In equation (2), it is assumed that $c = y^{def}$, that is, households consume all income y^{def} available in default states, with

$$y^{def} = \begin{cases} y & \text{if } y < \bar{y} \\ \hat{y} = \psi \cdot \bar{y} & \text{if } y > \bar{y} \end{cases}$$

where $\psi \in (0, 1)$ are output costs of default and $\bar{y} = E\{y\}$ is expected output. This implies that, in autarky, the output process is truncated because it is assumed that default entails some direct output

cost. The main motivation for doing this is to bring the default probability implied by the model in line with the data (see Arellano, 2008 and Section 2.2 for a discussion).

In equation (3), that is, in the non-default state, $c = y - q(d', y)d' + d$. The sovereign can borrow from international investors by selling one-period bonds d' at price $q(d', y)$, which is determined below, and promises to repay this by giving up one unit of consumption in the next period. Hence, d is the repayment due to previous borrowing (remember that $d < 0$ for indebted countries). Note that the lower is the price $q(d', y)$, the higher is the interest on debt.

Creditors are generally assumed to be risk neutral. Whenever they lend to the sovereign in the current period, they buy bonds d' at price $q(d', y)$ from the government. In the following period, the creditors may receive the face value of the bond depending on whether the government defaults or not. $\delta(d', y) = \Pr[V^D(y) > V^N(d, y)]$ is the default probability. Therefore, with this probability, the creditors get nothing, while, with probability $1 - \delta(d', y)$, they get the face value d' . Assuming a risk-free interest rate r , free market entry and zero profit on the creditor's side, the bond price thus equals

$$q(d', y) = \frac{1 - E\{\delta(d', y)\}}{1 + r} \quad (4)$$

We see that it depends on the default probability, $\delta(d', y)$, the current output state y , current asset holding in the economy d as well as the amount of new bonds d' sold by the government.

These equations are sufficient to characterize a simple default model. The *recursive equilibrium* of this economy is characterized by a set of value functions $\{V(d, y), V^D(y), V^N(d, y)\}$, a set of policy functions for households' consumption $c(d, y)$ as well as policy functions for the government's default decision D and optimal asset holdings d' such that

- Taking as given the government policies, households' consumption $c(d, y)$ satisfies the budget constraints (i.e. $c = y^{def}$ in autarky and $c = y - q(d', y)d' + d$ otherwise).
- Taking as given the bond price function $q(d', y)$, the governments policy functions d' (i.e. the desired amount of new borrowing per period) and default sets D satisfy the government's optimization problem.
- Bond prices $q(d', y)$ reflect the government's default probabilities and are consistent with the creditors' expected zero profit condition.

Numerical solution: the model is then solved numerically. Parameters are generally based on existing literature or calibrated to match specific facts. The output process is usually estimated according to the economy to be matched. Here, we assume that it is given by $y = \exp(z)$, where $z_t = \rho z_{t-1} + \epsilon_t$ and $\epsilon_t \sim N(0, \sigma_z^2)$. Our parameter choices (and the corresponding sources) are given in Table 1.

Because the sovereign uses generally non-linear present value functions to calculate the utility of not servicing or servicing its debt and one has to calculate the default probability endogenously, the solution of this maximization problem has to use non-linear techniques, too. The standard solution method used is the *discrete space technique*. This works as follows. After having defined the functional forms as well as the parameters, we need to define a state space for the sovereign's assets/debt with a lower and an upper bound. We define these bounds to be $\underline{d} = -0.25$ and $\bar{d} = 0.05$. A certain number of evenly spaced grid points are then taken from this space to be used as a proxy for all possible debt levels. As described below in the literature overview, it can be shown that this number of grid points has to be relatively large to avoid incorrect interest rate movements in the model.

In addition, a discrete state space has to be generated for output. We also need a matrix for transitional probabilities. For this purpose, it is usually assumed (as we have done also) that output follows an autoregressive process and a shock can move this level up or down. This shock, originating from a certain distribution function, can be converted to an n -state Markov chain using a quadrature-based method (see Tauchen and Hussey, 1991). This Markov chain (with dimension $n \times n$) can be interpreted

Table 1. Parameter Values.

What	Symbol	Value	Source
Risk aversion	σ	2.000	Arellano (2008)
Discount factor	β	0.953	Arellano (2008)
Risk-free interest rate	r	0.017	Arellano (2008)
Reentry probability	θ	0.252	Cuadra and Sapriza (2008)
Default output costs	ψ	0.950	Arellano (2008)
Output shock	ρ	0.850	Cuadra and Sapriza (2008)
	σ_z^2	0.025	Cuadra and Sapriza (2008)

Note: Values are taken from the literature without performing a separate empirical analysis because the presented model is intended to be an illustrative example and not to replicate actual/specific data.

as a ‘*matrix of transitional probabilities*’. At this juncture it should be noted again that if a sovereign defaults, output costs occur which lower the level described here further. For the illustrative example model here, we assume 22 output states and 200 asset states. Once these preparations have been made, the numerical solution of the model can begin.

In order to find the equilibrium allocations and the bond price schedule for debt, the following algorithm is generally used

- i. Discretize the state space as described above.
- ii. Start with a guess for the bond price schedule such that $q^0(d, y) = 1/(1 + r)$ for all d and y .
- iii. Use $q^0(d, y)$ to solve the sovereign’s problem recursively using value function iteration by taking the assumed price schedule for bonds as given and obtain the optimal policy functions for consumption $c(d, y)$, asset holdings $d'(d, y)$ and default sets D .
- iv. Given these policy functions, compute the probability of default $\delta(d', y)$.
- v. Update the price of bonds using equation (4).
- vi. Use the updated price of the bond $q^1(d, y)$ to repeat steps ii.–v. until the convergence criterion, $\max\{q^0(d, y) - q^1(d, y)\} < \Delta$, where Δ is a (very) small number, is met.

It is obvious that this solution method can be rather time consuming and computationally intensive even in small models. Given 22 output and 200 asset states, there are already 4400 states/combinations for which the solution just described has to solve the model. Hence, models with many state spaces as well as larger models may stretch even modern computers to their limits. We also see that linear-approximative solutions, which are commonly used in larger (ultimately linearized) DSGE models, cannot be applied here. Linearization means that they are not precise enough to calculate risk premiums effectively. Therefore, the main reasons why this idea has not yet been incorporated into larger models are likely to be the inaccuracy of linear solution methods as well as, in many cases, the complexity of many of the DSGE models used.⁶ As will be seen at the end of this paper, this should not stand in the way of continuing to develop the literature described below as it has great potential to provide answers to future questions.

Results: nearly all papers on this issue present the resulting bond price schedule as a function of assets for two values of the productivity shock (high and low), which we also do in Figure 1.

As the figure shows, the bond price is an increasing function of foreign assets, which can be explained as follows. It is evident from the model description that the utility for the representative household increases as consumption rises. If the sovereign repays debt, the utility for the representative household falls with outstanding government debt *ceteris paribus*. If the sovereign decides against

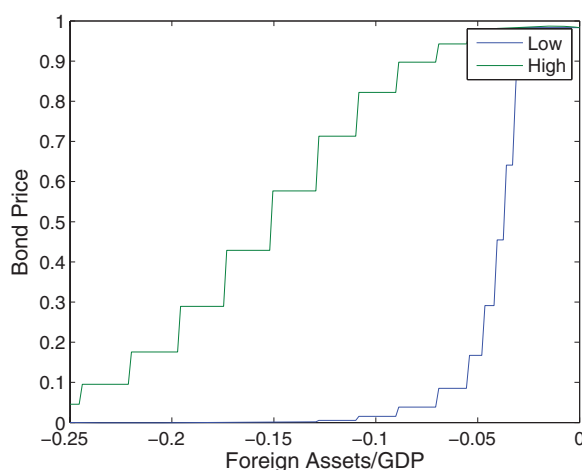


Figure 1. Bond Price Schedule.

repaying outstanding debt, then the utility for the representative household in that period increases. Hence, if investors were then to buy more sovereign bonds without considering the default risk, this would be an optimal time for the government to default in any period (see also Grossman and van Huyck, 1988). Of course, this does not happen. Thus, it is assumed that a sovereign default entails costs for the government: (1) Access to the capital markets is temporarily lost, meaning that the possibility of consumption smoothing is limited. In addition, it is assumed that if a sovereign defaults there will (2) also be unrest within the country which would lower productivity and therefore income. Just how long the government is excluded from the international capital markets is shown by an exogenous probability of regaining access.

The sovereign therefore has to make the following decision. If it pays down the debt, it lowers households' utility today but can continue to borrow and, in general, can expect higher output. If it chooses not to pay, it has to cope with productivity losses and a delay in smoothing consumption owing to exclusion from the capital market. The sovereign compares the expected present value of these two options in that period and decides whether to default. Figure 2 shows the value of the option to default or repay as a function of assets-to-GDP for high- and low-productivity levels. We observe that, for a given output realization, default is chosen for all levels of assets below a certain threshold, that is, when the value function becomes horizontal. In this case, the autarky is better than staying in the contract because financing costs become too high. When setting prices for bonds, international investors take the sovereign's decision into consideration. The more likely they believe the sovereign is to default on its debt, the less they are prepared to pay for a bond. Hence, as is shown, the price of bonds falls (i.e. the interest payments on government debt rise), the larger the outstanding government debt and the lower the expected productivity path of the country in question. In extreme cases, the price may fall to zero and interest payments thus rise infinitely. This would mean that no one wants to buy sovereign bonds of the country in question.

Furthermore, we can present some simulation results and the statistical properties of the model. In Table 2, we show the business cycle moments of selected macroeconomic variables for the simulated economy. These statistics are average values over 1000 simulations of 100 realizations each, drawn from a stationary distribution. The simulated series are logged and filtered. Figure 3 shows how the interest rate spread increases as output contracts in the economy.

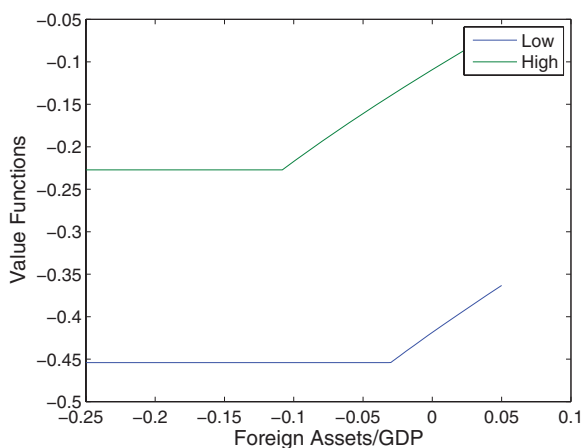


Figure 2. Value Function.

We see that this model class is able to match several stylized facts that can be observed in the data. These facts include, among others, that incentives to default are higher for more indebted countries and that default risk and interest rate spreads move countercyclically (see Figs. 1–3). It can also be seen that private consumption is positively correlated with output, while the trade balance is negatively correlated (see Table 2). However, we also observe some drawbacks of this model class. For example, they have difficulties in replicating the magnitude of the mean bond spreads and fare quite badly in replicating realistically high debt-to-GDP ratios. In order to show that this is not a feature of the simple model at hand, we also present the results of Arellano (2008) in order to compare our model results with hers. The data for Argentina is taken from Cuadra and Saprizza (2008). Despite these drawbacks, this class of models is, nevertheless, able to qualitatively fit the data as well as highlight some interesting aspects and mechanisms.

In the next section, we will describe recent developments in the literature of this model class in more detail. We will focus primarily on the qualitative findings of the literature and point out where there may be options for extensions.

Table 2. Simulation Results.

	Argentinean data	Arellano (2008)	Model results
Mean bond spread	10.25	3.58	3.65
Default probability	3.00	3.00	3.55
Mean debt-to-GDP ratio	48.79	5.95	4.70
Correlation (Spread-GDP)	-0.88	-0.29	-0.20
Correlation (Trade balance-GDP)	-0.64	-0.25	-0.32
Correlation (Consumption-GDP)	0.98	0.97	0.88

Note: The table shows selected simulated moments of the models and compares them to the ‘base model’ of Arellano (2008) as well as Argentinean data. The latter is obtained from Cuadra and Saprizza (2008).

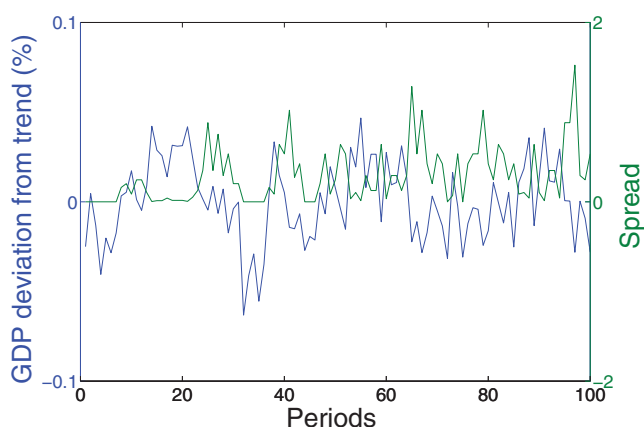


Figure 3. Dynamics of GDP and Spread. GDP is Shown in Percentage Deviations from its Trend. The Spread is Defined as $s = 1/q(d', y) - 1 - r$.

4. Developments in Quantitative Models of Sovereign Default

The base models: to the author's knowledge, Alfaro and Kanczuk (2005) were the first to integrate the purely theoretical studies on sovereign default into a quantitative model.⁷ In their model, it can be seen that, in addition to exclusion from the capital markets following a sovereign default in the country in question, further negative output effects themselves are a necessary condition for generating equilibria that contain both repayment and default situations at debt ratios that are seen as being more realistic, yet constant. The model presented in Alfaro and Kanczuk (2005) contains a 'good' and a 'bad' sovereign. A 'good' sovereign is one that attributes relatively high importance to future consumption and discounts it at the same rate as households, whereas a 'bad' sovereign is one that attributes a great deal of importance to current consumption and in the model always decides not to service its debt. After a default, lenders cannot verify which category the sovereign falls into and have to form expectations as to whether the sovereign is good or bad when purchasing and pricing new government bonds. Based on this assumption, the model is able to explain an empirical observation (at least as far as Latin American countries are concerned) as to why sovereigns do not default until they have experienced a series of negative shocks (i.e. not until after a sharp recession). A good sovereign uses a delay of this kind to signal its 'goodness' and thus to obtain better conditions on the capital markets in the future. In other words, the decision to default is not made until it is clear that there is no alternative (the authors call this phenomenon 'muddling-through equilibrium'). To use Grossman and Huyck's (1988) terminology, one would speak of an excusable default. Aided by numerical analysis, the authors also demonstrate that welfare is higher in all equilibria in which there is no default in any productivity state (measured as consumption equivalents as in Lucas, 1987).⁸

Alfaro and Kanczuk's (2005) model is relatively simple in the sense that it envisages only three productivity states, that the transitional matrix for these states is stipulated exogenously and that the equilibrium debt level is assumed to be exogenous and constant. The sovereign thus decides only whether to pay interest or not. By contrast, Cuadra and Saprizza (2008) endogenize government debt and assume that output follows a stochastic, normally distributed process. Furthermore, they assume that the economy under consideration has two different types of household, each of which derives utility from different public goods. These two types of household are represented by two different political

parties (e.g. left-wing or conservative governments) which – when in office – give preferential treatment to their own supporters. Unlike in the literature discussed in Section 2.1, both parties are not in power at the same time. One party governs but is at risk of being replaced by the opposition. Should this happen, the incumbent party becomes the opposition and the other party takes office. The transition probability is set exogenously and thus closely corresponds to Alfaro and Kanczuk's (2005) probability of being a good or bad sovereign. As the political parties have different preferences regarding the structure of public goods and as they know that this composition will change as soon as the other party takes office, the incentive for the incumbent party to give preferential treatment to its supporters and to finance this by borrowing increases. From the incumbent government's perspective, the costs of sovereign default fall in relative terms as it is fairly certain that they will have to be borne by the other party one could speak of a quasi-finite time horizon. This model structure improves the models' quantitative fit with the data and explains the interaction between political risk, government debt and interest rate spreads. Hatchondo *et al.* (2009), who differentiate between patient and impatient governments but not between public goods themselves, and D'Erasmus (2010), who additionally integrates debt renegotiation explicitly, come to similar conclusions. It can be demonstrated empirically, for example by Moser (2007), that political risk can actually impact on bond spreads and default probabilities.

Aguiar and Gopinath (2006) use a similar model, albeit without distinguishing between types of sovereign, in order to isolate transitory shocks from shocks to trend growth. They show that if transitory shocks exist, it is primarily the debt level which determines whether a sovereign defaults, and that, in model simulations, output shocks at a given, relatively low original debt level result in only a very low default rate. As outlined above, sovereigns decide to default if the discounted utility of the capital market autarky is larger than the discounted utility of servicing debts. The difference between these two discounted utilities is comparatively constant in the case of transitory shocks as such a shock has roughly the same effect on both utility functions, making the debt level the driving factor. However, the presence of trend shocks causes a greater response by the values of both discounted utilities as well as the difference between them so that, in this scenario, shocks have a much larger impact. This implies that, given the same initial level of debt, the insolvency rate is affected perceptibly more after a trend shock than compared to transitory one. A positive trend shock means that income rises today but rises even further tomorrow. By making it easier to access government debt, tomorrow's consumption can be brought forward to the present and the discounted utility of servicing debt thus rises at a relatively faster pace than in capital market autarky. The opposite is true in the case of negative trend shocks. Hence a trend shock has a much greater impact on the propensity to default than a transitory shock. From this we can derive that regular economic fluctuations should have virtually no impact on a sovereign's default decision, whereas long-term growth prospects are quite relevant.

An interesting aspect in this analysis is the finding that bailouts significantly increase the probability of default. Aguiar and Gopinath (2006) model bailouts as a transfer from an unmodelled third party to the country in question assuming that this third party guarantees part of the outstanding debt should the sovereign default. Lenders can thus purchase sovereign bonds up to this limit without incurring default risk as only debt in excess of this limit is subject to the risk of losses. This lowers the average interest rate on outstanding government debt and increases the incentive to borrow excessively because financing costs decrease. In comparison to a situation without bailouts, if corresponding output shocks exist, the risk of a country defaulting rises. This is a very simplified description of bailouts, which are usually *ad hoc* measures and not initially integrated into the system (see also Section 2.3). However, this could form the basis for integrating the analysis into a more realistic environment and also for modelling the bailout party which we will discuss in more detail below (see also Boz, 2011; Fink and Scholl, 2011).

Arellano (2008), whose working paper version of the article published in the *American Economic Review* in 2008 can probably be viewed as the basis for the other literature on this type of model,⁹ shows that the model is able to replicate most economic facts and the scenarios that led to the crisis

in Argentina rather well. Arellano (2008) illustrates another important aspect of this type of model. Since lenders take default probabilities into consideration, the interest rate they require rises, *ceteris paribus*, along with the level of public debt. This creates an endogenous borrowing limit, which the sovereign does not exceed voluntarily. In principle, the government would have an interest in increasing its borrowing so as to raise the current level of consumption, and would be able to do so subject to rising interest rates. However, the additional credit available on the market is comparatively expensive, meaning that additional current consumption can be funded only through very high losses in future consumption. Price and volume effects thus move in the opposite direction. Consequently, the government will not wish to borrow above the endogenous borrowing limit; this is essentially a kind of 'Laffer curve' effect.

In a model where investors additionally obtain information on the future output situation of an economy, Durdu *et al.* (2010) show that sovereign defaults can also occur in good times. This happens when investors view negative information obtained about an economy as credible and therefore raise the interest rates on government bonds, which means that it is no longer worthwhile for the government to service its debt. The information is modelled as a shock process that is correlated with the output process. Since this correlation is imperfect, however, the information obtained by investors does not necessarily reflect actual developments. Durdu *et al.* (2010) show that a corresponding calibration of the shock process for information can be used to replicate spread developments in industrial countries.

Risk aversion and contagion: Arellano (2008) also shows that more realistic interest rate spreads can be generated within the model framework if risk-neutral international investors include a stochastic discount factor in their calculations of the bond price, flexibly interpreted as somewhat similar to stochastic news shocks. However, this could also be derived endogenously in the case of risk-averse investors. Lizarazo (2010) explicitly models these risk-averse investors using a framework that is otherwise largely based on the model in Arellano (2008); as she illustrates, this allows realistic debt ratios to be calibrated much more easily than in the standard model and her model can better replicate the movements in yield spreads. In this framework, investors can decide whether to invest in T-Bills (which are assumed to be risk free) or in risky government bonds. The bond price now no longer depends solely on the economy's fundamentals but also on the financial wealth and risk aversion of the lender. This implies that the bond price can now be broken down into two components: a base premium that compensates the investors for the actual probability of default and the associated loss of income, and an *excess premium* that compensates them for taking the risk of default in the first place. The wealthier the investors, or the lower their risk aversion, the cheaper it is for a sovereign to borrow, because the excess premium, at least, falls.

Lizarazo (2009) explicitly integrates contagion of government crises from one country to others based on the model in Lizarazo (2010). This is not possible in Arellano's (2008) base model, which replicates risk-neutral investors. It is possible, however, if investors are assumed to be risk averse. In Lizarazo (2009), it is assumed that a representative risk-averse investor can purchase the bonds of several countries. The wealth of the investor now depends on the country composition of his bond portfolio and the respective default probabilities. If one of the countries defaults, the investor's wealth decreases. If he is risk averse, this will reduce his willingness to expose himself to other risky investments and, all other things being equal, he will invest more heavily in riskless T-Bills. A country whose fundamentals are essentially unchanged must now, *ceteris paribus*, pay a higher risk premium ('excess premium') as a result of the reduced wealth of the investor in order to convince him to purchase its bonds. A sovereign default or an elevated risk thereof in one country thus diminishes other countries' borrowing possibilities – even if the financial situation of these countries can essentially be regarded as secure/sound. Lizarazo (2009) describes this as the 'income' effect. Higher funding burdens in one country can therefore also increase other countries' default probabilities. However, a substitution effect also occurs. The investor wishes to restructure his portfolio because of the income effect. He can do this both via riskless but relatively low-yield T-Bills and the slightly higher yield government

bonds of other countries whose financial situation he considers to be 'sound enough'. Depending on his degree of risk aversion, he will hence wish to shift his bond portfolio away from 'unsound' and towards 'sound enough' countries, leading to a disproportionately sharp rise in the financing burdens of 'unsound' countries. Their default probabilities then increase as a result of this additional burden.¹⁰ For countries with fundamentals that are strong or regarded as such by the investor, the restructuring of the portfolio (i.e. the substitution effect) can then trigger a fall in their financing burden, thus enhancing their borrowing possibilities. This effect could also be described as a *flight to quality*. Which of the effects actually occurs depends to a large extent on the parameters used in the model. It is not possible to formally derive reliable theoretical relationships governing the threshold above which an investor considers a country to be sound or how strongly this and the other results are affected by increased risk aversion among investors (see also Lizarazo, 2010). In principle, however, it can be assumed that, in addition to the income effect, the substitution effect has a greater negative impact on a country's financial burdens the worse the country's fundamentals are, the greater the investor's risk aversion, the lower his wealth and the more exposed his existing portfolio is to the 'problem country' (which may choose/chooses to default) and to other 'unsound' countries. Simulations show that the model can successfully replicate the contagion effects of the Argentine crisis on Uruguay.

More complex fiscal policies: Cuadra *et al.* (2010) integrate more complex production and fiscal sectors into their model. They assume that households derive utility from private and public consumption goods and, at the same time, make a consumption-leisure choice, while output is produced using labour. The state generates revenue by taxing consumption, sets the level of government expenditure and new borrowing and decides whether or not to default. This significantly extends the sovereign's room for manoeuvre, and its decisions have more complex repercussions on the rest of the economy. With respect to the default decision, bond prices and risk spreads, the model can replicate the stylized facts just as well as the aforementioned models. Additionally, fiscal policy can be shown to act procyclically in this kind of model framework, that is, fiscal agents raise expenditure and cut taxes in good times. This is, at least for developing countries, a stylized fact, but it tends to be counterproductive from the point of view of wealth and consumption smoothing. During recessions, a government's debt financing conditions deteriorate owing to a higher risk of default and the associated rise in risk premiums. As households derive utility from the provision of public goods, the government then increasingly finances this expenditure via taxes, since debt financing is now relatively more expensive. However, higher taxes reduce the consumption of private goods (which causes utility losses) and lead, at the same time, to lower output. This is because higher taxes also induce households to reduce their supply of labour due to the consumption-leisure choice owing to the fact that the relative price ratio of consumption and leisure must correspond to the ratio of the marginal utilities. To partly compensate for the negative labour incentive, the government now reduces expenditure during the recession, too, so that taxes do not have to be raised as sharply.

In a two-period model, Cuadra *et al.* (2010) additionally investigate whether the results change if it is assumed that the private sector – in this case, households – can likewise borrow on foreign credit markets and choose to default. When setting the interest rate on government bonds, international investors now take account of both public and private sector debt. The same applies to interest rates on private debt instruments. The conclusions of the models in which only the sovereign can become indebted are, it transpires, essentially confirmed. However, the government's propensity to borrow can increase. If the economy is hit by a series of adverse shocks, it becomes more difficult for the sovereign to borrow because the default probability for the private sector also rises. In total, the overall probability of default hence increases. As outlined above, this increases the government's incentive to raise taxes. As the private sector can now, in principle, borrow abroad and can therefore carry out consumption smoothing itself, this incentive is actually strengthened. On the other hand, the private sector faces funding constraints similar to those of the government or even has to pay higher risk premiums, which means that, for the private sector, borrowing is not necessarily the best instrument

for smoothing consumption. A government seeking to maximize household utility may be inclined to raise taxes less sharply and take on more debt. The model's simulations show that the magnitude of the economic utility loss that a country faces following a default depends on whether it is the private sector, the government or both that decide not to fulfil the credit obligations. This initial comparatively simple and rather stylized step towards integrating the interactions between private and public sector debt into dynamic macro models could form the basis for more detailed analyses.

Negotiating to regain capital market access: Yue (2010) and D'Erasmus (2010) endogenize the probability of a country regaining access to the capital markets following a default – something which had previously been regarded as exogenous. They assume that a country explicitly negotiates its reentry into the capital markets and debt repayment. This takes the form of an one-shot Nash bargaining game, in which it is assumed that the debtor and the creditor negotiate how much of the outstanding debt is to be repaid. If this debt renegotiation fails, the creditor gets nothing and the country remains in permanent autarky. It is only once the negotiated repayment has been made that the defaulting country can regain access to the capital markets. These models thus combine both elements: the inclusion of endogenous default probabilities and endogenous debt recovery rates. The models can be shown to replicate the empirical facts of the other models while also being capable of reflecting the observable haircuts.

The incorporation of endogenous debt recovery rates influences the sovereign's default decision *ex ante*, too. As in the other models, the default probability increases, *ceteris paribus*, with rising government debt. However, the debt recovery rate following a default falls as debt rises.¹¹ When setting the bond price, creditors not only take the default probability into consideration but also form expectations regarding the debt recovery rate in the event of a default. The lower the expected recovery rate, the lower the price they are willing to pay for purchasing government bonds. The sovereign's incentives to borrow are now influenced by two countervailing effects. On the one hand, the incentive to borrow rises because the government might regain access to the capital markets in the event of a default more cheaply subject to a rather large haircut; on the other, the incentive to borrow decreases because this is taken into account by the investors when purchasing government bonds. In the baseline calibration of the model in Yue (2010), the former effect appears to be predominant, as it generates higher government debt. The same applies to D'Erasmus (2010), who, by integrating a reputation effect into a very similar model, is able to generate even higher debt ratios.

The delays in debt restructuring negotiations (in the case of Argentina, these took as long as 40 months) found in empirical data are generally regarded as evidence of inefficiencies (see also Section 2.3). Nonetheless, Bi (2008) shows that it can be beneficial for both the defaulting country and the debt holders to delay the negotiations (i.e. 'waiting' can be an efficient equilibrium outcome). In doing so, he builds on the literature discussed in the previous paragraphs and uses a model in which government debt restructuring *and* the probability of a government reentering the international capital markets are also negotiated following a default. Based on Merlo and Wilson's (1995, 1998) bargaining game, in each negotiation period borrowers or creditors are randomly selected to make a proposal regarding the size of the debt recovery rate. If the selected player decides to make a proposal, the other player can choose to accept it. The government then repays according to the agreement, regains access to the capital markets, and the negative output effects in the country 'disappear'. In all other cases, the game is repeated in the following period.¹² Depending on the volume of outstanding government debt and the output shock, it can be beneficial for both the defaulting country and the creditors to delay negotiations (i.e. to reject or pass). For the same reason as in the Nash bargaining games described above, the recovery rate falls when the debt-to-GDP ratio rises. In addition, it is easier for the country to make a higher repayment in good times. Given a temporary but fairly long-lasting negative output shock, it can be advantageous for both parties to wait. The creditor can hold out for a larger repayment because the size of the 'cake' will increase; the defaulting country can lessen the relative costs of repayment because the utility losses incurred by foregoing consumption have less of an impact in good

times. Interestingly, the debt recovery rate is affected by the identity of the proposer in the actual debt recovery period. If it is the defaulting country, debt recovery is lower than when the creditor makes the proposal. The debtor (creditor) will make a proposal that the creditor (debtor) is just about willing to accept. Although the creditor (debtor) would like to receive more (pay less), the expected probability of the player in question being able to improve the offer appears too low.

In a comprehensive paper, Benjamin and Wright (2009) present a new database covering 90 defaults by 73 countries that were settled in the period from 1989 to 2006 and containing extensive data on the relevant restructuring negotiations and their outcomes. They, too, embed the restructuring negotiations in the type of sovereign default model discussed above. Structurally, the negotiations are modelled in way similar to that in Bi (2008), that is, one player is randomly selected in each negotiation period to be the proposer, who is allowed to make an offer that the other player can either accept or reject. Unlike in Bi (2008), however, the players do not negotiate the debt recovery rate; instead, the selected player proposes a transfer of resources from the debtor to the creditor. This transfer (also when accepted by the creditor) can exceed the resources currently available to the debtor, requiring expected future income to be transferred as well. As this is not so simple to achieve, such proposals involve the sovereign borrowing new funds, which must be provided by the creditor. There is therefore a risk that the debtor country will default on this new government debt in a future period. In this structure, the creditor has two strategies: (1) he can demand a transfer and offer to lend new funds or (2) he can demand full repayment of the government debt. The debtor country can (1) repay the debt in full, (2) offer a transfer and request to borrow new funds or (3) offer nothing and remain in autarky. The other player can either accept or reject whichever proposal is made. If the proposal is rejected, the next round of bargaining begins. If a proposal is accepted, the debtor country is allowed to reaccess the capital markets. Creditors have an interest in maximizing the payments they receive, whereas the debtor country wants to keep them as low as possible. It therefore tends to prefer the option of funding a transfer through new borrowing over full repayment of the sovereign debt. Creditors, on the other hand, cannot be entirely confident that the debtor country will not default on this new debt. It can therefore be beneficial for both parties to wait for better times, when the probability of a default on the new debt falls and the expected payments to the creditor rise. The model is able to replicate the stylized facts calculated by Benjamin and Wright (2009) using the new database and explains why some debtor countries are more highly indebted when they exit such negotiations than when they entered them.

Delays in restructuring negotiations can thus represent an equilibrium which, depending on the interpretation, may be efficient because, in the underlying bargaining structure, both parties may be able to minimize/maximize their future loss/gain in utility. This is particularly true of Bi (2008), but such cases can also occur in Benjamin and Wright (2009). Nonetheless, the models described above do not take the coordination problems among creditors, mentioned in Section 2.3, into account. The model in Pitchford and Wright (2010), for example, shows that when creditors are uncoordinated the negotiations can be inefficiently delayed, as individual investors hold out for better payoffs or that individual investors free ride on negotiation costs. Bai and Zhang (2010) show that the structure of sovereign debt also plays a role. Before the 1990s, it was usually banks that acted as creditors, which made creditor coordination during government debt restructuring comparatively simple; since then, it has become much more difficult to coordinate creditors because governments issue bonds that can be traded on the secondary market. This delays restructuring and the government's reentry into the capital markets. However, these studies often have more of a focus on game theory and tend to be more concerned with restructuring negotiations than the macro modelling of sovereign defaults; they are therefore not discussed in more detail in this article. Putting the model findings just discussed here into an empirical perspective, we should emphasize the fact that, for example, Trebesch (2010) finds in a recent paper that government behaviour and political instability appear more important in explaining lengthy restructurings than holdouts, inter-creditor disputes, and litigation problems that are often the focus of the theoretical literature.

Debt maturity and debt dilution: all of the above-mentioned papers assume that sovereigns issue one-period bonds that must be fully refinanced in each period. In reality, however, governments generally issue bonds with differing maturities and also incur long-term debt. Building on the framework in Alfaro and Kanczuk (2005), which assumes a constant debt level, Alfaro and Kanczuk (2007) integrate one- and two-period bonds into their model. A country which can issue only one-period maturity bonds is compared with an otherwise identical country which can issue two-period maturity bonds. As outlined in more detail above, in Alfaro and Kanczuk (2005, 2007), investors form expectations regarding the type of government, 'good' or 'bad', they are confronted with and set the interest rates accordingly. The likelihood of the government type changing is mapped using a Markov chain. Information on the current type of government is thus comparatively indicative of the government type in the following period. However, uncertainty regarding the government type increases for periods in the more distant future. If investors consider the current government type to be 'good', it is relatively likely that this will still be the case in the following period. Nonetheless, the likelihood of this remaining so in later periods is lower. Alfaro and Kanczuk (2007) use this argument regarding uncertainty to explain why empirical data show that the interest rates paid on longer term bonds are usually higher than on short-term bonds. The opposite is true, however, if investors consider the current government to be 'bad'. Longer bond maturities lead to a fall in the volume of sovereign debt to be refinanced per period and, at least in bad times, in the financial burden of future sovereign debt due to the opposite uncertainty argument just described. This leads, on the one hand, to a decline in the utility of a government default; on the other, the costs of such a default also fall. The latter can be explained by the fact that, in the model, the sovereign is not excluded from the capital markets after a government default but instead faces 'only' a higher interest rate, and the rise in interest rates is smaller for long-term bonds than for short-term ones because of the opposite uncertainty argument. It is impossible to tell which of the two effects is predominant from a theoretical point of view. For most of the calibrations in Alfaro and Kanczuk (2007), however, the latter effect predominates, meaning that the propensity to borrow increases when bonds have long maturities.

The simplifying assumption of a constant debt level given longer term debt – as in Alfaro and Kanczuk (2007) – is relaxed by Hatchondo and Martinez (2009), who incorporate long-term debt and varying debt levels into Arellano's (2008) 'baseline framework'. A logical starting point is the assumption that the repayment of bonds issued in the current period (t) will mature in T periods. Since, given endogenous new borrowing, new bonds can be issued in each period, however, such an assumption means that the debt state space consists of the vector $(b_0, b_1, b_2, \dots, b_{T-1})$, where b_t denotes the volume of debt that must be repaid in t periods in the future. Because the solution method for solving these models is non-linear, the computational time and effort is immense or even unfeasible even for small T values, as there are at least T state variables that can themselves take on many different values. Hatchondo and Martinez (2009) therefore assume that the government issues bonds that promise an infinite stream of coupons which decreases at a constant rate. Depending on this rate, the model can replicate one-period bonds. This is the case if the rate is equal to one.¹³

In this model, the maturity structure of the outstanding bonds is exogenous and constant, but the level of government debt is variable. It is only possible to study the different effects that arise given different exogenously determined maturities. If the sovereign declares a default, it permanently discontinues payment of the concerned coupons to the investors, but can issue new coupons. Hatchondo and Martinez (2009) then look at the differences that arise when one-period and 4-year bonds are issued. They, too, show that the interest rates on 4-year bonds are higher, *ceteris paribus*, than those on one-period bonds. The argument is similar to that of the uncertainty argument in Alfaro and Kanczuk (2007). In addition, the paper addresses the fact that, unlike in the case of one-period bonds, holders of old debt face *debt dilution*. The longer the maturity, the less a government can commit to servicing the debt in the future. Furthermore, new borrowing increases a sovereign's probability of default – even for already/still outstanding government debt. These factors combined reduce the expected value

of the government debt for those who own old bonds, leading to debt dilution. The interest rate spread therefore rises even more strongly than can be justified by the uncertainty argument alone. In good times, and given a low level of sovereign debt, governments issuing one-period bonds within these models can borrow at almost the zero-profit price, that is, almost at the risk-free rate. The longer the maturity of the bond, the less possible this becomes. The welfare gains generated through long-term debt – resulting from the lower debt rollover and thus higher consumption per period¹⁴ – are reduced by this debt dilution argument. The model's simulations show that, although one-period bonds do not appear to be optimal from an *ex ante* welfare perspective, to maximize welfare, the maturity of the bonds should be kept relatively short.

Chatterjee and Eyigungor (2009) incorporate long-term bonds into their model using a very similar approach. They assume that bonds with a certain maturity can be issued in each period, but that unit bonds are infinitesimally small. These assumptions also prevent a disproportionately large rise in the state space, since the authors are able to calculate a sort of law of motion for bonds and thus still have only 'one' debt state space. The qualitative results are similar to those of Hatchondo and Martinez (2009).

As a thought experiment, Hatchondo *et al.* (2011) extend the model in Hatchondo and Martinez (2009). They assume the existence of a rule which requires that the government receive the consent of existing bondholders before issuing new debt. This could be viewed as an expanded form of the collective action clause. In order for the existing bondholders to consent to the government issuing debt, they must be compensated for the loss resulting from debt dilution (in the model: through a transfer). This reduces the interest rate spread required by creditors by lessening the possible future loss of income. While this is a very stylized analysis, such techniques could have a role to play in the design of adequate stabilization mechanisms, albeit, perhaps, in a different form.

A noteworthy limitation of the findings discussed in the quantitative literature of sovereign defaults is that it may downplay the potential benefits of long-term debt as it tends not to incorporate the perhaps self-fulfilling risk of a liquidity squeeze in its cost/benefit analysis of long-term debt. It may well be the case that long-term debt reduces the scope for self-fulfilling debt runs, for example, in a situation where the country is able and willing to repay its debt if it can spread repayment over time, but will default if it cannot rollover a sufficiently large share of its debt. Models with self-fulfilling features were quite popular in the international finance literature in the late 1990s/early 2000s. For example, Cole and Kehoe (2000) modelled such a self-fulfilling debt run.

Choosing the debt composition and where to borrow: Arellano and Ramanarayanan (2010) examine the question of governments' preferred maturity structure when they have bonds with different maturities at their disposal. Based on the model in Hatchondo and Martinez (2009), who compare the impact of differing exogenously assumed maturities, they endogenize the government's decision regarding the maturity structure if it can issue debt using two different bonds with different maturities. The government faces the following tradeoff: by issuing debt in the form of long-term bonds, it can hedge against shorter term fluctuations in the bond price and thus in the refinancing costs because (1) it must rollover only part of the outstanding government debt in each period and (2) the interest rate on the bonds is fixed when they are issued. By issuing short-term bonds, the government can, in a given period, obtain more liquidity at a cheaper interest rate because it avoids the above-mentioned commitment and debt dilution involved in long-term debt. However, owing to the rapid debt rollover, the financing costs in the following period are higher (and the discounted utility loss resulting from lower consumption in the near future falls). Given comparatively low spreads in good times, the government issues more long-term bonds, since its motives for hedging are more pressing. In bad times, it tends to issue more short-term debt, as additional liquidity is so important in these periods that it outweighs the near-term increase in financing costs. This continues to hold – at least up to a certain threshold – even when the interest rates on short-term bonds rise more sharply than those on long-term bonds owing to the increased short-term default risk. Owing to the concave utility function, additional current

consumption provides a stronger justification for future consumption losses in such periods because the marginal utility of each additional unit of consumption is relatively high. This mechanism can, at least partly, explain why governments take on more short-term debt in bad times (including shortly before a default) even though the spreads on short-term bonds rise more sharply than those on long-term bonds.

Boz (2011) analyses governments' decisions to borrow from private sector creditors or international financial institutions (IFIs) such as the International Monetary Fund (IMF). This borrowing takes the form of one-period bonds. Private sector lending is modelled, in principle, following the approach in Arellano (2008): the interest rate is determined endogenously according to the expected default probabilities. Loans from IFIs differ from private sector loans in three ways. First, IFI loans are enforceable. The stated reason for this is effective monitoring and the fact that the IFI has a preferred creditor status. Second, IFIs require an interest rate calculated from the sum of the risk-free rate and a charge that increases with the amount borrowed from the IFI. An IFI sets the interest rate purely on the basis of the outstanding level of debt owed to it, as this is sure to be serviced. Third, loans to a sovereign from an IFI are attached to conditionality arrangements regarding budgetary discipline. This conditionality is approximated by assuming that the sovereign has to switch to a higher discount factor (lower discount rate) when it is indebted to an IFI. It is thus assumed that fiscal policy becomes more 'patient' as soon as it borrows from an IFI. In a sense, Boz (2011) thus endogenizes the decision of the government to be a 'good' or 'bad' fiscal authority as defined by Alfaro and Kanczu (2005), Hatchondo *et al.* (2009) or D'Erasmus (2010). If the country decides to borrow from an IFI, it decides to adopt a 'good' fiscal policy. Since future consumption is valued more highly, budgetary policy automatically becomes 'more restrictive', making a default less likely *ceteris paribus*. The model is able to explain why borrowing on commercial credit markets is procyclical, whereas IFI lending is countercyclical. In good times, sovereigns prefer to borrow commercially because the low default probabilities keep the costs low. In bad times, financing conditions on commercial capital markets deteriorate. Unlike the private sector, which takes total indebtedness into account, an IFI will set interest rates purely on the basis of the debt owed to it; this makes it more worthwhile for a sovereign to borrow some of the funds that it requires from an IFI when times are bad. In relative terms, IFI lending becomes cheaper for the sovereign. From a theoretical perspective, it is unclear what effects the possibility of borrowing from IFIs has on a country's default probability. On the one hand, it can be expected to rise given that this debt, too, must be redeemed, reducing future consumption possibilities and increasing the incentive to default outlined above. On the other, the incentive to default falls because the sovereign becomes more 'patient' and places a higher value on future consumption which is implied, however, by the assumption of a lower discount rate. In simulations that are calibrated to developing countries, the former effect appears dominant.

A question similar to that in Boz (2011) is taken up by Fink and Scholl (2011). Building a model with a more complex fiscal policy in line with Cuadra *et al.* (2010), conditionality is included differently, however. They assume that, in addition to the consumption smoothing argument, the government also obtains utility from wasteful government spending in order to generate some kind of deficit bias in line with what is discussed in Section 2.1. Building on Aguiar and Gopinath (2006) and Scholl (2009), IFI financing is modelled as the country's possibility of obtaining an exogenously emerging transfer when the economic situation is bad enough, that is, when productivity is below and the debt level above some exogenous threshold. Conditionality enters as a constraint on fiscal policy limiting the size of government spending relative to private consumption. The sovereign can decide whether or not to accept the transfer and limit wasteful spending. The general findings are similar to those in Boz (2011) further showing that the presence of bailouts may generate more volatile interest rates. Additionally, the paper points to the possibility that IFI financing may generate a vicious circle in the form that there may be an incentive to over borrow in 'bad enough' economic situations in order to push the debt level beyond the threshold to obtain the transfer. Furthermore, their model allows us to rationalize why completion rates of conditional bailouts are low, which is shown, for example, by Mussa and

Savastano (2000) or Bird (2002). The sovereign will exit the bailout programme whenever his expected utility value of increasing wasteful spending is sufficiently high – even though the country will, then, temporarily be in autarky and the citizens' welfare may be decreased. The model is calibrated to Argentina and successfully mimics the empirical duration and frequency of bailout programmes.

As the recent debate in the euro area shows, issues related to assumptions of joint liability are no longer relevant solely to developing countries. This model class certainly gives room for expansions by, for example, including the funding of the financing organizations and relating them to euro area countries.

5. Integration of the Approaches into a RBC Model

To compare present value functions in the above-mentioned models – which inform the government's decision on whether to default – non-linear solution methods are needed. It is therefore difficult or even impossible to incorporate these decisions into the conventional, often larger scale DSGE models used by many institutions. Even where non-linear methods can be used, the specific solution method applied plays a role – at least from a quantitative point of view. This is illustrated by Hatchondo *et al.* (2010) in a rather technical analysis. The most commonly used solution method is the discrete state space technique (see Section 3.2), although this requires a large predetermined state space to be assumed in order to avoid erroneous interest rate movements. In simulations, however, this technique is comparatively inefficient, time intensive and – very importantly – prone to the parameterization of the grids. For example, Hatchondo *et al.* (2010) have shown that the correlation between spread and income reported in Aguiar and Gopinath (2006) may be reversed when using more accurate solution methods or significantly extending the state space. This, of course, casts doubt on some numerical solutions obtained. Other solution methods (such as Chebyshev polynomials or cubic splines) seem less vulnerable to parameterization or the choice of state space and, in some cases, allow more rapid calculation. As the discussion of these aspects is very technical, readers who would like more detailed information should refer to Hatchondo *et al.* (2010) and Zhu and Xie (2011).

Although it is difficult to transfer the modelling of a government's endogenous decision to conventional DSGE models, it does appear necessary – particularly in the current situation – to develop more complex default models so as to be able to replicate relevant current and future issues, such as restructuring as compared with an assumption of joint liability, adequacy of borrowing limits, etc. In particular, it would appear desirable to model the repercussions of a sovereign default on the domestic economy, which are, in most cases, built into the aforementioned models as an exogenous decline in domestic output. Mendoza and Yue (2011) present a model that can be viewed as an initial step in this direction. This model transcends the separation between RBC models, in which default risk is assumed to be an exogenous process, and the sovereign default models described here, which assume an exogenous output process. They combine the literature on 'sudden stops' – in which certain events can occur within a RBC model that lead to a sudden stop in the external financing possibilities of a country/sovereign – with the type of model discussed above, in which the sovereign decides whether or not to default.¹⁵

In their model, Mendoza and Yue (2011) assume that firms producing final goods do so using working capital and labour. Working capital consists of a combination of domestic and imported intermediate goods, which are imperfect substitutes. Imported intermediate goods must be paid for before production begins; this is funded, at least in part, through foreign loans. In the event of a sovereign default, it is assumed that the production sector also experiences financing difficulties on the capital markets (i.e. it loses access too), which implies that it can no longer purchase imported intermediate goods. Although imported intermediate goods can be replaced by domestic ones, they are imperfect substitutes, and this leads to a loss in efficiency. Unlike in the literature discussed above,

the default probability and the level of domestic productivity therefore interact. Lower productivity affects default probability positively as in the aforementioned models. Additionally, it now influences financing conditions negatively, that is, it generates larger spreads, including for the private sector. This, in turn, has negative repercussions for domestic productivity, more sharply increasing the incentives for a sovereign default. Because the intermediate products are modelled as Armington CES aggregators, the loss in output following a government default in this model is an endogenous, sharply rising and convex function of the productivity shock. Ultimately, this means that larger shocks have a greater impact on the default probability than small shocks and that output dynamics and default risk are determined jointly in this model. All of the previously discussed literature either assumes that productivity shocks have symmetric effects on output (and thus, implicitly, quasi-symmetric effects on default probability) or installs *ad hoc* asymmetries which are not explained endogenously to the model.¹⁶

In a recent paper, Padilla (2012) has taken up the approach just discussed to include a banking sector. In his paper, banks provide working capital loans to the private sector and also purchase government bonds. Whenever the government defaults, the banks' ability to lend to the private sector contracts due to the lack of repayment by the government. This triggers a 'credit crunch' which translates to a decline in output due to the decrease in working capital loans. Padilla (2012) assumes the banking sector to be 'quasi-domestic' (we will qualify this below). On the one hand, this acknowledges the high prevalence of government debt in financial intermediaries' balance sheets and illustrates how a sovereign default diminishes credit availability in the economy. On the other hand, the fact that banks are domestic makes the government consider the 'utility' of bankers when taking its default decision. All this makes the model less tractable, in general, and complicates generating equilibria with endogenous defaults. The main trick in solving this problem is to assume that banks receive an exogenous endowment each period to be able to provide at least some credit at all times, even when the government does not comply with its repayment obligations. This imposes the most important characteristics of international investors, who are usually assumed to provide loans to the government, on the domestic banking sector. They could, therefore, also be seen as being 'quasi-foreign'. The pricing kernel is analogous to the one present in the literature.

As Mendoza and Yue (2011) point out, adding capital makes the recursive contract with default option significantly harder to solve because it adds an additional endogenous state variable. Moreover, changes in the capital stock have been estimated to play a small role in output dynamics around financial crises (see Meza and Quintin, 2005; Mendoza, 2010). The relevance of capital accumulation in RBC/DSGE models and a recent paper by Bai and Zhang (2012), nevertheless, suggest that including capital may indeed be helpful for further developments. They develop a multi-country model where each country produces a homogeneous good that can be either consumed or invested, very much in line with the conventional RBC model. Additionally, they assume, very much in line with the literature discussed in this paper, that only non-contingent bonds with limited enforcement can be traded between financial intermediaries and countries. Financial intermediaries are assumed to be risk-neutral international investors, and countries can default on their debt. Furthermore, they include a resource cost for countries to enter international financial markets. This is supposed to reflect the degree of financial integration. The higher this resource cost is, the less integrated is the corresponding country. They find that international liberalization does not lead to an increase in international risk sharing, in terms of consumption smoothing, due to the fact that debt contracts are incomplete and repayment cannot be enforced. The presence of default risk makes borrowing more difficult, which also limits capital investment necessary for improving the economic situation. This hampers consumption smoothing possibilities. A decrease in the resource cost to enter international financial markets alleviates these frictions only to a small extent. Their paper thus gives a rationale for why the empirical literature on financial liberalization does not find as big an increase in risk sharing as would be predicted by the conventional RBC literature on the issue (see Kose *et al.*, 2009, for a discussion). Whereas Bai and Zhang (2012) focus is on the effects of financial liberalization on international risk sharing, Peña

(2011) assesses the ability of such a model to reproduce real business cycle properties of an economy that suffers from default risk. She also asks whether the inclusion of capital helps to reproduce the output dynamics around a default. As Peña (2011) shows, the model does this from a qualitative point of view. Nevertheless, it has difficulties in generating sufficiently high default rates and variability in sovereign spreads and consumption. The reason is that, by including capital, the economy's ability to smooth consumption without the help of international investors is improved. Still, the model helps to close the gap between the data, where default episodes are associated with significant downturns, and using models with exogenously given output processes, because it is able to reproduce these facts qualitatively.

While still stylized and – at least partly – assumption driven, the models discussed in this section are able to capture the feedback effects between default and output losses more explicitly. More specifically, they are able to illustrate three stylized facts: (1) the V-shaped dynamics of output around default events, (2) the negative correlation between interest rates on sovereign debt and output and (3) the disproportionately high government debt ratio when defaults take place. Conventional RBC/DSGE models are usually unable to explain points (1) and (3) given that sovereign defaults, if included at all, follow an exogenous process, whereas most of the literature discussed in this article cannot explain point (2), because the output process is modelled exogenously. Therefore, the papers discussed here represent a promising approach to expanding such models so that they encompass the conventional RBC approach and the conventional approach of the sovereign default literature.

6. Conclusions

Discussions about sovereign defaults will probably take on new dimensions in the course of the current crisis and are unlikely to remain restricted to developing countries. Expanding theory-based analytical tools such as RBC or DSGE models, which are used in larger or international institutions to investigate economic relationships or simulate certain political measures so that they can model sovereign defaults, would therefore be a welcome development. Formalizing aspects such as contagion effects, assumptions of joint liability, suitable insolvency legislation etc. could provide a useful contribution to the prospective debate on these issues.

Sovereign default literature – whose findings should at least be acknowledged in the current situation – came into being, above all, in response to government crises in a number of Latin American countries. The modelling techniques developed in this context could be helpful for refining the models that have been used until now.

From the literature discussed in this paper, it can be concluded that (1) despite the absence of international insolvency legislation, sovereigns have an interest in repaying debt owing to reputational damage and negative output effects, both aspects, however, included by assumption; (2) in the event of transitory shocks, debt level is the key factor determining the probability of default; in the event of trend shocks, the output level also plays a decisive role; (3) the political stability of a country reduces its probability of default; (4) bailouts or the possibility of borrowing from international financial institutions, such as the IMF, can increase the probability of default; (5) fiscal policy acts procyclically if it takes the probability of default and the impact on interest rates into account; (6) taking private sector debt into account does not change the above-mentioned qualitative statements, but can increase a government's propensity to borrow (in relative terms); (7) endogenising negotiations on debt recovery rates tends to increase the interest rates on sovereign debt, but a government's propensity to borrow can still rise; (8) it can be efficient for borrowers and creditors to delay negotiations on restructuring sovereign debt, although this is heavily dependent on the assumed bargaining game; (9) long-term bonds usually have a higher rate of interest than short-term ones; (10) contagion effects owing to risk-averse investors as well as information shocks affecting investors that impact on the interest rate can

even cause fundamentally sound governments to default and (11) the formal method of solving these models also plays a role in quantitative terms. These findings are results of the quantitative models reviewed in this paper. It should be noted here that, so far, not all the findings have resilient empirical support.

With respect to expanding the models used by larger or international institutions to include the aspect of sovereign defaults and approaches from the literature described above, the work of Mendoza and Yue (2011), in particular, lends itself to combining the RBC models with the default structure discussed in this paper. Expanding their model to include a multi-country setup as in Lizarazo (2009) would doubtless produce interesting findings regarding contagion effects or the design of a stabilization mechanism. Incorporating an information shock into such a model, as in Durdu *et al.* (2010), could also make the model easier to calibrate to the euro area. Financial markets then also look at other information alongside economies' fundamentals when setting interest rate spreads. Aguiar and Gopinath (2006) show how bailouts can be integrated into such systems, Hatchondo *et al.* (2010) incorporate a stylized type of collective action clause, Boz (2011) allows for international financing institutions' lending and, aspects of debt restructuring can be analysed using the proposed modelling approaches in Bi (2008), Benjamin and Wright (2009), D'Erasmus (2010) and Yue (2010). Also, the relationship between sovereign default risk and the banking system, for which the current European crisis provides fresh evidence, demands for further research, potentially along the lines of Padilla (2012). All in all, there should be a number of options for expanding this type of model so that it can be applied to the issues currently being debated in the euro area.

The extent to which all of this is actually feasible remains an open question, however, because the necessary solution methods are complex. The main difficulties are caused by the fact that the solution mechanisms used to date in DSGE models are linearized approximations of mostly very complex models which cannot really model a voluntary default. All in all, smaller models would probably need to be developed initially in order to analyse specific issues. The integration of nominal frictions and therefore monetary policy, as is usual in DSGE models, could also cause complications or perhaps be intractable. Nonetheless, this would allow analysing issues related to currency mismatches or comparing costs of monetization and default, both of which are important areas of future work. All in all, the author considers it worthwhile to pursue this course of action, as the sovereign default literature provides an informed basis for incorporating sovereign defaults into dynamic macro models.

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Notes

1. The following is a non-exhaustive list of fiscal policy issues that can be cited as examples: the effectiveness of fiscal stimulus programmes (see inter alia Almeida *et al.*, 2010; Coenen *et al.*, 2010, 2012; Hebous, 2011), a cost-benefit analysis of various consolidation measures (see inter alia Freedman *et al.*, 2009; Stähler and Thomas, 2012) and an evaluation of structural changes to tax legislation (see inter alia Coenen *et al.*, 2008).
2. Still, we should bear in mind that this literature abstracts from a situation in which, for example, a country, which is generally able and willing to repay debt, cannot do so because it is not able to rollover a sufficiently large amount of debt in some period due to self-fulfilling debt runs (see Cole and Kehoe, 2000).

3. This literature and the findings thereof are examined in depth in Mendoza (2006, 2010) and are therefore not described in detail in this paper.
4. For information on these aspects, see the discussions in Sargent and Wallace (1981), Kocherlakota and Phelan (1999), Leeper and Yun (2006) and McCallum and Nelson (2006).
5. If capital market effects play only a limited role in debt repayment, the debtor country may fear the imposition of sanctions by the creditor countries. These include political – or even military – pressure, imminent, or actual seizure of assets as well as constraints on trade relations. While military actions have in fact been carried out in the past, these do not seem likely today – at least in developed economies. Likewise, the threat of sovereign assets being seized is not expected (at least directly) to be a major incentive to repay debts (for a discussion of these aspects, see Mitzchner and Weidenmier, 2005; Tomz, 2007). However, a negative impact on trading has actually been identified, albeit not always for long periods of time (see Rose, 2005; Lanau, 2008; Borensztein and Panizza, 2010). Whether constraints on trade relationships *per se* are a reason to repay debts remains to be seen. Yet a debtor country's export-oriented companies are especially affected, a fact which could also be included in the domestic effects discussed below.
6. At this juncture it appears appropriate to note that there are now alternative approaches to the endogenization of risk premiums in larger dynamic macro models. However, these are less concerned with the sovereign's decision that it is not willing to repay debt. Instead, they assume that creditors believe that governments are no longer able to repay debt as of a certain debt ratio. As of this debt level, households are no longer willing to purchase government debt. The key term here is 'fiscal limits'. Basically, in this branch of the literature it is assumed that the sovereign credibly defines a certain path of primary surpluses, which then calculates the default probability to ensure that the transversality condition continues to hold. The default probability then generally determines how much of the outstanding government debt is repaid and can also be interpreted as an 'endogenous haircut'. A fiscal rule still ensures that long-term equilibrium is guaranteed, but now also takes the default probability into consideration. As this model class is not the main focus of the present paper, see Bi *et al.* (2010), Juessen *et al.* (2010) or Corsetti *et al.* (2012) for more details.
7. Of course, these issues were also pursued non-quantitatively (see, e.g. Bulow and Rogoff, 1989, as well as Cole *et al.*, 1995 and Cole and Kehoe, 1998, 2000 who also include the possibility of sunspot equilibria). However, as this paper focuses on more recent quantitative models, we do not discuss this point in detail.
8. Here, consumption equivalents as defined by Lucas (1987) would be interpreted as the percentage of steady-state consumption that households are prepared to surrender to enter a non-default equilibrium.
9. Hamann (2002) is an earlier work that, in principle, applied the same main idea of the model setup and similar solution methods but which has not yet gained too much attention.
10. This is also due to the fact that – as described in the base model (Lizarazo, 2010) – the incentives of a country with low output and/or a high level of debt to default rise more sharply than that of a higher output, lower debt country because the utility function of the private sector is concave. This behaviour is therefore rational from the investor's point of view.
11. The higher the outstanding sovereign debt of a defaulting country, the greater is the loss that creditors, as a whole, will suffer in the event of a total default. As they negotiate repayment with the defaulting sovereign, they accept a lower debt recovery rate when government debt is higher, although the recovery amount itself increases with rising debt. Ultimately, this is a composition effect resulting, among other things, from the Nash bargaining solution.
12. Merlo and Wilson (1995, 1998) use a game with complete information. It can be shown that, for this reason, a stationary sub-game perfect equilibrium implies that a proposal is only submitted if the other player accepts it. The formal solution is thus provided by the fixed point of a system of equations consisting of the relevant present value functions of propose/pass and accept/reject and of the recovery rates (selected as a proposal) resulting from maximization.

13. This can also be interpreted as if the debt issued by the government consisted of a portfolio of zero-coupon bonds of different maturities, where the portfolio weights decline geometrically with maturity. If the rate is equal to one, the entire weight is on the one-period bond.
14. Relatively higher consumption in 'bad times' results because the financial burdens rise disproportionately if debt is short term. In good times, the generally higher interest rate on long-term bonds can lead to a decline in consumption. Owing to falling marginal returns for consumption, however, higher consumption in bad times – at least up to a certain threshold for the maturity of the bonds – is the dominant concern from a wealth perspective.
15. For more information on the basic structure of these models, see Mendoza (2010). Mendoza (2006, 2010) provides an overview of the findings of this literature. As these models essentially fall into a different category, a detailed explanation of their workings would extend far beyond the scope of this paper; readers interested in learning more should refer to the aforementioned articles for an initial grounding in the subject.
16. It should be noted that, although output is endogenous (as was the case in previous work mentioned in the text, such as Aguiar and Gopinath, 2006, or Cuadra *et al.*, 2010 and Fink and Scholl, 2011, among others), the link between default and output in the setup by Mendoza and Yue (2011) is still not explained in a completely endogenous way, simply because in their model, when the sovereign defaults, it is assumed that the firms lose access to capital markets and thus cannot buy imported inputs to produce. Therefore, the model is still having an exogenous mechanism linking default and the performance of the private sector. Having such a link modelled in a more endogenous way would be a step forward in the literature.

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