Business Cycles & Monetary Policy

Session 5: Real Business Cycle Theory

ECO5021F: Advanced Macroeconomics
University of Cape Town

What will we do today?

Today we will discuss a model of short-run variations in aggregate output and employment.

Required reading #1

- Romer, D. (2019). **Chapter 5** (Real Business Cycle Theory)
- Note that this is an extensive chapter with many technical variations.
- Romer uses somewhat different structure and notation to focus on the analytical solution to such models.
- We will use a slightly different model and notation.
- Read the chapter with an overview in mind, to learn about the history and motivation of the models as well as the various variations in real (as opposed to nominal) features of the basic Walrasian real business cycle approach.

What will we do today?

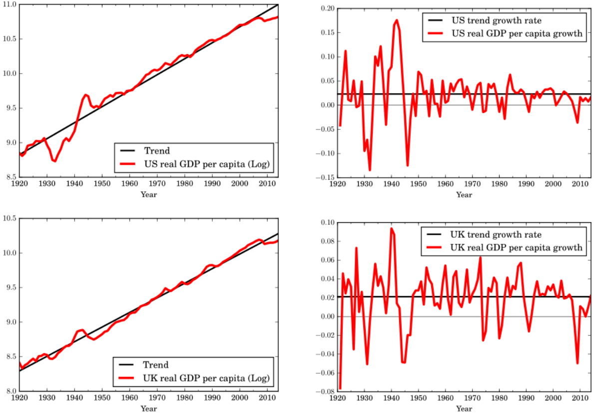
Required reading #2

- Kehoe, Midrigan and Pastorino (2018) Evolution of modern business cycle models: accounting for the Great Recession
- This is a very nice overview of the development of these models and how new adjustments manage to account for significant features of the Great Recession that followed the global financial crisis of 2008.
- Again, read with attention to get a feel of how these models are motivated and analysed.
- Focus on what the researchers want to achieve and how and to what extent each assumption changes the results in the required direction.
- This will be useful to build your intuition on how these models are conceived and updated with new features . . . and to not just swallow the critiques from ignoramuses or even notable academics!

Motivation

We want to understand:

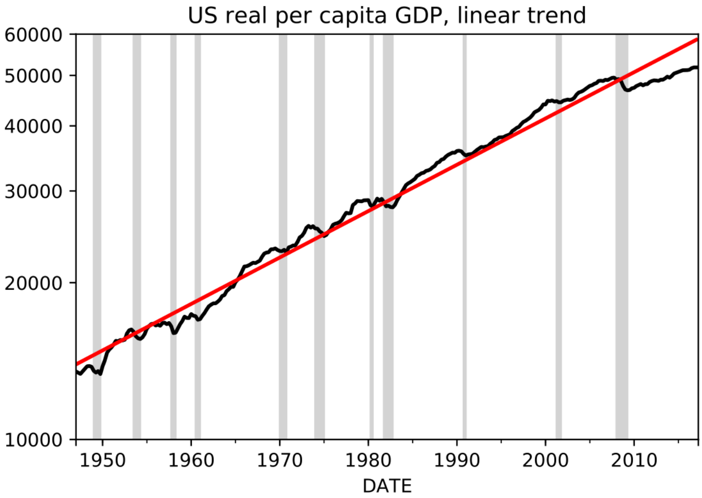
Long-run growth　　　　Business cycles



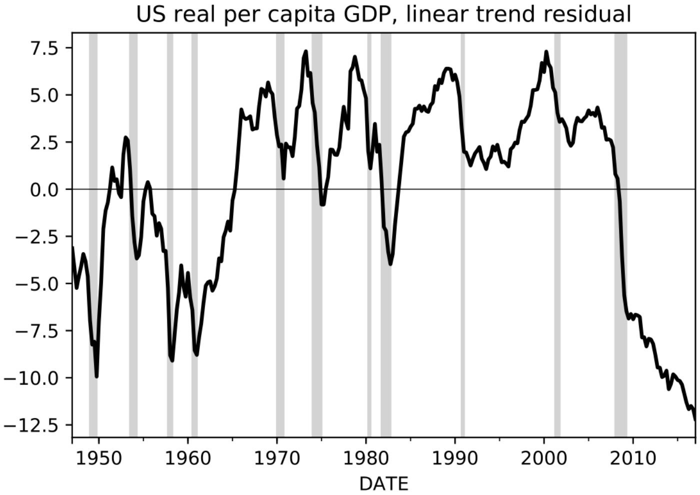
using tools of modern macroeconomic research

– microfounded general equilibrium models

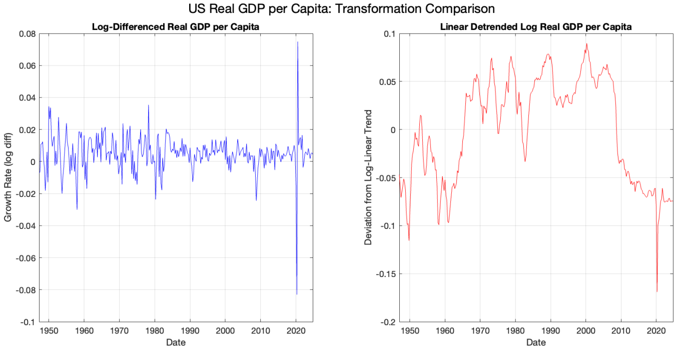
Trend vs Cycle: linear



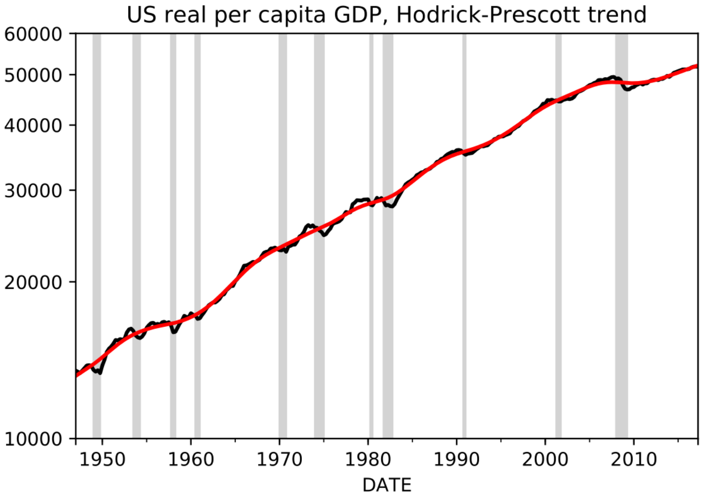
Trend vs Cycle: linear



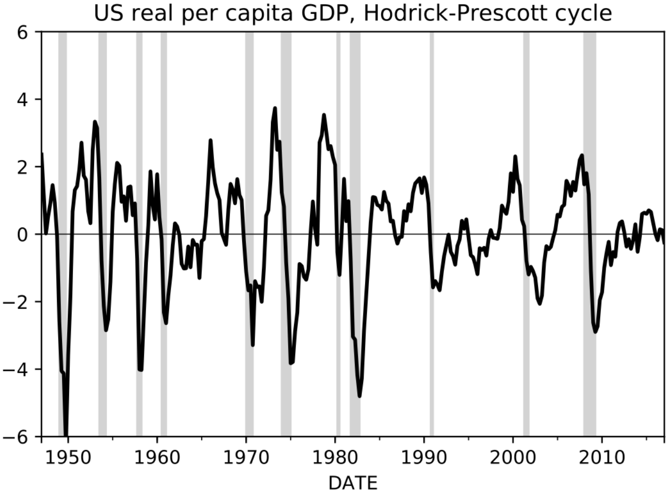
Trend vs Cycle: linear detrend



Trend vs Cycle: HP Filter



Trend vs Cycle: HP Filter



Hodrick-Prescott Filter

- This next section is useful for your tutorial (tbc)
- The Hodrick-Prescott filter is one of the most frequently used filters in macroeconomics
- Considered to be a technique to obtain a **smoothed estimate** of the long term trend
- Let y_t denote the logarithm of a time series variable, for $t = 1, 2 \ldots, T$
- The series is then made up of trend component, τ_t and a cyclical component c_t, such that $y_t = \tau_t + c_t$, assuming some irregular (noise) component $\varepsilon_t \sim \mathcal{N}(0, \sigma)$
- Given a chosen value of λ there is a trend component that will solve the following minimisation problem

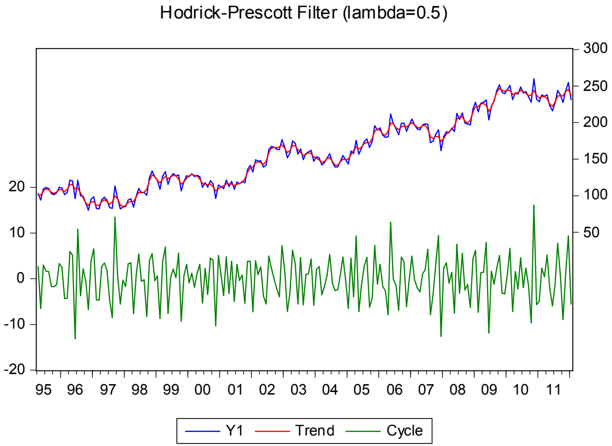
Hodrick-Prescott Filter

- The minimisation problem is,

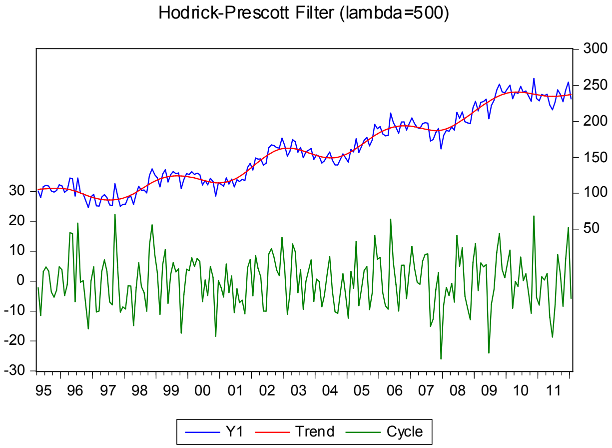
$$\min_{t}\left(\sum_{t=1}^{T}(\underbrace{y_t-\tau_t}_{c_t})^2+\lambda\sum_{t=2}^{T-1}\left[(\tau_{t+1}-\tau_t)-(\tau_t-\tau_{t-1})\right]^2\right)$$

- First term is the sum of the squared deviations from the trend, **penalises movements away from the trend** (cyclical component)
- Second term is a multiple, λ, of the sum of the **squares of the trend components second differences**
- This term penalises variations in the growth rate of the trend component
- Higher value of $\lambda \to$ higher penalty
- All you really have to know here is that λ controls the smoothness of the series estimate (i.e., λ penalises the trend for 'non-smoothness')
- If $\lambda \to \infty$: **straight line**
- If $\lambda \to 0$: **trend will lie on the series itself** because $y_t = \tau_t$ solves the minimisation problem

HP Filter (two-sided)



HP Filter (two-sided)



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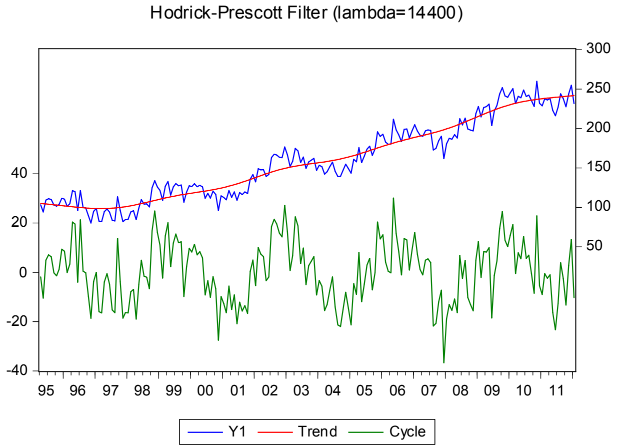
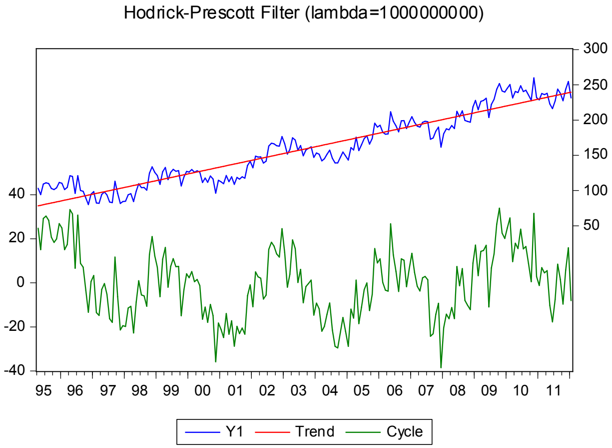


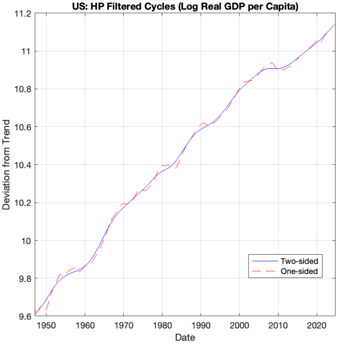
HP Filter (two-sided)

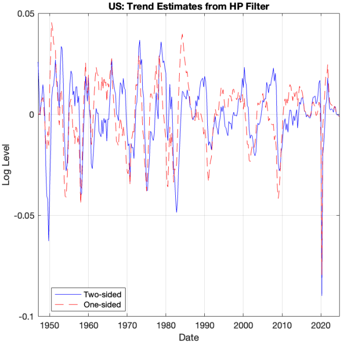


Drawbacks to the HP filter

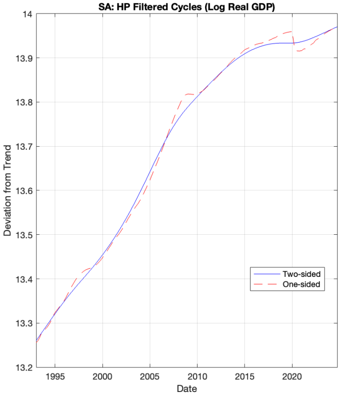
- **Remark 12** (Non-Causal Filters): Thou shalt not use a non-causal, i.e. two-sided or Baxter-King, filter
- See Section 4.1 in Pfeifer (2024): A Guide to Specifying Observation Equations for the Estimation of DSGE Models
- Hamilton vs. Hodrick
- When to Use the Hodrick-Prescott Filter

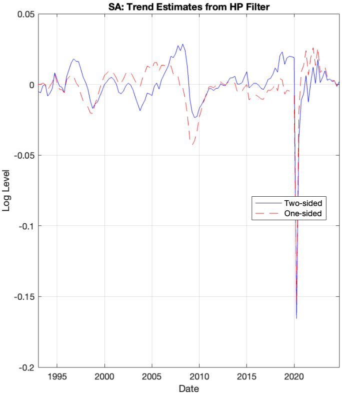
One-side vs. Two-sided HP filter





One-side vs. Two-sided HP filter



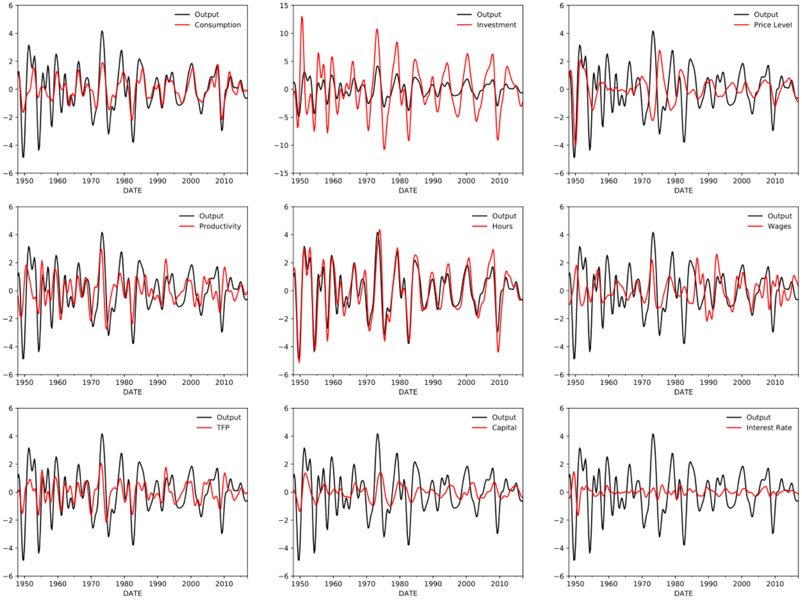


Goal of RBC models

- The goal of RBC models is to understand the sources and nature of aggregate fluctuations
- Before the theories are presented, we will identify some of the major facts about short-run fluctuations.

1. Fluctuations do not exhibit any simple regular or cyclical pattern
   - Movements **not regular** $\rightarrow$ economy is perturbed by disturbances of various types and sizes at random intervals which propagate through the economy
   - Major schools of economic thought differ on sources of shocks and propagation mechanisms
2. Fluctuations are distributed unevenly over components of output
3. Asymmetries in output movement
4. Magnitude of fluctuations change over time.

Business cycle facts: USA 1948-2016



Business cycle facts: USA 1948-2016

Here are some observations about the business cycle in the US.

- Consumption is coincident, procyclical and less volatile than output
- Investment is coincident, procyclical and more volatile than output
- Price level can be procyclical and countercyclical
- Productivity and TFP are both procyclical and leading output
- Hours are slightly more volatile than output with 1-2 quarters lag
- Real wage is procyclical when price level is countercyclical and countercyclical when price level is procyclical
- Capital stock is procyclical, mildly volatile and lags output
- Real interest rates are acyclical and the least volatile

Ideally, we would like our model to replicate these movements.

Business cycle facts: USA 1948-2016

		Std. Dev.	Rel. S. D.	Corr. w. y	Autocorr.
Output	y	1.65	1.00	1.00	0.85
Consumption	c	0.87	0.53	0.78	0.82
Investment	i	4.54	2.75	0.76	0.87
Capital	k	0.60	0.36	0.41	0.95
Hours	h	1.94	1.17	0.88	0.91
Interest rate	r	0.39	0.24	0.02	0.40
Wage	w	0.95	0.58	0.10	0.68
TFP	z	0.85	0.51	0.51	0.73
Productivity	$\frac{y}{h}$	1.07	0.64	0.40	0.71
Price level	P	0.90	0.55	-0.11	0.91

Early RBC research

- Early research extended Ramsey model to incorporate aggregate fluctuations, which requires two modifications.
  1. Introduce source of disturbances $\rightarrow$ without shocks model goes to BGP and grows smoothly
  2. Allow for variations in employment (i.e. introduce a labour market)
- Two extensions suggested for the source of disturbances: (i) shocks to technology and (ii) changes in government purchases
- Both of these shocks represent **real** disturbances!
- In order to allow for employment variations we can include hours worked in the households' utility function
  - Employment given by the intersection of labour demand and supply

Baseline RBC model (production)

- We will use a discrete-time variation of the Ramsey model.
- Production function is Cobb-Douglas,

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}, \quad 0 \leq \alpha \leq 1$$

- Capital stock in period $t+1$ is given by,

$$\begin{aligned} K_{t+1} &= K_t + I_t - \delta K_t \\ &= K_t + Y_t - C_t - G_t - \delta K_t, \quad \text{where} \quad Y_t = C_t + I_t + G_t \end{aligned}$$

- Labour and capital are paid their marginal products,

$$w_t = (1-\alpha)\left(\frac{K_t}{A_t L_t}\right)^{\alpha} A_t$$

$$r_t = \alpha\left(\frac{A_t L_t}{K_t}\right)^{1-\alpha} - \delta$$

Baseline RBC model (consumption)

- Representative household maximises expected value of

$$U = \sum_{t=0}^{\infty} e^{-\rho t} u(c_t, 1 - l_t) \frac{N_t}{H} d_t$$

  where $u(\bullet)$ is the instantaneous utility function, ρ is the discount rate, N_t is population and H is the number of households.
- Population grows exogenously at rate n: $\ln N_t = \bar{N} + nt$.
- Two arguments in the utility function $u(\bullet)$
  - Consumption per member of household, c_t
  - Leisure per member $(1 - l_t) \rightarrow$ difference between time endowment (1) and amount each member works (l_t).
- Specific functional form is **log-linear**: $u_t = \ln c_t + b \ln(1 - l_t)$.

Baseline RBC model (driving variables)

- Final assumption of the model concerns the behaviour of the driving variables: (i) technology and (ii) government purchases.

- In the **absence of shocks**, trend growth in technology, $\ln A_t$, would be $\bar{A} + g_t$. Likewise, trend growth in per capita government purchases, $\ln G_t$, would be $\bar{G} + (n + g)t$.

- We can, however, assume that technology (and/or government purchases) are subject to random disturbances:

$$\ln A_t = \bar{A} + g_t + \tilde{A}_t$$

where $\tilde{A}$ reflects departures from the trend and is assumed to follow an **AR(1)** process. In other words,

$$\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \varepsilon_{A,t}, \quad -1 \leq \rho_A \leq 1$$

where $\varepsilon_{A,t}$ are white noise disturbances. The equation shows that the effect of shocks disappear gradually over time.

Household behaviour

- For this section we will evaluate household behaviour at differing levels of complexity.
- We will consider three different cases.
  1. **Case 1**: Household lives for one period
  2. **Case 2**: Household lives for two periods
  3. **Case 3**: Household lives for two periods and there is uncertainty
- We will then extend this to a more general case, which is quite tricky.

Household behaviour: Case 1

- The budget constraint for this case is simple. Since the household (one member) does not save (i.e., no wealth), we have $c_t \leq w_t l_t$.
- This means that the optimisation problem is,

$$\mathcal{L} = \ln c_t + b\ln(1 - l_t) + \lambda_t(w_t l_t - c_t),$$

  where the household chooses c_t and l_t to maximize utility.
- First order conditions for this problem are then,

$$\mathcal{L}_c : \frac{1}{c_t} - \lambda_t = 0$$

$$\mathcal{L}_l : -b\left(\frac{1}{1 - l_t}\right) + \lambda_t w_t = 0$$

$$\mathcal{L}_\lambda : w_t l_t - c_t = 0$$

- Combining the first two equations, we have the following:

$$\frac{1}{l_t} = b\left(\frac{1}{1 - l_t}\right)$$

- Labour-leisure choice is independent of wage; only depends on the weight of leisure, b, in utility.

Household behaviour: Case 2

- Household lives for two periods
- Continue to assume that it has no initial wealth and that it has only one member
- In addition, assume that there is no uncertainty about the interest rate or the second-period wage.
- Budget constraint for this problem is now,

$$c_t + \frac{1}{1+r_{t+1}} c_{t+1} = w_t l_t + \frac{w_{t+1} l_{t+1}}{1+r_{t+1}}$$

- The Lagrangian for this problem is (note: $\beta = e^{-\rho}$),

$$\mathcal{L} = \ln c_t + b \ln(1-l_t) + \beta[\ln c_{t+1} + b \ln(1-l_{t+1})] +$$
$$\lambda_t \left(w_t l_t + \frac{w_{t+1} l_{t+1}}{1+r_{t+1}} - c_t - \frac{1}{1+r_{t+1}} c_{t+1} \right)$$

where the four choice variables are c_t, c_{t+1}, l_t and l_{t+1}.

Household behaviour: Case 2

- First order conditions for this problem are:

$$\mathcal{L}_{c_t} : \frac{1}{c_t} - \lambda_t = 0$$

$$\mathcal{L}_{c_{t+1}} : \beta \frac{1}{c_{t+1}} - \lambda_t \left(\frac{1}{1 + r_{t+1}} \right) = 0$$

$$\mathcal{L}_{l_t} : -b \left(\frac{1}{1 - l_t} \right) + \lambda_t w_t = 0$$

$$\mathcal{L}_{l_{t+1}} : -b\beta \left(\frac{1}{1 - l_{t+1}} \right) + \lambda_t \frac{w_t}{1 + r_{t+1}} = 0$$

- Combining the first two FOCs we get the **Euler equation**,

$$\frac{1}{c_t} = \beta(1 + r_{t+1}) \frac{1}{c_{t+1}}$$

- The third and fourth FOCs can be combined to form,

$$\frac{(1 - l_t)}{(1 - l_{t+1})} = \frac{1}{\beta(1 + r_{t+1})} \frac{w_{t+1}}{w_t}$$

- This equation shows that households will substitute labour across time.
- Response of labour supply to relative wages and interest rates is known as the **intertemporal substitution** in labour supply.

Household behaviour: Case 3

- For the third case there are two general approaches. Romer uses one method, I will stick to Lagrangians.

$$\mathcal{L} = \ln c_t + b\ln(1-l_t) + \beta\mathbb{E}_t[\ln c_{t+1} + b\ln(1-l_{t+1})] +$$
$$\lambda_t\left(w_t l_t + \mathbb{E}_t\left[\frac{w_{t+1}l_{t+1}}{1+r_{t+1}}\right] - c_t - \mathbb{E}_t\left[\frac{1}{1+r}c_{t+1}\right]\right)$$

- From the first order conditions, the Euler equation will be,

$$\frac{1}{c_t} = \beta\mathbb{E}_t\left[(1+r_{t+1})\frac{1}{c_{t+1}}\right]$$

- Cannot seperate the two terms on the right-hand side because c_{t+1} and r_{t+1} might be correlated.
- Combining FOCs for labour supply and consumption we get,

$$\frac{c_t}{(1-l_t)} = \frac{w_t}{b}$$

- Interesting to note that uncertainty does not enter into the labour choice.

Special case of the model

- The model we constructed cannot be solved analytically. So we will look at a **simplified version** of the model.
- Two changes: (i) eliminate government and (ii) 100% depreciation
- Evolution of capital stock and real interest rate become,

$$K_{t+1} = Y_t - C_t$$

$$1 + r_t = \alpha \left(\frac{A_t L_t}{K_t} \right)^{1-\alpha}$$

- To solve this type of model I will show you a method in class.

Special case of the model

- From the new method we determined that the following holds,

$$\frac{1}{c_t} = e^{-\rho}\mathbb{E}_t\left[\frac{1+r_{t+1}}{c_{t+1}}\right]$$

- Since $c_t = (1-s_t)Y_t/N_t$, we can rewrite this as,

$$\begin{aligned}
\frac{1}{(1-s_t)Y_t/N_t} &= e^{-\rho}\mathbb{E}_t\left[\frac{1+r_{t+1}}{(1-s_{t+1})Y_{t+1}/N_{t+1}}\right] \\
-\ln(1-s_t)Y_t/N_t &= -\rho + \ln\mathbb{E}_t\left[\frac{1+r_{t+1}}{(1-s_{t+1})Y_{t+1}/N_{t+1}}\right] \\
-\ln(1-s_t)Y_t/N_t &= -\rho + \ln\mathbb{E}_t\left[\frac{\alpha Y_{t+1}}{(1-s_{t+1})K_{t+1}Y_{t+1}/N_{t+1}}\right] \\
-\ln(1-s_t)Y_t/N_t &= -\rho + \ln\mathbb{E}_t\left[\frac{\alpha N_{t+1}}{(1-s_{t+1})s_t Y_t}\right]
\end{aligned}$$

- Finally this gives us,

$$\begin{aligned}
&-\ln(1-s_t) - \ln Y_t + \ln N_t \\
&= -\rho + \ln\alpha + \ln N_t + n - \ln s_t - \ln Y_t + \ln\mathbb{E}_t\left[\frac{1}{1-s_{t+1}}\right]
\end{aligned}$$

Special case of the model

$$\ln s_t - \ln(1 - s_t) = -\rho + \ln \alpha + n + \ln \mathbb{E}_t \left[\frac{1}{1 - s_{t+1}} \right]$$

- **Important**: The two state variables, A and K don't enter this equation (i.e. can get growth path without them)
- If s is constant at some value $\hat{s}$, then s_{t+1} is not uncertain and $\ln \mathbb{E}_t [1/1 - s_{t+1}]$ simply becomes $1/(1 - \hat{s})$.

- The result is that $\ln \hat{s} = \ln \alpha + n - \rho$ or $\hat{s} = \alpha e^{n-\rho}$.
- Model has a solution where the savings rate is constant.

Special case of the model

- From the new method we also determined that $c_t/(1-l_t) = w_t/b$.
- Since $c_t = C_t/N_t = (1-\hat{s})Y_t/N_t$, we can write

$$(1-\hat{s})Y_t/N_t/(1-l_t) = w_t/b$$
$$\ln(1-\hat{s})Y_t/N_t - \ln(1-l_t) = \ln w_t - \ln b$$

- Since the production function is Cobb-Douglas, we have that $w_t = (1-\alpha)Y_t/(l_t N_t)$, which yields,

$$\ln l_t - \ln(1-l_t) = \ln 1 - \alpha + \ln Y_t - \ln(1-\hat{s}) - \ln b$$

- Rewriting this, we get,

$$l_t = \frac{1-\alpha}{(1-\alpha) + b(1-\hat{s})}$$

which indicates to us that $l_t = \hat{l}$ is also a constant.

Special case of the model

- We need to assess output fluctuations from the dynamics of technology and behaviour of capital stock.
- If we take the natural logarithm of the production function, $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$ we have,

$$\begin{aligned}
\ln Y_t &= \alpha \ln K_t + (1-\alpha)(\ln A_t + \ln L_t) \\
&= \alpha \ln \hat{s} + \alpha \ln Y_{t-1} + (1-\alpha)(\ln A_t + \ln \hat{l} + \ln N_t) \\
&= \alpha \ln \hat{s} + \alpha \ln Y_{t-1} + (1-\alpha)(\ln A_t + \ln \hat{l} + \ln N_t) \\
&= \alpha \ln \hat{s} + \alpha \ln Y_{t-1} + (1-\alpha)(\bar{A} + gt + \tilde{A} + \ln \hat{l} + \bar{N} + nt)
\end{aligned}$$

- We can define a new variable $\tilde{Y}$ to be the deviation of the log of output $(\ln Y_t)$ from its "no shock" value.
- "No shock" value is the one that would occur if shocks were zero.

Special case of the model

- Decompose Y into a **trend and cyclical** component.
- We define $\bar{Y}_t$ to be the no-shock (trend) level of the log of output.
- This would mean that $\tilde{Y}_t \equiv \ln Y_t - \bar{Y}_t$ is the cyclical component.
- If all shocks were zero, then $\tilde{A}_t = 0$ and $\ln Y_{t-1} = \bar{Y}_{t-1}$.

- From this simplification (and the equation on the previous slide) we can get an equation for $\bar{Y}_t$.

$$\bar{Y}_t = \alpha \ln \hat{s} + \alpha \bar{Y}_{t-1} + (1-\alpha)(\bar{A} + gt) + (1-\alpha)(\ln \hat{l} + \bar{N} + nt)$$

- By extension we have an equation for $\tilde{Y}_t$, given by:

$$\begin{aligned} \tilde{Y}_t = \ln Y_t - \bar{Y}_t &= \alpha(\ln Y_{t-1} - \bar{Y}_{t-1}) + (1-\alpha)\tilde{A}_t \\ &= \alpha \tilde{Y}_{t-1} + (1-\alpha)\tilde{A}_t \end{aligned}$$

which is the equation in the textbook.

Special case of the model

- Since this equation holds each period it implies that

$$\tilde{Y}_t = \alpha\tilde{Y}_{t-1} + (1-\alpha)\tilde{A}_t$$
$$\tilde{Y}_{t-1} = \alpha\tilde{Y}_{t-2} + (1-\alpha)\tilde{A}_{t-1}$$

- This can then be rewritten in terms of $\tilde{A}_{t-1}$ as:

$$\tilde{A}_{t-1} = \frac{1}{1-\alpha}\left(\tilde{Y}_{t-1} - \alpha\tilde{Y}_{t-2}\right)$$

- We also know from our definition at the beginning that $\tilde{A}_t = \rho_A\tilde{A}_{t-1} + \varepsilon_{A,t}$. Combining these equations we have:

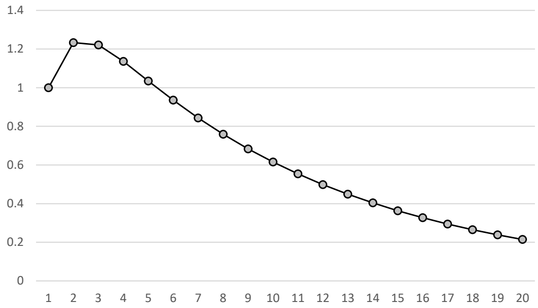
$$\begin{aligned}
\tilde{Y}_t &= \alpha\tilde{Y}_{t-1} + (1-\alpha)(\rho_A\tilde{A}_{t-1} + \varepsilon_{A,t}) \\
&= \alpha\tilde{Y}_{t-1} + (1-\alpha)\left[\frac{\rho_A}{1-\alpha}\left(\tilde{Y}_{t-1} - \alpha\tilde{Y}_{t-2}\right) + \varepsilon_{A,t}\right] \\
&= \alpha\tilde{Y}_{t-1} + \rho_A\left(\tilde{Y}_{t-1} - \alpha\tilde{Y}_{t-2}\right) + (1-\alpha)\varepsilon_{A,t} \\
&= (\alpha+\rho_A)\tilde{Y}_{t-1} - \alpha\rho_A\tilde{Y}_{t-2} + (1-\alpha)\varepsilon_{A,t}
\end{aligned}$$

- We see here that $\tilde{Y}_t$ can be written as a linear combination of two lags and a white noise disturbance term.

Special case of the model

$$\tilde{Y}_t = (\alpha + \rho_A)\tilde{Y}_{t-1} - \alpha\rho_A\tilde{Y}_{t-2} + (1-\alpha)\varepsilon_{A,t}$$

The combination of positive coefficient in the first lag and negative coefficient in the second lag can cause output to have a **hump-shaped** response to disturbances.



Special case of the model

- Despite output dynamics that seem to be acceptable, this special case does not match major features of fluctuations very well.
- Most obvious flaws are that (i) savings rate is constant and (ii) labour input does not vary over time.
- Constant savings rate implies consumption and investment are **equally volatile** (which is not true)
- We also know that employment and work hours are **strongly procyclical**.
- Model needs to be modified if we want to capture the major features of observed output movements.
- If we introduce depreciation back into the model, the model performs better.
- However, this is a more difficult case that we won't cover in detail (analytically); you will, however, numerically solve the general model in Dynare.

Solving the model in the General Case

- A common way of dealing with the intractability of solving a nonlinear system analytically is to **log-linearise** the model.
- That is, agents' decision rules and the equations of motion for the state variables are replaced by **first-order Taylor approximations** in the logs of the relevant variables around the path the economy would follow in the absence of shocks (i.e., the **steady-state**).
- We will discuss this section at a high-level.
- You will be provided all of the necessary resources to work through solving and simulating a version of `Hansen's basic RBC model`. You can do this tutorial individually or in groups.
- See RBC Tutorial Handout note on the course site for further instructions.

Solving the model: Euler Equation (example)

$$C_t^{-\eta} = \beta \mathbb{E}_t \left[C_{t+1}^{-\eta} R_{t+1} \right] \quad \Rightarrow \quad 1 = \beta \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\eta} R_{t+1} \right]$$

- **Intertemporal optimality condition** governing consumption and capital accumulation (via savings).
- Under *certainty equivalence*, assume: $\mathbb{E}[XY] \approx \mathbb{E}[X]\,\mathbb{E}[Y]$
- For the **steady-state** (drop "t" subscripts and solve):

$$1 = \beta \left[\left(\frac{C}{C} \right)^{\eta} R \right] \quad \Rightarrow \quad \beta = 1/R$$

- We will use this condition to calibrate β based on the sample mean of $R = (1 + r)$, which is typically around $0.99 \approx 1/1.01)$ for quarterly real interest rate of 1%.

Log-linearisation

- **Log-linearising** around steady state gives a linear expectational difference equation:

$$0 = E_t[\eta\,(c_t - c_{t+1})] + r_{t+1}$$

- In the **Dynare** code, the expectations operator does not appear:

```
(1/eta)*r(+1) = c(+1) - c;
```

  where `1/eta`$= 1/\eta$ is the elasticity of intertemporal substitution

Interpretation

System now expresses deviations from steady state: all variables (e.g. y_t, c_t) are log-deviations. Useful for computing IRFs and theoretical moments via `stoch_simul`.

- Read Uhlig's handout for examples on how to apply the Taylor series approximation and Uhlig's (short-cut) method.

Technology shock: technology, capital, & labour

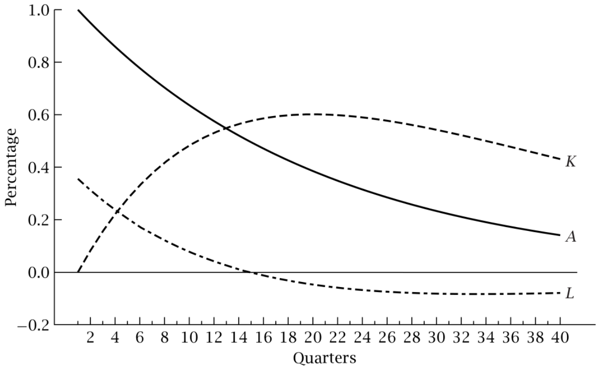


FIGURE 5.2 The effects of a 1 percent technology shock on the paths of technology, capital, and labor

- Each period corresponds to a quarter
- Baseline parameter values:
  $\alpha = \frac{1}{3}, g = 0.5\%, n = 0.25\%, \delta = 2.5\%, \rho_A = 0.95, \rho_G = 0.95$, and $\bar{G}, \rho$, and b such that $(G/Y)^* = 0.2, r^* = 1.5\%$, and $\ell^* = \frac{1}{3}$
  (See Problem 5.10 for the implications of these parameter values for the balanced growth path.)

Technology shock: output and consumption

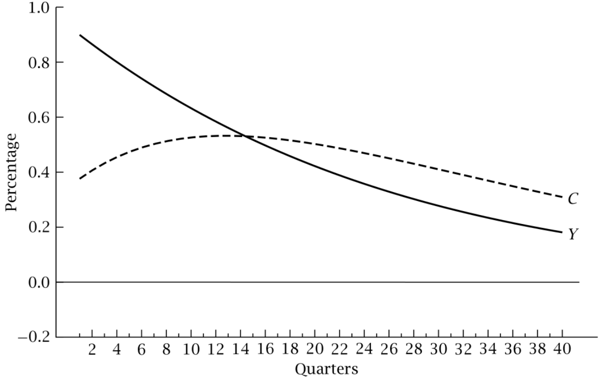


FIGURE 5.3 **The effects of a 1 percent technology shock on the paths of output and consumption**

Technology shock: wages and interest rate

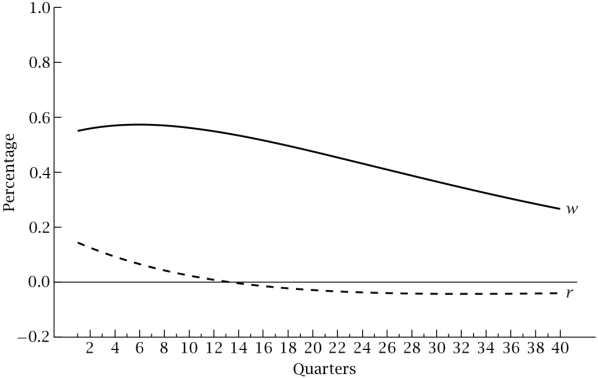


FIGURE 5.4 The effects of a 1 percent technology shock on the paths of the wage and the interest rate

The effects of changes in government purchases

- Intuitively, an increase in government purchases causes consumption to fall and labour supply to rise because of its negative wealth effects.
- And because the rise in government purchases is not permanent, agents also respond by decreasing their capital holdings.
- See Figures 5.5-5.7 in Romer. Can you replicate this effect in the Tutorial? Bonus marks will be awarded.

Empirical application: calibrating a RBC model

Key Steady-State Conditions (examples):

$$R = \frac{1}{\beta} \quad \text{(Euler equation)}$$

$$\frac{K}{Y} = \frac{\rho}{R - (1 - \delta)} \quad \text{(MPK = R)}$$

$$\frac{C}{Y} = 1 - \delta \cdot \frac{K}{Y} \quad \text{(from resource constraint)}$$

Interpretation:

- Capital-output and consumption-output ratios are pinned down by preferences and technology.
- Used to back out model parameters or to simplify coefficients in the linearised system.
- See Tutorial note for details.

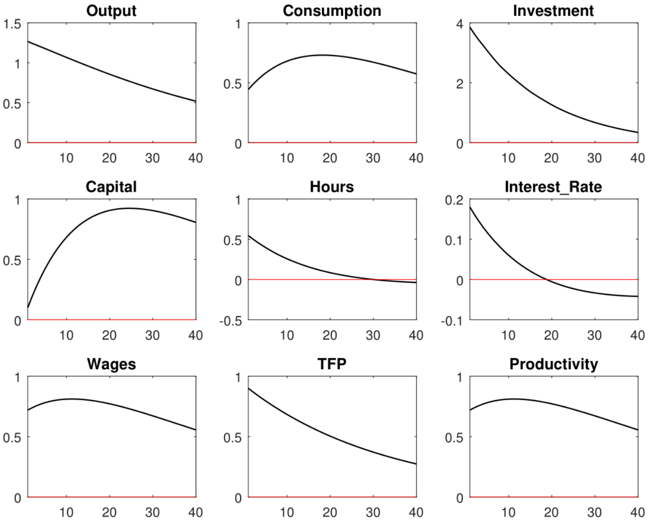
From theory to simulation: what dynare computes

- `stoch_simul(order=1)` solves the log-linearised model and computes:
  - Theoretical moments: std dev, autocorr, corr with output
  - Impulse Response Functions (IRFs)
- These moments form the basis for empirical assessment in Romer (Ch. 5.10).

Tutorial

Students compare model-implied moments to empirical data from their chosen country.

IRFs RBC Model (Matlab/Dynare)



Percent deviations from steady state values (for r percentage points)

Empirical application

- How do we decide if an RBC model matches the real world?
- Normally we look at some of the following things:
  1. Are relative amounts of cyclical variation in the variables correct?
  2. Does the degree of persistence in each variable match its actual persistence?
  3. Are the co-movements across variables realistic?
- The next slide shows how an RBC model compares to actual movements in the data, from **Plosser (1989)**.
- His article is in the *Journal of Economic Perspectives*, which is a great journal for less technical papers.

Plosser's evidence

Table 1
Summary Statistics 1954–1985

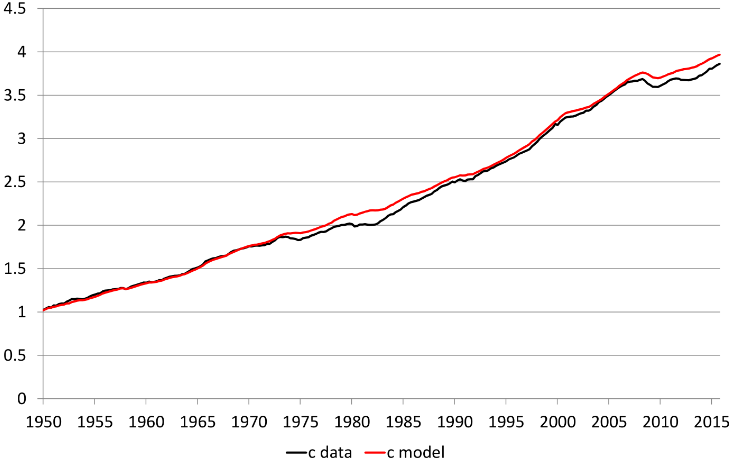
Variable	*Mean*	*Standard Deviation*	*Autocorrelation*[a] ρ_1	ρ_2	ρ_3	*Correlation With Output*	*Correlation With Actual*
				Panel A: Actual			
$\Delta \log(Y)$	1.55	2.71	.13	−.17	−.16	1.00	1.00
$\Delta \log(C)$	1.56	1.27	.39	.08	.05	.78	1.00
$\Delta \log(I)$	2.59	6.09	.14	−.28	−.19	.92	1.00
$\Delta \log(N)$	−0.09	2.18	.17	−.32	−.24	.81	1.00
$\Delta \log(w)$	0.98	1.80	.44	−.16	−.08	.59	1.00
				Panel B: Predicted			
$\Delta \log(Y)$	1.56	2.48	.30	.18	.14	1.00	.87
$\Delta \log(C)$	1.65	1.68	.55	.44	.37	.96	.76
$\Delta \log(I)$	1.37	4.65	.14	.00	−.02	.97	.72
$\Delta \log(N)$	−0.08	.89	.07	−.09	−.12	.87	.52
$\Delta \log(w)$	1.64	1.76	.51	.40	.33	.97	.65

[a]The approximate standard error of the estimated autocorrelations is .18.

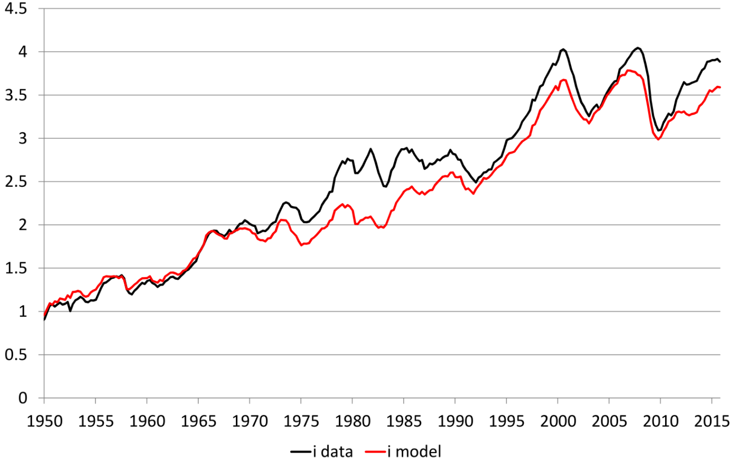
Model vs data updated

		Std. Dev.		Corr. w. y		Autocorr.	
		Data	Model	Data	Model	Data	Model
Output	y	1.65	1.64	1.00	1.00	0.85	0.72
Consumption	c	0.87	0.65	0.78	0.95	0.82	0.78
Investment	i	4.54	4.97	0.76	0.99	0.87	0.71
Capital	k	0.60	0.45	0.41	0.36	0.95	0.96
Hours	h	**1.94**	**0.69**	0.88	0.98	0.91	0.71
Wage	w	0.95	0.97	**0.10**	**0.99**	0.68	0.74
TFP	z	0.85	1.17	**0.51**	**1.00**	0.73	0.72
Productivity	$\frac{y}{h}$	1.07	0.97	**0.40**	**0.99**	0.71	0.74

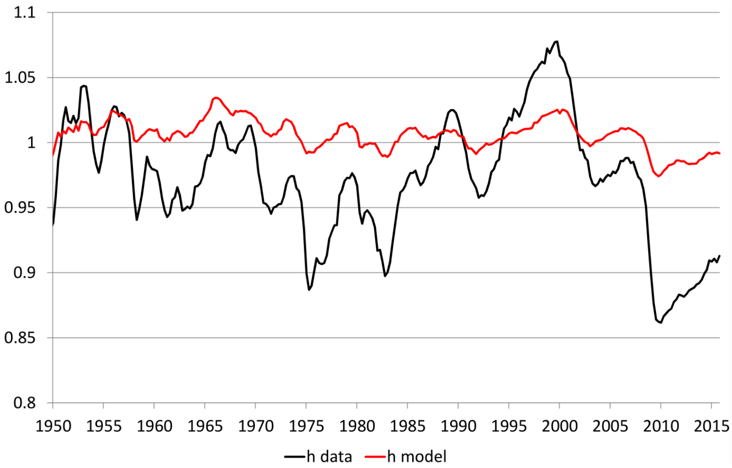
Model vs data: Consumption



Model vs data: Investment



Model vs data: Hours worked



Assessing RBC models

- Looking at the table we can see some problems emerge:
  - Consumption and wages are too highly correlated with GDP
  - Consumption, wages and output are too persistent
  - Much too little variation in employment
- The last point about variation in employment is worth discussing. Heavily debated in the literature.
  - RBC model explains employment fluctuations within a market clearing model (Walrasian)
  - The change in employment must be on the labour supply curve
  - If labour supply is **very elastic**, then we can have large changes in employment with small change in the wage
  - However, micro evidence shows that labour supply is quite inelastic! Complete opposite.
  - Simple RBC model can't reconcile large ΔL with $\Delta w \ldots$
- Basic RBC model does not seem to work for all cycles, but it might be useful for some $\rightarrow$ see, e.g., **Hamilton (2011)** and **Kehoe et al. (2018)**.