Investment

Macroeconomics

What will we do today?

- Which chapters?
  - **Chapter 9** (Investment) – Required
  - Podcast with Munger (in Econtalk) – Required
- Discuss different theories on investment

- Most recent Nobel Prize winner

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- **Angus Deaton**

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- (1945 -)

- Most recent Nobel Prize winner
- **Angus Deaton**
- (1945 -)
- Won the prize *for his analysis of consumption, poverty, and welfare*

What is investment and what is it not?

- What do you consider **investment** to be?
- Bankers, and probably most other people besides economists, would consider investment the purchase of financial assets
  - **Financial assets** $\rightarrow$ mutual funds, stocks, deposit accounts, etc.
- Macroeconomists reserve the term investment for transaction that increase the magnitude of **real aggregate capital**
  - Includes mainly the purchase (or production) of new real durable assets (e.g. factories, machines)
- Category of investment that receives the most attention is **business fixed investment** $\rightarrow$ purchase of new structures for production purposes (can be accumulated*)
  - However, two other important categories are inventory investment and residential investment

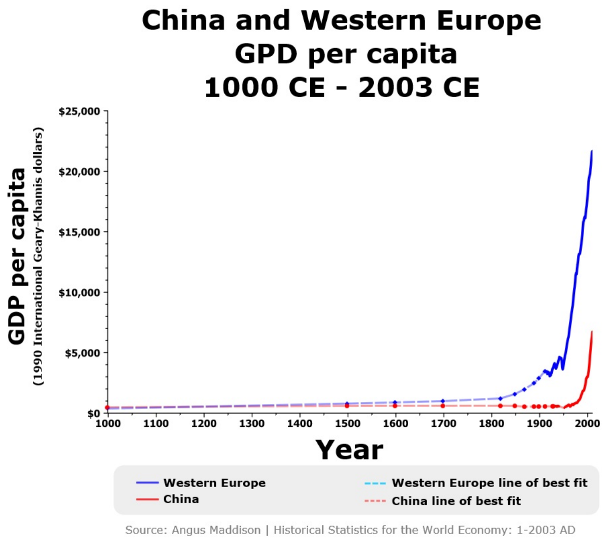
Reasons for studying investment

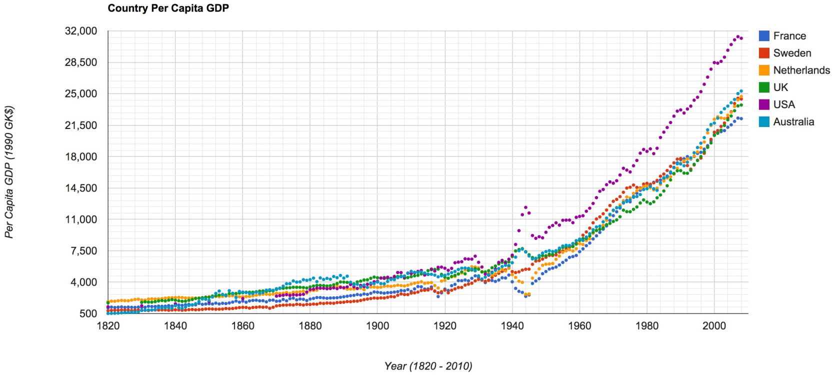
"The social object of skilled investment should be to defeat the dark forces of time and ignorance which envelope our future."

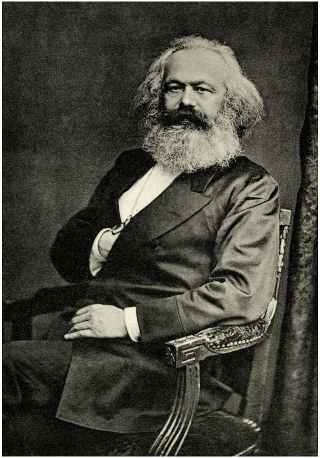
— **John Maynard Keynes**

- Two main reasons to study investment
  1. Investment matters for the **long-run prosperity** of society
  2. Investment matters for the **short run** (i.e. the business cycle)

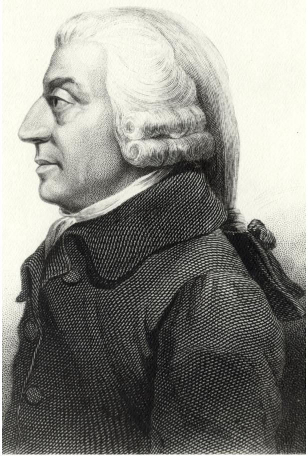
- In the **long-run** $\rightarrow$ investment is central to capitalist epoch
- Capitalism, with the institutions that make capitalism possible (e.g. private property, limited government, etc.)
- ...allows the accumulation of capital, which, in turn, allows the process of specialisation and trade to occur
- This idea is at the heart of the **industrial revolution!**


China and Western Europe
GPD per capita
1000 CE - 2003 CE
GDP per capita
(1990 International Geary-Khamis dollars)
$25,000
$20,000
$15,000
$10,000
$5,000
$0
1000
1100
1200
1300
1400
1500
1600
1700
1800
1900
2000
Year
Western Europe
China
Western Europe line of best fit
China line of best fit
Source: Angus Maddison | Historical Statistics for the World Economy: 1-2003 AD


Country Per Capita GDP
Per Capita GDP (1990 GK$)
32,000
28,500
25,000
21,500
18,000
14,500
11,000
7,500
4,000
500
1820
1840
1860
1880
1900
1920
1940
1960
1980
2000
Year (1820 - 2010)
France
Sweden
Netherlands
UK
USA
Australia



- **Karl Marx**
- Marx opens **the Capital** with the following statement,
- *The wealth of those societies in which the capitalist mode of accumulation prevails, presents itself as an immense accumulation of commodities . . .*
- Commodities $\rightarrow$ capital treated as a reproducible commodity in classical economics

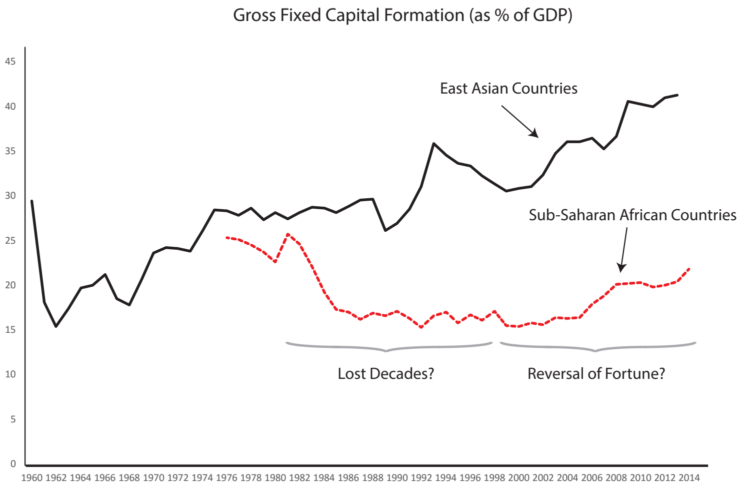


- **Adam Smith**
- Book II of the Wealth of Nations starts with,
- *As the accumulation of stock must, in the nature of things, be previous to the division of labour, so can labour be more and more subdivided in proportion only as **stock is previously more and more accumulated***
- *As the accumulation of stock is previously necessary for carrying this great improvement in the productive powers of labour, so that **accumulation naturally leads to this improvement***

Investment and observed economic progress

- Those regions where investment has lagged has remained underdeveloped
  - Falling investment rates in **SSA** during the 1970s, 80s and 90s
  - Increasing share thereof by government
  - As well as disinvestment from **SSA** (African capital flight)
- All of these were correlated with **SSA's** lost decades

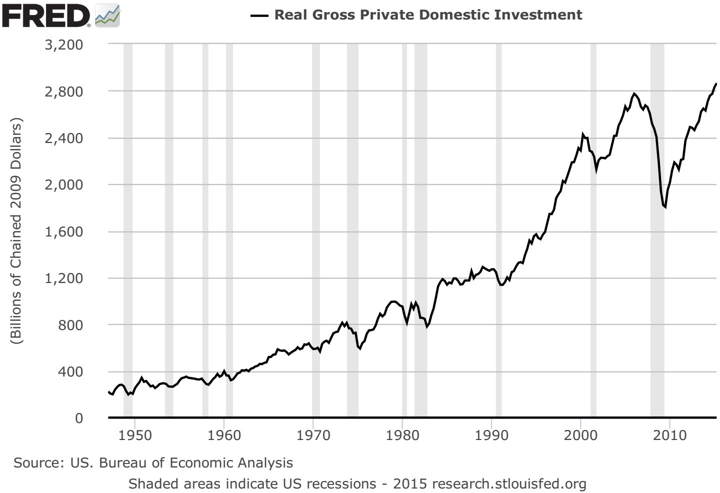
- Subsequent reversal of fortunes in many African countries have been associated with a return of investment



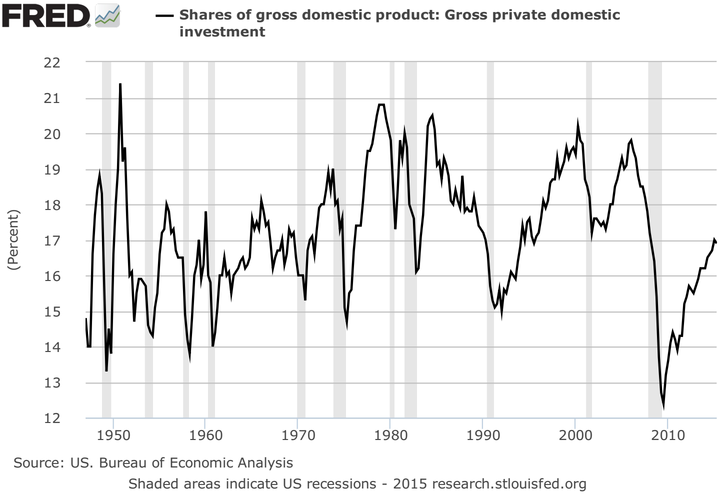
Gross Fixed Capital Formation (as % of GDP)
East Asian Countries
Sub-Saharan African Countries
Lost Decades?
Reversal of Fortune?
0
5
10
15
20
25
30
35
40
45
1960 1962 1964 1966 1968 1970 1972 1974 1976 1978 1980 1982 1984 1986 1988 1990 1992 1994 1996 1998 2000 2002 2004 2006 2008 2010 2012 2014

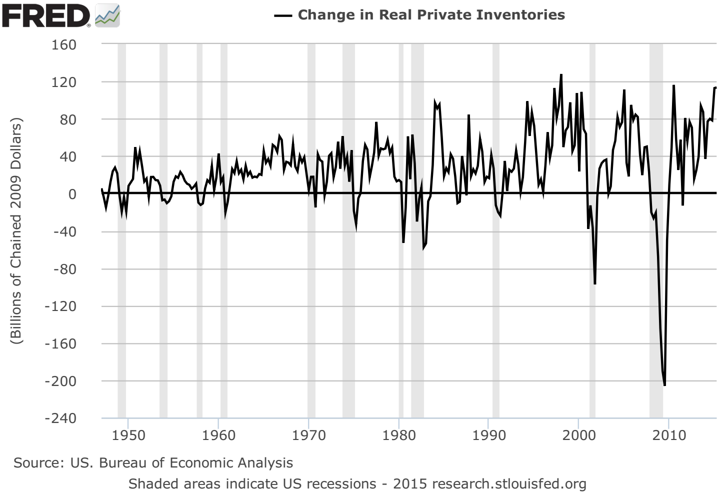
Investment matters in the short run

- Second reason for studying the systematic factors affecting investment $\rightarrow$ significant influence on the business cycle
- Even though investment expenditure is far smaller in relation to GDP when compared with consumption expenditure, **investment is far more volatile**
- Adds up to a **greater influence** on the expansionary and contractionary phases of economic activity



FRED
Real Gross Private Domestic Investment
(Billions of Chained 2009 Dollars)
3,200
2,800
2,400
2,000
1,600
1,200
800
400
0
1950
1960
1970
1980
1990
2000
2010
Source: US. Bureau of Economic Analysis
Shaded areas indicate US recessions - 2015 research.stlouisfed.org


FRED
Shares of gross domestic product: Gross private domestic investment
22
21
20
19
18
17
16
15
14
13
12
(Percent)
1950
1960
1970
1980
1990
2000
2010
Source: US. Bureau of Economic Analysis
Shaded areas indicate US recessions - 2015 research.stlouisfed.org


FRED
Change in Real Private Inventories
160
120
80
40
0
-40
-80
-120
-160
-200
-240
(Billions of Chained 2009 Dollars)
1950
1960
1970
1980
1990
2000
2010
Source: US. Bureau of Economic Analysis
Shaded areas indicate US recessions - 2015 research.stlouisfed.org

Modelling aggregate investment expenditure

- We are going to be discussing **stock and flow** approaches to modelling investment
- The stock approaches include,
  1. Accelerator theory
  2. Neo-classical investment theory
  3. Adjustment cost model (**q-theory**)
- There are also several flow approaches, but we will only discuss Tobin's flow model (**Tobin's q**)

Stock approaches

- Stock approaches are normally a **two-step procedure**
  1. Define an optimal level of capital stock K^*
  2. Define the cost of adjusting the existing capital stock to K^*

- An early investment model was proposed by **John Bates Clark**
- It is based on the notion of a production function with a **fixed capital output ratio** (in the short run)
  - If output changes, then so must the capital stock to maintain the capital output ratio
- Not really a theory, but an attempt to capture the **scale effect** (impact of AD) on aggregate investment
  - When proportions are fixed (K/Y), if the economy scales up $Y \uparrow$, then factor inputs need to adjust accordingly

Accelerator theory of investment

- Start with a simple relationship between the optimal capital stock, k^*, and real expected output y

$$k^* = \alpha y^{\alpha}, \quad \alpha > 0$$

- where α is the capital-output ratio. Then, specify an adjustment process where net investment is,

$$\Delta k_t = \beta(k^* - k_{t-1}), \quad 0 < \beta < 1$$

- Gross investment, after depreciation is then,

$$I_t = \underbrace{\Delta k_t + \delta k_{t-1}}_{\text{Net investment + Depreciation}}, \quad 0 < \delta < 1$$

Accelerator theory of investment

- Using a lag operator L where $Lk_t = k_{t-1}$, we can rewrite the equation for gross investment as,

$$\begin{aligned} I_t &= \Delta k_t + \delta k_{t-1} \\ &= k_t - Lk_t + \delta k_{t-1} \\ &= k_t(1 - L + \delta L) \\ &= k_t[1 - L(1-\delta)] \end{aligned}$$

- After some substitution and simplification we get:

$$I_t = \beta \alpha y^{\alpha}[1 - L(1-\delta)] + (1-\beta) I_{t-1}$$

- This gives a simple function if $\beta = 1$ and $\delta = 0$, which gives us the following estimable function,

$$I_t = \alpha y^{\alpha}(1 - L) = \alpha \Delta y$$

Accelerator theory of investment

- What this function $I_t = \alpha \mathbb{E}_t(y_{t+1} - y_t)$ shows is that investment is proportional to the increase in output in the coming period
- Investment depends on expected value of output (similar to the business cycle theory of **Keynes**)
- Expectations difficult to measure $\rightarrow$ solution was to have,

$$\mathbb{E}_t(y_{t+1} - y_t) = y_t - y_{t-1}$$

- Aligned with the **Keynesian idea** that when firms observed rise or decline in output $\rightarrow$ extrapolated that change into the future in determining their investment spending

Evaluation of accelerator model

- Empirical evidence suggests that accelerator effects are empirically important in many countries
- Model is an apparently successful way of incorporating **scale effects** in the investment model

- However, we also know that **relative price effects** are important in the allocation of resources
  - Subsequent developments in investment theory developed **relative price effects** $\rightarrow$ theory of the firm
  - Ever more sophisticated considerations of the costs and expected benefits associated with investment

Neoclassical investment theory

- Consider a firm that can rent productive capital at rate r_K
- Profits π at a point in time are given by,

$$\pi(k, \mathbf{X}) - r_K k$$

- where $\pi(\bullet)$ accounts for all other dimensions of optimisation over other relevant factors $\mathbf{X} = (X_1, X_2, \ldots, X_n)$

- Our necessary optimality condition is that,

$$\frac{\partial \pi(k^*, \mathbf{X})}{\partial k} = r_K$$

- The firm rents capital up to the point where marginal revenue product equals the rental cost of capital r_K
- Second order **sufficient condition** for optimality is satisfied,

$$\frac{\partial^2 \pi(k^*, \mathbf{X})}{\partial k^2} < 0$$

Neoclassical investment theory

- Let's try something you have done before
- Consider a **competitive firm** that produces using only K and L
- Revenue for this firm is PQ, where P is market price and Q the amount the firm produces
- Production function $\rightarrow Q = F(K, L)$
- In addition to capital cost, the firm incurs labour cost equal to wL
- Combining these elements we have,

$$\pi(K, P, w) - r_K K = P \cdot F(K, L) - r_K K$$

- Profit maximisation with respect to capital entails,

$$P \cdot F_K(K, L) = r_K K$$

Neoclassical investment theory

- Taking a derivative with respect to both sides of our FOC yields,

$$
\begin{aligned}
\frac{\partial \left[\frac{\partial \pi(k^*(r_K),\mathbf{X})}{\partial k}\right]}{\partial r_K} &= 1 \\
\frac{\partial \left[\frac{\partial \pi(k^*(r_K),\mathbf{X})}{\partial k}\right]}{\partial k} \cdot \frac{\partial k^*(r_K)}{\partial r_K} &= 1 \\
\frac{\partial^2 \pi(k^*(r_K),\mathbf{X})}{\partial k^2} \cdot \frac{\partial k^*(r_K)}{\partial r_K} &= 1 \\
\frac{\partial k^*(r_K)}{\partial r_K} &= \left(\frac{\partial^2 \pi(k^*(r_K),\mathbf{X})}{\partial k^2}\right)^{-1} < 0
\end{aligned}
$$

- Optimal capital stock inversely related to changes in rental cost of capital r_K

Neoclassical investment theory

- Now we move from '**rental cost**' to '**user cost**' of capital
- Rental price is rarely observe (since capital is rarely rented) $\rightarrow$ firms usually own most of their capital
- An empirical concept $\rightarrow$ user cost of capital was developed

- In the classical view the definition of the user cost of capital should include
  1. Opportunity cost of holding physical capital (rather than some other financial asset) $\rightarrow r(t)p_K(t)$
  2. Depreciation in value of capital at constant rate $\rightarrow \delta p_K(t)$
  3. Capital gains / losses $\rightarrow -\dot{p}_K(t) \equiv -\frac{\partial p_K(t)}{\partial t}$
- Combining all these components yields the user cost of capital,

$$r_K(t) = p_K(t)\left[r(t) + \delta - \frac{\dot{p}_K(t)}{p_K(t)}\right]$$

Difficulties with the baseline model

- Any discrete change in r_K implies a discrete change in k^*, which means two things for us,
  1. In continuous time, this implies an **infinite rate of investment** (stock of capital would jump to its new level)
  2. Since capital is allowed to jump, decisions about capital stock become static $\rightarrow$ determined by user cost of capital and **NOT forward looking**

- We have a few objections based on **empirical evidence**
  1. Investment rate can't be infinite $\rightarrow$ actual investment limited by current level of output in the economy (supply side constraint)
  2. **Expectations of future costs and returns** are crucial determinants of current investment

- We need to introduce something that slows down the adjustment of the capital stock in response to changes in the environment

Adjustment costs to capital stock

- To get capital to evolve smoothly $\rightarrow$ introduce **adjustment costs**
- These adjustment costs can either be **internal** or **external**

- Internal adjustment costs $\rightarrow$ firms face direct costs in adjusting their capital stock (costs of installing capital and training workers)

- External adjustment costs $\rightarrow$ firms face higher price of capital goods (inelastic supply of capital stock)

External adjustment cost

Adjustment cost model – the **q-theory**

- Model we develop here will consider internal adjustment costs
- Start with discrete time and move on to continuous $\rightarrow k_t$ vs. $k(t)$
- Firm operates in an industry with N identical firms. Assume that,
  1. Firm's profits are proportional to it's own capital stock k_t
  2. Firm's profits are declining in total capital stock of the industry $K_t = N \cdot k_t$
- With these assumptions we can write profit as,

$$\pi(K_t)k_t, \quad \pi'(K_t) < 0$$

- The downward sloping demand for the industry's product is reflected by the condition that $\pi'(K_t) < 0$

Adjustment cost model – the **q-theory**

- **Important**: Law of motion of capital, with no depreciation

$$k_t = k_{t-1} + I_t$$
$$\Delta k_t = I_t$$

- Assume that adjusting the firm's capital stock is costly
- Adjustment costs are expressed as a **convex function** of the change in the firm's capital stock,

$$C(\Delta k_t) \Rightarrow C(I_t), \quad C(0) = 0,\ C'(0) = 0, \text{ and } C''(\bullet) > 0$$

- Firm incurs positive adjustment costs when it changes its capital stock, either upward or downward
- Costs rise at an increasing rate $\rightarrow$ as **investment gets further from zero**

Adjustment cost model – the **q-theory**

- The objective function of the firm is,

$$\max_{I_t} \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} [\pi(K_t)k_t - I_t - C(I_t)]$$

- subject to $k_t = k_{t-1} + I_t$. Lagrangian is then,

$$\mathcal{L} = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} [\pi(K_t)k_t - I_t - C(I_t)] + \sum_{t=0}^{\infty} \lambda_t [k_{t-1} + I_t - k_t]$$

- Infinitely many time periods $\rightarrow$ infinitely many constraints
- In this equation, λ_t is the usual shadow price of the constraint, but in this model it is the key variable.

- Each Lagrange multiplier λ_t shows the marginal impact on the present value of the firm for an exogenous increase in k_t at time t
- Shows the marginal impact in **time period 0 dollars**

Adjustment cost model – the **q-theory**

- Define a new variable q such that q_t shows the marginal value of an increase in capital stock to the firm in **in time t dollars**

$$q_t = (1+r)^t \lambda_t$$

- We can use this to rewrite our Lagrangian,

$$\mathcal{L} = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} [\pi(K_t)k_t - I_t - C(I_t) + q_t(k_{t-1} + I_t - k_t)]$$

- The first order condition w.r.t I_t will be,

$$\left(\frac{\partial \mathcal{L}}{\partial I_t}\right) \quad \frac{1}{(1+r)^t}[-1 - C'(I_t) + q_t] = 0$$

$$1 + C'(I_t) = q_t$$

Adjustment cost model – the **q-theory**

$$1 + C'(I_t) = q_t$$

- How do we interpret this result?

- Cost of acquiring a unit of capital is equal to it's purchase price (**fixed at 1**) plus the marginal adjustment cost
- Firm invests to the point where the cost of acquiring capital equals the value of capital

Adjustment cost model – the **q-theory**

- The first order condition w.r.t k_t will be,

$$\left(\frac{\partial \mathcal{L}}{\partial k_t}\right) \quad \frac{1}{(1+r)^t}[\pi(K_t) - q_t] + \frac{1}{(1+r)^{t+1}} q_{t+1} = 0$$
$$\frac{1}{(1+r)^t}[\pi(K_t) - q_t] = -\frac{1}{(1+r)^{t+1}} q_{t+1}$$

- Multiply this expression by $(1+r)^{t+1}$ and rearrange

$$[\pi(K_t) - q_t] = -\frac{1}{(1+r)^1} q_{t+1}$$
$$(1+r)[\pi(K_t) - q_t] = -q_{t+1}$$
$$(1+r)\pi(K_t) - (1+r)q_t = -q_{t+1}$$
$$(1+r)\pi(K_t) = (1+r)q_t - q_{t+1}$$

- Define $\Delta q_t = q_{t+1} - q_t$. Rewrite RHS as $rq_t - \Delta q_t$, which yields,

$$\pi(K_t) = \frac{1}{(1+r)}(rq_t - \Delta q_t)$$

Adjustment cost model – the **q-theory**

$$\pi(K_t) = \frac{1}{(1+r)}(rq_t - \Delta q_t)$$

- How do we interpret this result?

- LHS of the equation is the marginal revenue product of capital
- RHS shows the opportunity cost of holding capital

- **In equilibrium**, returns to capital must equal the opportunity cost of holding the capital

Adjustment cost model – the **q-theory**

- We could rewrite this equation in the following way,

$$\pi(K_t) = \frac{1}{(1+r)}(rq_t - \Delta q_t)$$

$$\pi(K_t) = \frac{rq_t}{(1+r)} - \frac{\Delta q_t}{(1+r)}$$

$$\pi(K_t) = \frac{rq_t + q_t}{(1+r)} - \frac{q_{t+1}}{(1+r)}$$

$$\pi(K_t) + \frac{q_{t+1}}{(1+r)} = q_t$$

- Shows that **consistent valuation** requires that the value of the unit of capital in time period t is equal to the sum of the marginal revenue product of that capital in period t plus the value of capital in period $t+1$

Adjustment cost model – the **q-theory**

- Even though we have **consistency**, let us iterate forward to see if any complications emerge
- Suppose a firm has an additional unit of capital at period 0, this capital should raise profits in period t by $\pi(K_t)$
- We can write the amount that the capital contributes to the firm's objective function as,

$$MB = \lim_{T\to\infty}\left\{\sum_{t=0}^{T-1}\left[\frac{1}{(1+r)^t}\pi(K_t)\right]\right\}$$

- Consider what happens if we iterate $\pi(K_t) + \frac{q_{t+1}}{(1+r)} = q_t$ forward

Adjustment cost model – the **q-theory**

- Notice that we can write iterate forward from q_0

$$
\begin{aligned}
q_0 &= \pi(K_0) + \frac{q_1}{(1+r)} \\
&= \pi(K_0) + \frac{1}{(1+r)}\left(\pi(K_1) + \frac{1}{(1+r)}q_2\right) \\
&= \dots \\
&= \sum_{t=0}^{T-1}\left[\frac{1}{(1+r)^t}\pi(K_t)\right] + \frac{1}{(1+r)^T}q_T
\end{aligned}
$$

- Taking the limit as $T \to \infty$ yields,

$$
q_t = \lim_{T\to\infty}\left\{\sum_{t=0}^{T-1}\left[\frac{1}{(1+r)^t}\pi(K_t)\right] + \frac{1}{(1+r)^T}q_T\right\}
$$

- We are mainly interested in $\lim_{T\to\infty}\frac{1}{(1+r)^T}q_T$ for our analysis

Adjustment cost model – the **q-theory**

- We find that q_0 equals the contribution of an additional unit of capital to the firm's objective function if and only if,

$$\lim_{T\to\infty} \frac{1}{(1+r)^T} q_T = 0$$

- If this condition does not hold, then it means that some capital was not employed for profit $\rightarrow$ firm cannot be profit maximiser
- Mathematically, we have that if $\lim_{T\to\infty} \frac{1}{(1+r)^T} q_T > 0$, since $\lim_{T\to\infty} \frac{1}{(1+r)^T} = 0$ that $\lim_{T\to\infty} q_T = \infty$

Continuous form

- Let us try and rewrite things in continuous form $\rightarrow$ we will 'cheat' a bit because you haven't done Hamiltonians
- Suppose the jump between periods is not 1 but Δt
- Rewriting our first order condition wrt to investment yields,

$$q(t) = 1 + C'\left(\frac{k(t+\Delta t) - k(t)}{\Delta t}\right)$$

- Similarly, rewriting the FOC for $k(t)$ gives,

$$\pi(K(t)) = \left[\frac{1}{(1+r)}\right]^{\Delta t}\left(rq(t) - \frac{q(t+\Delta t) - q(t)}{\Delta t}\right)$$

- We can let $\Delta t \rightarrow 0$, which is the definition of continuous time

$$q(t) = 1 + C'\left(\dot{k}(t)\right) = 1 + C'\left(\frac{\dot{K}(t)}{N}\right)$$

$$\pi(K(t)) = rq(t) - \dot{q}(t)$$

Interpretation of q

- Let's get back to q and our interpretation of it

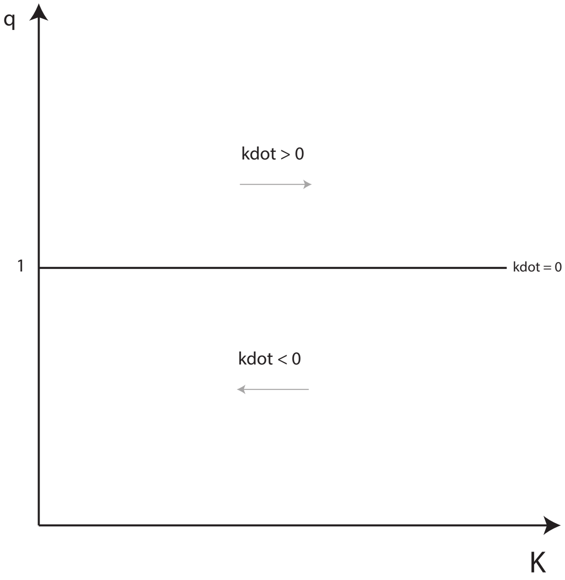
- q summarises all information we need to know with respect to the firm's investment decision
- When q is high, firm should invest $\rightarrow$ capital will justify the opportunity and adjustment cost
- Tobin realised that we can also interpret this value in an alternative way
  - Incorporate the information in financial markets

- We will now look at Tobin's q (**flow approach**)

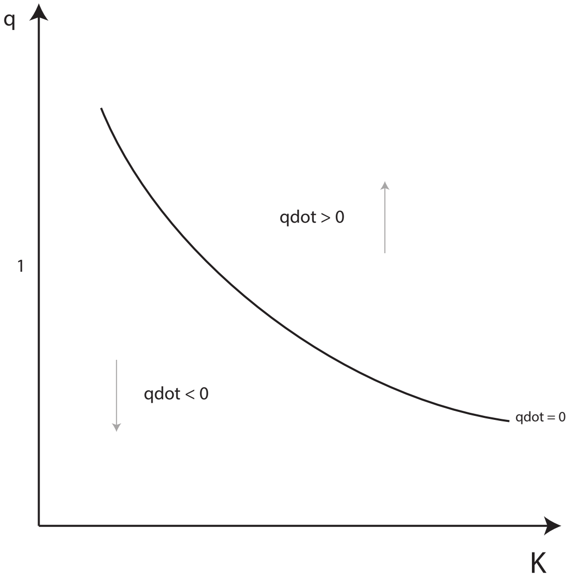
Tobin's flow model

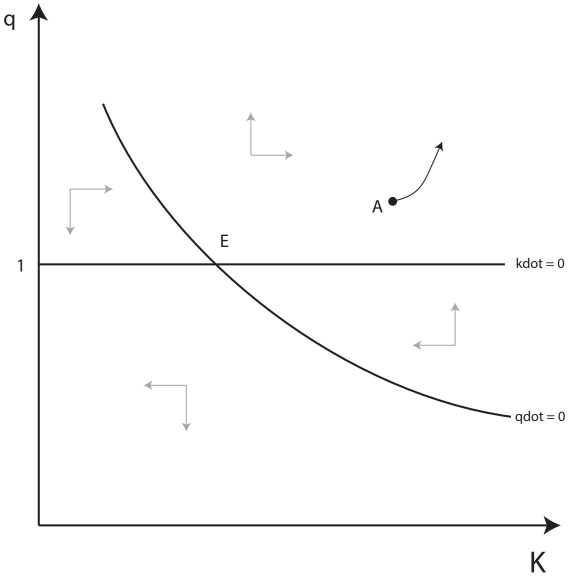
- Gives a direct explanation for the investment rate
- Incorporates financial markets
  - We have already seen that q captures the impact of higher capital on the firm's market value
  - When q is high, the firm will want to invest
  - What is high and low here?!?

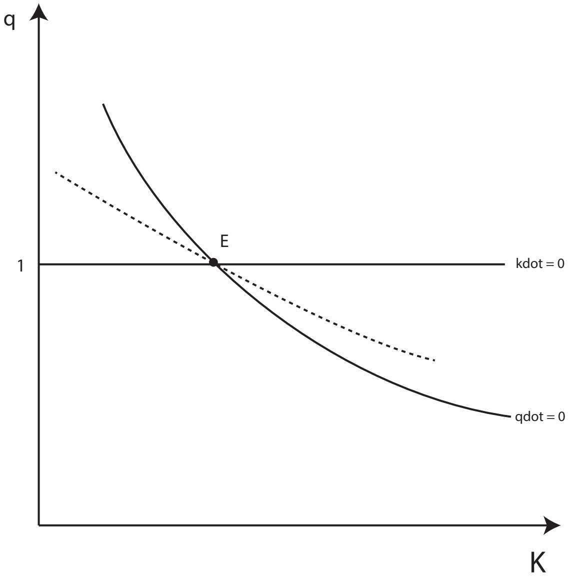
- q-theory argues that firms will want to invest when $q > 1$ and decrease capital stock when $q < 1$
- If $q > 1$ then a firm can buy one dollar's worth of capital and earn profits that have present value in excess of one dollar
- If $q < 1$ then present value of profits earned by installing new capital are less than the cost of capital $\rightarrow$ investment lowers profit
- In the case of $q < 1$ one can rather buy an existing firm than build a new one with capital
- The ratio of the market value of capital to the replacement cost of capital is called **Tobin's q**



q
kdot > 0
1
kdot = 0
kdot < 0
K


q
1
qdot > 0
qdot < 0
qdot = 0
K


q
A
E
1
kdot = 0
qdot = 0
K


q
E
1
kdot = 0
qdot = 0
K