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WHAT YOU MATCH DOES MATTER: THE EFFECTS OF DATA ON DSGE ESTIMATION

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SUMMARY

This paper explores the effects of using alternative combinations of observables for the estimation of Dynamic Stochastic General Equilibrium (DSGE) models. I find that the estimation of structural parameters describing the Taylor rule and sticky contracts in prices and wages is particularly sensitive to the set of observables. In terms of the model's predictions, the exclusion of some observables may lead to estimated parameters with unexpected outcomes, such as recessions following a positive technology shock. More importantly, two ways to assess different sets of observables are proposed. These measures favor a dataset consisting of seven observables.

1. INTRODUCTION

The search for a better understanding of the macroeconomic environment has forced economists to formulate and analyze complex models. Such complexity makes researchers rely more and more frequently on numerical solutions to their mathematical abstractions of reality. As a consequence, the policy implications of such models depend tremendously on the parameterization used to solve them. Most of this model-based policy analysis elaborates on the important contributions of Rotemberg and Woodford (1997) and Christiano *et al*. (2005, hereafter CEE).[1]

Although subsequent work in the spirit of CEE has adopted many of their economic assumptions (for example, staggered price and wage contracting, costly adjustment in investment, and habit formation), there are significant departures with respect to the econometric approach and, in particular, the data used to estimate the model. CEE, for example, infer the structural parameters in their model by using nine observable variables (output, consumption, investment, interest rates, productivity, profits, inflation, money growth, and real wages). Del Negro *et al*. (2004), Justiniano and Primiceri (2006), Levin *et al*. (2005), and Smets and Wouters (2007) estimate their models using data on output, consumption, investment, labor, real wages, interest rates, and inflation. Likewise, Altig *et al*. (2005, hereafter ACEL) opt for the same seven variables plus the price of investment.[2] Boivin and Giannoni (2006) use an even larger dataset including additional measures of personal consumption and inflation. Finally, authors like An and Schorfheide (2007), Fernandez-Villaverde and Rubio-Ramirez (2007a), and Rotemberg and Woodford (1997) choose smaller sets of observables.

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[1] Recent examples of this approach include Levin *et al*. (2005) and Schmitt-Grohe and Uribe (2004).

[2] These papers also differ in the econometric approach. For example, while Del Negro *et al*. (2004) and Justiniano and Primiceri (2006) use likelihood-based methods, CEE and Altig *et al*. (2005) choose a minimum-distance estimator.

At this stage it is unclear what effects the set of observables has on the estimated parameters and on the model's economic implications. Indeed, Fernandez-Villaverde and Rubio-Ramirez (2007b) emphasize this lack of knowledge: 'Unfortunately, we do not know much about the right choice of observables and how they may affect our estimation results.' But why should we be concerned with the choice of observables? To answer this question, recall from regression analysis that adding more observables may improve in-sample fitting at the cost of more imprecise estimation of parameters. On the other hand, omission of relevant variables may lead to unexpected results (biased coefficients in the classical sense). Consider, for example, the estimation of habit formation. Levin *et al*. (2005) estimate this parameter to be around 0.30, while Fernandez-Villaverde and Rubio-Ramirez (2007b), using a similar model and econometric approach, but a different set of observables, place habit formation in the neighborhood of 0.88. Issues like data availability and computational costs only compound the problem.

This paper systematically analyzes the effects of observables on the estimation of a fairly standard New Keynesian model. To that end, I employ Bayesian methods and different combinations of observables. Some of the datasets typically used in the related literature are considered. My discussion of the effects of variable omission is divided in two parts. I begin by analyzing the implications of permutations of the set of observables on the estimation of the Taylor rule coefficients and the parameters capturing persistence in the model. I center my discussion on those parameters because of their role in shaping fiscal and monetary policy, as recently emphasized in the literature (Schmitt-Grohe and Uribe, 2004). Next, I assess the overall effect of alternative observables based on two criteria: (1) the effect on the structural parameters and the model's economic predictions; and 2) the root mean square error (RMSE) of the out-of-sample forecast.

In terms of the estimated parameters, I find that the absence of observables (such as interest rates, inflation, and real wages) heavily influences the median estimates of the parameters capturing persistence in the model. For example, the posterior medians for habit formation and the smoothing parameter in the Taylor rule fluctuate in the ranges 0.67–0.97 and 0.24–0.93, respectively. Furthermore, the omission of consumption and real wages affects the estimation of the duration of the price spell in the model. With respect to impulse responses, I notice important departures between the predictions of the same model estimated using alternative combinations of the same set of observables. Among all variables, I find that the impulse responses of inflation, investment, labor, and real wages are particularly sensitive to the choice of observables. Take, for example, the responses to investment-specific shocks; excluding real wages from the estimation stage implies structural parameters such that the model predicts a strong recession following technology progress.

When we turn to ranking the alternative sets of observables, the proposed performance measures favor the dataset that includes output, consumption, investment, interest rates, total hours, inflation, and real wages. Specifically, this set delivers simultaneously small forecast errors (low RMSE), reasonable parameter estimates, and intuitive impulse responses. The same measures also suggest that the presence of the price of investment is solely justified on forecasting grounds since its inclusion has no significant effects on the model's estimates. Furthermore, I find that the exclusion of information about labor or investment has only minor implications.

The last part of this paper briefly discusses the role of observables in the identification of DSGE models and whether more observables are always better. Regarding identification, I find that the model's posterior is prone to displaying bimodality in the absence of certain observables such as consumption or real wages. Addressing the second point will inevitably take us to the delicate issue of model specification (Del Negro and Schorfheide, 2008). It will become clear that measurement errors play a crucial role in deciding whether to use more observables.

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The rest of this paper proceeds as follows. In Section 2, I show in the context of a simple macroeconomic model the potential pitfalls in the selection of observables. Section 3 then discusses the main features of the medium-scale DSGE model to be used in the rest of this paper. The econometric approach and data description are contained in Section 4. A discussion of the parameters' posterior distributions and the model's impulse responses is in Section 5. I outline the performance measures and ranking of the alternatives in Section 6. Finally, a short discussion about identification and observables, as well as concluding remarks, is provided in the last two sections of this paper.

2. AN ILLUSTRATIVE EXAMPLE

To understand how the choice of observables can influence the estimation of DSGE models, let us consider the following simple New Keynesian model from Woodford (2003, p. 246). The economy consists of a Phillips curve (PC), a Taylor Rule (TR), and an investment-savings equation (IS):

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} \tag{PC}$$

$$i_t = \bar{i} + \phi_\pi(\pi_t - \bar{\pi}) + \phi_x(x_t - \bar{x}) \tag{TR}$$

$$x_t = \mathbb{E}_t x_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \tag{IS}$$

where x_t, π_t and i_t correspond to output gap, inflation, and interest rates, respectively. Variables with a bar denote steady-state values. Straightforward manipulations imply that the model has a solution of the form

$$\underbrace{\begin{bmatrix} \pi_t \\ x_t \end{bmatrix}}_{s_t} = \underbrace{\begin{bmatrix} \beta^{-1} & -\kappa\beta^{-1} \\ \sigma(\phi_\pi - \beta^{-1}) & 1 + \sigma(\phi_x + \kappa\beta^{-1}) \end{bmatrix}}_{F} \underbrace{\begin{bmatrix} \pi_{t-1} \\ x_{t-1} \end{bmatrix}}_{s_{t-1}} + \Lambda_t \tag{TE}$$

Here, Λ_t contains constant terms and the driving process r_t^n. If, as in most of the recent literature, we opt to estimate the model via likelihood-based methods, the transition equation (TE) has to be supplemented with an observation equation of the form

$$Y_t = H \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} + v_t \tag{OE}$$

where Y contains the observables. Using the error decomposition method (Kim and Nelson, 1999), one can show that the log-likelihood of the model contains terms of the form

$$(Y_t - H \times F \times s_{t-1})'\Omega^{-1}(Y_t - H \times F \times s_{t-1})$$

for some covariance matrix Ω. From the last expression, it is quite clear that if, for example, the set of observables includes a measure of the output gap and inflation, all the coefficients in the model (β, κ, ϕ_π, ϕ, and σ) enter directly into the likelihood since H is a diagonal matrix. On the

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other hand, suppose we only include information about inflation. Then the matrix product in the likelihood is given by

$$H \times F = \begin{bmatrix} \beta^{-1} & -\kappa\beta^{-1} \\ 0 & 0 \end{bmatrix}$$

Hence the Taylor rule parameters, ϕ_x and ϕ_π, and the intertemporal elasticity of substitution, σ, do not enter directly in the likelihood. Our only hope to identify them is through their effect on past states $s_{t-1}, \ldots, s_1$. Similarly, omitting the output gap as an observable may complicate the identification of β and σ as they now enter the likelihood via products of the form $\sigma\beta^{-1}$, $\sigma\phi_x$, and $\sigma\phi_\pi$. To the extent that the system of equations (TE) is the data-generating process, this example illustrates the dangers behind ignoring observables for the estimation of DSGE models.

Using too many observables may also be counterproductive for we risk including variables that may be unrelated to the model. To see this point, suppose that we mistakenly use a random walk, ω_t, as our first observable. Furthermore, let us assume that its measurement equation has no error term, i.e., the first element of v_t is zero. Under these assumptions, the measurement equation equates inflation in the model with the random walk observable: $\omega_t = \pi_t$. Then a likelihood-based approach would try to make inflation in the model follow a driftless random walk

$$\mathbb{E}_t\pi_{t+1} = \pi_t$$

But equation (PC) implies that

$$\mathbb{E}_t\pi_{t+1} = \beta^{-1}\pi_t - \kappa\beta^{-1}x_t$$

For the last two equations to be satisfied, it must be the case that $\beta = 1$ and $\kappa = 0$, which is clearly at odds with the findings from the empirical Phillips curve literature (Gali and Gertler, 1999). Adding a measurement error may ameliorate the problem because it breaks the direct link between the random walk and inflation. Indeed, the measurement equation now dictates that $\omega_t = \pi_t + v_{1,t}$, where $v_{1,t}$ is the first element of the v_t. Clearly, the link becomes weaker as the persistence of the measurement error increases, which in turn makes the estimation of a discount factor less than one feasible.

This example shows that using observables for which the model has not been properly designed for also complicates the estimation of the structural parameters. In the rest of the paper I discuss more carefully the effects of the set of observables on the estimation of a medium-scale DSGE model.

3. MODEL

My formulation builds on ACEL, CEE, Schmitt-Grohe and Uribe (2004), and Del Negro *et al.* (2004). Since this type of environment has been extensively discussed in the literature, I provide here only a brief discussion that omits lengthy derivations, when possible. The main features of the model can be summarized as follows. The economy grows along a stochastic path; prices and wages are assumed to be sticky à la Calvo; preferences display external habit formation; investment is costly; and finally, there are five sources of uncertainty: neutral and capital embodied technology shocks, preference shocks, government expenditure shocks, and monetary shocks.

3.1. Firms

There is a continuum of monopolistically competitive firms indexed by $j \in [0, 1]$ each producing a final good out of capital services, k_j, and labor services, $L_{j,t}$. The technology function is given by

$$k_{j,t}^{\alpha}(S_t^L L_{j,t})^{1-\alpha} - S_t^* \psi$$

where ψ makes profits equal to zero in the steady state. S_t^* is the stochastic growth path of the economy (see below for its definition).[3] The neutral technology shock, S_t^L, grows at rate g_t^L which is assumed to follow the process

$$\ln g_t^L = (1 - \rho_{g^L}) \ln g_{ss}^L + \rho_{g^L} \ln g_{t-1}^L + \sigma_{g^L} \varepsilon_{g^L,t}$$

where $\varepsilon_{g^L,t}$ is distributed $\mathbb{N}(0,1)$.

Firms rent capital and labor in perfectly competitive factor markets. I assume that workers must be paid in advance. As a consequence, firms must borrow the wage bill, $W_t L_{j,t}$, from a financial intermediary. The loan plus the interest rate, R_t, must be repaid at the end of the period.

Firms choose prices to maximize the present value of profits; prices are set in a Calvo fashion; that is, each period, firms optimally revise their prices with an exogenous probability $1 - \xi_p$. If, instead, a firm does not re-optimize its price, then the price is updated according to the rule: $P_{j,t} = \pi_{t-1} P_{j,t-1}$, where π_{t-1} is the economy-wide inflation in the previous period. Consequently, an optimizing firm at time t sets prices according to the program

$$\max_{P_{j,t}} \mathbb{E}_t \sum_{n=0}^{\infty} (\xi_p \beta)^n \lambda_{t+n} \left[\frac{P_{j,t} \Pi_{\tau=0}^{n-1} \pi_{t+\tau}}{P_{t+n}} y_{t+n}(j) - mc_{t+n} y_{t+n}(j) \right]$$

Here, P_t is the price index, $y_t(j)$ is the aggregate demand for good type j, mc_t is firm j's marginal cost, β is the discount factor, and λ_t is the marginal utility of consumption at time t.

3.2. Households

The economy is populated by a continuum of households indexed by i. Every period households must decide how much to consume, work, and invest. In addition, they must choose the amount of money to be sent to a financial intermediary. I assume agents in the economy have access to complete markets; such an assumption is needed to eliminate wealth differentials arising from wage heterogeneity (CEE; Erceg *et al.*, 2000). Households maximize the expected present discounted value of utility

$$\mathbb{E}_0^i \sum_{t=0}^{\infty} \beta^t \left[S_t^u \log(C_{i,t} - bC_{t-1}) - \Phi \frac{L_{i,t}^{1+1/\gamma}}{1+1/\gamma} + \frac{\psi_m}{1-\zeta_m} \left(\frac{M_{i,t}}{S_t^* P_t} \right)^{1-\zeta_m} \right] \quad (1)$$

subject to

$$P_t C_{i,t} + \frac{P_t}{S_t^K}(I_{i,t} + a(x_t)K_{i,t}) + \mathcal{M}_{i,t} = R_t(\mathcal{M}_{i,t-1} - M_{i,t} + T_t) + R_t^K x_t K_{i,t} + W_{i,t} L_{i,t} + M_{i,t} + A_{i,t}$$

[3] The growth term is needed to have a well-defined steady state around which we can solve the model.

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and

$$K_{i,t+1} = (1-\delta)K_{i,t} + I_{i,t}\left(1 - \Gamma\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right)$$

Here, $\mathbb{E}_t^i$ is the time t expectation operator conditional on the information set of household i; S_t^u is a preference shock that follows the process $\log S_t^u = \rho_{Uc} \log S_{t-1}^u + \sigma_u \varepsilon_{u,t}$ with $\varepsilon_{u,t}$ distributed $\mathbb{N}(0,1)$; preferences display external habit formation measured by $b \in (0, 1)$; and Γ is a function reflecting the costs associated with adjusting the investment portfolio. This function is assumed to be increasing and convex satisfying $\Gamma = \Gamma' = 0$ and $\kappa \equiv \Gamma'' > 0$ in the steady state. $\mathcal{M}_{i,t-1}$ is household i's beginning-of-period-t stock of money, whereas T_t is a lump-sum transfer by the government. Households send the amount $\mathcal{M}_{i,t-1} - M_{i,t} + T_t$ to a financial intermediary where it earns the interest rate, R_t. Note that individual money holdings represented by $M_{i,t}$ are valuable as they provide utility. The stochastic trend, $S_t^* = S_t^L (S_t^K)^{\alpha/(1-\alpha)}$, in the money term is required to have a well-defined steady state. The term S_t^K is an investment specific shock whose growth rate obeys

$$\log g_t^K = (1 - \rho_{g^K}) \log g_{ss}^K + \rho_{g^K} \log g_{t-1}^K + \sigma_{g^K} \varepsilon_{g^K,t}$$

where $\varepsilon_{g^K,t}$ is distributed $\mathbb{N}(0,1)$.

As in ACEL, CEE, and Schmitt-Grohe and Uribe (2004), I assume that physical capital can be used at different intensities. Furthermore, using the capital with intensity x_t entails a cost $a(x_t)$, which satisfies $a(1) = 0$; $a''(1) > 0$; $a'(1) > 0$. For future reference, define $\varkappa_a = a''(1)$.

The term $A_{i,t}$ captures net payments from complete markets and government issued bonds, and profits from producers. The individual consumption good is assumed to be a composite made of differentiated goods indexed by j according to the aggregator

$$C_{i,t} = \left(\int_0^1 c_t(i, j)^{\frac{\zeta-1}{\zeta}} \mathrm{d}j\right)^{\frac{\zeta}{\zeta-1}}, \quad 1 \le \zeta < \infty$$

where $c(i, j)$ is the demand of household i for good type j. With this type of composite good, the demand for goods of type j is given by

$$c(i, j) = \left(\frac{P_{j,t}}{P_t}\right)^{-\zeta} C_{i,t}$$

Here, the nominal price index is $P_t = \left(\int_0^1 P_{j,t}^{1-\zeta} \mathrm{d}j\right)^{\frac{1}{1-\zeta}}$. Similarly, I assume that individual investment obeys $I_{i,t} = \left(\int_0^1 I_t(i, j)^{\frac{\zeta-1}{\zeta}} \mathrm{d}j\right)^{\frac{\zeta}{\zeta-1}}$. As with consumption, $I(i, j)$ denotes household i's demand for an investment good of type j.

3.3. Wage Setting

Following Erceg *et al.* (2000), I assume that each household is a monopolistic supplier of a differentiated labor service, $L_{i,t}$. Households sell these labor services to a competitive firm that

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aggregates labor and sells it to final firms. The technology used by the aggregator is

$$\widetilde{L}_t = \left[\int_0^1 L_{i,t}^{\frac{\zeta_w - 1}{\zeta_w}} \, \mathrm{d}j \right]^{\frac{\zeta_w}{\zeta_w - 1}}, \quad 1 \le \zeta_w < \infty$$

It is straightforward to show that the relationship between the labor aggregate and the wage aggregate, W_t, is given by

$$L_{i,t} = \left[\frac{W_t}{W_{i,t}} \right]^{\zeta_w} \widetilde{L}_t$$

To induce wage sluggishness, I assume that households set their wages in Calvo fashion. In particular, with exogenous probability ξ_w a household does not re-optimize wages each period. Hence, wages are set according to the rule of thumb $W_{i,t} = \pi_{t-1} W_{i,t-1}$. Similarly to the firms, households set wages according to the program

$$\max_{W_{i,t}} \mathbb{E}_t \sum_{n=0}^{\infty} (\xi_w \beta)^n \left[-\Phi \frac{L_{i,t+n}^{1+1/\gamma}}{1 + 1/\gamma} + \lambda_{t+n} \frac{W_{i,t} \Pi_{\tau=0}^{n-1} \pi_{t+\tau}}{W_{t+n}} \frac{W_{t+n}}{P_{t+n}} L_{i,t+n} \right]$$

The marginal utility of consumption, λ, is not indexed by i because of our assumption of complete markets.[4]

3.4. Government

The monetary authority sets the short-term interest rate according to a Taylor rule. In particular, the central bank smooths interest rates and responds to deviations of actual inflation from steady-state inflation, π, and deviations of output from its trend level, $(Y/S^*)_t$.

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_r} \left[\left(\frac{\pi_t}{\pi} \right)^{\phi_\pi} \left(\frac{Y_t / S_t^*}{(Y/S^*)_t} \right)^{\phi_y} \right]^{1-\rho_r} \exp(\sigma_m \varepsilon_{m,t}) \tag{2}$$

The term $\varepsilon_{m,t}$ is a random shock to the systematic component of monetary policy and is assumed to be standard normal; σ_m is the size of the monetary shock. Other authors have implemented similar Taylor rules (e.g., Del Negro *et al*., 2004; Justiniano and Primiceri, 2006).

As in the related literature (ACEL; CEE; Levin *et al*., 2005), I assume that the government has access to lump-sum taxes and debt. Accordingly, the government's budget constraint is

$$\mathcal{M}_t - \mathcal{M}_{t-1} + \frac{B_t^g}{1 + R_{g,t}} - B_{t-1}^g = T_t + G_t$$

where $\mathcal{M}$ is the aggregate stock of money and B^g corresponds to government debt whose price is $(1 + R_g)^{-1}$. As in Justiniano and Primiceri (2006), I assume that government purchases are

[4] The wage rule is not indexed by the stochastic trend because the model is solved taking into account the distortions induced by wage heterogeneity. See Schmitt-Grohe and Uribe (2004) and Fernandez-Villaverde and Rubio-Ramirez (2007b) for details.

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a stochastic fraction of output: $G_t = S_{g,t} Y_t$; and that the law of motion for S_g is $\log S_{g,t} = (1-\rho_g)\log S_g + \rho_g \log S_{g,t-1} + \sigma_g \varepsilon_{g,t}$, where $\varepsilon_{g,t}$ has a standard normal distribution.

3.5. Financial Intermediaries

Financial intermediaries receive money from two sources: households' deposits and transfers from the government, $\int(\mathcal{M}_{i,t-1} - M_{i,t})\mathrm{d}i + T_t$. All this money is lent to final-good firms so they can pay workers at the beginning of each period. Consequently, the clearing condition in the loan market is $\int(W_t L_{j,t})\mathrm{d}j = T_t + \int(\mathcal{M}_{i,t-1} - M_{i,t})\mathrm{d}i$.

4. ESTIMATION

I estimate the model using different sets of observables. Since the model allows for growth over time, I follow the standard practice of rescaling the growing variables by appropriate factors so that the new model is stationary. The resulting model is then solved using a linear approximation about the steady state. The baseline case considers seven variables: output, consumption, investment, real wages, labor, nominal interest rates, and inflation. Several recent DSGE papers have used those variables for estimation purposes; hence they seem a natural starting point.[5] Next, I successively suppress one observable variable at a time and re-estimate the model. The resulting median estimates and 90% probability intervals are reported in Table III.

To provide a deeper analysis, two additional cases are also considered. First, as in ACEL and Fernandez-Villaverde and Rubio-Ramirez (2007b), I include the price of investment as an extra observable variable. Second, I estimate the model using the following six observable variables: output, investment, real wages, inflation, interest rates, and the price of investment. This second case tries to relate my estimates to those coming from the estimation of small New Keynesian models (e.g., An and Schorfheide, 2007; Ireland, 2004; Rabanal and Rubio-Ramirez, 2005).[6]

A total of 10 alternative cases are evaluated. While this list of alternatives is far from exhaustive, it provides concrete evidence that the set of observables does indeed affect the estimated parameters, as well as the model's predictions. Furthermore, the results presented here are a useful blueprint for a researcher considering omitting a particular observable for estimation.

- **Data**

In the baseline case, the model is estimated using seven US variables: the growth rates of output, consumption, investment, and real wages ($\Delta \ln Y_t$, $\Delta \ln C_t$, $\Delta \ln I_t$, $\Delta \ln(w/P)_t$); and the levels of labor, nominal interest rates, and inflation ($\ln L_t$, i_t, π_t). The data correspond to that used in Justiniano and Primiceri (2006), which is extracted from the Haver Analytics database and spans 1954:III to 2004:IV. For inference purposes, I use the data up to 1999:IV. This shorter sample is chosen so that an out-of-sample forecast can be used as an additional measure of performance. The inclusion of the sample 2001–2004 has no significant impact on the results (see the working paper version).

[5] Examples include Del Negro *et al*. (2004) and Justiniano and Primiceri (2006).
[6] This choice of observables is also used by Fernandez-Villaverde and Rubio-Ramirez (2007b).

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The series are built as follows: real GDP results from dividing nominal GDP by population (16 years and older) and the GDP deflator. Real consumption is the sum of personal consumption of non-durables and services. Real investment consists of personal consumption expenditures of durables and gross private domestic investment. To obtain per capita measures, both real consumption and real investment are divided by population. The log of hours of all persons in the non-farm business sector divided by population corresponds to labor. Real wages result from dividing the nominal wage per hour in the non-farm business sector by the GDP deflator. Interest rates correspond to the effective Federal Funds Rate, while inflation is the quarterly log difference of the GDP deflator. Finally, when the price of investment is included in the estimation, I use Fisher's (2006) investment series.

- **Bayesian inference**

Following Schorfheide (2000), Smets and Wouters (2007), and Del Negro *et al*. (2004), I estimate the log-linearized version of the model using Bayesian methods. In particular, the posterior distribution of the structural parameters is characterized using a Markov chain Monte Carlo (MCMC) approach (for details of this algorithm, see the Appendix and the excellent surveys of An and Schorfheide, 2007, and Geweke, 2005). Since there are seven observable variables and only five structural shocks, I avoid stochastic singularity by following Sargent (1989) in including measurement errors to the state space representation used to estimate the model.[7],[8] These errors are assumed to be i.i.d. and distributed $\mathbb{N}(0, \sigma)$, where the scale can vary across the measurement equations (Section 6 elaborates on the implications of those errors on model specification). The results in the next sections are based on a Markov chain of 150,000 draws after discarding 20,000 replications from a burn-in phase. The convergence properties of the chains are discussed in the Appendix.

- **Priors**

A subset of the parameter space was fixed: $\alpha = 0.36$, $\delta = 0.025$, $\zeta = 6$, $\zeta_w = 21$, $S_g = 0.22$, $\zeta_m = 10.58$, $\psi_m = 0.055$. Initial estimation attempts showed that the posteriors of the elasticities of substitution, ζ and ζ_w, sat on top of their priors. Hence those parameters are fixed to the values used in CEE. A similar situation holds for the money demand parameters, ζ_m and ψ_m. The steady-state fraction S_g was set to match the average share of government expenditure in output in the sample. The parameter Φ is endogenously determined because I choose to estimate steady-state labor. The prior distributions for the remaining parameters are reported in Table I. These priors are loose and consistent with those typically used in the literature (see Del Negro *et al*., 2004; Levin *et al*., 2005; Justiniano and Primiceri, 2006). For future reference, let Ξ denote the set of all parameters to be estimated.

[7] Since we observe the exact values of interest rates, measurement errors were not included in the equation corresponding to interest rates in the state space representation.
[8] Each time an observable was removed, so was its measurement error from the estimation process.

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5. RESULTS

5.1. Posterior Distributions

Table II reports the median estimates for the elements in the set Ξ under the baseline case (estimation based on seven observables). Numbers in parentheses correspond to the 5th and 95th percentiles computed with the draws from the posterior simulator (a 90% probability interval). The shocks, σ_i, are expressed in percentage points. The absence of the price of investment as an observable implies that the two trends in the model, S^L and S^K, are not separately identified in the baseline case. Therefore, the steady-state growth rate of the investment-specific shock is set to one, $g^K = 1$. The last six entries in the third panel of Table II correspond to the scales of the measurement errors.

Broadly speaking, the estimates are in line with the results previously found in the literature (CEE; Smets and Wouters, 2007). For example, prices and wages are re-optimized on average every 5.5 and 3.3 quarters, respectively. The model displays significant habit formation, around 0.91, and some adjustment costs in investment, around 0.94. These values are consistent with the recent evidence reported in Fernandez-Villaverde and Rubio-Ramirez (2007). The estimated Taylor rule implies that the central bank actively responds to inflation and smoothes lagged interest rates with coefficients similar to those found in Justiniano and Primiceri (2006). The Frisch elasticity, γ, is well within the values used in related studies. CEE, for example, set this elasticity to 1. Note that the estimate for the growth rate of neutral technology, g^L, is lower than the quarterly average growth rate of output. This counterfactually low estimate seems to be driven by the interaction among inflation, interest rates, and the discount factor (more on this in Section 5). Finally, a quick comparison between priors and posteriors suggests that the data provide information about the structural parameters in the model (see Section 7 for additional details).

Next, I report the impact that the removal or addition of observables has on the structural parameters' median estimates, their probability intervals, and the model's impulse responses. Understanding the effects that each observable has on each single parameter and impulse response is a daunting task. In fact, each case can be easily the subject of a separate paper. The general equilibrium aspect of the model makes comparisons across sets of observables even harder. Hence I pursue a more modest approach and concentrate only on a reduced number of parameters, impulse responses, and correlations among variables. To facilitate the analysis, the parameter space is divided into four categories: sticky, Taylor rule, structural shock, and others. The median estimates and 90% probability intervals are reported in Table III (the complete set of results is available upon request).

5.2. Sticky Parameters

The recent DSGE literature has emphasized the crucial role of price stickiness and habit formation in capturing salient features of the data (ACEL and CEE) and in shaping optimal policies (Schmitt-Grohe and Uribe, 2004). Hence it seems natural to start the discussion with those parameters. Let us start by recalling that these parameters try to capture smoothness present in the data, specifically in inflation (ξ_p), real wages (ξ_w), and consumption (b). The values in Table III indicate that the duration of the wage contract can be on average as low as 1.5 quarters and as high as 5 quarters. Furthermore, the frequency of price changes ranges between 3.4 and 33 quarters.

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Table I. Priors densities for structural parameters

σ_m	σ_{g_L}	σ_{g_K}	σ_u	b	ξ_w	ξ_p	γ	ρ_R	ϕ_π
IG	IG	IG	IG	B	B	B	N	B	N
[2,2]	[2,2]	[2,2]	[2,2]	[0.5,0.1]	[0.5,0.1]	[0.5,0.1]	[1,0.15]	[0.75,0.1]	[1.70,0.3]
	ϕ_y	κ	$100(g_L - 1)$	$100(g_K - 1)$	$100(\pi - 1)$	L	$\varkappa_a$	β	
	G	N	N	N	N	N	N	B	
	[0.12,0.1]	[3,1]	[0.5,0.1]	[0.5,0.1]	[0.5,0.1]	[52.89,5]	[0.17,0.1]	[0.99,0.002]	
ρ_{g_L}	ρ_{g_K}	ρ_{Uc}		σ_{out}	σ_{cons}	σ_{invest}	σ_{labor}	σ_{wage}	σ_{inflat}
B	B	B		IG	IG	IG	IG	IG	IG
[0.5,0.15]	[0.5,0.15]	[0.5,0.15]		[0.05,0.03]	[0.05,0.03]	[0.05,0.03]	[0.05,0.03]	[0.05,0.03]	[0.05,0.03]

Note: IG, Inverse gamma; B, beta; N, normal; G, gamma. Mean and standard deviation in square brackets.

Table II. Estimated parameters baseline case

σ_m	σ_{g_L}	σ_{g_K}	σ_u	σ_{gov}	b	ξ_w	ξ_p	γ	ρ_R	ϕ_π
0.28	0.74	0.35	2.79	0.55	0.91	0.62	0.82	1.66	0.68	1.63
[0.25,0.31]	[0.65,0.83]	[0.28,0.43]	[2.04,3.90]	[0.37,0.79]	[0.87,0.94]	[0.54,0.69]	[0.80,0.85]	[1.10,2.45]	[0.62,0.73]	[1.49,1.77]
ϕ_y	κ	α	$100(g_L - 1)$	$100(\pi - 1)$		L	ζ	ζ_w	$\varkappa_a$	β
0.057	0.94	0.36	0.23	0.52		54.32	6	21	0.41	0.9943
[0.038,0.082]	[0.64,1.44]	[NA]	[0.17,0.32]	[0.37,0.71]		[51.49,58.21]	[NA]	[NA]	[0.26,0.57]	[0.993,0.996]
ρ_{g_L}	ρ_{g_K}	ρ_{Uc}	ρ_g	$100(g_K - 1)$	σ_{out}	σ_{cons}	σ_{invest}	σ_{labor}	σ_{wage}	σ_{inflat}
0.26	0.91	0.98	0.98	0	0.12	0.42	2.11	0.051	0.22	0.49
[0.60,0.78]	[0.86,0.78]	[0.96,0.99]	[0.96,0.99]	[NA]	[0.03,0.31]	[0.38,0.48]	[1.89,2.41]	[0.02,0.10]	[0.20,0.25]	[0.44,0.54]

Note: L, steady-state labor; π, steady-state inflation; g_L, growth rate of neutral technology g_k, growth rate of investment-specific shock.

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Table III. Estimated parameters

	σ_m	σ_{gL}	σ_{gK}	σ_u	σ_{gov}	b	ξ_w	ξ_p	γ
Baseline	0.28	0.74	0.35	2.79	0.55	0.91	0.62	0.82	1.66
	[0.25,0.31]	[00.65,0.83]	[00.28,0.43]	[02.04,3.90]	[00.37,0.79]	[00.87,0.94]	[00.54,0.69]	[0.80,0.85]	[01.10,2.45]
No labor	0.27	1.01	0.55	2.29	0.54	0.87	0.69	0.82	1.15
	[0.25,0.30]	[00.76,1.32]	[00.41,0.74]	[01.39,3.61]	[00.36,0.77]	[00.81,0.91]	[00.61,0.76]	[0.77,0.85]	[00.73,1.78]
No consump.	0.32	0.65	0.24	3.47	0.51	0.92	0.80	0.97	1.30
	[0.29,0.35]	[00.53,0.75]	[00.18,0.33]	[02.15,5.51]	[00.34,0.72]	[00.87,0.96]	[00.73,0.85]	[0.96,0.98]	[00.81,2.02]
No real wage	0.26	0.62	0.59	11.40	0.69	0.97	0.52	0.91	0.63
	[0.24,0.29]	[00.54,0.70]	[00.49,0.68]	[08.42,15.4]	[00.43,0.80]	[00.95,0.98]	[00.40,0.63]	[00.88,0.93]	[00.88,0.94]
No inflation	0.20	0.69	0.25	2.34	0.49	0.86	0.46	0.88	1.53
	[0.16,0.23]	[00.60,0.76]	[00.20,0.30]	[01.74,3.22]	[00.34,0.68]	[00.77,0.91]	[00.36,0.56]	[0.85,0.91]	[00.97,2.31]
No interest	0.43	0.62	0.36	4.10	0.48	0.91	0.73	0.81	0.59
	[0.28,0.64]	[00.55,0.70]	[00.27,0.45]	[02.66,6.06]	[00.33,0.69]	[00.88,0.94]	[00.67,0.78]	[0.77,0.84]	[00.38,0.96]
No output	0.26	0.41	3.18	4.10	0.60	0.94	0.34	0.85	1.06
	[0.24,0.29]	[00.35,0.49]	[02.75,3.65]	[02.89,5.80]	[00.41,0.86]	[00.90,0.96]	[00.28,0.53]	[0.84,0.87]	[00.68,1.61]
Price invest.	0.28	0.60	0.19	3.37	0.57	0.93	0.66	0.83	1.44
	[0.25,0.31]	[00.52,0.70]	[00.16,0.23]	[02.28,4.98]	[00.39,0.82]	[00.89,0.95]	[00.56,0.74]	[0.79,0.85]	[00.90,2.20]
No wage/con.	0.28	0.33	0.23	1.80	0.47	0.67	0.57	0.71	1.24
	[0.24,0.32]	[00.27,0.40]	[00.19,0.28]	[01.19,2.86]	[00.32,0.68]	[00.58,0.86]	[00.46,0.68]	[0.63,0.78]	[00.81,1.89]
No investment	0.27	0.46	0.44	3.15	1.47	0.89	0.50	0.79	1.46
	[0.24,0.30]	[00.38,0.55]	[00.34,0.53]	[02.27,4.43]	[01.07,1.89]	[00.84,0.93]	[00.38,0.61]	[0.75,0.82]	[00.94,2.18]

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Table III. (continued)

	ρ_R	ϕ_π	ϕ_y	κ	β	$100(g_L-1)$	$100(g_k-1)$	$100(\pi-1)$
Baseline	0.68	1.63	0.057	0.94	0.9943	0.23	*NA*	0.52
	[0.62,0.73]	[01.49,1.77]	[00.038,0.082]	[00.64,1.44]	[00.993,0.996]	[00.16,0.32]		[0.37,0.71]
No labor	0.76	1.67	0.042	1.44	0.9949	0.26	*NA*	0.56
	[0.71,0.81]	[01.53,1.81]	[00.02,0.07]	[00.89,2.26]	[00.993,0.996]	[00.19,0.34]		[0.39,0.75]
No consump.	0.70	1.77	0.0026	2.70	0.9908	0.23	*NA*	0.44
	[0.65,0.75]	[01.64,1.91]	[00.000,0.009]	[01.93,3.78]	[00.989,0.993]	[00.16,0.34]		[0.33,0.59]
No real wage	0.93	1.54	0.49	2.41	0.9809	0.27	*NA*	0.48
	[0.88,0.96]	[01.36,1.70]	[00.29,0.80]	[01.53,3.55]	[00.977,0.985]	[00.19,0.35]		[0.33,0.65]
No inflation	0.24	1.68	0.26	0.066	0.9779	0.38	*NA*	0.47
	[0.14,0.35]	[01.54,1.83]	[00.19,0.34]	[00.03,0.12]	[00.973,0.982]	[00.27,0.58]		[0.33,0.65]
No interest	0.83	1.70	0.077	0.74	0.9694	0.33	*NA*	0.53
	[0.75,0.90]	[01.56,1.85]	[00.04,0.144]	[00.53,1.03]	[00.964,0.975]	[00.25,0.41]		[0.38,0.73]
No output	0.72	1.56	0.032	6.22	0.9965	0.38	*NA*	0.49
	[0.67,0.76]	[01.44,1.69]	[00.02,0.045]	[05.05,7.44]	[00.995,0.997]	[00.31,0.54]		[0.35,0.67]
Price invest.	0.63	1.39	0.04	1.51	0.9950	0.10	0.14	0.45
	[0.58,0.69]	[01.26,1.53]	[00.03,0.06]	[02.33,0.96]	[00.993,0.996]	[00.05,0.18]	[00.07,0.23]	[00.32,0.62]
No wage/con.	0.60	1.27	0.030	2.91	0.9949	0.19	0.13	0.51
	[0.54,0.66]	[01.19,1.38]	[00.022,0.041]	[01.73,4.49]	[00.993,0.996]	[00.12,0.27]	[00.07,0.23]	[00.49,0.61]
No investment	0.74	1.61	0.067	3.99	0.9948	0.39	*NA*	0.49
	[0.69,0.79]	[01.47,1.76]	[00.039,0.118]	[02.88,5.28]	[00.993,0.996]	[00.28,0.50]		[0.35,0.67]

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According to the results in Table III, consumption and real wages are crucial for the estimation of habit formation. Dropping consumption as an observable removes an important source of identification for b, which in turn explains the low value reported here (0.67). Intuitively, the inference approach tries to improve the model's performance along some dimensions, say output and investment, at the expense of worsening the consumption block. This is now possible because consumption is not an object of interest from the econometric perspective. Indeed, the forecast exercise and the correlations reported in the next section confirm that the model estimated without consumption does well in all areas but consumption. Interestingly, there is a sizable decline in the stickiness of prices and wages ($\xi_p = 0.71$ and $\xi_w = 0.57$). This result in turn suggests that sluggishness is required in part for the model to explain the time series of consumption. In the absence of that observable, the model demands less persistence and thus the lower Calvo probabilities.

Observe that omitting real wages has important consequences for the sticky parameters. Without that observable, the wage contract parameter is only identified through its effects on inflation via the marginal costs. Recall that a simplified log-linear version of the Phillips curve (Woodford, 2003) requires

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-\xi_p)(1-\xi_p\beta)}{\xi_p} mc_t \tag{3}$$

where the marginal cost, mc_t, is a function of real wages. Hence sticky wages and sticky prices now compete to capture the time path of inflation. The identification strategy chooses a large degree of price stickiness (0.91) accompanied by moderate wage sluggishness (0.52). The low sticky wage contract is now feasible since real wages are not observed. Furthermore, we would expect wages to be disconnected with other variables in the model. This is precisely what we see in the correlations reported in Table IV.

Note also that ξ_p is identified via the Phillips curve, i.e., through the effects that inflation has on the model. Therefore, the absence of inflation eliminates the identifying source of the price contract parameter. The estimation procedure matches a high degree of price persistence (0.88) with frequent wage contract revisions (0.46). Of course, lower wage stickiness implies more volatile real wages, reducing its correlation with, say, consumption and output. Indeed, the results in the next section confirm that real wages are negatively correlated with consumption when the model is estimated without inflation.

5.3. Taylor Rule

Interest rates and inflation are two essential components of the Taylor rule (equation (2)). Hence it is hardly surprising that when we exclude any of those variables from the set of observables the estimates of the interest rate rule experience a notable change. As already discussed, without inflation the estimation strategy imposes a high degree of price persistence. This extra smoothness is compensated in part by driving down the interest-rate smoothing coefficient. The combined effect (high ξ_p and low ρ_R) seems to imply an inflation path resembling that predicted in the baseline case. Consequently, the inflation response coefficient, ϕ_π, is similar in the two cases (1.63 and 1.68, respectively).

Omitting real wages creates the most significant departures for the Taylor rule. For instance, the persistence component, ρ_R, is 36% larger in the absence of real wages (compare 0.93 with 0.68 in the baseline case). Furthermore, the output reaction coefficient, ϕ_y, is nine times bigger

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Table IV. Correlations in DGSE model and data

	y	c	i	w	Π	R	y	c	i	w	π	R
	Baseline (panel 1)						No real wage (panel 2)					
y	1.00	0.69	0.98	0.80	−0.33	−0.31	1.00	0.84	0.92	0.31	−0.41	−0.36
c		1.00	0.51	0.53	−0.44	−0.18		1.00	0.58	0.15	−0.44	−0.30
i			1.00	0.79	−0.26	−0.33			1.00	0.26	−0.21	−0.26
w				1.00	−0.43	−0.44				1.00	−0.27	−0.22
π					1.00	0.84					1.00	0.84
R						1.00						1.00
	No inflation (panel 3)						No real wage/consumption (panel 4)					
y	1.00	0.44	0.71	−0.29	−0.22	−0.57	1.00	0.29	0.94	0.36	−0.40	−0.34
c		1.00	−0.32	−0.85	−0.60	−0.42		1.00	0.15	0.35	−0.33	0.01
i			1.00	0.36	0.25	−0.26			1.00	0.23	−0.26	−0.18
w				1.00	0.29	0.27				1.00	−0.46	−0.38
π					1.00	0.69					1.00	0.82
R						1.00						1.00
	Baseline with $\xi p = 0.90$ (panel 5)						Baseline with $\xi w = 0.00$ (panel 6)					
y	1.00	0.69	0.97	0.83	−0.27	−0.27	1.00	0.81	0.97	0.78	−0.35	−0.39
c		1.00	0.49	0.60	−0.49	−0.25		1.00	0.65	0.84	−0.48	−0.28
i			1.00	0.79	−0.15	−0.25			1.00	0.64	−0.27	−0.42
w				1.00	−0.38	−0.44				1.00	−0.29	0.00
π					1.00	0.82					1.00	0.84
R						1.00						1.00
	Baseline with $b = 0.7$ (panel 7)						Data (panel 8)					
y	1.00	0.61	0.96	0.79	−0.26	−0.26	1.00	0.51	0.89	0.15	−0.31	−0.23
c		1.00	0.36	0.39	−0.14	0.12		1.00	0.24	0.15	−0.42	−0.19
i			1.00	0.79	−0.24	−0.34			1.00	0.11	−0.20	−0.19
w				1.00	−0.41	−0.43				1.00	−0.23	−0.19
π					1.00	0.84					1.00	0.68
R						1.00						1.00

Note: Correlations for output (y), consumption (c), investment (i), real wages (w), inflation (π), and interest rates (R).

than in the baseline case. To understand these results, recall that the structural shocks are more volatile when estimated without real wages (see next subsection). Hence the significant rise in the interest-rate smoothing coefficient as well as in the output reaction term of the Taylor rule (0.49) is needed to bring the model's volatility predictions more in line with the data.

5.4. Structural Shocks

The Taylor rule establishes a clear link between the monetary policy disturbance σ_m and interest rates. Therefore, it is hardly surprising that the size of that shock significantly increases when interest rates are not an observable. The absence of real wages has a notable effect on the size of the preference disturbance, σ_u. This shock is now almost four times larger than in the baseline case

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(compare 11.40 and 2.79). To understand this effect, temporarily assume that the model displays flexible wages. Then the optimal wage decision by households is

$$(\zeta_w - 1)S_t^u U_{c,t}\frac{W_t}{P_t} = \Phi\zeta_w L_t^{1/\gamma} \tag{4}$$

This Euler equation reveals that wages directly interact with the preference shock. Obviously, a larger preference shock tends to induce more volatility in consumption. Hence we can understand the large habit formation (0.97) as an attempt by the inference strategy to make consumption in the model comparable with the data. Continuing with the no-wage case, note the size the investment-specific, σ_{g^k}, and the government, σ_{gov}, shocks rise. Larger shocks increase the overall volatility predicted by the model. Hence we can interpret the larger coefficients in the Taylor rule as a way to diminish the discrepancy between the data and the model's predictions.

The volatility of the investment-specific shock is greatly amplified in the absence of output as an observable. Such an increase in σ_{g^k} is likely to be an attempt by the model to match the large volatility of investment present in the data. This is now feasible since the additional volatility induced in output is no longer a concern (recall that output is left out of the estimation). At the same time, we observe a sizeable decline in the size of the neutral technology shock.

5.5. Other Parameters

If the curvature of the investment cost function (κ) is the object of interest, the results in Table III show that that output, investment, and inflation strongly influence its estimated value. When the latter observable is omitted, the inference strategy places a low value for the volatility of the investment-specific shock. But less volatile disturbances induce less volatility on investment, which decreases the need for adjustment costs to match the investment series found in the data—hence the low estimate reported in Table III. However, there are other cases (price of investment or no inflation) in which a drop in the volatility of the investment shock is associated with an increase in the adjustment cost. In general, the costly investment coefficient seems to be very sensitive to the set of observables. Note that six out of 10 alternative cases delivered estimates that were statistically different from those of the baseline case.

The discount factor, β, obtains its lowest median estimate when the interest rate is excluded from the estimation. Such a result is expected since interest rates and the discount factor are linked through the Euler equation for money holdings $\mathcal{M}_t$

$$\lambda_t = \beta\mathbb{E}_t\lambda_{t+1}R_{t+1}/\pi_{t+1} \tag{5}$$

In steady state, this equation imposes $\beta = R^{-1}\pi g^*$. Consequently, the absence of interest rates leaves R and therefore β weakly identified in the model.[9] The discount factor also enters the Phillips curve (equation 3), which helps in understanding why that parameter is affected when inflation is left out of the set of observables.

The previous steady-state condition also stresses the link between interest rates and growth in steady state. In particular, note that product $\pi g^*\beta^{-1}$ determines the steady state nominal interest

[9] Note, however, that the absence of interest rates does not leave the discount factor unidentified. This parameter also influences the Phillips curves for prices and wages, and the investment decision equation. Hence inflation or real wages help in the identification of β.

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rate. Therefore, the estimation approach faces a trade-off when setting the growth of technology in the model. On the one hand, it has to match the average growth found in the data. On the other hand, the estimated growth rate must be compatible with the steady-state interest rate. The estimation strategy in the baseline scenario balances this trade-off with a low growth rate of technology—hence the results in Table III. When we omit the interest rates, the second trade-off vanishes, which contributes to the rise of the estimated growth rate of the economy (sixth row in Table III).

For the Frisch elasticity of labor supply, note that interest rates and real wages affect its estimation. In particular, the absence of former variable implies an elasticity almost three times smaller than in the baseline case. According to the labor decision equation (4), smaller values of the Frisch elasticity, γ, help to match volatile series of the marginal utility of consumption and real wages with smooth profiles of labor. The additional volatility in the former variables is a consequence of bigger shocks (σ_u and σ_m) and smaller wage contracts obtained when interest rates are omitted for estimation purposes (see Table III, row 6).

5.6. Impulse Responses

Now I turn to the implications that the different cases have on the model's economic predictions as summarized by impulse responses. As in the previous analysis, reporting a comprehensive analysis of each single impulse response requires a significant amount of space and is beyond the purpose of this paper.[10] Instead, I provide a brief discussion of general patterns emphasizing the idea that the variables used to estimate the model do indeed influence the model's outcome. Figures 1–3 present the impulse responses of the estimated models to one standard deviation of the structural shocks at time 0. Values in the vertical axes correspond to percentage deviations from the variables' steady-state trends. For all cases, the impulse responses are computed using the median estimates reported in Table III. For reference, starred lines indicate the predictions using the parameter estimates from the baseline case, while dotted lines correspond to the upper and lower bounds of a 90% probability interval for the baseline case.

Consistent with the point estimates in Table III, the impulse responses for the baseline case (starred lines) are broadly in line with the findings in the literature (see, for example, ACEL and Del Negro *et al*., 2004). From a quick look at the impulse responses, we learn that the largest departures occur for the impulse responses following monetary and investment-specific shocks (Figures 1 and 2). For the former disturbance, note that the model estimated without interest rates predicts strong responses of output, consumption, and investment. To understand this result, recall that in the absence of interest rates the estimated model requires a combination of large monetary shocks ($\sigma_m = 0.43$) and significant persistence in the Taylor rule ($\rho_R = 0.83$). Facing a large and persistent decline in interest rates, households understand that holding bank deposits will not be profitable in the near future. Hence households deplete their bank accounts to increase their consumption. Ultimately, output must significantly increase to meet the additional demand for goods. From Figure 1 we also find that the absence of inflation implies flat impulse responses associated with a monetary shock. This finding is driven by the small monetary shock ($\sigma_m = 0.20$) and the low persistence in the Taylor rule ($\rho_R = 0.24$).

[10] There are five structural shocks, seven relevant variables, and 10 sets of parameters. As a consequence, we would need to discuss a total of 350 impulse responses!

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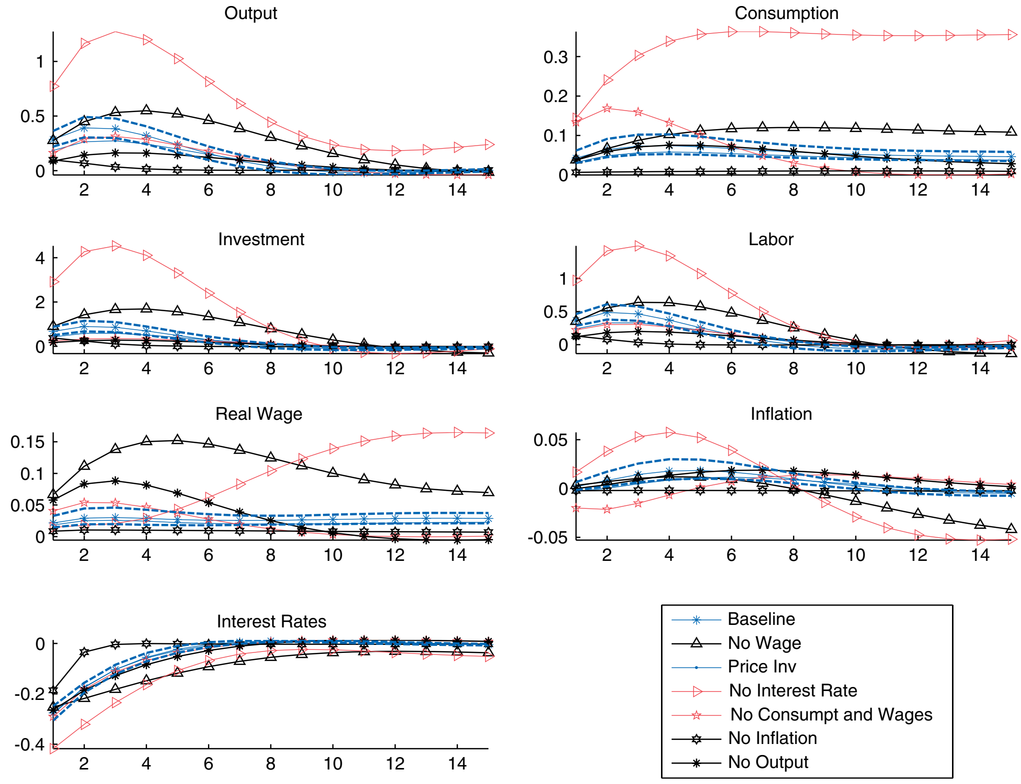

Figure 1. Responses to an expansionary monetary shock. This figure is available in color online at www.interscience.wiley.com/journal/jae

When we turn to the investment-specific impulse responses, the absence of real wages and consumption involves estimated parameters that make the model predict a persistent decline in output, investment, and labor (Figure 2). To understand this result, note that the econometric strategy estimates a very persistent process ($\rho_{g^K} = 0.97$). As a consequence, an investment-specific shock implies a persistent decline in the price of investment. The impulse response indicates that cheaper investment induces a strong contemporaneous wealth effect, which drives consumption up while contracting the labor supply and hence output.

In line with the previous analysis, the absence of real wages induces the largest deviations of the impulse responses after a neutral technology shock (Figure 3). This finding is a consequence of the more persistent process for the technology shock ($\rho_{g^L} = 0.80$ in the no-wage case versus $\rho_{g^L} = 0.40$ in the baseline case). With such a persistence disturbance, a technology shock significantly raises the productivity of labor, which pushes up the demand for labor as well as wages. Ultimately, higher wages invite households to increase their consumption and investment. Furthermore, the highly inertial response of inflation reflects the large duration of the price contract ($\xi_p = 0.91$). Without output as an observable, the estimation approach imposes a small volatility on the neutral technology shock—hence, the mild response of output and consumption reported in Figure (3).

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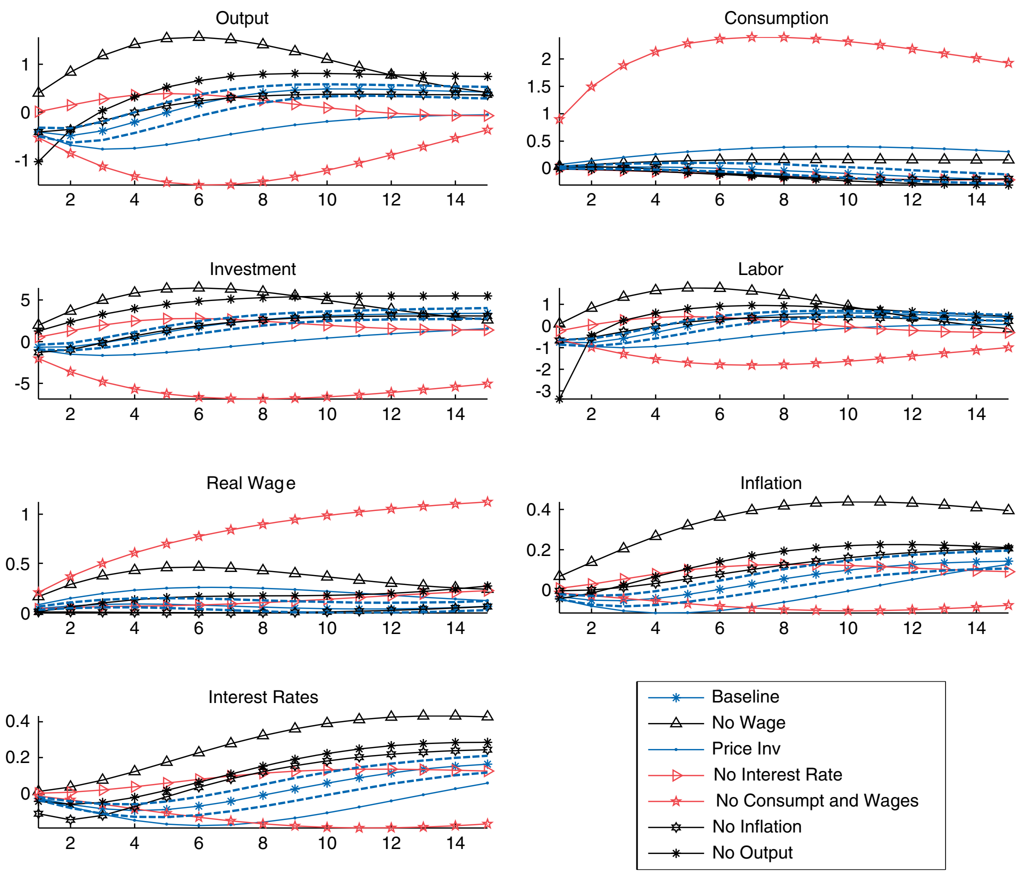

Figure 2. Responses to a positive investment-specific technology shock. This figure is available in color online at www.interscience.wiley.com/journal/jae

5.7. Correlations

To shed more light on the effects of observables, this section discusses the correlations implied by the estimated model for selected variables and sets of observables. Owing to space considerations, I discuss general aspects of the results, which are reported in the four uppermost panels in Table IV. According to the information there, the model under the baseline set of observables (panel 1) does a good job of replicating the correlations found in the data, which is particularly notable for output. Note also that wages display excessive co-movement with the other variables in the model relative to what we observe in the data (panel 8).

When the model is estimated without wages (panel 2), there is a significant decline in the correlation between real wages and the other variables. Such a result is expected since the drop in wage stickiness ($\xi_w = 0.52$) makes real wages more volatile and thus less correlated with the remaining variables in the model. Paradoxically, the correlations of real wages with the rest of variables in the model are more in line with those in the data. Omitting inflation (panel 3) induces a negative correlation between consumption and investment and consumption and real

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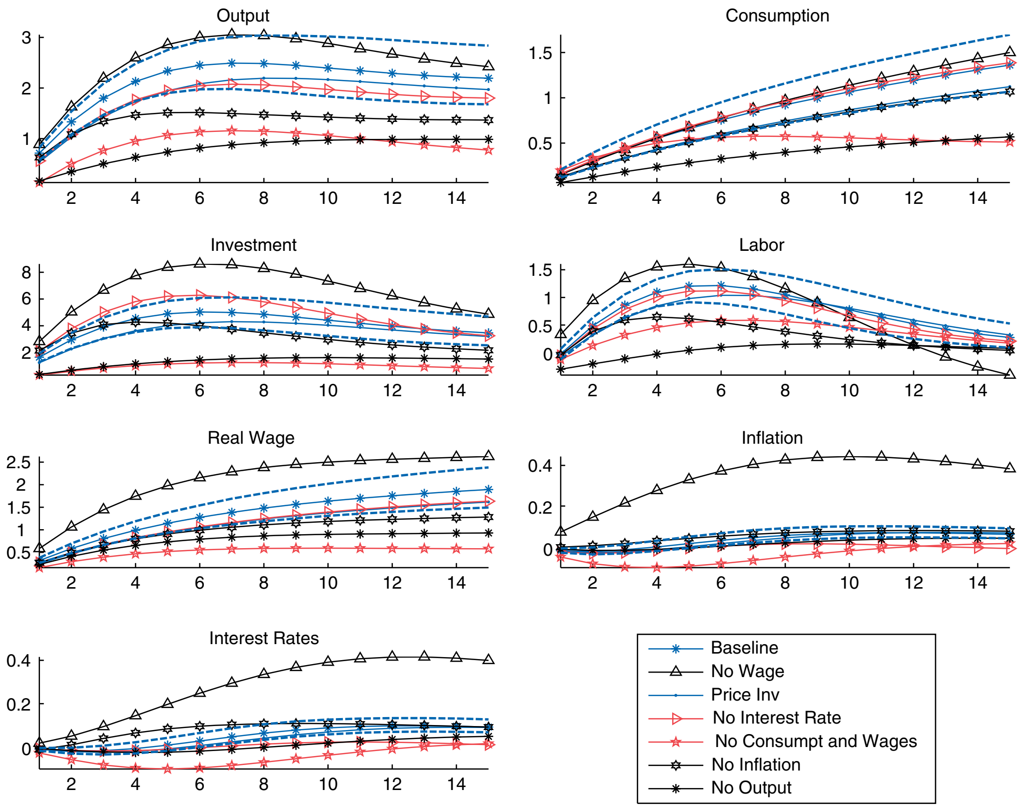

Figure 3. Responses to a positive neutral technology shock. This figure is available in color online at www.interscience.wiley.com/journal/jae

wages. Furthermore, investment is now positively correlated with inflation. These results suggest that even if the model adequately describes, say, inflation and real wages the model will fail explaining consumption and investment. We observe this trade-off in the out-of-sample exercise described in the next section.

Not surprisingly, if we leave consumption and wages outside the set of observables (panel 4), the model predicts a counterfactually low correlation of consumption with output, investment and in particular with interest rates. This result is likely driven by the relatively low habit formation estimate ($b = 0.67$), which induces higher consumption volatility and thus lower correlation. The large cost of capital adjustment ($\kappa = 2.91$) induces smoothness in investment, contributing to reduce its co-movement with other variables. Relative to the benchmark case, real wages are now less correlated with the rest of variables in the model, presumably because wages have become more volatile. The model can afford this additional variability since neither consumption nor real wages are used in the estimation.

The lower panels in Table IV display the correlations implied by the model using all but one of the parameters estimated with the baseline set of observables. For example, higher price stickiness (panel 5) makes inflation smoother and thus less correlated with other variables in the model.

Furthermore, wages and consumption become counterfactually more correlated (0.60 in the model versus 0.15 in the data).

With flexible wages (panel 6), consumption and real wages are linked directly via the labor decision equation (4)—hence the increase in the correlation between those two variables found in Table IV. When we recompute the correlations for the baseline scenario but with smaller habit formation (panel 7), note that consumption is less correlated with other variables in the model. Indeed, consumption is now predicted to be positively correlated with interest rates. To understand this result, recall that the Euler equation (5) implies

$$\beta\frac{R_t}{\pi_t} = \frac{\lambda_{t-1}}{\lambda_t} = \frac{c_t - bc_{t-1}}{c_{t-1} - bc_{t-2}}$$

Other things equal, a drop in habit formation causes contemporaneous consumption to co-move with the interest rates. Interestingly, the correlation between interest rates and inflation is not significantly affected in any of the specifications analyzed here.

Overall, the evidence in this section suggests that omitting a variable, say consumption, during estimation affects the part of the model pertaining to that variable. Therefore, the estimation procedure can use the parameters in that block, say habit formation and preference shocks, to improve the model's performance along other dimensions. Ultimately, the omitted observable becomes intrinsically uncorrelated with the remaining variables in the model.

6. RANKING THE ALTERNATIVES

The findings so far indicate that the set of observables plays a crucial role in the estimation of DSGE models as well as in determining the models' economic predictions. Choosing too few variables seems to deliver counterintuitive results, such as recessions following a positive investment-specific shock. On the other hand, there is the issue of data availability, the risk of model misspecification, and the computational burden.[11] Therefore, it seems desirable to have a way to assess the relative performance of alternative sets of observables. Posterior-odds comparisons, as in Geweke (2005), Schorfheide (2000) and Fernandez-Villaverde and Rubio-Ramirez (2004), become unfeasible due to the changing nature of the data. Here, I propose three ways to analyze the available combinations of observables.

Facing the results from the previous section, one possibility is to rank the combinations based on the estimated parameters. From micro-based studies (Bils and Klenow, 2002), we know that price contracts are adjusted on average every 6 months. Hence combinations of observables omitting real wages, consumption, or inflation could be ruled out since they imply too much price stickiness. On similar grounds, we could also eliminate the combination where output is absent since that implies adjustment costs in investment at odds with previous studies (Cooper and Haltiwanger, 2005). Leaving interest rates out of the estimation is unattractive because this alternative leads to excessive smoothness in the Taylor rule relative to that found in partial-information settings (see Clarida *et al*., 2000). Yet the presence of multimodality (next

[11] Each additional observable increases the size of the state space representation and, therefore, the time to estimate the model. One evaluation of the posterior distribution in the baseline case is on average 10% faster than in the price of investment case. This small difference becomes relevant when the posterior is evaluated hundred of thousands of times, as in the MCMC algorithm.

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section) makes choosing sets of observables based solely on estimated coefficients a risky business.

A second way to select among combinations of observables is to use impulse responses. In this case, we can discard the combination that excludes real wages and consumption (row 9, Table III) because the resulting estimated model predicts a counterfactual decline in output, investment, and labor after a positive investment-specific shock (the VAR evidence reported in ACEL suggests booms rather than recessions after such shocks). We can also discard the combination that omits inflation because it delivers negligible impulse responses after a monetary shock (CEE predicts strong responses). In a similar spirit, we can also use the predicted correlations to rule out sets of observables. Under this criterion, dropping inflation seems a bad choice as the model predicts a negative correlation between wages and output or wages and consumption, which is clearly at odds with the empirical correlations found in the data (see Table IV).

A more appealing option to rank the combinations is to study the out-of-sample forecasting performance of the model. To that end, the discussion is based on the following variables: the growth rates of output, consumption, investment, and real wages; and the levels of labor, inflation, and interest rates. The forecasting period corresponds to 2000:I–2004:IV; the choice is rather arbitrary and a consequence of the lack of guidance in the literature. The model was initially estimated over the sample 1954:III–1999:IV for each set of observables. The model was subsequently re-estimated every year. Table V presents the RMSE for three forecast horizons for each set of observables and each forecast variable. The statistics are computed using the posterior mode and normalized by the corresponding RSME obtained under the baseline case. Hence a number below one means better than the baseline case. The column labeled Ov reports the difference between the log determinant of the forecast error covariance matrix for each combination of observables and that of the benchmark case. This statistic is typically used as an indicative of overall forecasting performance (Smets and Wouters, 2007; Boivin and Giannoni, 2006). A number below zero means better than the baseline scenario.

The forecasting exercise reveals some interesting patterns. To begin with, the set of observables without real wages and consumption has the most significant impact on the forecasting performance. Its overall measure (column Ov) is the worst (largest relative to the benchmark) for all forecasting horizons. The worsening of the model's performance is quite notable for interest rates and inflation. The lack of correlation between interest rates and consumption and the excessive correlation between inflation and real variables (Table IV) already suggested that the model's forecast would be unsatisfactory.

Next, omitting inflation entails an estimated model with the second worst overall performance. Individual forecasts, in particular those for output, investment, and labor, are significantly affected by the absence of inflation. The correlations reported in the previous section already alerted us of the potentially poor performance of the model estimated without inflation. Indeed, Table IV indicates that investment and consumption are negatively correlated while interest rates and investment tend to co-move, which is clearly at odds with the data. To understand these findings, note that the adjustment cost parameter, κ, is estimated to be a very low number. Other things equal, investment and output become very volatile relative to the data, which in turn justifies the worsening of the forecasts reported in Table V.

Regarding individual forecasts, note that inflation, interest rates, investment, and labor are the most affected variables by the choice of observables. In four out of 10 cases, inflation's one-quarter-ahead forecast error worsened by more than any other variable. A similar pattern is found for the one- and three-year ahead forecasts. This result concurs with Boivin and Giannoni's (2006)

Table V. RMSE

	y	c	i	L	w	π	R	Ov
One-quarter-ahead forecast								
No labor	1.18	1.25	1.64	1.78	1.11	4.03	0.86	2.02
No real wage	1.05	1.42	1.00	1.74	1.34	3.39	1.45	2.69
Price investment	0.90	0.91	0.89	0.77	1.01	0.81	1.16	−0.02
No consumption	1.07	1.26	0.91	0.89	1.06	0.82	1.80	0.95
No interest rate	1.39	2.61	1.18	2.62	1.13	4.53	2.41	1.72
No wage/consump.	1.74	1.24	0.93	3.36	1.39	5.57	3.12	5.36
No inflation	6.38	2.12	10.3	9.45	1.67	1.57	1.76	3.65
No output	1.52	1.06	1.11	3.98	0.97	0.87	1.08	0.77
No investment	1.17	1.10	0.94	1.02	0.95	0.88	1.28	2.08
One-year-ahead forecast								
Nolabor	1.87	1.32	2.33	1.06	1.01	5.13	0.81	2.06
No real wage	1.42	0.99	1.42	0.85	1.04	3.36	1.55	2.72
Price investment	1.05	0.98	1.05	0.90	0.97	0.84	1.09	−0.02
No consumption	1.12	1.31	1.01	0.76	1.03	0.91	1.44	0.97
No interest rate	1.18	2.82	2.13	1.52	1.11	4.15	2.25	1.72
No wage/consump.	2.20	1.45	1.43	1.73	1.04	6.08	2.05	5.28
No inflation	8.03	2.23	11.4	6.07	1.01	2.89	3.32	4.04
No output	1.09	1.01	0.90	1.62	0.96	0.81	1.08	0.67
No investment	1.34	1.16	1.15	0.95	0.97	0.94	1.24	2.12
Three-year-ahead forecast								
No labor	2.30	1.22	3.05	0.62	1.01	5.41	0.68	1.96
No real wage	1.60	1.22	1.66	0.64	1.27	3.43	1.53	2.87
Price investment	1.08	0.98	1.11	1.00	1.00	1.07	1.03	0.00
No consumption	1.17	1.31	1.11	0.87	1.00	1.06	1.11	0.86
No interest rate	0.96	2.35	2.63	0.99	1.02	2.89	2.02	1.72
No wage/consump.	2.29	1.31	1.51	1.39	1.30	8.04	0.95	5.11
No inflation	6.26	1.97	9.09	2.77	1.13	2.64	2.50	3.94
No output	1.06	1.08	0.99	0.84	1.00	0.99	0.97	0.66
No investment	1.58	1.23	1.41	1.04	0.98	1.42	1.22	2.23

finding that forecasting inflation significantly benefits from the inclusion of additional observables such as alternative measures of inflation and economic activity.

Table V also reveals that omitting output has the smallest impact on the forecasts. We can rationalize this result by noticing that output and investment are highly correlated in the data (Table IV). Furthermore, the correlation of investment with nominal variables and real wages tends to be a good proxy of the correlation displayed by output with those variables. Hence the absence output as an observable seems to be compensated by the presence of investment. A similar argument explains why the model is not seriously affected by omitting consumption.

Based on the out-of-sample forecast performance, the estimated model with the price of investment as an observable has the edge at the one-quarter and one-year ahead forecasts. In particular, investment, inflation, and labor benefit the most from including the additional observable. To understand the superior forecast, note that investment and output display different trends in the data. Hence the presence of an additional trend in the model helps to track more closely the time profile of investment (more on this in the next section). This extra trend distorts the growth rate in the economy, g^*, and hence the estimate for the steady interest rate (recall $R = \beta^{-1}\pi^*$), which explains why the baseline case delivers a better forecast for interest rates.

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As we move to the three-year-ahead forecast, predicting inflation becomes more imprecise for most combinations of observables. Furthermore, based on the overall measure, we are indifferent between the baseline set of observables and the one which includes the price of investment.

6.1. Role of Measurement Errors

The forecasting exercises show that omitting observables may distort the economic predictions of DSGE models. Furthermore, the better forecasting performance of the model estimated with the price of investment suggests that larger datasets are perhaps desirable. If the model is correctly specified, as in the example in Section 2, additional observables can be beneficial as they may help to overcome weak identification. However, as we increase the number of observables, we risk pushing the model along dimensions for which it has not been properly designed. Consequently, addressing the issue of whether more observables is better inevitably takes us to the delicate topic of model specification.

An and Schorfheide (2007) and Del Negro and Schorfheide (2008) forcefully argue that DSGE models are intrinsically misspecified. These authors further argue that incorporating measurement errors is one way to ameliorate misspecification (see example in Section 2). Their argument and the results in the previous sections invite us to conjecture that more observables are indeed desirable as long as they are accompanied by measurement errors. To explore this hypothesis, I repeat the out-of-sample forecast exercise for some variables and some combinations of observables when measurement errors have been either removed or kept fixed during estimation.

The first exercise consists on re-estimating the model with the price of investment as an observable but with its measurement error set to zero. [12] Presumably, the lack of measurement errors will exacerbate any misspecification along the part of the model related to investment, which in turn should affect the model's forecasting performance. Table VI(a) reports the forecast errors for this new scenario relative to those from the benchmark formulation. The one-quarter-ahead forecast worsens for all variables but consumption and inflation. Furthermore, the overall fitting measure indicates that the model with price of investment as an observable does not outperform the baseline specification. More important, the model's overall performance declines as we move to longer forecasting horizons. Moreover, the baseline scenario definitely does a better job in predicting variables like investment, labor, interest rates, and output. For the remaining variables the evidence is mixed, although the baseline set of observables has a slight advantage.

To understand the previous result, recall that the price of investment is a relatively volatile variable (its annualized volatility is 1.9%). In the absence of measurement errors, the structural

Table VI(A). RSME, no measurement errors

	Price of investment							
	y	c	i	L	w	π	R	Ov
One-quarter-ahead	1.01	0.93	1.04	1.38	1.03	0.81	1.19	0.00
One-year-ahead	1.04	1.01	1.11	1.15	0.96	0.77	1.20	0.04
Three-year-ahead	0.87	0.93	1.07	1.03	1.00	1.04	1.12	0.06

[12] Let $\Delta p_{i,t}$ and $\Delta \widetilde{p}_{i,t}$ denote the growth rate of price of investment in the data and in the model, respectively. Then the measurement equation for this variable without measurement error establishes that $\Delta p_{i,t} = \Delta \widetilde{p}_{i,t}$.

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Table VI(B). RSME with fixed errors

	y	c	i	L	w	π	R	Ov
One-quarter-ahead forecast								
Baseline	1.03	1.46	1.16	1.57	1.01	1.08	1.79	0.74
Price investment	3.46	1.33	1.47	3.41	1.06	0.86	1.20	2.49
No real wage	1.86	1.39	2.45	2.04	1.08	3.27	1.45	2.45
No interest rate	0.85	1.11	0.97	1.00	1.02	1.94	6.59	2.02
No wage/consump.	1.10	1.28	1.02	1.16	1.21	3.09	1.66	3.46
No inflation	4.18	1.23	7.05	5.42	1.77	1.42	3.01	4.90
One-year-ahead forecast								
Baseline	1.02	1.74	0.99	1.06	0.98	1.09	1.51	0.81
Price investment	2.59	1.86	1.44	2.03	1.03	1.00	1.31	2.29
No real wage	2.31	1.79	2.83	1.74	1.23	5.10	1.48	2.77
No interest rate	0.80	1.20	0.88	0.89	0.96	1.77	3.27	1.83
No wage/consump.	1.29	1.27	1.11	0.99	1.11	3.47	1.49	3.52
No inflation	4.62	1.41	6.43	3.41	1.09	2.22	3.14	5.22
Three-year-ahead forecast								
Baseline	1.32	1.24	1.18	0.74	0.98	1.05	1.27	0.81
Price investment	1.04	2.08	1.07	1.16	1.01	1.43	1.20	2.21
No real wage	1.44	2.17	2.15	1.43	1.15	6.04	1.60	2.72
No interest rate	0.89	1.19	0.88	0.89	0.97	1.13	2.14	1.51
No wage/consump.	1.25	1.34	1.14	0.93	1.10	4.05	1.42	3.67
No inflation	3.27	1.42	4.03	1.69	1.24	2.07	2.33	5.23

disturbances must become more volatile to capture the price of investment's variability. But this excess volatility influences other variables in the model and thus affects the parameter estimates. For example, the cost of investment adjustment is 6.16, while the volatility of the government shock, σ_{gov}, increases to 3.29, i.e., six times larger than in the baseline formulation. In other words, the estimated parameters are adjusted to explain the price of investment at the expense of worsening the fit of other variables in the model.

For the next exercise, the model is re-estimated but this time with the volatilities of measurement errors calibrated to capture 10% of the fluctuations observed in the data except for investment and real wages, whose errors explain 20%.[13],[14] By fixing the measurement errors, we try to expose potential misspecifications in the model. Ultimately, we expect that the model's forecast worsens when we include an observable for which the model is not properly specified (see Section 2). Table VI(b) presents the new forecasting indicators relative to those in the baseline specification in which the measurement errors are estimated.

The results in the first row indicate that the predictive power of the model estimated with the baseline choice of observables deteriorates when the measurement errors are fixed. Labor and interest rates are particularly sensitive, which suggests model specification is problematic along those dimensions (see also footnote 14). In contrast, the low RSME for output indicates that the model is not severely misspecified at least for that variable. Note that the forecasting of investment and real wages does not deteriorate by much, thanks to larger measurement errors used during the

[13] Using the notation from the previous footnote, let the measurement equation for the observable x be $x_t = \widetilde{x}_t + \varepsilon_t^x$. Then the volatility of the error term, σ_ε, is calibrated to 10% of the standard deviation of x.

[14] If the measurement errors are calibrated to explain 10% of the variability of wages and investment, I find a strong decline in the forecasting performance of the model. This finding hints that misspecification may be somehow severe along those dimensions.

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Table VII. Estimated parameters and multimodality

	σ_m	σ_{gL}	σ_{gK}	σ_u	σ_{gov}	b	ξ_w	ξ_p	γ
No labor (−952.8)	0.27	1.01	0.55	2.29	0.54	0.87	0.69	0.82	1.15
	[0.25,0.30]	[00.76,1.32]	[00.41,0.74]	[01.39,3.61]	[00.36,0.77]	[00.81,0.91]	[00.61,0.76]	[0.77,0.85]	[00.73,1.78]
No labor (−964.7)	0.27	0.81	1.31	5.02	0.53	0.93	0.76	0.89	1.11
	[0.25,0.30]	[00.57,1.13]	[00.94,1.69]	[02.31,10.5]	[00.37,0.74]	[00.89,0.96]	[00.68,0.83]	[0.85,0.91]	[00.68,1.72]
No cons (−938.3)	0.32	0.65	0.24	3.47	0.51	0.92	0.80	0.97	1.30
	[0.29,0.35]	[00.53,0.75]	[00.18,0.33]	[02.15,5.51]	[00.34,0.72]	[00.87,0.96]	[00.73,0.85]	[0.96,0.98]	[00.81,2.02]
No cons (−938.4)	0.33	0.49	0.35	1.21	0.50	0.54	0.43	0.96	1.30
	[0.30,0.37]	[00.39,0.61]	[00.26,0.48]	[00.79,1.93]	[00.35,0.73]	[00.38,0.74]	[00.32,0.54]	[0.94,0.97]	[00.80,2.10]
No wage (−994.0)	0.26	0.62	0.59	11.4	0.69	0.97	0.52	0.91	0.63
	[0.24,0.29]	[00.54,0.70]	[00.49,0.68]	[08.42,15.4]	[00.43,0.80]	[00.95,0.98]	[00.40,0.63]	[0.88,0.93]	[00.88,0.94]
No wage (−994.9)	0.27	0.77	0.36	3.39	0.53	0.92	0.60	0.82	1.60
	[0.25,0.30]	[00.70,0.86]	[00.29,0.44]	[02.39,4.95]	[00.36,0.75]	[00.89,0.95]	[00.50,0.68]	[0.79,0.84]	[01.06,2.36]
No interest (−1030)	0.43	0.62	0.36	4.10	0.48	0.91	0.73	0.81	0.59
	[0.28,0.64]	[00.55,0.70]	[00.27,0.45]	[02.66,6.06]	[00.33,0.69]	[00.88,0.94]	[00.67,0.78]	[0.77,0.84]	[00.38,0.96]
No interest (−1031)	1.09	0.69	0.29	2.19	0.49	0.73	0.47	0.80	1.95
	[0.61,1.73]	[00.62,0.77]	[00.24,0.35]	[01.72,2.90]	[00.33,0.67]	[00.61,0.84]	[00.38,0.55]	[0.77,0.82]	[01.62,1.89]

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Table VII. (continued)

	ρ_R	ϕ_π	ϕ_y	κ	β	$100(g_L - 1)$	$100(\pi - 1)$
No labor (−952.8)	0.76	1.67	0.042	1.44	0.9949	0.26	0.56
	[0.71,0.81]	[01.53,1.81]	[00.02,0.07]	[00.89,2.26]	[00.993,0.996]	[00.19,0.34]	[00.39,0.75]
No labor (−964.7)	0.96	1.62	0.43	2.99	0.9892	0.29	0.48
	[0.94,0.98]	[01.47,1.77]	[00.24,0.70]	[01.92,4.28]	[00.985,0.992]	[00.21,0.38]	[00.33,0.65]
No cons (−938.3)	0.70	1.77	0.0026	2.70	0.9908	0.23	0.44
	[0.65,0.75]	[01.64,1.91]	[00.000,0.009]	[01.93,3.78]	[00.989,0.993]	[00.16,0.34]	[00.33,0.59]
No cons (−938.4)	0.68	1.75	0.043	3.24	0.9948	0.46	0.38
	[0.62,0.74]	[01.61,1.90]	[00.026,0.069]	[01.97,4.81]	[00.993,0.996]	[00.35,0.57]	[00.26,0.52]
No wage (−994.0)	0.93	1.54	0.49	2.41	0.9809	0.27	0.48
	[0.88,0.96]	[01.36,1.70]	[00.29,0.80]	[01.53,3.55]	[00.977,0.985]	[00.20,0.35]	[00.34,0.65]
No wage (−994.9)	0.69	1.64	0.053	1.03	0.9931	0.26	0.50
	[0.64,0.74]	[01.51,1.79]	[00.036,0.078]	[00.71,1.54]	[00.991,0.995]	[00.18,0.35]	[00.35,0.68]
No interest (−1030)	0.83	1.70	0.077	0.74	0.9694	0.33	0.53
	[0.75,0.90]	[01.56,1.85]	[00.04,0.144]	[00.53,1.03]	[00.964,0.975]	[00.25,0.42]	[00.38,0.73]
No interest (−1031)	0.09	1.75	0.20	0.087	0.9745	0.34	0.50
	[0.02,0.25]	[01.62,1.89]	[00.13,0.29]	[00.04,0.15]	[00.970,0.978]	[00.25,0.43]	[00.36,0.68]

estimation of the model. Overall, these results indeed suggest that measurement errors improve the model's fit.

Table VI(b) also shows that when we fix the measurement errors the baseline formulation performs significantly better than the other sets of observables. Notably, the inclusion of the price of investment now hurts the model's predictive power for any forecasting horizon. Output and labor are particularly sensitive to the use of that additional observable. These results and those in Table VI(a) imply that the model may not adequately capture the dynamics of the price of investment. Therefore, including this variable as an observable is beneficial from a forecasting perspective insofar as its measurement equation includes an error term.

The combination which omits the interest rate is the second best alternative. To understand this result, recall that its measurement equation did not contain an error term from the beginning (footnote 7). Therefore, the absence of interest rates does not significantly damage the model's forecast because any misspecification related to interest rates was already embedded in the model. Finally, excluding inflation or consumption and real wages still delivers the worst forecasting performance.

We leave this section with two important lessons. First, we confirm our suspicion that more observables is better as long as the model in correctly specified. Second, measurement errors help to mask potential misspecifications in the model and thus contribute to improve its forecasting properties. This second result is particularly notable for the price of investment.

7. OBSERVABLES AND POSTERIOR MULTIMODALITY

An and Schorfheide (2007) have shown in the context of a small DSGE model that the posterior is prone to displaying multiple modes. Furthermore, they find that usual convergence checks (Brooks and Gelman, 1998) fail to detect the irregularities surrounding the posterior. The discussion in Section 2 suggests that the absence of observables may distort the posterior, increasing the chances of multimodality. Hence it seems necessary to check whether such a possibility arises for the model discussed in Section 3. Based on the explorative algorithm outlined in the Appendix,

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I find that four of the alternative combinations of observables induce bimodality in the model's posterior. Table VII contains the estimated parameters and probability intervals for the two modes (the highest mode is the one reported in Table III). The number in parentheses corresponds to the posterior kernel, $\ln[\mathbf{L}(y;\Theta)p(\Theta)]$, where $\mathbf{L}$ is the likelihood and p is the prior.

Perhaps the most striking result comes in the no-consumption case. Note that although the posterior modes are almost identical (-938.33 and -938.47), the consumption-related parameters (b and σ_{Uc}) and the sticky wage coefficient (ξ_w) significantly differ between the two modes. In the low mode, the structural shocks are less volatile; e.g., the preference shock is almost a third of that in the high mode. The lower volatility seems to drive the smaller habit formation and sticky wages required by the model. In the terminology of Canova and Sala (2006), we can say that the vectors of coefficients in the high and low modes are observationally equivalent, i.e., $\ln[\mathbf{L}(y;\Theta_H)p(\Theta_H)] = \ln[\mathbf{L}(y;\Theta_L)p(\Theta_L)]$ and $\ln[\mathbf{L}(y;\Theta_L)p(\Theta_L)] > \ln[\mathbf{L}(y;\widetilde{\Theta})p(\widetilde{\Theta})]$ for any other vector of parameters $\widetilde{\Theta}$ (the H and L subindices refer to the high and low mode, respectively).

A similar situation emerges from the no-wage case (fifth and sixth rows in Table V). As before, the low mode is characterized by less volatile shocks, specifically preference and government shocks. With smaller volatility there is less need to induce smoothness in the model, which implies the decline in habit formation and in the length of price contracts. Additionally, the Frisch elasticity is more than twice the size of that in the high mode. Such a result is expected since the absence of real wages affects identification of the labor sector. When we turn to the no-labor case, the two modes are not so close (-952.81 and -964.71). Further, the discrepancies between the estimated parameters are less dramatic. The exception is the output reaction parameter in the Taylor rule, which is now one order of magnitude smaller in the low mode. Finally, omitting interest rates from the estimation generates two modes characterized by significant departures of the discount factor and the smoothing coefficient in the Taylor rule (see Section 5.5 for intuition on this finding).

One potential explanation for the bimodal distributions is that the chains are not long enough to fully describe the posterior contour. In the working paper version, I find that even chains with as many as 1.5 million draws fail to explore the two modes (at least for the no-real-wage and no-consumption cases). This finding echoes the evidence reported in An and Schorfheide (2007).

In summary, the absence of observables contributes to the emergence of bimodal posterior distributions. This bimodality particularly affects the posteriors of habit formation and sticky wages. Furthermore, the exclusion of consumption or real wages exacerbates the problem.

8. CONCLUSION

The observable variables used to estimate DSGE models have more importance than previously thought. On the one hand, using too many observables raises the issue of model misspecification. It is found that the inclusion of measurement errors partially alleviates the problem. On the other hand, the exclusion of variables like real wages, consumption, or inflation may severely influence not only the model's forecasting abilities but also the median estimates of parameters such as those describing sticky contracts in prices and wages, and habit formation.

Based on the model's forecasting performance and its impulse responses, I find that a suitable set of observable variables should consist of seven variables: output, consumption, investment, real wages, total labor, interest rates, and inflation. Furthermore, as long as measurement errors are used to compensate for any model misspecification, the results in this paper suggest that the

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presence of the price of investment is solely justified on forecasting grounds. Note that its inclusion leaves practically unaffected the parameter estimates or the model's impulse responses. Finally, information about labor and investment appear to be of little substance for the model's estimation.

As noted in Section 4, the set of observables used in this paper is far from exhaustive. There are additional cases that may be worth pursuing in future research. Examples include use of financial data or estimation using ratios of variables (as in ACEL). In an open-economy framework, the possibilities are even larger as data from exchange rates, current account, and trade flows can be incorporated into the estimation.[15]

APPENDIX: MCMC ALGORITHM

Let $p(\Theta)$ and $\mathbf{L}(Y_T;\Theta)$ be the prior distribution of the parameter vector Θ and the likelihood of the data conditional on the parameter vector, respectively. I use the data, a state-space representation of the model, and the Kalman filter to evaluate the posterior distribution $p(\Theta|Y_T)$. The results reported in this paper are based on 150,000 draws $\Theta_{(n)}$ from the posterior distribution generated using a random walk Metropolis–Hasting algorithm. At each iteration n, a candidate parameter vector $\widetilde{\Theta}$ is drawn from the distribution $\mathbb{N}\left(\Theta_{(n-1)}, c_o^2 \sum\right)$ and the acceptance ratio, r, is computed:

$$r = \frac{p(Y_T|\widetilde{\Theta})p(\widetilde{\Theta})}{p(Y_T|\widetilde{\Theta}_{(n-1)})p(\widetilde{\Theta}_{(n-1)})}$$

The new draw $\widetilde{\Theta}$ is kept with probability min $(r,1)$ and rejected otherwise.

To characterize the variance of the jumping distribution, $c_o^2 \sum$, I proceed as follows. First, I apply Christopher Sims' *csminwel* code to compute the mode of the posterior distribution. To that end, 5000 draws from the prior distribution are used to evaluate the posterior. The 30 draws achieving the highest posteriors are the initial points for Sims' maximization code. This procedure leaves us with 30 potential candidates. To increase the probability of detecting the global maximum, I require that the optimization routine reaches the highest mode at least three times. If this is not the case, I increase the number of initial points in intervals of 10. In practice, I find that 50 initial conditions are enough for Sims' code to find the mode at least three times for all combinations of observables reported in this paper. Second, I compute the inverse Hessian at the mode and use it as the variance of the jumping distribution. Third, the constant c_o is set to achieve an acceptance rate close to 0.35, a value typically suggested in the literature (An and Schorfheide, 2007; Casella and Robert, 2004; Fernandez-Villaverde and Rubio-Ramirez, 2004).

To assess convergence of the resulting algorithm, I follow two approaches. First, as is common practice in the literature, I run three chains each of size 150,000 (the first 10,000 iterations are discarded). Each chain starts at a different random draw from the jumping distribution centered at the mode. All chains generated very similar results for the recursive means of each estimated parameter. Second, to further confirm convergence, I compute Brooks and Gelman's (1998) interval and variance potential scale reduction factors (PSRF) for each parameter and dataset. According to Brooks and Gelman (1998), factors below 1.1 are indicative of convergence. For all parameters and datasets, the variance PSRF factors were less than 1.005, while the interval factors were less than 1.05.[16]

[15] For example, Adolfson *et al.* (2007) use 10 variables to estimate an open economy model for the European area.
[16] Owing to space limitations, I do not report the results, which are available upon request.

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The detection of multimodality in the posterior distributions is based on the following algorithm. The maximization routine previously discussed delivers 30 potential modes. A vector of parameters is ruled to be a potential second mode if its posterior mode is sufficiently close to the highest mode (less than 10 log points away), and, more importantly, if its value repeats itself at least twice among the potential candidates. The second modes reported in Table V are those satisfying these conditions and whose resulting MCMC chains clear the convergence tests. There were situations, like the no-wage no-consumption case, in which the candidate mode was a false hit (the MCMC chain quickly moved towards the highest mode).

We should note, however, that the algorithm to detect multimodality is far from perfect. For example, it is quite possibly that a second mode shows up because the optimization routine runs out of iterations. To reduce the risk of such an outcome, I rule a second potential mode as a true mode if the optimization routine reaches that mode from two different starting points.

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