Supplementary Material:
The Impact of Oil Shocks in a Small Open Economy NK-DSGE Model for an Oil-Importing Country: The Case of South Africa

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Supplementary Material

S.1 The linearized system

S.1.1 Aggregate demand

$$
\begin{aligned}
\hat{c}_t^h &= \gamma_c(\eta_c - \eta_o)(\hat{rer}_t - \hat{\psi}_t^f) - (\gamma_c(\eta_c - \eta_o) + \eta_o)\hat{pr}_t^h + \hat{c}_t && \text{(S.1)}\\
\hat{c}_t^f &= (1-\gamma_c)(\eta_c - \eta_o)\hat{pr}_t^h + (\gamma_c - \eta_c)(\hat{rer}_t - \hat{\psi}_t^f) + \hat{c}_t && \text{(S.2)}\\
\hat{o}_t^c &= -\eta_o \hat{pr}_t^o + \hat{c}_t && \text{(S.3)}\\
\hat{c}_t &= \frac{1}{(1+\phi)}\hat{c}_{t+1} + \frac{\phi}{(1+\phi)}\hat{c}_{t-1} - \frac{(1-\phi)}{\sigma_c(1+\phi)}(\hat{i}_t^b - \hat{\pi}_{t+1} + \hat{\mu}_t^b) && \text{(S.4)}\\
\hat{rer}_t &= \frac{\sigma_c}{1-\phi}(\hat{c}_t - \phi\hat{c}_{t-1}) - \frac{\sigma_c^*}{1-\phi^*}(\hat{c}_t^* - \phi^*\hat{c}_{t-1}^*)\ . && \text{(S.5)}
\end{aligned}
$$

Eq.S.1 domestic consumption of home goods; Eq.S.2 domestic consumption of foreign goods; Eq.S.3 domestic consumption of oil; Eq.S.4 Euler eqn; Eq.S.5 is the international risk sharing condition (where $\hat{c}_t^* = \hat{y}_t^*$).[1]

S.1.1.1 Investment schedule

$$
\hat{v}_t - \hat{k}_t = \beta E_t(\hat{v}_{t+1} - \hat{k}_{t+1}) + \frac{\beta R^k}{\kappa_v} E_t(\hat{r}_{t+1}^k) + \frac{\sigma_c}{\kappa_v}(\hat{c}_t - \hat{c}_{t+1})\ , \qquad \text{(S.6)}
$$

where $R^k = 1/\beta - (1-\delta)$.

S.1.2 Aggregate supply & inflation

S.1.2.1 (real) wage setting equation:

$$
\begin{aligned}
\hat{w}_t &= \Omega\beta E_t \hat{w}_{t+1} + \Omega\hat{w}_{t-1} + \Omega\Omega^*(\hat{mrs}_t - \hat{w}_t)\\
&\quad + \Omega\beta E_t\hat{\pi}_{t+1} - \Omega\hat{\pi}_t - \Omega\theta_w\beta\gamma_w\hat{\pi}_t + \Omega\gamma_w\hat{\pi}_{t-1}\ .
\end{aligned}
$$

The real wage ($\hat{w}_t = w_t - p_t$) setting equation can be re-written in nominal wage inflation form as:[2]

$$
\hat{\pi}_t^w - \gamma_w\hat{\pi}_{t-1} = \beta E_t \hat{\pi}_{t+1}^w - \theta_w\beta\gamma_w\hat{\pi}_t + \Omega^*(\hat{mrs}_t - \hat{w}_t), \qquad \text{(S.8)}
$$

where $\Omega^* = \frac{(1-\theta_w)(1-\theta_w\beta)}{\theta_w(1+\xi^w\sigma_n)}$, $\Omega = \frac{1}{(1+\beta)}$, and $\hat{mrs}_t = \frac{\sigma_c}{1-\phi}(\hat{c}_t - \phi\hat{c}_{t-1}) + \sigma_n\hat{n}_t$.

[1] The UIP condition holds from the Euler equations of the domestic and foreign sectors: $\hat{i}_t^b = \hat{i}_t^{b*} + E_t[\Delta\hat{\varepsilon}_{t+1}] + \hat{\Phi}_t$, which implies that the real exchange rate equates the marginal utilities of consumption between the domestic and foreign households.

[2] Log-linearizing the optimality condition for wage-setting and solving for $\tilde{w}_t$ gives the optimal reset wage equation:

$$
\tilde{w}_t = \frac{(1-\theta_w\beta)}{(1+\xi^w\sigma_n)} E_t \sum_{i=0}^{\infty} (\theta_w\beta)^i \Big(\chi mrs_{t+i} + \xi^w\sigma_n w_{t+i} + p_{t+i} - \gamma_w\pi_{t+i-1}\Big) \qquad \text{(S.7)}
$$

where $\chi \equiv \frac{W}{MRS^s\mu^w}$. Combining (S.7) with the log-linearized wage index equation gives the aggregate sticky real wage ($\hat{w}_t = w_t - p_t$) equation, which we can re-write in nominal wage inflation form as Eq. S.8.

S.1.2.2 Domestic production and inflation (for consumption goods)

$$\hat{\pi}_t^h = \frac{\gamma_p}{(1+\gamma_p\beta)}\hat{\pi}_{t-1}^h + \frac{\beta}{(1+\gamma_p\beta)}E_t\hat{\pi}_{t+1}^h + \kappa_h(\hat{mc}_t^h + \hat{\xi}_t^p), \tag{S.9}$$

where $\hat{mc}_t^h = \hat{\lambda}_t$ is the real marginal cost of production, and $\kappa_h = \frac{(1-\theta_h)(1-\theta_h\beta)}{\theta_h(1+\gamma_p\beta)}$.

$$\hat{\lambda}_t = (\hat{w}_t - \hat{pr}_t^h) - (\hat{y}_t^h - \hat{n}_t) \tag{S.10}$$
$$\hat{\lambda}_t = \hat{r}_t^k - (\hat{y}_t^h - \nu\hat{k}_t - \hat{yx}_t) \tag{S.11}$$
$$\hat{\lambda}_t = (\hat{pr}_t^o - \hat{pr}_t^h) - (\hat{y}_t^h - \nu\hat{o}_t^h - \hat{yx}_t) \ , \tag{S.12}$$

where $\hat{yx}_t = \vartheta(1-\nu)\hat{k}_t + (1-\vartheta)(1-\nu)\hat{o}_t^h$.

$$\hat{y}_t^h = \hat{a}_t + \alpha\hat{n}_t + (1-\alpha)\vartheta\hat{k}_t + (1-\alpha)(1-\vartheta)\hat{o}_t^h \ , \tag{S.13}$$

S.1.2.3 Imported inflation (for foreign consumption goods)

$$\hat{\pi}_t^f = \beta E_t[\hat{\pi}_{t+1}^f] + \kappa_f\hat{\psi}_t^f, \tag{S.14}$$

where $\kappa_f = \frac{(1-\theta_f)(1-\theta_f\beta)}{\theta_f}$, and $\hat{\psi}_t^f$ measures the l.o.p gap:[3]

$$\begin{aligned} \hat{\psi}_t^f &= \hat{\varepsilon}_t + \hat{p}_t^{f*} - \hat{p}_t^f, \\ &= \hat{rer}_t - \hat{pr}_t^f \ . \end{aligned} \tag{S.15}$$

S.1.2.4 Inflation aggregation equations

From the inflation aggregation equations we have:

$$\hat{\pi}_t^z = (1-\gamma_c)\hat{\pi}_t^h + \gamma_c\hat{\pi}_t^f$$
$$\hat{\pi}_t = (1-\gamma_o)\hat{\pi}_t^z + \gamma_o\hat{\pi}_t^o \tag{S.16}$$
$$\therefore \hat{\pi}_t = (1-\gamma_o)(1-\gamma_c)\hat{\pi}_t^h + (1-\gamma_o)\gamma_c\hat{\pi}_t^f + \gamma_o\hat{\pi}_t^o. \tag{S.17}$$

Eq. S.16 and Eq. S.17 can be re-written as (see, also, Medina & Soto, 2005)

$$\hat{\pi}_t^z = \hat{\pi}_t - \frac{\gamma_o}{(1-\gamma_o)}(\hat{pr}_t^o - \hat{pr}_{t-1}^o) \tag{S.18}$$
$$0 = \gamma_o\hat{pr}_t^o + (1-\gamma_o)(1-\gamma_c)\hat{pr}_t^h + (1-\gamma_o)\gamma_c(\hat{pr}_t^f). \tag{S.19}$$

S.1.2.5 Evolution of relative prices

$$\hat{pr}_t^h = \hat{pr}_{t-1}^h + \hat{\pi}_t^h - \hat{\pi}_t \tag{S.20}$$
$$\hat{pr}_t^f = \hat{pr}_{t-1}^f + \hat{\pi}_t^f - \hat{\pi}_t \tag{S.21}$$
$$\hat{pr}_t^o = \hat{rer}_t + \hat{pr}_t^{o*} + \hat{\psi}_t^o \tag{S.22}$$
$$\hat{rer}_t = \hat{rer}_{t-1} + \Delta\hat{\varepsilon}_t + \hat{\pi}_t^{f*} - \hat{\pi}_t \tag{S.23}$$
$$\hat{\pi}_t^o = \hat{pr}_t^o - \hat{pr}_{t-1}^o + \hat{\pi}_t \ , \tag{S.24}$$
$$\hat{s}_t = \hat{pr}^f - \hat{pr}^h \ , \tag{S.25}$$
$$\hat{w}_t = \hat{w}_{t-1} + \hat{\pi}_t^w - \hat{\pi}_t \ , \tag{S.26}$$

[3] $\hat{rer}_t = \hat{\varepsilon}_t + \hat{p}_t^{f*} - \hat{p}_t$ and $\hat{pr}_t^f = \hat{p}_t^f - \hat{p}_t$.

where $\hat{pr}_t^{o*}$ (the relative (real) foreign price of oil, $\hat{p}_t^{o*} - \hat{p}_t^{f*}$) and $\hat{\psi}_t^o$ (deviations from l.o.p on relative (real) domestic price of oil, $\hat{pr}_t^o$), are AR(1) processes.[4] Eq.S.23 is the equation of motion for the relative purchasing power parity condition[5] Here, we can think of nominal exchange rate changes ($\Delta\hat{\varepsilon}_t$) as the price adjustment mechanism that maintains equilibrium between foreign and domestic goods markets. We can derive an equation for oil inflation in nominal dollar (i.e., foreign currency) terms:

$$\hat{\pi}_t^{o*} = \hat{pr}_t^{o*} - \hat{pr}_{t-1}^{o*} + \hat{\pi}_t^{f*} \ , \tag{S.27}$$

where $\hat{pr}_t^{o*}$ is a stochastic process capturing shocks to the price of oil relative to the foreign price level. Eq.S.27 therefore capture both changes in real oil price movements and the endogenous evolution of price, productivity and risk premium shocks from the foreign economy.

S.1.2.6 Evolution of capital

$$\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \delta\hat{v}_t \tag{S.28}$$

S.1.2.7 Policy rule

$$\hat{i}_t^b = \rho_i \hat{i}_{t-1}^b + (1-\rho_i)\kappa_\pi \hat{\pi}_t + (1-\rho_i)\kappa_y(\hat{y}_t - \hat{y}_{t-1}) + \epsilon_t^i \ . \tag{S.29}$$

S.1.3 Foreign economy

We assume a large open economy for the foreign market. This allows us to specify the foreign rate, $\hat{i}_t^{b*}$, foreign inflation $\hat{\pi}_{t+1}^* = \hat{\pi}_{t+1}^{f*}$, and foreign consumption $\hat{y}_t^* = \hat{c}_t^*$ according to the standard 3-equation New-Keynesian model, namely: an IS curve, a Phillips curve, and a Taylor-type policy rate rule.

$$\hat{y}_t^* = \frac{1}{(1+\phi^*)}\hat{y}_{t+1}^* + \frac{\phi^*}{(1+\phi^*)}\hat{y}_{t-1}^* - \frac{(1-\phi^*)}{\sigma_c^*(1+\phi^*)}(\hat{i}_t^{b*} - E_t[\hat{\pi}_{t+1}^*] + \hat{\mu}_t^{b*}) \tag{S.30}$$

$$\hat{\pi}_t^* = \frac{\gamma^*}{(1+\gamma^*\beta)}\hat{\pi}_{t-1}^* + \frac{\beta}{(1+\gamma^*\beta)}E_t[\hat{\pi}_{t+1}^*] + \kappa_* \hat{mc}_t^*, \tag{S.31}$$

where $\hat{mc}_t^*$ is the real marginal cost of production, and $\kappa_* = \frac{(1-\theta_*)(1-\theta_*\beta)}{\theta_*(1+\gamma^*\beta)}$.

$$\hat{mc}_t^* = \left(\frac{\sigma_c^*}{1-\phi^*} + \sigma_n^*\right)\hat{y}_t^* - \left(\frac{\sigma_c^*\phi^*}{1-\phi^*}\right)\hat{y}_{t-1}^* - (1+\sigma_n^*)\hat{a}_t^*, \tag{S.32}$$

$$\hat{i}_t^{b*} = \rho_{i*}\hat{i}_{t-1}^{b*} + (1-\rho_{i*})\kappa_\pi^* \hat{\pi}_t^* + (1-\rho_{i*})\kappa_y^*(\hat{y}_t^* - \hat{y}_{t-1}^*) + \epsilon_t^{i*}, \tag{S.33}$$

S.1.4 Aggregate equilibrium

$$\begin{aligned} \hat{y}_t^h &= \frac{C^h}{Y^h}\hat{c}_t^h + \frac{C^{h*}}{Y^h}\hat{c}_t^{h*} \\ &= \frac{C^h}{Y^h}\hat{c}_t^h + \frac{(1-C^h)}{Y^h}(\hat{y}_t^* - \xi^{f*}(\hat{pr}_t^h - \hat{rer}_t)) \ , \end{aligned} \tag{S.34}$$

[4]Specification of stochastic processes in Eq.(S.22) is important. It depends on how we treat the price of oil in estimation: if it enters as $\hat{pr}_t^{o*}$ then we can separate the shocks; if we introduce as $\hat{pr}_t^o$, then we must combine them. As we are interested in foreign real oil price shocks we opt for the former.

[5]Derived from $\hat{rer}_{t+1} = \hat{rer}_t + (\hat{i}_t^b - \hat{\pi}_{t+1}) - (\hat{i}_t^{b*} - \hat{\pi}_{t+1}^{f*} + \hat{\Phi}_t)$, where $(\hat{i}_t^b - \hat{\pi}_{t+1})$ and $(\hat{i}_t^{b*} - \hat{\pi}_{t+1}^{f*})$ are the domestic and foreign real interest rates on bonds, i.e., the Fisher equations; $\hat{\Phi}_t = \hat{\mu}_t^{b*} - \hat{\mu}_t^b$

where ξ^{f*} is the foreign price elasticity of demand for domestic goods (i.e., the change in foreign demand for domestic goods given the foreign price of domestic goods relative to the foreign price of foreign goods).

$$\begin{aligned}
\hat{y}_t &= \frac{C}{Y}\hat{c}_t + \frac{V}{Y}\hat{v}_t + \frac{X}{Y}\hat{x}_t - \frac{M}{Y}\hat{m}_t && \text{(S.35)} \\
\hat{x}_t = \hat{c}_t^{h*} &= \hat{y}_t^* - \xi^{f*}(\hat{pr}_t^h - \hat{rer}_t) && \text{(S.36)} \\
\hat{m}_t &= \frac{C^f}{M}\hat{c}_t^f + \frac{O}{M}\hat{o}_t && \text{(S.37)} \\
\hat{o}_t &= \frac{O^c}{O}\hat{o}_t^c + \frac{O^h}{O}\hat{o}_t^h \ , && \text{(S.38)}
\end{aligned}$$

where $O/M = (M - C^f)/M$.

S.1.5 Exogenous shocks

We include 8 shocks in the model. The oil shock to the foreign real price of oil follows as: $\hat{pr}_t^{o*} = \rho_{o*}\hat{pr}_{t-1}^{o*} + \epsilon_t^{o*}$. For the domestic economy, the monetary policy shock (ϵ_t^i), as given in Eq. S.29, is i.i.d, whereas the domestic technology shock and domestic price markup shock follow AR(1) processes: $\hat{a}_t = \rho_a\hat{a}_{t-1} + \epsilon_t^a$; $\hat{\xi}_t^p = \rho_p\hat{\xi}_{t-1}^p + \epsilon_t^p$. The foreign economy follows with an i.i.d monetary policy shock (ϵ_t^{i*}) and the following supply shock: $\hat{a}_t^* = \rho_{a*}\hat{a}_{t-1}^* + \epsilon_t^{a*}$. In addition, the risk premium shocks on domestic-currency assets relative to the policy rate and for foreign-currency borrowing abroad (equivalent to a negative demand shocks) are described as follows: $\hat{\mu}_t^{b*} = \rho_b\hat{\mu}_{t-1}^{b*} + \epsilon_t^{b*}$ and $\hat{\mu}_t^b = \rho_b\hat{\mu}_{t-1}^b + \epsilon_t^b$.

S.2 Data and sources

Data sources retrieved from the Federal Reserve Bank of St. Louis (FRED), the South African Reserve Bank (SARB), US. Energy Information Administration, Eurostat and OECD.stat:

1. Consumer Price Index of All Items in the United States [CPIAUCSL], United Kingdom [GBRCPIALL], Euro area [EZCCM086NEST], Japan [JPNCPIALL] and in South Africa [ZAFCPIALL] retrieved from FRED (Copyright, 2017, OECD)

2. Real Gross Domestic Product by Expenditure for the United States [GDPC1], United Kingdom [GBQ661S], Euro area [EURSCAB1GQEA19], Japan [JPQ661S] and South Africa [ZAQ661S], retrieved from FRED (Copyright, 2017, OECD)

3. Interest Rates, Government Securities, 3-Month Treasury Bills for United States [Gs3M], United Kingdom [GBM193N], Euro Area [EZQ193N], Japan [JPM193N], and South Africa [ZAM193N] retrieved from FRED (Copyright, 2017, IMF and Eurostat)

4. Population: United States (Civilian Noninstitutional Population) [CNP16OV], Japan (15 and over) [JPQ647S], United Kingdom (Total) [POPNC; GBQ647S], and Euro area (Total) [POPNC; EZQ647S] (Copyright, 2016, OECD)

5. SARB, Balance of payments statistics [KBP5000L - KBP5010L]

6. SARB, Final consumption expenditure by households: Total (PCE) [KBP6007L]

7. SARB, Gross fixed capital formation (Investment) [KBP6009L]

8. Crude Oil Prices: Brent - Europe, Dollars per Barrel, Quarterly, Not Seasonally Adjusted [MCOILBRENTEU]

9. World Development Indicators, Fuel imports (% of merchandise imports), South Africa [TM.VAL.FUEL.ZS.UN]

S.3 Tables & Figures

Table S.3.1: Implied steady-state values from the model

Parameter	Description	Value
$1/\beta - (1-\delta)$	Return on capital	0.040
$\frac{V}{Y}/\delta$	Capital-output ratio	6.670
$(1 - \frac{V}{Y} - \frac{X}{Y} + \frac{M}{Y})$	Total consumption-output ratio	0.795
$(1 - \frac{X}{Y^h})$	Domestic consumption-production ratio	0.720
$(1 - \frac{O}{M})$	Consumption of foreign goods to total imports	0.840
$(1 - \frac{O^c}{O})$	Firm's usage share of fuel imports	0.250

Table S.3.2: Relative Root Mean Square Errors (RMSEs)

Horizon	1	2	3	4	5	6	7	8	Avg.
2008:Q2–2017:Q2									
Output	0.918**	0.909***	0.995	1.016	1.000	1.017	1.035	1.016	0.988
Total (Headline) Inflation	3.172	1.059	0.938**	1.045	1.001	1.043	1.089	1.041	1.299
Nominal Interest Rate	1.003	1.107	1.093	1.047	1.029	1.036	1.051	1.066	1.054
2008:Q2–2011:Q2									
Output	0.859**	0.747***	0.920*	0.965	0.965	0.984	1.060	1.034	0.942
Total (Headline) Inflation	3.232	1.105	0.916*	1.033	1.025	1.107	1.126	1.098	1.330
Nominal Interest Rate	0.762***	1.128	1.148	1.050	1.007	1.007	1.034	1.050	1.023

Notes: A RMSE ratio < 1 means that the *oil* model outperforms the *no oil* model.
***, **, and * indicate significance at the 1%, 5% and 10% levels.

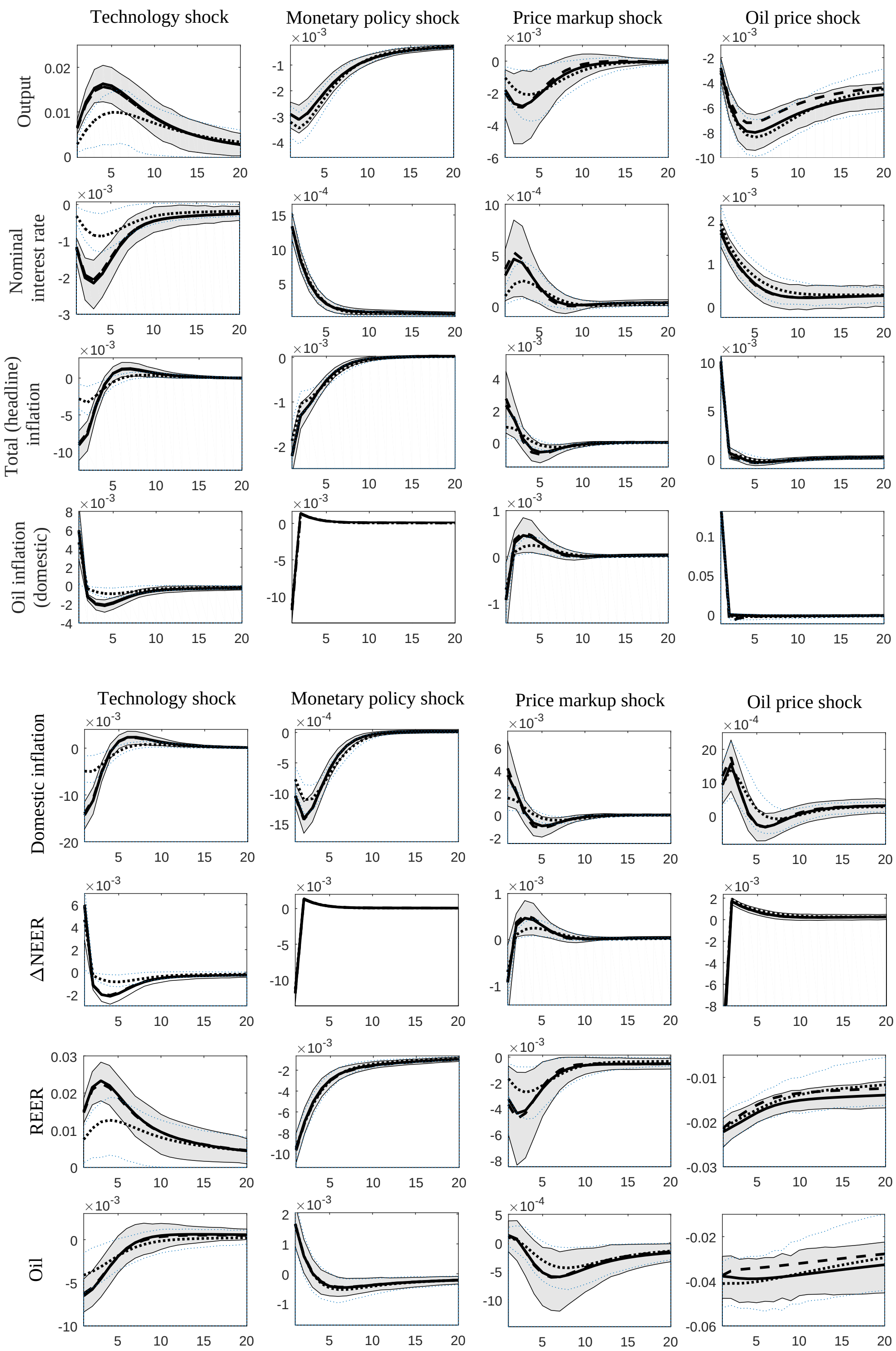

Figure S.3.1: Impulse responses for *oil* mod.1 (solid line), mod.2 (dashed line), and mod.3 (dotted line). 90% highest posterior density interval included for mod.1 and mod.3.

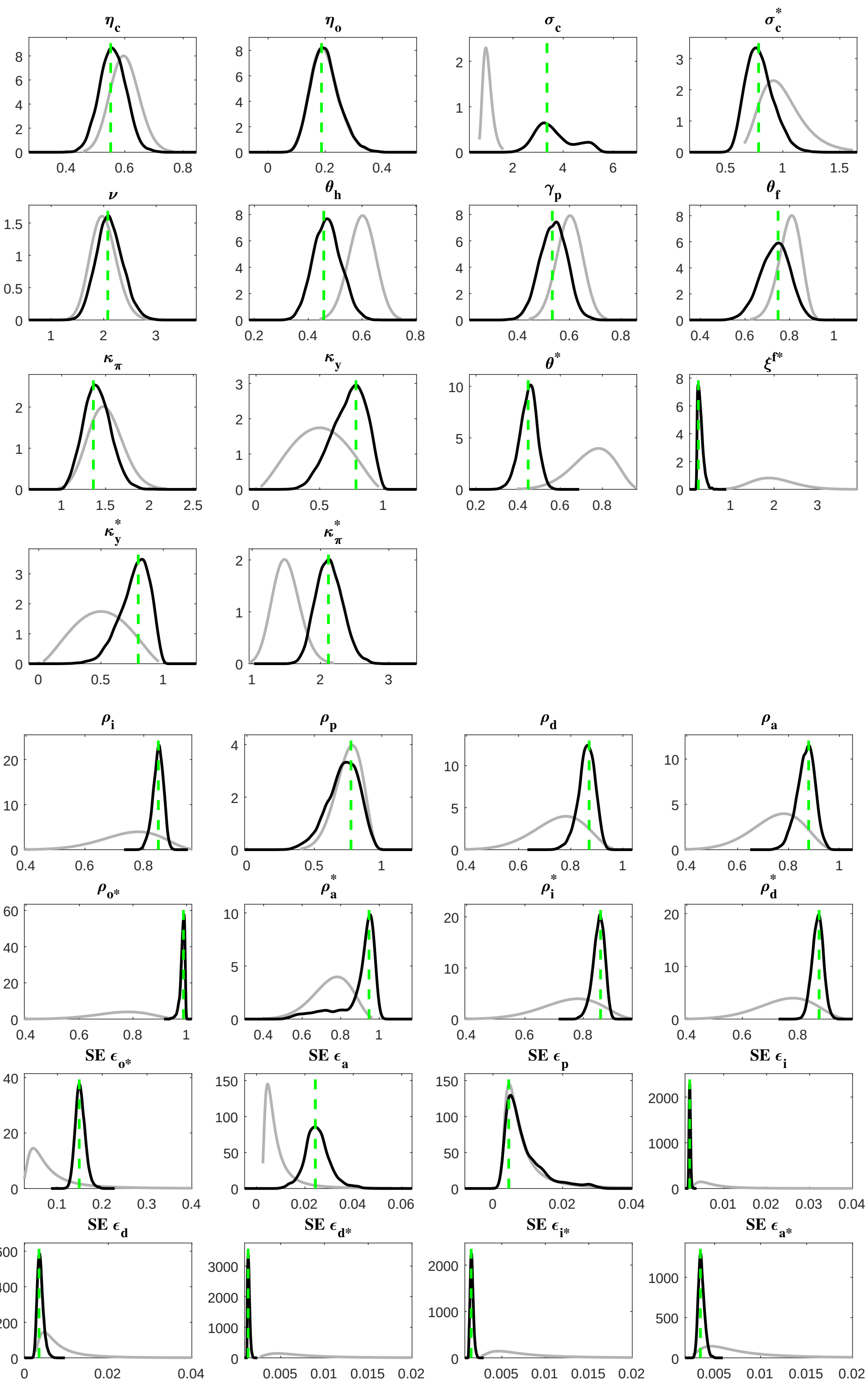

Figure S.3.2: Prior and Posterior distributions of the estimated parameters.