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Evaluating the forecasting power of an open-economy DSGE model when estimated in a data-Rich environment

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ABSTRACT

This paper examines the inferences and forecasting benefits that can be made when one incorporates a large quantity of economic time series into international structural macroeconomic models. I estimate a close variation of Adolfson et al. (2007a, 2008) small open-economy dynamic stochastic general equilibrium (DSGE) model in a data-rich environment and evaluate its predictive performance of the Canadian macroeconomy. The data set I use in the paper includes Canadian, American, Asian and European macro-financial data. I compare the forecasting performance of the DSGE model estimated in a data-rich environment (DSGE-DFM) to the forecasts generated by the DSGE model estimated in its traditional fashion and forecasts generated by other reduced form forecasting models. I find that an open-economy DSGE model estimated in a data-rich environment significantly out performs its regularly estimated DSGE counterpart. Further, DSGE-DFM forecasts that incorporate real-time data are similar or better to the Bank of Canada's Staff Economic Projections for GDP, consumption, investment, and trade statistics. In addition, the DSGE-DFM model of this paper is useful in forecasting both the real and nominal exchange rate in the short and medium-term.

1. Introduction

New open-economy macroeconomics (NOEM) have generated significant empirical and theoretical innovations in international macroeconomic modeling. Estimated dynamic stochastic general equilibrium (DSGE) models became the marquee policy analysis tools for most central banks around the world because of their structural foundations and their competitive forecasting performance against other reduced-form forecasting models such as vector autoregressions (Gurkaynak et al., 2013) and judgement based forecasts by experts (Kolasa et al., 2012). However, many of the DSGE models' forecasts that have been evaluated empirically are centered around a closed-economy and have had significant declines in their predictive power following the Great Recession. (Del Negro and Schorfheide, 2013).

Computational gains have led to additional estimation techniques to become possible for medium scale models that were previously thought to be computationally burdensome. One example includes the estimation of DSGE models using a high dimensional data vector, first introduced by Boivin and Giannoni (2006), and often referred to as DSGE dynamic factor model estimation or DSGE-DFM estimation for short. Gelfer (2019) found that a closed-economy DSGE model with financial frictions for the United States estimated in the DSGE-DFM fashion was able to significantly out-forecast modern

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DSGE models not estimated in a data-rich environment and the Survey of Professional Forecasters (SPF), in regard to core macroeconomic growth variables and many labor and financial metrics.

In this paper, I examine the inferences and forecasting benefits that can be made when one incorporates the DSGE-DFM estimation technique of Boivin and Giannoni (2006) and Gelfer (2019) into a small open-economy DSGE model with capital, analogous to the model used in Adolfson et al. (2007a, 2008). The data set I use in the paper to estimate the DSGE-DFM model includes 93 series of Canadian, American, Asian and European macro-financial data. To my knowledge, this is the first time an open-economy DSGE model has been estimated in this fashion. Given the construction of traditional open-economy DSGE model estimation (henceforth, DSGE-Reg) researchers were only able to evaluate the forecasts and dynamics of a few macroeconomic series, while the DSGE-DFM model of this paper will be able to look at the dynamics and forecasts of such series as domestic employment by sector, domestic credit measures and international financial series.

After estimating the open-economy DSGE-DFM model for Canada, I find that many of the structural parameter estimates are more in line with firm level survey evidence and that many of the exogenous shocks are estimated to be far less volatile and less persistent than the open-economy DSGE-Reg model of this paper. As a result, many of the theoretical foundations and co-movements the structural model is built upon still hold when I compare the impulse response functions (IRF's) of the DSGE-Reg model to the IRF's of the DSGE-DFM model, but the magnitudes and inertia of the model's variables are very different.

For example, the DSGE-DFM model's variance decomposition attributed to international shocks for domestic GDP growth and its components is much larger when compared to the DSGE-Reg model of this paper and other open-economy DSGE models in general. Further, the risk premium shock (FX shock) of the DSGE-DFM model plays a significant role in the variance of domestic inflation and the policy rate in the long-run. This result is what will drive the more accurate forecasts and international co-movements that I see in the DSGE-DFM model of this paper. The large amount of real-time global macro-financial data that is incorporated in the open-economy DSGE-DFM model can help identify the global and FX shocks, which in return help identify the dynamics of the domestic economic variables.

Open-economy DSGE models have been known to have three drawbacks connected to them. First, the strong theoretical structure entrenched in them has not resulted in improvement in the forecast accuracy for domestic macroeconomic variables. In some cases open-economy DSGE model forecasts of the domestic variables can be less accurate than their closed-economy DSGE model counterpart (Kolasa and Rubaszek, 2018). Second, open-economy DSGE models shed little light on the short and medium term dynamics of real and nominal exchange rates. They often result in the inability of open-economy DSGE models to out-perform a naive random walk forecast for the real and nominal exchange rate in the short to medium term for a variety of countries (Ca'Zorzi et al., 2017). Third, small open-economy models fail to generate the cross-correlation that empirically exists between the domestic economy and its international trading partners when it comes to the business cycle and price co-movements (Corbo and Strid, 2020; De Walque et al., 2017; Justiniano and Preston, 2010).

The contribution of this paper is to provide an evaluation of whether open-economy DSGE models estimated in a data-rich environment can be successful in mitigating any of these three key issues of open-economy DSGE models that are estimated in the traditional fashion. DSGE-DFM estimation has the potential to address these three as some of the additional data used in the estimation may be informative about the exogenous shocks or other state variables. For example, some exogenous shocks or other state variables, that are assumed to be unobserved, may be partially observed by a combination of other domestic and international data sets. Utilizing the information from additional series could result in more accurate estimations of the model's parameters and states and thus creating more accurate conclusions about the model's dynamics. The key insight of this paper is that in fact, an open-economy DSGE-DFM model can mitigate all three of these issues.

First, like Kolasa and Rubaszek (2018), I find that the open-economy DSGE-Reg model does not produce better forecasts for GDP growth, consumption growth, investment growth, interest rates and inflation when compared to the Bank of Canada's Staff Economic Projections (SEP) or other reduced from forecasting models. However, the open-economy DSGE-DFM model that incorporates real-time domestic and international macro-financial data was able to significantly out-forecast both the SEP and the open-economy DSGE-Reg model for domestic growth variables, export growth and import growth in regard to both the short and medium term horizon.

Second, in accordance with Ca'Zorzi et al. (2017), I find that the open-economy DSGE-Reg model of this paper is able to slightly out-forecast a naive random walk for the real exchange rate, but not the nominal exchange rate. However, the open-economy DSGE-DFM model of this paper is able to out-forecast a naive random walk in regard to both the real exchange rate and the nominal exchange rate when it incorporates real-time international macro-financial data that would have been currently available to the econometrician.

Third, in contrast to Justiniano and Preston (2010), I find that there are significant and empirically validated co-movements in the DSGE-DFM model of this paper between the United States and Canada. When I evaluate the model implied correlation between the Canadian economy and the U.S. economy, I find that GDP growth and its components resemble empirical correlations seen in the data. The DSGE-DFM model is still not able to produce any policy rate correlation between the United States and Canada and a smaller than empirically-observed correlation for inflation between the two countries. However, when I input values for United States macro-financial data into the model that correspond to an empirical generic U.S. recession of the last 70 years, I am able to generate the probabilistic co-movements and correlations between Canada's and the United States levels of inflation, policy rates and expenditure growths.

I choose to use Canada as the domestic country in the open-economy model for various reasons. First, Canada is included in the forecasting applications of both Kolasa and Rubaszek (2018) and Ca'Zorzi et al. (2017). Second, I will be able to directly address the findings of Justiniano and Preston (2010) in this paper. Finally, the extensiveness of the Bank of Canada's SEP collected and analyzed by Champagne and Sekkel (2018) allows me to compare the model's forecasts to real-time expert-based forecasts for a number of macroeconomic variables.

The remainder of this paper is structured as follows. Section 2 briefly explains the features of the open-economy DSGE model and presents its linearized equations. Section 3 outlines the estimation technique used to incorporate the large set of international economic and financial series. Section 4 discusses the priors for the state-space and structural parameters as well as an overview of the data series used in the estimation. Section 5 presents the estimation results and the resulting dynamics of the DSGE-DFM model. Section 6 examines the forecasting performance of the DSGE-DFM model against its DSGE-Reg counterpart, other reduced form models, and the Bank of Canada's Staff Economic Projections for various macroeconomic variables. In addition, the forecasts of the real and nominal exchange rates in the DSGE-DFM model are compared against random walk forecasts and other small and large data forecasting models. Section 7 examines the model implied correlations between Canada and the United States. It also considers the impact a U.S. recession that is calibrated using historical averages would have on Canada's macroeconomy. Finally, section 8 concludes and discusses future extensions.

2. The structural DSGE model

The structural DSGE model I use in the paper is an extension of the Smets and Wouters (2003, 2007) New Keynesian model as well as the Adolfson et al. (2007a, 2008); Lubik and Schorfheide (2007) and Seneca (2010) open-economy models. It features a domestic sector and a world sector, endogenous capital, price and wage setting frictions and indexation as well as capital utilization and investment adjustment costs. In this section, I outline the agents of the open-economy DSGE model and I present the linearized equations of the model around the steady state that I use to produce my results.

2.1. General outline of the model

The model involves a number of exogenous shocks, economic agents, and market frictions. The agents include an exogenous world economy, domestic households, domestic capital producers, domestic firms and government agencies.

World Economy is specified as an exogenous simple New Keynesian micro-founded closed-economy model without capital. The world sector has price stickiness, price indexation, habit persistence and purchases and sells consumption goods from and to the domestic sector. The short term nominal interest rate is determined by the foreign monetary authority, which is assumed to follow a generalized Taylor Rule. The world economy is subject to demand, price and monetary policy shocks. In addition, the world and domestic economy share a permanent technology shock that alters the steady-state growth path of production.

Households supply household-specific labor to employment agencies. Households maximize a CRRA utility function over an infinite horizon with additively separable utility in consumption, leisure and money. Households earn income form wages, rented capital, and interest payments from domestic and foreign bonds. Utility from consumption includes a habit persistence measure. The final consumption good that enters the domestic household utility function is an aggregate of home and foreign goods. Households are subject to an exogenous preference shock and risk premium shock. These can be viewed as a shock in the consumer's consumption and savings decision and international bond investment decision.

Firms supply domestic goods and exports in a monopolistically competitive market. Firms produce differentiated goods, decide on labor and capital inputs, and set domestic prices and export prices in a Calvo (1983) manner. As with wages, those firms unable to change their prices, are able to partially index them to past inflation rates. Firms face four types of exogenous shocks, the first is a permanent global productivity shock, the second is a temporary technology shock that affects their production ability. The third is a price mark-up shock which captures the degree of competitiveness in the domestic and foreign goods market as well as the labor input market. Finally, firms are also subject to an asymmetric trade shock.

Capital Producers are competitive firms that control the creation of new capital (Investment), a process that requires both the newly bought consumption output and the previous stock of capital in the economy. The investment procedure is subject to adjustment costs and capital producers are subject to investment shocks that affect the marginal efficiency of investment as in Justiniano et al. (2011).

Domestic Government Agencies are composed of a monetary authority and a fiscal authority. The short term nominal interest rate is determined by the monetary authority, which is assumed to follow a generalized Taylor Rule and is subject to monetary policy shocks. The fiscal authority sets fiscal policy and is subject to exogenous government spending shocks.

2.2. Log linear equations

The model is linearized around the non-stochastic steady state and then solved using the Sims (2002) method. This solution is the transition equation in the state-space set-up of Section 3. Variables denoted with a time script are defined as log deviations around the steady state. Variables denoted without a time script are steady state values. If variables are stationary $\left(y_t = log\left(\frac{Y_t}{Y}\right)\right)$ and if variables are non-stationary $\left(y_t = log\left(\frac{Y_t}{Z_t}\right) - log\left(\frac{Y}{Z}\right)\right)$ where Z_t is the stochastic growth path determined by permanent technology shocks. In all, the open-economy model is reduced to 32 equations and 15 exogenous shocks all of which are listed in this subsection.

2.2.1. World sector

The world sector IS curve is a combination of the following two equations:

$$\frac{(1-H^*)}{1-\beta H^*}\lambda_t^* = (1-H^*)(z_{y^*,t} - H^*\beta E_t\left[z_{y^*,t-1}\right]) - (1-(\beta H^*)^2)y_t^* + H^*(y_{t-1}^* + \beta E_t\left[y_{t+1}^*\right] - \zeta_t + \beta E_t[\zeta_{t+1}]) \tag{1}$$

$$\lambda_t^* = R_t^* + E_t\left[\lambda_{t+1}^*\right] - E_t[\zeta_{t+1}] + E_t\left[\pi_{t+1}^*\right] \tag{2}$$

where λ_t^* is the marginal utility of output in the foreign household utility function, y_t^*, R_t^* and π_t^* are world output, world nominal interest rate and world inflation rate respectively. While ζ_t is a permanent global technology shock and $z_{y^*,t}$ is foreign demand shock. β is the global discount rate and H^* is the habit persistence in foreign output.

The world sector yields a Keynesian Phillips curve and linearized Taylor Rule equal to:

$$\pi_t^* = \frac{1}{1+\chi^*\beta}(\beta E_t\left[\pi_{t+1}^*\right] + \chi^*\pi_{t-1}^* + \kappa^* y_t) + \mu_{\pi^*,t} \tag{3}$$

$$R_t^* = \rho_r^* R_{t-1}^* + (1-\rho_r^*)[\phi_{\pi^*}\pi_t^* + \phi_{y^*}y_t^*)] + \varepsilon_{R^*,t} \tag{4}$$

where χ^* is world inflation indexation, κ^* is the slope of the world Phillips curve and $\mu_{\pi^*,t}$ and $\varepsilon_{R^*,t}$ are world price and monetary policy shocks.

2.2.2. Resource identities and factor demands

The open-economy's resource constraint takes the form:

$$y_t = C_y c_t + I_y i_t + G_y z_{G,t} + \frac{I_y r^k}{\delta} u_t + nx_t \tag{5}$$

where C_y, I_y and G_y are the steady state ratios of consumption, investment and government expenditures. While r^k is the real rental rate of capital, δ is the depreciation rate and u_t is the utilization rate of capital. nx_t is net exports and $z_{G,t}$ is an exogenous government purchase shock of domestic goods. The definitions of net exports, exports and imports are all defined below.

$$nx_t = \alpha(ex_t - im_t - t_t - z_{D,t}) \tag{6}$$

$$ex_t = -\eta P_{H,t}^* + y_t^* \tag{7}$$

$$im_t = -\eta P_{F,t} + C_y c_t + I_y i_t + G_y z_{G,t} + \frac{I_y r_k}{\delta} u_t \tag{8}$$

where t_t is the terms of trade relation[1], α is the degree of openness to foreign goods and η is the elasticity of substitution between domestically produced and imported goods.[2] $P_{H,t}^*$ and $P_{F,t}$ denote the relative price for exported goods and imported goods relative to the final good. $z_{D,t}$ is a net exports shock the reflects international differences in terms of trade.

The domestic economy's resource constraint and aggregate production takes the following forms:

$$y_t^H = (1-\alpha)\left(-\eta P_{H,t} + C_y c_t + I_y i_t + G_y z_{G,t} + \frac{I_y r_k}{\delta} u_t\right) + \alpha ex_t \tag{9}$$

$$y_t^H = \psi_H k_t + (1-\psi_H)(z_{H,t} + L_t) \tag{10}$$

where $y_t^H$is domestic production and k_t ad L_t are domestic effective capital and labor with ψ_H being the proportion of capital in the domestic production function. Aggregate domestic production is subject to temporary labor augmenting technology shocks denoted by $z_{H,t}$.

The domestic factor input and marginal costs for domestic firms are:

$$r_t^k = w_t + L_t - k_t \tag{11}$$

$$mc_t = (1-\psi_H)w_t + \psi_H r_t^k - (1-\psi_H)z_{H,t} \tag{12}$$

[1] As in Ca'Zorzi et al. (2017); Justiniano and Preston (2010); Seneca (2010) and Kolasa and Rubaszek (2018), terms of trade is included in equation 5, the open-economy resource constraint equation. Using an alternative modeling assumption as many do in the open-economy DSGE literature has little impact on the results reported in the paper.

[2] The same value for the elasticities of substitution between domestic and imported goods in both the domestic and world economy is assumed.

2.2.3. Domestic investment and consumption

The relationship between the stock of capital k^s_t and effective capital k_t is

$$k_t = u_t + k^s_t - \zeta_t \tag{13}$$

while the capital accumulation equation is given by

$$k^s_{t+1} = (1-\delta)(k^s_t - \zeta_t) + \delta(i_t + z_{I,t}) \tag{14}$$

where $z_{I,t}$ is an investment shock to the marginal efficiency of capital creation.

The capital utilization condition captures capital utilization costs in the parameter λ_u. Its condition states:

$$r^k_t = \frac{1}{\lambda_u} u_t \tag{15}$$

The linearized investment transition equation is given by:

$$i_t = \frac{1}{1+\beta}\left(\beta E_t[i_{t+1} + \zeta_{t+1}] + i_{t-1} - \zeta_t + \frac{1}{\lambda_I}(q_t + z_{I,t})\right) \tag{16}$$

where λ_I capture investment adjustment costs. A large λ_I implies that adjusting the investment schedule is costly. q_t is the domestic relative price of capital and is characterized by:

$$q_t = -(R_t - E_t[\pi_{t+1}]) + (1 - \beta + \beta\delta)E_t\left[r^k_{t+1}\right] + (\beta - \beta\delta)E_t[q_{t+1}] \tag{17}$$

The linearized consumption transition equation that includes habit persistence (h) and a consumption shock ($z_{C,t}$) is given by:

$$\begin{aligned} c_t = {} & \frac{h}{1+h}c_{t-1} + \frac{1}{1+h}E_t[c_{t+1}] - \frac{1-h}{1+h}(R_t - E_t[\pi_{t+1}]) \\ & - \frac{1-h}{1+h}(E_t[z_{C,t+1}] - z_{C,t}) + \frac{1}{1+h}E_t[\zeta_{t+1}] - \frac{h}{1+h}\zeta_t \end{aligned} \tag{18}$$

2.2.4. Trade and exchange rate

The real exchange rate (s_t) is given by

$$s_t = s_{t-1} + \pi_{\epsilon,t} + \pi^*_t - \pi_t \tag{19}$$

where $\pi_{\epsilon,t}$ is the nominal depreciation rate of the domestic currency price of one unit of foreign currency. The linearized terms of trade relation equation is:

$$t_t = P_{F,t} - P^*_{H,t} - s_t \tag{20}$$

where $P^*_{H,t}$ is the relative price of exports to the world sector in terms of the final good.

The evolution of the domestic country's real net asset position ($b^*_{H,t}$) is:

$$b^*_{H,t+1} = \frac{1}{\beta}b^*_{H,t} + nx_t \tag{21}$$

The linearized uncovered interest rate parity condition is given by:

$$s_t = E_t[s_{t+1}] - (R_t - E_t[\pi_{t+1}]) + \left(R^*_t - E_t\left[\pi^*_{t+1}\right]\right) - \phi_B b^*_{H,t} - z_{B_t} \tag{22}$$

where ϕ_B and z_{B_t} are a risk premium scale parameter and exogenous shock respectively.

2.2.5. New keynesian philips curve

The domestic economy has four linearized New Keynesian Phillips curves. One for wage inflation ($\pi_{w,t}$) :

$$\begin{aligned} \pi_{w,t} = {} & \beta E_t[\pi_{w,t+1}] + \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w(1+\psi\epsilon_w)}\Bigg(\psi L_t + \frac{1}{1-h}(c_t - hc_{t-1}) - \\ & z_{C,t} + \frac{h}{1-h}\zeta_t - w_t + \mu_{w,t}\Bigg) + \gamma_W(\pi_{t-1} - \beta\pi_t) + (1-\beta\rho_\zeta)\zeta_t \end{aligned} \tag{23}$$

where ψ is the CRRA coefficient on Labor in the domestic household's utility function. In addition to wages there is a linearized New Keynesian Phillips curve for domestic price inflation ($\pi_{H,t}$), export price inflation ($\pi^*_{H,t}$) and import price inflation ($\pi_{F,t}$):

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \gamma_H(\pi_{t-1} - \beta\pi_t) + \frac{(1-\beta\theta_H)(1-\theta_H)}{\theta_H}(mc_t - P_{H,t} + \mu_{H,t}) \tag{24}$$

$$\pi^*_{H,t} = \beta E_t\left[\pi^*_{H,t+1}\right] + \gamma_{H^*}(\pi^*_{t-1} - \beta\pi^*_t) + \frac{(1-\beta\theta_{H^*})(1-\theta_{H^*})}{\theta_{H^*}}\left(mc_t - P^*_{H,t} - s_t + \mu_{H^*,t}\right) \tag{25}$$

$$\pi_{F,t} = \beta E_t[\pi_{F,t+1}] + \gamma_F(\pi_{t-1} - \beta\pi_t) + \frac{(1-\beta\theta_F)(1-\theta_F)(1-\psi_F)}{\theta_F(1-\psi_F+\psi_F\epsilon_F)}(\eta_{mc}y_t^* - P_{F,t} + s_t + \mu_{F,t}) \tag{26}$$

where θ_ℓ denotes the degree of price stickiness parameter and γ_ℓ is the degree of price indexation for wages, domestic prices, export prices and import prices. $\mu_{\ell,t}$ is a price mark up shock to a particular market, $\ell = \{w, H, H^*, F\}$.

2.2.6. Price definitions

The linearized domestic CPI identity is equal to:

$$(1-\alpha)P_{H,t} + \alpha P_{F,t} = 0 \tag{27}$$

Wage inflation is defined as:

$$\pi_{w,t} - \pi_t = w_t - w_{t-1} + \zeta_t \tag{28}$$

Domestic good inflation is defined as:

$$\pi_{H,t} - \pi_t = P_{H,t} - P_{H,t-1} \tag{29}$$

Export good inflation is defined as:

$$\pi^*_{H,t} - \pi^*_t = P^*_{H,t} - P^*_{H,t-1} \tag{30}$$

Import good inflation is defined as:

$$\pi_{F,t} - \pi_t = P_{F,t} - P_{F,t-1} \tag{31}$$

2.2.7. Monetary policy and exogenous shocks

Monetary policy is conducted using the following linearized Taylor Equation that determines the nominal interest rate:[3]

$$R_t = \rho_r R_{t-1} + (1-\rho_r)[\phi_\pi \pi_t + \phi_y y_t + \phi_s(s_t - s_{t-1})] + \varepsilon_{R,t} \tag{32}$$

In all, the open-economy model has 15 exogenous shocks, 13 of which are AR(1) processes, the lone exceptions being the global monetary policy shock and domestic monetary shock which are simply white noise shocks. Included, in the remaining 13 shocks are two foreign sector shocks, four mark-up shocks, four component demand shocks, a domestic technology shock, a risk premium shock and a global productivity shock that alters the global steady state growth path. All processes are assumed to be i.i.d. with mean zero and standard deviation σ_i and autocorrelation parameters ρ_i, where $i = \{z^*_y, \mu_{\pi^*}, \mu_W, \mu_H, \mu_{H^*}, \mu_F, z_C, z_I, z_G, z_D, z_H, z_B, \zeta, \varepsilon_{R^*}, \varepsilon_R\}$

3. Estimation

I estimate the model in two different ways. The first method estimates the model using specific data that match particular states in the model, I call this DSGE-Reg estimation. The second method estimates the model using specific data that match particular states and other data series that have no direct connection to the model's endogenous variables, I call this method DSGE-DFM estimation. I use the common approach in the literature and estimate each method in two stages. In the first stage, I estimate the exogenous world sector's structural parameters. In the second stage, I estimate the small open-economy's structural parameters, taking as given the posterior mean values of the world sector's parameters from the first stage. In the remaining subsections, I give an overview of both the DSGE-Reg and DSGE-DFM estimation methods.

3.1. Regular DSGE estimation

The state space representation of the solved model consists of a transition equation, which is calculated by solving the linearized system of the given model one wishes to evaluate for a given set of structural model parameters (θ):[4]

$$S_t = G(\theta)S_{t-1} + H(\theta)\nu_t \quad \text{where } \nu_t \sim NID(0, I_m) \tag{33}$$

and the measurement equation:

$$X_t^{reg} = \Lambda S_t + \epsilon_t \quad \text{where } \epsilon_t \sim NID(0, R^{reg}) \tag{34}$$

Here X_t^{reg} are the economic data sets and Λ is a matrix that matches the observed data to the definitions of the model's state variables S_t.[5] The matrices $G(\theta)$ and $H(\theta)$ are functions of the model's structural parameters (θ) and ϵ_t is a vector of i.i.d. shocks representing measurement error between the model and the data series used in estimation. Exact measurement equations for equation 34 can be found in Appendix A.

[3] The interest setting rule includes a central bank response (ϕ_s) to changes in the real exchange rate. This corresponds to findings by Lubik and Schorfheide (2007) and Alstadheim et al. (2013). However others (Dong, 2013) and Champagne et al. (2018) have found that the response has declined since the global financial crisis. Given the mixture of findings surrounding, ϕ_s, I include it in the rate setting rule but estimate it with a tight prior centered around zero.

[4] For more detail on Bayesian DSGE-Reg estimation techniques please see An and Schorfheide (2007).

[5] A complete list of the DSGE model's state variables in S_t can be found in the online appendix.

3.2. DSGE-DFM Estimation

Bayesian estimation of a DSGE model in a data-rich environment incorporates the state space model discussed above with a few modifications.

The set up for DSGE-DFM estimation is characterized by equations (35)-(37).

$$S_t = G(\theta)S_{t-1} + H(\theta)\nu_t \text{ where } \nu_t \sim NID(0, I_m) \tag{35}$$

$$X_t = \Lambda S_t + e_t \tag{36}$$

$$e_t = \Psi e_{t-1} + \epsilon_t \text{ where } \epsilon_t \sim NID(0, R) \tag{37}$$

Here e_t follows an AR(1) process and is often referred to as measurement error. The matrix X is J x T where J is the number of data series used in estimation and T is the number of observations for each series. The Matrix Λ is now no longer assumed to be known by the econometrician, but instead is estimated within the MCMC routine. The matrices Ψ and R that govern the measurement error's stationary processes for each series are assumed to be diagonal and are also estimated within the MCMC routine.

The measurement equation (36) has the following structure:

$$\begin{bmatrix} \textit{World GDP Growth} \\ \textit{World Inflation} \\ \vdots \\ ------ \\ [\textit{Int'l Macro/Fin}] \\ [\textit{Expenditure Components}] \\ [\textit{Labor Market}] \\ [\textit{Yields}] \\ [\textit{Credit Market}] \\ [\textit{Price/Wage Indexes}] \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & \ldots & 1 \\ 0 & 0 & 4 & \ldots & 0 \\ -- & -- & ---- & --- & -- \\ [\lambda_{I_1}] & [\lambda_{I_2}] & [\lambda_{I_3}] & \ldots & [\lambda_{I_n}] \\ [\lambda_{E_1}] & [\lambda_{E_2}] & [\lambda_{E_3}] & \ldots & [\lambda_{E_n}] \\ [\lambda_{L_1}] & [\lambda_{L_2}] & [\lambda_{L_3}] & \ldots & [\lambda_{L_n}] \\ \vdots & \vdots & \vdots & \ldots & \vdots \end{bmatrix} \begin{bmatrix} y_t^* \\ y_{t-1}^* \\ \pi_t^* \\ \vdots \\ \zeta_t \end{bmatrix} + \begin{bmatrix} e_{t,1} \\ e_{t,2} \\ \vdots \\ e_{t,J} \end{bmatrix}$$

where X_t is partitioned into core series and non-core series separated by the dashed line. The core series are series that are only allowed to load on particular variables of the state vector, S_t, to which there is a known relationship between series and state(s). For instance, World GDP growth to $y_t^* - y_{t-1}^* + \zeta_t$. Further, the factor loading coefficient of each core variable that corresponds to a particular known state definition is assumed to be perfectly tight. This is represented by the1′s and 4 in the Λ matrix. This anchors the estimated states of the DSGE model and ensures that they don't drift too far away from their theoretical foundation.

The non-core series consists of the remaining data sets not in the core series and are grouped into subgroups. These series are allowed to "load" on all or certain states (structure determined by the econometrician) in the state vector. Non-core series may have up to n (where n is the number of elements in S_t) non-zero elements for their corresponding row in Λ unlike the core series whose corresponding row in Λ may only have non-zero elements relating to their related state definition.

Following the work of Boivin and Giannoni (2006) and Gelfer (2019), a Metropolis-within-Gibbs algorithm is used to estimate the state space parameters, $\Gamma = [\Lambda,\ \Psi,\ R]$, and the structural DSGE parameters θ. The adaptive Metropolis-within-Gibbs algorithm used follows the following steps:

1. Specify Initial values of $\theta^{(0)}$, and $\Gamma^{(0)}$, $\Gamma = \{\Lambda,\ \Psi,\ R\}$
2. Repeat for g=1... G
   2.1 Solve the DSGE model numerically and obtain $G(\theta^{(g-1)})$ and $H(\theta^{(g-1)})$
   2.2 Draw from $p(\Gamma|G(\theta^{(g-1)}), H(\theta^{(g-1)}); X_{1:T})$
      2.2.1 Generate unobserved states $S^{1:T,(g)}$ from $p(S^{1:T}|\Gamma^{(g-1)}, G(\theta^{(g-1)}), H(\theta^{(g-1)}); X_{1:T})$ using the Carter-Kohn forward-backward algorithm
      2.2.2 Generate state-space parameters $\Gamma^{(g)}$ from $p(\Gamma|S^{1:T,(g)}; X_{1:T})$ by drawing from a set of known conditional densities $[R^{(g)}|\Lambda^{(g-1)}, \Psi^{(g-1)}; S^{1:T,(g)}]$, $[\Lambda^{(g)}|R^{(g)}, \Psi^{(g-1)}; S^{1:T,(g)}]$, $[\Psi^{(g)}|\Lambda^{(g)}, R^{(g)}; S^{1:T,(g)}]$.
   2.3 Draw DSGE parameters $\theta^{(g)}$ from $p(\theta|\Gamma, S^{1:T}; X_{1:T})$ using adaptive Metropolis Hastings using the proposal equation of $\theta^* = \theta^{(g-1)} + \bar{c}\,\varepsilon_\ell$ where $\varepsilon_\ell \sim NID(0, \Sigma^{-1})$
   2.4 Calculate acceptance rate of proposed θ for 1 to g draws. If the acceptance rate is lower than target acceptance rate decrease $\bar{c}$ by w (iff $\bar{c} > w$). If acceptance rate is greater than target acceptance rate increase $\bar{c}$ by w. This target acceptance rate adaption can be implemented every n^* iterations of g. In addition the condition $w \to 0$ as $g \to \infty$ must be satisfied
3. Return $\{\theta^{(g)},\ \Gamma^{(g)}\}_{g=1}^{G}$

Table 1
Data Series used in Estimation of the World Sector.

Data Set	Transform	Data Set	Transform
DSGE-Reg Estimation			
World Aggregate GDP Growth	0		
World Aggregate Inflation	0		
World Aggregate Policy Rate	0		
DSGE-DFM Estimation			
World Aggregate GDP Growth	0	United States 10-year Treasury	0
World Aggregate Inflation	0	Germany 10-year Government Bond Rate	0
World Aggregate Policy Rate	0	UK 10-year Government Bond Rate	0
United States GDP Growth	0	France 10-year Government Bond Rate	0
EU GDP Growth	0	Italy 10-year Government Bond Rate	0
China GDP Growth	0	Spain 10-year Government Bond Rate	0
Mexico GDP Growth	0	Switzerland 10-year Government Bond Rate	0
Japan GDP Growth	0	Japan 10-year Government Bond Rate	0
United States Inflation	0	United States Major Stock Index Growth	0
EU Inflation	0	China Major Stock Index Growth	0
China Inflation	0	Japan Major Stock Index Growth	0
Mexico Inflation	0	Mexico Major Stock Index Growth	0
Japan Inflation	0	EU ETF Stock Index Growth	0
United States Policy Rate	0		
EU Policy Rate	0		
Japan Policy Rate	0		
UK Policy Rate	0		
Mexico Policy Rate	0		

Table 2
Countries and Weights used to determine the World Sector Aggregates.

Country	Weight in World Sector
United States	0.753
EU	0.122
China	0.061
Mexico	0.027
Japan	0.025
South Korea	0.012

The algorithm must first be initialized with $\theta^{(0)}$, $\Gamma^{(0)}$ and Σ^{-1}. The values of $\theta^{(0)}$ are retrieved by taking the mean of $P(\theta|X^{reg})$ when estimated as described in Section 3.1. Once $\theta^{(0)}$ is obtained it is then used to calculate $S^{1:T,(0)}$. The estimated states are then used to run line-by-line OLS for each series in X to generate initial values of $\Gamma^{(0)}$. Σ^{-1} is the inverse Hessian of the DSGE model evaluated at its posterior mode under DSGE-Reg estimation.

The applied algorithm is based on 1,000,000 draws, discarding an initial burn-in period of 250,000 iterations. The calibrations regarding the adaptive step include the acceptance target rate which is set at 27%, an initial $\bar{c}$ which is set to 0.1, the adaptive jump size w which is set at 0.005 and an adjustment rate n^* which is set at 25. The adjustment rate n^* determines how many iterations take place between changing $\bar{c}$ as described in step 2.4.[6]

4. Data and priors

In total I estimate the model using 93 Canadian and international quarterly data series that cover the time period of 1995Q1 t0 2017Q4.[7] The first stage (World sector) is estimated with 3 data series in the DSGE-Reg estimation and 31 in the DSGE-DFM estimation method. For estimating the entire open-economy DSGE-Reg model a total of 16 data series are used and a total of 93 data series are used to estimated the entire open-economy DSGE-DFM model.

Let's first discuss the series used in estimating the DSGE-Reg and DSGE-DFM structural parameters of the exogenous world sector. Table 1 lists the different data sets used in estimating the world sector in both fashions respectively.

To calculate world aggregates, I use a trade volume weighted average of Canada's five largest trading partners and the EU from 2010 to 2017. The weights and countries used to determine the world aggregates are listed in Table 2. These weights are determined by calculating the average yearly share of export and import values for the six countries from 2010 to 2017. The world's aggregate GDP growth, inflation rate and policy rate is a weighted average of real GDP growth, annualized GDP

[6] For more detail on the Bayesian DSGE-DFM estimation technique please see Gelfer (2019).

[7] A 3-month average is used to obtain quarterly data from monthly series, including interest rates, level of employment, level of total credit and price index level.

Table 3
Data Series used in Estimation of the Canadian DSGE-DFM model.

Data Set	Trans	RT	Data Set	Trans	RT
DSGE-DFM Estimation					
Core-Series					
World Aggregate GDP Growth	0	-	CAN Import Growth	γ	-
World Aggregate Inflation	0	-	CAN Hours Growth	0	-
World Aggregate Policy Rate	0	-	CAN Wage Inflation	0	-
CAN GDP Growth	γ	-	CAN Export Inflation	0	-
CAN Consumption Growth	γ	-	CAN Import Inflation	0	-
CAN Investment Growth	γ	-	CAN CPI Inflation	0	-
CAN Government Growth	γ	-	CAN Real FX Rate Growth	0	-
CAN Export Growth	γ	-	CAN Policy Rate	0	-
International Series					
US GDP Growth	0	-	Mexico Policy Rate	0	RT
EU GDP Growth	0	-	United States 10-year Treasury	0	RT
China GDP Growth	0	-	Germany 10-year Govt Bond Rate	0	RT
Mexico GDP Growth	0	-	UK 10-year Govt Bond Rate	0	RT
Japan GDP Growth	0	-	France 10-year Govt Bond Rate	0	RT
US Inflation	0	-	Italy 10-year Govt Bond Rate	0	RT
EU Inflation	0	-	Spain 10-year Govt Bond Rate	0	RT
China Inflation	0	-	Switzerland 10-year Govt Bond Rate	0	RT
Mexico Inflation	0	-	Japan 10-year Govt Bond Rate	0	RT
Japan Inflation	0	-	US Major Stock Index Growth	0	RT
US Policy Rate	0	RT	China Major Stock Index Growth	0	RT
EU Policy Rate	0	RT	Japan Major Stock Index Growth	0	RT
Japan Policy Rate	0	RT	Mexico Major Stock Index Growth	0	RT
UK Policy Rate	0	RT	EU ETF Stock Index Growth	0	RT
Expenditure Components					
CAN Industrial Production	1	RT2	CAN Residential Investment	1	-
CAN Durable Cons Goods	1	-	CAN Non-Residential Structure Inv	1	-
CAN Semi-Durable Cons Goods	1	-	CAN Machinery and Equipment Inv	1	-
CAN Non-Durable Cons Goods	1	-	CAN Intellectual Property Inv	1	-
CAN Consumption Services	1	-	CAN Inventory Investment	1	-
CAN Business Investment	1	-			
Employment					
CAN Total Employees	1	RT2	CAN Professional Services Employees	1	RT2
CAN Agricultural Employees	1	RT2	CAN Business Services Employees	1	RT2
CAN FFMO Employees	1	RT2	CAN Educational Services Employees	1	RT2
CAN Utilities Employees	1	RT2	CAN Health care Services Employees	1	RT2
CAN Construction Employees	1	RT2	CAN Information Services Employees	1	RT2
CAN Manufacturing Employees	1	RT2	CAN Food Services Employees	1	RT2
CAN Wholesale & Retail Employees	1	RT2	CAN Public Employees	1	RT2
CAN Transportation Employees	1	RT2	CAN Unemployment Rate	0	RT2
CAN FIRE Employees	1	RT2			
Yields and Spreads					
Consumer Loan Rate	0	RT	CAN 2-year to 3-month Treas Spread	0	RT
Prime Corporate Rate	0	RT	CAN 5 to 2-year Treasury Spread	0	RT
Overnight Money Market Rate	0	RT	CAN 7 to 2-year Treasury Spread	0	RT
CAN 3-month Treasury Rate	0	RT	CAN 10 to 2-year Treasury Spread	0	RT
CAN 6 to 3-month Treas Spread	0	RT	CAN Mortgage Rate	0	RT
CAN 1-year to 3-month Treas Spread	0	RT			
Credit Markets					
CAN Total Bus Credit Outstanding	1	RT2	CAN Outstanding Business Bonds	1	RT2
CAN Total Bus Loans Outstanding	1	RT2	CAN Mortgage Credit Outstanding	1	RT2
CAN Total Commercial Paper	1	RT2	CAN Consumer Credit Outstanding	1	RT2
CAN Housing Starts	2	RT2			
Other Prices					
CAN Core CPI Inflation	0	-	USD/CAD Exchange Rate Growth	0	RT
CAN GDP Deflator Inflation	0	-	Oil Price Inflation	0	RT2

deflator growth and the policy rate for the selected trading partners. The world aggregate policy rate does not include the Bank of China's policy rate.

Table 3 lists all the data used for the Canadian DSGE-Reg and DSGE-DFM models. For both the DSGE-Reg and DSGE-DFM estimation methods, all Canadian GDP growth, expenditure growth components and hours worked growth are in per capita terms using the total population of Canada. Canadian wage inflation is defined as the percentage change in the nominal hourly wage of all employees. Export, import and CPI inflation is defined as the percentage change in the GDP price deflator for exports, imports and the percentage change in the CPI price index respectively. The Real foreign exchange rate growth is defined as the percentage change of the effective foreign exchange rate of Canada weighed by trading partners.

Table 4
Priors for DSGE-DFM Γ Parameters.

	Description	Distribution	Mean	Std
Γ Parameters				
$\Psi_{i,i}$	AR(1) coef. of measurement error	Normal	0	1
$R_{i,i}$	Variance of measurement error	Inv. Gamma	0.001	3*
$\Lambda_{i,j}$	Factor loadings of Non-core data sets	Normal	0	$R_{i,i}I$
$\Lambda_{i,j}$	Factor loadings of Core data sets	Normal	1	0

In all the DSGE-DFM model is estimated using the 16 core data series in the DSGE-Reg model while also adding sub-aggregate GDP expenditure components, total and sub-aggregate employment data, various yields and yield spreads, credit market statistics and multiple measures of inflation to its X_t data set. As is common in the dynamic factor model literature, all non-core series sample standard deviation is normalized to 1. Measurement equations for all core series and the transformation code applied to each series can be found in Appendix A. The data sources of all data used in the estimations can be found in the online appendix.

4.1. Structural and state-Space parameter priors

The structural parameter marginal priors are in accordance to ToTEM II priors by Dorich et al. (2013). Some structural parameters are fixed including the global discount rate, share of capital, depreciation rate, and the component's steady state share to total Canadian output. The steady state GDP share G_y of government purchases is calibrated to the average proportion of government purchases of Canadian GDP over the sample period (0.24). A domestic price mark-up of 1.15, a depreciation rate of 0.02 and a capital share of production of 0.3 implies a steady state share of investment to GDP (I_y) of 0.19. This is just below the average share of investment to Canadian GDP over the sample period (0.193). The parameter α is calibrated to 0.31 to math the import share to Canadian GDP found in the data over the sample period. β is calibrated to correspond to a steady state real interest rate of 3%. The parameter γ is calibrated to correspond to an annual per capita real GDP growth rate of 1.5%. This is the average of Canadian per capita GDP over the sample period and slightly lower than the aggregate world per capita real GDP growth over the sample period. A complete list of calibrated structural parameters as well as the prior mean, standard deviation and description of the estimated structural parameters can be found in Table C.1 and Table C.2.

The priors for the state space parameters include the elements of Λ and the diagonal elements of Ψ and R. The elements of Λ can be separated between core and non-core elements. Core series only have a non-zero row elements of Λ whose prior is perfectly tight around the modeled state definition. Each non-core series corresponding row elements of Λ has a multivariate normal prior centered around zero.

The prior for each i^{th} row of the non-core series follows the work of Boivin and Giannoni (2006) and Gelfer (2019), who use a Normal-Inverse-Gamma prior distribution for $(\Lambda_i, R_{i,i})$. So that, $R_{i,i} \sim IG_2(.001, 3)$ and the prior mean of factor loadings for the i^{th} row is given by $\Lambda_i|R_{i,i} \sim N(0, R_{i,i}I)$ where the mean is a vector of zeros and I is the identity matrix. The prior for the i^{th} measurement equation's autocorrelation parameter, $\Psi_{i,i}$ is $N(0, 1)$ for all rows. The autocorrelation parameter prior is truncated to values inside the unit circle to ensure all error processes are stationary. All prior distributions for the state space parameters are summarized in Table 4. The estimated measurement error parameters of both models and a discussion about their importance to the results of this paper is found in Appendix B.

5. Estimation results

This section is divided between estimation results and the economic inferences that DSGE-DFM estimation can tell us about how economic series and sectors not directly modeled in the structural DSGE model react to structural economic shocks. Posterior estimates of the structural parameter are discussed first. Second, impulse response functions (IRF's) of the world sector and domestic sector are presented and discussed along with the diagnostic inferences they bring when comparing economic events that are driven by different types of structural shocks. This will help us understand why the DSGE-DFM model is able to create superior forecasts compared to other models for so many macroeconomic variables.

The posterior estimates for the structural parameters for both estimation techniques are tabulated in Tables C.3 and C.4. A few observations emerge. First, in the world sector the persistence of world productivity growth shocks is estimated to be almost non-existent in the DSGE-DFM model compared to the DSGE-Reg model. With mean ρ_ζ estimated to be 0.21 in the DSGE-DFM model compared to 0.85 in the DSGE-Reg model. This implies that changes in the steady state growth path of output are much less persistent but more volatile when a model is estimated in a data-rich environment. Additionally, the New Keynesian Phillips curve is estimated to be flatter in the DSGE-DFM model. All remaining parameters in the world sector show little difference between the two estimation techniques.

The price and wage Calvo estimates for wages, domestic prices and export prices are lower in the DSGE-DFM model, suggesting less nominal rigidities in pricing behavior. These smaller, yet still significant, price and wage rigidities are more in line with the findings of Amirault et al. (2006) who surveyed monthly price changes by business type and found that the median price duration is about every 3 months. However, the price stickiness of exports is estimated to be larger in the DSGE-DFM model, suggesting exports are more likely to face pricing frictions.

The parameters that govern habit formation substantially increase in the DSGE-DFM estimation for the world sector and decrease in the domestic sector when compared to their estimates under DSGE-Reg estimation. Taylor Rule policy parameters are found to be slightly more responsive to the exchange rate and slightly less responsive to inflation when estimated in the data-rich environment. All of these findings will account for different trade, inflation and consumption dynamics when I compare the core macroeconomic variables between the two estimation techniques.[8]

Many of the parameters linked to the exogenous shocks of the model become smaller across the estimation techniques of the DSGE model. The standard deviation coefficients on consumption and investment shocks are estimated to be significantly lower in the DSGE-DFM technique. Price and wage mark-up shocks are estimated to be slightly less persistent, but have much lower volatility estimates in the DSGE-DFM estimation technique. The presence of other price and wage indexes, including oil prices, may drive this result as different inflation dynamics are needed to describe them.

5.1. Impulse response functions

In this subsection, I illustrate some of the key economic mechanisms around the model's equilibrium. I do so with the aid of impulse response functions (IRF's) which I report in Figures 1 to 7. DSGE-DFM estimation allows for economic series not directly corresponding to any endogenous variables in the DSGE model to be related to the model's exogenous shocks and endogenous variables. This allows IRF's to be generated for many economic series whose IRF's do not exist outside of structural VAR estimation. The figures plot the median IRF and highest 95% and lowest 5% posterior density band for the DSGE-DFM model in blue and the median IRF and highest 95% and lowest 5% posterior density band for the DSGE-Reg model in red.[9]

For all IRF's figures, the Canadian (domestic) economy's response is plotted. The top panel plots the model implied percent deviation away from steady state for annualized real per capita GDP growth, the percent deviation away from trend for Industrial Production and the percent deviation of annualized CPI and core CPI inflation. The second row plots the model implied percent deviation away from steady state for annualized real per capita Consumption, Investment, Export and Import growth. The third row plots the implied percent deviation away from the linear trend of per capita total, construction, manufacturing and wholesale & retail trade employment. The fourth row plots the implied percent deviation away from the linear trend of business credit, business loans, consumer credit and mortgage credit. The final row plots the basis points deviation away from steady state for the policy rate, the prime rate, the term spread between 10 year and 2 year Canadian treasuries and the average mortgage rate. For datasets used in both DSGE-DFM and DSGE-Reg estimation both IRF's are plotted with the DSGE-DFM model in blue and the DSGE-Reg model in red. All expenditure growth rates and inflation are in annualized percentage.

Figure 1 presents responses to a positive one standard deviation monetary policy shock. There is little difference in core variables between the estimation techniques with the policy interest rate rising by 25 basis point and GDP growth and CPI inflation both falling as a result. Employment in the construction sector contracts the most as mortgage rates rise and outstanding mortgage credit falls. Due to the different structural parameters between the estimation techniques investment growth and import growth fall by a greater extent in the DSGE-DFM model.

Figure 2 plots responses to a one standard deviation global productivity shock that temporarily increase the steady state growth path. The higher estimated σ_{ζ} and lower estimated ρ_{ζ} in the DSGE-DFM model creates bigger increases and faster returns to steady state growth in GDP and expenditure growth from the shock compared to the DSGE-Reg model. As theory would suggest, employment increases across all sectors and household credit expands while business credit falls slightly. One difference between the two estimated models is that the policy rate decreases from the shock in the DSGE-Reg model while it increases in the DSGE-DFM model.

As shown in Figure 3, a one standard deviation positive demand shock in the world sector increases domestic exports and inflation in both models. The higher inflation causes the monetary authority to raise interest rates, decreasing domestic consumption and investment. The export boom, causes employment to rise in the manufacturing sector but the resulting higher mortgage rate causes employment to fall in the construction sector resulting in a neutral movement in total employment.

A one standard deviation positive consumption shock plotted in Figure 4, provides the classic trade-off of consumption and investment in both models. Domestic inflation is less impacted by the positive consumption shock in the DSGE-DFM model partly due to the lower θ_H so the policy rate stays unchanged in the DGSE-DFM model compared to an increase in the DSGE-Reg model. The consumption shock corresponds to an increase in consumer credit, imports and employment in the wholesale and retail trade sector as one would expect.

[8] The estimated value of the elasticity of substitution of foreign and domestic goods shows little difference between the estimation techniques and is estimated at a value of 0.4 for both. This is well below the prior that most open-economy DSGE models are estimated with but is in line with other recent open-economy DSGE models estimates for Canada. Kabaca (2016).

[9] The plotted IRF's for each data series do not account for any measurement error uncertainty. The density bands represent model implied uncertainty around the dynamics generated by sampling the posterior estimates of the structural parameters θ and Λ.

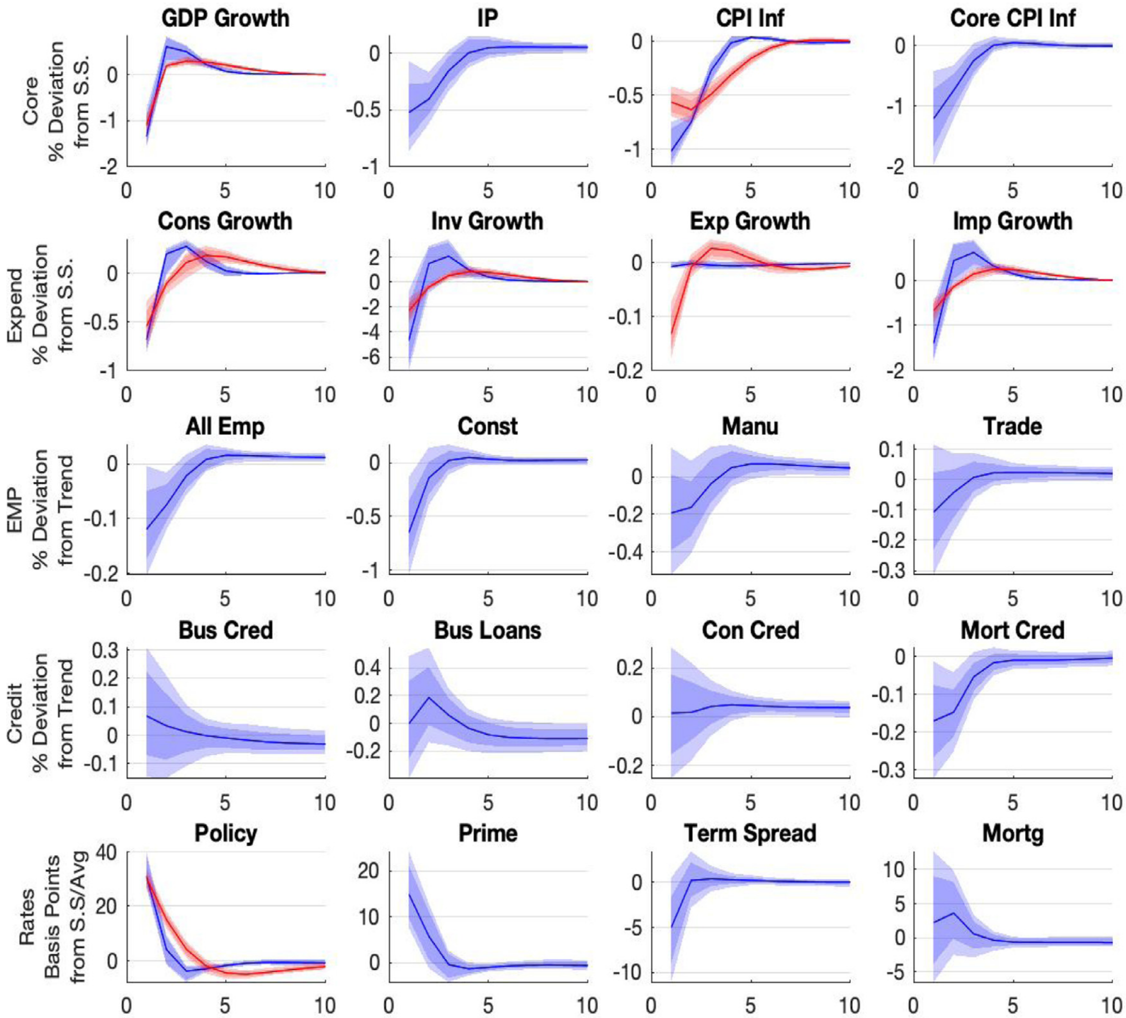

Fig. 1. Domestic Monetary Policy Shock (ε_R). The top panel plots the model implied percent deviation away from steady state for annualized real per capita GDP growth, the percent deviation away from trend for Industrial Production and the percent deviation of annualized CPI and core CPI inflation. The second row plots the model implied percent deviation away from steady state for annualized real per capita expenditure component growth. The third row plots the implied percent deviation away from the linear trend of per capita total and sector employment. The fourth row plots the implied percent deviation away from the linear trend of outstanding credit. The final row plots the basis points deviation away from steady state for the various interest rates. For datasets used in both DSGE-DFM and DSGE-Reg estimation both IRF's are plotted with the DSGE-DFM model in blue and the DSGE-Reg model in red. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

There are a few small differences between the models with respect to a one standard deviation net exports shock as illustrated in Figure 5. Domestic inflation is more stable, again in part to a lower θ_H in the DSGE-DFM model causing the policy rate to fall by a lesser amount relative to the policy rate fall in the DSGE-Reg model. In the DSGE-DFM model, export growth increases by a lesser amount and import growth is delayed and smaller after the net exports shock compared to the responses of the DSGE-Reg model. Similar results are found when looking at IRF's created by an investment shock in Figure 6.

Figure 7 plots responses to a one standard deviation positive domestic price shock. The positive domestic price shock causes GDP growth and Industrial production to fall and inflation and interest rates to rise. Employment falls in the capital intensive sectors of construction and manufacturing as investment falls by a larger amount than does consumption in both models. The exchange rate adjustment is smaller in the DSGE-DFM model as exports fall by a smaller amount compared to the DSGE-Reg model. Employment in wholesale and retail trade falls by a smaller amount when compared to employment in the construction and manufacturing sectors.

The remaining IRF's of the other eight exogenous shocks can be found in Appendix C along with the IRF's of the world sector that also plot the responses for international countries' GDP growth, inflation, policy rate, long-run interest rate and stock price growth from exogenous global shocks.

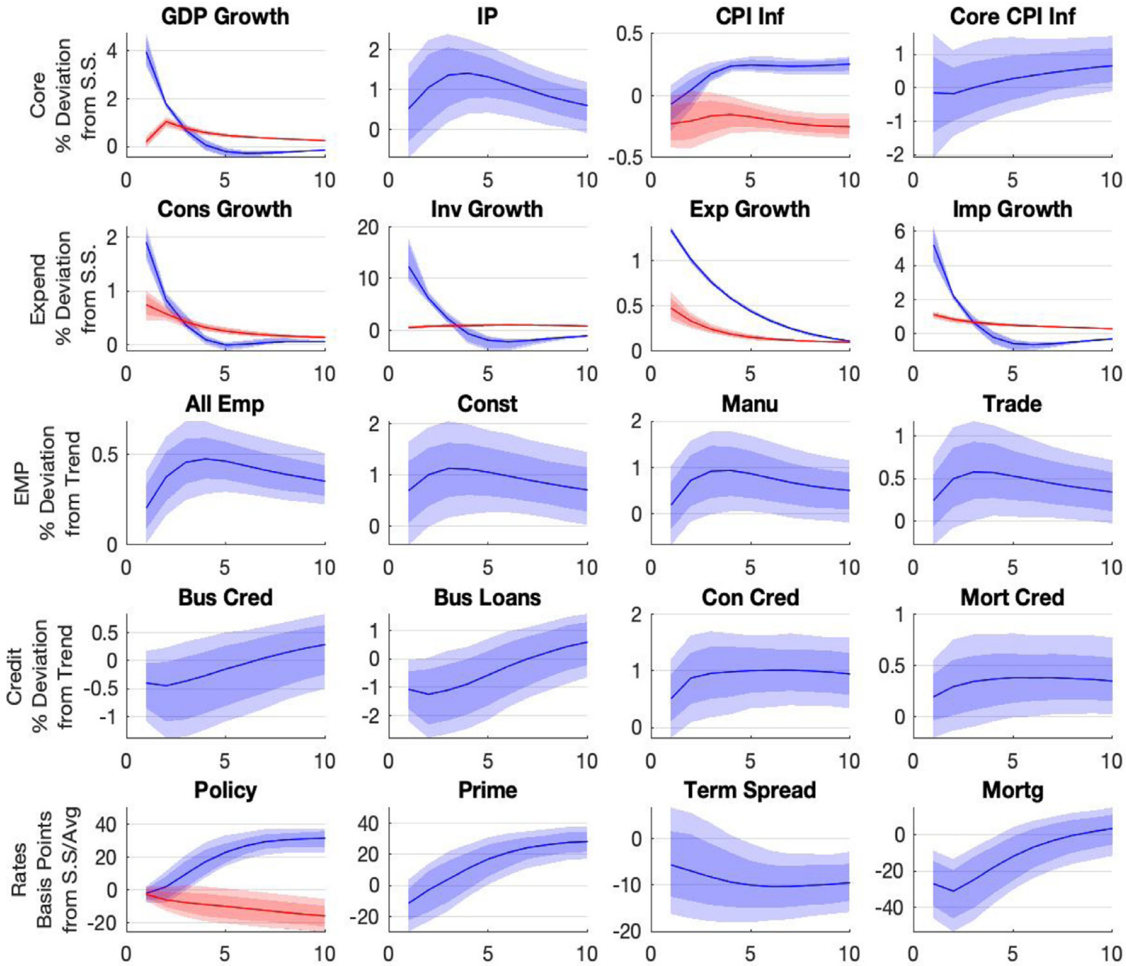

Fig. 2. Global Productivity Shock (ζ). The top panel plots the model implied percent deviation away from steady state for annualized real per capita GDP growth, the percent deviation away from trend for Industrial Production and the percent deviation of annualized CPI and core CPI inflation. The second row plots the model implied percent deviation away from steady state for annualized real per capita expenditure component growth. The third row plots the implied percent deviation away from the linear trend of per capita total and sector employment. The fourth row plots the implied percent deviation away from the linear trend of outstanding credit. The final row plots the basis points deviation away from steady state for the various interest rates. For datasets used in both DSGE-DFM and DSGE-Reg estimation both IRF's are plotted with the DSGE-DFM model in blue and the DSGE-Reg model in red. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

5.2. Variance decomposition

The unconditional contribution of each of the structural shocks or grouping of the structural shocks[10] to the forecast error variance decomposition (FEVD) of the core macroeconomic data series used in both estimation techniques at various horizons (short-run: 1 year, medium-run: 2.5 years, long-run: 10 years) are shown in Tables C.5-–C.7. The FEVD in the tables is calculated using the posterior means of all estimated parameters of the DSGE-Reg and DSGE-DFM models. The FEVD for the DSGE-Reg model for each data set is the top row, while the FEVD for the DSGE-DFM model for each data set is the bottom row.

Open-economy DSGE-Reg models have been found to rely heavily on import/export cost-push shocks (μ_F, μ_{H*}) and risk premium shocks (z_B) to explain variance in the exchange rate, export growth and import growth at all horizons. However, these shocks explain little variation in domestic GDP growth, inflation and interest rates at all horizons. Further, in open-economy DSGE-Reg models, global economic shocks provide only a small source of variation in the domestic sector's real and nominal macroeconomic variables at all horizons.[11]

[10] Historical shock decompositions for the last half of the sample period for each of the data series used in DSGE-DFM estimation can be found in the online appendix.

[11] Adolfson et al. (2007a); Ca'Zorzi et al. (2017); Gibbs et al. (2018); Justiniano and Preston (2010); Lubik and Schorfheide (2007); Ratto et al. (2009); Rees et al. (2016).

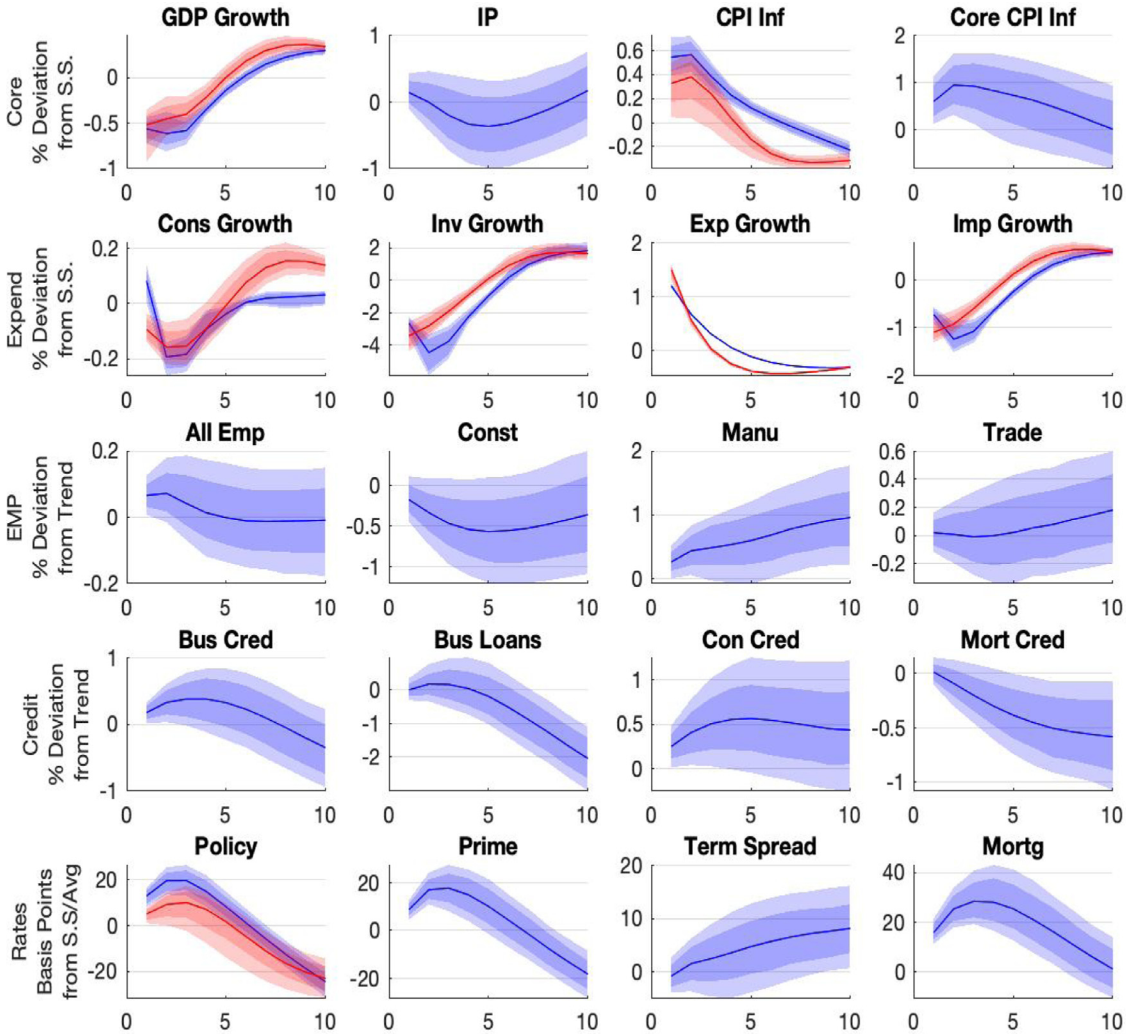

Fig. 3. World Demand Shock (z_{y^*}). The top panel plots the model implied percent deviation away from steady state for annualized real per capita GDP growth, the percent deviation away from trend for Industrial Production and the percent deviation of annualized CPI and core CPI inflation. The second row plots the model implied percent deviation away from steady state for annualized real per capita expenditure component growth. The third row plots the implied percent deviation away from the linear trend of per capita total and sector employment. The fourth row plots the implied percent deviation away from the linear trend of outstanding credit. The final row plots the basis points deviation away from steady state for the various interest rates. For datasets used in both DSGE-DFM and DSGE-Reg estimation both IRF's are plotted with the DSGE-DFM model in blue and the DSGE-Reg model in red. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

When I examine the variance decomposition of the DSGE-Reg open economy model of this paper I see that the model confers with the above cited literature. The FEVD of global shocks is less than 9% for domestic GDP growth and all component growth in the short-run and long-run. In the medium run, import/export cost-push shocks and trade shocks account for 67% of the variance in the real exchange rate but only about 35% of the variation in the policy rate.

When I compare the DSGE-Reg and DSGE-DFM open economy model's of this paper I see a clear pattern emerge. At the short-run and medium-run horizons, variance in GDP growth is attributed much more (57% to 5%) to global shocks in the DSGE-DFM model compared to the DSGE-Reg model, which attributes the variance of GDP growth to mostly mark-up and investment shocks. The same is true for domestic consumption growth and investment growth variance decomposition. Global shocks account for 54% and 37% of consumption and investment growth variance in the DSGE-DFM model compared to 9% and 4% in the DSGE-Reg model. As a result, the DSGE-DFM model see a significant decline in variance attributed to mark-up shocks, risk premium shock and individual component shocks for consumption and investment growth compared to the DSGE-Reg model. A similar result is found for export and import growth between the two models.

Turning to short-run domestic inflation and interest rate dynamics, I see similar results across the two estimation techniques. Mark-up shocks attributing about 50% of the variance in domestic CPI inflation for both models. Investment shocks are slightly less influential on CPI inflation and the policy rate in the DSGE-DFM model. While policy shocks, global shocks

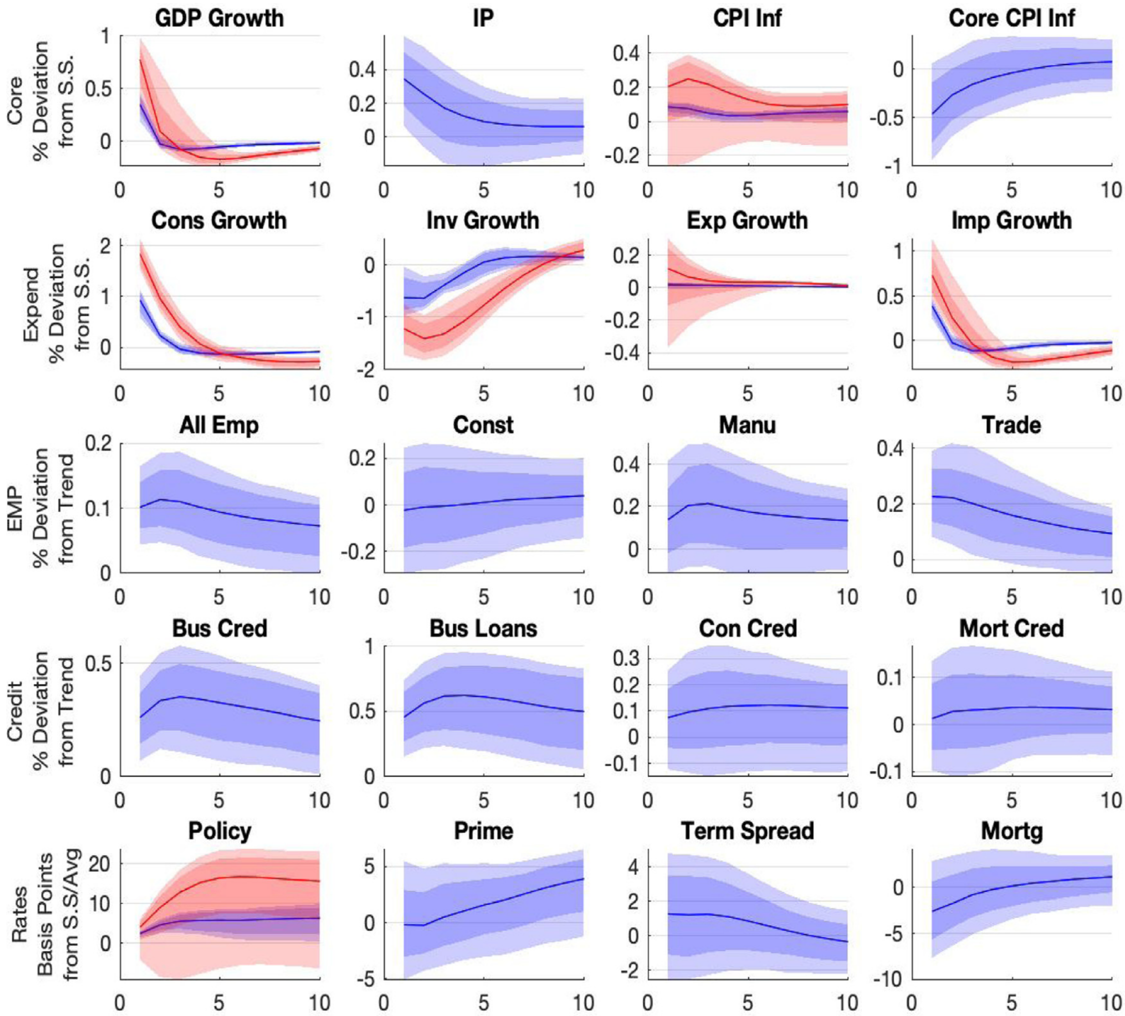

Fig. 4. Consumption Shock (z_C). The top panel plots the model implied percent deviation away from steady state for annualized real per capita GDP growth, the percent deviation away from trend for Industrial Production and the percent deviation of annualized CPI and core CPI inflation. The second row plots the model implied percent deviation away from steady state for annualized real per capita expenditure component growth. The third row plots the implied percent deviation away from the linear trend of per capita total and sector employment. The fourth row plots the implied percent deviation away from the linear trend of outstanding credit. The final row plots the basis points deviation away from steady state for the various interest rates. For datasets used in both DSGE-DFM and DSGE-Reg estimation both IRF's are plotted with the DSGE-DFM model in blue and the DSGE-Reg model in red. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

and trade shocks are slightly more influential compared to the DSGE-Reg model. Of note, short-run and long-run export inflation is mostly explained by the exogenous measurement error component, suggesting the structural DSGE model itself is missing a key modeling component when it comes to export price dynamics.[12] We also see that short-run and medium-run wage inflation variation are more attributed to global shocks and less attributed to mark-up shocks in the DSGE-DFM model.

In the long-run global shocks still attribute a dominant share to GDP growth and its components in the DSGE-DFM model. This added empirical connection between the world sector and the domestic sector, that is generated with DSGE-DFM estimation, helps explain the forecasting advantages and correlations between Canada's macroeconomy and the global macroeconomy presented in the next two sections. Further, the long-run variance decomposition attributed to the risk premium and net export shock becomes quite large for CPI inflation, wage inflation, import inflation and the policy rate, 77%, 65%, 59% and 82% respectively. This suggests that the risk premium shock and net exports shock is motivating long-run variation on inflation and interest rates while having a relatively small impact on the variance of the real exchange rate

[12] A further discussion on this can be found in the online appendix

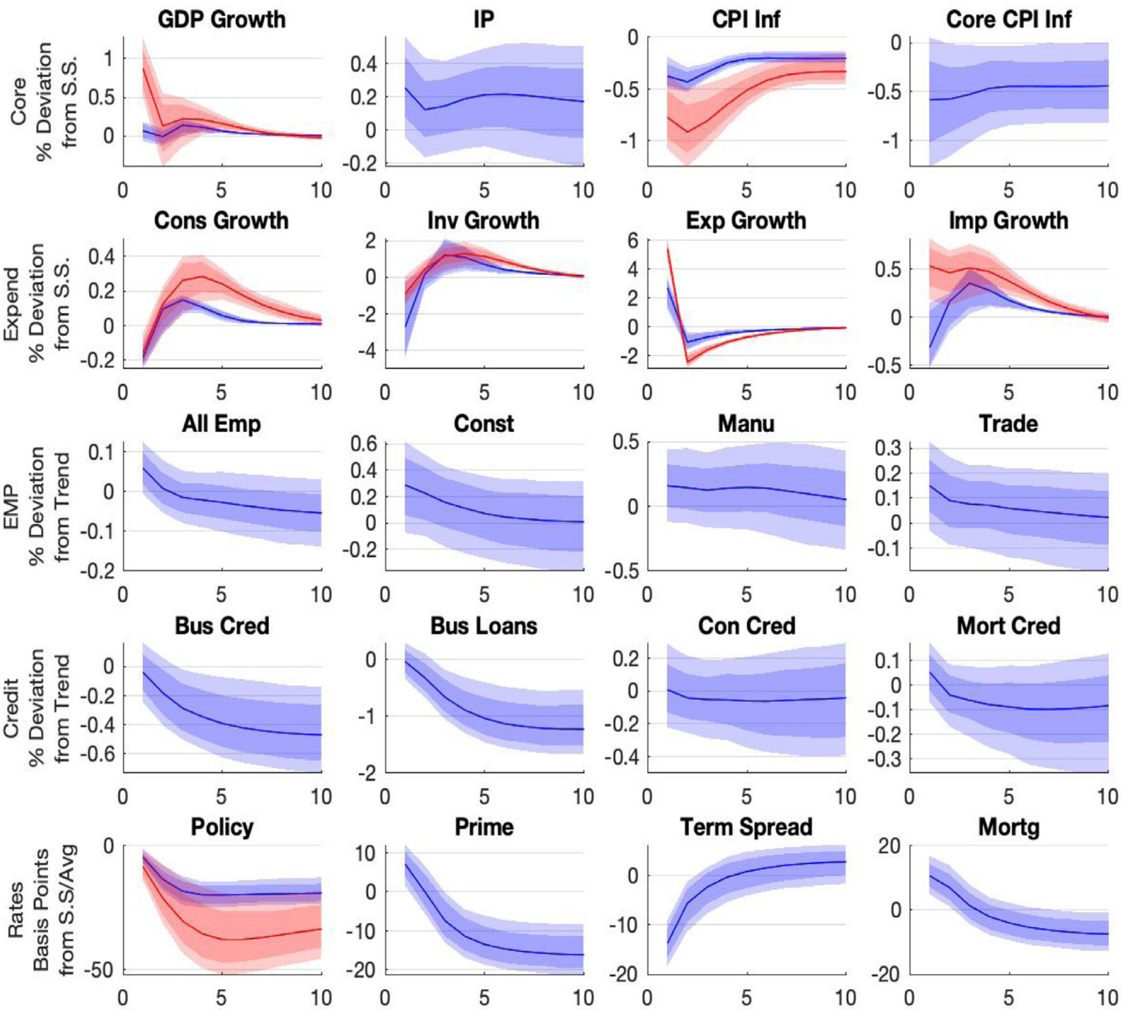

Fig. 5. Net Exports Shock (z_D). The top panel plots the model implied percent deviation away from steady state for annualized real per capita GDP growth, the percent deviation away from trend for Industrial Production and the percent deviation of annualized CPI and core CPI inflation. The second row plots the model implied percent deviation away from steady state for annualized real per capita expenditure component growth. The third row plots the implied percent deviation away from the linear trend of per capita total and sector employment. The fourth row plots the implied percent deviation away from the linear trend of outstanding credit. The final row plots the basis points deviation away from steady state for the various interest rates. For datasets used in both DSGE-DFM and DSGE-Reg estimation both IRF's are plotted with the DSGE-DFM model in blue and the DSGE-Reg model in red. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

itself in the long-run (35%). This finding which differs with the previous open-economy DSGE literature helps explain the forecasting advantages that DSGE-DFM estimation can have in regard to forecasting the real and nominal exchange rates.

6. Forecast evaluation

In this section, I perform a similar exercise as Ca'Zorzi et al. (2017); Kolasa and Rubaszek (2018) and Champagne et al. (2018) of comparing the forecast errors of the different models and of the Bank of Canada's Staff Economic Projections (SEP). I chose to compare the model forecasts to the SEP for several reasons. First, Champagne et al. (2018) find that the SEP forecast are significantly more accurate than those from standard econometric models and other professional forecasts in the short-run. Second, some hard and soft indicators for the current quarter are available to the staff when the forecasts are prepared, this will allow for a fair comparison when real-time DSGE-DFM forecasts are evaluated in Section 6.1. Third, between 2005 and 2015, the SEP forecasts are mainly produced with ToTEM II, an open-economy, New Keynesian DSGE model (Dorich et al., 2013) that has a richer sectoral and financial market framework in its model than does the DSGE model of this paper. All the above reasons suggest that if the SEP forecasts are significantly out-performed by any of the models in this paper there is good reason to believe that the quality of these forecasts are near the frontier of forecasting accuracy.

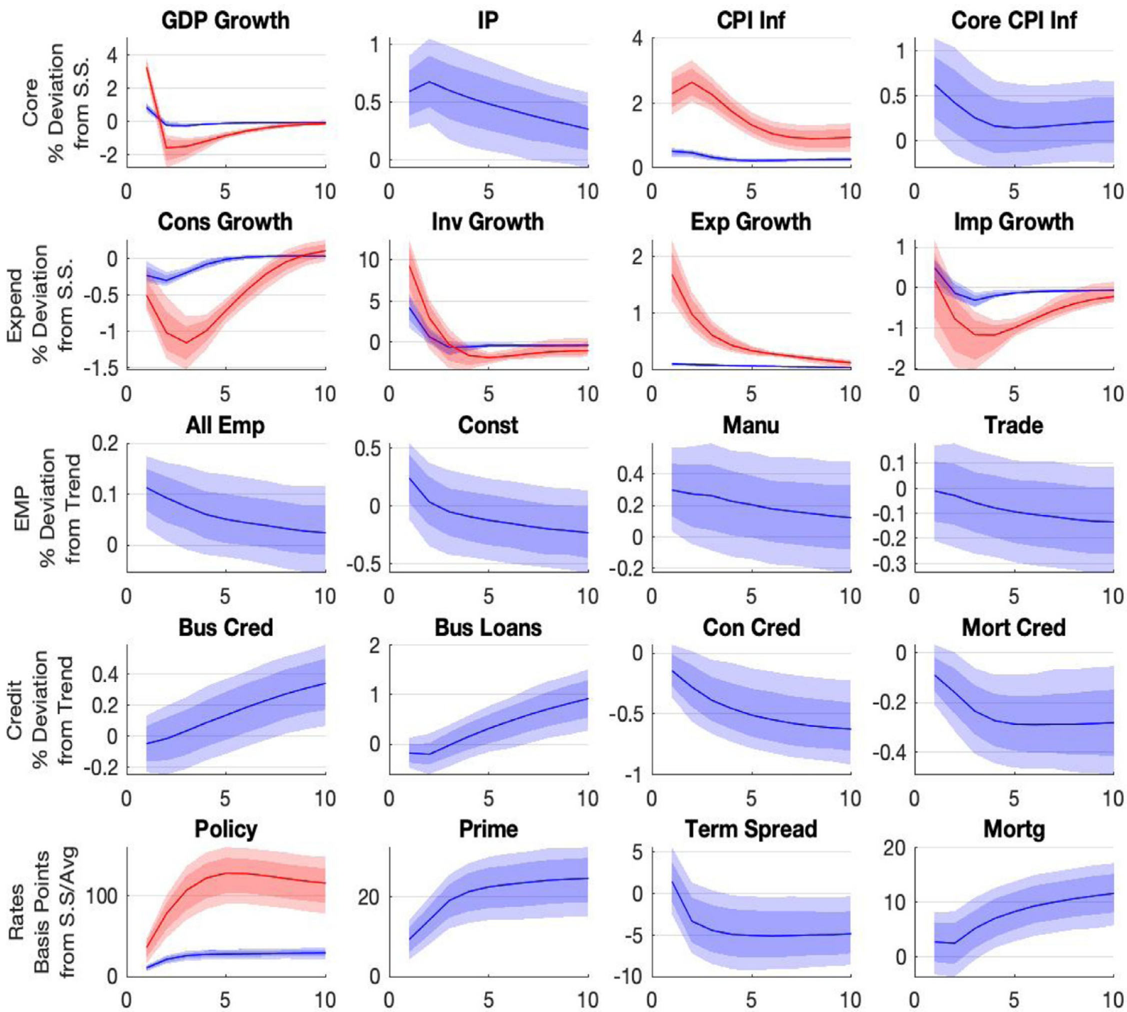

Fig. 6. Investment Shock (z_I). The top panel plots the model implied percent deviation away from steady state for annualized real per capita GDP growth, the percent deviation away from trend for Industrial Production and the percent deviation of annualized CPI and core CPI inflation. The second row plots the model implied percent deviation away from steady state for annualized real per capita expenditure component growth. The third row plots the implied percent deviation away from the linear trend of per capita total and sector employment. The fourth row plots the implied percent deviation away from the linear trend of outstanding credit. The final row plots the basis points deviation away from steady state for the various interest rates. For datasets used in both DSGE-DFM and DSGE-Reg estimation both IRF's are plotted with the DSGE-DFM model in blue and the DSGE-Reg model in red. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

I compare the accuracy of the forecasts of the SEP to the DSGE models and other econometric models by using root mean squared errors (RMSE) calculated using equations 38 through 41 for x equal to annualized GDP growth and expenditure component growth as well annualized CPI inflation, annualized Core CPI inflation and the Policy Rate. In the RMSE calculations $t = 1$ corresponds with the time period of 2007Q3 and $\hat{T}$ equals 33 and corresponds with the time period 2015Q3.

$$RMSE_{\tau=1,2,\ldots 8} = \sqrt{\frac{1}{\hat{T}}\sum_{t=1}^{\hat{T}}(x_{t+\tau}^{Forecast} - x_{t+\tau}^{Actual})^2} \tag{38}$$

$$RMSE_{1st\ year} = \sqrt{\frac{1}{\hat{T}}\sum_{t=1}^{\hat{T}}\left[\left(\frac{\sum_{h=1}^{4} x_{t+h}^{Forecast}}{4}\right) - \left(\frac{\sum_{h=1}^{4} x_{t+h}^{Actual}}{4}\right)\right]^2} \tag{39}$$

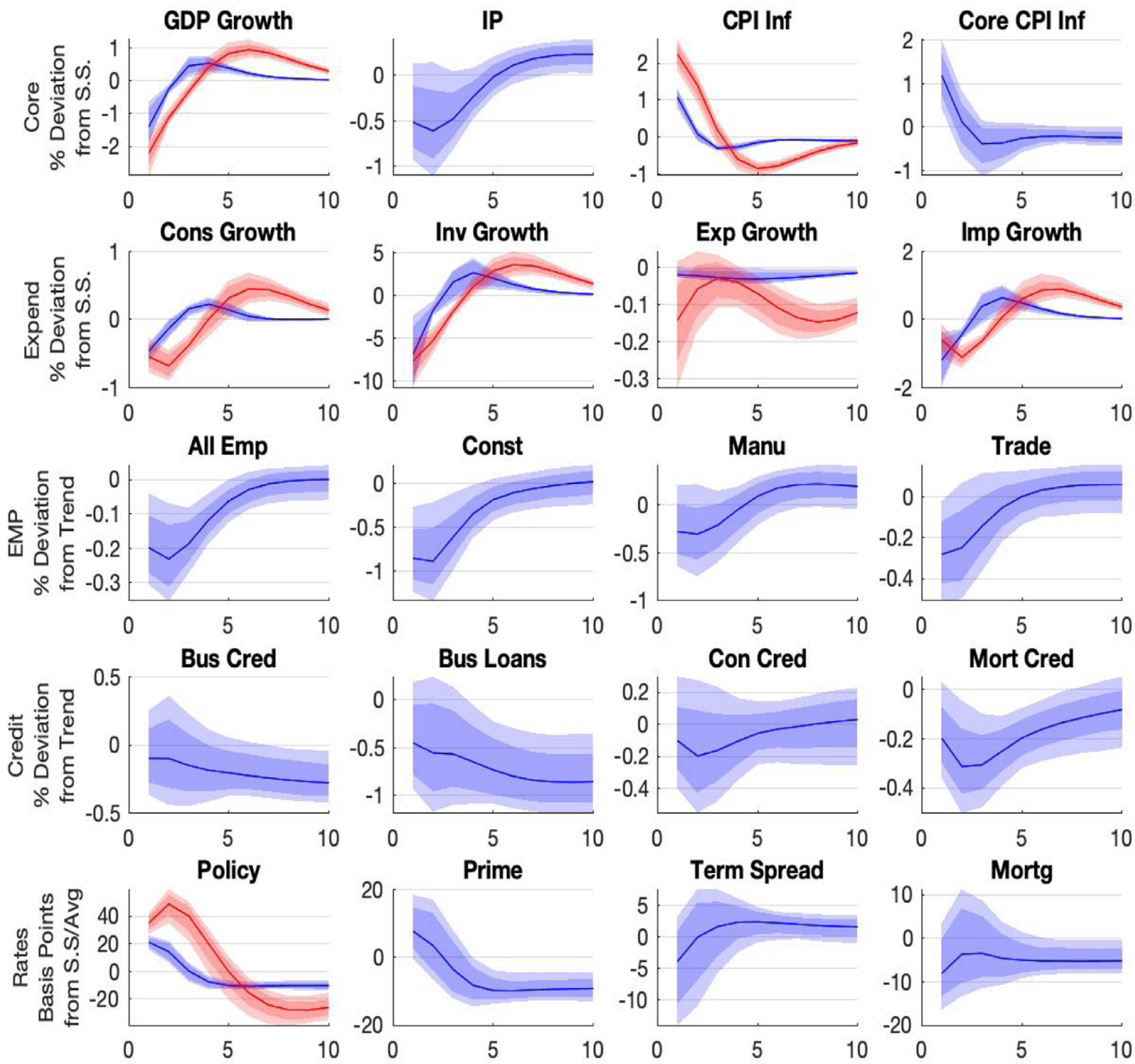

Fig. 7. Domestic Price Shock (μ_H). The top panel plots the model implied percent deviation away from steady state for annualized real per capita GDP growth, the percent deviation away from trend for Industrial Production and the percent deviation of annualized CPI and core CPI inflation. The second row plots the model implied percent deviation away from steady state for annualized real per capita expenditure component growth. The third row plots the implied percent deviation away from the linear trend of per capita total and sector employment. The fourth row plots the implied percent deviation away from the linear trend of outstanding credit. The final row plots the basis points deviation away from steady state for the various interest rates. For datasets used in both DSGE-DFM and DSGE-Reg estimation both IRF's are plotted with the DSGE-DFM model in blue and the DSGE-Reg model in red. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$$RMSE_{2nd\,year} = \sqrt{\frac{1}{\hat{T}}\sum_{t=1}^{\hat{T}}\left[\left(\frac{\sum_{h=5}^{8} x_{t+h}^{Forecast}}{4}\right) - \left(\frac{\sum_{h=5}^{8} x_{t+h}^{Actual}}{4}\right)\right]^2} \tag{40}$$

$$RMSE_{2\,year\,Avg} = \sqrt{\frac{1}{\hat{T}}\sum_{t=1}^{\hat{T}}\left[\left(\frac{\sum_{h=1}^{8} x_{t+h}^{Forecast}}{8}\right) - \left(\frac{\sum_{h=1}^{8} x_{t+h}^{Actual}}{8}\right)\right]^2} \tag{41}$$

Table 5
Relative RMSE for CAN Expenditure Growth.

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
GDP Growth								
DSGE-Reg	3.31	2.48	1.29	**0.94**	1.11	2.55	**0.98**	1.96
DSGE-World-DFM	1.75	1.49	**0.97**	**0.91**	1.02	1.25	**0.89**	**0.96**
DSGE-DFM	1.62	1.46	**0.99**	**0.91**	1.00	1.17	**0.88**	**0.98**
Consumption Growth								
DSGE-Reg	**0.53**	**0.56**	1.24	1.41	1.28	**0.84**	1.50	1.65
DSGE-World-DFM	**0.52**	**0.50**	**0.91**	**0.81**	**0.63**	**0.60**	**0.61**	**0.64**
DSGE-DFM	**0.53**	**0.48**	**0.84**	**0.78**	**0.65**	**0.54**	**0.57**	**0.48**
Investment Growth								
DSGE-Reg	1.15	1.17	1.08	**0.93**	**0.94**	1.18	**0.83**	1.02
DSGE-World-DFM	**0.96**	**0.94**	**0.93**	**0.95**	1.02	**0.82**	**0.94**	**0.72**
DSGE-DFM	**0.94**	**0.97**	**0.97**	**0.94**	**0.95**	**0.94**	**0.82**	**0.80**
Government Growth								
DSGE-Reg	**0.82**	1.16	1.01	**0.66**	**0.78**	1.05	**0.63**	**0.81**
DSGE-World-DFM	**0.85**	1.22	1.05	**0.72**	**0.84**	1.14	**0.72**	**0.95**
DSGE-DFM	**0.73**	1.07	**0.96**	**0.67**	**0.82**	**0.91**	**0.66**	**0.78**

Notes: The table shows the ratios of the RMSE from a given model in comparison to the SEP forecasts for a given variable. Values below one indicate that forecasts from the model are on average more accurate than the SEP.

Table 6
Relative RMSE for CAN Trade Growth, CPI Inflation, and Policy Rate.

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
Export Growth								
DSGE-Reg	1.41	1.34	1.05	1.04	**0.93**	1.32	1.11	1.32
DSGE-World-DFM	1.24	1.31	1.06	1.05	**0.95**	1.31	1.13	1.41
DSGE-DFM	1.20	1.26	1.05	1.05	**0.95**	1.20	1.13	1.30
Import Growth								
DSGE-Reg	1.32	1.16	1.13	**0.96**	1.05	1.13	**0.93**	1.10
DSGE-World-DFM	1.24	1.12	1.08	1.00	1.00	1.06	1.00	1.04
DSGE-DFM	1.27	1.17	1.10	**0.97**	**0.95**	1.15	**0.85**	**0.99**
Policy Rate								
DSGE-Reg	6.14	2.66	1.79	1.72	1.71	2.19	1.72	1.83
DSGE-World-DFM	6.95	2.50	1.26	1.03	**0.94**	1.81	1.01	1.18
DSGE-DFM	7.76	2.80	1.55	1.34	1.20	2.12	1.30	1.48
CPI Inflation								
DSGE-Reg	3.09	1.77	1.90	2.17	2.49	3.45	4.45	6.24
DSGE-World-DFM	2.27	1.06	**0.82**	**0.82**	**0.95**	1.38	1.35	1.77
DSGE-DFM	2.31	1.11	1.09	1.19	1.33	1.69	2.25	2.90

Notes: The table shows the ratios of the RMSE from a given model in comparison to the SEP forecasts for a given variable. Values below one indicate that forecasts from the model are on average more accurate than the SEP.

Each forecast is generated by 500,000 simulations, 5000 draws from the posterior parameter distribution and each parameter draw is simulated using 100 draws of future structural shocks for 8 quarters. In all simulations, the zero lower bound is established using shadow monetary policy shocks using an algorithm outlined by Holden and Paetz (2012).

The forecast evaluation window looks at the accuracy of forecasts from 2007Q4 to 2015Q4.[13] I first estimate the models from 1995-2007Q3 and generate forecasts for 2007Q4. When the new actualized data are revealed in 2007Q4, the new values are inserted into the Kalman filter and are used to forecast 2008Q1. The process continues until the model is re-estimated once a year when all Q3 data for a given year would have been available to the econometrician.

I start by estimating three different model specifications and compare their forecasting accuracy to the forecasts from the SEP. The first model I evaluate is the DSGE-Reg model I have discussed previously. The second model is a DSGE-DFM model estimated only using international data and the domestic core data series, denoted DSGE-World-DFM and the last is the DSGE-DFM model previously discussed. The relative RMSE for different variables and different time horizons of the pseudo out-of-sample forecast are presented and discussed below.

Table 5 reports the relative RMSE by normalizing them against the RMSE for the SEP using equation 38 and for different τ, the four quarter rolling forecast average using equations 39 and 40, and the 8 quarter ahead rolling forecast average using equation 41 for real GDP, consumption, investment, and government expenditure growth. While Table 6 reports the relative RMSE for real export and import growth, CPI inflation and the Canadian Policy rate. Any relative RMSE above one implies

[13] The forecast evaluation window ends at 2015Q4 because the SEP forecasts are released with a five year lag, making 2015Q4 the latest publicly available release at the time this paper was written.

that the model's RMSE is above the RMSE for the SEP and any number below one implies that the model's RMSE is lower than the RMSE for the SEP. The actual RMSE's of the SEP for all variables are reported in Table C.8.

A few patterns emerge. First, the DSGE-Reg model has significantly[14] higher RMSE compared to the data-rich DSGE models and the SEP for all variables besides consumption growth for τ less than 5. For $\tau = 6$ and beyond the RMSE's for the DSGE-Reg model are on par with the SEP forecasts. This is a finding consistent with Del Negro et al. (2013); Gervais and Gosselin (2014) and Gelfer (2019) where DSGE-Reg models for the United States and Canada can compete with many forecast models for τ greater than four but cannot for τ less than four.

Introducing the international data sets into the estimation method (DSGE-World-DFM) outlined in Table 3 shrinks the RMSE's for all growth variables compared to the DGSE-Reg model but is still significantly higher than the RMSE's of the SEP forecasts for all horizons of τ when it comes to export/import growth, policy rate and inflation forecasts. However, the RMSE's for investment and consumption growth are significantly decreased with the extra data in estimation and even lower than the SEP's RMSEs for τ less than and equal to four and the four quarter averages for year one and two.

When the Canadian data series are introduced into the estimation method (DSGE-DFM), the RMSE's are lower for export growth and government growth and remain similar to the RMSE's of the DSGE-World-DFM model for all other variables for τ less than and equal to four. The introduction of the Canadian data series brings the RMSE's for the average four quarter growth rate of the second year of forecasts below the RMSE's of the SEP for all growth components. Even given the declines of the RMSE's for growth variables, the RMSEs do not fall for the forecasted Bank of Canada policy rate when the international series are introduced into the estimation method, In addition, RMSE's remain significantly higher than the SEP's for CPI inflation. This is mainly due to the fact that early economic data for the current quarter are available to the staff when the SEP forecasts are prepared, something I control for in the next subsection.

6.1. Real-time forecasting performance

In addition to the DSGE-DFM models outperforming the DSGE-Reg models and many of the SEP's forecasts throughout the forecast evaluation window (2007Q4-2015Q4), I also see that the "predictive performance enhancing" effects of large data sets used in DSGE-DFM estimation can be amplified if available real time data are used when forecasting with the DSGE-DFM model.

I introduce real-time forecasting using two different approaches. The first approach, denoted DSGE-DFM-RT, assumes that the econometrician would use all real-time financial data (international and domestic) available for the current quarter. For example, in generating the 2007Q4 forecasts the econometrician would know all non-financial data realizations up to and including 2007Q3 and all financial data up to and including 2007Q4.

The second approach, denoted DSGE-DFM-RT 2, uses the same assumption but also assumes that the econometrician would have the first monthly value for all data series in the model that are reported at a monthly frequency. The econometrician would then forecast the remaining two monthly observations for each partially reported data series using a simple AR(2) forecasting model to be able to generate the quarterly observation for all partially reported monthly frequency variables. For example, in generating the 2007Q4 forecasts the econometrician would know all non-financial data realizations up to and including 2007Q3 and all financial data up to and including 2007Q4 and all monthly frequency variables up to 2007Q3 and the monthly observation for October 2007.[15] They would then generate forecasts for November and December 2007 for a forecasted observation for 2007Q4. The real-time data used for both the DSGE-DFM-RT and DSEG-DFM-RT 2 models are defined in Table 3 under the column RT. All data denoted RT in the column is also used in the DSGE-DFM-RT 2 model.

The relative RMSE's for all models using real-time data are reported in Tables 7 and 8. I include the relative RMSE's for the DGE-DFM model without any real-time data for comparison. First, the RMSE's are lowered for both real-time models for all variables for τ less than and equal to four and remain the same for all τ greater than four. This suggest that the addition of the real-time data affect only the very short-run dynamics of the structural DSGE model, while leaving the dynamics for periods 4 through 8 relatively unchanged.

In addition, the relative RMSE's of the DSGE-DFM-RT 2 model are lowered for almost every variable and time horizon measure. The additional data used in the estimation and the use of real-time data have the biggest impact in reducing the RMSE's for investment growth, export growth and import growth. With the addition of the real-time data the RMSE's of the DSGE-DFM-RT 2 are lower than the RMSE of the SEP for consumption, investment, government, export and import growth for all τ. Further, the addition of the real-time data significantly decrease the RMSE's for the policy rate, CPI inflation and core CPI inflation. Finally, the RMSE for one period ahead forecasted GDP growth are lowered by almost 80% compared to the RMSE of the DSGE-Reg model but are still higher then the RMSE of the SEP.

[14] Diebold-Mariano test statistics are computed for various model comparisons and can be found in Tables C.9 and C.10 of the appendix.

[15] These real-time quarterly forecasts would have been generated in March, June, September, December of each year, analogous to when the SEP forecasts would also have been generated.

Table 7
Relative RMSE for CAN Expenditure Growth with Real-time Data.

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
GDP Growth								
DSGE-DFM	1.62	1.46	**0.99**	**0.91**	1.00	1.17	**0.88**	**0.98**
DSGE-DFM-RT	1.84	1.30	**0.97**	**0.92**	**0.99**	1.09	**0.88**	**0.97**
DSGE-DFM-RT 2	1.34	1.21	**0.94**	**0.92**	**0.97**	1.03	**0.88**	**0.95**
Consumption Growth								
DSGE-DFM	**0.53**	**0.48**	**0.84**	**0.78**	**0.65**	**0.54**	**0.57**	**0.48**
DSGE-DFM-RT	**0.55**	**0.46**	**0.87**	**0.79**	**0.66**	**0.51**	**0.57**	**0.52**
DSGE-DFM-RT 2	**0.46**	**0.44**	**0.87**	**0.79**	**0.64**	**0.49**	**0.57**	**0.52**
Investment Growth								
DSGE-DFM	**0.94**	**0.97**	**0.97**	**0.94**	**0.95**	**0.94**	**0.82**	**0.80**
DSGE-DFM-RT	**0.82**	**0.95**	**0.97**	**0.94**	**0.94**	**0.85**	**0.84**	**0.78**
DSGE-DFM-RT 2	**0.69**	**0.89**	**0.96**	**0.95**	**0.94**	**0.83**	**0.84**	**0.76**
Government Growth								
DSGE-DFM	**0.73**	1.07	**0.96**	**0.67**	**0.82**	**0.91**	**0.66**	**0.78**
DSGE-DFM-RT	**0.79**	1.09	**0.96**	**0.67**	**0.81**	**0.97**	**0.66**	**0.82**
DSGE-DFM-RT 2	**0.70**	1.08	**0.97**	**0.67**	**0.81**	**0.93**	**0.66**	**0.80**

Notes: The table shows the ratios of the RMSE from a given model in comparison to the SEP forecasts for a given variable. Values below one indicate that forecasts from the model are on average more accurate than the SEP.

Table 8
Relative RMSE for CAN Trade Growth, CPI Inflation, and Policy Rate with Real-time Data.

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
Export Growth								
DSGE-DFM	1.20	1.26	1.05	1.05	**0.95**	1.20	1.13	1.30
DSGE-DFM-RT	**0.95**	**0.99**	**0.86**	**0.85**	**0.95**	**0.95**	**0.88**	1.05
DSGE-DFM-RT 2	**0.93**	**0.99**	**0.87**	**0.85**	**0.96**	**0.94**	**0.88**	1.05
Import Growth								
DSGE-DFM	1.27	1.17	1.10	**0.97**	**0.95**	1.15	**0.85**	**0.99**
DSGE-DFM-RT	**0.85**	**0.91**	**0.88**	**0.71**	**0.96**	**0.92**	**0.82**	**0.92**
DSGE-DFM-RT 2	**0.77**	**0.92**	**0.88**	**0.71**	**0.96**	**0.91**	**0.82**	**0.92**
Policy Rate								
DSGE-DFM	7.76	2.80	1.55	1.34	1.20	2.12	1.30	1.48
DSGE-DFM RT	1.21	1.67	1.23	1.14	1.06	1.37	1.12	1.17
DSGE-DFM-RT 2	1.26	1.69	1.23	1.12	1.05	1.36	1.10	1.15
CPI Inflation								
DSGE-DFM	2.31	1.11	1.09	1.19	1.33	1.69	2.25	2.90
DSGE-DFM-RT	1.49	1.13	1.01	1.08	1.27	1.51	2.04	2.45
DSGE-DFM-RT 2	1.33	1.12	1.02	1.06	1.24	1.46	1.99	2.40
Core CPI Inflation								
DSGE-DFM	1.72	1.13	1.21	1.23	1.31	1.47	1.41	1.72
DSGE-DFM-RT	1.50	1.21	1.17	1.16	1.21	1.44	1.28	1.60
DSGE-DFM-RT 2	1.38	1.14	1.16	1.15	1.16	1.34	1.26	1.53

Notes: The table shows the ratios of the RMSE from a given model in comparison to the SEP forecasts for a given variable. Values below one indicate that forecasts from the model are on average more accurate than the SEP.

6.2. Comparing DSGE-DFM to other forecasting models

In addition to comparing the forecasts generated by the DSGE-DFM model against the SEP forecasts, I also evaluate the forecast accuracy of the DSGE-DFM forecasts to other reduced form forecasting models. During the great moderation, structural closed-economy DSGE models of the United States were found to out-forecast VARs and Bayesian VARs in terms of short-term and medium term output, inflation and short term interest rates (Edge and Gurkaynak, 2010; Gurkaynak et al., 2013) and Wolters (2015).

However, during the Great Recession and its recovery this result seems to have only held for certain DSGE models, most notably the closed economy DSGE models with financial frictions (Del Negro et al., 2013) and Cai et al. (2019). Further, Gelfer (2019) found that closed-economy DSGE-DFM models that have financial frictions amplify the forecasting accuracy of DSGE models around the financial crisis and the Great Moderation for all key macroeconomic variables outside of inflation. In this subsection, I compare the forecasting results of the open-economy DSGE-DFM and DSGE-Reg models against forecasts generated by reduced form time-series models to see if the above mentioned results hold for an international DSGE-DFM model.

Like the previous literature I choose to compare the DSGE-DFM model to standard VARs of the form of equation 42.

$$X_t^{VAR} = \sum_{i=1}^{n} \hat{\Lambda}_i X_{t-i}^{VAR} + \hat{e}_t \tag{42}$$

where n is the number of lags[16] and X_t^{VAR} encompasses the same sixteen data series used to estimate the DSGE-Reg model of this paper.

I also choose to add a dynamic factor model (DFM) to the model evaluation pool because Stock and Watson (2011) have found that over the 3–6 month time horizon DFMs out perform many simple and more complex forecasting models. The principle behind a DFM is that there exists a handful of latent factors f_t inside the economy that power the co-movements among macroeconomic variables. These latent factors are believed to be extractable using a large set of macroeconomic time series. This is analogous to the theory around the DSGE-DFM model but imposes no structural foundation around the forecasting model.

I use the DFM linear/Gaussian state space set-up of equations 43–(44) outlined in Stock and Watson (2011) to estimate the parameters of the DFM model.

$$X_t^{DFM} = \hat{\lambda} f_t + \hat{e}_t \tag{43}$$

$$f_t = \hat{\Psi} f_{t-1} + \omega_t \tag{44}$$

where N is the number of series used in estimation and q is the number of extracted latent factors[17] and $\hat{\lambda}$ is a N x q matrix of factor loadings. The q x q transition matrix, $\hat{\Psi}$, oversees the VAR dynamics of the q latent factors. The 93 series of X_t^{DFM} are identical to the series used in the DSGE-DFM model of this paper. As is common in the dynamic factor model literature, all series sample standard deviations are normalized to 1.

The relative RMSE's for forecasts of the open-economy DSGE-Reg model, vector autoregression model, dynamic factor model, dynamic factor model using real-time data and the open-economy DSGE-DFM model using real time data are reported in Tables 9 and 10. I see that similar to previous literature, the DSGE-Reg model of this paper produces similar or better forecasts, in terms of accuracy when compared to forecasts that are produced by a VAR for GDP growth, its expenditures growth, inflation and the policy rate. This result is magnified as τ gets larger. However, the RMSE's are still significantly higher than the SEP forecasts for all τ less than and equal to six for these variables.

Evaluating the accuracy of the forecasts generated by the dynamic factor model, I see that they are more accurate than both the VAR and DSGE-Reg forecasts for all growth variables, inflation and the policy rate. This is true for all measures of τ. These results are amplified when forecasts from the dynamic factor model incorporate real-time financial and monthly data for the current quarter as done in the same way as discussed in the previous subsection. However, the forecasts generated by the open-economy DSGE-Reg model for export and import growth are determined to be more accurate than those generated by the DFM model with or without real-time data. This suggests that the theoretical structure of the open-economy DSGE-Reg model is providing insight into the future dynamics of international trade variables. Nevertheless, the RMSE's produced by the forecasts of the DFM and DFM RT 2 models are still higher than the RMSE's of the SEP for GDP, export, import growth and the policy rate for most τ's.

Finally, I see that the RMSE's of the open economy DSGE-DFM model that utilizes real-time data are the lowest compared to all of the models and for all variables besides inflation in regard to all values of τ. In addition, the relative RMSE's for the DSGE-DFM RT 2 model are below one for consumption, investment, government purchase, export and import growth for all τ and GDP for τ larger than two. This signifies that the open-economy DSGE-DFM model produces more accurate forecasts than those produced by the expert based SEP. However, the DSGE-DFM model is not able to forecast the dynamics associated with domestic inflation, domestic core inflation or the policy rate as well as the SEP does. In fact the only model, that is able to out-forecast the SEP for τ larger than one when it comes to domestic inflation and core inflation is the DFM model that utilizes real time data.

In summary, I see that the DSGE-DFM model that incorporates real-time financial and employment data (DSGE-DFM-RT 2) is able to significantly out-forecast its DSGE-Reg counterpart and other reduced from models for all expenditure growth series, the policy rate and inflation rate series for many horizons inside two years. Further, the DSGE-DFM-RT 2 model is able to compete with and in some cases significantly out-forecast the SEP projections of the Bank of Canada. For the forecast evaluation period, the SEP forecasts are predominantly produced by ToTEM, a large-scale stylized multi-sector DSGE model with a modeled financial sector. This suggests that the DSGE-DFM model which does have financial data but not a modeled financial sector is able to compete with DSGE models that do have a modeled financial sector even during the global financial crisis period and its proceeding recovery.

I evaluate this point further by comparing the DSGE-DFM model to two other models, one which uses the Wu and Xia (2016) shadow rate for the U.S. and the E.U. to calculate the world aggregate policy rate. This model is denoted as DSGE-Reg-Shadow and will allow the DSGE-Reg model to account for some of the unconventional monetary policy seen

[16] The number of lags is endogenously chosen by whichever lag produces the lowest AIC for $n = \{1, 2, 3\}$.

[17] The DFM model in this paper sets $q = 6$ to match the work done in Stock and Watson (2012).

Table 9
Relative RMSE for CAN Expenditure Growth.

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
GDP Growth								
DSGE-Reg	3.31	2.48	1.29	**0.94**	1.11	2.55	**0.98**	1.96
DSGE-Reg-AR1	2.61	1.88	1.10	**0.98**	1.09	1.78	1.06	1.44
VAR(n)	2.44	2.62	1.16	1.10	1.05	2.00	1.38	1.97
DFM	1.73	1.76	1.09	**0.98**	1.02	1.60	1.06	1.40
DFM-RT 2	1.76	1.63	1.04	**0.96**	**0.98**	1.47	**0.99**	1.24
DSGE-DFM	1.62	1.46	**0.99**	**0.91**	1.00	1.17	**0.88**	**0.98**
DSGE-DFM-RT 2	1.34	1.21	**0.94**	**0.92**	**0.97**	1.03	**0.88**	**0.95**
Consumption Growth								
DSGE-Reg	**0.53**	**0.56**	1.24	1.41	1.28	**0.84**	1.50	1.65
DSGE-Reg-AR1	**0.55**	**0.61**	1.26	1.43	1.03	**0.89**	1.43	1.69
VAR(n)	1.04	**0.91**	1.26	1.18	**0.75**	1.15	**0.86**	1.44
DFM	**0.67**	**0.64**	1.02	**0.90**	**0.65**	**0.79**	**0.74**	**0.99**
DFM-RT 2	**0.61**	**0.59**	**0.94**	**0.82**	**0.67**	**0.68**	**0.65**	**0.79**
DSGE-DFM	**0.53**	**0.48**	**0.84**	**0.78**	**0.65**	**0.54**	**0.57**	**0.48**
DSGE-DFM-RT 2	**0.46**	**0.44**	**0.87**	**0.79**	**0.64**	**0.49**	**0.57**	**0.52**
Investment Growth								
DSGE-Reg	1.15	1.17	1.08	**0.93**	**0.94**	1.18	**0.83**	1.02
DSGE-Reg-AR1	**0.97**	1.03	**0.99**	**0.97**	1.03	**0.96**	**0.96**	**0.81**
VAR(n)	1.85	1.98	1.45	1.25	1.25	1.54	1.45	1.77
DFM	1.02	1.17	1.08	1.00	1.00	1.25	**0.99**	1.18
DFM-RT 2	1.03	1.12	1.03	**0.99**	**0.97**	1.18	**0.98**	1.08
DSGE-DFM	**0.94**	**0.97**	**0.97**	**0.94**	**0.95**	**0.94**	**0.82**	**0.80**
DSGE-DFM-RT 2	**0.69**	**0.89**	**0.96**	**0.95**	**0.94**	**0.83**	**0.84**	**0.76**
Government Growth								
DSGE-Reg	**0.82**	1.16	1.01	**0.66**	**0.78**	1.05	**0.63**	**0.81**
DSGE-Reg-AR1	**0.87**	1.24	1.06	**0.68**	**0.80**	1.17	**0.67**	**0.91**
VAR(n)	**0.94**	1.38	**0.88**	**0.71**	**0.90**	**0.91**	**0.60**	**0.68**
DFM	**0.73**	1.04	**0.89**	**0.64**	**0.79**	**0.85**	**0.61**	**0.73**
DFM-RT 2	**0.77**	1.06	**0.90**	**0.66**	**0.78**	**0.87**	**0.63**	**0.75**
DSGE-DFM	**0.73**	1.07	**0.96**	**0.67**	**0.82**	**0.91**	**0.66**	**0.78**
DSGE-DFM-RT 2	**0.70**	1.08	**0.97**	**0.67**	**0.81**	**0.93**	**0.66**	**0.80**

Notes: The table shows the ratios of the RMSE from a given model in comparison to the SEP forecasts for a given variable. Values below one indicate that forecasts from the model are on average more accurate than the SEP.

around the globe during the forecast evaluation window. The other model is a DSGE-DFM model that does not incorporate any financial data series or Canadian credit data series in its X_t vector (DSGE-DFM-NoFin). This DSGE-DFM model is estimated and forecasted with just international economic data and sub-aggregate expenditure and employment data for Canada. The relative RMSE's for both of these models and the RMSE's for the DSGE-Reg, DSGE-DFM, DSGE-DFM RT 2 models are presented in Table C.11.

There are two clear patterns that emerge. First, using the shadow rate as a proxy for the world aggregate policy rate does little to the forecasting results. The relative RMSE's all have similar values to the relative RMSE's of the DSGE-Reg model. Second, only including sub-aggregate component and employment data (DSGE-DFM-NoFin) can significantly decrease the relative RMSE's for all time horizons and bring them relatively close to the DSGE-DFM's relative RMSE's. The additional real-time financial data does make a difference in forecasting investment, import and export growth. The table suggests that financial data can enhance the forecasting results but the data-rich estimation method itself is accounting for the majority of the superior forecasting results seen in this paper.

6.3. Forecasts of the real and nominal exchange rate

It has been shown by Ca'Zorzi et al. (2017) that open-economy DSGE models provide little predictive power when it comes to the short and medium term dynamics of real and nominal exchange rates. These models often do not posses the ability to out-forecast a naive random walk for the real and nominal exchange rate in the short to medium term. In this subsection, I compare the forecasts of reduced form models, open-economy DSGE-Reg and DSGE-DFM models against the naive random walk (RW) forecast for both the real and nominal exchange rate.

The relative RMSE's for forecasts of the open-economy DSGE-Reg model, vector autoregression model, dynamic factor model, dynamic factor model using real-time data and the open-economy DSGE-DFM model not utilizing and utilizing real time data are reported in Tables 11 and 12. Any relative RMSE above one implies that the model's RMSE's is above the RMSE's for the RW benchmark and any number below one implies that the model's RMSE's is lower than the RMSE's for the RW benchmark. The RMSE's for the real exchange rate are computed using the index level of the real broad effective exchange rate for Canada. The level forecasts are generated by the model's forecasts for real FX growth, these are then inputed into the past quarter's index level to generate a forecast path of the index level.

Table 10
Relative RMSE for CAN Trade Growth, CPI Inflation, and Policy Rate.

	$\tau=1$	$\tau=2$	$\tau=4$	$\tau=6$	$\tau=8$	1^{st} year	2^{nd} year	2 year Avg
Export Growth								
DSGE-Reg	1.41	1.34	1.05	1.04	**0.93**	1.32	1.11	1.32
DSGE-Reg-AR1	1.32	1.31	1.07	1.05	**0.94**	1.34	1.13	1.43
VAR(n)	2.14	1.64	1.38	1.12	**0.88**	1.82	1.50	2.33
DFM	1.27	1.47	1.15	1.11	**0.99**	1.58	1.41	1.94
DFM-RT 2	1.20	1.34	1.08	1.07	**0.97**	1.35	1.23	1.56
DSGE-DFM	1.20	1.26	1.05	1.05	**0.95**	1.20	1.13	1.30
DSGE-DFM-RT 2	**0.93**	**0.99**	**0.87**	**0.85**	**0.96**	**0.94**	**0.88**	1.05
Import Growth								
DSGE-Reg	1.32	1.16	1.13	**0.96**	1.05	1.13	0.93	1.10
DSGE-Reg-AR1	1.27	1.23	1.16	1.06	1.09	1.22	1.17	1.41
VAR(n)	1.19	1.68	1.66	1.17	1.07	1.64	1.36	2.09
DFM	1.39	1.42	1.24	1.01	**0.94**	1.45	**0.97**	1.46
DFM-RT 2	1.30	1.29	1.15	**0.99**	**0.93**	1.26	**0.91**	1.08
DSGE-DFM	1.27	1.17	1.10	**0.97**	**0.95**	1.15	**0.89**	**0.99**
DSGE-DFM-RT 2	**0.77**	**0.92**	**0.88**	**0.71**	**0.96**	**0.91**	**0.82**	**0.92**
Policy Rate								
DSGE-Reg	6.14	2.66	1.79	1.72	1.71	2.19	1.72	1.83
DSGE-Reg-AR1	6.47	2.75	1.79	1.53	1.40	2.25	1.50	1.68
VAR(n)	8.69	3.37	1.62	1.24	1.11	2.32	1.20	1.43
DFM	8.65	2.14	1.16	1.03	**0.90**	1.53	**0.98**	1.05
DFM-RT 2	5.86	2.11	1.27	1.06	**0.92**	1.64	1.02	1.16
DSGE-DFM	7.76	2.80	1.55	1.34	1.20	2.12	1.30	1.48
DSGE-DFM-RT 2	1.26	1.69	1.23	1.12	1.05	1.36	1.10	1.15
CPI Inflation								
DSGE-Reg	3.09	1.77	1.90	2.17	2.49	3.45	4.45	6.24
DSGE-Reg-AR1	2.66	1.48	1.54	1.47	1.41	2.75	2.78	4.32
VAR(n)	3.01	1.21	**0.84**	1.05	**0.97**	1.46	1.57	2.06
DFM	2.33	1.07	**0.95**	**0.86**	**0.94**	1.50	1.45	2.13
DFM-RT 2	2.23	1.03	**0.94**	**0.86**	**0.90**	1.42	1.41	2.01
DSGE-DFM	2.31	1.11	1.09	1.19	1.33	1.69	2.25	2.90
DSGE-DFM-RT 2	1.33	1.12	1.02	1.06	1.24	1.46	1.99	2.40
Core CPI Inflation								
DFM	1.55	**0.87**	**0.90**	**0.90**	**0.86**	**0.92**	**0.82**	**0.82**
DFM-RT 2	1.55	**0.87**	**0.88**	**0.88**	**0.85**	**0.91**	**0.78**	**0.76**
DSGE-DFM	1.72	1.13	1.21	1.23	1.31	1.47	1.41	1.72
DSGE-DFM-RT 2	1.38	1.14	1.16	1.15	1.16	1.34	1.26	1.53

Notes: The table shows the ratios of the RMSE from a given model in comparison to the SEP forecasts for a given variable. Values below one indicate that forecasts from the model are on average more accurate than the SEP.

Table 11
Relative RMSE for the real exchange rate.

	$\tau=1$	$\tau=2$	$\tau=3$	$\tau=4$	$\tau=6$	$\tau=8$
Real Exchange Rate Level						
VAR(n)	1.16	1.40	1.49	1.58	1.53	1.49
DFM	1.05	1.09	1.13	1.15	1.22	1.32
DFM-RT 2	**0.15**	**0.67**	**0.86**	1.00	1.09	1.14
DSGE-Reg	1.02	**0.99**	**0.94**	**0.90**	**0.85**	**0.85**
DSGE-Reg-AR1	1.06	1.03	1.00	**0.95**	**0.94**	**0.98**
DSGE-DFM	1.01	1.01	**0.99**	**0.99**	1.01	1.04
DSGE-DFM-RT 2	**0.17**	**0.62**	**0.81**	**0.90**	**0.95**	**0.97**

Notes: The table shows the ratios of the RMSE from a given model in comparison to the forecasts generated by a random walk model. Values below one indicate that forecasts from the model are on average more accurate than the RW benchmark.

Examining the table we are able to see the Ca'Zorzi et al. (2017) finding that an open-economy DSGE-Reg model for Canada is able to out-forecast the RW benchmark for the real exchange rate in regards to τ greater than two.[18] The DSGE-Reg model also posses more accurate dynamics of the real exchange rate when compared to the reduced-form models of the VAR and dynamic factor model. In comparison the open-economy DSGE-DFM model that does not utilize real-time data is right around the benchmark RW for all τ in regards to the real exchange rate. However, when real-time data is utilized

[18] A Similar result was found for open an economy DSGE-Reg model for the EU area by Christoffel et al. (2010).

Table 12
Relative RMSE for the nominal exchange rates.

	$\tau=1$	$\tau=2$	$\tau=3$	$\tau=4$	$\tau=6$	$\tau=8$
Modeled Nominal Exchange Rate Level						
VAR(n)	1.31	1.60	1.73	1.84	1.81	1.85
DFM	1.08	1.15	1.19	1.24	1.39	1.54
DFM-RT 2	**0.40**	**0.68**	**0.90**	1.02	1.21	1.30
DSGE-Reg	1.04	1.00	**0.96**	**0.92**	**0.87**	**0.89**
DSGE-Reg-AR1	1.11	1.11	1.09	1.08	1.15	1.25
DSGE-DFM	1.01	1.01	1.00	**0.99**	1.05	1.17
DSGE-DFM-RT 2	**0.35**	**0.64**	**0.80**	**0.87**	**0.97**	1.07
USD/CAD Level						
VAR(n)	1.18	1.44	1.55	1.65	1.62	1.60
DFM	1.05	1.10	1.13	1.16	1.26	1.36
DFM-RT 2	**0.10**	**0.64**	**0.84**	0.96	1.10	1.16
DSGE-Reg	1.02	1.00	**0.95**	**0.89**	**0.81**	**0.77**
DSGE-Reg-AR1	1.08	1.07	1.05	1.03	1.05	1.10
DSGE-DFM	1.01	1.01	1.00	1.00	1.05	1.12
DSGE-DFM-RT 2	**0.04**	**0.60**	**0.78**	**0.86**	**0.95**	1.01
SEP	**0.06**	**0.52**	**0.76**	**0.89**	1.01	1.00

Notes: The table shows the ratios of the RMSE from a given model in comparison to the forecasts generated by a random walk model. Values below one indicate that forecasts from the model are on average more accurate than the RW benchmark.

Table 13
Contemporaneous Correlation in DGSE-DFM for Canada and U.S. Series.

Series	Data	$Cor(\varepsilon_{\zeta}, \varepsilon_{\zeta^*}) = 1$	$Cor(\varepsilon_{\zeta}, \varepsilon_{\zeta^*})) = .5$	$Cor(\varepsilon_{\zeta}, \varepsilon_{\zeta^*})) = 0$
CAN GDP & US GDP Growth	0.51	0.51	0.28	0.11
CAN Cons & US GDP Growth	0.39	0.59	0.38	0.14
CAN Inv & US GDP Growth	0.35	0.36	0.28	0.20
CAN Exp & US GDP Growth	0.50	0.30	0.05	-0.08
CAN Imp & US GDP Growth	0.54	0.41	0.27	0.19
CAN Inflation & US Inflation	0.42	0.10	0.10	0.10
CAN Policy & US Policy	0.92	0.04	0.05	0.08

The table reports the contemporaneous correlations between Canadian series and US series found in the data and implied by the DSGE-DFM model given three different calibrations around the correlation of global and domestic permanent technology shocks ε_{ζ} and ε_{ζ^*}.

the model is able to significantly outperform the RW benchmark for τ less than four and holds on to those gains for τ greater than four. The dynamic factor model also outperforms the RW benchmark for the real exchange rate for τ less than four when it utilizes real-time data but fails to hold its forecasting gains for τ greater than four.

I evaluate forecasts for the nominal exchange rate using two different definitions. The "Modeled Nominal Exchange Rate" compares the RMSE's for forecasts of the index level of the broad effective exchange rate for Canada. These forecasts are generated by the model's forecasts for nominal FX growth as defined by the model. These growth forecasts are then inputed into the previous quarters index level to generate a forecast path of the index level. I also compute the RMSE's for forecasts of the USD/CAD exchange rate.[19]

I see that a similar pattern holds. The open-economy DSGE-Reg and DSGE-DFM model is at best comparable to the RW benchmark for τ less than four. However, when the open-economy DSGE-DFM model utilizes real-time data the results change significantly to the better.[20] Additionally, the DSGE-DFM-RT 2 model is on par with the SEP forecasts for the USD/CAD nominal exchange rate for all τ. A finding that gives open-economy DSGE-DFM models credibility when it comes to their accuracy in foretelling the future path of both the real and nominal exchange rates.

7. Co-movement of world economy and domestic economy

It has been shown that small open-economy models fail to empirically account for the cross-correlation that exists between the domestic economy and its international trading partners when it comes GDP growth, expenditure growth, interest rate and price co-movements (Justiniano and Preston, 2010). A conclusion I also observe when I examine the model implied cross-correlations of the domestic sector and the world sector of the DSGE-Reg model of this paper. However, the DSGE-DFM model of this paper is able to produce empirically verified cross-correlations between U.S. GDP and Canadian GDP and its components.

[19] For models that do not explicitly incorporate the USD/CAD exchange rate (DSGE-Reg and VAR(n)), it is defined as $(1 + \%\Delta RER_t + \pi_t^* - \pi_t)(USD/CAD)_{t-1}$.

[20] Diebold-Mariano test statistics are computed for various model comparisons against the random walk and can be found in Tables C.12 of the appendix.

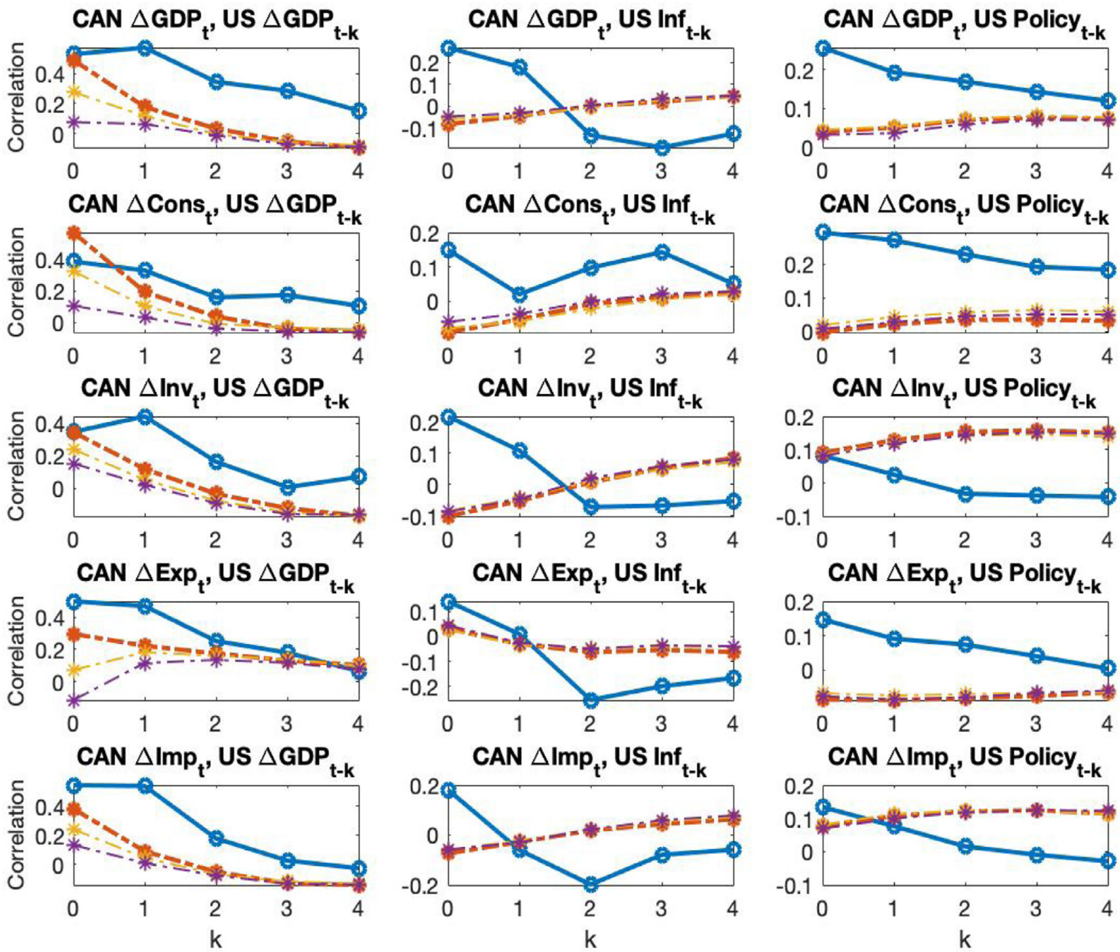

Fig. 8. Data and DSGE-DFM cross-correlations for Canada and U.S. Note: The solid blue line reports the sample correlation found in the data, while the dashed red line reports the correlation found in the simulated DSGE-DFM model. The x-axis represent the k lags of U.S. variables. The figure also plots the correlation found in the simulated model with different levels of technology shock correlation between Canada and the world. Correlations of 0.5 and 0 are plotted in gold and purple. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Figure 8 gives the cross-correlations between Canadian data and lagged U.S. data as well as the cross-correlations seen between the two countries in the simulated DSGE-DFM model for various expenditure series. When I evaluate the model implied correlation between the Canadian economy and the U.S. economy based on 10,000 simulations, I find that GDP growth and its components do resemble those seen in the actual data. For example, the contemporaneous cross-correlations between Canadian GDP, consumption, investment, export and import growth with U.S. GDP growth seen in the sample data are: 0.51, 0.39, 0.35, 0.50 and 0.54 respectively. While the contemporaneous cross-correlations between the Canadian series and U.S. GDP growth in the simulated DSGE-DFM model are: 0.51, 0.59, 0.36, 0.30 and 0.41 respectively. The cross-correlations between these five Canadian series and U.S. inflation and the U.S. policy rate are all near zero for the simulated DSGE-DFM model. However, this is in line with those seen in the data as they are all below 0.25 and most below 0.15.

However, the simulated DSGE-DFM model is still not able to produce any policy rate correlation between the United States and Canada and a smaller than empirically observed cross-correlation for inflation between the two countries. This can be seen in the plots of Figure 9. The contemporaneous cross-correlation between Canadian inflation and the U.S. inflation is 0.42 in the data and only 0.10 in the simulated model. Further, the contemporaneous cross-correlation between the policy rates of Canada and the U.S. is 0.92 in the data and only 0.04 in the simulated model. This lack of co-movement between interest rates and inflation is not unique to this paper as Adolfson et al. (2007a,b); Alpanda and Aysun (2014); De Walque et al. (2005); Justiniano and Preston (2010) and De Walque et al. (2017) all obtain similar results in other estimated small open-economy DSGE models.[21] Further, the short-run policy rate and inflation rate can still move together if multiple U.S. series move in a logical fashion. A point I illustrate in the next subsection.

[21] See Corbo and Strid (2020), for a further discussion on the issue and what can be done to mitigate it.

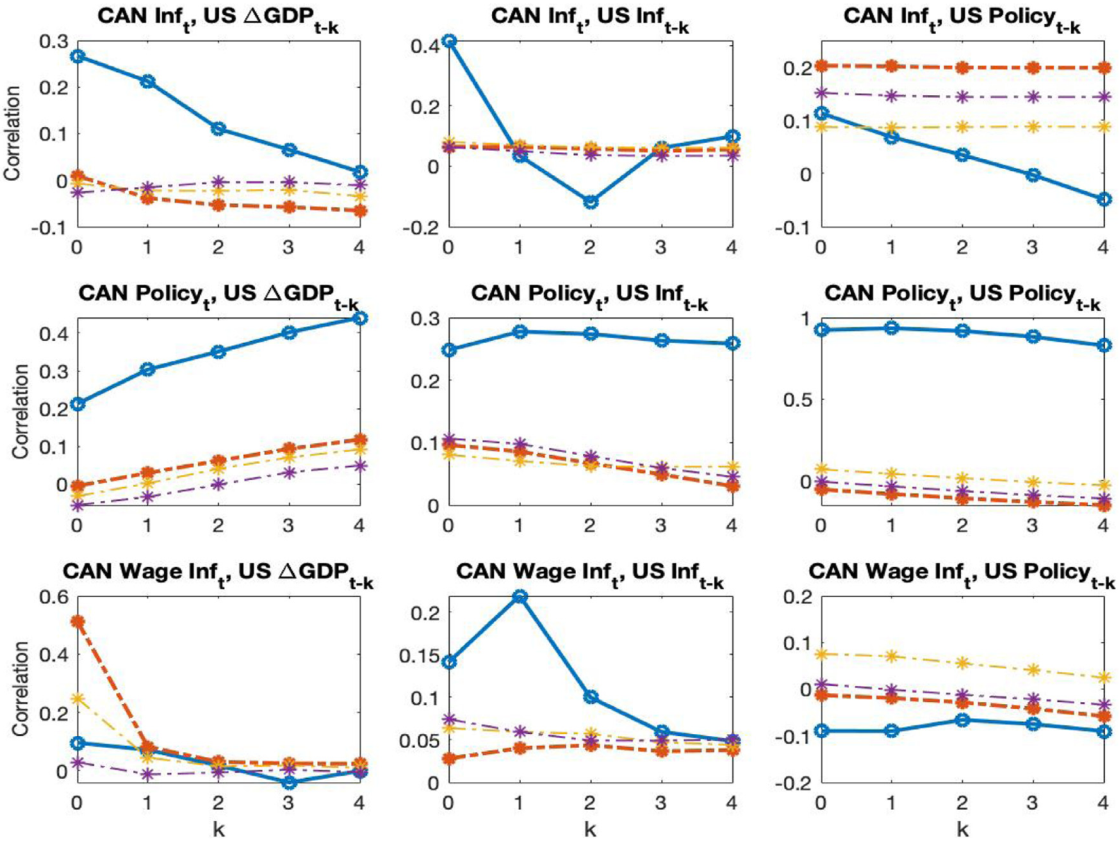

Fig. 9. Data and DSGE-DFM cross-correlations for Canada and U.S.

The open-economy DSGE-DFM model of this paper assumes that there is a common global technology growth shock (ζ) that effects the steady state growth path of both the world sector and domestic sector with perfect correlation. Given this assumption it is important to examine, how much is this common shock responsible for the cross correlations discussed above. To answer such a question, I replace ζ with ζ^* in the world sector in equations 1 and 2. I assume ζ is still a permanent technology growth shock to the domestic sector. I assume that ζ and ζ^* are still governed by the same estimated AR(1) and standard deviation coefficients as in the benchmark model. With this assumption I can compare how much the common shock is driving the correlation between Canada and the United States seen in the model by adjusting the cross-correlation between the two i.i.d technology shocks ε_ζ and ε_{ζ^*}.

I first bring the correlation down to zero, plotted with the purple dashed line in Figures 8 and 9. I see that the correlation between Canadian GDP growth and U.S. GDP growth declines to about 0.1. However, correlation between Canadian consumption, investment and imports to U.S. GDP declines only marginally. The biggest decline in correlation occurs in Canadian export growth and U.S. GDP growth where the correlation falls to below zero. We see little differences in the model correlations with regards to Canadian inflation and interest rates with U.S. GDP growth, inflation and interest rates when common technology shock correlation is brought down to zero.

In addition, if I examine the model's implied correlation when the cross-correlation of the technology shocks in the model is calibrated at 0.5, I see that correlation between Canadian GDP growth and U.S. GDP growth is reduced to 0.28, while correlation between Canadian consumption growth and investment growth with U.S. GDP growth is at the empirically valid values 0.38 and 0.28. However the correlation between Canadian export growth and U.S. GDP growth is zero when the cross-correlation of the technology shocks is calibrated at 0.5. The results suggest that the common technology shock is important but not exclusively so in producing the match between the model's implied correlations and the correlations seen in the data when it comes to Canadian expenditure growth and U.S. GDP growth.

In summary, the model on its own exhibits empirically verified cross-correlations between U.S. GDP growth and Canadian GDP growth and its component's growth but not for inflation or the policy rate. However, these co-movements for inflation and the policy rate between the two countries can exist in the short-term when multiple U.S. series are calibrated as I show in the next subsection.

Table 14
Median Statistics of Post-WWII U.S. Recession.

Series	Median Effect
Annualized GDP Growth	-2.19%
Annualized Inflation Change	-0.45%
Annualized Policy Rate Change	-1.30%
Annualized LR Interest Rate Change	-0.51%
Stock Index Price Growth	-1.5%

Table 15
Estimates of Standardized World Sector Shocks from U.S. Recession Inputs .

	Shock	Median	5%	95%
ε_{R^*}	World Interest Rate shock	-2.1	-3.2	-1.0
μ_{π^*}	World Price shock	-0.6	-1.8	0.6
ζ	Global Productivity shock	-1.0	-1.8	-0.3
z_{y^*}	World Demand shock	-3.0	-6.2	0.3

Table A.1
Data Series used in Estimation of the Canadian DSGE-Reg model.

Data Set	Transform	Data Set	Transform
DSGE-Reg Estimation			
World Aggregate GDP Growth	0	CAN Import Growth	γ
World Aggregate Inflation	0	CAN Hours Growth	0
World Aggregate Policy Rate	0	CAN Wage Inflation	0
CAN GDP Growth	γ	CAN Export Inflation	0
CAN Consumption Growth	γ	CAN Import Inflation	0
CAN Investment Growth	γ	CAN CPI Inflation	0
CAN Government Growth	γ	CAN Real FX Rate Growth	0
CAN Export Growth	γ	CAN Policy Rate	0

7.1. Cross-country correlations of a calibrated event

A major advantage to the DSGE-DFM estimation technique is that it permits us to consider the economic effects of structural shocks outside the scope of the standard variables of GDP, prices, and short-term interest rates. Further, given the amount of data series and the international scope of the data set used, DSGE-DFM estimation can help us estimate the impact international, country-specific events and financial volatility have on the modeled domestic economy.

The model is able to quantify the average effect that certain international events would have on the Canadian economy if we assume a starting point for the DSGE-DFM model and input the average macroeconomic estimates of certain global events. For example, I can assume the model is at steady state, calculate the average effects of U.S. data during the start of a U.S. recession to estimate the most likely combination of global shocks to push the U.S. economy to the calibrated recession state. The estimated combination of global shocks can then be used to calculate the IRF's of the Canadian economy from the combination of global shocks. In many ways it is a reverse IRF. Instead of shocking the model away from steady state and seeing how the data responds, I exogenously shock the global data and use the Kalman filter to calculate how the global structural shocks "responded".

To create a "Generic U.S. Recession", I calculate the median quarterly effect of GDP growth, change in inflation, change in the U.S. policy rate, change in the long run interest rate and the return of the U.S. stock index during the first 6 months of the ten post-WWII recessions as determined by the NBER. Table 14 reports the estimates.

GDP contracts by 2.2%, inflation measured by the GDP deflator falls by 45 basis points, the policy rate is cut by 130 basis points, the 10 year U.S. Treasury falls by 51 basis points, and the stock market index contracts by 1.5%. After subtracting the sample average growth rate from GDP growth and stock price growth, it is these values I input into fictional X^{World} data set to calculate the filtered structural shocks of the world sector to push the world sector out of a steady state and into a state where U.S. GDP growth is 4.69% below average annualized growth, U.S. inflation, the U.S. policy rate, and the U.S. 10-year treasury are 45, 130, and 51 annualized basis points away from their sample averages respectively and the U.S. major stock index price growth is 4% below its average growth rate.

Table 15 reports the estimates and ranges of world sector structural shocks to create the scenario described in Table 14. The most likely combination of shocks to push the U.S. economy away from a global structural steady state include a large negative demand shock, a modest negative shock to global productivity, a marginal negative price shock and a significant monetary policy shock that lowers interest rates.

Figure 10 plots the economic response of the world economy and the Canadian economy to such a combination of shocks. The dark lines plot the median IRFs from the combination of shocks while the lighter shaded regions plot the one and two

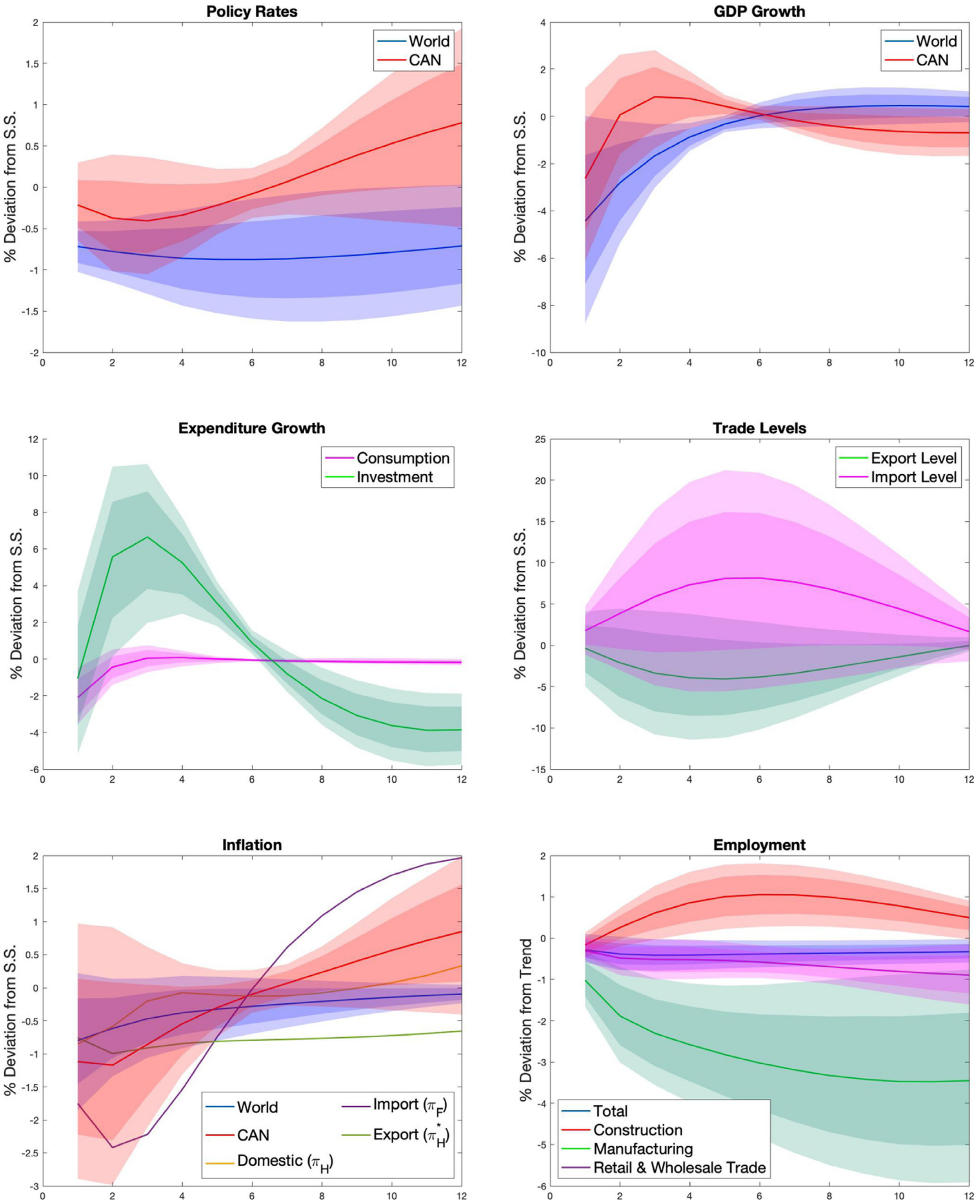

Fig. 10. CAN and World Response to Generic U.S. Recession The figure plots the annualized percent deviation from steady state in GDP growth, consumption growth and investment growth, policy rates, different measures of inflation, the percent deviation from steady state in trade levels and the percent deviation from trend for total and sectoral employment in response to a combination of global shocks outlined in Table 15.

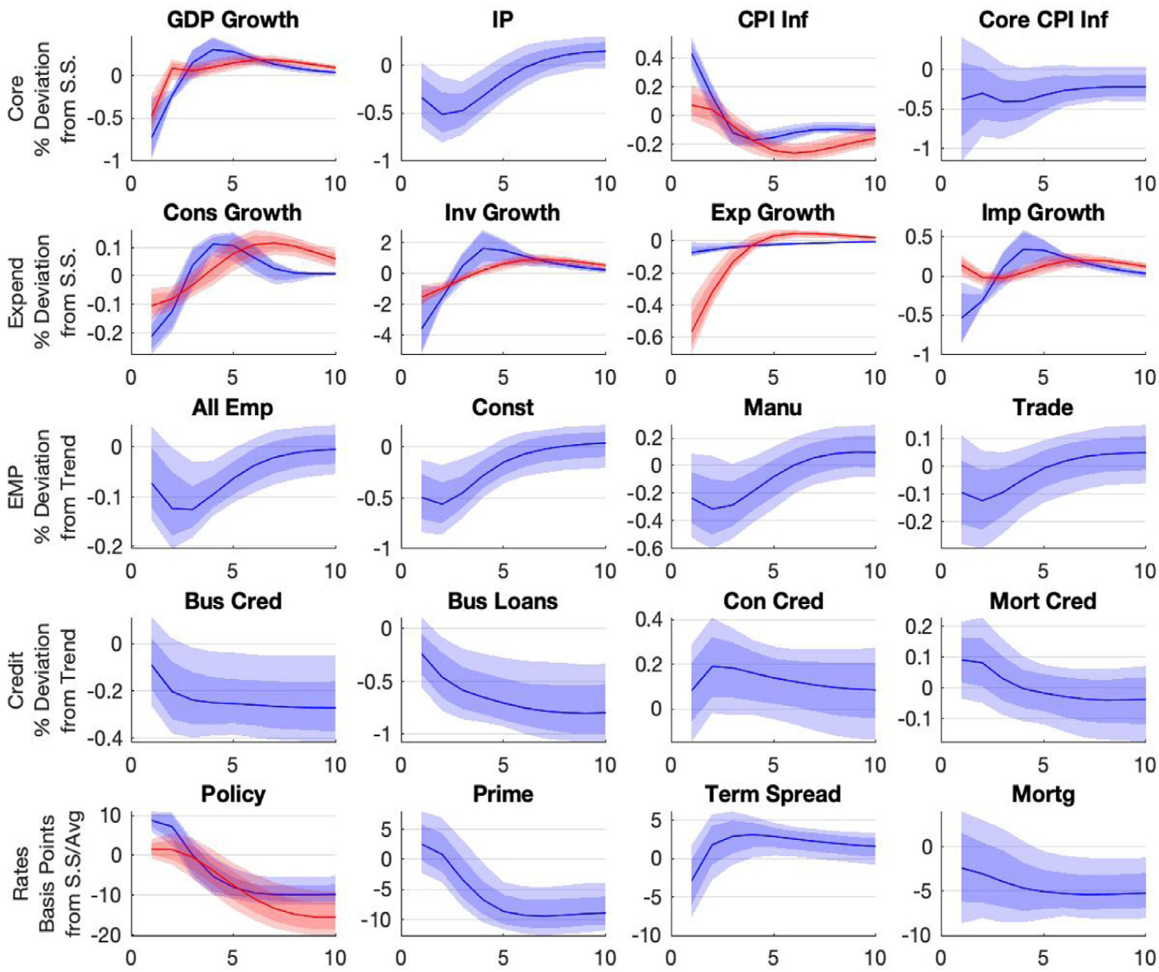

Fig. C.1. Wage Shock (μ_w).

Table A.2
Data Transformation Rubric.

Code	Description
0	Demeaned
1	Linear detrended Log() per capita
2	Detrended Log()
γ	Calibrated steady state γ is subtracted from per capita growth rate

standard deviations of the IRFs. The posterior density band take into account two types of uncertainties by sampling the DSGE-DFM posterior estimates (parameter uncertainty) and sampling the different combinations of world sector shocks that "created" the U.S. recession (world sector state uncertainty calculated by the Kalman filter).

A U.S. recession calibrated to its post WWII averages for GDP growth, inflation, interest rate change and stock market growth declines, would probabilistically result in negative Canadian GDP growth and World GDP growth, with the world declining by a larger amount than Canada does.[22] The policy rate for Canada and global central banks would both fall. World interest rates would fall in the resulting first year, with Canadian rates falling by 50 basis points while global rates would fall by 100 basis points on average. The Canadian policy rate would recover by about 6 quarters after the shocks, while global interest rates would remain below average for the medium term.

[22] In addition, consumption growth would first fall in Canada but recover quickly, while investment growth would first fall as well, but significantly recover, due to the lower interest rate and inflation environment. The interest rate differential between Canada and the world, strengthens the Canadian dollar and stimulates imports and shrinks exports a few quarters after the initial combination of shocks. The resulting decline of exports and increase in investment growth and imports leads to an eventual increase in import inflation. It also has heterogenous effects in the Canadian labor market.

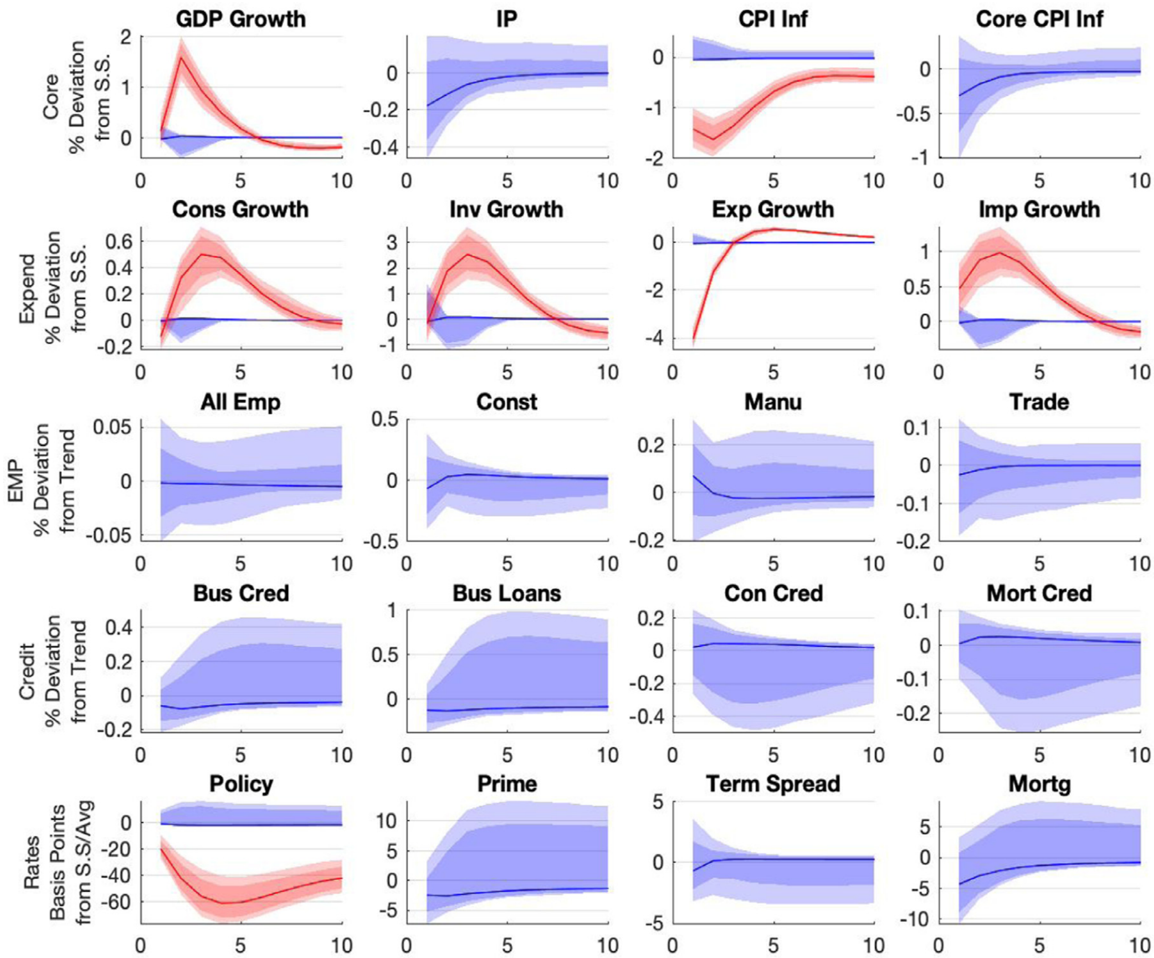

Fig. C.2. Export Price Shock (μ_{H^*}).

The calibrated event would create an estimated correlation of the two countries' policy rates of 0.76 and an estimated correlation of 0.70 for the countries' inflation rates for the first six quarters. The estimated co-movement of the policy rates and inflation rates show that if there is enough co-movement in other non-modeled data series, the DSGE-DFM model will be able to produce the correct co-movements in both, even though the ergodic behavior implied by the model does not.[23]

8. Conclusion

In this paper, an open-economy New Keynesian dynamic stochastic general equilibrium (DSGE) model similar to the Lubik and Schorfheide (2007) and Adolfson et al. (2007a) models is estimated using a large set of Canadian and international macroeconomic and financial series following the work of Boivin and Giannoni (2006) and Gelfer (2019). I denote a model estimated in this way as a DSGE-DFM model. I then conduct similar exercises to Ca'Zorzi et al. (2017); Kolasa and Rubaszek (2018) and Champagne et al. (2018), comparing the root mean squared errors (RMSE's) of the open-economy DSGE-DFM model to the RMSE's of the open-economy DSGE-Reg model and other forecasting models. I find that the DSGE-DFM model is better in capturing the dynamics of many economic series including GDP, investment, consumption, export and import growth, as well as exchange rates around the time of the Great Recession and its ensuing recovery than is the open-economy DSGE model estimated in a traditional sense or other reduced-from forecasting models.

Further, the open-economy DSGE-DFM model that uses real-time data to generate its forecasts produces significantly better forecasts of GDP, consumption, investment, export and import growth when compared to the Bank of Canada's Staff Economic Projections (SEP). In addition, the DSGE-DFM model of this paper that incorporates real-time data significantly out-forecasts a random walk and all other reduced-form models in regard to the short-term and medium-term path of both the

[23] If the above calibrated event were put into the DSEG-Reg model the estimated correlations between CAN's policy rate and inflation level and the World's policy rate and inflation level rate would only be equal to 0.25 and 0.2 respectively.

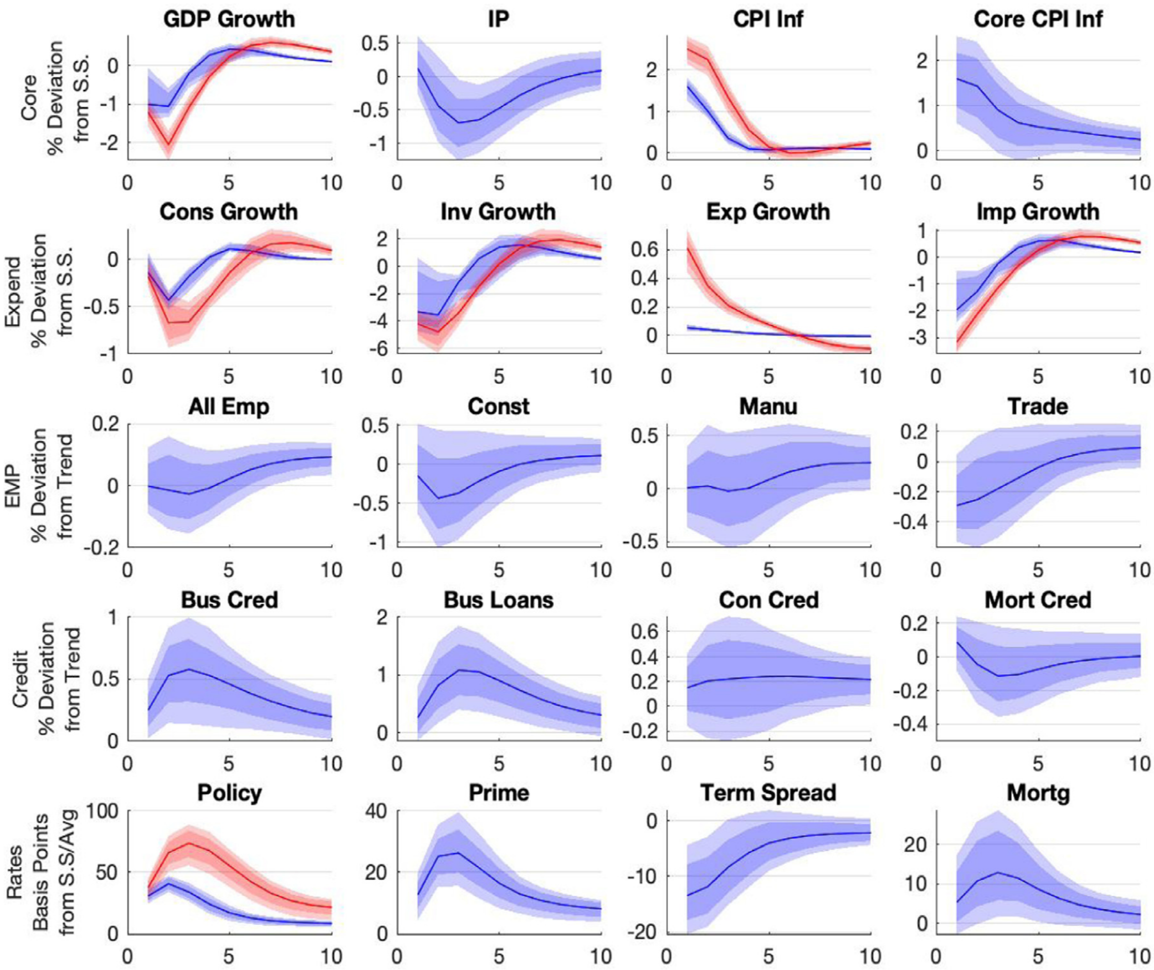

Fig. C.3. Import Price Shock (μ_F).

real and nominal exchange rate. An accomplishment that other open-economy DSGE models do not posses (Ca'Zorzi et al., 2017).

In addition, I see that the open-economy DGSE-DFM model does exhibit empirically valid correlations between the United States and Canada mending the cross-country linkage issue found by Justiniano and Preston (2010) for GDP and its components. The Canadian open-economy DSGE-DFM model in this paper is able to produce cross-country correlation between the U.S. and Canada in regards to inflation and policy rates when calibrated U.S. data for GDP growth, inflation and interest rates are inputed for a given time period into the model. In summary, a generic U.S. recession calibrated to its post WWII averages for GDP growth, inflation change, interest rate change and asset price declines for the U.S. that is inputed in the data vector of the model, would probabilistically result in negative Canadian GDP growth, negative employment growth, a decline in the Canadian policy rate and inflation rate in the model. A finding that is consistent with what is seen in cross-country data for the U.S. and Canada.

The continuing advancements in computational programming and the ever growing number of international macroeconomic and financial series available allows DSGE-DFM estimation to be a bountiful area of future empirical research and open-economy model building.

Appendix A. Measurement Equations and Data

The following lists the measurement equations for the DSGE-Reg model.

$$World\ GDP\ Growth = 100(y_t^* - y_{t-1}^* + \zeta_t) \tag{A.1}$$

$$World\ Inflation = 400(\pi_t^*) \tag{A.2}$$

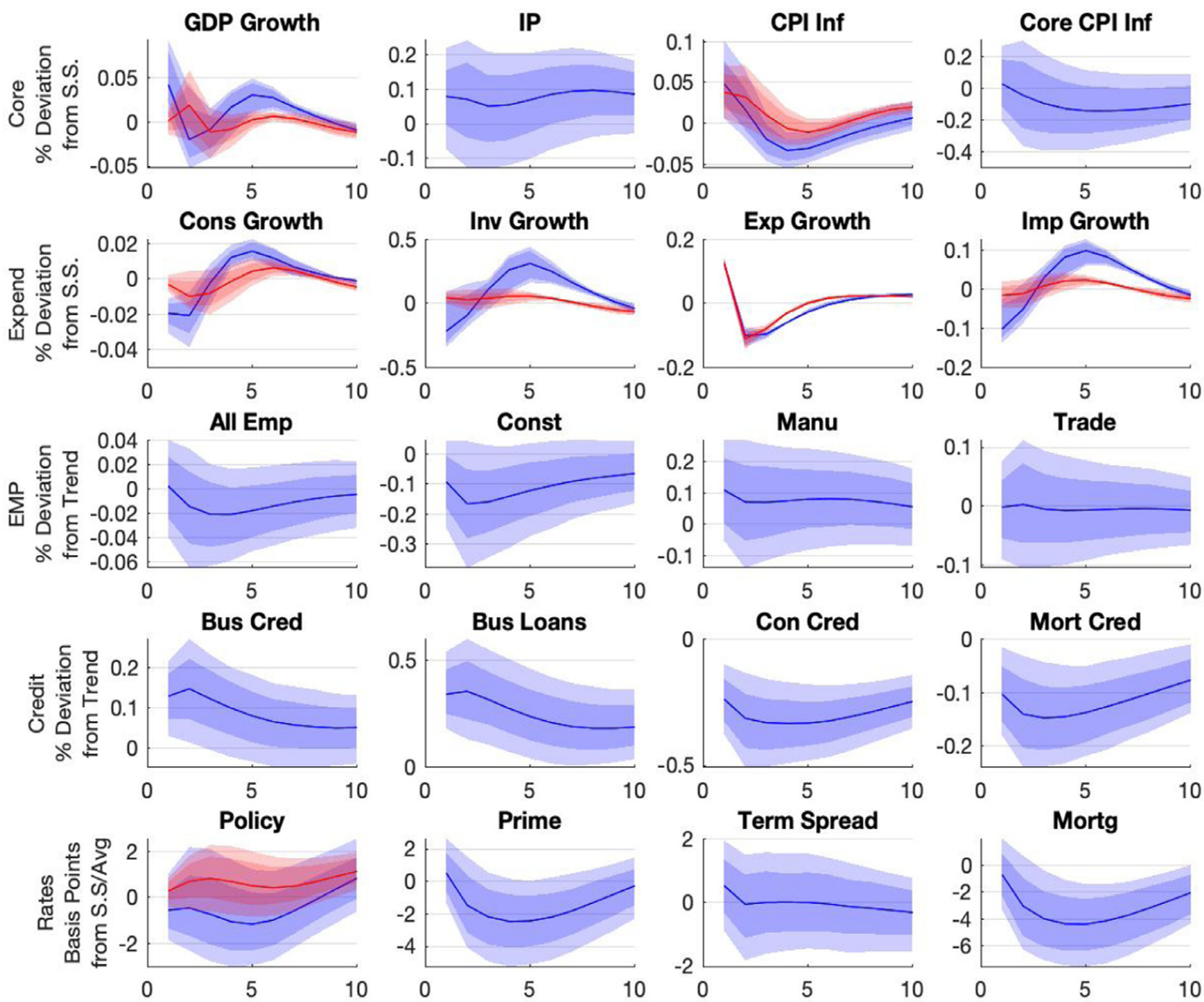

Fig. C.4. World Price Shock (μ_{π^*}).

$$World\ Policy\ Rate = 400(R_t^*) \tag{A.3}$$

$$CAN\ GDP\ Growth = 100(y_t - y_{t-1} + \zeta_t) \tag{A.4}$$

$$CAN\ Consumption\ Growth = 100(c_t - c_{t-1} + \zeta_t) + \epsilon_{c,t} \tag{A.5}$$

$$CAN\ Investment\ Growth = 100(c_t - c_{t-1} + \zeta_t) + \epsilon_{i,t} \tag{A.6}$$

$$CAN\ Government\ Growth = 100(z_{G_t} - z_{G,t-1} + \zeta_t) + \epsilon_{g,t} \tag{A.7}$$

$$CAN\ Export\ Growth = 100(ex_t - ex_{t-1} + \zeta_t) + \epsilon_{ex,t} \tag{A.8}$$

$$CAN\ Import\ Growth = 100(im_t - im_{t-1} + \zeta_t) + \epsilon_{im,t} \tag{A.9}$$

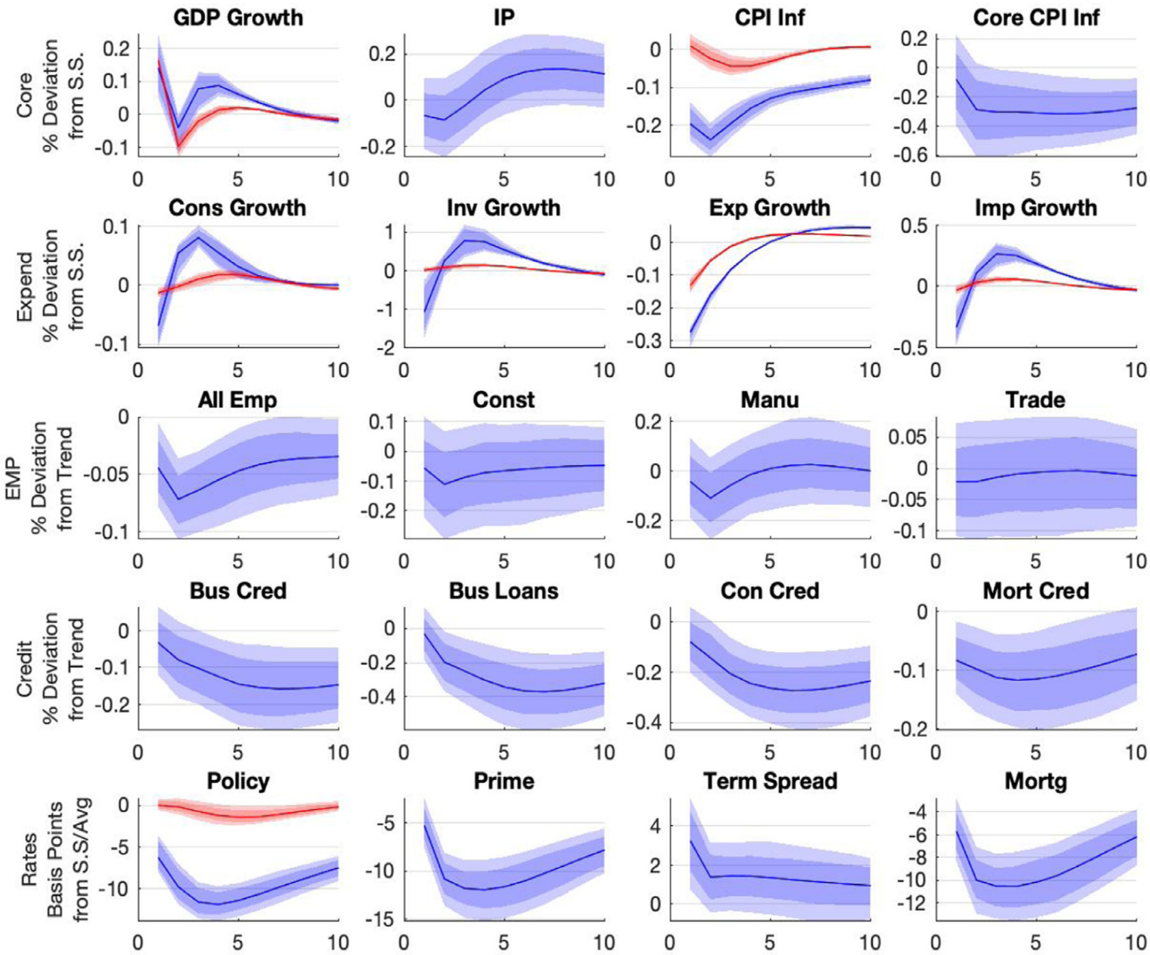

Fig. C.5. World Interest Rate Shock (ε_{R^*}).

$$CAN\ Hours\ Growth = 100(L_t - L_{t-1}) + \epsilon_{L,t} \tag{A.10}$$

$$CAN\ Wage\ Inflation = 100(\pi_{w,t}) \tag{A.11}$$

$$CAN\ Export\ Inflation = 100(\pi^*_{H,t}) \tag{A.12}$$

$$CAN\ Import\ Inflation = 100(\pi_{F,t}) \tag{A.13}$$

$$CAN\ CPI\ Inflation = 400(\pi_t) \tag{A.14}$$

$$CAN\ Real\ FX\ Growth = -100(s_t - s_{t-1}) \tag{A.15}$$

$$CAN\ Policy\ Rate = 400(R_t) \tag{A.16}$$

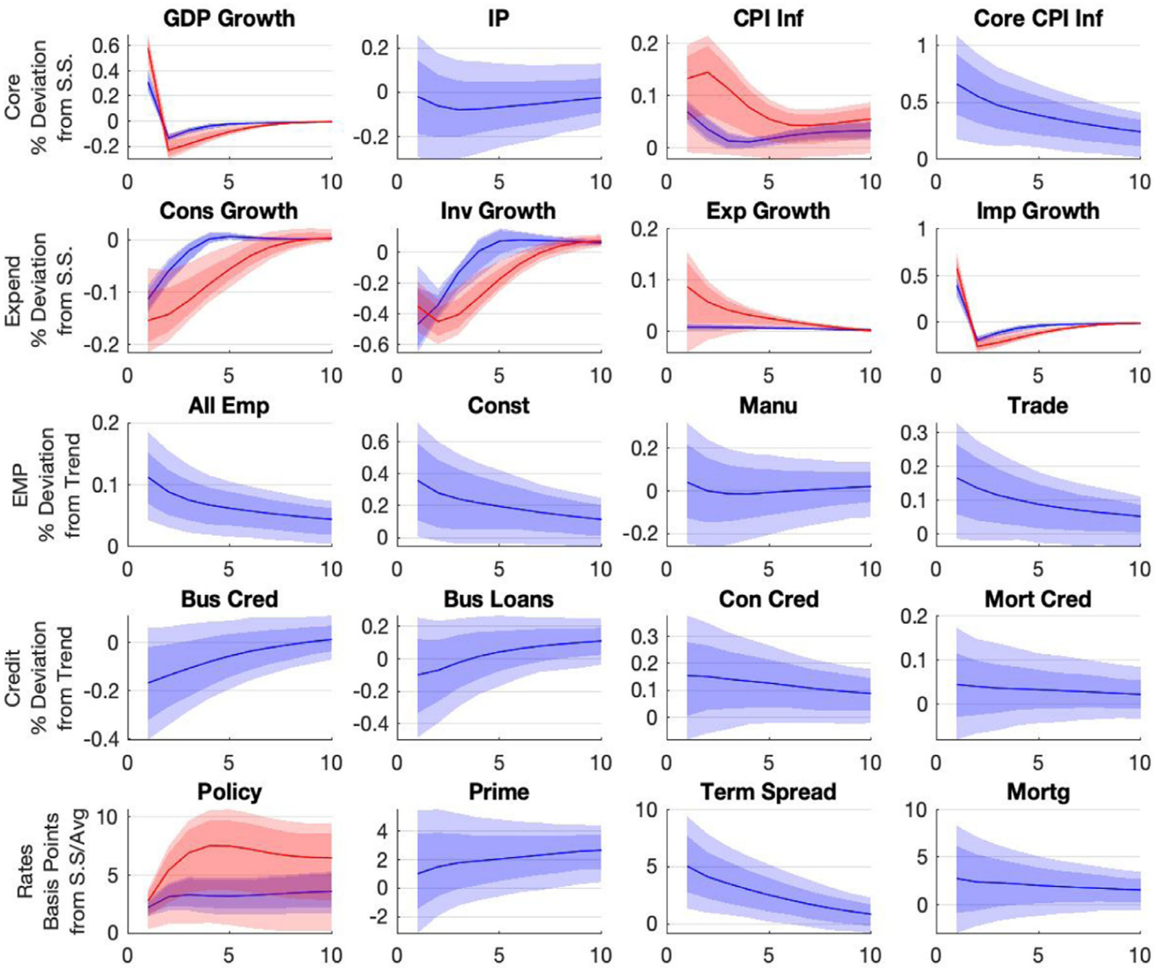

Fig. C.6. Government Shock (z_G).

Appendix B. DSGE-Reg vs DSGE-Reg-AR1 vs DSGE-DFM

Given that the DSGE-DFM model includes an AR(1) component in the measurement equation, but the DSGE-Reg model does not, an assessment must be conducted to determine how much the added and persistence measurement error in the DSGE-DFM model is driving the superior forecasting results of the DSGE-DFM model. To perform such an assessment, I estimate an additional DSGE-Reg model that allows for AR(1) measurement error in the same way and the same priors as does the DSGE-DFM model. I denote this model, DSGE-Reg-AR1.

Examining the estimated measurement error parameters for all three models in Table B.1, I see that the estimated persistence in the measurement error (Ψ) for the DSGE-Reg-AR1 model is low for all variables except Export Inflation. Thus, the unconditional Forecast Error Variance Decomposition (FEVD) for all series (but three) is below 7% in the DSGE-Reg-AR1 model. Further, the estimated measurement error variance (R) is very similar across the DSGE-Reg-AR1 model and the DSEG-Reg model for series that were assumed to have a non-zero measurement error in the DSGE-Reg model. In comparison, the DSGE-DFM model has larger estimates for R and Ψ for many of the series and thus a high share of unconditional FEVD, especially for export/import growth, export inflation and the real exchange rate. This implies that the most important determinant of the measurement error estimates and significance is not the estimation assumption but rather the amount of data that is used in estimation.

Further, when analyzing the relative forecasting RMSE of the DSGE-Reg and DSGE-Reg-AR1 models, I see that the relative RMSEs are indeed lower in the DSEG-Reg-AR1 model for five of the eight variables when τ is less than four. However, the DSGE-Reg-AR1 model exhibits higher relative RMSEs for five of the eight variables when τ is greater than four. Moreover, the relative DSGE-DFM model produces lower relative RMSEs for all variables except the policy rate for the vast majority of horizons. Also reported are the relative RMSEs of the DSGE-DFM-RT 2 model. This model is clearly the dominant model, as 52 of the 64 reported relative RMSEs are the lowest for each respected variable and horizon and ten of the remaining

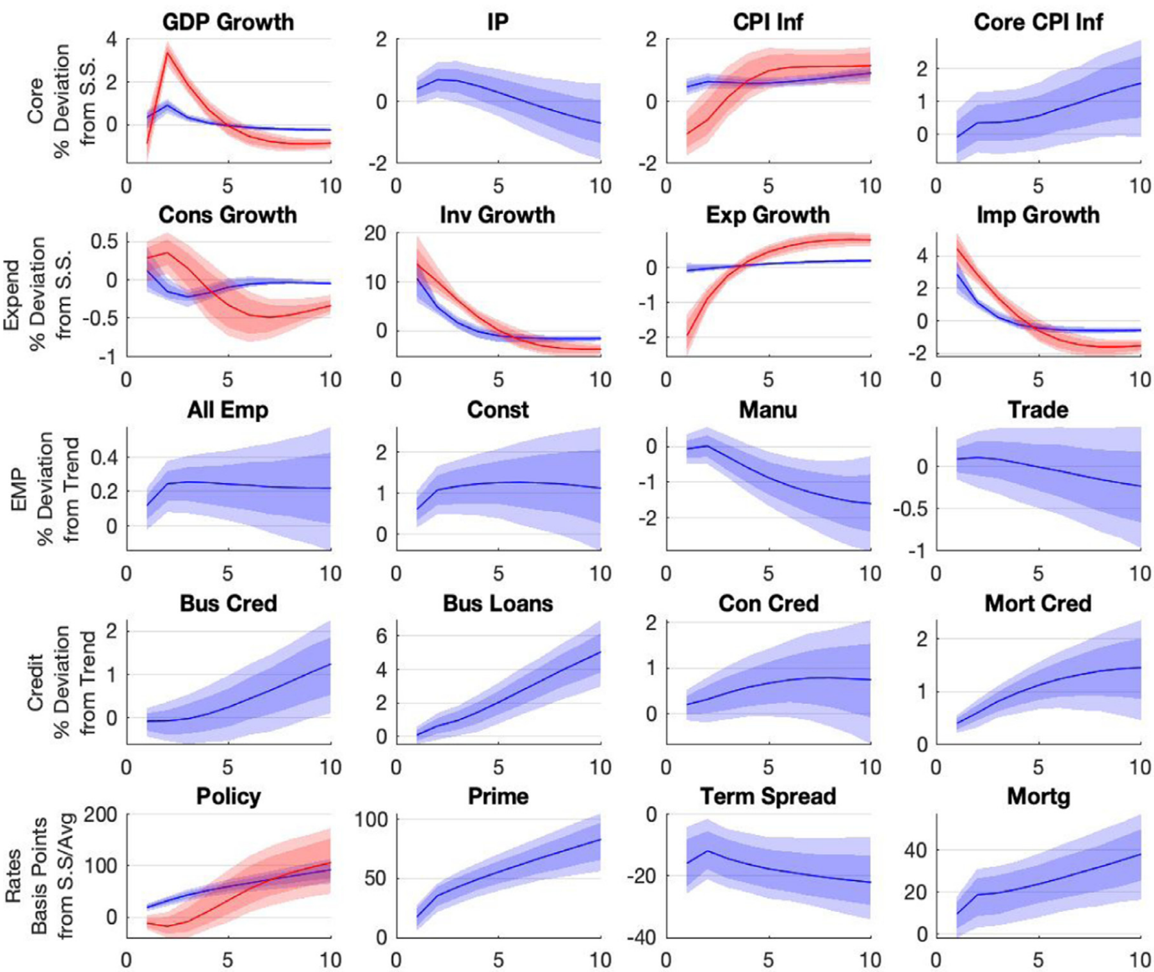

Fig. C.7. Risk Premium Shock (z_B).

Table B.1
Measurement Error Parameter Estimates and Unconditional FEVD.

	DSGE-Reg			DSGE-Reg-AR1			DSGE-DFM		
	Ψ	R	FEVD	Ψ	R	FEVD	Ψ	R	FEVD
World GDP Growth	0	0	0.0	0.00	0.01	0.0	0.01	0.09	16.1
World Inflation	0	0	0.0	0.00	0.00	0.0	0.03	0.00	0.0
World Int Rate	0	0	0.0	0.00	0.00	0.0	0.02	0.00	0.0
GDP Growth	0	0	0.0	-0.10	0.04	3.2	0.28	0.12	5.4
Consumption Growth	0	0.03	2.1	-0.01	0.03	4.8	-0.08	0.07	13.3
Investment Growth	0	1.55	2.5	0.00	1.65	6.6	0.00	2.61	5.3
Gov't Growth	0	0.06	6.7	0.18	0.06	6.8	0.52	0.17	14.8
Export Growth	0	0.55	9.5	-0.03	0.75	16.9	0.15	1.72	61.8
Import Growth	0	0.53	7.5	0.20	0.73	18.9	0.23	1.65	34.2
Hours Growth	0	0.03	0.8	0.02	0.03	1.8	-0.14	0.09	7.3
Wage Inflation	0	0	0.0	0.67	0.03	3.7	0.29	0.10	0.5
Export Inflation	0	0	0.0	0.24	0.56	85.3	0.25	1.78	80.9
Import Inflation	0	0	0.0	0.18	0.39	5.8	0.17	1.07	4.4
CPI Inflation	0	0	0.0	0.01	0.25	0.9	-0.02	0.00	0.0
Real FX Growth	0	0	0.0	0.00	0.02	0.0	0.15	2.31	39.4
Policy Rate	0	0	0.0	0	0	0.0	0	0	0.0

The table reports the median estimates of Ψ, R and the unconditional FEVD attributed to the measurement error for the DSGE-Reg, DSGE-Reg-AR1 and DSGE-DFM models.

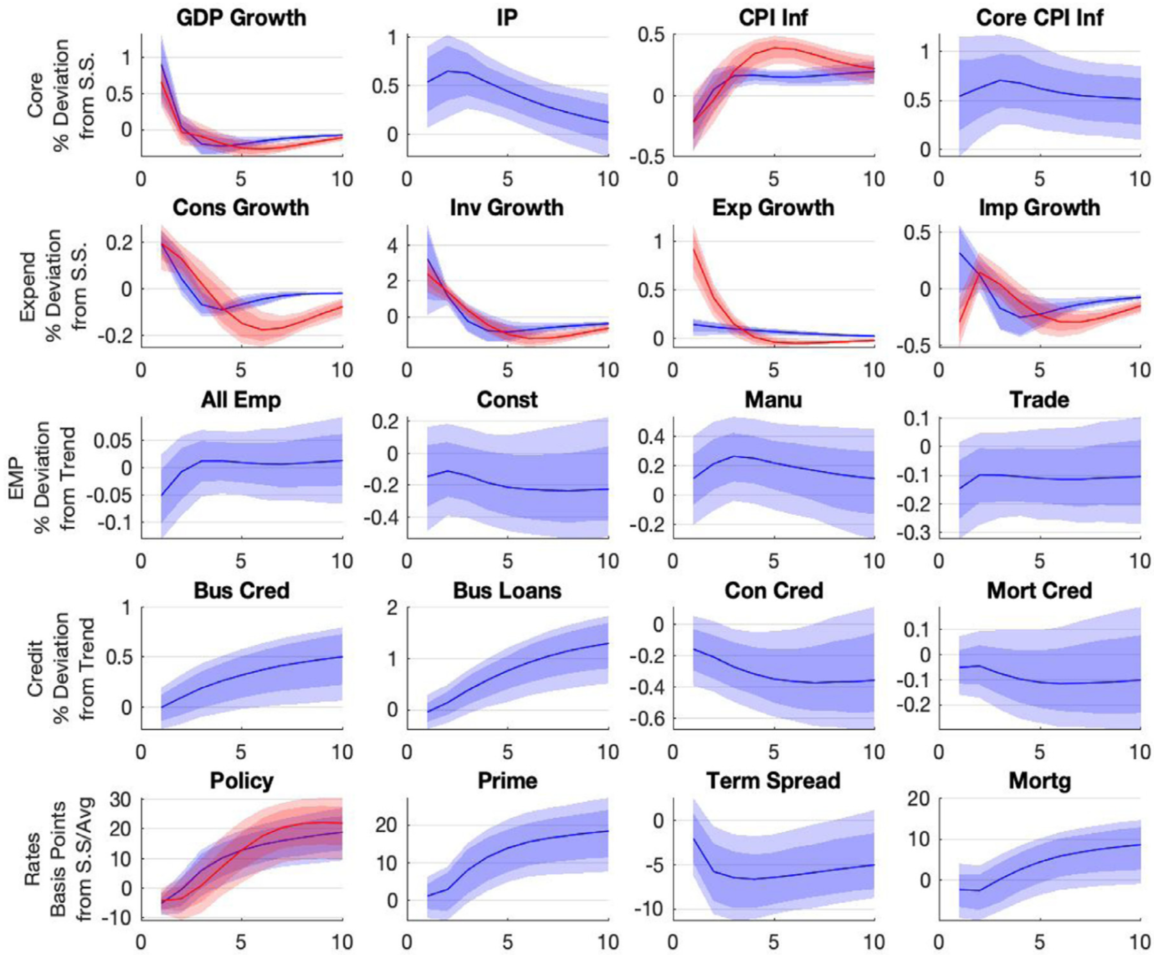

Fig. C.8. Domestic Technology Shock (z_H).

twelve just being marginally higher than the relative RMSEs of the DSGE-DFM model that does not incorporate real-time financial and employment data.

Table B.3 surveys the DM test statistics between the DSGE-Reg and DSGE-AR1 models. I see that the added measurement error does significantly enhance the forecasting precision for short-term CPI inflation, GDP and investment growth but provides no enhancement for GDP and investment growth for τ greater than 2 and no enhancement for any other variables for all horizons. In summary, the added measurement error and persistence in measurement error provides at best only a fraction of the enhanced forecasting ability that data-rich DSGE estimation (DSGE-DFM) offers.

Appendix C

Tables and Figures

Table B.2
Relative RMSE for CAN Macroeconomic Variables.

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
GDP Growth								
DSGE-Reg	3.31	2.48	1.29	**0.94**	1.11	2.55	**0.98**	1.96
DSGE-Reg-AR1	2.61	1.88	1.10	**0.98**	1.09	1.78	1.06	1.44
DSGE-DFM	1.62	1.46	**0.99**	**0.91**	1.00	1.17	**0.88**	**0.98**
DSGE-DFM-RT 2	1.34	1.21	**0.94**	**0.92**	**0.97**	1.03	**0.88**	**0.95**
Consumption Growth								
DSGE-Reg	**0.53**	**0.56**	1.24	1.41	1.28	**0.84**	1.50	1.65
DSGE-Reg-AR1	**0.55**	**0.61**	1.26	1.43	1.03	**0.89**	1.43	1.69
DSGE-DFM	**0.53**	**0.48**	**0.84**	**0.78**	**0.65**	**0.54**	**0.57**	**0.48**
DSGE-DFM-RT 2	**0.46**	**0.44**	**0.87**	**0.79**	**0.64**	**0.49**	**0.57**	**0.52**
Investment Growth								
DSGE-Reg	1.15	1.17	1.08	**0.93**	**0.94**	1.18	**0.83**	1.02
DSGE-Reg-AR1	**0.97**	1.03	**0.99**	**0.97**	1.03	**0.96**	**0.96**	**0.81**
DSGE-DFM	**0.94**	**0.97**	**0.97**	**0.94**	**0.95**	**0.94**	**0.82**	**0.80**
DSGE-DFM-RT 2	**0.69**	**0.89**	**0.96**	**0.95**	**0.94**	**0.83**	**0.84**	**0.76**
Government Growth								
DSGE-Reg	**0.82**	1.16	1.01	**0.66**	**0.78**	1.05	**0.63**	**0.81**
DSGE-Reg-AR1	**0.87**	1.24	1.06	**0.68**	**0.80**	1.17	**0.67**	**0.91**
DSGE-DFM	**0.73**	1.07	**0.96**	**0.67**	**0.82**	**0.91**	**0.66**	**0.78**
DSGE-DFM-RT 2	**0.70**	1.08	**0.97**	**0.67**	**0.81**	**0.93**	**0.66**	**0.80**
Export Growth								
DSGE-Reg	1.41	1.34	1.05	1.04	**0.93**	1.32	1.11	1.32
DSGE-Reg-AR1	1.32	1.31	1.07	1.05	**0.94**	1.34	1.13	1.43
DSGE-DFM	1.20	1.26	1.05	1.05	**0.95**	1.20	1.13	1.30
DSGE-DFM-RT 2	**0.93**	**0.99**	**0.87**	**0.85**	**0.96**	**0.94**	**0.88**	1.05
Import Growth								
DSGE-Reg	1.32	1.16	1.13	**0.96**	1.05	1.13	0.93	1.10
DSGE-Reg-AR1	1.27	1.23	1.16	1.06	1.09	1.22	1.17	1.41
DSGE-DFM	1.27	1.17	1.10	**0.97**	**0.95**	1.15	**0.89**	**0.99**
DSGE-DFM-RT 2	**0.77**	**0.92**	**0.88**	**0.71**	**0.96**	**0.91**	**0.82**	**0.92**
Policy Rate								
DSGE-Reg	6.14	2.66	1.79	1.72	1.71	2.19	1.72	1.83
DSGE-Reg-AR1	6.47	2.75	1.79	1.53	1.40	2.25	1.50	1.68
DSGE-DFM	7.76	2.80	1.55	1.34	1.20	2.12	1.30	1.48
DSGE-DFM-RT 2	1.26	1.69	1.23	1.12	1.05	1.36	1.10	1.15
CPI Inflation								
DSGE-Reg	3.09	1.77	1.90	2.17	2.49	3.45	4.45	6.24
DSGE-Reg-AR1	2.66	1.48	1.54	1.47	1.41	2.75	2.78	4.32
DSGE-DFM	2.31	1.11	1.09	1.19	1.33	1.69	2.25	2.90
DSGE-DFM-RT 2	1.33	1.12	1.02	1.06	1.24	1.46	1.99	2.40

Notes: The table shows the ratios of the RMSE from a given model in comparison to the SEP forecasts for a given variable. Values below one indicate that forecasts from the model are on average more accurate than the SEP.

Table B.3
DM Test Statistics for DSGE-Reg vs DSGE-Reg-AR1.

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
GDP Growth								
DSGE-Reg vs DSGE-Reg-AR1	**3.4**	**2.9**	1.1	-0.9	0.2	**2.0**	-0.7	1.2
Consumption Growth								
DSGE-Reg vs DSGE-Reg-AR1	-0.9	-1.1	-0.1	-0.1	1.9	-0.4	0.3	-0.1
Investment Growth								
DSGE-Reg vs DSGE-Reg-AR1	**2.8**	**2.2**	1.1	-0.7	**-2.0**	**3.6**	-0.9	1.7
Government Growth								
DSGE-Reg vs DSGE-Reg-AR1	**-2.9**	**-3.0**	**-5.6**	-1.7	-1.2	**-4.3**	-1.8	**-3.0**
Export Growth								
DSGE-Reg vs DSGE-Reg-AR1	1.2	1.2	-0.3	-0.4	-1.0	-0.2	-0.3	-0.7
Import Growth								
DSGE-Reg vs DSGE-Reg-AR1	0.7	-1.3	-0.4	-1.4	-1.4	-0.9	-1.4	-1.5
Policy Rate								
DSGE-Reg vs DSGE-Reg-AR1	-0.8	-0.3	0.0	0.7	1.1	-0.2	0.9	0.5
CPI Inflation								
DSGE-Reg vs DSGE-Reg-AR1	**2.3**	1.5	0.9	1.3	**2.1**	1.0	1.8	1.1

Note: The table shows Diebold-Mariano test statistic. Negative test statistics imply the RMSE are lower for the model on the left. Values in bold are significant at the 95% level for the model on the right and values in bold and red are significant at the 95% level for the model on the right.

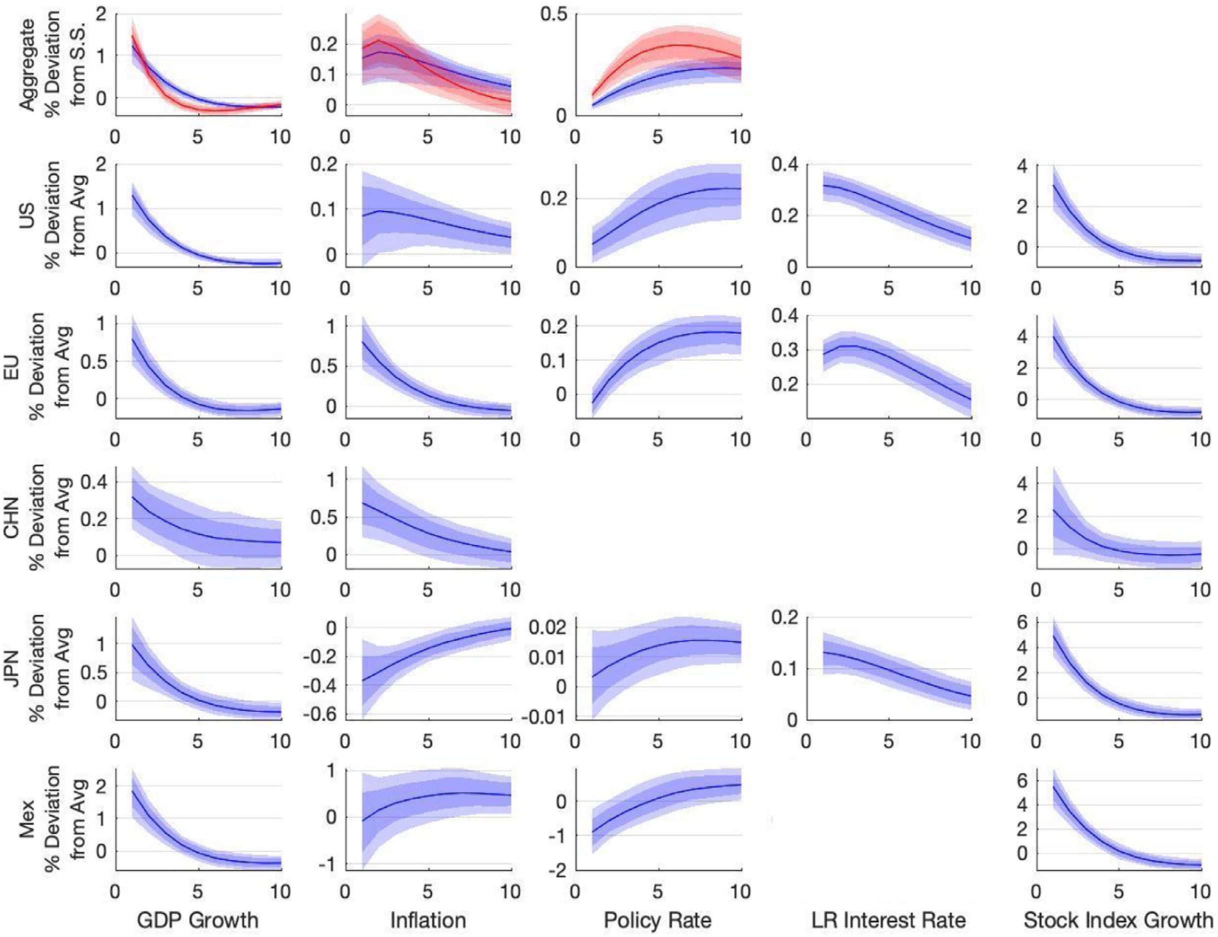

Fig. C.9. World Sector Demand Shock (z_{y^*}).

Table C.1
Calibrated Parameters.

	Description	Value
World Sector		
β	Discount rate	0.9925
γ	World productivity growth rate	0.375
π^*	S.S. level of world inflation	1.00
Domestic Sector		
C_y	S.S. consumption proportion of CAN output	0.57
I_y	S.S. investment proportion of CAN output	0.19
G_y	S.S. government proportion of CAN output	0.24
α	Share of imports of CAN output	0.31
δ	Depreciation rate	0.02
ψ_H	Share of capital in CAN production	0.3
ψ_F	Share of capital in World production	0.3
λ_w	Degree of wage markup	1.25
λ_H	Degree of domestic price markup	1.15
λ_F	Degree of import price markup	1.15
π	S.S. level of domestic inflation	1.00
NX	S.S. level of net exports	0

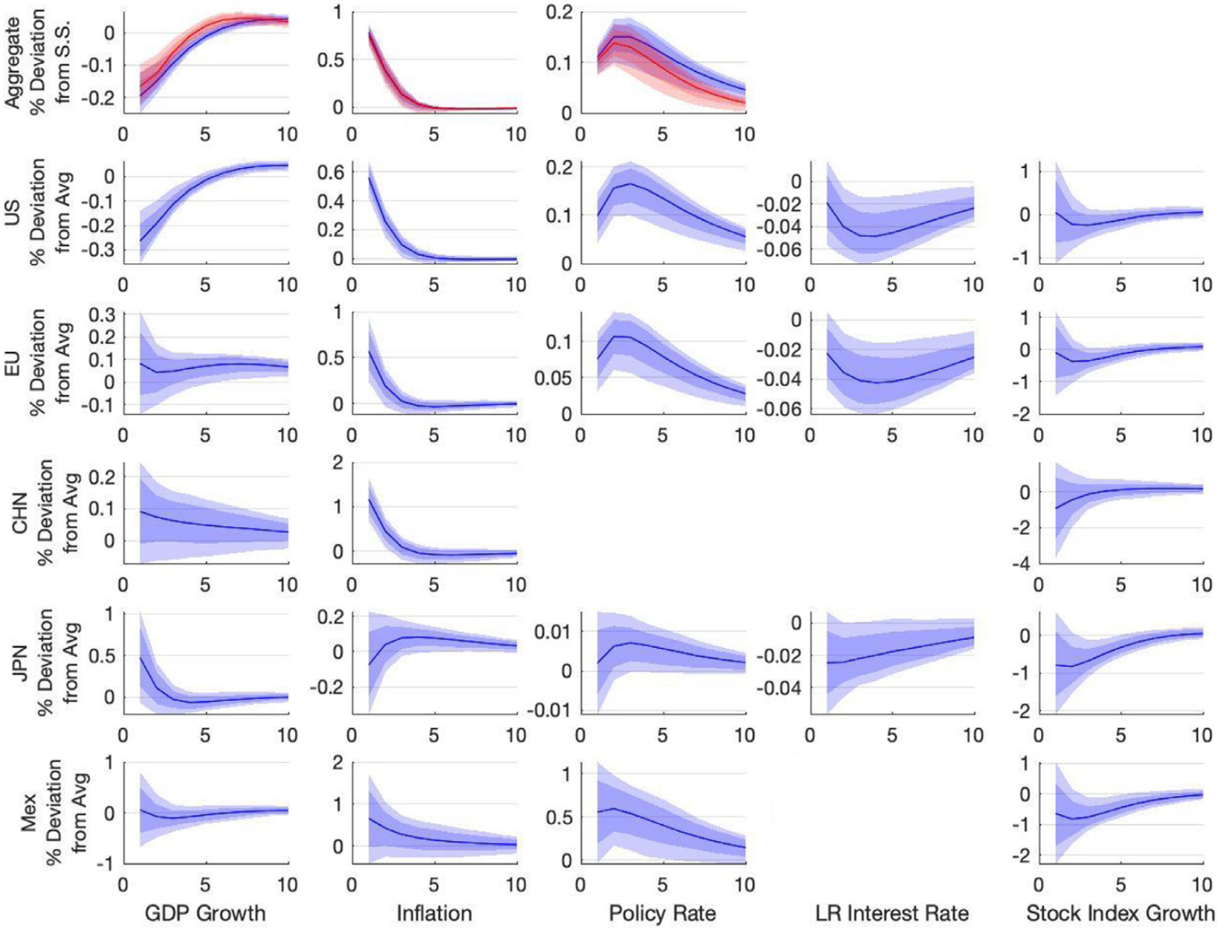

Fig. C.10. World Sector Price Shock (μ_{π^*}).

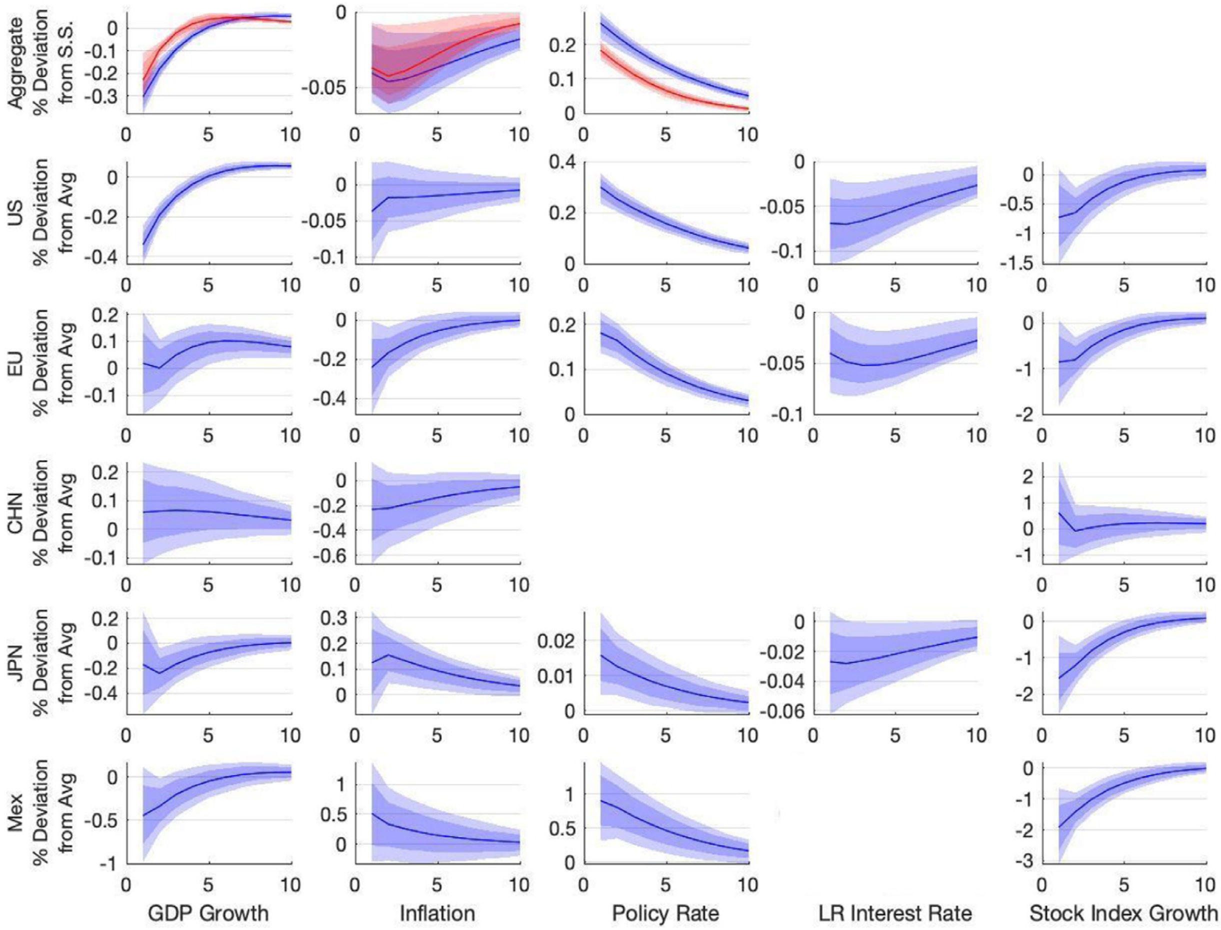

Fig. C.11. World Sector Interest Rate Shock (ε_{R^*}).

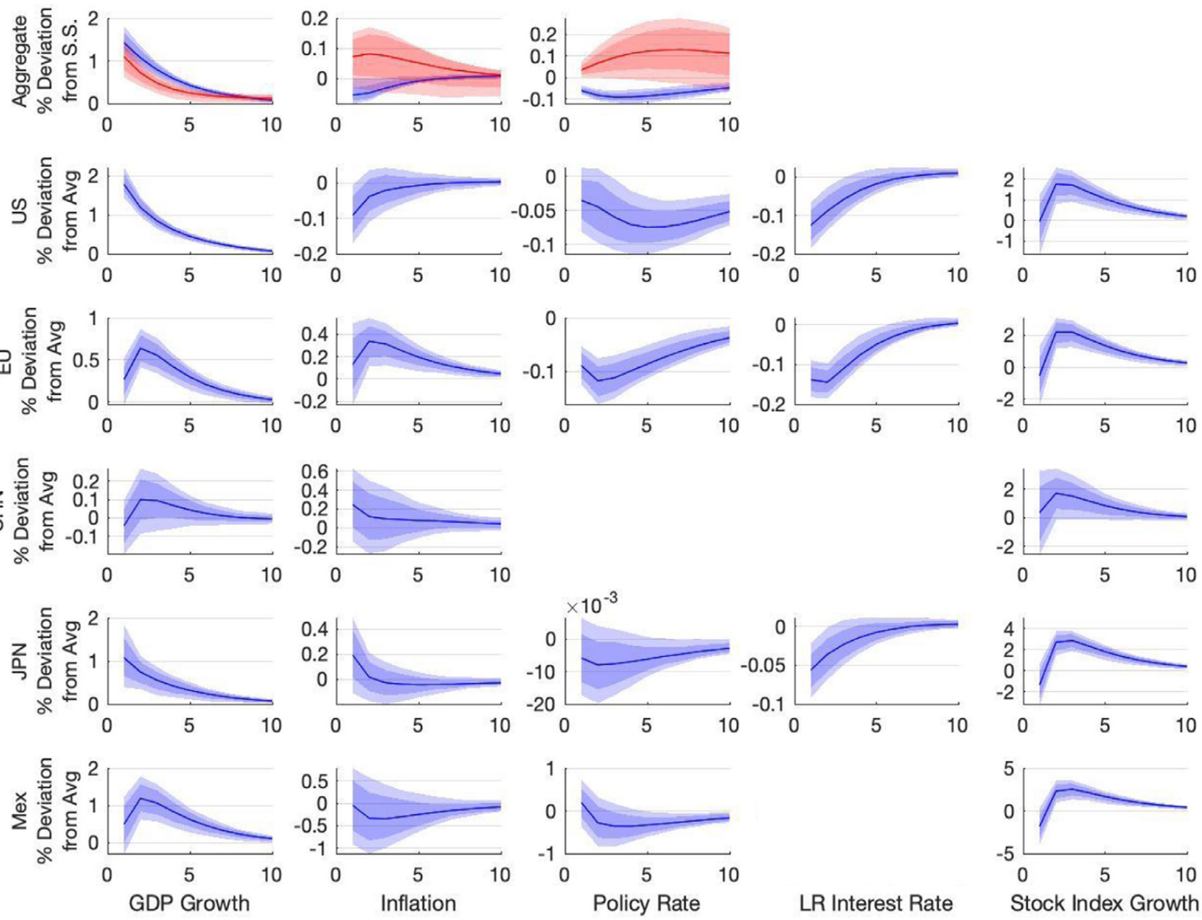

Fig. C.12. World Sector Technology Shock (ζ).

Table C.2
Priors for DSGE Structural Parameters.

	Description	Dist	Mean	Std		Description	Dist	Mean	Std
World Sector									
χ^*	World inflation indexation	Beta	0.3	0.1	$\rho_{\mu_{\pi^*}}$	AR(1) coef. on world price shock	Beta	0.5	0.2
κ^*	World Philips curve slope	Gam	2	0.8	$\rho_{z_{y^*}}$	AR(1) coef. on world demand shock	Beta	0.5	0.2
H^*	World habit persistence	Beta	0.5	0.15	ρ_{ζ}	AR(1) coef. on global prod growth	Beta	0.5	0.2
ϕ_{pi^*}	Taylor Rule coef. on World inflation	Norm	1.5	0.1	$\sigma_{\mu_{\pi^*}}$	Std. of world price shock	IG	0.5	0.4
ϕ_{y^*}	Taylor Rule coef. on World output	Norm	0.25	0.05	$\sigma_{z_{y^*}}$	Std. of world demand shock	IG	0.5	0.4
ρ_r^*	Lagged coef. World Taylor Rule	Beta	0.75	0.1	σ_{ζ}	Std. of global prod. growth shock	IG	0.5	0.4
					σ_{R^*}	Std. of world monetary policy shock	IG	0.5	0.4
Domestic Economy									
Price Parameters									
γ_w	Wage inflation indexation	Beta	0.5	0.1	θ_w	Calvo wage stickiness	Beta	0.75	0.1
γ_H	Domestic inflation indexation	Beta	0.5	0.1	θ_H	Calvo domestic price stickiness	Beta	0.75	0.1
γ_{H^*}	Export inflation indexation	Beta	0.5	0.1	θ_{H^*}	Calvo export price stickiness	Beta	0.75	0.1
γ_F	Import inflation indexation	Beta	0.5	0.1	θ_F	Calvo import price stickiness	Beta	0.75	0.1
Structural Parameters									
h	Habit consumption	Beta	0.5	0.2	ψ	CRRA coef. on labor	Gam	1.5	0.3
λ_I	Investment adjustment cost	Norm	5	1	λ_u	Capital utilization costs	Beta	0.2	0.05
ϕ_B	Risk premium elasticity	IG	0.03	0.05	η	Elasticity of sub. of home goods	Gam	0.9	0.1
η_{mc}	World markup above marginal cost	Gam	1.5	0.3					
Taylor Rule Parameters									
ϕ_π	Taylor Rule coef. on inflation	Gam	1.5	0.25	ϕ_y	Taylor Rule coef. on output	Gam	0.25	0.05
ϕ_s	Taylor Rule coef. on FX growth	Norm	0	0.05	ρ_r	Taylor Rule coef. on lagged rate	Beta	0.75	0.1
AR(1) Coefficients									
ρ_{μ_w}	AR(1) coef. on wage shock	Beta	0.5	0.05	ρ_{μ_H}	AR(1) coef. on domestic price shock	Beta	0.5	0.05
$\rho_{\mu_{H^*}}$	AR(1) coef. on export price shock	Beta	0.5	0.05	ρ_{μ_F}	AR(1) coef. on import price shock	Beta	0.5	0.05
ρ_{z_H}	AR(1) coef. on domestic prod. shock	Beta	0.7	0.1	ρ_{z_D}	AR(1) coef. on export shock	Beta	0.7	0.1
ρ_{z_I}	AR(1) coef. on investment shock	Beta	0.7	0.1	ρ_{z_C}	AR(1) coef. on consumption shock	Beta	0.7	0.1
ρ_{z_G}	AR(1) coef. on gov't growth shock	Beta	0.7	0.1	ρ_{z_B}	AR(1) coef. on risk premium shock	Beta	0.7	0.1
Std. Coefficients									
σ_{μ_w}	Std. of wage shock	IG	0.5	0.4	σ_{μ_H}	Std. of domestic price shock	IG	0.5	0.4
$\sigma_{\mu_{H^*}}$	Std. of export price shock	IG	0.5	0.4	σ_{μ_F}	Std. of import price shock	IG	0.5	0.4
σ_{z_H}	Std. of domestic prod. shock	IG	0.5	0.4	σ_{z_D}	Std. of export shock	IG	0.5	0.4
σ_{z_I}	Std. of investment shock	IG	0.5	0.4	σ_{z_C}	Std. of consumption shock	IG	0.5	0.4
σ_{z_G}	Std. of gov't growth shock	IG	0.5	0.4	σ_{z_B}	Std. of risk premium shock	IG	0.5	0.4
σ_r	Std. of monetary policy shock	IG	0.15	0.1					

Table C.3
Posterior Estimates for DSGE-Reg Parameters.

	Mean	5%	95%		Mean	5%	95%
World Sector							
χ^*	0.24	0.12	0.39	$\rho_{\mu_{\pi^*}}$	0.30	0.11	0.50
κ^*	1.52	0.79	2.51	$\rho_{z_{y^*}}$	0.85	0.76	0.92
H^*	0.66	0.54	0.77	ρ_{ζ}	0.85	0.63	0.95
ϕ_{pi^*}	1.43	1.26	1.60	$\sigma_{\mu_{\pi^*}}$	0.11	0.09	0.14
ϕ_{y^*}	0.27	0.20	0.34	$\sigma_{z_{y^*}}$	2.17	1.52	2.98
ρ_r^*	0.89	0.87	0.91	σ_{ζ}	0.17	0.09	0.31
				σ_{R^*}	0.05	0.04	0.06
Domestic Economy							
Price Parameters							
γ_w	0.43	0.29	0.59	θ_w	0.38	0.31	0.45
γ_H	0.60	0.44	0.75	θ_H	0.63	0.58	0.68
γ_{H^*}	0.49	0.33	0.66	θ_{H^*}	0.62	0.58	0.66
γ_F	0.61	0.46	0.76	θ_F	0.38	0.32	0.44
Structural Parameters							
h	0.70	0.57	0.82	ψ	1.28	0.92	1.69
λ_I	1.06	0.66	1.81	λ_u	0.23	0.15	0.32
ϕ_B	0.02	0.01	0.03	η	0.39	0.37	0.41
η_{mc}	1.19	0.83	1.6				
Taylor Rule Parameters							
ϕ_π	1.53	1.38	1.70	ϕ_y	0.15	0.11	0.20
ϕ_s	-0.04	-0.09	0.01	ρ_r	0.89	0.87	0.91
AR(1) Coefficients							
ρ_{μ_w}	0.54	0.45	0.63	ρ_{μ_H}	0.64	0.58	0.70
$\rho_{\mu_{H^*}}$	0.62	0.57	0.68	ρ_{μ_F}	0.72	0.67	0.77
ρ_{z_H}	0.71	0.61	0.79	ρ_{z_D}	0.68	0.59	0.76
ρ_{z_I}	0.94	0.87	0.97	ρ_{z_C}	0.88	0.75	0.96
ρ_{z_G}	0.95	0.93	0.97	ρ_{z_B}	0.94	0.92	0.97
Std. Coefficients (x 100)							
σ_{μ_w}	4.54	3.15	6.10	σ_{μ_H}	5.26	4.28	6.37
$\sigma_{\mu_{H^*}}$	9.03	7.60	10.54	σ_{μ_F}	5.05	4.06	6.05
σ_{z_H}	1.17	1.03	1.32	σ_{z_D}	1.58	1.38	1.79
σ_{z_I}	1.73	1.06	3.22	σ_{z_C}	2.08	1.39	3.08
σ_{z_G}	0.82	0.71	0.94	σ_{z_B}	0.81	0.60	1.04
σ_r	0.10	0.09	0.12				

Table C.4
Posterior Estimates for DSGE-DFM Parameters.

	Mean	5%	95%		Mean	5%	95%
World Sector							
χ^*	0.18	0.09	0.30	$\rho_{\mu_{\pi^*}}$	0.34	0.18	0.51
κ^*	0.65	0.30	1.12	$\rho_{z_{y^*}}$	0.89	0.86	0.93
H^*	0.76	0.73	0.80	ρ_ζ	0.21	0.11	0.33
ϕ_{pi^*}	1.52	1.37	1.69	$\sigma_{\mu_{\pi^*}}$	0.11	0.09	0.14
ϕ_{y^*}	0.10	0.05	0.15	$\sigma_{z_{y^*}}$	2.22	1.80	2.81
ρ_r^*	0.90	0.88	0.92	σ_ζ	0.99	0.88	1.11
				σ_{R^*}	0.07	0.06	0.08
Domestic Economy							
Price Parameters							
γ_w	0.35	0.19	0.48	θ_w	0.20	0.12	0.25
γ_H	0.59	0.43	0.69	θ_H	0.30	0.26	0.33
γ_{H^*}	0.34	0.22	0.44	θ_{H^*}	0.85	0.80	0.89
γ_F	0.29	0.19	0.43	θ_F	0.40	0.32	0.45
Structural Parameters							
h	0.50	0.45	0.54	ψ	0.62	0.42	0.89
λ_I	0.28	0.08	0.62	λ_u	0.23	0.17	0.31
ϕ_B	0.01	0.01	0.02	η	0.41	0.39	0.42
η_{mc}	1.29	1.07	1.51				
Taylor Rule Parameters							
ϕ_π	1.33	1.27	1.40	ϕ_y	0.19	0.16	0.23
ϕ_s	-0.10	-0.14	-0.06	ρ_r	0.84	0.82	0.87
AR(1) Coefficients							
ρ_{μ_w}	0.61	0.54	0.69	ρ_{μ_H}	0.62	0.58	0.70
$\rho_{\mu_{H^*}}$	0.54	0.46	0.61	ρ_{μ_F}	0.59	0.54	0.64
ρ_{z_H}	0.89	0.86	0.92	ρ_{z_D}	0.68	0.60	0.78
ρ_{z_I}	0.95	0.94	0.97	ρ_{z_C}	0.85	0.71	0.92
ρ_{z_G}	0.87	0.83	0.92	ρ_{z_B}	0.96	0.94	0.98
Std. Coefficients (x 100)							
σ_{μ_w}	1.06	0.79	1.40	σ_{μ_H}	0.98	0.78	1.19
$\sigma_{\mu_{H^*}}$	0.49	0.22	2.08	σ_{μ_F}	3.77	3.70	3.83
σ_{z_H}	0.35	0.24	0.46	σ_{z_D}	0.76	0.50	1.09
σ_{z_I}	0.30	0.22	0.41	σ_{z_C}	0.54	0.41	0.68
σ_{z_G}	0.56	0.45	0.69	σ_{z_B}	0.26	0.20	0.34
σ_r	0.14	0.12	0.15				

Table C.5
FEVD-Short run (h=4). .

	Mark-Ups	Domestic Tech	Global	Policy	Cons	Inv	Trade	Measure
World GDP Growth	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	82.0	0.0	0.0	0.0	0.0	18.0
World Inflation	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
World Int Rate	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
CAN GDP Growth	31.1	0.9	4.8	3.2	1.0	29.0	30.0	0.0
	18.0	2.9	57.1	9.4	0.3	2.8	3.5	6.1
CAN Cons Growth	19.9	0.5	8.9	3.1	34.2	26.9	3.4	3.0
	6.9	0.6	54.0	7.5	11.3	2.5	2.4	14.9
CAN Investment Growth	25.8	1.3	4.0	1.1	1.1	13.2	49.9	3.6
	20.2	2.6	36.6	6.8	0.2	3.5	26.5	3.6
CAN Govt Growth	0.0	0.0	9.5	83.5	0.0	0.0	0.0	7.0
	0.0	0.0	64.6	20.4	0.0	0.0	0.0	15.0
CAN Export Growth	24.1	1.3	3.7	0.0	0.0	5.5	54.3	10.8
	0.0	0.1	13.6	0.0	0.0	0.1	20.8	65.3
CAN Import Growth	29.3	0.1	7.3	1.5	0.9	5.0	44.2	11.7
	10.7	0.3	40.3	4.2	0.2	0.6	12.3	31.2
CAN Hours Growth	32.1	30.9	0.4	2.6	1.7	16.1	15.2	1.0
	33.9	2.3	14.5	14.3	0.9	4.0	21.8	8.3
CAN Wage Inflation	50.2	1.0	4.7	4.8	0.1	29.8	9.3	0.0
	12.8	0.1	59.8	9.7	0.0	0.9	10.0	6.6
CAN Export Inflation	61.2	3.4	2.9	0.1	0.1	13.9	18.4	0.0
	0.4	0.9	1.9	0.0	0.0	0.6	0.6	95.6
CAN Import Inflation	46.5	1.4	2.4	0.7	0.2	29.7	19.1	0.0
	40.1	1.6	14.6	2.2	0.1	2.9	6.3	32.2
CAN CPI Inflation	51.5	0.4	0.8	2.0	0.3	37.1	8.0	0.0
	47.3	1.0	10.0	18.0	0.1	6.2	17.3	0.0
CAN Real FX Growth	16.7	2.6	3.1	1.1	0.1	26.7	49.6	0.0
	5.7	5.0	7.6	3.5	0.1	2.9	32.3	42.8
CAN Policy Rate	42.7	0.1	0.6	1.8	0.6	49.2	4.9	0.0
	26.8	1.0	12.0	5.7	0.5	11.8	42.2	0.0

Note: The FEVD is calculated using the posterior means of all estimated parameters of the DSGE-Reg and DSGE-DFM models. The FEVD for the DSGE-Reg for each data set is the top row and the FEVD for the DSGE-DFM for each data set is the bottom row. Total FEVD may not add up to 100 because of rounding. Mark-ups include $(\mu_w, \mu_H, \mu_{H^*}, \mu_F)$ shocks, Global include $(z_{y^*}, \mu_{\pi^*}, \varepsilon_{R^*}, \zeta)$ shocks, Policy include (ε_R, z_G) shocks, Trade include (z_B, z_D) shocks.

Table C.6
FEVD-Medium run (h=10). .

	Mark-Ups	Domestic Tech	Global	Policy	Cons	Inv	Trade	Measure
World GDP Growth	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	83.3	0.0	0.0	0.0	0.0	16.7
World Inflation	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
World Int Rate	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
CAN GDP Growth	33.2	1.2	5.9	2.7	1.0	25.9	30.1	0.0
	20.0	3.0	55.5	8.8	0.3	2.7	3.9	5.7
CAN Cons Growth	21.8	1.1	8.7	2.7	28.6	25.6	9.3	2.3
	7.3	0.7	52.9	7.3	12.2	2.5	2.7	14.5
CAN Investment Growth	29.5	1.8	5.1	1.0	1.0	11.7	47.1	2.9
	22.1	2.7	36.7	6.1	0.2	3.3	25.6	3.2
CAN Govt Growth	0.0	0.0	12.0	81.2	0.0	0.0	0.0	6.8
	0.0	0.0	64.2	20.8	0.0	0.0	0.0	15.0
CAN Export Growth	23.8	1.2	4.5	0.0	0.0	5.5	54.9	10.0
	0.0	0.1	15.3	0.0	0.0	0.1	21.0	63.5
CAN Import Growth	28.6	0.5	8.2	1.2	0.9	6.0	45.8	8.7
	12.4	0.5	40.1	3.9	0.2	0.7	13.5	28.8
CAN Hours Growth	36.7	27.1	0.7	2.5	1.6	15.3	15.3	0.8
	36.1	2.2	15.4	13.0	0.8	3.7	21.3	7.5
CAN Wage Inflation	44.3	1.5	3.5	3.5	0.1	35.0	12.1	0.0
	13.2	0.4	53.7	8.6	0.0	1.9	16.4	5.8
CAN Export Inflation	56.5	3.0	2.8	0.1	0.1	13.0	24.7	0.0
	0.4	1.1	3.4	0.0	0.0	0.8	3.4	90.7
CAN Import Inflation	38.8	1.3	3.4	0.6	0.2	27.9	27.9	0.0
	32.2	1.5	17.2	1.7	0.1	3.0	19.5	24.9
CAN CPI Inflation	42.8	1.1	1.7	1.5	0.3	35.9	16.7	0.0
	33.4	1.8	10.3	12.3	0.2	6.3	35.5	0.0
CAN Real FX Growth	16.6	2.6	3.0	1.1	0.1	26.0	50.6	0.0
	5.6	4.8	7.5	3.4	0.1	2.8	34.4	41.3
CAN Policy Rate	24.0	1.1	1.3	0.7	0.8	52.5	19.6	0.0
	9.1	2.5	13.4	1.4	0.5	9.5	63.7	0.0

Note: The FEVD is calculated using the posterior means of all estimated parameters of the DSGE-Reg and DSGE-DFM models. The FEVD for the DSGE-Reg for each data set is the top row and the FEVD for the DSGE-DFM for each data set is the bottom row. Total FEVD may not add up to 100 because of rounding. Mark-ups include $(\mu_w, \mu_H, \mu_{H^*}, \mu_F)$ shocks, Global include $(z_{y^*}, \mu_{\pi^*}, \varepsilon_{R^*}, \zeta)$ shocks, Policy include (ε_R, z_G) shocks, Trade include (z_B, z_D) shocks.

Table C.7
FEVD-Long run (h=40). .

	Mark-Ups	Domestic Tech	Global	Policy	Cons	Inv	Trade	Measure
World GDP Growth	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	83.7	0.0	0.0	0.0	0.0	16.3
World Inflation	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
World Int Rate	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
CAN GDP Growth	31.5	1.1	6.0	2.6	0.9	24.2	33.7	0.0
	19.3	3.0	55.0	8.5	0.3	2.6	5.6	5.5
CAN Cons Growth	20.1	1.0	8.8	2.5	28.6	24.1	12.9	2.1
	7.0	0.7	51.6	7.0	12.0	2.5	5.4	13.9
CAN Investment Growth	26.8	1.7	5.3	0.9	1.1	11.1	50.4	2.5
	21.1	2.6	37.6	5.8	0.2	3.2	26.5	3.1
CAN Govt Growth	0.0	0.0	12.3	81.0	0.0	0.0	0.0	6.7
	0.0	0.0	64.1	20.9	0.0	0.0	0.0	15.0
CAN Export Growth	22.9	1.2	4.6	0.0	0.0	5.3	56.2	9.6
	0.0	0.1	15.9	0.0	0.0	0.1	21.6	62.2
CAN Import Growth	25.6	0.5	8.1	1.0	0.8	5.4	51.0	7.5
	11.8	0.4	40.3	3.8	0.2	0.6	15.3	27.5
CAN Hours Growth	36.4	26.8	0.8	2.4	1.6	15.2	15.9	0.8
	35.5	2.1	16.1	12.8	0.8	3.7	21.7	7.4
CAN Wage Inflation	15.6	1.0	2.8	1.0	0.3	28.0	51.3	0.0
	4.1	1.2	22.8	2.3	0.1	2.4	65.3	1.6
CAN Export Inflation	51.5	2.7	2.8	0.0	0.1	12.0	30.7	0.0
	0.4	1.1	4.4	0.0	0.0	0.8	10.7	82.6
CAN Import Inflation	25.6	0.9	3.1	0.4	0.2	23.1	46.6	0.0
	12.8	1.1	14.7	0.7	0.1	2.3	58.8	9.7
CAN CPI Inflation	18.6	0.9	2.5	0.6	0.2	26.9	50.3	0.0
	5.1	1.5	11.6	1.7	0.1	2.9	77.1	0.0
CAN Real FX Growth	16.3	2.5	3.1	1.1	0.1	25.5	51.4	0.0
	5.4	4.7	7.6	3.3	0.1	2.7	35.7	40.4
CAN Policy Rate	7.9	0.8	2.2	0.2	0.5	30.3	58.2	0.0
	1.3	1.5	11.8	0.1	0.2	3.1	81.9	0.0

Note: The FEVD is calculated using the posterior means of all estimated parameters of the DSGE-Reg and DSGE-DFM models. The FEVD for the DSGE-Reg for each data set is the top row and the FEVD for the DSGE-DFM for each data set is the bottom row. Total FEVD may not add up to 100 because of rounding. Mark-ups include $(\mu_w, \mu_H, \mu_{H^*}, \mu_F)$ shocks, Global include $(z_{y^*}, \mu_{\pi^*}, \varepsilon_{R^*}, \zeta)$ shocks, Policy include (ε_R, z_G) shocks, Trade include (z_B, z_D) shocks.

Table C.8
RMSE for Bank of Canada's Staff Economic Projections (SEP).

	$\tau=1$	$\tau=2$	$\tau=4$	$\tau=6$	$\tau=8$	1^{st} year	2^{nd} year	2 year Avg
GDP Grwth	1.53	1.91	3.17	3.11	1.90	1.50	1.73	1.16
Cons Growth	3.08	3.21	1.89	1.49	1.69	1.83	1.27	0.92
Inv Growth	14.39	14.82	16.06	16.76	13.41	9.62	9.10	6.75
Gov't Growth	4.16	2.88	3.24	4.50	3.69	2.39	3.44	2.49
Export Growth	8.03	8.12	10.65	10.07	8.52	5.03	4.13	2.83
Import Growth	7.59	9.06	9.84	10.22	7.77	6.28	5.47	3.41
Policy Rate	0.07	0.32	0.95	1.48	1.79	0.47	1.53	0.98
CPI Inflation	0.82	1.88	1.98	1.75	1.51	0.85	0.83	0.51
Core CPI Inflation	0.42	0.64	0.65	0.67	0.72	0.41	0.51	0.34

Table C.9
DM Test Statistics for CAN Expenditure Growth.

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
GDP Growth								
DSGE-Reg vs DSGE-DFM	**3.8**	**2.5**	1.2	0.8	1.5	1.8	1.2	1.3
DSGE-DFM vs DSGE-DFM-RT 2	1.4	1.1	0.7	-0.6	1.3	0.9	0.1	0.5
DFM-RT 2 vs DSGE-DFM-RT 2	1.5	1.0	1.2	1.1	0.2	1.2	1.5	**2.6**
SEP vs DSGE-Reg	**-4.2**	**-2.4**	-1.2	0.8	-1.5	-1.7	0.1	-1.2
SEP vs DSGE-DFM	**-3.0**	-1.8	0.7	1.7	0.1	-1.1	1.2	0.2
SEP vs DSGE-DFM-RT 2	-1.6	**-2.2**	0.8	**2.1**	0.9	-0.6	1.4	1.3
Consumption Growth								
DSGE-Reg vs DSGE-DFM	-0.0	1.8	**5.3**	**2.8**	**5.5**	**3.4**	**3.7**	**2.9**
DSGE-DFM vs DSGE-DFM-RT 2	1.3	1.0	-1.1	-0.4	1.5	1.1	-0.1	-1.4
DFM-RT 2 vs DSGE-DFM-RT 2	1.2	1.2	1.6	0.6	0.5	1.1	1.6	**2.0**
SEP vs DSGE-Reg	1.5	0.9	-1.8	-1.5	-1.0	0.4	-1.4	-1.3
SEP vs DSGE-DFM	1.6	1.1	1.1	**6.4**	**2.0**	1.2	1.5	1.4
SEP vs DSGE-DFM-RT 2	1.7	1.1	1.0	**3.7**	**2.0**	1.2	1.5	1.3
Investment Growth								
DSGE-Reg vs DSGE-DFM	**2.3**	1.8	1.8	-0.4	-0.1	**2.7**	0.0	**2.2**
DSGE-DFM vs DSGE-DFM-RT 2	**2.2**	0.9	0.5	-0.1	0.2	1.1	-0.3	0.9
DFM-RT 2 vs DSGE-DFM-RT 2	**2.4**	1.4	**2.0**	0.8	0.4	1.7	0.8	1.0
SEP vs DSGE-Reg	-1.2	-1.1	-0.8	1.0	0.7	-1.1	0.9	-0.1
SEP vs DSGE-DFM	0.6	0.5	0.6	1.5	1.8	0.6	1.4	1.2
SEP vs DSGE-DFM-RT 2	1.8	1.1	1.3	1.3	1.4	1.2	1.2	1.3
Government Growth								
DSGE-Reg vs DSGE-DFM	1.5	0.9	1.0	-0.6	-1.4	1.1	-1.1	0.7
DSGE-DFM vs DSGE-DFM-RT 2	0.5	-1.5	-1.5	-0.3	1.1	-1.2	-0.5	-0.9
DFM-RT 2 vs DSGE-DFM-RT 2	1.2	-0.4	**-2.1**	-0.2	-0.7	-0.7	-0.6	-0.7
SEP vs DSGE-Reg	1.0	-0.7	-0.0	1.4	1.7	-0.2	1.4	0.7
SEP vs DSGE-DFM	1.4	-0.4	0.3	1.4	1.4	0.3	1.3	0.7
SEP vs DSGE-DFM-RT 2	1.8	-0.5	0.2	1.4	1.5	0.2	1.3	0.7

Note: The table shows Diebold-Mariano test statistic. Negative test statistics imply the RMSE are lower for the model on the left. Values in bold are significant at the 95% level for the model on the right and values in bold and red are significant at the 95% level for the model on the right

Table C.10
DM Test Statistics for CAN Trade Growth, CPI, and Policy Rate.

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
Export Growth								
DSGE-Reg vs DSGE-DFM	1.4	1.0	1.7	-0.5	-1.6	1.2	-0.6	1.2
DSGE-DFM vs DSGE-DFM-RT 2	1.7	1.4	1.4	0.3	-0.3	1.1	0.5	1.2
DFM-RT 2 vs DSGE-DFM-RT 2	1.3	**4.7**	1.1	1.2	0.9	1.5	1.8	**2.1**
SEP vs DSGE-Reg	-1.2	-1.2	-0.6	-0.8	1.5	-1.1	-1.2	-1.0
SEP vs DSGE-DFM	-0.9	-1.3	-0.5	-0.8	1.0	-1.0	-1.0	-0.9
SEP vs DSGE-DFM-RT 2	-0.4	-1.3	-0.4	-0.8	0.9	-0.9	-1.0	-0.8
Import Growth								
DSGE-Reg vs DSGE-DFM	0.6	-0.1	**2.9**	-0.3	1.4	-0.4	0.4	0.8
DSGE-DFM vs DSGE-DFM-RT 2	**2.5**	1.2	-0.4	1.3	-0.0	1.6	**10.9**	0.5
DFM-RT 2 vs DSGE-DFM-RT 2	**2.1**	1.5	1.5	**2.2**	-0.7	1.4	1.2	1.0
SEP vs DSGE-Reg	-1.1	-0.7	-1.0	1.5	-0.7	-1.1	0.7	-0.7
SEP vs DSGE-DFM	-0.9	-0.9	-0.8	1.4	0.9	-1.4	**5.3**	0.0
SEP vs DSGE-DFM-RT 2	0.1	-0.8	-0.8	1.8	1.7	-0.9	**3.1**	0.1
Policy Rate								
DSGE-Reg vs DSGE-DFM	-1.4	-0.2	0.4	0.8	1.0	0.1	1.0	0.6
DSGE-DFM vs DSGE-DFM-RT 2	**2.0**	1.6	1.5	1.4	1.4	1.5	1.5	1.4
DFM-RT 2 vs DSGE-DFM-RT 2	**2.9**	1.5	0.2	-0.2	-0.4	1.0	-0.3	0.0
SEP vs DSGE-Reg	**-2.4**	**-3.9**	**-2.0**	**-2.1**	**-2.0**	**-2.8**	**-2.4**	**-2.0**
SEP vs DSGE-DFM	**-2.0**	-1.5	-1.3	-1.0	-0.7	-1.4	-1.0	-1.1
SEP vs DSGE-DFM-RT 2	-0.8	-1.3	-0.9	-0.6	-0.2	-1.2	-0.5	-0.7
CPI Inflation								
DSGE-Reg vs DSGE-DFM	**2.3**	**2.5**	1.7	1.6	1.5	**2.0**	**2.0**	1.4
DSGE-DFM vs DSGE-DFM-RT 2	1.3	-0.3	1.2	1.4	1.3	1.3	1.4	1.3
DFM-RT 2 vs DSGE-DFM-RT 2	**2.4**	-0.7	-0.7	-1.0	-0.8	-0.2	-0.9	-0.4
SEP vs DSGE-Reg	**-3.6**	**-3.6**	**-2.2**	**-2.5**	**-2.7**	**-2.7**	**-3.3**	**-2.3**
SEP vs DSGE-DFM	-1.4	-0.4	-0.7	-0.7	-0.7	-1.3	-1.5	-1.2
SEP vs DSGE-DFM-RT 2	-1.5	-0.4	-0.2	-0.3	-0.6	-1.5	-1.4	-1.2

Note: The table shows Diebold-Mariano test statistic. Negative test statistics imply the RMSE are lower for the model on the left. Values in bold are significant at the 95% level for the model on the right and values in bold and red are significant at the 95% level for the model on the right

Table C.11
Relative RMSE for CAN Macroeconomic Variables.

	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 6$	$\tau = 8$	1^{st} year	2^{nd} year	2 year Avg
GDP Growth								
DSGE-Reg	3.31	2.48	1.29	0.94	1.11	2.55	**0.98**	1.96
DSGE-Reg-Shadow	3.41	2.30	1.24	**0.99**	1.18	2.31	1.08	1.73
DSGE-DFM-NoFin	1.81	1.36	1.03	**0.93**	1.02	1.15	**0.93**	1.07
DSGE-DFM	1.62	1.46	**0.99**	**0.91**	1.00	1.17	**0.88**	**0.98**
DSGE-DFM-RT 2	1.34	1.21	**0.94**	**0.92**	**0.97**	1.03	**0.88**	**0.95**
Consumption Growth								
DSGE-Reg	**0.53**	**0.56**	1.24	1.41	1.28	**0.84**	1.50	1.65
DSGE-Reg-Shadow	**0.53**	**0.58**	1.33	1.45	1.33	**0.86**	1.54	1.68
DSGE-DFM-NoFin	**0.53**	**0.50**	**0.90**	**0.82**	**0.60**	**0.58**	**0.59**	**0.62**
DSGE-DFM	**0.53**	**0.48**	**0.84**	**0.78**	**0.65**	**0.54**	**0.57**	**0.48**
DSGE-DFM-RT 2	**0.46**	**0.44**	**0.87**	**0.79**	**0.64**	**0.49**	**0.57**	**0.52**
Investment Growth								
DSGE-Reg	1.15	1.17	1.08	**0.93**	**0.94**	1.18	**0.83**	1.02
DSGE-Reg-Shadow	1.12	1.13	1.08	**0.93**	**0.93**	1.15	**0.82**	**0.96**
DSGE-DFM-NoFin	1.02	**0.93**	**0.95**	**0.92**	**0.93**	**0.83**	**0.80**	**0.68**
DSGE-DFM	**0.94**	**0.97**	**0.97**	**0.94**	**0.95**	**0.94**	**0.82**	**0.80**
DSGE-DFM-RT 2	**0.69**	**0.89**	**0.96**	**0.95**	**0.94**	**0.83**	**0.84**	**0.76**
Government Growth								
DSGE-Reg	**0.82**	1.16	1.01	**0.66**	**0.78**	1.05	**0.63**	**0.81**
DSGE-Reg-Shadow	**0.85**	1.24	1.04	**0.68**	**0.81**	1.14	**0.67**	**0.89**
DSGE-DFM-NoFin	**0.73**	1.05	**0.94**	**0.65**	**0.78**	**0.88**	**0.62**	**0.73**
DSGE-DFM	**0.73**	1.07	**0.96**	**0.67**	**0.82**	**0.91**	**0.66**	**0.78**
DSGE-DFM-RT 2	**0.70**	1.08	**0.97**	**0.67**	**0.81**	**0.93**	**0.66**	**0.80**
Export Growth								
DSGE-Reg	1.41	1.34	1.05	1.04	**0.93**	1.32	1.11	1.32
DSGE-Reg-Shadow	1.44	1.37	1.07	1.05	**0.93**	1.39	1.14	1.46
DSGE-DFM-NoFin	1.14	1.28	1.05	1.05	**0.94**	1.24	1.13	1.34
DSGE-DFM	1.20	1.26	1.05	1.05	**0.95**	1.20	1.13	1.30
DSGE-DFM-RT 2	**0.93**	**0.99**	**0.87**	**0.85**	**0.96**	**0.94**	**0.88**	1.05
Import Growth								
DSGE-Reg	1.32	1.16	1.13	**0.96**	1.05	1.13	**0.93**	1.10
DSGE-Reg-Shadow	1.33	1.19	1.18	1.01	1.09	1.17	1.09	1.30
DSGE-DFM-NoFin	1.27	1.13	1.07	**0.98**	**0.93**	1.07	**0.88**	**0.89**
DSGE-DFM	1.27	1.17	1.10	**0.97**	**0.95**	1.15	**0.89**	**0.99**
DSGE-DFM-RT 2	**0.77**	**0.92**	**0.88**	**0.71**	**0.96**	**0.91**	**0.82**	**0.92**
Policy Rate								
DSGE-Reg	6.14	2.66	1.79	1.72	1.71	2.19	1.72	1.83
DSGE-Reg-Shadow	8.07	3.89	2.81	2.63	2.64	3.37	2.66	2.85
DSGE-DFM-NoFin	7.64	2.66	1.40	1.18	1.05	1.97	1.14	1.33
DSGE-DFM	7.76	2.80	1.55	1.34	1.20	2.12	1.30	1.48
DSGE-DFM-RT 2	1.26	1.69	1.23	1.12	1.05	1.36	1.10	1.15
CPI Inflation								
DSGE-Reg	3.09	1.77	1.90	2.17	2.49	3.45	4.45	6.24
DSGE-Reg-Shadow	3.36	2.03	2.37	2.87	3.49	4.22	6.06	8.28
DSGE-DFM-NoFin	2.28	1.13	1.06	1.12	1.15	1.70	1.98	2.71
DSGE-DFM	2.31	1.11	1.09	1.19	1.33	1.69	2.25	2.90
DSGE-DFM-RT 2	1.33	1.12	1.02	1.06	1.24	1.46	1.99	2.40

Note: The table shows the ratios of the RMSE from a given model in comparison to the SEP forecasts for a given variable. Values below one indicate that forecasts from the model are on average more accurate than the SEP.

Table C.12
DM Test Statistics for the real and nominal exchange rates.

	$\tau = 1$	$\tau = 2$	$\tau = 3$	$\tau = 4$	$\tau = 6$	$\tau = 8$
Real Exchange Rate						
RW vs VAR(n)	-1.3	**-3.1**	**-2.4**	**-2.8**	**-2.7**	**-2.9**
RW vs DFM	-1.2	**-2.0**	**-5.1**	**-3.3**	-1.6	-1.6
RW vs DFM-RT 2	**2.8**	**2.2**	**2.0**	0.9	-0.9	-0.9
RW vs DSGE-Reg	-0.3	0.1	0.5	0.7	0.8	0.9
RW vs DSGE-Reg-AR1	-0.8	-0.3	0.0	0.3	0.3	0.1
RW vs DSGE-DFM	-0.5	-0.4	0.0	0.2	-0.1	-0.4
RW vs DSGE-DFM-RT 2	**2.8**	**2.5**	**3.2**	**7.1**	**5.3**	0.3
Modeled Nominal Exchange Rate						
RW vs VAR(n)	-1.8	**-3.4**	**-3.1**	**-3.3**	**-3.1**	**-3.0**
RW vs DFM	-1.6	**-2.2**	**-3.3**	**-2.9**	**-2.2**	**-2.1**
RW vs DFM-RT 2	**2.7**	**2.0**	1.1	-0.6	**-2.4**	**-2.0**
RW vs DSGE-Reg	-0.4	-0.0	0.2	0.4	0.4	0.4
RW vs DSGE-Reg-AR1	-1.2	-0.7	-0.5	-0.3	-0.4	-0.5
RW vs DSGE-DFM	-0.2	-0.1	0.0	0.1	-0.2	-0.6
RW vs DSGE-DFM-RT 2	**3.0**	**2.4**	**2.7**	1.0	0.2	-1.0
USD/CAD						
RW vs VAR(n)	-1.4	**-3.0**	**-2.7**	**-3.0**	**-2.8**	**-3.1**
RW vs DFM	-1.4	-1.9	**-3.1**	**-2.6**	-1.7	-1.7
RW vs DFM-RT 2	**2.6**	**2.2**	1.9	0.8	-1.0	-1.0
RW vs DSGE-Reg	-0.3	0.1	0.4	0.7	0.8	0.9
RW vs DSGE-Reg-AR1	-1.1	-0.6	-0.3	-0.1	-0.2	-0.3
RW vs DSGE-DFM	-0.3	-0.2	-0.0	0.0	-0.2	-0.6
RW vs DSGE-DFM-RT 2	**2.6**	**2.4**	**2.7**	**2.4**	0.6	-0.2
RW vs SEP	**2.6**	**2.6**	**2.2**	1.1	-0.1	0.0

Note: The table shows Diebold-Mariano test statistic. Negative test statistics imply the RMSE are lower for the RW model. Values in bold are significant at the 95% level, while values is red and bold are significant at the 95% level for the RW model.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.jedc.2021.104177

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