THE
ECONOMIC
JOURNAL

The Economic Journal, **123** (*February*), F4–F30. Doi: 10.1111/ecoj.12010 © 2013 The Author(s). The Economic Journal © 2013 Royal Economic Society. Published by Blackwell Publishing, 9600 Garsington Road, Oxford OX4 2DQ, UK and 350 Main Street, Malden, MA 02148, USA.

FISCAL FATIGUE, FISCAL SPACE AND DEBT SUSTAINABILITY IN ADVANCED ECONOMIES*

Atish R. Ghosh, Jun I. Kim, Enrique G. Mendoza, Jonathan D. Ostry and Mahvash S. Qureshi

How high can public debt rise without compromising fiscal solvency? We answer this question using a stochastic model of sovereign default in which risk-neutral investors lend to a government that displays 'fiscal fatigue', whereby its ability to increase primary balances cannot keep pace with rising debt. As a result, the government faces an endogenous debt limit beyond which debt cannot be rolled over. Using data for 23 advanced economies over the period 1970–2007, we find evidence of a fiscal reaction function with these features, and use it to compute 'fiscal space', defined as the difference between current debt ratios and the estimated debt limits.

A critical question confronting the world economy today is how much fiscal adjustment advanced economies need to restore public debt sustainability and regain some room for fiscal manoeuvre – 'fiscal space'. Financial bailouts, stimulus spending and lower revenues in the Great Recession have all contributed to produce some of the highest ratios to GDP of public debt and primary deficits seen in advanced economies in the past 40 years (Figure 1). But, while the financial crisis clearly worsened the public debt positions of most advanced economies, the rise in debt ratios was not especially dramatic when viewed against the infinite-horizon intertemporal budget constraint. Nevertheless, in a number of cases such as Greece, Ireland, Italy, Portugal and Spain, borrowing costs have increased sharply, causing these sovereigns severe financing difficulties, and undermining their solvency.[1] Understanding why financing costs shoot up rapidly from essentially risk-free rates to prohibitively costly interest rates, and why previously sustainable debt ratios can suddenly look unsustainable, is crucial to both returning these countries to fiscal solvency and to providing early warning to others at risk of exhausting their fiscal space.

In this article, we develop a new framework for assessing debt sustainability in advanced economies. Specifically, we seek to determine a 'debt limit' beyond which fiscal solvency is in doubt (with 'fiscal space' defined as the distance between the current debt level and this limit). Our model incorporates a sovereign borrower whose behaviour can be represented in reduced form by a fiscal reaction function that describes how the primary balance responds to changes in debt and risk-neutral

* Corresponding author: Atish R. Ghosh, IMF Research Department, HQ1-09-612, 700 19th Street NW, Washington, DC 20431, USA. Email: aghosh@imf.org

We thank Olivier Blanchard, Henning Bohn, David Romer, two anonymous referees and the editor, Wouter den Hann, as well as participants at the IMF's 11th Annual Jacques Polak Research Conference, the Econometric Society Australasian Meeting 2011 and Stockholm School of Economics Research Seminar Series, for helpful comments and suggestions. Any errors are the authors' responsibility. The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board or its management.

[1] Between end-2008 and end-2009, public debt ratios in Greece, Ireland, Italy, Portugal and Spain rose by an average of 14% of GDP to reach 89% of GDP, while bond yields rose by an average of only two basis points (bp) – with the spread to Germany actually declining from 135 bps to 128 bps. Between end-2009 and end-2010, public debt ratios in these countries rose by a further 13% of GDP, while yields rose almost 300 bps, and the spread to Germany widened to 448 bps.

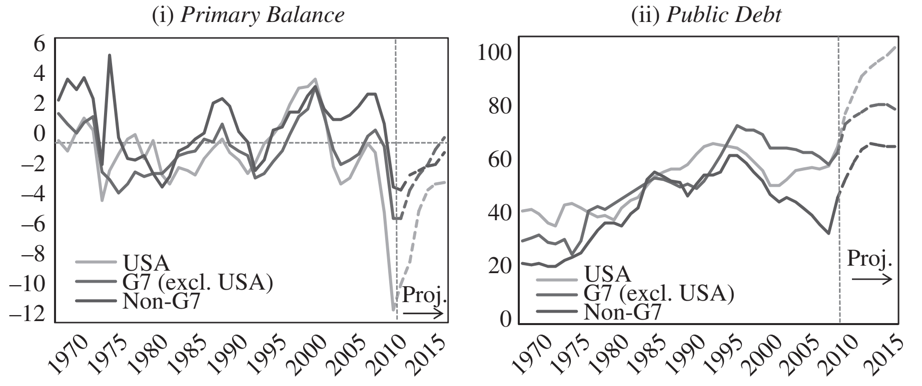

Fig. 1. *Primary Balance and Public Debt of Advanced Economies*, 1970–2015 (% of GDP)
Notes. Median value for the corresponding year: G7 comprises: Canada, France, Germany, Italy, Japan, UK and US; Non-G7 advanced countries in the sample include: Australia, Austria, Belgium, Denmark, Finland, Greece, Korea, Iceland, Israel, Netherlands, New Zealand, Norway, Portuga, Spain and Sweden.
Source: IMF's WEO database.

creditors who arbitrage the expected return on government debt with the safe interest rate, taking account of the possibility that the government may default because it is unable to repay.

Our analysis begins from the premise that governments in advanced economies usually behave responsibly, increasing primary (i.e. non-interest) surpluses in response to rising debt service so as to stabilise the public debt-to-GDP ratio at a reasonable level. This is an empirically relevant premise consistent with the findings of Bohn (2008) for the US, and Mendoza and Ostry (2008) for subsets of industrial and emerging economies. Large shocks – such as wars or the fiscal fallout of financial crises – may cause temporary deviations from this (implicit or explicit) primary balance 'rule'. As long as the subsequent increase in the primary balance is sufficient to offset the higher interest bill, however, the debt ratio will again converge to its long-run value.

Of course, it cannot literally be true that the primary balance would always increase enough to offset the interest bill, because, at sufficiently high levels of debt, this would require primary balances that exceed GDP.[2] If the primary balance (eventually) exhibits 'fiscal fatigue' such that it does not keep pace with higher interest payments as debt rises, then – even assuming a constant interest rate so as to abstract from the endogeneity of the risk premium on government debt – there will be a debt level above which the debt dynamics become explosive and the government will necessarily default. In fact, default will occur before this point because the rising risk premium, as default becomes imminent, exacerbates the debt dynamics. In particular, as the probability of default rises, so will the risk premium, making it less likely that the primary surplus will suffice to meet the interest bill and

[2] Empirically, we find below that while fiscal effort is generally increasing in the debt level, it eventually peters out as it becomes increasingly difficult to keep raising taxes or cutting non-interest expenditures.

raising the probability of default further.[3] Eventually, the 'fixed point' problem of a higher probability of default leading to a larger risk premium, in turn leading to a higher probability of default, has no solution at a finite interest rate. At this point (which we term the debt limit), the government loses market access, is unable to rollover its debt and is forced to default. To determine this point, we solve simultaneously for the probability of default, the interest rate faced by the sovereign and the debt limit.

Our theoretical framework, motivated by Bohn (1998, 2008), thus departs fundamentally from earlier work in pinning down the concept of debt limit. While Bohn shows that a sufficient condition for the government to satisfy its intertemporal budget constraint is that the primary balance always reacts positively to lagged debt, this can be thought of as a weak sustainability criterion that does not, for example, rule out an ever-increasing debt-to-GDP ratio (and thus the need for a primary surplus that eventually exceeds GDP).[4] A stricter sustainability criterion, which we adopt here, is that public debt should be expected to converge to some finite proportion of GDP. If the primary balance is always a constant proportion of lagged debt, then a sufficient condition for this stricter definition is that the responsiveness of the primary balance be greater than the interest rate–growth rate differential.[5] But, once we allow for the possibility of 'fiscal fatigue' whereby the primary balance eventually responds more slowly to rising debt than the interest rate–growth rate differential, there will in general be a finite debt limit. Moreover, as debt approaches this debt limit, the cost of financing will shoot up from the risk-free rate to a prohibitively high interest rate within a very narrow range of debt ratios. The model also allows us to analyse the effects of unanticipated fiscal shocks (such as Greece's large revisions to its primary balance), which lower the debt limit, making a previously sustainable level of debt unsustainable.

Applying our framework empirically to a sample of 23 advanced economies over the period 1970–2007, we find strong support for the existence of a non-linear reduced-form relationship between the primary balance and (lagged) public debt that exhibits the fiscal fatigue characteristic. Specifically, the relationship is well approximated by a cubic function: at low levels of debt there is no, or even a slightly negative, relationship between the primary balance and debt. As debt increases, the primary balance also increases but the responsiveness eventually weakens and then actually decreases at very

[3] There is a large body of literature on sovereign default, although most of these papers consider strategic default by developing seven country governments (see the survey by Eaton and Fernandez (1995) and the recent quantitative studies along the lines of Arellano (2008) and Aguiar and Gopinath (2006)). Here, we model default, not as an explicit strategic decision by the government, but rather as a result of the government's inability to rollover debt in the face of rising interest payments, given its primary balance reaction function (Cottarelli *et al.*, 2010).

[4] Bohn (2007) shows that satisfying the government budget constraint requires only that there is some (arbitrarily high but finite) degree of differencing at which the time series of the debt-to-GDP ratio becomes stationary – which is always satisfied in the data. For this reason, he concludes that this sustainability criterion (and the associated stationarity tests on which it rests) is uninteresting, and suggests that examining the behavioural response of the primary balance may be a more fruitful way of establishing debt sustainability; this is the tack taken here.

[5] Intuitively, the debt-to-GDP ratio grows autonomously at a rate given by the interest rate–output growth rate differential; if the response of the primary balance to rising debt is stronger than this differential, then the primary adjustment will offset the autonomous dynamics and the debt ratio will converge to a finite ratio.

high levels of debt. This relationship is robust to the addition of a long list of conditioning variables and to a variety of estimation techniques.[6]

Combining the empirical estimates of the primary balance reaction function with actual interest rate data or with endogenous interest rates obtained from the model, we gauge each country's debt limit and corresponding fiscal space. Our results indicate a precarious fiscal situation for several eurozone periphery countries – notably for Greece, Italy and Portugal, with Ireland and Spain also constrained in their available fiscal space. To the extent that our primary balance reaction function is based on pre-global financial crisis data, our estimates provide early warning for the recent fiscal troubles encountered by these countries, and as such, the financing difficulties that they have been experiencing are consistent with our empirical analysis. Among other countries, Japan has in all likelihood exhausted its fiscal space, whereas Iceland, the US and the UK are also constrained in their degree of fiscal manoeuvre. By contrast, Australia, Korea, New Zealand and the Nordic countries appear to have the most fiscal space to deal with unexpected shocks.

Our contribution to the literature is thus three-fold. First, we provide a simple, intuitive definition of a debt limit (and corresponding fiscal space) that has the reasonable properties of increasing in the country's average fiscal effort and decreasing in the interest rate-growth rate differential. Second, our model can help explain why governments can suddenly lose financing access and why revisions to market estimates about the size of possible shocks to the primary balance (even if the shocks are not realised) can abruptly push a country into a situation of unsustainable debt dynamics. Third, we provide empirical estimates of available fiscal space for advanced economies, which provide early warning for them. Moreover, our approach can quantify the extent to which policy and institutional changes could help to increase the fiscal space of these countries.

In what follows, Section 1 develops the theoretical framework and derives the debt limit for a deterministic example, with the general stochastic case covered in Appendix A. Section 2 presents estimation results for the fiscal reaction function, and reports fiscal space estimates for our sample of countries. Section 3 concludes.

1. Theoretical Framework

1.1. *Sovereign Debtor and Creditors*

We consider a credit relationship between a sovereign borrower and a large number of atomistic lenders. The government budget constraint is standard:

$$d_{t+1} - d_t = (r_t - g)d_t - s_{t+1}, \tag{1}$$

where d is one-period debt (as a share of GDP) at the end of the period, g is the growth rate of real GDP, which is assumed to be exogenous and constant, s is the primary balance

[6] While we believe this article is the first to explore the debt sustainability implications of a non-linear response of primary balance to rising debt, we are not the first to find a non-linear response. For example, using a quadratic specification for the US (over 1916–95 and 1792–2003), Bohn (1998, 2008) considers how fiscal behaviour varies with debt levels, but finds that the primary surplus is more responsive to increases in debt at higher debt levels. By contrast, Abiad and Ostry (2005) and Mendoza and Ostry (2008), who look at international evidence (and a more recent sample), find that the response of primary balance to debt weakens at higher debt levels.

(as a percentage of GDP), and r is the real interest rate on debt contracted in period t and due in period $t + 1$.[7] The interest rate is endogenous and thus greater than or equal to the risk-free interest rate, r^*, which we assume to be exogenously given (in equilibrium, the interest rate will be an increasing function of the probability of default). We make three assumptions regarding the behaviour of the sovereign debtor and its creditors.

1.1.1. *Assumption I. Fiscal reaction function under fiscal fatigue*

Drawing on Bohn's approach to analyse fiscal sustainability, we postulate the following reduced form for the government's fiscal reaction function (i.e. the response of the primary balance to lagged public debt):

$$s_{t+1} = \mu + f(d_t) + \varepsilon_{t+1} \tag{2}$$

where μ captures all systematic determinants of the primary balance other than lagged debt (these can include the standard cyclical effects from output or government outlays but also fiscal effects of changes in inflation, real exchange rates, external deficits etc.).[8] $f(d)$ is the response of the primary balance to lagged debt, which is a continuous function with the properties defined below, and ε is an independent and identically distributed (i.i.d.) shock to the primary balance with the distribution function $G(\varepsilon)$ defined over the finite support $[-\bar{\varepsilon}, \bar{\varepsilon}]$ with $\bar{\varepsilon} > 0$. We assume that $G(\varepsilon)$ satisfies standard properties such that $G'(-\varepsilon) = G'(\varepsilon) \leq G'(0)$, $G''(\varepsilon < 0) \geq 0$ and $G''(\varepsilon > 0) \leq 0$. Assuming a finite support is plausible since the primary deficit or surplus cannot exceed 100% of GDP (and, in practice, seldom exceeds more than a few percentage points of GDP).

We deviate from Bohn's standard treatment of the fiscal reaction function as a linear function to capture the idea of fiscal fatigue. Accordingly, the function $f(d)$ is assumed to be continuously differentiable and have the property that there exists a debt ratio $d^m > \bar{\varepsilon}$ such that:

$$\mu + f(d^m) - \bar{\varepsilon} \geq (r^* - g)d^m \text{ and } f'(d) < r^* - g \quad \forall d > d^m \tag{3}$$

Hence, at d^m and with the worst primary balance shock, debt is non-increasing if $r = r^*$ (which indeed turns out to be the case in equilibrium at d^m) and, for any higher debt ratio, the response of the primary balance is lower than the growth-adjusted interest rate.

[7] While the analysis could be enriched by allowing the output growth rate to be a function of the level of debt and/or the real interest rate, $g = g(r, d)$, this complicates the theoretical analysis (because of the potentially non-linear effect of the feedback between debt, interest rates and growth in the model's equilibrium) but the empirical evidence on the relationship between debt/sovereign borrowing costs and long-run growth is mixed at best. For example, Reinhart and Rogoff (2010) and Checherita and Rother (2010) find that higher debt affects growth negatively but Schclarek (2004) and Panizza and Presbitero (2012) find no relationship between debt and growth in advanced economies. Likewise, the evidence on the effect of fiscal contractions (larger primary surpluses) on growth is ambiguous: IMF (2010) argues that fiscal contractions reduce growth, while Rzonca and Cizkowicz (2005) and Alesina and Ardagna (2010) find evidence that fiscal adjustment is expansionary.

[8] As in Bohn (2008), this approach to solvency analysis only requires the period-by-period government budget constraint in (1) and a reduced-form fiscal reaction function like (2) that splits the determinants of the primary balance into its response to lagged debt and 'the rest'. It does not require further structural treatment of the determinants of government revenues or outlays, or of the fiscal implications of features like the currency composition of the public debt, seigniorage revenue, or external deficits, other than those captured in μ or in $f(d)$. This does not mean that these features are irrelevant for debt sustainability, only that they are already captured in the reduced form reaction function. The drawback, of course, is that one cannot gauge the particular contributions of each of these various mechanisms in driving the behaviour summarised by $f(d)$.

1.1.2. *Assumption II. Default rule*

The government defaults if and only if debt exceeds the debt limit, $\bar{d}$, where the latter is defined as the maximum debt level at which the government can rollover its maturing debt and finance the primary deficit at a finite interest rate. The default rule of the sovereign thus takes the form:

$$D_{t+1} = \begin{cases} 1 & \text{if } d_{t+1} > \bar{d} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where D is an indicator equal to one if the government defaults and zero otherwise.[9]

1.1.3. *Assumption III. Risk-neutral creditors*

Creditors are atomistic and risk neutral, lending to the government under the following assumptions:

(*i*) There is less than unit probability that government debt (as a proportion of GDP) is on an explosive path (i.e. increasing without bound regardless of all future realisations of ε).

(*ii*) There exists a finite interest rate that compensates risk-neutral lenders for the (endogenous) risk of default:

$$1 + r^* = (1 - p_{t+1})(1 + r_t) + p_{t+1}\theta(1 + r^*) \tag{5}$$

where p_{t+1} is the probability of default in the next period (when current debt matures) and θ is an assumed recovery value in the event of default.[10] This arbitrage condition implies the standard result from sovereign default models that, as long as the default probability is positive but less than unity, the default risk premium is a positive, increasing and convex function of default probability.[11]

(*iii*) If there are multiple interest rates that satisfy the above arbitrage condition, then creditors are assumed to choose the lowest such interest rate.[12]

1.2. *Derivation of the Debt Limit*

The rational expectations equilibrium of the model is defined by sequences of interest rates and public debt such that the government satisfies its budget constraint, behaves

[9] In principle, the debt limit could vary over time with the government's ability-to-pay. The model could be readily modified to incorporate time varying ability-to-pay as long as μ is non-stochastic.

[10] For given p_{t+1}, there are infinitely many combinations of θ and r_t that satisfy the arbitrage condition. We therefore fix the recovery value, θ, exogenously. The permissible range of the recovery value needs to be bounded above at a level 'sufficiently' less than unity to ensure that the government has the ability to pay the recovery value in the event of default. A convenient and sufficient upper bound is to assume that $0 \leq \theta \leq \bar{\theta} = 1 - \bar{\varepsilon}/(1 + r^*)d^m$. This assumption ensures that the debt limit under uncertainty will lie below the debt limit under no uncertainty (see Appendix A) but still allows for large recovery values. For example, $\bar{\theta}$ is above 90% if $\bar{\varepsilon}$ and d^m are 5% and 60% of GDP respectively.

[11] Equation (5) can also be used to consider a situation of partial default through inflation. Specifically, recovery values of zero and 100% give the two extreme cases of default with no repayment, and effectively no default, respectively. When the recovery value is between zero and 100%, the complement of the recovery value (i.e. the fraction not recovered, $1 - \theta$) can be viewed as the fraction partially defaulted upon by eroding the real value of the debt through surprise inflation.

[12] In the case of two interior solutions, the higher one is not sensible in that the interest rate (and the associated default probability) falls, not rises, as debt increases. See Appendix A.

according to the reduced-form reaction function and its default rule and lenders satisfy their arbitrage condition. Assumptions I–III are sufficient to guarantee the existence of a finite debt limit, $\bar{d}$, such that if debt were to ever exceed that limit, it would tend to rise without bound; consequently, creditors would cut off access to the market, the interest rate would become infinite and the government would necessarily default. Here, we present a heuristic treatment of a simple deterministic example in which there are no shocks to the primary balance; Appendix A considers the more general stochastic case in which there are shocks to the primary balance, proves the existence of the debt limit and derives its properties.

What makes the deterministic case particularly easy to solve is that the market will lend at the risk-free rate until the debt limit is reached, at which point it will charge an infinite interest rate. As the debt limit is the maximum debt at which the government can just honour its obligations, it must be the largest root of the following equation:

$$\mu + f(\bar{d}) = (r^* - g)\bar{d}. \tag{6}$$

While both sides of (6) are increasing in d, by the assumption in (3), the left-hand side rises more slowly than the right-hand side for $d > d^m$. Hence, beyond $\bar{d}$, the primary balance is never sufficient to meet the interest payment, debt grows continuously and the government necessarily defaults, which is the definition of the debt limit.

For this deterministic example, the equilibrium level of debt, the debt limit and the interest payment schedule can all be illustrated by means of a diagram. In Figure 2, the thick black curve represents the non-linear fiscal reaction function, which satisfies the properties defined in Assumption I, except that there is no stochastic shock to the primary balance (such shocks would imply upward or downward shifts of the curve). The interest payment schedule is a straight line with slope given by $(r^* - g)$ as long as the market lends at the risk-less rate, becoming vertical thereafter when the government is shut out of the market. In general, there will be two stationary equilibria (ignoring the intersection that would occur at $d < 0$). The first is given by the lower intersection between $\mu + f\ (d)$ and the $(r^* - g)d$ schedule. This lower intersection, denoted d^*, is the long-run public debt ratio to which the economy converges conditionally (as long as debt does not cross the limit $\bar{d}$). It is readily verified that this equilibrium is dynamically stable.[13]

The second stationary equilibrium corresponds to the debt limit implied by the model, and is given by the higher intersection between $\mu + f(d)$ and the $(r^* - g)d$ schedule. Thus, convergence to d^* is only conditional, because if debt were to ever exceed the higher equilibrium, then it would not return to d^*. If debt were to exceed this limit, there is no finite interest rate that would compensate creditors for the probability of default (which is unity) because, beyond the debt limit, the primary balance is never sufficient to cover the interest payment (even at the risk-free interest rate) so the debt ratio would increase without bound. Recognising this, creditors

[13] For comparison, Bohn's standard linear reaction function with positive response coefficient has a unique, stable stationary equilibrium if the response coefficient exceeds the growth-adjusted real interest rate. If the response coefficient is smaller, solvency is still maintained but with 'mildly explosive' paths for debt and deficit (see Proposition 2 in Bohn (2008)). Extensions to include quadratic terms would have either a single stable steady state (if the quadratic term is non-negative) or one stable and one unstable steady state (if the quadratic term is negative).

demand an infinite interest rate beyond the debt limit – effectively shutting the government out of the market.

The general stochastic case, as discussed in Appendix A, is rather more complicated because of the joint endogeneity of the risk premium and the default probability. The market will lend at the risk-free rate up to some point $\hat{d}$, charging a risk premium that reflects the (now, non-zero) probability of default thereafter. The higher interest rate, however, makes it less likely that, given the stochastic shock, the primary surplus will be sufficient to meet the interest payment, raising the likelihood of default. The higher default probability in turn raises the risk premium; eventually, this 'fixed point' problem has no solution at a finite interest rate – and that is the point that defines the debt limit.

In Figure 2, the stochastic case is depicted by an upward bending, convex interest payment schedule beyond $\hat{d}$ that becomes vertical as debt exceeds its limit $\bar{d}$.[14] It can be shown that the debt limit in the stochastic case ($\bar{\varepsilon} > 0$) is necessarily lower than the debt limit in the corresponding deterministic case ($\bar{\varepsilon} = 0$). Finally, it is noteworthy that the convex segment of the interest rate schedule occurs over a short interval less than the support of the shock $\bar{\varepsilon}$. This result obtains because, with one-period debt, the probability of default is zero until debt is within the worst-possible overall deficit from

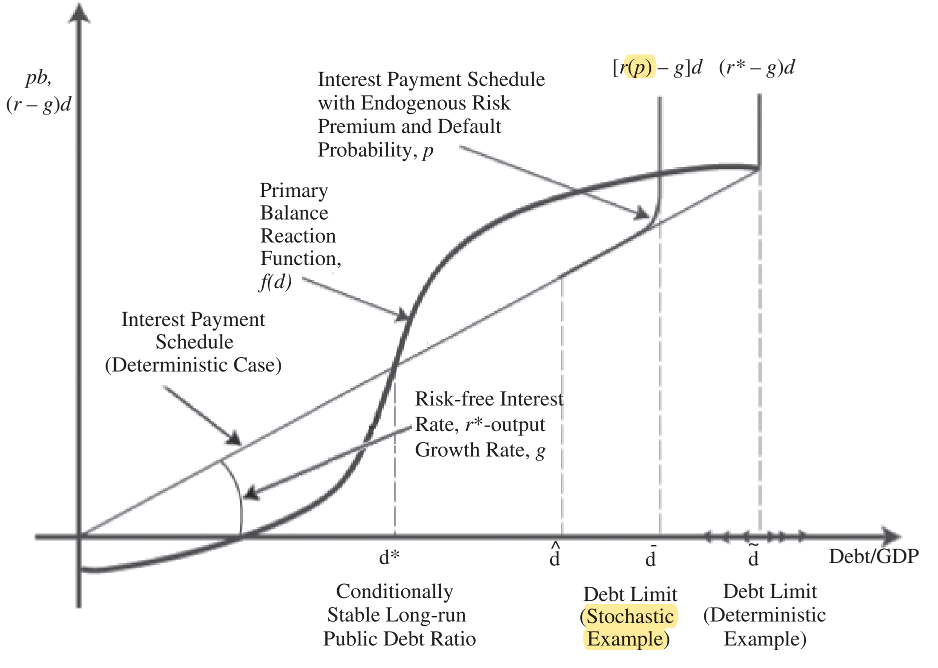

Fig. 2. *Determination of Debt Limit in Deterministic Example*

[14] In the general stochastic case, at the debt limit, the fiscal reaction function (abstracting from the stochastic shock) could lie above, at or below the interest payment schedule (which is yet to become vertical), depending on the precise shape of the fiscal reaction function, the support of the shock and the recovery rate in the event of default. For most parameter values (and all empirically relevant cases considered below), it lies above the interest payment schedule (as drawn in Figure 2). In the deterministic case, the point at which the interest payment schedule becomes vertical is necessarily where the risk-free interest payment schedule intersects the fiscal reaction function.

the debt limit. As $\hat{d}$ lies in the region where the overall balance is in surplus if there were no shock to the primary balance, the worst-possible overall deficit is less than the support of the shock. While the relationship is more complicated with multi-period debt, this result suggests that debt levels initially considered safe could suddenly become unsustainable particularly if $\bar{\varepsilon}$ is small. In general, the precise shape of the interest rate schedule depends on the distribution of the shock to the primary balance, the recovery rate in the event of default and the curvature of the primary balance reaction function.

1.3. *Properties of the Debt Limit*

The debt limit obtained above has four important properties (here we give a heuristic treatment based on Figure 2; Appendix A provides a more formal treatment). First, from Figure 2, a decrease in the economy's output growth rate or an increase in the risk-free interest rate rotates the $(r^* - g)d$ schedule counterclockwise, reducing $\bar{d}$ (while raising the long-run debt ratio to which the economy conditionally converges, d^*). Second, greater willingness to undertake fiscal adjustment (an upward shift of the intercept of the fiscal reaction function or the steepening of this schedule as a function of lagged debt) increases the debt limit (while reducing d^*). Third, the equilibrium at the debt limit is dynamically unstable: a positive (and even moderately negative) shock to the primary balance at that point would likely bring the debt back towards the long-run debt ratio, d^*; a (large) negative shock will push the government to default. Fourth, a re-evaluation of the support of shocks to the primary balance (e.g. a symmetric widening of the support $\bar{\varepsilon}$) leads to an immediate steepening of the interest rate schedule and a lowering of the debt limit, even if the shock is not actually realised (i.e. if the market learns that the shocks to the primary balance could be larger than previously believed, then this itself would be sufficient to lower the debt limit).[15] Although greater uncertainty implies both downside risk and upside potential, given the nature of a debt contract, upside potential cannot fully offset downside risk. Large positive shocks to the primary balance yield no additional return to creditors (because the interest rate is predetermined), whereas large negative shocks can cause default. Therefore, downside risk matters more than upside potential even if creditors are risk neutral (*a fortiori* if they are risk averse). If debt were close to $\bar{d}$, this could result in a formerly sustainable level of debt becoming unsustainable, triggering default. In this model, therefore, a data revision that leads to higher debt being reported to the market could have two effects – in Figure 2, a rightward jump of the debt level reflecting the new debt that is being recognised and, if this triggered a revision in market perceptions about the support of the shocks to the primary balance,

[15] The exception to this is when, in the stochastic case, the interest payment schedule lies above the fiscal reaction function when it becomes vertical, which only occurs if the recovery value is sufficiently high. In this case, at the debt limit, the interest payment is greater than the expected primary balance and, assuming $G(\varepsilon)$ is symmetric, the probability of default exceeds 0.5. This is akin to 'gambling for redemption': default can be avoided only if a positive shock occurs. As any negative shock, regardless of size, would result in default while only a sufficiently large positive shock helps avoid default, a mean preserving increase in the support of ε would (in this case) make default less likely and raise the debt limit (see Appendix A); however, as noted above, for most parameter values, this case does not obtain.

a counter-clockwise rotation of the interest rate schedule. Both effects would bring the government closer to default – a phenomenon that resonates with the recent experience of some European countries.

2. Empirical Implementation

This Section applies the theoretical framework developed above to a sample of 23 advanced economies using data for the period 1970–2007. The data includes gross public debt and the primary fiscal balance as shares of GDP as well as a set of control variables discussed below; data details are provided in online Appendix B.[16] We proceed in three steps:

(*i*) estimate the primary balance reaction function;
(*ii*) determine the appropriate interest rate–growth rate differential; and
(*iii*) calculate each country's debt limit, and associated fiscal space (defined as the difference between current debt ratios and the computed debt limit).

2.1. *Fiscal Reaction Function*

A first question is whether the 'fiscal fatigue' behaviour postulated above holds in the simple bivariate relationship between the primary balance and lagged debt. The scatter plot in Figure 3 (drawn as the sample average or median primary balance for a given range of debt, 0–10% of GDP, 11–20% of GDP, etc.) suggests that the postulated non-linear fiscal fatigue behaviour is indeed plausible: at very low debt ratios, there is

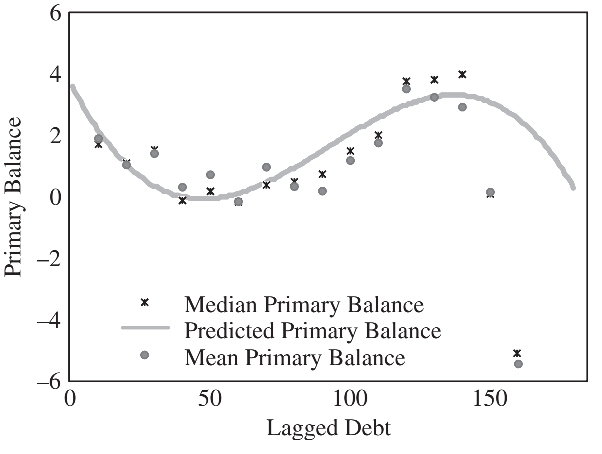

Fig. 3. *Primary Balance and Public Debt,* 1970–2007 (% of GDP)
Notes: Mean and median primary balance computed for 23 advanced economies over the specified debt range. Fitted values obtained after controlling for output gap and government expenditure gap.
Source: IMF's WEO database and authors' estimates.

[16] We use gross rather than net debt as the latter is not always comparable across countries due to different definitions or treatment of asset components, particularly, those of the social security system.

little (or even a slightly negative) relationship between lagged debt and the primary balance. As debt increases, the primary balance rises but the responsiveness eventually begins to weaken and then actually decreases at very high levels of debt (although the downward sloping portion is driven by relatively few observations).[17]

The fitted line in the Figure is based on the empirical analysis below, which confirms that fiscal fatigue is a robust, statistically significant feature of the data that holds even controlling for other determinants of the fiscal reaction function.

The estimation results for the fiscal reaction function relating primary balances to lagged debt (allowing for a cubic function to capture the two inflexion points in the curvature of the response) and various control variables, as well as country-specific fixed effects, are presented in Table 1. Estimates for two sample periods (1970–2007; 1985–2007) are reported, where data availability for the latter period allows for the inclusion of a richer set of structural variables as determinants of the primary balance.

Following the literature (for example, Roubini and Sachs, 1988; Galí and Perotti, 2003; Abiad and Ostry, 2005; Bohn, 2008; Mendoza and Ostry, 2008), we include a range of variables such as the output gap to control for the effect of business cycles, the government expenditure gap to measure the effect of temporary fluctuations in government outlays (such as military spending), fuel and non-fuel commodity prices, trade openness and the average inflation rate in the previous three years to examine the possible effects of inflation (such as bracket-creep effects or greater fiscal effort to counter the effects of higher interest rates accompanying higher inflation) on the fiscal balance.

The institutional variables consist of a political stability index – which is a composite measure of the institutional capacity as well as government stability in the country – with higher values indicating lower risk; a fiscal rules index, which indicates if the country has any type of fiscal rule (balanced budget, expenditure, revenue or debt) in a given year; and a dummy variable indicating whether the country had an International Monetary Fund (IMF)-supported programme, as a proxy for international influence on fiscal behaviour. To take account of the country's demographic structure, we also include the current and projected age-dependency ratios.

Estimation of the fiscal reaction function raises some econometric issues stemming from the dependence of lagged debt on past values of the primary balance. From the debt dynamics equation, lagged debt is necessarily correlated with the unobserved country-specific determinants of the primary balance – that is, countries able to generate higher primary balances on average because of favourable fixed effects, would tend to have lower levels of public debt. This negative association between debt and time-invariant country-specific features, if not properly accounted for, could generate a downward bias in the estimated response of primary balance to lagged debt. In our estimation, we control for this source of endogeneity bias through the inclusion of country fixed effects in the fiscal reaction function. Furthermore, to the extent that there is persistence in the error term (ε), the debt dynamics would render lagged debt in (2) endogenous – even when

[17] The downward sloping segment of the primary balance reaction function at higher debt levels is an empirical feature, that is, both across and within individual countries (for which we observe high debt levels), the primary balance generally declines with higher debt at very high debt levels. Possibly governments are running into Laffer curve effects as they seek to raise higher revenues, or the public's tolerance for expenditure cuts and tax hikes decreases as debt levels begin to appear almost insurmountably high.

Table 1

Estimation Results for the Fiscal Reaction Function, 1970–2007

	1970–2007		1985–2007	
Sample specification	(1)	(2)	(3)	(4)
Lagged debt	−0.2080***	−0.2249***	−0.0805	−0.0864
	(0.059)	(0.061)	(0.076)	(0.070)
Lagged debt_square	0.0032***	0.0034***	0.0016*	0.0017*
	(0.001)	(0.001)	(0.001)	(0.001)
Lagged debt_cubic	−0.00001***	−0.00001***	−0.00001*	−0.00001**
	(3.0e−06)	(3.0e−06)	(3.0e−06)	(3.0e−06)
Output gap	0.4974***	0.4910***	0.4849***	0.4408***
	(0.047)	(0.046)	(0.053)	(0.053)
Government expenditure gap	−0.1847***	−0.1837***	−0.1831***	−0.1826***
	(0.047)	(0.045)	(0.052)	(0.047)
Trade openness		0.0908*		0.1461***
		(0.050)		(0.054)
Inflation		3.4005		4.6201**
		(2.519)		(2.008)
Oil price†		8.7747***		9.5288***
		(3.216)		(3.244)
Age dependency				−0.0717
				(0.101)
Future age dependency				−0.0154
				(0.067)
Non-fuel commodity price†				3.0049
				(8.362)
Political stability				0.0678**
				(0.030)
IMF arrangement				−1.1421
				(0.999)
Fiscal rules				0.3000
				(0.347)
Observations	642	642	496	491
Number of countries	23	23	23	23
R^2	0.282	0.316	0.304	0.405
AR (1) coefficient	0.791	0.760	0.819	0.749

Notes. Dependent variable is general government primary balance to GDP (in per cent); in all specifications, country-specific fixed effects are included and error term is assumed to follow an AR (1) process; robust standard errors reported in parentheses; ***, ** and * denote significance at 1%, 5% and 10% levels respectively.

†Applies to oil and non-oil commodities exporters only.

country fixed effects are included – and induce a downward bias in its estimated coefficient. To address this potential bias, we allow for serial correlation in the error term – modelling ε as an AR(1) process (that is, $\varepsilon_{t+1} = \alpha\varepsilon_t + v_t$).

The results presented in Table 1 indicate that the coefficients of the cubic functional form – capturing the increasing but slowing response of the primary balance to lagged debt – are statistically significant. In fact, based on the estimated coefficients, the marginal response of the primary balance to lagged debt starts to decline at debt levels of around 90–100% of GDP, becoming negative as the debt ratio approaches about 150% of GDP. This is in contrast to the findings of Bohn (1998, 2008) for the US, which show that fiscal effort increases with the debt level but supports the results of Mendoza and Ostry (2008) who find that sustainability is less assured in advanced economies when public debt is high than when it is moderate. The estimated response of primary balance at moderate debt levels (in the range of 0.02–0.06 percentage points) is also consistent with the results of the latter but the empirical approach adopted here is preferable because instead of an *ad hoc* classification of countries into high and low-debt groups (according to the debt ratio in relation to the sample mean), the cubic specification allows more flexibility and helps pin down the debt range where the sovereign risks moving into insolvency territory.

The estimated coefficients of other determinants included in the fiscal reaction function are also plausible and broadly in line with previous studies. For example, the primary balance responds positively to the output gap. Temporary increases in government outlays as captured by the government expenditure gap variable, affect the primary balance negatively. More open economies exhibit better fiscal performance, as do countries with stronger institutions (as proxied by the political stability variable), and fuel exporters when oil prices rise.[18] The non-linearity of the reduced-form fiscal reaction function and the response of the primary balance to other structural and economic determinants are robust to a variety of different estimation methods discussed in online Appendix B.[19]

Our empirical analysis, while including fixed effects for individual countries to capture differences in average primary balances, is based on panel estimates of the primary balance reaction function to lagged debt. As with any panel estimation, this raises questions about slope homogeneity. But, the issue is more complex here because the cubic 'fiscal fatigue' behaviour is postulated to be a feature of the entire range of debt (from 0 to about 180% of GDP), whereas individual countries' debt ratios may be observed over only a portion of that range. Our analysis for countries that, in the sample, we do not observe at high debt ratios must therefore rest on the assumption that these countries – if their debt were to increase – would behave similarly (in terms of fiscal fatigue) as countries that we do observe at high debt ratios.

While this is – and can only be – a conjecture, three observations suggest that it is a reasonable conjecture. First, our sample consists only of advanced economies, which share similar characteristics and is therefore relatively homogeneous. Second, the contrary assumption – that some countries would never exhibit fiscal fatigue and would always increase the primary balance in line with rising debt – seems implausible, if only because the primary surplus cannot exceed GDP. Third, slope homogeneity across countries cannot be rejected (in almost every case) over the debt ranges (low-to-moderate; moderate-to-high) in which they are observed. Specifically, when observed at low-to-moderate debt ranges, countries (including those that eventually have high debt) react similarly with a positive, linear coefficient to lagged debt (see online Appendix B for detailed results).[20] Likewise, when observed at moderate-to-high debt

[18] Norway is the only country classified as an oil exporter in the sample.

[19] The estimation results for the fiscal reaction function are also robust to alternative specifications with additional possible determinants such as total interest expenditure (to GDP), average debt maturity (in years) and a dummy variable allowing for different behaviour of eurozone members following euro adoption – all of which have statistically insignificant effects on the primary balance. Moreover, interaction terms between (lagged) debt and the institutional quality and fiscal rules variables to allow for non-linear relationships with the primary balance (such that weak institutions have a larger negative effect at higher debt levels) also turn out to be statistically insignificant.

[20] Of course, countries do differ in their average primary balances, as captured by the country fixed effects and other structural determinants of the primary balance.

ranges, countries (including those that, in the sample, have some low debt observations) exhibit similar fiscal fatigue as captured by the cubic specification. In other words, countries in the sample behave similarly at common debt ratios at which they are observed – providing some confidence that they would behave similarly at debt ratios at which (some) are not observed.[21]

2.2. *Fiscal Space*

To perform our calculations of fiscal space, we also need estimates of the relevant interest rates on public debt. We consider two approaches to obtain these estimates. The first, which we label 'market approach', is to use market interest rates on government debt, based on the assumption that the market rate reflects the perceived probability of default. Table 2 reports the interest rate–growth rate differentials obtained using two variants of this approach. The first variant is the historical average (over the past 10 years to cover a sufficiently long time span and smooth out fluctuations) of the implied nominal interest rate on government debt (interest payments divided by end-period debt) relative to the growth rate of nominal GDP. The second variant replaces historical averages with the IMF projections of long-term government bond yields and for GDP growth obtained from IMF (2010). With minor exceptions, projected interest rate–growth differentials are considerably less favourable than historical differentials, reflecting the expectation of both higher interest rates and slower real GDP growth (Table 2).[22] The use of (historical or projected) market interest rates may overestimate the true debt limit implied by the model because it ignores the fact that the interest rate will rise sharply as debt approaches its limit and default risk increases. An alternative approach, therefore, is to exploit the stochastic version of the model to derive the risk premium (and therefore the interest rate) endogenously. The resulting fiscal space estimates are reported in Appendix A; in fact, they are not very different from those reported here, essentially because the model implies that the interest rate will only rise sharply when debt is close to the debt limit.

2.2.1. *Point estimates*

Table 3 reports the debt limits, $\bar{d}$, as well as the long-run debt ratio to which each country's public debt conditionally converges, d^*, calculated using the country's fitted primary balance function obtained from the results for the recent period (1985–2007) reported in Table 1 (column 4) and both the historical and projected market interest rates.[23] While there is, of course, significant variation across countries (reflecting the country-specific fixed effects and interest rate–growth rate differentials), d^* ranges between 0% and about 100% of GDP – with a median of 50% of GDP – using historical interest rates and the estimated $\bar{d}$ ranges between 150% and 250% of GDP with a

[21] This observation is also in line with the strong evidence of homogeneity (for 42 of 56 industrial and developing countries) of the fiscal response coefficient reported in Mendoza and Ostry (2008).

[22] The projected interest rate-growth rate differential is taken as the average over the next 5 years to get a medium-term perspective (such that the output gap is closed).

[23] For the calculation of $\bar{d}$ and d^*, output and government expenditure gaps are assumed to be closed. We also compute fiscal space estimates based on the 1970–2007 sample and find the results to be very similar.

Table 2

Public Debt and Interest Rate–Growth Rate Differential (%)

Country	Debt/GDP 2007	Debt/GDP 2009	Debt/GDP 2015†	Interest rate–growth rate differential Historical‡	Interest rate–growth rate differential Projected†
Australia	9.4	15.5	20.9	0.1	1.2
Austria	59.5	67.3	77.3	1.4	0.8
Belgium	82.8	97.3	99.9	1.2	2.1
Canada	65.0	82.5	71.2	1.7	0.4
Denmark	34.1	47.3	49.8	3.2	0.1
Finland	35.2	44.0	76.1	0.0	1.4
France	63.8	77.4	94.8	0.8	0.5
Germany	65.0	72.5	81.5	2.6	1.5
Greece	95.6	114.7	158.6	−1.5	2.2
Iceland	29.3	105.1	86.6	−1.4	4.1
Ireland	24.9	64.5	94.0	−5.8	3.2
Israel	78.1	77.8	69.9	0.1	0.2
Italy	103.4	115.8	124.7	1.4	1.7
Japan	187.7	217.7	250.0	2.0	1.0
Korea	29.6	32.6	26.2	−0.7	−2.3
Netherlands	45.5	59.7	77.4	0.5	0.6
New Zealand	17.4	26.1	36.1	1.1	2.5
Norway	58.6	53.6	53.6	−3.4	−0.7
Portugal	63.6	77.1	98.4	−0.6	2.2
Spain	36.1	55.2	94.4	−2.4	2.6
Sweden	40.5	40.9	37.6	−0.5	−0.7
UK	44.1	68.2	90.6	0.4	1.3
US	62.1	83.2	109.7	0.3	1.6
Median	58.6	68.2	81.5	0.3	1.3
Mean	57.9	73.7	86.1	0.0	1.1

Source: IMF's WEO database.
† IMF (2010) projections. Interest rate–growth rate differential is based on the long-term government bond yield (average for 2010–14).
‡ Average of 1998–2007 based on the implied interest rate on public debt.

median of 190%. Using the projected interest rate–growth rate differentials, the corresponding median d^* is 63% of GDP and the median $\bar{d}$ is 183% of GDP.

In a few cases (Greece, Iceland, Italy, Japan, Portugal) and depending on the interest rate used, no estimate of d^* (or of $\bar{d}$) is reported. This is because, given these countries' estimated primary balance reaction function and assumed interest rate–growth rate differential, public debt would not be expected to converge to a finite steady-state debt ratio (it follows that there is no maximum debt level below which convergence occurs, so it is not meaningful to calculate $\bar{d}$ either).[24] For Italy and Japan, d^* does not exist even using the historical interest rate–growth rate differential, implying that their debt ratios had not been on a convergent path even prior to the crisis (and indeed, especially for Japan, the public debt ratio has been rising steadily over much of the sample period).[25] For Iceland,

[24] In terms of Figure 2, the fiscal reaction function always lies below the interest rate schedule, implying no intersection.

[25] This holds even if net debt rather than gross debt is used in the analysis for Japan. For Italy, estimating the fiscal reaction function over a more recent period (1990–2007) – that excludes the 1980s, when Italy's fiscal performance was especially weak – yields a finite ratio, d^*, to which debt should converge.

Table 3

Long-run d^* and $\bar{d}$ under Projected Market Interest Rate-Growth Rate (% of GDP), 1985–2007

Country	Debt (end-2015)	d^* Historical	d^* Projected	$\bar{d}$ Historical	$\bar{d}$ Projected
Australia	20.9	0.0	0.0	203.9	193.2
Austria	77.3	63.9	54.3	179.7	187.3
Belgium	99.9	60.3	76.3	182.0	168.4
Canada	71.2	110.8	82.6	152.3	181.1
Denmark	49.8	0.0	0.0	175.7	208.7
Finland	76.1	0.0	0.0	200.4	184.5
France	94.8	94.8	89.8	170.9	176.1
Germany	81.5	94.5	71.0	154.1	175.8
Greece	158.6	80.5	…	196.5	…
Iceland	86.6	0.0	…	213.5	…
Ireland	94.0	0.0	90.7	245.7	149.7
Israel	69.9	79.7	82.1	184.8	182.4
Italy	124.7	…	…	…	…
Japan	250.0	…	…	…	…
Korea	26.2	0.0	0.0	217.2	229.2
Netherlands	77.4	50.2	50.7	190.5	190.1
New Zealand	36.1	0.0	0.0	201.0	186.4
Norway	53.6	0.0	0.0	263.2	249.2
Portugal	98.4	77.1	…	191.6	…
Spain	94.4	0.0	94.8	218.3	153.9
Sweden	37.6	0.0	0.0	203.5	204.9
UK	90.6	79.6	94.9	182.0	166.5
US	109.7	78.7	101.2	183.3	160.5
Median	81.5	50.2	62.6	191.6	183.4
Mean	86.1	41.4	49.3	195.7	186.0

Source: IMF (2010) and authors' calculations.
$\bar{d}$ is the debt limit, above which debt grows without bound given the country's historical primary balance behaviour; d^* is the long-run average proportion of GDP to which public debt converges conditional on not exceeding $\bar{d}$; … indicates that given the fiscal reaction function and the interest rate-growth differential, the public debt dynamics are not on a sustainable path to converge to a finite stable steady-state debt ratio; 0.0 indicates that convergence is achieved at a negative d^*, implying a positive government asset position. All results are based on the estimated fiscal reaction function for 1985–2007 reported in Table 1 (column 4).

there is a finite long-run debt ratio using either the historical or the model-implied interest rate (see Appendix A) but not with projected market interest rates. For Greece and Portugal, d^* exists only using the historical interest rate–growth rate differential – not at projected or model-implied interest rates. As such, the financing difficulties that these countries have recently been experiencing should not come as a surprise because, as indicated by the results, based on their past fiscal behaviour but at the current interest rate –growth rate differential, their debt ratio would not converge to a finite level.

It is worth noting, however, that these fiscal space estimates are based on the projected values of the set of structural variables included in $f(d)$. Any change in these values would shift the predicted fiscal reaction function and hence affect the debt limit and the corresponding fiscal space estimates. This is a particularly useful feature of our approach as it allows us to quantify the impact of any structural changes on fiscal sustainability – or, conversely, identify what structural improvements can put the country's debt dynamics back on a sustainable path. Thus, for example, using the estimates reported in column (4) of Table 1 and assuming that the institutional capacity (that is, the political stability index) in all countries is increased to the maximum value in the sample, we find that the available fiscal space increases – albeit to different degrees – in almost all economies.[26] Among the countries with no available fiscal space, however, Iceland is the only country for which an improvement in institutional capacity leads to an appreciable increase in fiscal space.[27]

2.2.2. *Likelihood of fiscal space*

As mentioned above, our definition of fiscal space is simply the difference between the debt ratio projected for 2015 and the debt limit, $\bar{d}$. Although the model takes account of shocks to the primary balance, these shocks are assumed to be temporary columns (3)–(5) and agents in the model are assumed to know the average primary balance reaction function, $\mu + f(d)$. In practice, however, the primary balance reaction function must be estimated econometrically and the empirical estimates will be subject to standard errors. Because these standard errors essentially imply permanent upward or downward shifts in $\mu + f(d)$, they affect the debt limit by the annuity value of the shift in the primary balance and could have a correspondingly large effect on the standard error of the debt limit. This makes intuitive sense: the sustainable level of debt will be much larger (smaller) if the government is able to maintain permanently, say, a 1% of GDP higher (lower) primary balance than estimated.[28]

To take account of this estimation uncertainty, we compute the probability of a given amount of fiscal space; that is, one minus the p-value for the null hypothesis that the difference between the debt limit and the projected debt level for 2015 is no larger than a specified amount of fiscal space. Table 4 presents these probabilities for positive fiscal space and fiscal space larger than 50% and 100% of GDP in columns (3)–(5) respectively.[29] The results show that generally countries with a fiscal space point estimate of larger than 50% of GDP have a 100% probability of positive fiscal space. For example, the point estimate of Austria's fiscal space is about 110% of GDP (its estimated $\bar{d}$ is 187 – with a standard error of 13 – while its projected 2015 debt is 77% of GDP). The results for its probability of fiscal space indicate that we can reject the null of no available fiscal space with a p-value of 0.00; reject the null of <50% of GDP of fiscal space with a p-value of 0.00; but that we fail to reject the null of <100% of GDP of fiscal space (as the p-value is 0.77).

Consistent with the discussion above, we fail to reject the null hypothesis that Greece, Iceland, Italy, Japan and Portugal have no fiscal space (it follows that we fail to reject even more strongly the hypotheses that these countries have <50% or 100% of GDP of fiscal space). For Belgium, France, Ireland, Spain, the UK and the US, we reject

[26] The highest score for the political stability index in the sample is for Finland. The results for this exercise are available upon request.

[27] For Greece, Italy, Japan and Portugal, $\bar{d}$ increases but remains lower than the projected debt level for 2015.

[28] To put this in perspective, if the primary balance that the government can run is 1% of GDP larger than estimated, then, with an interest rate-growth rate differential of 1% per year, $\bar{d}$ increases by 100% of GDP.

[29] The standard errors and probabilities are computed using 10,000 bootstrapped parameter estimates of the fiscal reaction function and calculation of the corresponding debt limit, which yields the standard error of the debt limit (and hence of fiscal space).

Table 4

Estimated Fiscal Space (% of GDP) *and Probability of Given Fiscal Space* (%) *under Projected Market Interest Rates,* 1985–2007

	Point estimate (1)	SE (2)	Pr (FS > 0) (3)	Pr (FS > 50) (4)	Pr (FS > 100) (5)
Australia	172.2	12.0	100.0	100.0	100.0
Austria	109.9	13.0	100.0	99.7	23.4
Belgium	68.5	10.2	100.0	76.5	0.1
Canada	109.9	14.5	100.0	99.9	28.0
Denmark	158.9	10.5	100.0	100.0	100.0
Finland	108.4	13.5	100.0	99.6	26.5
France	81.3	14.0	100.0	85.1	0.9
Germany	94.3	13.9	100.0	96.4	4.2
Greece	…	32.9	0.5	0.0	0.0
Iceland	…	72.5	46.9	33.2	0.3
Ireland	55.7	14.0	100.0	62.0	0.1
Israel	135.5	13.3	100.0	100.0	31.1
Italy	…	40.0	5.3	0.1	0.0
Japan	…	23.1	0.0	0.0	0.0
Korea	203.0	12.6	100.0	100.0	100.0
Netherlands	112.7	12.2	100.0	100.0	30.1
New Zealand	150.3	13.7	100.0	100.0	99.7
Norway	195.5	13.4	100.0	100.0	100.0
Portugal	…	31.3	3.0	1.3	0.0
Spain	59.6	13.5	100.0	52.3	0.1
Sweden	167.3	11.4	100.0	100.0	100.0
UK	76.0	13.8	100.0	82.5	0.6
US	50.8	13.6	99.9	23.2	0.0
Median	109.9		100.0	96.4	4.2
Mean	117.2		80.7	70.1	32.4

Source: Authors' estimates.
Point estimate refers to the difference between $\bar{d}$ at projected interest rate and IMF (2010) projections of the debt ratio at end-2015 (given in Table 3); … indicates that given the fiscal reaction function and the interest rate-growth differential, the public debt dynamics are not on a sustainable path to converge to a finite stable steady-state debt ratio. Standard error of fiscal space obtained from the distribution of $\bar{d}$, which is obtained from 10,000 bootstrapped parameter estimates of the fiscal reaction function (and calculation of the corresponding $\bar{d}$ using projected market interest rates).

the null of no available fiscal space but fail to reject the null of <50% (and 100)% of GDP of remaining fiscal space. Finally, for Australia, Korea and some of the Nordic countries (Denmark, Norway as well as Sweden), we reject the null of <100% of GDP of fiscal space.

In assessing the results reported in Tables 3 and 4, several points should be borne in mind. First, the estimates of fiscal space depend on the debt level projected for 2015 and the debt limit, which depends on the country's historical track record of primary balances. Thus, even a country with relatively high debt currently may enjoy additional fiscal space (and thus scope for dealing with unexpected shocks) if, in the past, it has acted fiscally responsibly, adjusting the primary balance once the shock has passed. Second, the reported estimates of fiscal space are against projected debt levels in 2015 and do not take account of possible contingent liabilities. Contingent liabilities, such as unfunded pension obligations and potential bank bailouts, by increasing actual debt can worsen our fiscal space estimates substantially. Moreover, the debt limit – on which the fiscal space estimates are predicated – is by no means a ‘desirable’ or ‘optimal’ level of debt. For a variety of reasons, including possible rollover risk, governments will generally want to ensure they do not exhaust their fiscal space and that their debt remains well below its calculated limit.[30] Third, as discussed in the theoretical section, a key feature of the model is that a revision in market estimates of the support of the possible shock to the primary balance – even if the shock does not occur – can trigger a rise in the market interest rate, potentially undermining debt sustainability. Fourth, although not evident in the reported estimates of fiscal space, simulations to calculate the fiscal space under the model-implied interest rate suggest that the increase from the risk-less to the risky interest rate tends to happen quite abruptly as debt reaches its limit (in fact, with one-period debt, the jump from the risk-less interest rate to complete market shut-out will happen within one support of the primary balance shock below the debt limit; see Appendix A, Proof 6). Thus, a country may be able to borrow at low (‘risk-free’) interest rates even as its public debt increases and then suddenly find itself facing much steeper interest rates or even shut out of the capital markets. The last two properties resonate with the experience of some Southern European countries in the recent crisis and underscore the importance of maintaining sufficient fiscal space.

3. Conclusion

This article contributes to the burgeoning literature on public debt sustainability by proposing a new framework that conceptualises the notion of fiscal space – defined as the difference between the country’s current debt level and its debt limit, where the latter is the debt level beyond which fiscal solvency fails. Our concept of the debt limit, which takes explicit account of fiscal fatigue in a stochastic setting, is intuitively appealing and has several sensible properties. In particular, an improvement in a country’s structural characteristics or economic growth rate raises the debt limit, while the occurrence (or recognition of the possibility) of a negative fiscal shock could push an otherwise sustainable debt level into unsustainable territory.

We apply our framework empirically to a sample of 23 advanced economies over 1970–2007 and find robust empirical support for the fiscal fatigue characteristic. Specifically, we find that the marginal response of the primary balance to lagged debt is non-linear, remaining positive at moderate debt levels but starting to decline when debt reaches around 90–100% of GDP. Combining these estimates, including country fixed effects, with estimates of the interest rate–growth rate differential, we find that the resulting debt limits and corresponding fiscal space vary considerably across countries. For countries in our sample, the estimated debt limit ranges between 150% and 250% of GDP, whereas the fiscal space estimates indicate limited or no available room for fiscal manoeuvre for Greece, Iceland, Italy, Japan and Portugal, and ample space for Australia, Korea and the Nordic countries.

The probability estimates of positive fiscal space present a similar picture with several countries including Australia, Korea and the Nordics projected to have the highest

[30] On runs and rollover risk in the sovereign debt context, Chatterjee and Eyigungor (2011).

probability of positive fiscal space, and a number of Southern European countries, and Japan and Iceland the lowest. In addition, the UK and the US also appear to be constrained in their room for fiscal manoeuvre. To the extent that our primary balance reaction function is based on pre-global financial crisis data, our estimates provide early warning for the recent fiscal troubles encountered by some of these advanced countries, and as such, the financing difficulties that they have been experiencing are consistent with our empirical analysis. Moreover, our model helps in understanding the factors that contributed to the fiscal challenges these countries face, and can be used to identify the structural improvements that can help put their debt dynamics back on a sustainable path.

The estimates presented here do not, however, take into account liquidity/rollover risks or the possible realisation of contingent liabilities. Furthermore, while our model is essentially a one-period model that incorporates an endogenous interest rate, exploring the implications of longer term debt and/or stochastic endogenous growth are possible avenues for future research. From a policy perspective, however, our estimates can be used to flag cases where fiscal consolidation may be urgently needed to ensure that debt remains on a sustainable path and that shocks, including data revisions, do not derail sustainability.

Appendix A. Stochastic Model: Derivation of Debt Limit and Fiscal Space Estimates

This Appendix derives the debt limit in the general case where the primary balance is subject to stochastic shocks, characterises the key properties of this limit, and reports the resulting debt limit and fiscal space estimates when the model-endogenous interest rate is used.

A.1. *Derivation of Debt Limit*

A.1.1. *Probability of default given debt limit*

In any period, the probability that the government defaults in the following period, p_{t+1}, is simply the probability that debt exceeds the debt limit:

$$p_{t+1} = \mathrm{pr}(d_{t+1} > \bar{d}) = \mathrm{pr}(\varepsilon_{t+1} < H_t) = G(H_t), \tag{A.1}$$

where $H_t = (r_t - g)d_t - \mu - f(d_t) - (\bar{d} - d_t)$. The arbitrage condition (5) can be rewritten as:

$$1 + r_t = (1 + r^*)\left(\frac{1 - \theta p_{t+1}}{1 - p_{t+1}}\right). \tag{A.2}$$

Exploiting the time recursive structure of the problem to drop time subscripts, we combine (A.1) and (A.2) to obtain the default probability (for a given $\bar{d}$) as the solution to the fixed-point problem:

$$p = z(p; d, \bar{d}), \tag{A.3}$$

where

$$z(p; d, \bar{d}) = \begin{cases} 0 & \text{if } H(p; d, \bar{d}) \leq -\bar{\varepsilon} \\ 1 & \text{if } H(p; d, \bar{d}) > \bar{\varepsilon} \\ G[H(p; d, \bar{d})] & \text{otherwise} \end{cases}$$

$$H(p; d, \bar{d}) = [(r^* - g)d - \mu - f(d)] - (\bar{d} - d) + (1 - \theta)(1 + r^*)d[p/(1 - p)].$$

It is possible to show that at least one corner solution to (A.3) always exists, which is $p = 1$. There may be multiple interior solutions to (A.3), but Assumption III (*iii*) ensures a unique equilibrium, given by the lowest interior solution.

A.1.2. *Bounds on debt limit*

Before turning to the derivation of the debt limit, it is useful to establish its upper and lower bounds. The upper bound, $\bar{d}_2$, is given implicitly by the largest root of:

$$\mu + f(\bar{d}_2) + \bar{\varepsilon} = (r^* - g)\bar{d}_2. \tag{A.4}$$

The left-hand side of (A.4) is the largest primary surplus (i.e. the surplus under the best realisation of the shock) that the government can achieve at the debt level, $\bar{d}_2$. The right-hand side is the minimum interest payment (minimum because it assumes the market charges the risk-free rate despite any default risk) payable at a debt ratio $\bar{d}_2$. Beyond $\bar{d}_2$, therefore, even the best realisation of the stochastic shock forever onwards would be insufficient to meet the interest payment, and debt would increase without bound.

The lower bound on the debt limit, $\bar{d}_1$, is given implicitly by the largest root of:

$$\mu + f(\bar{d}_1) - \bar{\varepsilon} = (r^* - g)\bar{d}_1. \tag{A.5}$$

The left-hand side of (A.5) is the smallest primary surplus (i.e. the primary surplus under the worst realisation of the shock) at the debt level $\bar{d}_1$, and this primary surplus is just sufficient to meet the interest payment when creditors (correctly) charge the risk-free rate. As debt is non-increasing for all realisations of the shock, there is no risk of default, and therefore creditors are justified in charging the risk-free interest rate. It is readily shown (see Proof 1) that $\bar{d}_2 > \bar{d}_1 > d^m$.

A.1.3. *Determination of the debt limit*

Stochastic case To determine $\bar{d}$, first we note that for a given debt limit within the bounds defined above, there is a threshold debt ratio $\hat{d} < \bar{d}$ that corresponds to the maximum debt ratio that yields the corner solution $p = 0$ to (A.3) – that is, the largest root of $z(0; d, \bar{d})$. Thus, $\hat{d}$ guarantees that the government will be able to repay next period, and the range of debt ratios $(\hat{d}, \bar{d}]$ is the range in which debt with default risk is traded. By construction, the default probability in the next period is zero for $d \leq \hat{d}$, hence the interest rate charged by the market will be the risk-free rate by Assumption III (*iii*).[31] It is readily verified that $\hat{d} \geq \bar{d}_1$ because $\hat{d}$ only guarantees that the government will be able to repay one period ahead, while $\bar{d}_1$ guarantees repayment for any future sequence of shocks.

As debt rises above $\hat{d}$, the market charges a positive risk premium and the interior solution to (A.3) determines the equilibrium probability of default and the interest rate. We denote the interior solution (the lowest one in case of multiple solutions) by $p^* = p^*(d, \bar{d}) \in (0, 1)$. Note that p^* and the associated finite interest rate are a legitimate equilibrium solution only for $d > \hat{d}$ because otherwise the corner solution $p = 0$ and the risk-free interest rate must be the equilibrium solution by Assumption III (*iii*). Moreover, the debt limit must be larger than $\hat{d}$ if p^* exists because it is defined as the maximum level of debt the government can roll-over at a finite interest rate. Assuming that p^* exists, it can be easily shown that

$$\partial p^*/\partial d > 0 \quad \text{and} \quad \partial p^*/\partial \bar{d} < 0, \tag{A.6}$$

which is intuitive: the default probability rises with actual debt, but falls as the debt limit rises.

[31] Given the non-linearity of $f(d)$, $p = 0$ may not be supported as the fixed-point solution at some levels of debt below $\hat{d}$. Nevertheless, the market will charge the risk-free interest rate because it foresees that the same interest rate is consistent with higher debt levels.

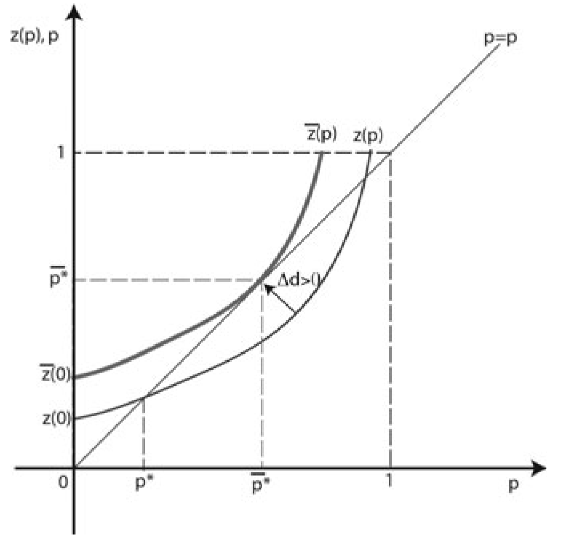

Fig. A1. *Determination of the Probability of Default*

Given the government's default rule, in the rational expectations equilibrium, the debt limit must satisfy the following two equilibrium conditions simultaneously:

$$\begin{aligned}&(i) \quad p^* \text{ must exist for } d \in (\hat{d}, \bar{d}\,)\\&(ii) \quad p^* \text{ must not exist for } d > \bar{d}.\end{aligned} \tag{A.7}$$

The first condition implies that if debt remains at or below the debt limit, there are always risk-neutral lenders who are willing to lend to the government at a positive finite risk premium. The second implies that $p = 1$ must be the only solution if debt exceeds the debt limit. In this case, no finite risk premium can fully compensate for the associated default risk and, as a result, the government effectively faces an infinite interest rate (or a complete loss of market access).[32] As the primary balance is finite given that the debt limit and the support of the shock are both finite, debt dynamics become explosive and the government necessarily defaults whenever debt exceeds the debt limit – which is consistent with the assumed default rule. These two conditions essentially pin down the debt limit in equilibrium.

Denoting the interior solution at $d = \bar{d}$ by $\bar{p}^* = p^*(\bar{d}, \bar{d})$, condition (*ii*) in (A.7), together with the first result in (A.6), imply that $\bar{p}^*$ must be the maximum interior solution. Figure A1, illustrates this point. For a given debt limit and some initial debt ratio below the limit (and above $\hat{d}$), there would in general be two (or more) interior solutions to (A.3), the lowest of which is p^*. As d increases, p^* also increases while the other (upper) interior solution decreases. As d increases further, the two interior solutions converge to each other before collapsing into a single interior solution, which occurs at the tangency point between $z(p; d, \bar{d})$ and the 45 degree line. The interior solution at this tangency point is the maximum interior solution that the given debt limit can sustain: any further increase in debt will lead to the corner solution $p = 1$, resulting in an infinite interest rate. But, this is the very definition of debt limit. At the tangency point, therefore, actual debt must coincide with $\bar{d}$ or, equivalently, $\bar{p}^*$ is determined by the tangency point.

[32] Flood and Marion (2006) follow a similar approach to analyse how emerging market sovereign borrowers can be shut out of international capital markets in a relatively short period of time.

Formally, we require:

$$(i) \quad \bar{p}^* = \bar{z}(\bar{p}^*)$$
$$(ii) \quad \partial\bar{z}(p)/\partial p|_{\bar{p}^*} = 1, \tag{A.8}$$

where we denote $\bar{z}(p) = z(p; \bar{d}, \bar{d})$ for convenience. It is straightforward to show that such a $\bar{p}^*$ will exist provided two boundary conditions are satisfied:

$$-\bar{\varepsilon} < F(\bar{d}) < \bar{\varepsilon} \quad \text{and} \quad (\bar{d}/d^m)\bar{\varepsilon} < W(\bar{d}) < \left\{\frac{1 - G[F(\bar{d})]}{G[F(\bar{d})]}\right\} \cdot [\bar{\varepsilon} - F(\bar{d})], \tag{A.9}$$

where

$$F(d) = (r^* - g)d - \mu - f(d)$$
$$W(d) = (1 - \theta)(1 + r^*)d.$$

The first condition in (A.9) can be further reduced to yield:

$$\bar{d}_1 \le \bar{d} < \bar{d}_2, \tag{A.10}$$

where $\bar{d}_1$ and $\bar{d}_2$ are defined above, and $d^m \le \bar{d}_1 < \bar{d}_2$ (see Proof 1).

Deterministic case Substituting from (A.3), the equilibrium conditions for the debt limit in the deterministic case (which we denote $\bar{d}_{\bar{\varepsilon}=0}$ to distinguish it from the debt limit in the stochastic case) can be written:

$$(i)\ H(0; d, \bar{d}_{\bar{\varepsilon}=0}) \le 0 \quad \text{for} \quad d \le \bar{d}_{\bar{\varepsilon}=0} \quad \text{and} \quad (ii)\ H(1; \bar{d}_{\bar{\varepsilon}=0}, \bar{d}_{\bar{\varepsilon}=0}) > 0 \quad \text{for} \quad d > \bar{d}_{\bar{\varepsilon}=0}. \tag{A.11}$$

As (ii) holds trivially, $\bar{d}_{\bar{\varepsilon}=0}$ is essentially determined by (i). It can be easily shown that $H(0; \bar{d}_{\bar{\varepsilon}=0}, \bar{d}_{\bar{\varepsilon}=0}) = F(\bar{d}_{\bar{\varepsilon}=0})$ where $F(d)$ is as defined in (A.9). Given this result, $\bar{d}_{\bar{\varepsilon}=0}$ is obtained as the largest root of $F(d)$ as shown in (6). It is possible that $H(0; d, \bar{d}_{\bar{\varepsilon}=0}) > 0$ for some $d < \bar{d}_{\bar{\varepsilon}=0}$ due to the non-linearity of $f(d)$. But, the market will charge the risk-free interest rate because it is assured that debt is sustainable up to and including $\bar{d}_{\bar{\varepsilon}=0}$.

A.2. *Key Characteristics of the Debt Limit*

Proof 1. $d^m \le \bar{d}_1 < \bar{d}_{\bar{\varepsilon}=0} < \bar{d}_2$.

By property (3), $F(d^m) \le -\bar{\varepsilon}$ and $F'(d) > 0, \forall d > d^m$. We show for the proof that $F(d^m) \le F(\bar{d}_1) < F(\bar{d}_{\bar{\varepsilon}=0}) < F(\bar{d}_2)$. By definition, $F(\bar{d}_1) = -\bar{\varepsilon}, F(\bar{d}_{\bar{\varepsilon}=0}) = 0$, and $F(\bar{d}_2) = \bar{\varepsilon}$. This completes the proof.

Proof 2. $\hat{d} \ge \bar{d}_1$.

Substituting $p = 0$ into (A.3) yields the following inequality:

$$B(d) = F(d) + \bar{\varepsilon} - (\bar{d} - d) \le 0,$$

where $d \le \bar{d}$. Note that $B(\hat{d}) = 0$ by definition, and that by property (3), $B'(d) > 0$ for all $d > d^m$. As $\bar{d} \ge \bar{d}_1$ and $F(\bar{d}_1) + \bar{\varepsilon} = 0$, it follows that $B(\bar{d}_1) \le 0$. This completes the proof.

Proof 3. $\partial p^*/\partial d > 0$ and $\partial p^*/\partial \bar{d} < 0$ if p^* exists.

Assuming that p^* exists, we first show that

$$\partial z/\partial d = G'(H)\,[1 + (1 - \theta)(1 + r^*)p/(1 - p) + F'(d)] > 0,$$
$$\partial z/\partial \bar{d} = -G' < 0,$$
$$\partial z/\partial p = G'(H)\, W(d)/(1 - p)^2 > 0,$$

where the first inequality follows from Proof 1. Total differentiating (A.3) and rearranging terms yields:

$$\partial p^*/\partial d = (\partial z/\partial d)/[1 - (\partial z/\partial p)] \text{ and } \partial p^*/\partial \bar{d} = (\partial z/\partial \bar{d})/[1 - (\partial z/\partial p)].$$

For the proof, we show that $\partial z/\partial p < 1$ at $p = p^*$. As $\partial z/\partial d > 0$ and $z(0; \hat{d}, \bar{d}) = 0$ by definition, it readily follows that $z(0; d, \bar{d}) > 0$ for $d \in (\hat{d}, \bar{d})$. This result, together with $\partial z/\partial p > 0$, ensures that $\partial z/\partial p < 1$ at $p = p^*$ (the lowest interior solution), and $\partial z/\partial p > 1$ at the upper interior solution in case of two interior solutions. This completes the proof.

Proof 4. Tangency condition for the debt limit $(\partial \bar{z}(p)/\partial p = 1)$.

The equilibrium condition states that p^* must exist for $d \leq \bar{d}$ and must not exist for $d > \bar{d}$. This condition can be satisfied only if the fixed-point solution jumps from p^* to $p = 1$ at the very

Table A1

Estimated $\bar{d}$ (% of GDP), *Fiscal Space* (% of GDP) *and Probability of Given Fiscal Space* (%) *at Model-Implied Interest Rates,* 1985–2007

			Fiscal space				
Country	Debt (end 2015)	d^*	Point estimate	Std. error	Pr (FS > 0)	Pr (FS > 50)	Pr (FS > 100)
Australia	20.9	0.0	181.8	13.1	100.0	100.0	100.0
Austria	77.3	55.1	93.3	14.1	100.0	99.6	15.0
Belgium	99.9	53.7	72.2	13.6	100.0	82.5	0.4
Canada	71.2	75.2	101.9	14.5	100.0	99.6	32.7
Denmark	49.8	0.0	146.1	13.1	100.0	100.0	99.3
Finland	76.1	0.0	90.9	13.1	100.0	98.7	16.9
France	94.8	92.7	64.9	13.1	100.0	80.8	0.1
Germany	81.5	63.6	88.5	14.0	100.0	98.6	6.9
Greece	158.6	…	…	39.7	2.1	0.3	0.0
Iceland	86.6	0.0	70.7	49.4	87.7	87.1	2.0
Ireland	94.0	42.9	63.6	12.4	100.0	85.9	0.4
Israel	69.9	65.0	114.0	14.7	100.0	99.8	55.6
Italy	124.7	…	…	26.5	3.1	0.0	0.0
Japan	250.0	…	…	2.1	0.0	0.0	0.0
Korea	26.2	0.0	194.1	13.3	100.0	100.0	100.0
Netherlands	77.4	58.0	91.2	14.1	100.0	99.5	12.6
New Zealand	36.1	0.0	161.5	13.4	100.0	100.0	100.0
Norway	53.6	0.0	179.9	12.4	100.0	100.0	100.0
Portugal	98.4	…	…	57.8	81.3	43.7	0.0
Spain	94.4	70.2	74.0	12.6	100.0	80.1	0.6
Sweden	37.6	0.0	130.2	15.1	100.0	100.0	96.1
UK	90.6	75.5	75.5	13.9	100.0	94.9	2.0
US	109.7	77.6	63.4	14.2	100.0	60.3	0.1
Median	81.5	53.7	91.2	13.9	100.0	98.6	6.9
Mean	86.1	38.4	108.3	18.3	85.8	78.8	32.2

Notes. $\bar{d}$ is the debt limit, above which debt grows without bound given the country's historical primary balance behaviour; the estimates of are obtained assuming a recovery rate of 90% in the event of default; d^* is the long-run average debt ratio to which the economy converges conditional on not exceeding $\bar{d}$; … indicates that given the fiscal reaction function and the interest rate–growth differential, the public debt dynamics are not on a sustainable path to converge to a finite stable steady-state debt ratio; 0 indicates that convergence is achieved at a negative d^*, implying a positive government asset position. All results are based on the estimated fiscal reaction function reported in Table 1 (col. 4). Point estimate refers to the difference between and IMF's WEO (2010) projections of the debt ratio at end 2015. Standard error of fiscal space obtained from the distribution of stochastic using 10,000 bootstrapped parameter estimates of the fiscal reaction function.
Source. $\bar{d}$ IMF's WEO database and authors' calculations.

moment when debt exceeds the debt limit or, equivalently, only if $\partial p^*/\partial d = \infty$ at $d = \bar{d}$. It follows from Proof 3 that $\partial \bar{p}^*/\partial d = \infty$ iff $\partial \bar{z}/\partial p = 1$. This completes the proof.

Proof 5. $\bar{d} < \bar{d}_{\bar{\varepsilon}=0}$.

The assumption on the recovery value (see footnote 10) imposes the following restiction on the value of $W(d)$:

$$W(d) \geq (1-\bar{\theta})(1+r^*)d = (d/d^m)\bar{\varepsilon}. \tag{A.12}$$

For the proof, we show that this condition for $W(d)$ is violated if $\bar{d} \geq \bar{d}_{\bar{\varepsilon}=0}$. Suppose that $\bar{d} \geq \bar{d}_{\bar{\varepsilon}=0}$. It follows from Proof 1 that $F(\bar{d}) \geq 0$, which in turn implies that $G[F(\bar{d})] \geq 1/2$. Given these results, it is straightforward to show

$$\left\{\frac{1-G[F(\bar{d})]}{G[F(\bar{d})]}\right\} \cdot [\bar{\varepsilon} - F(\bar{d})] < \bar{\varepsilon} < (\bar{d}/d^m) \cdot \bar{\varepsilon}.$$

This completes the proof.

Proof 6. $0 \leq (\bar{d} - \hat{d}) < \bar{\varepsilon}$.

The expression $B(\hat{d}) = 0$ shown in Proof 2 can be rewritten as

$$(\bar{d} - \hat{d}) = F(\hat{d}) + \bar{\varepsilon}.$$

It suffices for the proof to show that $-\bar{\varepsilon} \leq F(\hat{d}) \leq 0$. Proof 1 and 5 immediately imply that $-\bar{\varepsilon} = F(\bar{d}_1) \leq F(\hat{d}) < F(\tilde{d}) = 0$. This completes the proof.

Proof 7. Equilibrium properties of the debt limit.

For the proof, we show that

$$\partial\bar{d}/\partial r^* < 0, \quad \partial\bar{d}/\partial g > 0, \quad \partial\bar{d}/\partial\mu > 0, \quad \text{and} \quad \partial\bar{d}/\partial\bar{\varepsilon} = \begin{cases} \leq 0 & \text{if } \bar{p}^* \leq 0.5 \\ > 0 & \text{otherwise} \end{cases},$$

where in the last inequalities, $\partial\bar{\varepsilon} > 0$ refers to a mean-preserving spread (i.e. a symmetric widening of the support). Total differentiating $p = \bar{z}(p)$ while exploiting the tangency condition for the debt limit yields:

$$\partial\bar{d}/\partial x = -(\partial\bar{z}/\partial x)/(\partial\bar{z}/\partial\bar{d}),$$

where $x = \{r^*, g, \mu, \bar{\varepsilon}\}$. It is straightforward to show

$$\partial\bar{z}/\partial r^* > 0, \quad \partial\bar{z}/\partial g < 0, \quad \partial\bar{z}/\partial\mu < 0, \quad \text{and} \quad \partial\bar{z}/\partial\bar{d} > 0.$$

To complete the proof, we show that

$$\partial\bar{z}/\partial\bar{\varepsilon} = \begin{cases} \geq 0 & \text{if } \bar{p}^* \leq 0.5 \\ < 0 & \text{otherwise} \end{cases}.$$

As $\bar{z}(\bar{p}^*) \equiv G[H(\bar{p}^*; \bar{d}, \bar{d})] = \bar{p}^*$ in equilibrium, it is readily verified that $H(\bar{p}^*; \bar{d}, \bar{d}) \leq 0$ if $\bar{p}^* \leq 1/2$, and $H(\bar{p}^*; \bar{d}, \bar{d}) > 0$ otherwise. Then, the assumed properties of $G(\boldsymbol{\varepsilon})$ ensure that $\tilde{G}(H) \geq G(H)$ if $H \leq 0$, and $\tilde{G}(H) < G(H)$ otherwise, where $\tilde{G}$ is a mean-preserving spread of G. This completes the proof.

A.3. *Debt Limit and Fiscal Space Estimates Using Model-Implied Interest Rates*

In the text, we report the debt limit and fiscal space estimates when market interest rates are used to calculate the interest rate-growth rate differential. Here, we replace the market interest rates by the interest rate implied by the model itself. The main drawback of this approach is that it requires various assumptions about the risk-free interest rate, the distribution (and support) of the shocks to the primary balance and the recovery rate in the event of default. We assume a triangular distribution for the shocks to primary balance, calibrated for each country based on the residuals obtained from the primary balance reaction function in the estimation sample and assume a recovery rate of 90% in the event of default.[33] The estimated d^* and $\bar{d}$ as well as the resulting fiscal space obtained from the model-implied interest rate, reported in Table A1, are similar to the estimates based on market interest rates (Tables 3 and 4), although, in most cases somewhat lower (which is consistent with the intuition that estimates using market-based interests rate will likely overstate the available fiscal space because they do not necessarily take full account of the sharp rise in interest rates as fiscal space is exhausted).

International Monetary Fund
University of Pennsylvania

Additional Supporting Information may be found in the online version of this article:

Appendix B: Sensitivity Analysis of the Estimated Fiscal Reaction Function.

References

Abiad, A. and Ostry, J.D. (2005). 'Primary surpluses and sustainable debt levels in emerging market countries', Policy Discussion Paper No. 05/6, International Monetary Fund.

Aguiar, M. and Gopinath, G. (2006). 'Defaultable debt, Interest rates and the current account', *Journal of International Economics*, vol. 69(1), pp. 64–83.

Alesina, A. and Ardagna, S. (2010). 'Large Changes in fiscal policy: taxes versus spending', in (J.R. Brown, ed.), *Tax Policy and the Economy*, vol. 24, pp. 35–68, Chicago, IL: University of Chicago Press.

Arellano, C. (2008). 'Default risk and income fluctuations in emerging economies', *American Economic Review*, vol. 98(3), pp. 690–712.

Benjamin, D. and Wright, M. (2009). 'Recovery before redemption? A theory of delays in sovereign debt renegotiations', Working Paper 15/2009, Center for Applied Macroeconomic Analysis.

Bohn, H. (1998). 'The behaviour of US public debt and deficits', *Quarterly Journal Economics*, vol. 113(3), pp. 949–63.

Bohn, H. (2007). 'Are stationarity and cointegration restrictions really necessary for the intertemporal Budget Constraint?', *Journal of Monetary Economics*, vol. 54(7), pp. 1837–47.

Bohn, H. (2008). 'The sustainability of fiscal policy in the United States', in (R. Neck and J.E. Sturm, eds.), *Sustainability of Public Debt*, pp. 15–49. Cambridge, MA: MIT Press.

Checherita, C. and Rother, P. (2010). 'The impact of high and growing government debt on economic growth: an empirical investigation for the euro area', Working Paper 1237, European Central Bank.

Chatterjee, S. and Eyigungor, B. (2011). 'Maturity, indebtedness, and default risk', Working Paper 11-33, Federal Reserve Bank of Philadelphia.

Cottarelli, C., Forni, L., Gottschalk, J. and Mauro, P. (2010). 'Default in today's advanced economies: unnecessary, undesirable, and unlikely', IMF Staff Position Note SPN/10/12, International Monetary Fund.

Eaton, J. and Fernandez, R. (1995). 'Sovereign debt', Working Paper 5131, National Bureau of Economic Research.

Flood, R. and Marion, N.P. (2006). 'Getting shut out of the international capital markets: it doesn't take much', Working Paper WP/06/144, International Monetary Fund.

Galí, J. and Perotti, R. (2003). 'Fiscal policy and monetary integration in Europe', *Economic Policy*, vol. 18(37), pp. 533–72.

[33] The theoretical model presented in this Appendix does not assume any specific shape of the distribution of primary balance shocks, except that the distribution is defined over a finite support. Empirically, however, assuming a uniform distribution for primary balance shocks is unrealistic: large (positive or negative) shocks are less likely than small shocks, while using a truncated normal distribution would involve subjectivity in determining the range/bounds within which the probability mass is centred. We therefore adopt a triangular distribution as a simple, analytically tractable finite-support distribution. We construct the finite support of the shock ($\bar{\varepsilon}$) by using the country-specific average of the worst five negative residuals. The risk-free interest rate is assumed to be common to all countries in the sample, and is taken as the average over 2003–07 (a period when default risk in advanced economies was generally considered to have been low) of the effective real interest rate on government debt. It is growth adjusted using the 5-year country-specific average of the IMF projected real GDP growth rate. Benjamin and Wright (2009) estimate the recovery rate for upper-middle income countries to be about 60–100%; hence, for advanced economies we assume an (eventual) average recovery rate of 90%.

IMF (2010). *World Economic Outlook: October 2010*, Washington, DC: International Monetary Fund.
Mendoza, E.G. and Ostry, J.D. (2008). 'International evidence on fiscal solvency: is fiscal policy responsible?', *Journal of Monetary Economics*, vol. 55(6), pp. 1081–93.
Panizza, U. and Presbitero, A.F. (2012). 'Public debt and economic growth: is there a causal effect?', Working Paper 168, Institute of Public Policy and Public Choice.
Reinhart, C. and Rogoff, K. (2010). 'Growth in a time of debt', *American Economic Review*, vol. 100(2), pp. 573–8.
Roubini, N. and Sachs, J. (1988). 'Political and economic determinants of budget deficits in the industrial democracies', Working Paper 2682, National Bureau of Economic Research.
Rzonca, A. and Cizkowicz, P. (2005). 'Non-Keynesian effects of fiscal contraction in new member states', Working Paper no. 519, European Central Bank.
Schclarek, A. (2004). 'Debt and economic growth in developing and industrial countries' Department of Economics Working Paper 2005:34, Lund University.