The Baseline New Keynesian Model

Extra Notes for Dynare Code

Contents:

Recommended readings

- **Walsh**, C. E., 2017. **Monetary Theory and Policy**, Ch. 2: *Money-in-the-Utility Function* and Ch. 8: *New Keynesian Monetary Economics*. 4^{th} Ed. The MIT Press.
- **Gali**, J., 2015. **Monetary Policy, Inflation, and the Business Cycle: An introduction to the New Keynesian Framework and Its Applications**, Ch. 3: *The Basic New Keynesian Model* and Ch. 4: *Monetary Policy Design in the Basic New Keynesian Model*. 2^{nd} Ed. Princeton University Press.

Introduction: the RBC revolution

The *RBC revolution* (Kydland and Prescott, 1982) has both conceptual and methodological impacts (Gali, 2015):

1. Conceptual: RBC theory asserts
   a) the efficiency of business cycles;
   b) the importance of technology shocks as the source of economic fluctuations;
   c) the limited role of monetary factors.
2. Methodological: use DSGE model as a central tool for macroeconomic analysis; evaluate models by calibration and simulation.

Introduction: from RBC models to NK models

The NK model has more solid *micro-foundations* than its Keynesian ancestor, and it is more useful than its RBC predecessor.

The main properties of the NK model are:

- Monopolistically competitive firms;
- Nominal rigidities (prices & wages);
- Short-run non-neutrality of money.

Differences with respect to RBC models:

- Business cycles are inefficient; i.e., the economy's response to shocks is inefficient in the short-run;
- Nominal rigidities generate short-run non-neutrality of monetary policy, which allows for potential welfare-enhancing interventions by the monetary authority.

Introduction: from RBC models to NK models

The **NK** model, linked to the **IS-LM-PC** model, takes the basic **RBC** model with **money-in-utlity** and introduces the assumption of monopolistically competitive goods market and price stickiness.

Three key modifications from the RBC model:

1. Simplified such that endogenous variations in the capital stock ignored ($Y_t = \xi_{z,t} N_t$)
   - Even though little observed relationship between capital and output in business cycle dynamics, BGG(1999), CEE(2005) show variable capital utilisation costs on inflation important; it also means we ignore investment.
2. Single good replaced by a continuum of differentiated goods (monopolisitic competition)
3. Monetary policy rule for nominal interest rate setting: nominal quantity of money therefore endogenously determined to achieve desired i_t (or vice versa).

The Baseline NK model

This session presents the 'baseline' New Keynesian Model in the literature, in which imperfect competition and price stickiness are embedded in a general equilibrium model.

Main features of the baseline New Keynesian model:

- **Households:** consume goods, supply labour, and hold money and bonds;
- **Final good firms:** produce final good Y_t using intermediate goods $Y_{j,t}$ as the only input, firms are perfectly competitive;[1]
- **Intermediate goods firms:** hire labour to produce and sell differentiated intermediate goods $Y_{j,t}$ in monopolisitcally competitive markets;
- **Monetary authority:** controls monetary policy.

[1] Walsh p.331 discusses a slightly different conceptual approach to derive the demand function for good j, but with analogous results.

The Model: Households

The representative household chooses $\{C_t, N_t, M_t, B_t\}$ to maximise lifetime utility:

$$E \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t)^{1-\eta_c}}{1-\eta_c} + \xi_{m,t} \frac{a(M_t/P_t)^{1-\eta_m}}{1-\eta_m} - \frac{(N_t)^{1+\eta_n}}{1+\eta_n} \right] \tag{1}$$

subject to the budget constraint (BC):

$$\frac{(1+i_{t-1})B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \frac{W_t}{P_t} N_t + T_t + D_t = C_t + \frac{B_t}{P_t} + \frac{M_t}{P_t} \tag{2}$$

where D_t denotes the real dividends (profits) received from ownership of intermediate goods firms.
$\xi_{m,t}$ is a money demand shock and a is the weight of real money balances on utility (we set both to 1 for now).

The Model: Households

The following conditions, in addition to the BC, must hold in equilibrium

- FOC for hours worked:

$$\frac{N_t^{\eta_n}}{C_t^{-\eta_c}} = \frac{W_t}{P_t} \tag{3}$$

Intratemporal optimality condition setting MRS btw leisure and cons. $=$ real wage.

- FOC for bond holdings:

$$C_t{}^{-\eta_c} = \beta(1+i_t)E_t\left[\frac{P_t}{P_{t+1}}C_{t+1}{}^{-\eta_c}\right] \tag{4}$$

Euler eqn for the optimal intertemporal allocation of consumption.

- FOC for money holdings:

$$\frac{\left(\frac{M_t}{P_t}\right)^{-\eta_m}}{C_t^{-\eta_c}} = \frac{i_t}{1+i_t} \tag{5}$$

Intratemporal optimality condition setting MRS btw money and cons. $=$ opp. cost of holding money.

The Model: Final Goods Firm

A representative final-goods firm produces the composite final good Y_t using a continuum of intermediate goods $Y_{j,t}$ according to the Dixit and Stiglitz (1977) CES production function:[2]

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\varphi_{p,t}-1}{\varphi_{p,t}}} dj \right)^{\frac{\varphi_{p,t}}{\varphi_{p,t}-1}} . \tag{6}$$

The firm minimizes its costs:[3]

$$\min_{Y_{j,t}} \int_0^1 P_{j,t} Y_{j,t} dj \tag{7}$$

given the production constraint:

$$Y_t \leq \left(\int_0^1 Y_{j,t}^{\frac{\varphi_{p,t}-1}{\varphi_{p,t}}} dj \right)^{\frac{\varphi_{p,t}}{\varphi_{p,t}-1}} \tag{8}$$

[2] The integral here represents a continuum of intermediate goods indexed by $j \in [0, 1]$. The elasticity of substitution $\varphi_{p,t}$ is time-varying, following an AR(1) stochastic process: called a price markup shock.

[3] Regardless of the level of Y_t, it will always be optimal for the firm to purchase the combination of individual goods that minimizes the cost of achieving this level of the composite good.

The Model: Final Goods Firm

The Lagrangian for the firm is given by the following expression:

$$L = \int_0^1 P_{j,t} Y_{j,t} dj + \mu_t \left[Y_t - \left(\int_0^1 Y_{j,t}^{\frac{\varphi_{p,t}-1}{\varphi_{p,t}}} dj \right)^{\frac{\varphi_{p,t}}{\varphi_{p,t}-1}} \right] \tag{9}$$

The first order condition with respect to $Y_{j,t}$ is:

$$P_{j,t} = \left(\frac{Y_{j,t}}{Y_t} \right)^{\varphi_{p,t}} P_t \tag{10}$$

or

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\varphi_{p,t}} Y_t \tag{11}$$

and the price index:

$$P_t = \left(\int_0^1 P_{j,t}^{1-\varphi_{p,t}} dj \right)^{1/1-\varphi_{p,t}} \tag{12}$$

The Model: Intermediate Goods Firm

- A representative intermediate goods firm j produces $Y_{j,t}$ according to the following production function:[4]

$$Y_{j,t} = \xi_{z,t} N_{j,t} \ , \tag{13}$$

where technology, $\xi_{z,t}$, follows an exogenous stochastic process.

- The market demand for $Y_{j,t}$:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\varphi_{p,t}} Y_t \tag{11}$$

- Facing Calvo-type stickiness: in each time period, only a random fraction $(1 - \theta_p)$ of firms can reset prices.

[4] For simplicity, we ignore capital stock K_t. (13) implies constant returns to scale.

The Model: Intermediate Goods Firm

Marginal Cost

To derive the marginal cost for the firm, first consider the firm's cost minimization, in real terms, s.t. (13):

$$\min_{N_{j,t}} \left(\frac{W_t}{P_t}\right) N_{j,t} + \lambda_t (Y_{j,t} - \xi_{z,t} N_{j,t}) \ . \tag{14}$$

The FOC implies:

$$\frac{W_t}{P_t} = \lambda_t \xi_{z,t} \quad \Rightarrow \quad \frac{W_t}{P_t} = \lambda_t \left(\frac{Y_{j,t}}{N_{j,t}}\right) \tag{15}$$

Multiplying $N_{j,t}$ on both sides of (15) gives us the firm's cost function:

$$\frac{W_t}{P_t} N_{j,t} = \lambda_t Y_{j,t} \ , \tag{16}$$

where $\lambda_t = \frac{W_t}{P_t} / \xi_{z,t}$ can be treated as the firm's real marginal cost (MC_t).

The Model: Intermediate Goods Firm

The pricing decision with sticky prices

Following Calvo (1983), in each time period only a random fraction $1-\theta_p$ of intermediate good firms have an opportunity to reset prices. Assuming a CES aggregate of the average price level, the aggregate price index is given by:

$$P_{j,t}^{1-\varphi_{p,t}} = \theta_p P_{j,t-1}^{1-\varphi_{p,t}} + (1-\theta_p)(P_{j,t}^{*})^{1-\varphi_{p,t}}, \qquad 0 \leq \theta_p \leq 1, \qquad (17)$$

where $P_{j,t-1}$ is previous price level, and $P_{j,t}^{*}$ is the average price chosen by those who have the chance to reset the prices.

The Model: Intermediate Goods Firm

The pricing decision with sticky prices

In each time period, the representative intermediate-good firm chooses $P^*_{j,t}$ to maximize its life-time profit:

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \left[\left(\frac{P^*_{j,t}}{P_{t+i}} - MC_{t+i} \right) Y_{j,t,t+i} \right] , \tag{18}$$

where the subjective discount factor is $\beta^i \Lambda_{t,t+i} = \beta^i \left(\frac{C_{t+i}}{C_t} \right)^{-\eta_c}$

The Model: Intermediate Goods Firm

The pricing decision with sticky prices

The FOC for the optimal $P^*_{j,t}$:

$$\frac{P^*_{j,t}}{P_t} = \left(\frac{\varphi_{p,t}}{\varphi_{p,t}-1}\right)\frac{E_t\sum_{i=0}^{\infty}(\theta_p\beta)^i\Lambda_{t,t+i}\left[MC_{t+i}(\frac{P_t}{P_{t+i}})^{\varphi_{p,t}}Y_{t+i}\right]}{E_t\sum_{i=0}^{\infty}(\theta_p\beta)^i\Lambda_{t,t+i}\left[(\frac{P_t}{P_{t+i}})^{\varphi_{p,t}-1}Y_{t+i}\right]}\,. \qquad (19)$$

We can now drop *j* because all firm's face the same decision rule.

When prices are *sticky* ($0 < \theta_p < 1$), the optimal price is a markup over a weighted average of current and expected future nominal marginal costs. Weights depend on expected demand in future and on how quickly firms discount profits.

The Model: Intermediate Goods Firm

The New Keynesian Phillips Curve
Log-linearising the price index (17) and the pricing decision (19) gives:

$$\begin{aligned} \hat{p}_t &= \theta_p \hat{p}_{t-1} + (1-\theta_p)\hat{p}_t^* && (20) \\ \hat{p}_t^* &= (1-\theta_p)\beta\Sigma_{i=0}^{\infty}(\theta_p\beta)^i E_t(\hat{p}_{t+i} + \widehat{mc}_{t+i}) && (21) \end{aligned}$$

where (21) has the same interpretation as (19).

Combining (20) and (21) gives us the NKPC:[5]

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \widehat{mc}_t \quad (22)$$

where $\kappa = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}$

[5] For the detailed derivation see the appendix section 8.6 in Walsh (2017).

The Model: The New Keynesian Phillips Curve

Remarks: compared to the traditional NKPC ($\pi_t = \pi_{t-1} + \alpha \tilde{y}_t^{ad} + \xi_t$), the NKPC has the following properties:

1. derived explicitly from agents' optimization problems;
2. inflation process is forward-looking;
3. marginal cost is the 'correct' (micro-founded) transmission mechanism for the inflation process (not depend on an ad hoc measure of the output gap $\tilde{y}_t^{ad}$);

But: We can also relate real MC ($\widehat{mc}_t$) to the output gap $\tilde{y}_t = \hat{y}_t - \hat{y}_t^f$:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \tilde{\kappa}(\tilde{y}_t) \ , \tag{23}$$

where $\tilde{\kappa} = (\eta_n + \eta_c)\kappa$ (show below).

The Model: The New Keynesian Phillips Curve

The pricing decision with flexible prices

When $\theta_p = 0$, (19) becomes:

$$\frac{P_t^*}{P_t} = \frac{\varphi_{p,t}}{\varphi_{p,t} - 1} MC_t = \psi_t MC_t \ , \tag{24}$$

where each firm sets its price equal to a markup over its nominal marginal cost: $P_t^* = \psi_t P_t MC_t$.[6]

When prices are *flexible* all firms can charge the same price: $P_t^* = P_t \ \Rightarrow \ MC_t = 1/\psi_t$.[7]

[6] Note: (1) the markup ψ is inversely related to the price elasticity of demand φ_p; (2) in Walsh (2017), the price elasticity of demand is assumed constant, $\psi_{t=0}$.

[7] Under a flexible equilibrium with perfect competition: $\varphi_p = \infty$ and $\psi = 1 \Rightarrow MC = 1$.

The Model: The New Keynesian Phillips Curve

Recall $MC_t = \lambda_t = \frac{W_t}{P_t}/\xi_{z,t}$ from (15), we have, under flexible prices and assuming no time-varying price elasticity:

$$\frac{W_t^f}{P_t} = \frac{\xi_{z,t}}{\psi} \tag{25}$$

Combining (25) and the FOC for labour supply for the household (3), we have:

$$\frac{W_t^f}{P_t} = \frac{\xi_{z,t}}{\psi} = (N_t^f)^{\eta_n}(C_t^f)^{\eta_c} \tag{26}$$

which gives us ***the labour market equilibrium***:

$$\xi_{z,t} = \psi(N_t^f)^{\eta_n}(C_t^f)^{\eta_c} \tag{27}$$

Log-linearising (27), $\eta_n \hat{n}_t^f + \eta_c \hat{c}_t^f = \hat{\xi}_{z,t}$, and the production function, $\hat{y}_t^f = \hat{n}_t^f + \hat{\xi}_{z,t}$, and the resource constraint, $\hat{y}_t^f = \hat{c}_t^f$, and combining, gives us the ***flexible price equilibrium output***:

$$\hat{y}_t^f = \frac{1+\eta_n}{\eta_c+\eta_n}\hat{\xi}_{z,t} \tag{28}$$

where $\hat{\xi}_{z,t}$ follows an exogenous stochastic AR(1) process with i.i.d innovations.

Nonlinear Equilibrium Conditions

- Aggregate resource constraint:

$$Y_t = C_t \tag{29}$$

- Euler equation for Bonds:

$$R_t \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\eta_c} \right] = 1 \qquad ((4))$$

- Aggregate production function:

$$Y_t = \xi_{z,t} N_t \qquad ((13))$$

- Labor market equilibrium:[8]

$$MC_t = \frac{N_t^{\eta_n} C_t^{\eta_c}}{\xi_{z,t}}, \qquad ((27))$$

- Demand for money:

$$C_t^{\frac{\eta_c}{\eta_m}} \left(\frac{i_t}{1+i_t} \right)^{-\frac{1}{\eta_m}} = \frac{M_t}{P_t} \qquad ((5))$$

- Aggregate price:

$$P_t^{1-\varphi_{p,t}} = \theta_p P_{t-1}^{1-\varphi_{p,t}} + (1-\theta_p)(P_t^*)^{1-\varphi_{p,t}}, \qquad 0 \leq \theta_p \leq 1, \qquad ((17))$$

- Price setting equation for P_t^* ... ((19))

[8] note: $MC_t = \lambda_t = W_t/P_t/\xi_{z,t}$, but we subst W_t/P_t out of equilibrium. In flex-price equilibrium, $MC_t = 1$.

The Log-Linearised Equilibrium Conditions

$$\hat{y}_t = \hat{c}_t \tag{30}$$

$$\hat{y}_t = \hat{n}_t + \hat{\xi}_{z,t} \tag{31}$$

$$\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\eta_c}\hat{r}_t \tag{32}$$

$$\hat{m}_t = \frac{\eta_c}{\eta_m}\hat{c}_t - (\frac{1}{\eta_m})(\frac{1}{1+i})\hat{i}_t \tag{33}$$

$$\hat{i}_t = \hat{r}_t + E_t\hat{\pi}_{t+1} \tag{34}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\hat{mc}_t \tag{35}$$

$$\hat{mc}_t = (\eta_n + \eta_c)\hat{y}_t - (1+\eta_n)\hat{\xi}_{z,t} \tag{36}$$

$$\hat{\xi}_{z,t} = \rho_z\hat{\xi}_{z,t-1} + \varepsilon_t^z \ , \quad \varepsilon_t^z \sim \mathcal{N}(0,\sigma^2) \quad \text{i.i.d.} \tag{37}$$

To complete the model we need to add a **monetary policy reaction function**, such as a Taylor-type interest rate rule or a money supply rule. But first . . .

- **Note:** $\hat{m}_t$ in (33) is assumed to be *real* money balances; nominal money supply is fully endogenous and adjusts immediately to price level shifts in this baseline NK model.
- We can introduce the price level as follows:

$$\hat{m}_t - \hat{p}_t = \frac{\eta_c}{\eta_m}\hat{c}_t - (\frac{1}{\eta_m})(\frac{1}{1+i})\hat{i}_t$$

- Doing so requires including the definition of inflation in our list of equilibrium conditions:

$$\hat{\pi}_t = \hat{p}_t - \hat{p}_{t-1}$$

- Notice that the price level is indeterminate! You get the same result modelling money either way!
- In the appendix, I show you a simple alternative way to introduce a non-superficial role for money in the standard NK model. Can you introduce this in your Dynare code example?

The Flex-Price Equilibrium

We may also be interested in the flexible price equilibrium ($\pi_t = 0$) as our efficient benchmark:

$$
\begin{aligned}
\hat{y}_t^f &= \hat{c}_t^f && (38)\\
\hat{y}_t^f &= \hat{n}_t^f + \hat{\xi}_{z,t} && (39)\\
\hat{c}_t^f &= E_t \hat{c}_{t+1}^f - \frac{1}{\eta_c} \hat{r}_t^n && (40)\\
\hat{m}_t^f &= \frac{\eta_c}{\eta_m} \hat{c}_t^f - (\frac{1}{\eta_m})(\frac{1}{1+r}) \hat{r}_t^n && (41)\\
\hat{y}_t^f &= \frac{(1+\eta_n)}{(\eta_n + \eta_c)} \hat{\xi}_{z,t} && (42)
\end{aligned}
$$

This also allows as to model the natural interest rate (r_t^n) and output gap ($\tilde{y}_t = \hat{y}_t - \hat{y}_t^f$). Note: since $\pi = 0$ in steady-state, $\frac{1}{1+r} = \frac{1}{1+i} = \beta$.

The Policy Authority

- To complete the **New Keynesian** model we add a monetary policy reaction function that follows a **Taylor-type interest rate rule**:

$$\hat{i}_t \quad = \quad \rho_i \hat{i}_{t-1} + (1 - \rho_i)(\kappa_\pi \hat{\pi}_t + \kappa_y \tilde{y}_t) + \xi_{i,t} \ , \tag{43}$$

  where ρ_i captures policy rate smoothing, $\{\kappa_\pi, \kappa_y\}$ the weights on inflation and the output gap, and $\xi_{i,t}$ captures exogenous monetary policy changes.
- Next, we can simplify the model further by combining equations and, finally, we can ignore the demand for money equation since money is fully endogenous when we use the Taylor rule.
- In your **Tutorial** you will compare an interest rate rule to a money supply rule.
- You can also compare a range of different rules (see, e.g., p.8 in Benchimol & Fourcans, 2019)

The Baseline (3-Equation) NK Model

The following three equations represent the equilibrium condition for a well-specified general equilibrium model.

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \frac{1}{\eta_c}(\hat{i}_t - E_t\hat{\pi}_{t+1} - r_t^n) \tag{44}$$

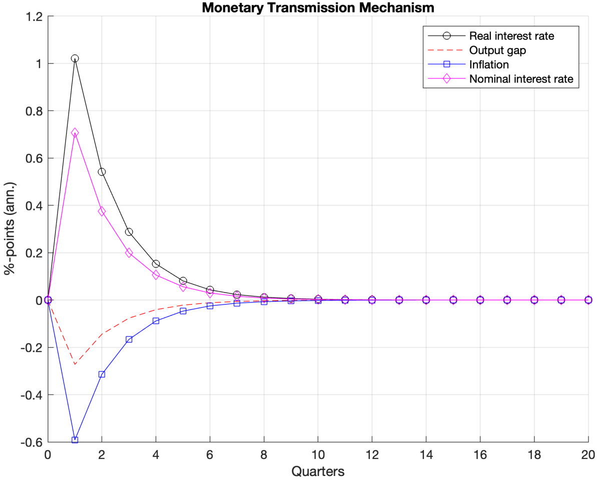
$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \tilde{\kappa}\tilde{y}_t \tag{23}$$

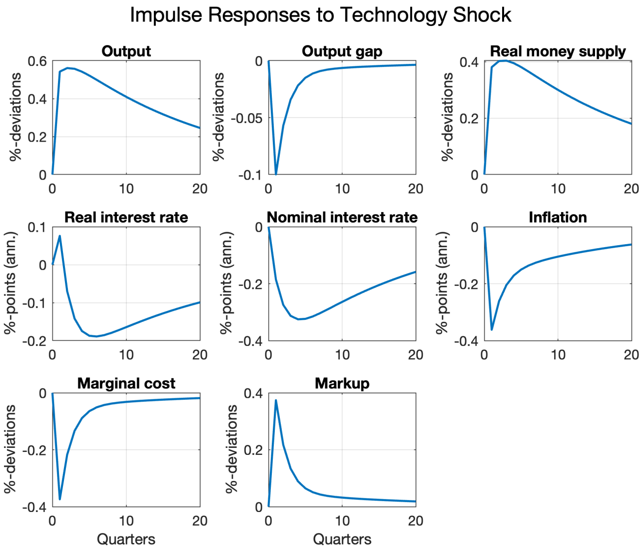
where, $r_t^n = \eta_c[E_t(\hat{y}_{t+1}^f - \hat{y}_t^f)] = \eta_c[E_t\Delta\xi_{z,t+1}(\frac{1+\eta_n}{\eta_c+\eta_n})]$, from (36).

- (44) : AD, an expectational forward-looking IS curve; derived from the Euler condition for the representative household's decision problem.
- (23) : AS, the NKPC; derived from the pricing decisions of individual firms.
- (43) : MP, demand-side stabilisation policy
  (Romer assumes no interest rate smoothing ($\rho_i = 0$): $\hat{i}_t = \rho_\pi\hat{\pi}_t + \rho_y\tilde{y}_t + \xi_{i,t}$).

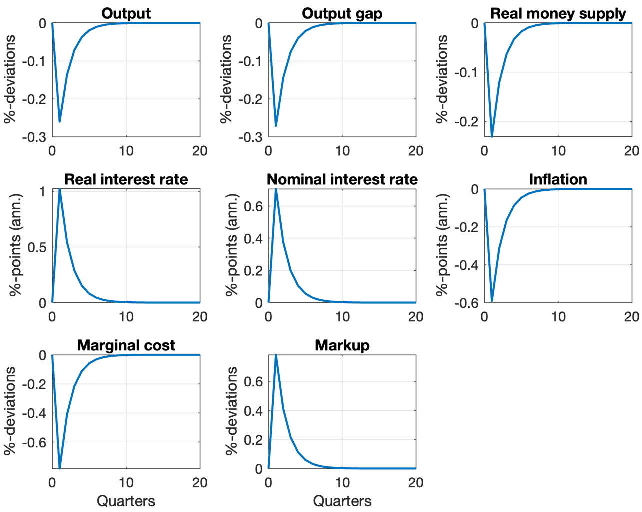
Monetary Transmission Mechanism

Monetary policy alters real rate: short-run non-neutrality of monetary policy




Impulse Responses to Technology Shock
Output
Output gap
Real money supply
Real interest rate
Nominal interest rate
Inflation
Marginal cost
Markup
%-deviations
%-points (ann.)
Quarters
0
10
20
0.6
0.4
0.2
-0.05
-0.1
0.1
-0.1
-0.2
-0.4

Impulse Responses to Monetary Policy Shock



Optimal Monetary Policy in NK Models

- Distortions caused by price stickiness lead to short-run non-optimal fluctuations in relative prices.
- This price dispersion in the intermediate goods sector generates a welfare loss.
- The central bank therefore dislikes output gaps and inflation, and setting $\pi_t = \tilde{y}_t = 0$ will eliminate price distortions from the Phillips curve
- We assume that the central bank seeks to minimize the following quadratic loss function:

$$\min_{\pi_t, \tilde{y}_t} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{\pi}_t^2 + \omega \tilde{y}_t^2 \right)$$

subject to (23): $\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \tilde{\kappa} \tilde{y}_t$

where $\omega = \tilde{\kappa}/\varphi_p$ and φ_p is the price elasticity of demand.

The ‘Divine Coincidence’ in the Baseline NK Model

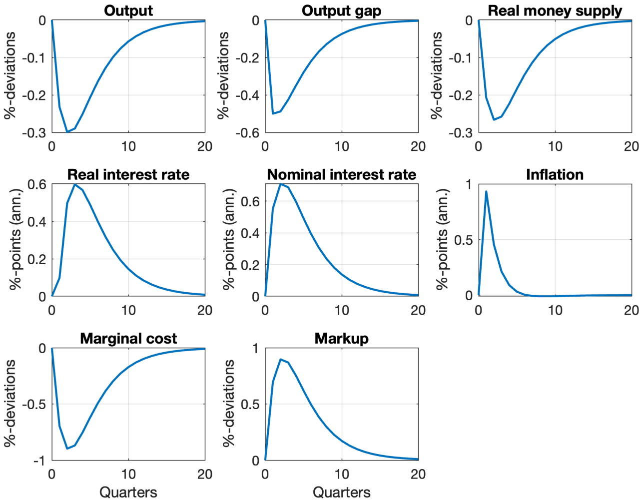
- This standard loss function representation can be derived from a quadratic approximation of household welfare.
    - in other words, the policy that minimises this loss function the most maximises lifetime household welfare; the global maximum is hence achieved when $\pi_t = \tilde{y}_t = 0$.
    - In the baseline New Keynesian model, the divine coincidence refers to the result that stabilizing inflation also stabilizes the output gap. That is, monetary policy faces no tradeoff between output stabilization and inflation stabilization — both are achieved simultaneously by targeting inflation. This is the ‘divine coincidence’ coined by Blanchard and Gali (2007)
- This representation also requires a sufficiently small utility weight on real money balances ($a \to 0$).
- Collard and Dellas (2005) show that welfare rankings are robust to a relative risk aversion coefficient greater than one ($\eta_c > 1$), and that the assumption of a separable utility function makes a negligible difference.

Optimal Policy Rules: Discretion vs Commitment

- Minimising the loss function subject to the NKPC constraint gives the optimal policy rule under discretion: $\tilde{y}_t = -(\tilde{\kappa}/\omega)\hat{\pi}_t$.
- In the baseline NK model presented, no welfare gain arises from commitment relative to discretion.
- For example, there is no gain from commitment to a price level target, $\tilde{y}_t = -(\tilde{\kappa}/\omega)\hat{p}_t$, over period-by-period policy discretion.[9]
- Indeed, it is always possible for the central bank to eliminate output gaps and inflation under any nominal (e.g., money demand or preference) or technology shock in this baseline model.
- One way to force an output-inflation trade-off is to introduce a cost-push (price markup) shock to (23): assuming that $\varphi_{p,t}$ is time-varying allows us to define the markup ψ_t as a exogenous stochastic AR(1) process: $\hat{\xi}_{p,t} = \rho_p \hat{\xi}_{p,t-1} + \epsilon_t^p$

[9] With $\beta \approx 1$, the optimal commitment policy becomes price-level targeting! Can you show this mathematically?

Impulse Responses to Cost-Push (Markup) Shock



Efficient Benchmark: Modified Taylor Rule

- Now, consider an implementable optimal simple rule that mimics the derived optimal policy:

$$\hat{i}_t = \hat{r}_t^n + \kappa_\pi \hat{\pi}_t + \kappa_y \tilde{y}_t$$

- When the central bank hits its objectives: $\hat{\pi}_t = \tilde{y}_t = 0 \Rightarrow \hat{i}_t = \hat{r}_t^n$
- This characterizes a neutral interest rate policy in the absence of shocks.
- We interpret the estimated rule for the Great Moderation as reflecting feasible conduct, whereas this rule serves as the normative benchmark.
- **Tutorial**: run optimal simple rules scenarios using `osr` in `Dynare`.

Appendix: alternative baseline NK model with money

Hollander & Christensen (2022)

$$\text{Fisher relation}: \quad i_t = E_t\left[\pi_{t+1}\right] + \left[r_t^n + \eta_c\left(E_t\left[\tilde{y}_{t+1}\right] - \tilde{y}_t\right)\right] \tag{45}$$

$$\text{Money demand}: \quad m_t - p_t = \frac{\eta_c}{\eta_m} y_t - \frac{1}{\eta_m} i_t + \xi_{m_d,t} \tag{46}$$

$$\text{Euler equation}: \quad r_t = r_t^n + \eta_c\left(E_t\left[\tilde{y}_{t+1}\right] - \tilde{y}_t\right) \tag{47}$$

$$\text{Natural rate}: \quad r_t^n = \eta_c\left(E_t\left[y_{t+1}^n\right] - y_t^n\right) \tag{48}$$

$$\text{Money supply (broad)}: \quad m_t = \left(1 + \phi_{rr}\right) h_t \tag{49}$$

$$\text{Money supply (base)}: \quad h_t = \rho_h h_{t-1} - v_h\left(i_t - i_t^T\right) + \xi_{m_s,t} \tag{50}$$

$$\text{Policy target rate}: \quad i_t^T = \rho_i i_{t-1}^T + \left(1 - \rho_i\right)\left(\kappa_\pi \pi_t + \kappa_y \tilde{y}_t\right) + \xi_{1,t} \tag{51}$$

$$\text{NK Phillips curve}: \quad \pi_t = \beta E_t\left[\pi_{t+1}\right] + \tilde{\kappa}\tilde{y}_t \tag{52}$$

$$\text{Output gap}: \quad \tilde{y}_t = y_t - y_t^n \tag{53}$$

$$\text{Natural output}: \quad y_t^n = \left(1 + \eta_n\right) / \left(\eta_c + \eta_n\right) \xi_{z,t} \tag{54}$$

$$\text{Inflation definition}: \quad \pi_t = p_t - p_{t-1} \tag{55}$$

Key differences: for your `Dynare` code simply add (49), (50), and redefine (51) as a new variable. You can assume $\phi_{rr} = 0$, so that $m_t = h_t$ and you can ignore (49). For a money supply rule: simply embed $(1 - \rho_h)\left(\kappa_\pi \pi_t + \kappa_y \tilde{y}_t\right)$ in (50); set $v_h = 0$ for strict money rule $\rightarrow$ (51) redundant; $v_h > 0$ is a combination policy.

Appendix: Derivation of $p^*_{j,t}$

Substituting (11) into (18), yields:

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \left[\frac{P^*_{j,t}}{P_{t+i}} \left(\frac{P^*_{j,t}}{P_{t+i}}\right)^{-\varphi_{p,t}} Y_{t+i} - \frac{MC_{t+i}}{P_{t+i}} \left(\frac{P^*_{j,t}}{P_{t+i}}\right)^{-\varphi_{p,t}} Y_{t+i} \right] \tag{56}$$

Note: MC_{t+i}, here, is the nominal marginal cost ($P_{t+i}\lambda_{t+i}$). FOC gives:

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \left[(1-\varphi_{p,t}) \left(\frac{P^*_{j,t}}{P_{t+i}}\right)^{-\varphi_{p,t}} \frac{Y_{t+i}}{P_{t+i}} + \varphi_{p,t} \frac{MC_{t+i}}{P_{t+i}} \left(\frac{P^*_{j,t}}{P_{t+i}}\right)^{-\varphi_{p,t}-1} \frac{Y_{t+i}}{P_{t+i}} \right] = 0 \tag{57}$$

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \left[(1-\varphi_{p,t}) \left(\frac{P^*_{j,t}}{P_{t+i}}\right)^{1-\varphi_{p,t}} Y_{t+i} + \varphi_{p,t} \frac{MC_{t+i}}{P_{t+i}} \left(\frac{P^*_{j,t}}{P_{t+i}}\right)^{-\varphi_{p,t}} Y_{t+i} \right] = 0 \tag{58}$$

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \left[\left(\frac{P^*_{j,t}}{P_{t+i}}\right)^{1-\varphi_{p,t}} Y_{t+i} + \frac{\varphi_{p,t}}{1-\varphi_{p,t}} \frac{MC_{t+i}}{P_{t+i}} \left(\frac{P^*_{j,t}}{P_{t+i}}\right)^{-\varphi_{p,t}} Y_{t+i} \right] = 0 \tag{59}$$

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \left[\left(\frac{P^*_{j,t}}{P_{t+i}}\right)^{1-\varphi_{p,t}} Y_{t+i} + \frac{\varphi_{p,t}}{1-\varphi_{p,t}} MC_{t+i} \frac{P^*_{j,t}}{P_{t+i}} \left(\frac{P^*_{j,t}}{P_{t+i}}\right)^{-\varphi_{p,t}} \frac{1}{P^*_{j,t}} Y_{t+i} \right] = 0 \tag{60}$$

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \left[\left(\frac{P^*_{j,t}}{P_{t+i}}\right)^{1-\varphi_{p,t}} Y_{t+i} + \frac{\varphi_{p,t}}{1-\varphi_{p,t}} MC_{t+i} \left(\frac{P^*_{j,t}}{P_{t+i}}\right)^{1-\varphi_{p,t}} \frac{1}{P^*_{j,t}} Y_{t+i} \right] = 0 \tag{61}$$

The optimal $P^*_{j,t}$:

$$P^*_{j,t} = \frac{E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \left[\frac{\varphi_{p,t}}{\varphi_{p,t}-1} MC_{t+i} \left(\frac{1}{P_{t+i}}\right)^{1-\varphi_{p,t}} Y_{t+i} \right]}{E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \left[\left(\frac{1}{P_{t+i}}\right)^{1-\varphi_{p,t}} Y_{t+i} \right]} \tag{62}$$

Appendix Derivations: A. FOCs with (external) habit formation; B. Calvo pricing with price indexation (the hybrid NKPC)

Appendix A: FOCs
Household
In the NKDSGE model, the representative household chooses $\{C_t, N_t, M_t, B_t\}$ to maximize the utility function:

$$E\sum_{t=0}^{\infty}\beta^t\left[\frac{(C_t-H_t)^{1-\eta c}}{1-\eta c}+\xi_{m,t}\frac{(M_t/P_t)^{1-\eta m}}{1-\eta m}-\frac{(N_t)^{1+\eta n}}{1+\eta n}\right], \tag{A.1}$$

where external habit formation is defined as $H_t = hC_{t-1}$. This means that the household does not take lagged consumption into consideration when optimising.
where $0 < \beta < 1, \quad 0 < h < 1, \quad \eta c, \eta m > 0, \quad \eta n \geq 0.$
subject to the budget constraint:

$$C_t+\frac{B_t}{P_t}+\frac{M_t}{P_t}\leq\frac{W_t}{P_t}N_t+\frac{B_{t-1}(1+i_{t-1})}{P_t}+\frac{M_{t-1}}{P_t}+\frac{D_t}{P_t} \tag{A.2}$$

The resulting Bellman's equation is as follows:

$$V(M_{t-1},B_{t-1})=\max\left[\frac{(C_t-H_t)^{1-\eta c}}{1-\eta c}+\xi_{m,t}\frac{(M_t/P_t)^{1-\eta m}}{1-\eta m}-\frac{(N_t)^{1+\eta n}}{1+\eta n}+\beta E_t V(M_t,B_t)\right]-\lambda_t\,[] \tag{Eqa2}$$

Substituting C_t from (A.2) and solving this problem yields the following first order conditions (FOC). Note: we can now substitute in $H_t = hC_{t-1}$.
FOC for hours worked:

$$\frac{\partial V(M_{t-1},B_{t-1})}{\partial N_t} = 0 \tag{A.3}$$

$$(C_t-hC_{t-1})^{-\eta c}\frac{W_t}{P_t}-N_t^{\eta n} = 0 \tag{A.4}$$

$$(C_t-hC_{t-1})^{-\eta c}\frac{W_t}{P_t} = N_t^{\eta n} \tag{A.5}$$

$$\frac{N_t^{\eta n}}{(C_t-hC_{t-1})^{-\eta c}} = \frac{W_t}{P_t} \tag{A.6}$$

The FOC for bond holdings is given as follows:

$$\frac{\partial V(M_{t-1}, B_{t-1})}{\partial B_t} = 0 \tag{A.7}$$

$$(C_t - hC_{t-1})^{-\eta c}(-\frac{1}{P_t}) + \frac{\partial \beta E_t V(M_t, B_t)}{\partial B_t} = 0 \tag{A.8}$$

The associated envelope condition is:

$$\frac{\partial V(M_{t-1}, B_{t-1})}{\partial B_{t-1}} = (C_t - hC_{t-1})^{-\eta c} \frac{1 + i_{t-1}}{P_t} \tag{A.9}$$

Updating (A.9) and combining with (A.8) yields

$$(C_t - hC_{t-1})^{-\eta c}(-\frac{1}{P_t}) + \beta E_t[(C_{t+1} - hC_t)^{-\eta c} \frac{1 + i_t}{P_{t+1}}] = 0 \tag{A.10}$$

$$\beta E_t[(C_{t+1} - hC_t)^{-\eta c} \frac{1 + i_t}{P_{t+1}}] = (C_t - hC_{t-1})^{-\eta c}(\frac{1}{P_t}) \tag{A.11}$$

$$(1 + i_t)\beta E_t[(C_{t+1} - hC_t)^{-\eta c} \frac{p_t}{P_{t+1}}] = (C_t - hC_{t-1})^{-\eta c} \tag{A.12}$$

$$\beta E_t[(C_{t+1} - hC_t)^{-\eta c} \frac{1 + i_t}{1 + \pi_{t+1}}] \frac{1}{(C_t - hC_{t-1})^{-\eta c}} = 1 \tag{A.13}$$

$$R_t \beta E_t\left[(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}})^{-\eta c}\right] = 1 \tag{A.14}$$

where R_t is the gross real rate of return, $R_t = 1 + r_t$.

FOC for money holdings can be derived as follows:

$$\frac{\partial V(M_{t-1},B_{t-1})}{\partial M_t} = 0 \tag{A.15}$$

$$(C_t - hC_{t-1})^{-\eta c}(\frac{-1}{P_t}) + \xi_{m,t}(\frac{M_t}{p_t})^{-\eta m}(\frac{1}{P_t}) + \frac{\partial \beta E_t V(M_t,B_t)}{\partial M_t} = 0 \tag{A.16}$$

The associated envelope condition is:

$$\frac{\partial V(M_{t-1},B_{t-1},K_{t-1})}{\partial M_{t-1}} = (C_t - hC_{t-1})^{-\eta c}(\frac{1}{P_t}) \tag{A.17}$$

Updating (A.17) and combining with (A.16), we have:

$$(C_t - hC_{t-1})^{-\eta c}(\frac{-1}{P_t}) + \xi_{m,t}(\frac{M_t}{p_t})^{-\eta m}(\frac{1}{P_t}) + \beta E_t[(C_{t+1} - hC_t)^{-\eta c}(\frac{1}{P_{t+1}})] = 0 \tag{A.18}$$

$$(C_t - hC_{t-1})^{-\eta c} - \beta E_t[(C_{t+1} - hC_t)^{-\eta c}(\frac{P_t}{P_{t+1}})] = \xi_{m,t}(\frac{M_t}{p_t})^{-\eta m} \tag{A.19}$$

Using (A.12):

$$(C_t - hC_{t-1})^{-\eta c} - (\frac{1}{1+i_t})(C_t - hC_{t-1})^{-\eta c} = \xi_{m,t}(\frac{M_t}{p_t})^{-\eta m} \tag{A.20}$$

$$\frac{i_t}{1+i_t}(C_t - hC_{t-1})^{-\eta c} = \xi_{m,t}(\frac{M_t}{p_t})^{-\eta m} \tag{A.21}$$

$$(C_t - hC_{t-1})^{\frac{\eta c}{\eta m}}(\frac{i_t}{1+i_t})^{-\frac{1}{\eta m}}\xi_{m,t}^{\frac{1}{\eta m}} = \frac{M_t}{P_t} \tag{A.22}$$

Final goods firm
A representative final-goods firm produces the final good Y_t using intermediate goods $Y_{j,t}$ according to the CES production function:

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\varphi_{p,t}-1}{\varphi_{p,t}}} dj\right)^{\frac{\varphi_{p,t}}{\varphi_{p,t}-1}} \tag{A.23}$$

The firm minimizes its costs:

$$min \int_0^1 P_{j,t} Y_{j,t} dj \tag{A.24}$$

given the production constraint:

$$Y_t \leq \left(\int_0^1 Y_{j,t}^{\frac{\varphi_{p,t}-1}{\varphi_{p,t}}} dj\right)^{\frac{\varphi_{p,t}}{\varphi_{p,t}-1}} \tag{A.25}$$

The Lagrangean for the firm is given by the following expression:

$$L = \int_0^1 P_{j,t} Y_{j,t} dj + \lambda\left[Y_t - \left(\int_0^1 Y_{j,t}^{\frac{\varphi_{p,t}-1}{\varphi_{p,t}}} dj\right)^{\frac{\varphi_{p,t}}{\varphi_{p,t}-1}}\right] \tag{A.26}$$

The first order condition with respect to $Y_{j,t}$ is:

$$\begin{aligned}
P_{j,t} - \lambda \frac{\partial Y_t}{\partial Y_{j,t}} &= 0 && \text{(A.27)}\\
P_{j,t} - \lambda \frac{\varphi_{p,t}}{\varphi_{p,t}-1}\left(\int_0^1 Y_{j,t}^{\frac{\varphi_{p,t}-1}{\varphi_{p,t}}} dj\right)^{\frac{1}{\varphi_{p,t}-1}} \left(\frac{\varphi_{p,t}-1}{\varphi_{p,t}}\right) Y_{j,t}^{-1/\varphi_{p,t}} &= 0 && \text{(A.28)}\\
P_{j,t} - \lambda Y_t^{-\varphi_{p,t}} Y_{j,t}^{\varphi_{p,t}} &= 0 && \text{(A.29)}\\
P_{j,t} &= \lambda \left(\frac{Y_{j,t}}{Y_t}\right)^{\varphi_{p,t}} && \text{(A.30)}
\end{aligned}$$

To solve for the Lagrange multiplier λ, consider that given Euler's theorem, profits in final-goods sector must equal to zero in equilibrium:

$$P_t Y_t = \int_0^1 P_{j,t} Y_{j,t} dj \tag{A.31}$$

This condition implies that the equilibrium price for the final-good is:

$$P_t = \frac{1}{Y_t} \int_0^1 P_{j,t} Y_{j,t} dj \tag{A.32}$$

Using (A.27), yields:

$$\begin{aligned}
P_{j,t} &= \lambda \frac{\partial Y_t}{\partial Y_{j,t}} && \text{(A.33)} \\
\frac{\partial Y_{j,t}}{\partial Y_t} P_{j,t} &= \lambda && \text{(A.34)} \\
\frac{1}{Y_t} \int_0^1 \frac{\partial Y_{j,t}}{\partial Y_t} \frac{Y_t}{Y_{j,t}} P_{j,t} Y_{j,t} dj &= \lambda && \text{(A.35)} \\
\frac{1}{Y_t} \int_0^1 P_{j,t} Y_{j,t} dj &= \lambda && \text{(A.36)}
\end{aligned}$$

Combining (A.32) and (A.36), we obtain:

$$\lambda = P_t \tag{A.37}$$

Substituting (A.37) into (A.30), we obtain:

$$P_{j,t} = \left(\frac{Y_{j,t}}{Y_t}\right)^{\varphi_{p,t}} P_t \tag{A.38}$$

or

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\varphi_{p,t}} Y_t \tag{A.39}$$

Substituting (A.39) into (A.31), we obtain:

$$\begin{aligned}
P_t Y_t &= \int_0^1 P_{j,t}\left(\frac{P_{j,t}}{P_t}\right)^{-\varphi_{p,t}} Y_t dj && \text{(A.40)}\\
P_t &= \int_0^1 P_{j,t}^{1-\varphi_{p,t}}\left(\frac{1}{P_t}\right)^{-\varphi_{p,t}} dj && \text{(A.41)}\\
P_t P_t^{-\varphi_{p,t}} &= \int_0^1 P_{j,t}^{1-\varphi_{p,t}} dj && \text{(A.42)}\\
P_t &= \left(\int_0^1 P_{j,t}^{1-\varphi_{p,t}} dj\right)^{1/1-\varphi_{p,t}} && \text{(A.43)}
\end{aligned}$$

Appendix B: Calvo-type sticky prices and indexation (hybrid NKPC)

Following Calvo (1983), in each time period only a random fraction $1-\theta_p$ of intermediate-good firms have the opportunities to reset prices (this fraction is independent from the previous period). In addition, the model assumes that those firms who cannot reset the prices simply index to lagged inflation as in Smets and Wouters (2007):

$$P_{j,t} = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p} P_{j,t-1}, \qquad 0 \leq \gamma_p \leq 1 \tag{B.1}$$

where γ_p is the degree of price indexation, a fraction of price setters index their prices to the lagged inflation.

Assuming a CES aggregate of the average price level, the aggregate price index is given by:

$$P_{j,t}^{1-\varphi_{p,t}} = \theta_p[(\frac{P_{t-1}}{P_{t-2}})^{\gamma_p} P_{j,t-1}]^{1-\varphi_{p,t}} + (1-\theta_p)(P_{j,t}^*)^{1-\varphi_{p,t}}, \qquad 0 \leq \theta_p \leq 1, \quad 0 \leq \gamma_p \leq 1 \tag{B.2}$$

where $P_{j,t-1}$ is previous price level, and $P_{j,t}^*$ is the average price chosen by those who have the chance to reset the prices.

In each time period, the representative intermediate-good firm chooses $P_{j,t}^*$ to maximize its profit:

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} [(\frac{P_{j,t}^*}{P_{t+i}} - \frac{MC_{t+i}}{P_{t+i}}) Y_{j,t,t+i}] \tag{B.3}$$

Where

$$\Lambda_{t,t+i} = (\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}})^{-\eta c} \tag{B.4}$$

Substituting (A.39) into (B.3), yields:

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} [\frac{P_{j,t}^*}{P_{t+i}} (\frac{P_{j,t}^*}{P_{t+i}})^{-\varphi_{p,t}} Y_{t+i} - \frac{MC_{t+i}}{P_{t+i}} (\frac{P_{j,t}^*}{P_{t+i}})^{-\varphi_{p,t}} Y_{t+i}] \tag{B.5}$$

FOC w.r.t. $p^*_{j,t}$:

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \Big[(1 - \varphi_{p,t}) (\frac{P^*_{j,t}}{P_{t+i}})^{-\varphi_{p,t}} \frac{Y_{t+i}}{P_{t+i}} + \varphi_{p,t} \frac{MC_{t+i}}{P_{t+i}} (\frac{P^*_{j,t}}{P_{t+i}})^{-\varphi_{p,t}-1} \frac{Y_{t+i}}{P_{t+i}} \Big] = 0 \quad \text{(B.6)}$$

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \Big[(1 - \varphi_{p,t}) (\frac{P^*_{j,t}}{P_{t+i}})^{1-\varphi_{p,t}} Y_{t+i} + \varphi_{p,t} \frac{MC_{t+i}}{P_{t+i}} (\frac{P^*_{j,t}}{P_{t+i}})^{-\varphi_{p,t}} Y_{t+i} \Big] = 0 \quad \text{(B.7)}$$

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \Big[(\frac{P^*_{j,t}}{P_{t+i}})^{1-\varphi_{p,t}} Y_{t+i} + \frac{\varphi_{p,t}}{1-\varphi_{p,t}} \frac{MC_{t+i}}{P_{t+i}} (\frac{P^*_{j,t}}{P_{t+i}})^{-\varphi_{p,t}} Y_{t+i} \Big] = 0 \quad \text{(B.8)}$$

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \Big[(\frac{P^*_{j,t}}{P_{t+i}})^{1-\varphi_{p,t}} Y_{t+i} + \frac{\varphi_{p,t}}{1-\varphi_{p,t}} MC_{t+i} \frac{P^*_{j,t}}{P_{t+i}} (\frac{P^*_{j,t}}{P_{t+i}})^{-\varphi_{p,t}} \frac{1}{P^*_{j,t}} Y_{t+i} \Big] = 0 \quad \text{(B.9)}$$

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \Big[(\frac{P^*_{j,t}}{P_{t+i}})^{1-\varphi_{p,t}} Y_{t+i} + \frac{\varphi_{p,t}}{1-\varphi_{p,t}} MC_{t+i} (\frac{P^*_{j,t}}{P_{t+i}})^{1-\varphi_{p,t}} \frac{1}{P^*_{j,t}} Y_{t+i} \Big] = 0 \quad \text{(B.10)}$$

The optimal $P^*_{j,t}$:

$$P^*_{j,t} = \frac{E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \Big[\frac{\varphi_{p,t}}{\varphi_{p,t}-1} MC_{t+i} (\frac{1}{P_{t+i}})^{1-\varphi_{p,t}} Y_{t+i} \Big]}{E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \Big[(\frac{1}{P_{t+i}})^{1-\varphi_{p,t}} Y_{t+i} \Big]} \quad \text{(B.11)}$$

Marginal cost and the output gap
Linearising (25):

$$\hat{mc}_t = \hat{w}_t - \hat{p}_t - \hat{\xi}_{z,t} \quad \text{(B.12)}$$

Linearising the FOC for labor (3):

$$\hat{w}_t - \hat{p}_t = \eta_n \hat{n}_t + \eta_c \hat{c}_t \quad \text{(B.13)}$$

Replacing $\hat{c}_t$ by $\hat{y}_t$ according to the linearized aggregate resource constraint in (B.13) and substituting into (B.12):

$$\hat{mc}_t = \eta_n \hat{n}_t + \eta_c \hat{y}_t - \hat{\xi}_{z,t} \tag{B.14}$$

Substituting the linearize production function $\hat{y}_t = \hat{n}_t + \hat{\xi}_{z,t}$ into (B.14), we have:

$$\hat{mc}_t = \eta_n(\hat{y}_t - \hat{\xi}_{z,t}) + \eta_c \hat{y}_t - \hat{\xi}_{z,t} \tag{B.15}$$

$$\hat{mc}_t = (\eta_n + \eta_c)\hat{y}_t - (1 + \eta_n)\hat{\xi}_{z,t} \tag{B.16}$$

$$\hat{mc}_t = (\eta_n + \eta_c)[\hat{y}_t - (\frac{1 + \eta_n}{\eta_n + \eta_c})\hat{\xi}_{z,t}] \tag{B.17}$$

$$\hat{mc}_t = (\eta_n + \eta_c)[\hat{y}_t - \hat{y}_t^f] = (\eta_n + \eta_c)[\tilde{y}_t] \tag{B.18}$$

where the final step uses the flexible equilibrium output (28) and the definition for the output gap.

Price indexation and the hybrid NKPC

When a fraction θ_p of non-reoptimising firms index prices to past inflation, the log-linearized New Keynesian Phillips Curve becomes (using (B.18)):

$$\hat{\pi}_t = \frac{\beta}{1 + \beta\gamma_p} E_t \hat{\pi}_{t+1} + \frac{\gamma_p}{1 + \beta\gamma_p} \hat{\pi}_{t-1} + \frac{\tilde{\kappa}}{1 + \beta\gamma_p} \tilde{y}_t \tag{B.19}$$

where $\tilde{\kappa} = (\eta_n + \eta_c)\kappa$ and $\kappa = \frac{(1-\theta_p)(1-\theta_p\beta)}{\theta_p}$.

Equation (B.19) is the **hybrid NKPC** with backward-looking indexation ($\gamma_p > 0$), capturing inflation inertia observed in the data. Setting $\gamma_p = 0$ gives us the forward-looking NKPC is recovered (22):

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \tilde{\kappa}\tilde{y}_t \tag{(22)}$$