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Sovereign default risk premia, fiscal limits, and fiscal policy

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ABSTRACT

We develop a closed economy model to study the interactions among sovereign risk premia, fiscal limits, and fiscal policy. The fiscal limits, which measure the government's ability to service its debt, arise endogenously from dynamic Laffer curves. The state-dependent distributions of fiscal limits depend on the growth of lump-sum transfers, the size of the government, the degree of countercyclical policy responses, and economic diversity. The country-specific fiscal limits imply that the market perceives the riskiness of sovereign debt issued by different countries to be different, which is consistent with the observation that developed countries are downgraded at different levels of debt. A nonlinear relationship between sovereign risk premia and the level of government debt emerges in equilibrium, which is in line with the empirical evidence that once risk premia begin to rise, they do so rapidly. Nonlinear simulations show that fiscal austerity measures that aim to balance the government budget in the short run fail to contain the default risk premium, even with sizeable cuts in government purchases; but a long-term plan for fiscal reform, if it credibly changes the market's expectation about future fiscal policies, can alleviate the rising risk premium.

1. Introduction

Developed countries are facing unprecedented fiscal challenges. In the wake of deteriorating public finances, rating agencies downgraded the sovereign debt of several developed countries during the period of 2009–2011. Spain was downgraded from AAA to AA, Ireland from AAA to BBB+, Portugal from AA to BBB−, and Greece from A all the way to CC.[1] The spread between 10-year Greek government bonds and equivalent German bonds widened to 13.7 percentage points in August 2011. Historical data shows that developed countries have frequently been penalized when concerns arise about the riskiness of government debt. Fig. 1 depicts the sovereign downgrades of some developed countries since 1975.[2] There is also tremendous diversity in the levels of debt at which downgrades have occurred. Fig. 2 compares six countries in three groups: New Zealand and Canada, Italy and Belgium, and Sweden and Japan. The rating of New Zealand government debt was reduced by three notches from AAA to AA− as its gross debt climbed from 58% to 75% of the GDP. The Canadian government, on the other hand, was able to keep its AAA rating until its debt hit 90% of GDP.

The theoretical analysis of fiscal policy in developed countries has been largely abstracted from sovereign default risk by assuming that sovereign debts are always honored, which may be an innocuous assumption in normal times. The financial markets, however, have brought fiscal concerns to the front page in the current fiscal crisis in the Euro area. More broadly, government debt in developed countries, buoyed by anti-crisis measures, is projected to rise from an average of

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[1] All sovereign ratings in this paper are from Standard & Poor's.

[2] Cantor and Packer (1996) show that the bond spreads broadly share the relative rankings of sovereign credit risks made by Moody's Investors Service and Standard and Poor's.

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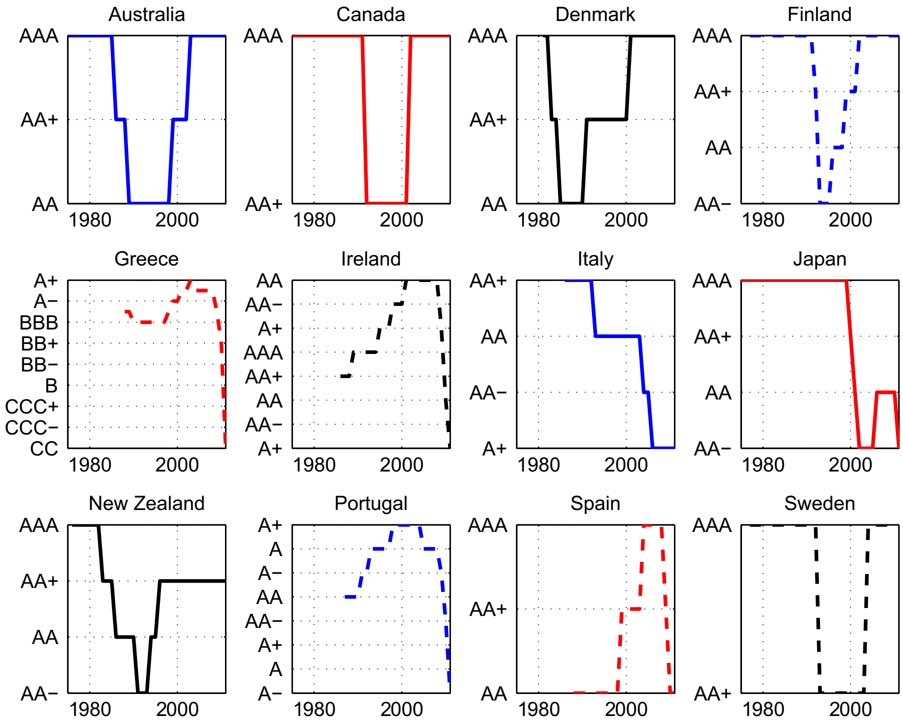

Fig. 1. Sovereign downgrades in some developed countries from 1975 to 2010 (Standard & Poor's).

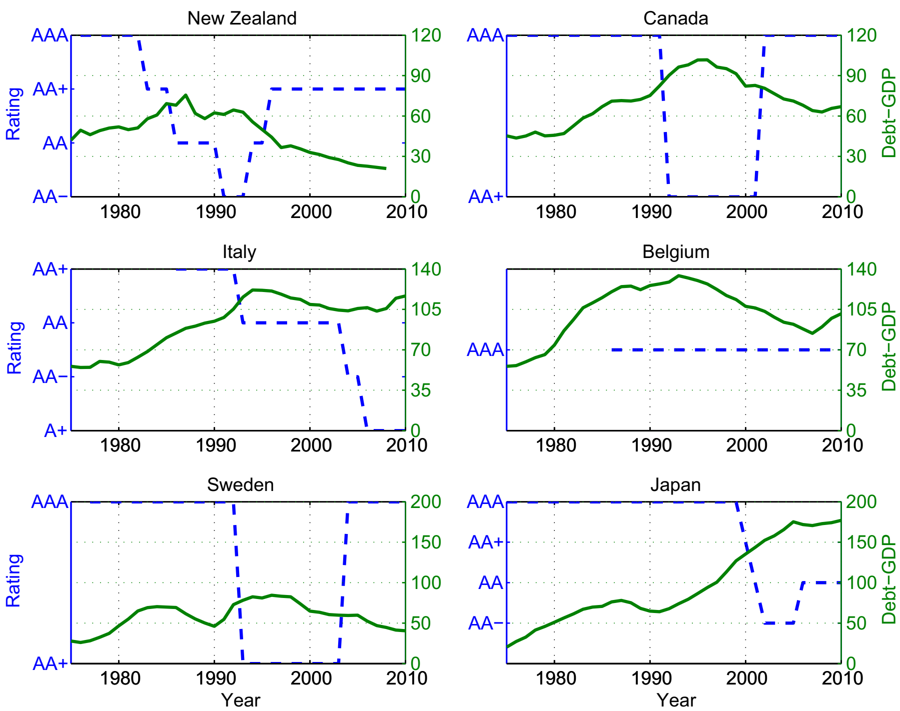

Fig. 2. Comparison of sovereign downgrades (dashed blue line, measured to the left axis) and government debt-GDP ratio (solid green line, measured to the right axis) in selected countries from 1975 to 2010. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

about 73% of GDP in 2007 to about 108% of GDP in 2015. Age-related spending in many developed countries will further raise government indebtedness in the coming decades. It is, therefore, important to understand the interaction between sovereign default risk and fiscal policy and, furthermore, to discuss what kind of fiscal policies can contain the default risk. One way to do so is through the use of a dynamic stochastic general equilibrium (DSGE) framework, which is at the heart of this paper. Using a nonlinear model, we analyze how the maximum level of debt that the government is able to service, which we call the *fiscal limit*, may depend on macroeconomic fundamentals, and what the quantitative impact of sovereign default risk upon the economy may be.

We consider a closed economy in which the government finances lump-sum transfers and an exogenous level of purchases by collecting distorting taxes and issuing non-state-contingent debts. The government raises the tax rate when the debt level goes up. Laffer curves, nevertheless, arise endogenously from distorting taxes—if the tax rate is on the "slippery" side of the Laffer curve, then the government is unable to raise more tax revenue through higher tax rates. The lump-sum transfers follow a Markov regime-switching process, with one regime being stationary while the other explosive. If the government stays in the explosive-transfer regime for too long, the debt can rise to such a level that tax rate may eventually reach the peak of Laffer curve and the government will be unable to repay its debt in full amount. Even if the current tax rate is not there yet, a positive probability of eventually hitting the peak of Laffer curve in the future can prompt forward-looking households to demand a higher default risk premium on sovereign debt today.

The maximum level of debt that the government is able to service, which we call the *fiscal limit*, is defined as the sum of the discounted maximum fiscal surplus in all future periods. Given the persistence of exogenous disturbances, the fiscal limit depends on the current state of the economy, for instance, whether the productivity level is high or low, and whether the transfers are in the stationary or explosive regime. For a given state of the economy, on the other hand, there exists a set of maximum levels of debt that government is able to service, which depend on random disturbances hitting the economy in the future. The fiscal limit, therefore, is state-dependent and stochastic. At each period, an effective fiscal limit is drawn from the state-dependent distribution. If the level of government debt surpasses the effective fiscal limit, then the government reneges on a fraction of its debt and the realized default rate follows an empirical distribution that is computed from historical data. Using the state-dependent distributions of fiscal limits and the empirical distribution of default rates, households can decide the quantity of government debts that they are willing to purchase and the price at which they are willing to pay.

A couple of results emerge. First, the distributions of fiscal limits depend on the underlying macroeconomic fundamentals. Potentially explosive transfers can, in some states, keep the fiscal surplus very low for extended periods, generating distributions of fiscal limits with fatter tails in the lower bound. Everything else, including the current levels of transfers and debt, being equal, a country with upward-trending transfers faces a higher default probability than the country with stationary transfers. Strong countercyclical spending policies, which arise from large automatic stabilizers or discretionary countercyclical fiscal policies, can produce more dispersed distributions of fiscal limits, as they exacerbate the deterioration of government budgets when tax revenue is low. A government with a heavy burden of transfer payments or government purchases faces lower fiscal surplus and significantly lower fiscal limits. An economy that is more vulnerable to exogenous shocks faces higher uncertainty about the government's ability to service its debt and, therefore, more dispersed distributions of fiscal limits. To sum up, the country-specific distributions imply that the market perceives the riskiness of sovereign debt issued by different countries to be different, which is consistent with the observation that developed countries are downgraded at different levels of debt, as illustrated in Fig. 2.

Second, the default risk premium is a nonlinear function of the level of government debt. It begins to emerge as the level of debt approaches the lower bound of the state-dependent distribution of fiscal limits. Nonlinearity between the sovereign risk premia and government indebtedness is widely identified in empirical literature, see Alesina et al. (1992), Bernoth et al. (2006), among others. In addition, the distributions being state-dependent implies that an economic downturn can significantly compound a fiscal crisis. A lower level of productivity not only slashes the tax revenue, forcing the government to borrow more, but also shifts down the state-dependent distribution of fiscal limits, raising the default probability even if the debt level stayed the same. Higher government debt, in conjunction with the lower distributions of fiscal limits, bids up the sovereign borrowing cost, which further deteriorates the government budget. Such a self-fulling mechanism can spur a risk premium hike in a recession.

Last but not least, nonlinear simulations show that fiscal austerity measures that aim to balance the government budget in the short run, unfortunately, do not change households' expectation about fiscal policy in the long run and, therefore, fail to contain the default risk premium, even with sizeable cuts in government purchases. A long-term plan for fiscal reform, on the other hand, if it credibly changes the market's expectation about future fiscal policies, can alleviate the rising risk premium.

In the literature of sovereign default in emerging market economies, Uribe (2006) shows that default is inevitable if the monetary authority implements an inflation target and the tax policy is exogenous, as both monetary and fiscal policy are 'active' in the sense of Leeper (1991). Eaton and Gersovitz (1981) and Arellano (2008) model sovereign default as an outcome of optimal and strategic decision by the government. The predicted level of government debt at which the sovereign default occurs, however, is much lower than the debt level at which sovereign risk premia are observed in developed countries.

A closely related paper is by Juessen et al. (2011), which also computes the government's debt repayment capacity using Laffer curves. Their paper focuses on how default risk premia depend on output fluctuations. By keeping the government spending and tax rate constant, the government has to issue more debt when an economic downturn slashes the tax revenue. If starting at a high level of debt, a severe recession can further raise the debt to such a level that the transversality condition is violated and the government has to partially default. The authors show that sizeable risk premium can emerge if the output is sufficiently volatile. Our paper complements (Juessen et al., 2011) but differs in the following ways. The government can raise tax rate with rising debt in our model, which captures the tax policies in developed countries, as illustrated in Fig. 3. Sovereign defaults, nevertheless, can still emerge due to potentially explosive transfers. If the transfers stay in the explosive regime for too long, the debt level can steadily rise from the steady state to such a point that the government will be unable to repay its debt in full amount. By attacking the cause of sustained rises in government debt in developed countries, our model provides a framework to discuss fiscal austerity measures in the short run versus fiscal reforms in the long run. In addition, a richer setup of our model allows us to rank the quantitative impact of different macroeconomic fundamentals upon the distributions of fiscal limits and, therefore, the sovereign risk premium.

The remainder of the paper is organized in four sections. Section 2 presents the model, and Section 3 discusses the state-dependent distributions of fiscal limits. The nonlinear model is solved in Sections 4 and 5 closes the paper.

2. Model

In this section, we lay out a closed economy model in which the fiscal limit, a measurement of the government's ability to service its debt, arises endogenously from dynamic Laffer curves. With linear production technology, the output is

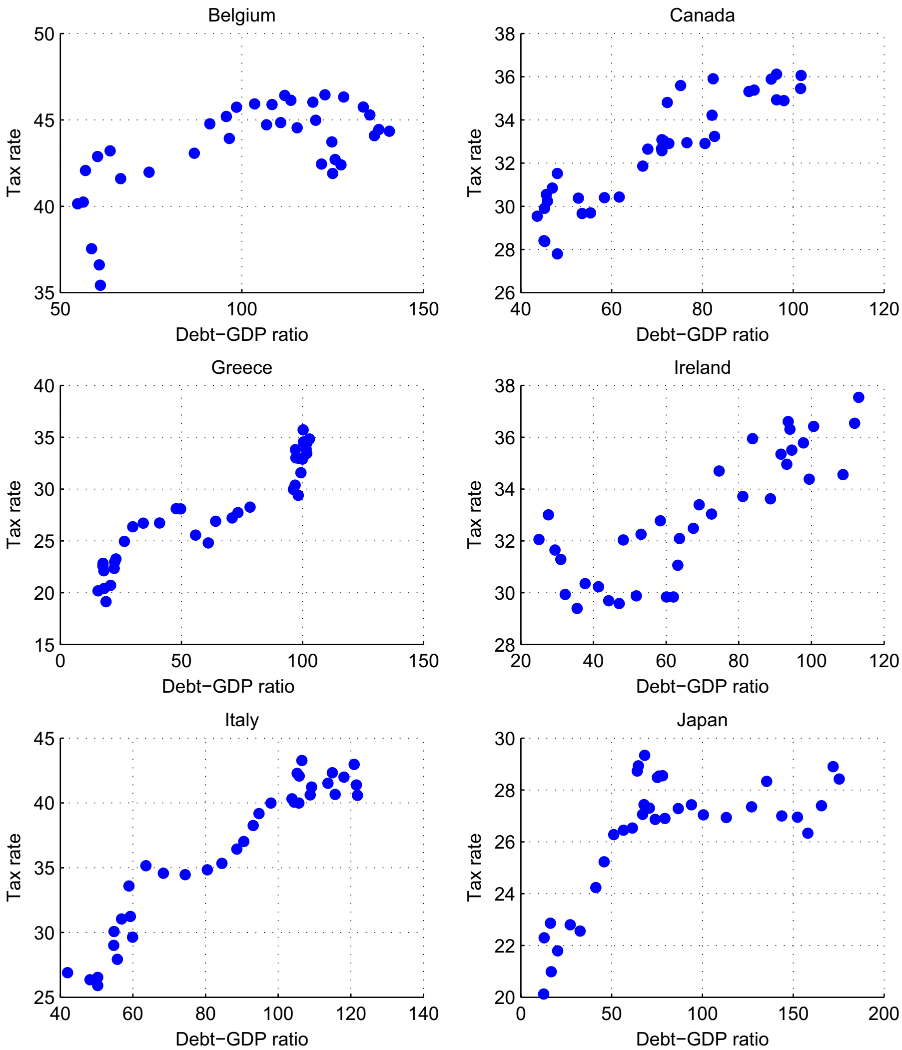

Fig. 3. Debt-tax scatter plot for highly indebted countries from 1975 to 2010.

determined by the level of productivity (A_t) and the labor supply ($1-L_t$). The household consumption (c_t) and government purchases (g_t) satisfy the aggregate resource constraint:

$$c_t+g_t=A_t(1-L_t) \tag{1}$$

where the level of productivity follows an $AR(1)$ process with A representing the steady-state technology level:

$$\ln\frac{A_t}{A}=\rho^A\ln\frac{A_{t-1}}{A}+\varepsilon_t^A,\quad \varepsilon_t^A\sim\mathcal{N}(0,\sigma_A^2) \tag{2}$$

2.1. Government

The government finances lump-sum transfers to households (z_t) and exogenous and unproductive purchases by collecting tax revenue and issuing one-period bonds (b_t).

The tax revenue is raised through a time-varying tax rate (τ_t) on labor income. We assume that the government follows a simple tax rule, as specified in Eq. (3), and refer to the parameter of γ as the "tax adjustment parameter". A larger γ means that the government is more willing to retire debt by raising the tax rate[3]:

$$\tau_t-\tau=\gamma(b_t^d-b)\quad(\gamma>0) \tag{3}$$

This tax rule is an abstraction designed to capture a key feature of tax policy whereby fiscal authorities tend to increase the tax rate when government debt rises. Fig. 3 provides debt-tax scatter plots for highly indebted countries that reached or went beyond a debt-GDP ratio of 100% during the period of 1970 and 2007.[4] With debt-GDP ratios on the horizontal axis and tax rates on the vertical axis, the positive correlations between the two illustrate that governments tend to raise taxes when debt climbs up. The willingness to do so, which is reflected by the steepness of each cluster, nevertheless, varies across countries.

The processes of the government purchases and the lump-sum transfers are specified to capture the trends and fluctuations of government expenditures observed in most OECD countries between 1970 and 2007. First, we assume that the lump-sum transfers are countercyclical, while the government purchases follow an exogenous $AR(1)$ process. Both assumptions follow from the estimation, shown in Fig. 4 that the elasticity of detrended real transfers with respect to detrended real GDP per worker (η^z) is negative for all the countries in our dataset, varying from -0.093 to -2.22, while the estimated elasticity of detrended real government purchases with respect to detrended real GDP per worker (η^g) is inconclusive across countries, varying from $+2.5$ to -2.5.[5] The government purchases, therefore, are specified as:

$$\ln\frac{g_t}{g}=\rho^g\ln\frac{g_{t-1}}{g}+\varepsilon_t^g,\quad \varepsilon_t^g\sim\mathcal{N}(0,\sigma_g^2) \tag{4}$$

where g represents the government purchases at the steady state.

Second, we assume that the trend of lump-sum transfers follow a Markov regime-switching process, with one regime being stationary and the other explosive. This assumption follows the observation that many developed countries have transferred or are still transferring a rising share of GDP from the government to households since 1970, illustrated in Fig. 6. The transfer outcomes, which are ultimately political decisions, depend on voters' preferences, the budget process, constitutional rules, and other underlying political factors. Without resorting to a structural political economy model, we specify the transfers to be a regime-switching process and allow potentially explosive transfers. Forward-looking agents may still be willing to purchase sovereign debts issued by countries with growing transfers, if they expect that regime to be reverted. If the transfers stay in the explosive regime for too long, however, the debt may rise to such a level that the government will be unable to repay its debt in full amount even if it follows a tax rule, as specified in Eq. (3), due to the dynamic Laffer curves that arise endogenously from distorting taxes. A positive probability of eventually hitting the peak of Laffer curve in the future can spur sovereign default fears today even if the current tax rate is not at the peak of Laffer curve yet. Section 3 provides a further discussion of the fiscal limit.

The transfers can move from the stationary regime to the explosive regime, dictated by a regime-switching index rs_t^z. Depending on the transition matrix, there can be prolonged periods during which rising transfer steadily raise government debt. In both regimes, the transfers are countercyclical, measured by the parameter $\zeta^z=\eta^z z/A<0$:

$$z_t\equiv z(rs_t^z,A_t)=\begin{cases} z+\zeta^z(A_t-A) & \text{if } rs_t^z=1 \\ \mu^z z_{t-1}+\zeta^z(A_t-A) & \text{if } rs_t^z=2 \end{cases}$$

with $\mu^z>1$ and rs_t^z following a transition matrix of $\begin{pmatrix} p_1^z & 1-p_1^z \\ 1-p_2^z & p_2^z \end{pmatrix}$.

Let q_t be the price of the bond in units of consumption at time t. For each unit of bond, the government promises to pay the household one unit of consumption in the next period. However, the bond contract is not enforceable. At time t, the

[3] Replacing b_t^d with b_{t-1} will not change the results materially.

[4] Appendix A provides a detailed data description.

[5] The key results of this paper do not change even if we assume both lump-sum transfers and government purchases are countercyclical, see Bi and Leeper (2010) for a case study of Sweden.

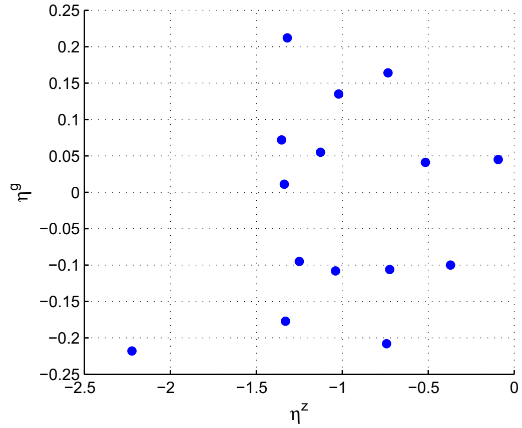

Fig. 4. Estimated elasticity of detrended real transfers with respect to detrended productivity (η^z) versus that of detrended real government purchases with respect to detrended productivity (η^g) from 1975 to 2007.

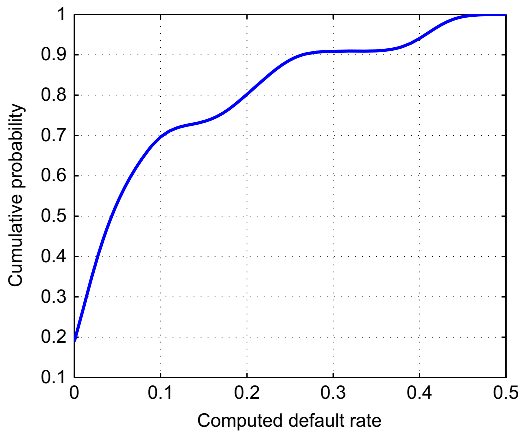

Fig. 5. Empirical distribution of default rates (Ω) computed in Table 1.

government may partially default on its liability (b_{t-1}) by a fraction of Δ_t, with the post-default government liability being denoted as b_t^d:

$$\tau_t A_t(1-L_t)+b_t q_t = \underbrace{(1-\Delta_t)b_{t-1}}_{b_t^d} + g_t + z_t \tag{5}$$

2.2. Default scheme

The default scheme at each period depends on the effective fiscal limit (b_t^*), which follows a conditional distribution ($\mathcal{B}^*(A_t, g_t, rs_t^z)$).[6] The conditional distributions arise endogenously from the distorting taxes and Section 3 provides a further discussion of the fiscal limit. If what the government owes at the beginning of period t is less than the effective fiscal limit, then it repays its debt at full amount and no default occurs; otherwise, the government partially defaults, and the stochastic default rate follows an empirical distribution Ω.

$$\Delta_t = \begin{cases} 0 & \text{if } b_{t-1} < b_t^* \\ \delta_t & \text{if } b_{t-1} \geq b_t^* \end{cases}$$

where $b_t^* \sim \mathcal{B}^*(A_t, g_t, rs_t^z)$ and $\delta_t \sim \Omega$.

Since few sovereign default has been observed in developed countries in the post-war period by the time this paper is written, the distribution Ω is computed from the sovereign debt defaults and restructures observed in the emerging

[6] Cochrane (2011) emphasizes the impact of the fiscal limit upon monetary policy in the discussion of monetary and fiscal policy in the financial crisis of 2008–2009.

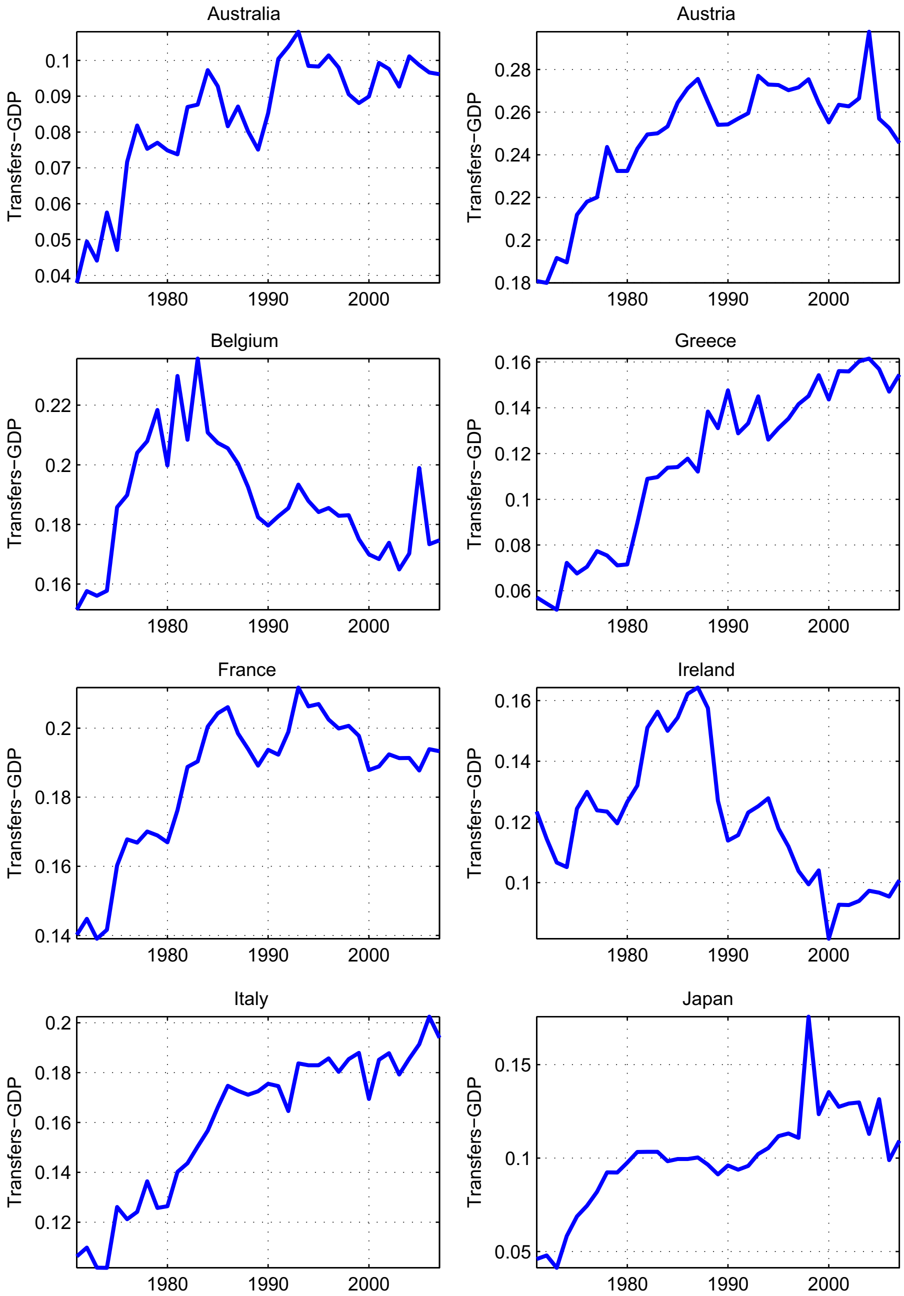

Fig. 6. Countries that have had or are still having growing transfers between 1970 and 2007.

market economies during the period of 1998–2005. We assume that households in our model price sovereign bonds based on this historical information.

The assumptions of a partial default and a stochastic default rate reflect the following observations. First, the government often defaults on a subset of its total debt. Moody's (2009) reports the total amounts of defaulted debt during the rated sovereign bond defaults since 1983 and Panizza (2008) provides a thorough dataset on the public debt in developing countries, based on which I compute the claimed default rate, defined as the share of defaulted debt over the total public debt. As shown in Table 1, the share is far below 100%, varying from 1% in Venezuela to 57% in Argentina. Second, Reinhart and Rogoff (2009), looking at the long historical data, show that creditors can often get a significant share of what they are owed after contentious debt negotiations. As reported in Table 1, Sturzenegger and Zettelmeyer (2008) estimated the haircuts in sovereign debt restructures in emerging market economies for the period between 1998 and

Table 1
Computed default rates in emerging countries.
Data source: Moody's (2009), Panizza (2008), Sturzenegger and Zettelmeyer (2008).

Country	Time of default	Defaulted debt over total public debt	Haircuts	Actual default rate
Venezuela	July 1998	0.01	~0	~0
Russia	August 1998	0.37	0.5–0.63	0.185–0.233
Ukraine	September 1998	0.09	0.3	0.027
	January 2000	0.075	0.28	0.021
Pakistan	July 1999	0.035	0.35	0.0123
Ecuador	August 1999	0.46	0.4	0.188
Peru	Sep 00	0.2	~0	~0
Argentina	November 2001	0.57	0.72	0.41
Uruguay	May 2003	0.53	0.13	0.07
Dominican Republic	April 2005	0.13	0.05	0.0065
Belize	December 2006	0.267	0.3	0.08

2005, which varied from 5% in Dominican republic to 72% in Argentina.[7] Finally, the last column of Table 1 shows that the actual default rates, which are the product of the claimed default rates and the hair cuts, vary across countries.

The actual default rate, instead of the claimed default rate or haircut, is consistent with the default rate δ_t in our model. Since the model is abstracted from debt renegotiation, we assume that the realized default rate is stochastic and households price the sovereign bond using the empirical distribution of default rates, plotted in Fig. 5. It shows that the stochastic default rate falls between 0 and 0.1 with 70% probability, between 0 and 0.3 with 90% probability, and between 0 and 0.5 with 100% probability.

2.3. Household

With access to the sovereign bond market, a representative household chooses consumption (c_t), leisure (L_t), and bond purchases (b_t) according to,

$$\max \quad E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, L_t) \tag{6}$$

$$\text{s.t.} \quad A_t(1-\tau_t)(1-L_t)+z_t-c_t = b_t q_t-(1-\Delta_t)b_{t-1} \tag{7}$$

with policies (τ_t, z_t, Δ_t) taken as given. E_t is the mathematical expectation conditional on the information available at time t, including the sovereign default information. The utility function ($u(c,L)$) is strictly increasing in terms of consumption and leisure. β is the discount factor.

The household's first-order condition requires that the marginal rate of substitution between consumption and leisure equates to the after-tax wage. The government bond price in Eq. (9) reflects the household's expectation about the probability and magnitude of sovereign default in the next period:

$$\frac{u_L(t)}{u_c(t)} = A_t(1-\tau_t) \tag{8}$$

$$q_t = \beta E_t\left((1-\Delta_{t+1})\frac{u_c(t+1)}{u_c(t)}\right) \tag{9}$$

The optimal solution to the household's maximization problem must also satisfy the following transversality condition:

$$\lim_{j\to\infty} E_t \beta^{j+1} \frac{u_c(t+j+1)}{u_c(t)}(1-\Delta_{t+j+1})b_{t+j} = 0. \tag{10}$$

3. Distribution of fiscal limit

This section first explains how sovereign defaults can emerge in equilibrium in this model, and then discusses how the distributions of fiscal limits depend on the underlying macroeconomic fundamentals.

[7] In Table 1, the haircuts in Venezuela and Peru are approximated to zero because both countries cured the default within a short period of time by meeting the payment.

3.1. Laffer curve and fiscal limit

The proportional tax on labor income distorts a household's behavior as it lowers the after-tax wage and induces household to work less. Depending on the existing rate, an increase in the tax rate on labor income may or may not raise tax revenue. In general, higher rates increase the revenue when the existing tax rate is low and, on the other hand, reduce the revenue when the existing rate is high, which is the basis for the Laffer curve.

Laffer curves are usually dynamic in the sense that the shape of the Laffer curve depends on the state of the economy.[8] In our model, for given levels of productivity and government purchases (A_t,g_t), a tax rate exists such that higher rates do not raise more revenue. This point is the peak of the dynamic Laffer curve, denoted as $\tau^{max}(A_t,g_t)$, at which the government can collect the maximum level of tax revenue for the given state, denoted as $T^{max}(A_t,g_t)$.

In addition, the government's ability to service its debt also depends on the size of government purchases and lump-sum transfers, which are political decisions emerged from conflicts and compromises among parties with different ideologies, see Persson and Svensson(1989) and Alesina et al. (1990) among many other papers. Without resorting to a structural political economy model, we specify the processes of government purchases and transfers as to capture the trends and fluctuations of government expenditures observed in the data. As explained in Section 2.1, the government purchases are specified as an exogenous $AR(1)$ process, and the lump-sum transfers follow a Markov regime-switching process with one regime being stationary and the other featuring an upward trend.

Drazen (2000) observes that "the growth in transfer payments has been one of the most dramatic changes in government expenditures in the last 60 years." In our model, the potentially explosive transfers, in conjunction with dynamic Laffer curves, play the crucial role in generating the sovereign default risk premium. In linearized models, were the transfers always stationary, a positive feedback from government debt to taxes, like the tax rule specified in Eq. (3), can keep the sovereign debt from exploding unless the tax adjustment parameter is too small, see Leeper (1991). Such guarantee no longer holds, however, if the transfers follow the Markov regime-switching process as specified in Section 2.1. Rising transfers raise the debt level and, under the specified tax rule, also the tax rate. If the transfers stay in the explosive regime for too long, the debt can rise to such a level that the tax rate will eventually hit the peak of Laffer curve. Even if the current tax rate is not there yet, the positive possibility of eventually hitting the peak of Laffer curve may prompt forward-looking households to ask for a higher risk premium on sovereign bond today. The higher the possibility is, the higher the default risk premium they demand.

To the end, we define the maximum level of debt that the government is able to pay back, or the fiscal limit, as the sum of the discounted maximum fiscal surplus in all future periods conditional on the existing state. The state-dependent distribution of fiscal limit is defined as:

$$\mathcal{B}^*(A_t,g_t,rs_t^z) = E_t \sum_{j=0}^{\infty} \beta^j \frac{u_c^{max}(A_{t+j},g_{t+j})}{u_c^{max}(A_t,g_t)} (T^{max}(A_{t+j},g_{t+j}) - g_{t+j} - z(rs_{t+j}^z,A_{t+j})) \tag{11}$$

u_c^{max} represents the marginal utility of consumption when the tax rate is at the peak of the Laffer curve (τ^{max}). T^{max} is defined in Eq. (B.5) in Appendix B. Given the structural parameters of the model and the specifications of shock processes, the unique mapping between the peak of dynamic Laffer curve and the exogenous state of the economy determines the state-dependent distribution of the fiscal limit. The *conditional* distribution implies that households' expectation about the government's ability to pay back its debt hinges on the current state of economy, for instance, whether current transfers are stationary or explosive, and whether the current productivity level is high or low.

As specified in Section 2.2, we assume the effective fiscal limit (b_t^*) is a draw from the conditional distribution, instead of always equaling to the mean of the distribution. As shown in the numerical analysis below, the state-dependent distribution can be quite dispersed, especially when the current transfers are in the explosive regime. The distribution tells us the probability that a particular debt level can be supported by taxing income at the peak of the Laffer curves given the current state and the stochastic processes for transfers, government purchases and productivity. At any point of the distribution, therefore, default is possible, but in the left tail it would require a run of bad shocks to the economy, while in the right tail it becomes possible even with a run of good shocks. If a debt level of b^{**} is associated with a probability of p^{**} in the distribution, then it implies that with the probability p^{**} a run of bad shocks may occur such that the debt level of b^{**} becomes unsustainable. In our model, households' expectations of default follow this distribution $b_t^* \sim \mathcal{B}^*(A_t,g_t,rs_t^z)$.

3.2. Quantitative analysis of fiscal limit

As shown in Eq. (11), the conditional distribution of fiscal limits depends on the underlying parameters, such as the regime-switching process for transfers, the size of the government, and shock processes. In this section, we start with a baseline case with only productivity and government purchase shocks, which is a standard setup in the related literature, while abstracting from explosive and countercyclical transfers. We then add or change one parameter at a time, while keeping all other parameters the same as in the baseline case, to understand the quantitative impact of macroeconomic fundamentals upon the distribution of fiscal limits.

[8] Trabandt and Uhlig (forthcoming) compute Laffer curves for the United States and 15 European countries using a neoclassical model.

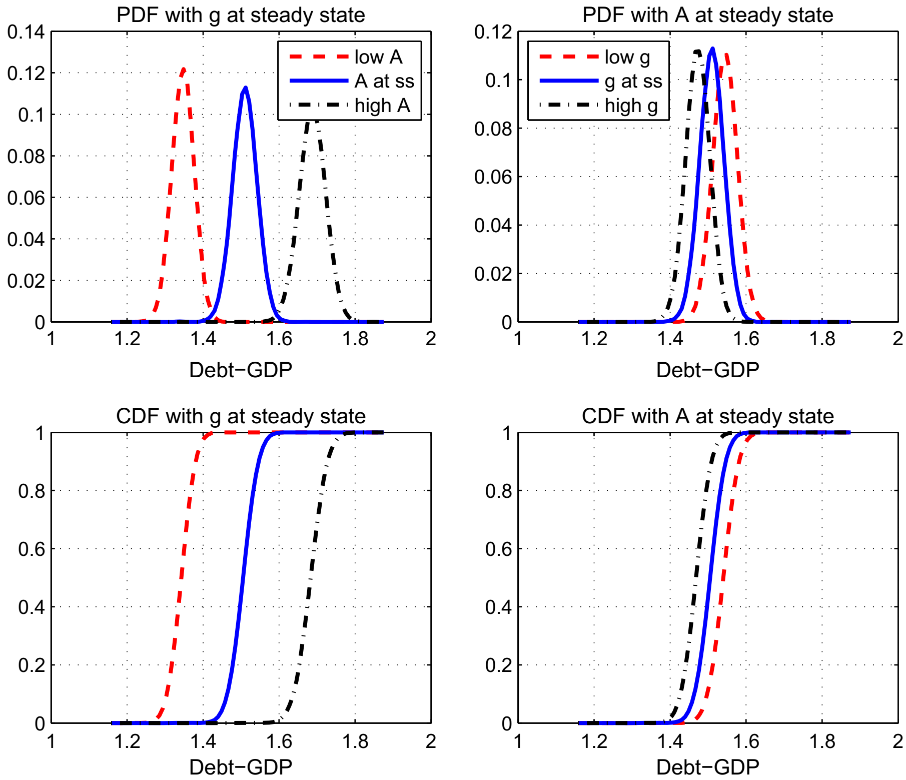

Fig. 7. State-dependent distributions of fiscal limits in the baseline case with only productivity and government purchase shocks: the top panels show the probability density functions (PDFs), while the bottom panels show the corresponding cumulative density functions (CDFs).

3.2.1. *Baseline case*

The model is calibrated at annual frequency. The discount rate is 0.95 and the net interest rate is 5.26%.[9] The utility function is assumed to be $u(c,L) = \log c + \phi \log L$. The leisure preference parameter (ϕ) is calibrated in such a way that households spend 25% of time working and the Frisch elasticity of labor supply is 3. The total amount of time and the productivity level at the steady state are normalized to 1.

In the baseline case, we abstract from the stochastic transfers by setting $z_t = z$, while calibrating other parameters to the average over all countries in the sample between 1971 and 2007. The average government purchase-GDP ratio (g/y) is 21.3%, the average transfers-GDP ratio (z/y) is 15.7%, and the average tax rate is 0.362. Using a HP filter, both the productivity and the government purchase shocks have an average persistence of 0.553 and an average standard deviation of 0.02.

We simulate the state-dependent distributions of fiscal limits using Markov Chain Monte Carlo method, which is described in Appendix B. In Fig. 7, the top panels show the probability density functions (PDFs), while the bottom panels show the corresponding cumulative density functions (CDFs).[10] Given that the effective fiscal limit follows the state-dependent distribution, the CDFs can be interpreted as the probability of sovereign default at different levels of debt. It is clear that the state of the economy can have a significant impact upon the default probability.

The left panels compare the state-dependent distributions at different productivity levels while keeping the government purchases at the steady state. The productivity level changes the tax revenue directly and, therefore, has a significant impact on the distributions. An 8% reduction in the productivity level (dashed lines in the left panels) can shift the distribution down by 20% of the steady-state output, and the impact is symmetric with a rise in productivity level (dash-dotted lines). For instance, at the debt ratio of 150%, the default probability is 0.5 if the productivity level is at the steady state, and 0 if the productivity level is 8% higher than the steady state, but 1 if the level is 8% lower instead.

The right panels compare the state-dependent distributions at different levels of government purchases while keeping the productivity level at the steady state. An 8% reduction in the government purchases (solid lines), equaling to 1.7% of the steady-state output, shift up the distribution by about 4%.

[9] In the related literature, Uribe (2006) uses an annual real interest rate of 6%.

[10] In the model, all the fiscal variables are specified in levels (x_t) instead of the shares of output (x_t/y_t). In the graphs, however, the debt levels are scaled by the steady-state output (y) to provide a meaningful illustration.

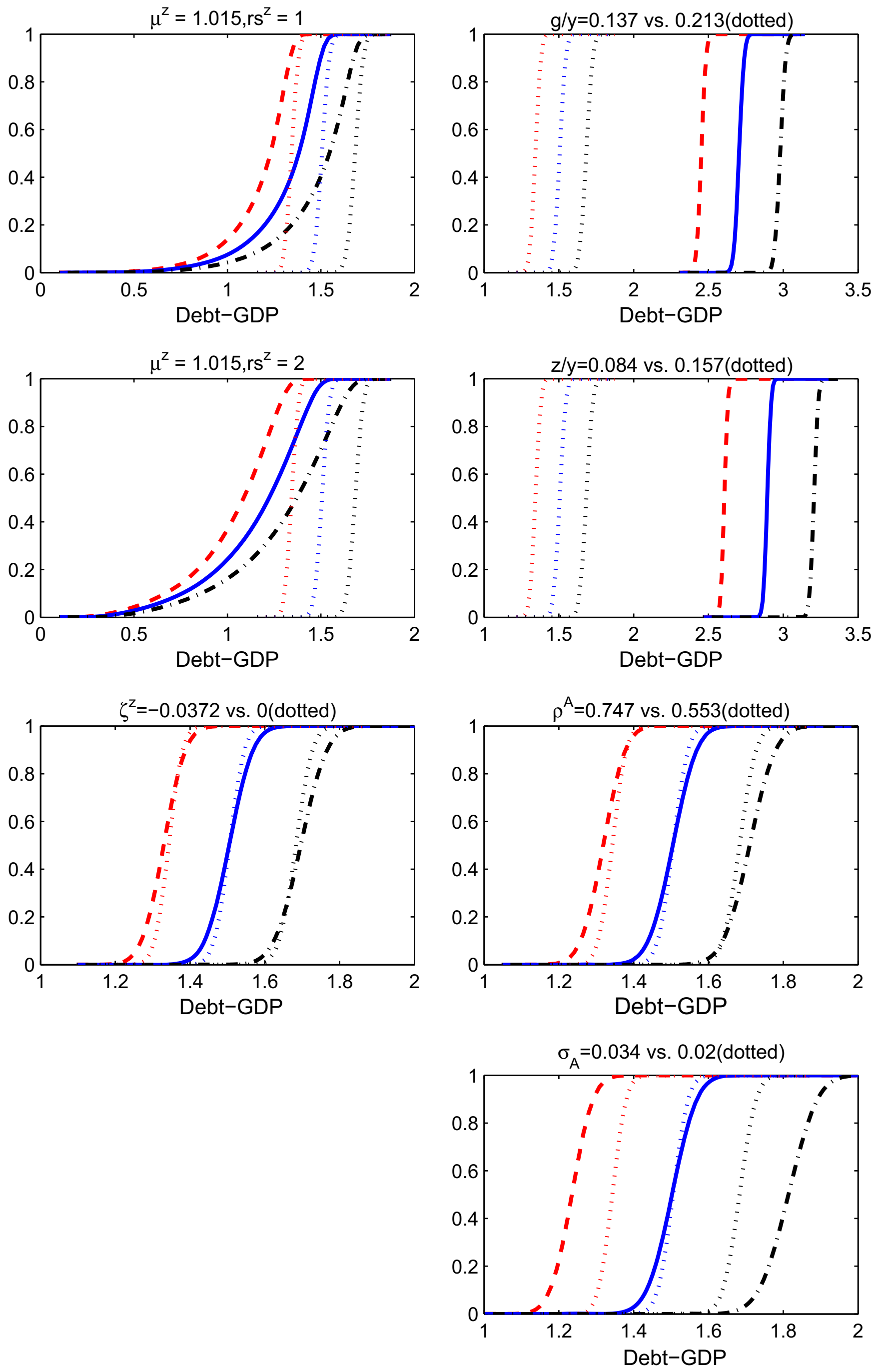

Fig. 8. Comparison of the state-dependent distributions of fiscal limits: the dotted lines in each panel represent the CDFs in the baseline case and are identical to those in the bottom left panel of Fig. 7; the solid, dashed, and dash-dotted lines in each panel represent the state-dependent distributions under different productivity levels for the experiments as calibrated in Table 2.

3.2.2. Macroeconomic fundamentals

In order to understand the quantitative impact of macroeconomic fundamentals upon the distributions, we change or add only one parameter in each of the following experiments while keeping other parameters the same as in the baseline case, and then plot the resulting distributions against the baseline case in Fig. 8. The dotted lines in each panel represent the CDFs in the baseline case and are identical to those in the bottom left panel of Fig. 7, providing a benchmark for different experiments. On the other hand, the solid, dashed, and dash-dotted lines in each panel represent the

Table 2
Different calibrations for fiscal limit comparisons.

Parameter	Baseline	Transfers		Government size		Shock process	
ρ^A	0.553					0.747	0.553
σ_A	0.02					0.02	0.034
g/y	0.213			0.137	0.213		
z/y	0.157			0.157	0.084		
ζ^z	0	0	-0.0871				
μ^z	n.a.	1.015	n.a.				

state-dependent distributions under different productivity levels for the corresponding experiment. Table 2 summarizes the calibrations for key parameters in the following experiments.

Explosive transfers: We first explore the impact of potentially explosive transfers by setting $\mu^z = 1.015$ and $p_1^z = p_2^z = 0.95$, while keeping other parameters the same as the baseline case. The regime-switching parameters are chosen such that transfers, if staying in the explosive regime, rise by 5.5% of GDP within 20 years, which is within the empirical range, as shown in Fig. 6.

The left panel of the first row shows the state-dependent distributions when the current transfers are in the stationary regime, which, nevertheless, have much fatter tails in the lower bound than those in the baseline case. The possibility that future transfers may switch to the explosive regime implies that future fiscal surplus could be significantly lower than those in the baseline case, constraining the government's ability to service its debt today even if the current transfers are stationary. The lower the regime-switching parameter p_1^z is, the more likely the future transfers may switch to the explosive regime, and, therefore, the more dispersed the distributions become. The left panel of the second row illustrates a similar comparison except that the current transfers are already in the explosive regime, which leads to even fatter tails than those in the first row. Given the regime persistence, future transfers are more likely to stay in the explosive regime if current transfers are explosive, instead of stationary.

The transfer growth (μ^z) and the regime persistence (p_1^z,p_2^z) are ultimately political decisions. The transfer policy outcomes depend on voters' preferences, the budget process, constitutional rules, and other underlying political factors, which are likely to be embedded in investors' pricing decisions. For instance, Standard & Poor's (2008) states that "… the stability, predictability, and transparency of a country's political institutions are important considerations in analyzing the parameters for economic policymaking."

Countercyclical transfers: In the left panel of the third row, while keeping all the other parameters the same as in the baseline case, we change the response of transfers with respect to productivity (ζ^z) from 0 to -0.0871, which is the estimate in Sweden, implying a decrease of productivity by 10% raises the transfers by 3.5% of GDP. Strong countercyclical fiscal transfers, arising from a large automatic stabilizer or discretionary countercyclical policy, can aggravate the volatility of the fiscal limit, as the government transfers more resources to households in the state with lower tax revenue. The distributions become more dispersed, but the magnitude is much less pronounced than that with explosive transfers.

Government size: The lowest average share of government purchases over GDP is 13.7% in Switzerland, compared to an average of 21.3% across all countries in the sample. The right panel of the first row shows that the distributions rise dramatically by 120% of the steady-state output when the government purchase share drops from 21.3% to 13.7%. Lower government purchases raise fiscal surplus at each period and, therefore, significantly raise the fiscal limit. Similar dramatic changes are observed in the right panel of the second row, as the lump-sum transfers change from 15.7% in the baseline case to 8.4%, which is the average ratio in Australia, the lowest share among all countries in the sample.

Shock process: In right panels of the third and bottom rows, we compare the distributions when the persistence and the standard deviation of the productivity shock change. In the third row, when the shock persistence changes from 0.553 in the baseline case to 0.747, which is calibrated to Spanish data, the distributions become slightly more dispersed. In the bottom row, the standard deviation of productivity changes from 0.02 in the baseline case to 0.034, which is calibrated to the economy of New Zealand. The productivity level that is lower than the steady state by 4 standard deviations shifts down the distribution by 30% of the steady-state output, instead of 20% in the baseline case. The economy may face a more dispersed distribution of fiscal limits if the productivity shock is more persistent or volatile.[11]

3.2.3. Country analysis

The country-specific distributions, which depend on the underlying macroeconomic fundamentals, imply that the market perceives the riskiness of sovereign debt issued by different countries to be different, which is consistent with the observation that developed countries are downgraded at different levels of debt, as illustrated in Fig. 2. In this section, we briefly discuss three groups of countries: Canada and New Zealand, Belgium and Italy, and Japan and Sweden.

Canada versus New Zealand: The two countries have been treated differently by the rating agencies, as illustrated in the top row of Fig. 2. From 1983 to 1992, the rating of New Zealand government debt was reduced by three notches from AAA to AA− as its gross debt climbed from 58% to 75% of GDP. The Canadian government, on the other hand, was able to keep

[11] The persistence and the variance of government purchases do not have much of an impact on the distributions.

its AAA rating until the gross debt hit 90% of GDP in 1992. The rating was reduced by one notch to AA+ and then stayed at this level until 2001.

An important reason for this discrepancy is that the economy of New Zealand is less diversified than that of Canada. For instance, Standard & Poor's (2007) sees the economic structure of New Zealand as relatively narrow and heavily reliant on agriculture, reporting that agriculture accounted for close to 7% of New Zealand's GDP and 58% of its export receipts. This type of economic structure makes New Zealand particularly vulnerable to international commodity price fluctuations and global economic slowdowns. During the period between 1970 and 2007, the standard deviation of the detrended real GDP per worker in New Zealand is about twice as large as that of Canada.

Belgium versus Italy: Both countries accumulated massive public debt in the 1990s, well above 100% of the GDP, as shown in the middle row of Fig. 2. However, only the Italian government received persistent downgrades, while Belgium's rating has been stable. One possible explanation is that Belgium demonstrated strong political will to be fiscally responsible by making great strides in reforming the welfare program and reducing transfers by about 10% of the GDP since 1980. In contrast, a high level of public debt has been sustained in Italy since 1980 in spite of fiscal consolidation attempts that have occurred periodically in the country. Fig. 6 compares the paths of transfers in both countries.

Japan versus Sweden: A repeated warning from the rating agencies is that Sweden's large general government sector constrains its fiscal flexibility and puts Sweden in an unfavorable position relative to its peers (see Standard & Poor's, 1997, 2009). On the other hand, Standard & Poor's (2001) claims that Japan could still maintain fiscal flexibility in the medium term when the government gross debt climbed to 140% of the GDP in 2001. One possible explanation is that the two countries are very different in terms of average tax rates, average government spending shares, average transfer shares, and the elasticities of transfers with respect to productivity.

4. Sovereign risk premia: nonlinear solution

In this section, we calibrate the model to the economy of Greece.[12] Due to the existence of fiscal limits, the model cannot be solved through linearization. Instead, we resort to the monotone mapping method and solve the nonlinear model.

4.1. Method

The complete model consists of a system of nonlinear equations, including the household's optimization conditions, constraints, specifications of the fiscal policy and default scheme, shock processes, and the transversality condition. The solution method, based on Coleman (1991) and Davig (2004), conjectures candidate decision rules that reduce the equilibrium conditions to a set of expectation first-order difference equations. The solution consists of one function that maps the current state $\psi_t = \{b_{t-1}, b_t^*, \delta_t, A_t, g_t, z_{t-1}, rs_t^z\}$ into the end-of-period government debt $b_t = f^b(\psi_t)$.

The decision rule for government debt, $f^b(\psi_t)$, is found by substituting the conjectured rule into the complete model. The core equation of the model is

$$\frac{(1-\Delta_t)b_{t-1}+g_t+z_t-\tau_t A_t(1-L(\psi_t))}{f^b(\psi_t)} = \beta E_t\left\{(1-\Delta(\psi_{t+1}))\frac{u_c(\psi_{t+1})}{u_c(\psi_t)}\right\} \tag{12}$$

where $\psi_{t+1} = (f^b(\psi_t), b_{t+1}^*, \delta_{t+1}, A_{t+1}, g_{t+1}, z_t, rs_{t+1}^z)$. The left-hand side of the expression is from the government budget constraint, while the right-hand side is from the household's optimization condition. Appendix C describes the nonlinear method in details.

After finding the decision rule for government debt $(f^b(\psi_t))$, the pricing rule, $q_t = f^q(\psi_t)$, can be solved using the government budget constraint. The interest rate on government bonds can also be solved using $R_t = 1/q_t$, denoted as $f^R(\psi)$.

4.2. Calibration to the economy of Greece

The fiscal parameters are calibrated to the Greek data from 1971 to 2007. At the steady state, the government purchases are set to 16.7% of GDP, the lump-sum transfers to 13.34% of GDP, and the tax rate to 0.32, leaving the government debt to be 40% of GDP. The estimated elasticity of detrended real transfers with respect to detrended real GDP per worker (η^z) is -0.45, equivalent to ζ^z of -0.015. The tax adjustment parameter (γ) is estimated through linear regression of tax rates over the government debt-GDP ratios. For the period of 1971–2007, the estimate is 0.37, implying the government is willing to raise tax rate by about 1 percentage point responding to an increase of government debt by 10% of GDP. Using a HP filter, the productivity shock has a persistence of 0.45 and a standard deviation of 0.0328, while the government purchase shock has a persistence of 0.426 and a standard deviation of 0.0294.

The transfers follow a Markov regime-switching process. In the explosive regime, the transfer growth (μ^z) is calibrated to 1.015 to match the observation that transfers from the Greek government to the private sector have risen by 10% of GDP within 40 years. The regime persistence p_1^z and p_2^z is calibrated to 0.975, implying that the probability of switching between the two regimes is 1/40. Table 3 reports the parameter calibrations.

[12] One caveat of my model is that it is abstracted from banking sectors, which have played a crucial role in shaping the crises in Ireland and Spain.

4.3. Decision rule

As explained in Section 4.1, the pricing rule of the interest rate maps a 7-dimension state space to a 1-dimension state space, $R_t = f^R(b_{t-1}, b_t^*, \delta_t, A_t, g_t, z_{t-1}, rs_t^z)$. Given the direct mapping from $(b_{t-1}, b_t^*, \delta_t)$ to b_t^d, we can plot the pricing rule of the interest rate as a function of $(b_t^d, A_t, g_t, z_{t-1}, rs_t^z)$ and the reason is following. At the beginning of each period, the realizations for productivity, government purchases and transfer regime determine the state-dependent distribution of fiscal limit $\mathcal{B}^*(A_t, g_t, rs_t^z)$, from which the effective fiscal limit (b_t^*) is drawn. If what the government owes at the beginning of period (b_{t-1}), which is predetermined at the previous period, surpasses the effective fiscal limit, then a stochastic default rate (δ_t) is drawn from the empirical distribution (Ω); otherwise, the government pays back its debt in full amount. The post-default liability (b_t^d), in conjunction with the exogenous state (A_t, g_t, rs_t^z), determines the tax rate, household consumption and labor supply. In addition, forward-looking households decide the bond price and how much bond to purchase based on the available information $(b_t^d, A_t, g_t, rs_t^z)$.

By keeping the government purchases and productivity at the steady state, the top panel of Fig. 9 compares the response of net interest rate (r_t) to the current government liability (b_t^d) under different transfer regime. The solid line represents the response in interest rate when the current transfers are in the stationary regime, while the dashed line

Table 3
Calibration to the Greek economy.

Parameter	Value
Discount rate (β)	0.95
Leisure (L)	0.75
Persistence of productivity (ρ^A)	0.45
Standard deviation of productivity (σ_A)	0.0328
Persistence of government purchases (ρ^g)	0.426
Standard deviation of government purchases (σ_g)	0.0294
Response of taxes to debt (γ)	0.37
Spending-GDP ratio (g/y)	0.167
Transfers-GDP ratio (z/y)	0.134
Tax rate (τ)	0.32
Elasticity of transfers to productivity (ζ^z)	−0.015
Transfer growth (μ^z)	1.015
Regime-switching parameters (p_1^z/p_2^z)	0.975/0.975

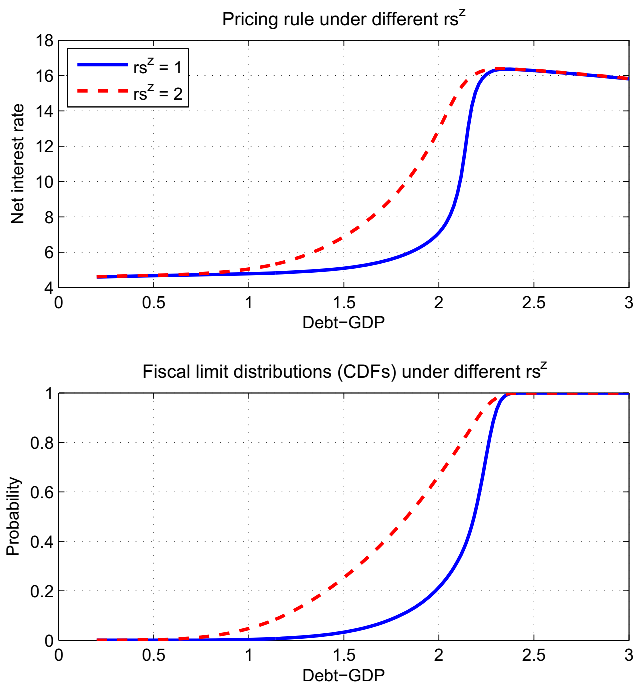

Fig. 9. The pricing rule of net interest rate $r(b_t^d) = 100(R(b_t^d) - 1)$: top panel compares the pricing rules at different transfer regime while other exogenous states are at steady state; bottom panel shows the corresponding state-dependent distributions of fiscal limits.

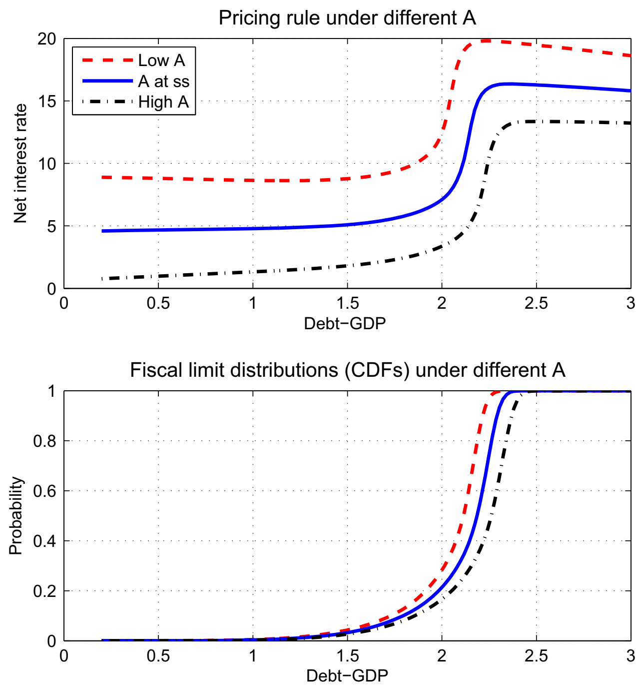

Fig. 10. The pricing rule of net interest rate $r(b_t^d) = 100(R(b_t^d) - 1)$: top panel compares the pricing rules at different level of productivity while other exogenous states are at steady state; bottom panel shows the corresponding state-dependent distributions of fiscal limits. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

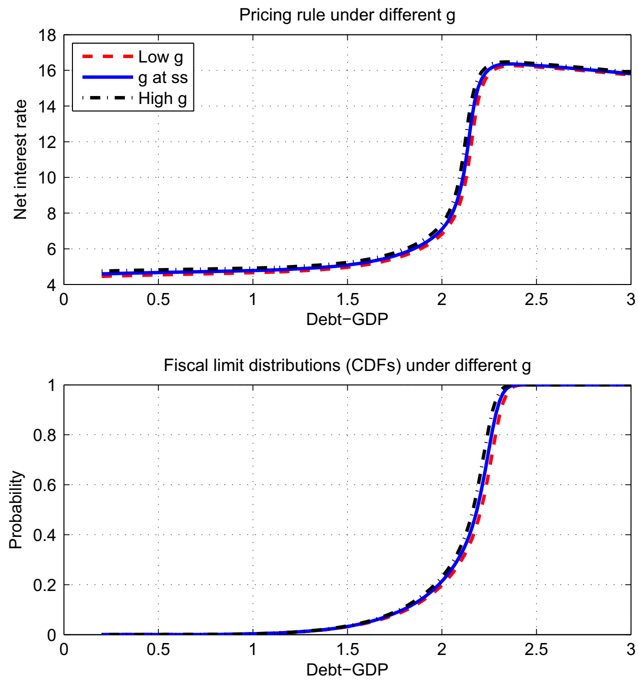

Fig. 11. The pricing rule of net interest rate $r(b_t^d) = 100(R(b_t^d) - 1)$: top panel compares the pricing rules at different level of government purchases while other exogenous states are at steady state; bottom panel shows the corresponding state-dependent distributions of fiscal limits.

shows the case when transfers are in the explosive regime. The bottom panel shows the state-dependent distributions of fiscal limits at the corresponding transfer regime.

Two findings emerge from the comparison. First, the net interest rate rises with the current government liability in a nonlinear way. The financial market starts to demand a risk premium on the government bond as the probability of sovereign default rises

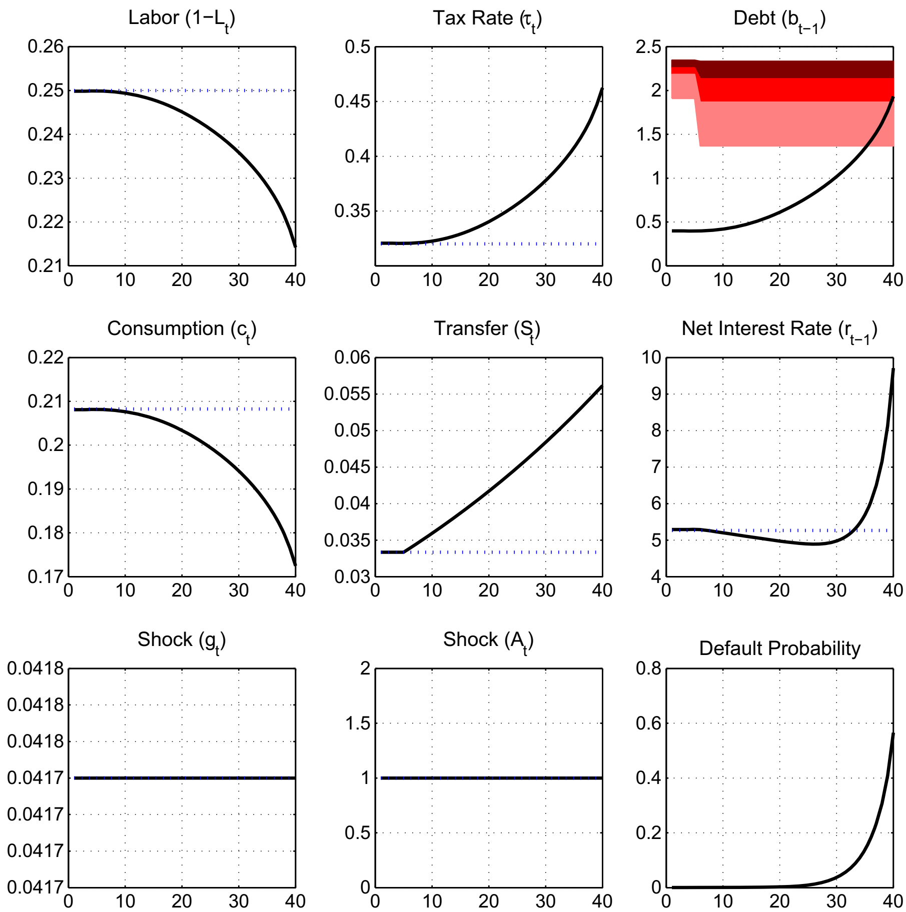

Fig. 12. Nonlinear simulations with growing transfers: transfers switch to and stay at the explosive regime for 35 years. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

above zero. The nonlinear increase of interest rate with the current government liability follows closely, but is not identical to, the state-dependent distribution of fiscal limit at time *t*. Everything else being equal, the higher the current government liability, the more debt it has to issue for next period, and the more likely sovereign default will occur then. The size of default risk premium also depends on the expected default rate, which is 9.47% given the empirical distribution in Fig. 5. Second, the interest rate starts to rise at a much lower level of debt if the current transfers are in the explosive, instead of stationary, regime. When the debt ratio reaches to 200%, household demand a return of 7% if the transfers are stationary; but would demand a return of 12% if the government runs rampant transfer policy, even though the productivity is at the steady-state level.

The top panel of Fig. 10 compares the response of the net interest rate to the government liability at different levels of productivity, and the bottom panel shows the corresponding state-dependent distributions of fiscal limit. Again, the interest rate rises nonlinearly with the current government liability. The productivity level has a substantial impact on the risk-free interest rate due to the intertemporal substitution effect. The lower the productivity level, the higher the interest rate, i.e. the dashed red line lies above the dashed-dotted black line even when the debt level is close to the steady-state level. In addition, a lower productivity level shifts down the state-dependent distribution and, therefore, raise the sovereign default probability at a given level of government debt.

The top panel of Fig. 11 illustrates a similar comparison of interest rate at different levels of government purchase, and the bottom panel shows the corresponding distributions of fiscal limits. Again, the interest rate rises sharply as the government liability enters the lower bound of the distribution of fiscal limits. However, different levels of government purchases do not have much of an impact on the interest rates, as the dashed, solid and dashed-dotted lines are very close, which is in stark contrast to the case with different transfer regime or with different productivity level. The reasons are following. First, while regime switches in transfers change the households' expectation about fiscal policy in the long run, shocks on government purchases are short-lived, which is reflected in the state-dependent distributions of fiscal limits. Second, government purchases change the level of government debt in two opposite directions: higher purchases drive up the government liability on one hand; but, on the other hand, they crowd out private consumption and force the household to work more due to an income effect, and, therefore, a higher output raises the tax revenue and helps to finance government purchases. Overall, different government purchases do not have a significant impact on the interest rate.

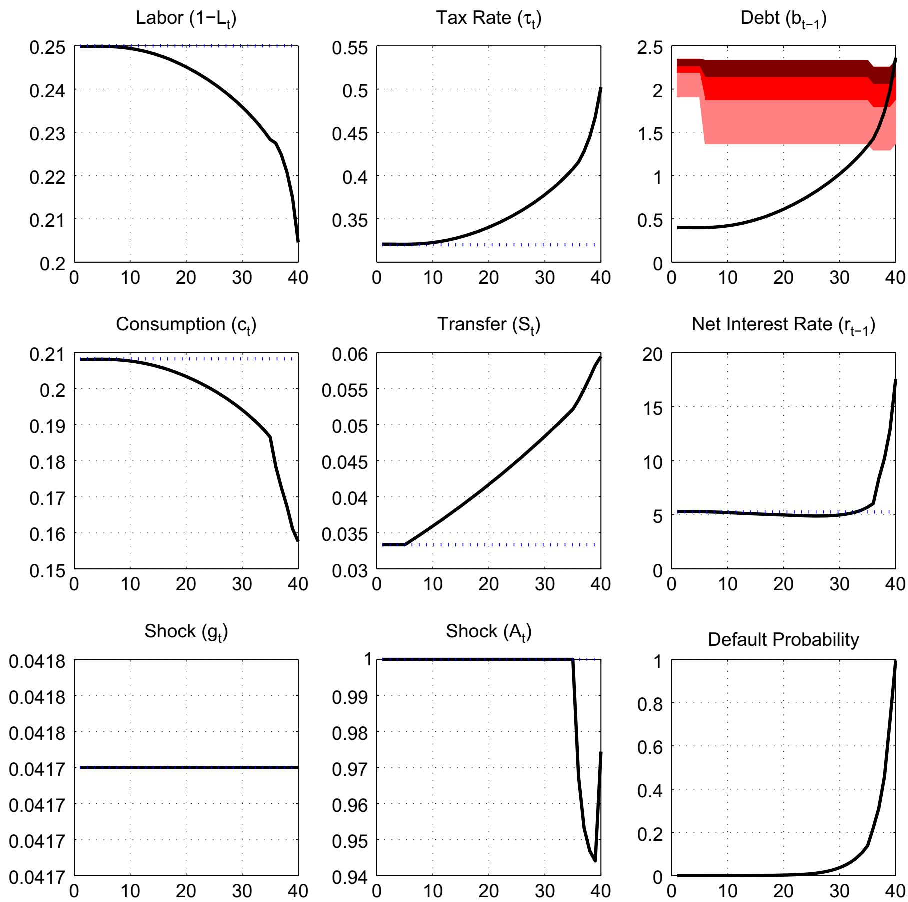

Fig. 13. Nonlinear simulations of economic downturn with growing transfers: lower productivity levels from $t=35$–40.

To sum up, the regime of transfers has a much more pronounced impact on risk premium than the level of productivity and government purchases.

4.4. Nonlinear simulation

The nonlinear simulations in this section are designed to capture the fiscal woes in Greece and to discuss whether different fiscal policies could alleviate the sovereign default risk.

We first consider a baseline case in which the government allows the transfers to grow for an extended period to capture the upward-trending transfers in Greece during the past several decades. The government responds to growing transfers by raising tax rates, which, nevertheless, is not sufficient to stabilize the debt level. We then consider a significant economic downturn. When the economy is hit by a severe recession, tax revenue plummets and the debt level climbs up even further, spurring a rise in sovereign default risk. We finally discuss two possible responses from the government to the rising cost of sovereign borrowing. In the first scenario, the government adopts fiscal austerity measures by cutting its purchases; in the second scenario, the government provides a credible plan to reform the transfer system and, therefore, stabilize the transfer flow.

Baseline case: Fig. 12 illustrates the baseline case. The economy is at its steady state until the period $t=5$, then the transfers switch to and stay at the explosive regime for the next 35 years. In the top right panel, the shaded areas show how the state-dependent distributions of fiscal limits change over time, with the horizontal axis representing the time horizon and the vertical axis showing the debt level scaled by the steady-state output. The light red area covers the debt range within which the cumulative probability rises from 0.15 to 0.5, the red area is for the probability range from 0.5 to 0.8, and the dark red area from 0.8 to 0.99. The time-varying distributions become more dispersed at the period $t=5$ when the transfers switch to the explosive regime. For instance, at the debt level of 190% of steady-state GDP, the cumulative probability is 0.15 in the stationary transfer regime, but jumps to above 0.5 in the explosive regime.

Growing transfers raise the fiscal deficit and, therefore, the debt level. Tax rates go up, which, nevertheless, is not sufficient to contain the rising debt. Higher distorting taxes lower the labor supply and the consumption, and also keep the interest rate low for an extended period. The latter is because forward-looking households expect higher taxes and, therefore, higher marginal utility of consumption in the future due to growing transfers, which motivates them to save more today and lowers

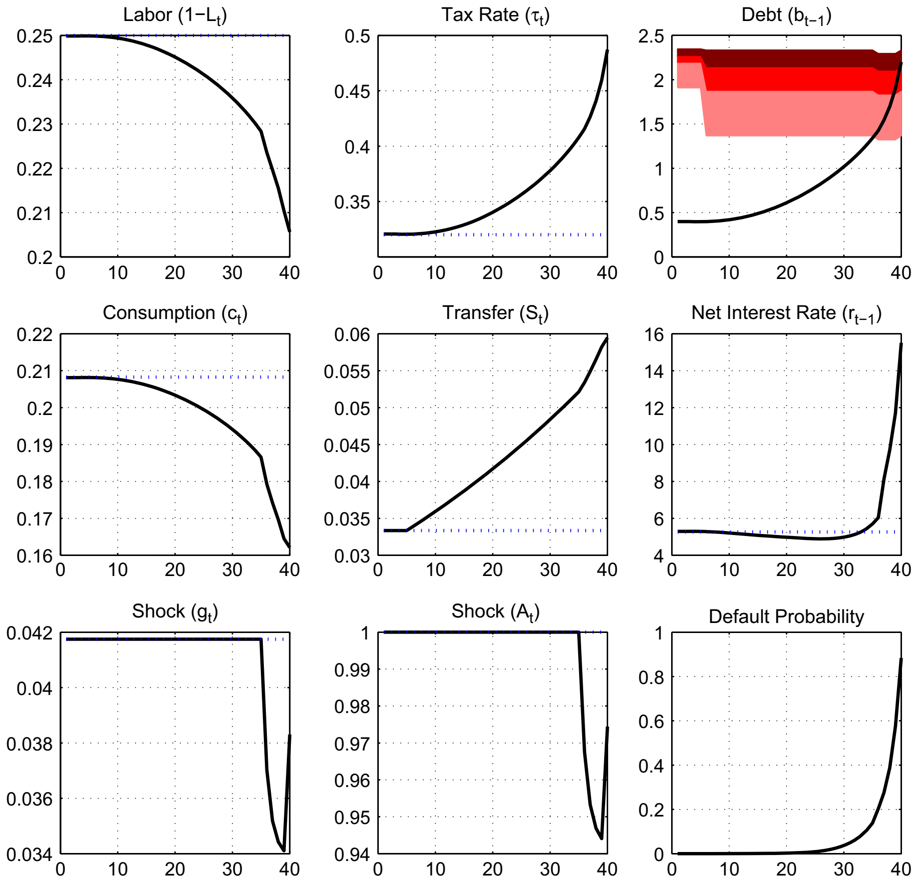

Fig. 14. Nonlinear simulations with fiscal austerity measures in the short run: lower government purchases from $t=35$–40.

the borrowing cost for the government. The right plot on the second row shows that the interest rate can be lower than the steady-state level for 30 years even the debt ratio is rising steadily from 40% all the way to 110%. Households are willing to continue purchasing the government debt because they expect the growing-transfer regime to be reverted. Rising debt, however, eventually comes at a price. As the debt level shoots up from 120% to 150% and then to 200%, the interest rate jumps from 5.2 to 6.2 and then to 9.5 percentage points, and the default probability rises from less than 0.1 all the way to 0.55.

The stunning message from Fig. 12 is that the path of interest rate can be highly nonlinear. Even when government debt rises up steadily, interest rate can stay low for an extended period due to intertemporal consumption smoothing. Default risk premia, however, start to emerge when the debt level reaches to such a point that sovereign default becomes possible. Once risk premia begin to rise, they do so rapidly.

Economic downturn: The productivity level stays at the steady state in the baseline case. Now we consider a severe recession on top of the growing transfers. As illustrated in Fig. 13, the economy receives a sequence of negative productivity shocks. The simulated path of productivity is summarized in the following table, for instance, the level is 3.25% lower than the steady state at the period $t=35$.

	$t=35$	$t=36$	$t=37$	$t=38$	$t=39$	$t=40$
Productivity	−3.25%	−4.67%	−5.31%	−5.6%	−2.56%	−1.16%

With a lower productivity, households work and consume less. The interest rate rises partly because the household substitutes away from future consumption and partly because the default probability rises sharply. Comparing to the baseline case in Fig. 12, the economic downturn intensifies the fiscal woes through two channels. A lower level of productivity shifts down the distribution of fiscal limits, raising the default probability even if the debt level stayed the same. At the same time, it also slashes tax revenue and, therefore, the government has to borrow more to roll over its debt. Higher government debt, in conjunction with lower distributions of fiscal limits, bids up the sovereign borrowing cost, which further deteriorates the government budget. Under such a self-fulling mechanism, a persistent economic downturn, which lasts for 6 years and cuts the productivity level by 5.6% at the valley, can raise the debt ratio by 100%, as shown in Fig. 12. To the end, the interest rate reaches 17 percentage points which is more than

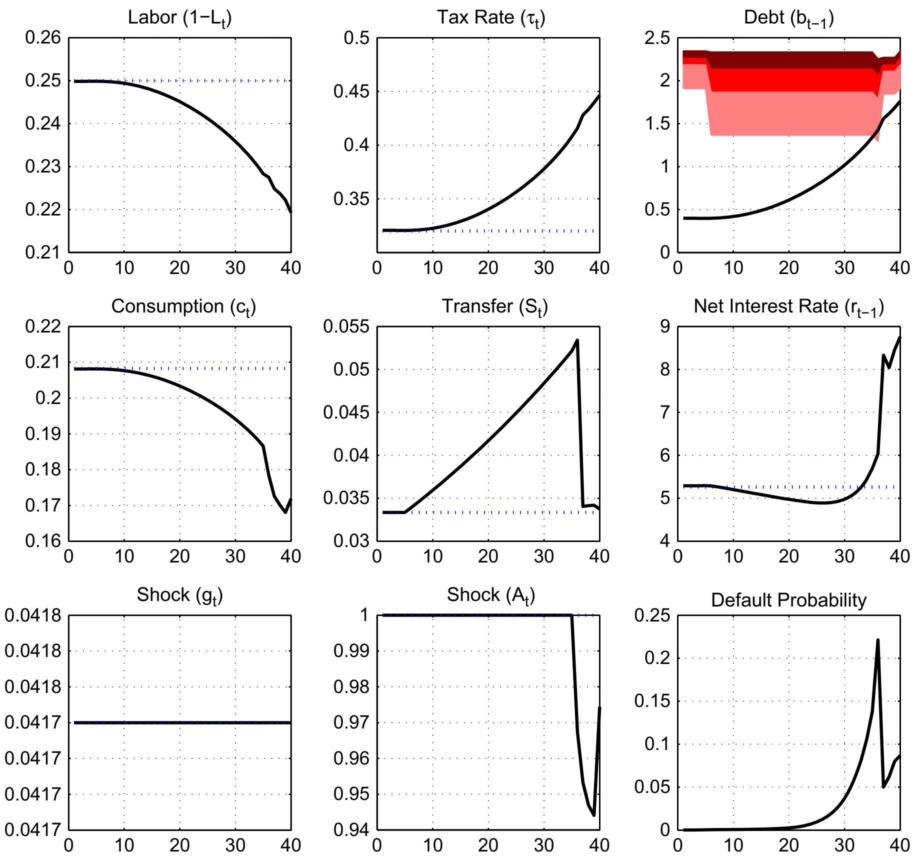

Fig. 15. Nonlinear simulations with long-term fiscal reform: transfers switch to the stationary regime at $t=35$.

7 percentage points higher than the baseline case, and sovereign default becomes unavoidable as the probability jumps close to 1.

Different fiscal responses: The government may attempt to stabilize the debt level and alleviate the sovereign default risk. Fig. 14 shows the scenario in which the government attempts to contain the default risk by cutting its purchases (g_t). The following table summarizes the simulated paths for the government purchases and the productivity. The government begins to cut its purchases at the start of its economic downturn, for instance, the purchases are 11.3% lower than the steady-state level at the period $t=35$.

	$t=35$	$t=36$	$t=37$	$t=38$	$t=39$	$t=40$
Productivity	−3.25%	−4.67%	−5.31%	−5.59%	−2.56%	−1.16%
Government purchases	−11.3%	−15.7%	−17.5%	−18.3%	−8.25%	−3.6%

Comparing Figs. 13 and 14, the sizeable cuts in government purchases reduce the debt ratio by 20%, but have a negligible impact in containing the default risk, as the interest rate drops down by a mere 2 percentage points and the default probability is still as high as 0.85. Such a marginal impact is consistent with the pricing rules in Fig. 11: although a lower level of government purchases does reduce the fiscal deficit, the temporary reduction does not change households' expectation about fiscal policy in the long run and, therefore, do not shift down the distributions of fiscal limits much, against which households price the sovereign debt.

An alternative scenario is the government has enough political power and willingness to switch the lump-sum transfers away from the explosive path to the stationary regime, as illustrated in the middle panel of the second row in Fig. 15. Following the same logic as in the case of economic downturn, the switch of transfer regime helps to contain the default risk through two channels. A lower level of transfers helps to balance the government budget. The tax revenue still drops in an economic downturn, and as a result, the debt level continues to increase, but do so at a much slower pace when comparing to Fig. 13. More importantly, the regime switch of transfers changes the households' expectation about transfers and fiscal surplus in the future, shifting up the state-dependent distributions of fiscal limit today by a substantial margin. As shown in the right top panel, the debt level that corresponds to a default probability of 0.15, or the bottom of the light red area, goes up by 50% of the steady-state output. As a result, the default probability drops from more than 0.2 to less than 0.1, even when the debt level continues to increase.

These two experiments highlight that fiscal austerity measures that aim to balance the government budget in the short run, unfortunately, may not be able to contain the default risk premium, even with sizeable cuts as shown in Fig. 14. On the other hand, a long-term plan for fiscal reform, if credibly changes the market's expectation about future fiscal policies, may be able to alleviate the rapidly rising risk premium.

5. Conclusion

This paper presents a general equilibrium framework in which the fiscal limits, which measure the government's ability to service its debt, arise endogenously from dynamic Laffer curves. The state-dependent distributions of fiscal limits depends on underlying economic fundamentals, such as the growth of transfers, the size of the government, the degree of the countercyclical fiscal policy, and economic diversity. The country-specific distributions imply that the market perceives the riskiness of sovereign debt issued by different countries to be different, which is consistent with the observation that developed countries are downgraded at different levels of debt. Due to the existence of fiscal limits, the model produces a nonlinear relationship between the default risk premia and the level of government debt. Even when government debt rises up steadily, interest rate can stay low for an extended period. Default risk premia, however, start to emerge when the debt level reaches to a point that sovereign default becomes possible. Once risk premia begin to rise, they do so rapidly.

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Appendix A. Data appendix

For the developed countries, the fiscal data is from the OECD Economic Outlook No. 84 (2009) for the period between 1971 and 2010 (due to data restriction, the data of Greek government debt is from European Commission (2009)). The sample includes Australia, Austria, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, Greece, Ireland, Italy, Japan, Netherlands, Norway, New Zealand, United Kingdom, and United States. The average tax rate is defined as the ratio of the total tax revenue over the GDP, including social security, indirect and direct taxes. The government purchases are government final consumption of expenditures. Lump-sum transfers are defined as the sum of social security payments, net capital transfers and subsidies. Using a Hodrick and Prescott (1997) filter, we detrend the data of the real GDP per worker from Penn World Table Version 6.2 (see Heston et al., 2009) and estimate the shock process of productivity.[13] The elasticity of lump-sum transfers with respect to productivity (η^z) is estimated using the detrended data of real lump-sum transfers and real GDP per worker. The elasticity of government purchases with respect to productivity (η^z) is estimated using the detrended data of real government expenditures and real GDP per worker. The estimation excludes Switzerland, Germany, Spain and United Kingdom due to data restrictions.

For the default data in the emerging market economies, the claimed amount of default debt is from Moody's (2009), the total public debt is from Panizza (2008), and the haircuts are estimates from Sturzenegger and Zettelmeyer (2008).

Appendix B. Simulation procedure for fiscal limits

In this model, the choices of household consumption and labor supply only depend on the income tax rate and the exogenous state variables (A_t,g_t). Assume the utility function is $u(c,L) = \log c + \phi \log L$. The household first-order conditions can be written as:

$$1-L_t = \frac{A_t(1-\tau_t)+\phi g_t}{A_t(1+\phi-\tau_t)} \tag{B.1}$$

$$c_t = \frac{(A_t-g_t)(1-\tau_t)}{1+\phi-\tau_t} \tag{B.2}$$

[13] The real GDP per worker is only available until 2007.

The tax revenue (T_t) is

$$T_t=\tau_t\frac{A_t(1-\tau_t)+\phi g_t}{1+\phi-\tau_t}=(1+2\phi)A_t-\phi g_t-\left(A_t(1+\phi-\tau_t)+\frac{(1+\phi)\phi(A_t-g_t)}{1+\phi-\tau_t}\right) \tag{B.3}$$

The tax revenue reaches to the maximum level (T_t^{max}) when the tax rate reaches the peak point of the Laffer curve (τ_t^{max}):

$$\tau_t^{max}=1+\phi-\sqrt{\frac{(1+\phi)\phi(A_t-g_t)}{A_t}} \tag{B.4}$$

$$T_t^{max}=(1+2\phi)A_t-\phi g_t-2\sqrt{(1+\phi)\phi A_t(A_t-g_t)} \tag{B.5}$$

Since there exists a unique mapping between the exogenous state space (A_t,g_t) to τ_t^{max} and T_t^{max}, the conditional distribution of fiscal limit ($\mathcal{B}^*(A_t,g_t,rs_t^z)$) can be obtained using Markov Chain Monte Carlo simulation:

1. First, for each simulation i, we randomly draw the shocks for productivity (A_{t+j}), government purchases (g_{t+j}), and the transfer regime (rs_{t+j}^z) for 200 periods conditional on the starting state (A_t,g_t,rs_t^z). Assuming that the tax rate is always at the peak of the dynamic Laffer curves, we compute the paths of all other variables following the household first-order conditions and the budget constraints, and the discounted sum of maximum fiscal surplus as specified below:

$$B_i^*(t)=\sum_{j=0}^{\infty}\beta^j\frac{u_c^{max}(A_{t+j},g_{t+j})}{u_c^{max}(A_t,g_t)}(T^{max}(A_{t+j},g_{t+j})-g_{t+j}-z(rs_{t+j}^z,A_{t+j})) \tag{B.6}$$

2. Second, we repeat the simulation for 100,000 times and obtain the conditional distribution of $\mathcal{B}^*(A_t,g_t,rs_t^z)$ using the simulated $B_i^*(t)$ ($i=1,\ldots,100,000$).
3. Finally, we repeat the first and second steps for all possible exogenous states (A_t,g_t,rs_t^z) within the discretized state space.

Appendix C. Nonlinear computational method

Other than the end-of-period government debt b_t, all other variables are either exogenous or can be computed in terms of the current state ($\psi_t=\{b_{t-1},b_t^*,\delta_t,A_t,g_t,z_{t-1},rs_t^z\}$):

$$\tau_t-\tau=\gamma((1-\Delta_t)b_{t-1}-b) \tag{C.1}$$

$$c_t=\frac{(A_t-g_t)(1-\tau_t)}{1+\phi-\tau_t} \tag{C.2}$$

$$\ln\frac{g_t}{g}=\rho^g\ln\frac{g_{t-1}}{g}+\varepsilon_t^g \tag{C.3}$$

$$\ln\frac{A_t}{A}=\rho^A\ln\frac{A_{t-1}}{A}+\varepsilon_t^A \tag{C.4}$$

$$z_t=\begin{cases} z+\zeta^z(A_t-A) & \text{if } rs_t^z=1\\ \mu^z z_{t-1}+\zeta^z(A_t-A) & \text{if } rs_t^z=2\end{cases}$$

$$\Delta_t=\begin{cases} 0 & \text{if } b_{t-1}<b_t^* \quad (b_t^*\sim\mathcal{B}^*(A_t,g_t,rs_t^z))\\ \delta_t & \text{if } b_{t-1}\geq b_t^* \quad (\delta_t\sim\Omega)\end{cases}$$

The decision rule for government debt, $b_t=f^b(\psi_t)$, is solved in the following steps:

1. Define the grid points by discretizing the state space ψ_t. Make an initial guess of the decision rule f_0^b over the state space.
2. At each grid point, solve the following core equation and obtain the updated rule f_i^b using the given rule f_{i-1}^b:

$$\frac{(1-\Delta_t)b_{t-1}+g_t+z_t-\tau_tA_t(1-L(\psi_t))}{f_i^b(\psi_t)}=\beta E_t\left\{(1-\Delta(\psi_{t+1}))\frac{u_c(\psi_{t+1})}{u_c(\psi_t)}\right\} \tag{C.5}$$

   where $\psi_{t+1}=(f_{i-1}^b(\psi_t),b_{t+1}^*,\delta_{t+1},A_{t+1},g_{t+1},z_t,rs_{t+1}^z)$. The integral in the right-hand side is evaluated using numerical quadrature.
3. Check convergence of the decision rule. If $|f_i^b-f_{i-1}^b|$ is above the desired tolerance (set to $1e-6$), go back to step 2; otherwise, f_i^b is the decision rule.

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