Asset Price Bubbles

Honours Macroeconomics Problem Set

October 18, 2021

Consider a stock that pays dividends of D_t and whose price is P_t in period t. Consumers are risk neutral with discount rate r, so their objective function is:

$$E_t\left[\sum_{s=0}^{\infty}\frac{C_{t+s}}{(1+r)^s}\right]$$

1. Use a calculus of variations argument (similar to the one in the notes and textbook) to show that equilibrium requires $P_t = E_t\left[\frac{D_{t+1}+P_{t+1}}{(1+r)}\right]$ (assume that if the stock is sold, it happens after the dividend for that period is paid out.)

2. Iterate the expresion in (1) forward to derive an expression for P_t in terms of only future dividends and the interest rate, using the following *no-bubbles* condition:
$$\lim_{s\to\infty} E_t\left[\frac{P_{t+s}}{(1+r)^s}\right] = 0$$
and the law of iterated expectations: $E_t\left[E_{t+1}\left[x_{t+s}\right]\right] = E_t\left[x_{t+s}\right]$.

3. Give a clear description of the intuitive meaning of the *no-bubbles* condition.

4. Now we relax the *no bubbles* assumption

   (a) **Deterministic bubbles:** Suppose that P_t equals the expression you derived in (2) plus $(1+r)^t b$ where $b > 0$. Show that this expression still satisfies the consumer's optimality condition $P_t = E_t\left[\frac{D_{t+1}+P_{t+1}}{(1+r)}\right]$ and interpret what this means. [Hint: start with the new expression, lead it forward one period and take conditional expectation E_t]

   (b) **Stochastic Bursting bubbles:** (Blanchard, 1979) Suppose that P_t equals the expression you derived in (2) plus q_t, where
$$q_t = \begin{cases} \frac{(1+r)q_{t-1}}{\alpha} & \text{with probability } \alpha \\ 0 & \text{with probability } (1-\alpha) \end{cases}$$

i. Again show that this new expression for P_t still satisfies $P_t = E_t\left[\frac{D_{t+1}+P_{t+1}}{(1+r)}\right]$

ii. If there is a bubble in period t (i.e. that $q_t > 0$), what is the probability that the bubble has burst by period $t+s$ (i.e. that $q_{t+s} = 0$)?

iii. What is the limit of this probability in (ii) as $s \to \infty$. Interpret this result.