Problem Set - RBC model - Memo

Set-up

Consider the standard RBC model covered in class, but assume that there is a constant proportional tax levied on income from capital. Denote the tax rate by τ. The objective function is given by:

$$\mathcal{U} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + b \ln \left(1 - L_t\right)\right]$$

Where C_t is consumption, L_t is labour supply, $\beta \in (0,1)$ is the subjective discount factor and $b > 0$ a parameter. Consumers own their labour resources as well as the capital stock in the economy. Each unit of labour supplied earns wages W_t and each unit of capital earns before tax return of r_t. Capital K_t depreciates at rate $\delta > 0$ per period. Denote investment by I_t. The government consumes its tax income G_t at no benefit to the consumer.

Question 1

1. Give the government budget constraint.

- Constant proportional tax τ levied on capital income, $r_t K_t$.
- The government consumes its tax income G_t.

$$\begin{aligned} \text{Expenses} &= \text{Income} \\ G_t &= \tau r_t K_t \end{aligned}$$

Question 2

2. Why is government income time varying while the tax rate is constant?

$$G_t = \tau r_t K_t$$

It depends on the time varying level of capital, K_t, and capital equilibrium return, r_t. In steady state, everything will be constant.

Question 3

3. Derive the inter-temporal budget constraint of the consumer in terms of consumption, capital stock and labour.

$$\mathcal{U} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + b \ln (1 - L_t)\right]$$

- Consumers own their labour resources. Each unit of labour supplied earns wages W_t.
- Consumers own the capital stock in the economy. Each unit of capital earns before tax return of r_t. Capital K_t depreciates at rate $\delta > 0$ per period.
- Constant proportional tax τ levied on capital income, $r_t K_t$.

Budget constraint: Expenses = Income

- Expenses: $C_t + I_t$
- Income: $W_t L_t + (1 - \tau) r_t K_t$
- Capital stock K_t depreciates at rate δ each period. The capital accumulation equation (often called the law of motion of capital) is given by:

$$\begin{aligned} K_{t+1} &= (1 - \delta) K_t + I_t \\ I_t &= K_{t+1} - (1 - \delta) K_t \end{aligned}$$

The full budget constraint in period t of the household:

$$\begin{aligned} \text{Expenses} &= \text{Income} \\ C_t + I_t &= W_t L_t + (1 - \tau) r_t K_t \\ C_t + K_{t+1} - (1 - \delta) K_t &= W_t L_t + (1 - \tau) r_t K_t \\ C_t + K_{t+1} &= W_t L_t + (1 + (1 - \tau) r_t - \delta) K_t \end{aligned}$$

Question 4

4. Set up the Lagrangian and derive the Euler equation.

$$\begin{aligned} \mathcal{L}(C_t, L_t, K_{t+1}) = \mathbb{E}_0 \sum_{t=0}^{\infty} \quad & \left\{ \beta^t \left[\ln C_t + b \ln (1 - L_t)\right] \right. \\ & \left. + \lambda_t \left[W_t L_t + (1 + (1 - \tau) r_t - \delta) K_t - C_t - K_{t+1}\right] \right\} \end{aligned}$$

FOC wrt C_t :

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0$$

$$\beta^t C_t^{-1} - \lambda_t = 0 \tag{1}$$

$$\beta^t C_t^{-1} = \lambda_t \tag{2}$$

$$\beta^{t+1} C_{t+1}^{-1} = \lambda_{t+1} \tag{3}$$

Question 4

FOC wrt K_{t+1} :

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0$$

$$\lambda_t = \mathbb{E}_t \left[\left(1 + (1-\tau) r_{t+1} - \delta\right) \lambda_{t+1} \right] \tag{4}$$

Combining the first order conditions allows us to find the standard optimality conditions, i.e. the Euler equation, which determines the optimal evolution of consumption.
Combine equations 2, 3 and 4:

$$C_t^{-1} = \mathbb{E}_t \left[\beta \left(1 + (1-\tau) r_{t+1} - \delta\right) C_{t+1}^{-1} \right]$$

Question 5

$$C_t^{-1} = \mathbb{E}_t \left[\beta \left(1 + (1-\tau) r_{t+1} - \delta\right) C_{t+1}^{-1} \right]$$

5. In steady state, consumption and capital returns are constant and there is no uncertainty. Find the steady state level of the return on capital in terms of the parameters of the model and determine how it is affected by an increase in the tax rate, τ?

$$C_t = C^* \ \forall \ t$$

$$C^{*-1} = \beta \left(1 + (1-\tau) r^* - \delta\right) C^{*-1}$$

$$\beta \left(1 + (1-\tau) r^* - \delta\right) = 1$$

$$r^* = \frac{1}{(1-\tau)} \left(\frac{1}{\beta} - 1 + \delta \right)$$

With an increase in the tax rate, $0 < \tau < \tau' < 1$, the steady state rate of return on capital rises:

$$r^* = \uparrow \frac{1}{(1-\tau')} \left(\frac{1}{\beta} - 1 + \delta \right)$$

An increase in the tax rate increases the steady state return of capital:
→ makes sense, since less capital will be held in steady state
→ diminishing marginal returns: lower capital means higher return