The Benchmark New Keynesian Model

Dynamic Economic Theory (871)

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Contents:

Readings

- Walsh, C. E., 2010. Monetary Theory and Policy, 3rd Ed. The MIT Press. (CH 8, or CH5 in the 2nd Ed.)
- Gali, J., 2008. Monetary Policy, Inflation, and the Business Cycle: An introduction to the New Keynesian Framework. Princeton University Press. (CH 3)

Introduction

RBC vs. NKM

The *RBC revolution* (Kydland and Prescott, 1982) has both conceptual and methodological impacts (Gali, 2008):

- RBC theory claims: 1. the efficiency of business cycles; 2. the importance of technology shocks as the source of economic fluctuations; 3. the limited role of monetary factors;
- Methodologically, use DSGE model as a central tool for macroeconomic analysis; evaluate models by calibration and simulation.

Introduction

RBC vs. NKM

The *NKM* has more solid micro foundations than its Keynesian ancestor, and it is more useful than its RBC predecessor. The main properties of the NKM are:

- Monopolistically competitive firms;
- Nominal rigidities (prices & wages);
- Short-run non-neutrality of monetary policy.

Differences with respect to RBC models:

- Business cycles are inefficient; i.e., the economy's response to shocks is inefficient in the short-run;
- Nominal rigidities cause(?) short-run non-neutrality of monetary policy, which justifies(?) potential welfare-enhancing interventions by the monetary authority (namely, on the AD side).

Introduction

Basic MIU model combines with the assumption of monop. comp. goods mrkt and price stickiness $\Rightarrow$ basis of the simple linear NK model $\rightarrow$ linked to AS-IS-LM model.

3 key modifications to MIU model $\Rightarrow$ "benchmark" NK model:

1. endogenous variations in the capital stock ignored
   - $\because$ little observed relation. btw capital and output in BC dynamics
   - BUT BGG(1999), CEE(2005) show variable capital utilisation costs on inflation NB!
2. single good replaced by a continuum of differentiated goods (monop. comp.)
3. MP rule for nominal interest rate setting: nominal Q of money $\therefore$ endog. deter. to achieve desired i_t (vice versa).

Introduction

This session presents the so called "benchmark" New Keynesian Model (NKM) in the literature, in which imperfect competition and price stickiness are embedded in a general equilibrium model.

Main features of the NKM:

- Households: consume goods, supply labour, and hold money and bonds;
- Final good firms: produce final good Y_t using intermediate goods $Y_{j,t}$ as the only input, firms are perfectly competitive;[1]
- Intermediate goods firms: hire labour to produce and sell differentiated intermediate goods $Y_{j,t}$ in MC markets;
- Monetary authority: controls monetary policy.

[1] Walsh p.331 discusses a slightly different conceptual approach to derive the demand function for good j, but with analogous results.

The Model

Household

The representative household chooses $\{C_t, N_t, M_t, B_t\}$ to max her utility function:

$$E\sum_{t=0}^{\infty}\beta^t\left[\frac{(C_t)^{1-\eta_c}}{1-\eta_c}+\frac{(M_t/P_t)^{1-\eta_m}}{1-\eta_m}-\frac{(N_t)^{1+\eta_n}}{1+\eta_n}\right] \tag{1}$$

s.t. the budget constraint (BC):

$$\frac{(1+i_{t-1})B_{t-1}}{P_t}+\frac{M_{t-1}}{P_t}+\frac{W_t}{P_t}N_t+T_t+D_t=C_t+\frac{B_t}{P_t}+\frac{M_t}{P_t} \tag{2}$$

where D_t denotes the real dividends received from ownership of intermediate goods firms.

The Model

Household

The following conditions, in addition to the BC, must hold in equil$\equiv$

- FOC for hours worked:

$$\frac{N_t^{\eta_n}}{C_t^{-\eta_c}} = \frac{W_t}{P_t} \tag{3}$$

Intratemporal optimality condition setting MRS btw leisure and cons. $=$ real wage.

- FOC for bond holdings:

$$C_t^{-\eta_c} = \beta(1+i_t)E_t\left[\frac{P_t}{P_{t+1}}C_{t+1}^{-\eta_c}\right] \tag{4}$$

Euler eqn for the optimal intertemporal allocation of consumption.

- FOC for money holdings:

$$\frac{\left(\frac{M_t}{P_t}\right)^{-\eta_m}}{C_t^{-\eta_c}} = \frac{i_t}{1+i_t} \tag{5}$$

Intratemporal optimality condition setting MRS btw money and cons. $=$ opp. cost of holding money.

The Model

Final Goods Firm

A representative final-goods firm produces the composite final good Y_t using a continuum of intermediate goods $Y_{j,t}$ according to the Dixit and Stiglitz (1977) CES production function:[2]

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\varphi_{p,t}-1}{\varphi_{p,t}}} dj \right)^{\frac{\varphi_{p,t}}{\varphi_{p,t}-1}} . \tag{6}$$

The firm minimizes its costs:[3]

$$\min_{Y_{j,t}} \int_0^1 P_{j,t} Y_{j,t} dj \tag{7}$$

given the production constraint:

$$Y_t \leq \left(\int_0^1 Y_{j,t}^{\frac{\varphi_{p,t}-1}{\varphi_{p,t}}} dj \right)^{\frac{\varphi_{p,t}}{\varphi_{p,t}-1}} \tag{8}$$

[2] The integral here represents a continuum of intermediate goods indexed by $j \in [0, 1]$.

[3] Regardless of the level of Y_t, it will always be optimal for the firm to purchase the combination of individual goods that minimizes the cost of achieving this level of the composite good.

The Model

Final Goods Firm

The Lagrangian for the firm is given by the following expression:

$$L = \int_0^1 P_{j,t} Y_{j,t} dj + \mu_t \left[Y_t - \left(\int_0^1 Y_{j,t}^{\frac{\varphi_{p,t}-1}{\varphi_{p,t}}} dj \right)^{\frac{\varphi_{p,t}}{\varphi_{p,t}-1}} \right] \tag{9}$$

The first order condition with respect to $Y_{j,t}$ is:

$$P_{j,t} = \left(\frac{Y_{j,t}}{Y_t} \right)^{\varphi_{p,t}} P_t \tag{10}$$

or

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\varphi_{p,t}} Y_t \tag{11}$$

and the price index:

$$P_t = \left(\int_0^1 P_{j,t}^{1-\varphi_{p,t}} dj \right)^{1/1-\varphi_{p,t}} \tag{12}$$

The Model

Intermediate Goods Firm

- A representative intermediate goods firm j produces $Y_{j,t}$ according to the following production function:[4]

$$Y_{j,t} = \xi_{z,t} N_{j,t} \tag{13}$$

- The market demand for $Y_{j,t}$:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\varphi_{p,t}} Y_t \qquad ((11))$$

- Facing Calvo-type stickiness: in each time period, only a random fraction $(1 - \theta_p)$ of firms can reset prices.

[4] For simplicity, we ignore capital stock K_t. Eq. 13 exhibits constant returns to scale.

The Model

Intermediate Goods Firm

Marginal Cost

To derive the MC for the firm, 1st consider the firm's cost minimization, in real terms, s.t. Eq.(13):

$$\min_{N_{j,t}} \left(\frac{W_t}{P_t} \right) N_{j,t} + \lambda_t (Y_{j,t} - \xi_{z,t} N_{j,t}) \; . \tag{14}$$

The FOC implies:

$$\frac{W_t}{P_t} = \lambda_t \xi_{z,t} \tag{15}$$

$$or \quad \frac{W_t}{P_t} = \lambda_t \left(\frac{Y_{j,t}}{N_{j,t}} \right) . \tag{16}$$

Multiplying $N_{j,t}$ on both sides of (16) gives us the firm's cost function:

$$\frac{W_t}{P_t} N_{j,t} = \lambda_t Y_{j,t} \; , \tag{17}$$

where $\lambda_t = \frac{\frac{W_t}{P_t}}{\xi_{z,t}}$ can be treated as the firm's real marginal cost (MC_t).

The Model

Intermediate Goods Firm
The pricing decision with sticky prices

Following Calvo (1983), in each time period only a random fraction $1-\theta_p$ of intermediate good firms have an opportunity to reset prices. Assuming a CES aggregate of the average price level, the aggregate price index is given by:

$$P_{j,t}^{1-\varphi_{p,t}} = \theta_p P_{j,t-1}^{1-\varphi_{p,t}} + (1-\theta_p)(P_{j,t}^{*})^{1-\varphi_{p,t}}, \qquad 0 \leq \theta_p \leq 1, \tag{18}$$

where $P_{j,t-1}$ is previous price level, and $P_{j,t}^{*}$ is the average price chosen by those who have the chance to reset the prices.

The Model

Intermediate Goods Firm

The pricing decision with sticky prices

In each time period, the representative intermediate-good firm chooses $P^*_{j,t}$ to maximize its profit:

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \big[\big(\frac{P^*_{j,t}}{P_{t+i}} - MC_{t+i} \big) Y_{j,t,t+i} \big] \tag{19}$$

The discount factor is $\beta^i \Lambda_{t,t+i}$, where:

$$\Lambda_{t,t+i} = \big(\frac{C_{t+i}}{C_t} \big)^{-\eta_c} \tag{20}$$

The FOC for the optimal $P^*_{j,t}$:

$$Q_{j,t} = \frac{P^*_{j,t}}{P_t} = \left(\frac{\varphi_{p,t}}{\varphi_{p,t} - 1} \right) \frac{E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \left[MC_{t+i} \left(\frac{P_t}{P_{t+i}} \right)^{\varphi_{p,t}} Y_{t+i} \right]}{E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \left[\left(\frac{P_t}{P_{t+i}} \right)^{\varphi_{p,t} - 1} Y_{t+i} \right]} \ , \tag{21}$$

and $\because$ all firm's face the same decision rule, we drop j:

$$Q_t = \frac{P^*_t}{P_t} = \frac{\varphi_{p,t}}{\varphi_{p,t} - 1} E_t \sum_{i=0}^{\infty} \Phi_{t,t+i} MC_{t+i} \ . \tag{22}$$

The Model

Intermediate Goods Firm

The pricing decision with sticky prices

Under a flexible adjust. equilibrium ($\theta_p = 0$), (22) becomes:

$$\frac{P_t^*}{P_t} = \frac{\varphi_{p,t}}{\varphi_{p,t} - 1} MC_t = \psi_t MC_t \tag{23}$$

This is a standard result in a model of monopolistic competition, each firm sets its price P_t^* equal to a markup, $\frac{\varphi_p}{\varphi_p - 1}$ over its nominal marginal cost $P_t MC_t$ (Walsh, 2010: 334). That is, $P_t^* = \psi P_t MC_t$.[5]

- When prices are *flexible* all firms can charge the same price: $P_t^* = P_t \Rightarrow MC_t = \frac{1}{\psi}$. Here, the markup ψ is inversely related to the price elasticity of demand φ_p.[6]
- When prices are *sticky* ($0 < \theta_p < 1$), the optimal price is a markup over a weighted average of current and expected future nominal marginal costs. Weights depend on expected demand in future and on how quickly firms discount profits: see Eq.(22).

[5] Note: we assume from here that the price elasticity of demand is constant, $\psi_{t=0}$, as in Walsh (2010).

[6] Under a flexible equilibrium with perfect competition: $\varphi_{p,t} = \infty$ and $\psi = 1. \Rightarrow MC_t = 1$.

The Model

Intermediate Goods Firm

The pricing decision with sticky prices

Recall $MC_t = \lambda_t = \frac{\frac{W_t}{P_t}}{\xi_{z,t}}$ from (15), we have:

$$\frac{W_t}{P_t} = \frac{\xi_{z,t}}{\psi} \tag{24}$$

Combining (24) and the FOC for labor supply for the household (3), we have:

$$\frac{W_t}{P_t} = \frac{\xi_{z,t}}{\psi} = \frac{N_t^{\eta_n}}{C_t^{-\eta_c}} \tag{25}$$

which gives us ***the labor market equilibrium***:

$$\xi_{z,t} = \psi \frac{N_t^{\eta_n}}{C_t^{-\eta_c}} \tag{26}$$

Combining the linearized (26), $\eta_n \hat{n}_t^f + \eta_c \hat{c}_t^f = \hat{\xi}_{z,t}$, and the linearized conditions of production function, $\hat{y}_t^f = \hat{n}_t^f + \hat{\xi}_{z,t}$, and resource constraint, $\hat{y}_t^f = \hat{c}_t^f$, gives us the ***flexible price equilibrium output***:

$$\hat{y}_t^f = \frac{1+\eta_n}{\eta_c + \eta_n} \hat{\xi}_{z,t} \tag{27}$$

The Model

Intermediate Goods Firm

The new Keynesian Phillips curve

The linearized (21):

$$\hat{p}_t^* = (1-\theta_p)\beta\Sigma_{i=0}^{\infty}(\theta_p\beta)^i E_t(\hat{p}_{t+i} + \hat{mc}_{t+i}) \tag{28}$$

(28) has the same interpretation as (21): in equilibrium, the optimal price has to be equal to the weighted average of current and future marginal costs.[7] The optimal price is also weighted by the probability that this price will hold in the last period $(1-\theta_p)$.
Linearizing the price index (18):

$$\hat{p}_t = \theta_p\hat{p}_{t-1} + (1-\theta_p)\hat{p}_t^* \tag{29}$$

Combining the last two equations gives us the new Keynesian Phillips curve (NKPC):[8]

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\hat{mc}_t \tag{30}$$

where $\kappa = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}$

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[7] In equilibrium, all firms that reset the price face the same demand curve and choose the same price, therefore $p_{j,t} = p_t$.

[8] For the detailed derivation see the appendix section 8.6 in Walsh (2010).

The Model

Intermediate Goods Firm

The new Keynesian Phillips curve

Remarks:

Compare to the traditional ad hoc Keynesian Philips curve ($\pi_t = \pi_{t-1} + \alpha \tilde{y}_t^{ad} + \xi_{z,t}$), the NKPC has the following properties:

1. derived explicitly from agents' optimization problems;
2. Inflation process is forward-looking;
3. marginal cost is the correct driving variable for inflation process (not depend on an ad hoc measure of output gap $\tilde{y}_t^{ad}$);

But: We can also relate real MC (λ_t) to the output gap $\tilde{y}_t = \hat{y}_t - \hat{y}_t^f$:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \tilde{\kappa}(\tilde{y}_t) , \tag{31}$$

where $\tilde{\kappa} = (\eta_n + \eta_c)\kappa$.

Symmetric Equilibrium

- Aggregate resource constraint:

$$Y_t = C_t \tag{32}$$

- Euler equation for Bonds:

$$R_t \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\eta_c} \right] = 1 \tag{(4)}$$

- Aggregate production function:

$$Y_t = \xi_{z,t} N_t \tag{(13)}$$

- Labor market equilibrium:[9]

$$\lambda_t \xi_{z,t} = \frac{N_t^{\eta_n}}{C_t^{-\eta_c}} \ , \tag{(26)}$$

- Euler equation for money:

$$C_t^{\frac{\eta_c}{\eta_m}} \left(\frac{i_t}{1+i_t} \right)^{-\frac{1}{\eta_m}} = \frac{M_t}{P_t} \tag{(5)}$$

- Aggregate price:

$$P_t^{1-\varphi_{p,t}} = \theta_p P_{t-1}^{1-\varphi_{p,t}} + (1-\theta_p)(P_t^*)^{1-\varphi_{p,t}}, \qquad 0 \leq \theta_p \leq 1, \tag{(18)}$$

- Price setting equation for P_t^* ((21))

[9] note: $\lambda_t = \frac{W_t/P_t}{\xi_{z,t}}$, but we subst W_t/P_t out of equilibrium. In flex-price equilibrium, $\lambda_t = 1$.

The Complete Linearized Model[10]

$$\hat{y}_t = \hat{c}_t \tag{33}$$

$$\hat{y}_t = \hat{n}_t + \hat{\xi}_{z,t} \tag{34}$$

$$\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\eta_c} \hat{r}_t \tag{35}$$

$$\hat{m}_t = \frac{\eta_c}{\eta_m} \hat{c}_t - \left(\frac{1}{\eta_m}\right)\left(\frac{1}{i}\right)\hat{i}_t \tag{36}$$

$$\hat{i}_t = \hat{r}_t + E_t \hat{\pi}_{t+1} \tag{37}$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \hat{mc}_t \tag{38}$$

$$\hat{mc}_t = (\eta_n + \eta_c)\hat{y}_t - (1+\eta_n)\hat{\xi}_{z,t} \tag{39}$$

[10] Note: to complete the model, we still need to add monetary policy rule, such as a Taylor-type interest rate rule or money supply rule.

The linearized IS curve

Remarks:

The following two equations represent the equilibrium condition for a well-specified general equilibrium model.

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \frac{1}{\eta_c}(\hat{i}_t - E_t\hat{\pi}_{t+1}) + r_t^n \tag{40}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \tilde{\kappa}\tilde{y}_t \tag{(31)}$$

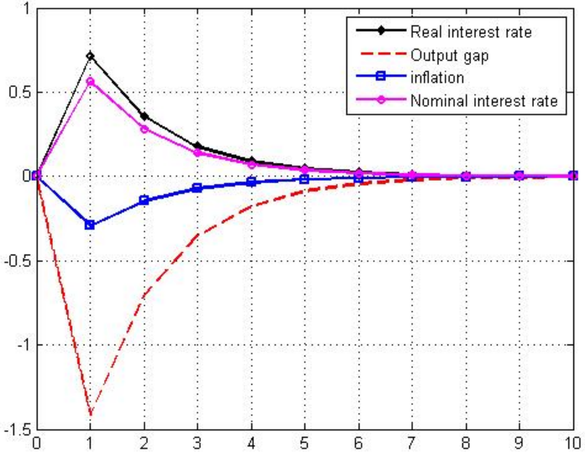
where, $r_t^n = E_t(\hat{y}_{t+1}^f - \hat{y}_t^f) = E_t\Delta\xi_{z,t+1}(\frac{1+\eta_n}{\eta_c+\eta_n})$.

- (40) : AD, an expectational forward-looking IS curve; derived from the Euler condition for the representative household's decision problem;
- (31): AS, the NKPC; derived from the pricing decisions of individual firms;
- (40) & (31), together with a monetary policy equation (eg. $\hat{i}_t = \rho_\pi\hat{\pi}_t + \rho_y\tilde{y}_t + \xi_{i,t}$) will give us the so called benchmark dynamic NKM (3-equation NKM).

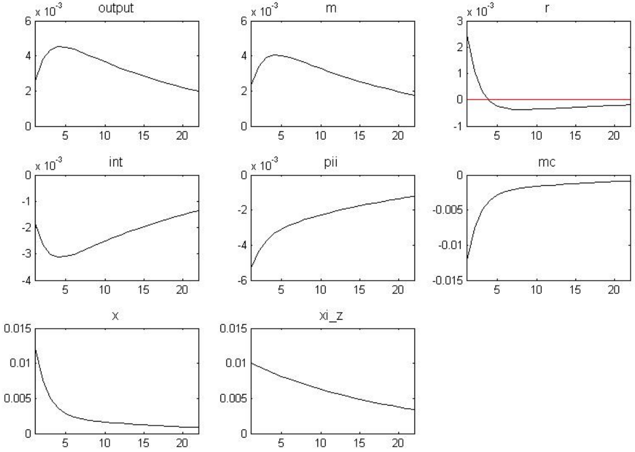
Monetary Transmission Mechanism

Remarks:

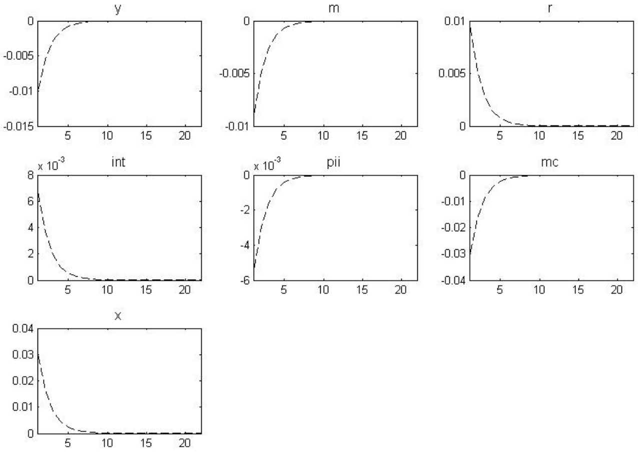
Monetary policy (simple interest rate rule) alters real rate (short-run non-neutral of monetary policy)!



Tech shock



MP shock


y
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-0.005
-0.01
-0.015
5
10
15
20
m
0
-0.005
-0.01
r
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0.005
0
int
x 10^-3
8
6
4
2
0
pii
x 10^-3
0
-2
-4
-6
mc
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-0.01
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-0.03
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x
0.04
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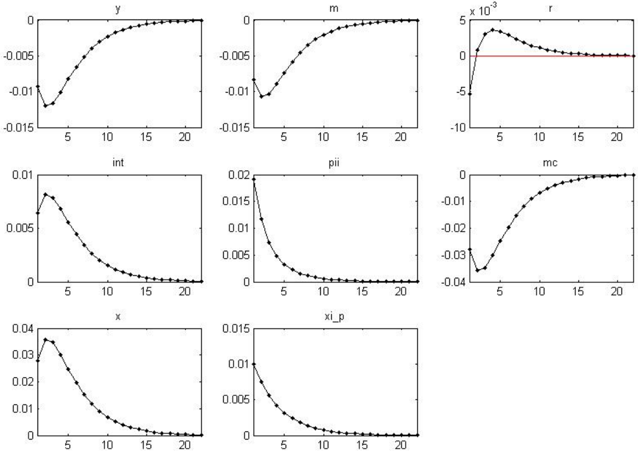
0

Inflation shock



Derivation of $p^*_{j,t}$

Substituting (11) into (19), yields:

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} [\frac{P^*_{j,t}}{P_{t+i}} (\frac{P^*_{j,t}}{P_{t+i}})^{-\varphi_{p,t}} Y_{t+i} - \frac{MC_{t+i}}{P_{t+i}} (\frac{P^*_{j,t}}{P_{t+i}})^{-\varphi_{p,t}} Y_{t+i}] \tag{41}$$

Note: MC_{t+i}, here, is the nominal marginal cost ($P_{t+i}\lambda_{t+i}$). FOC gives:

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \Big[(1-\varphi_{p,t}) (\frac{P^*_{j,t}}{P_{t+i}})^{-\varphi_{p,t}} \frac{Y_{t+i}}{P_{t+i}} + \varphi_{p,t} \frac{MC_{t+i}}{P_{t+i}} (\frac{P^*_{j,t}}{P_{t+i}})^{-\varphi_{p,t}-1} \frac{Y_{t+i}}{P_{t+i}} \Big] = 0 \tag{42}$$

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \Big[(1-\varphi_{p,t}) (\frac{P^*_{j,t}}{P_{t+i}})^{1-\varphi_{p,t}} Y_{t+i} + \varphi_{p,t} \frac{MC_{t+i}}{P_{t+i}} (\frac{P^*_{j,t}}{P_{t+i}})^{-\varphi_{p,t}} Y_{t+i} \Big] = 0 \tag{43}$$

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \Big[(\frac{P^*_{j,t}}{P_{t+i}})^{1-\varphi_{p,t}} Y_{t+i} + \frac{\varphi_{p,t}}{1-\varphi_{p,t}} \frac{MC_{t+i}}{P_{t+i}} (\frac{P^*_{j,t}}{P_{t+i}})^{-\varphi_{p,t}} Y_{t+i} \Big] = 0 \tag{44}$$

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \Big[(\frac{P^*_{j,t}}{P_{t+i}})^{1-\varphi_{p,t}} Y_{t+i} + \frac{\varphi_{p,t}}{1-\varphi_{p,t}} MC_{t+i} \frac{P^*_{j,t}}{P_{t+i}} (\frac{P^*_{j,t}}{P_{t+i}})^{-\varphi_{p,t}} \frac{1}{P^*_{j,t}} Y_{t+i} \Big] = 0 \tag{45}$$

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \Big[(\frac{P^*_{j,t}}{P_{t+i}})^{1-\varphi_{p,t}} Y_{t+i} + \frac{\varphi_{p,t}}{1-\varphi_{p,t}} MC_{t+i} (\frac{P^*_{j,t}}{P_{t+i}})^{1-\varphi_{p,t}} \frac{1}{P^*_{j,t}} Y_{t+i} \Big] = 0 \tag{46}$$

The optimal $P^*_{j,t}$:

$$P^*_{j,t} = \frac{E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \Big[\frac{\varphi_{p,t}}{\varphi_{p,t}-1} MC_{t+i} (\frac{1}{P_{t+i}})^{1-\varphi_{p,t}} Y_{t+i} \Big]}{E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \Lambda_{t,t+i} \Big[(\frac{1}{P_{t+i}})^{1-\varphi_{p,t}} Y_{t+i} \Big]} \tag{47}$$

THE END