



# Firm-specific capital, nominal rigidities and the business cycle<sup>☆</sup>

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## ABSTRACT

This paper formulates and estimates a three-shock U.S. business cycle model. The estimated model accounts for a substantial fraction of the cyclical variation in output and is consistent with the observed inertia in inflation. This is true even though firms in the model re-optimize prices on average once every 1.8 quarters. The key feature of our model underlying this result is that capital is firm-specific. If we adopt the standard assumption that capital is homogeneous and traded in economy-wide rental markets, we find that firms re-optimize their prices on average once every 9 quarters. We argue that the micro implications of the model strongly favor the firm-specific capital specification.

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## 1. Introduction

Macroeconomic data indicate that inflation is inertial. To account for this inertia, macro modelers embed assumptions that are either implausible on a priori grounds or directly in conflict with micro data. For example, in many new-Keynesian macroeconomic models, firms index non-optimized prices to lagged inflation. These models account for inflation inertia by assuming that firms re-optimize their prices every six quarters or even less often.<sup>1</sup> Other new-Keynesian models don't allow for indexing to lagged inflation. In estimated versions of these models, firms change prices once every two years or

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<sup>1</sup> For example, Eichenbaum and Fisher (2007) find that estimated versions of Calvo pricing models with indexing to lagged inflation imply that firms re-optimize prices roughly once every six quarters. Smets and Wouters' (2003) estimated model implies that firms re-optimize prices on average once every nine quarters.

less often.<sup>2</sup> This property contrasts sharply with findings in Bills and Klenow (2004), Golosov and Lucas (2007) and Klenow and Kryvtsov (2008) who argue that firms change prices more frequently than once every two quarters.<sup>3</sup>

In this paper we formulate and estimate a model which is consistent with the evidence of inertia in inflation, even though firms re-optimize prices on average once every 1.8 quarters.<sup>4</sup> In addition our model accounts for the dynamic response of 10 key U.S. macro time series to monetary policy shocks, neutral technology shocks and capital embodied shocks.<sup>5</sup>

In our model aggregate inflation is inertial despite the fact that firms re-optimize prices frequently. The inertia reflects that when firms do re-optimize prices, they change prices by a small amount. Firms change prices by a small amount because each firm's short run marginal cost curve is increasing in its own output.<sup>6</sup> This positive dependency reflects our assumption that in any given period, a firm's capital stock is pre-determined. In standard equilibrium business cycle models a firm's capital stock is not pre-determined and all factors of production, including capital, can be instantly and costlessly transferred across firms. These assumptions are empirically unrealistic but are defended on the grounds of tractability. The hope is that these assumption are innocuous and do not affect major model properties. In fact these assumptions matter a lot.

In our model, a firm's capital is pre-determined and can only be changed over time by varying the rate of investment. These properties follow from our assumption that capital is completely firm-specific. Our assumptions about capital imply that a firm's marginal cost curve depends positively on its output level.<sup>7</sup> To see the impact of this dependence on pricing decisions, consider a firm that contemplates raising its price. The firm understands that a higher price implies less demand and less output. A lower level of output reduces marginal cost, which other things equal, induces a firm to post a lower price. Thus, the dependence of marginal cost on firm-level output acts as a countervailing influence on a firm's incentives to raise price. This countervailing influence is why aggregate inflation responds less to a given aggregate marginal cost shock when capital is firm-specific.

Anything, including firm-specificity of some other factor of production or adjustment costs in labor, which causes a firm's marginal cost to be an increasing function of its output works in the same direction as firm-specificity of capital. This fact is important because our assumption that the firm's entire stock of capital is predetermined probably goes too far from an empirical standpoint.

We conduct our analysis using two versions of the model analyzed by Christiano, Eichenbaum, and Evans (CEE henceforth, 2005): in one, capital is homogeneous whereas in the other, it is firm-specific. We refer to these models as the homogeneous and firm-specific capital models, respectively. We show that the only difference between the log-linearized equations characterizing equilibrium in the two models pertains to the equation relating inflation to marginal costs. The form of this equation is identical in both models: the change in inflation at time  $t$  is equal to discounted expected change in inflation at time  $t + 1$  plus a reduced form coefficient,  $\gamma$ , multiplying time  $t$  economy-wide average real marginal cost. The difference between the two models lies in the mapping between the structural parameters and  $\gamma$ . In non-linear framework, however, it is not true that the solutions to the homogeneous and firm-specific capital models are observationally equivalent with respect to macro data.<sup>8</sup>

In the homogeneous capital model,  $\gamma$  depends only on agents' discount rates and on the fraction,  $1 - \xi_p$ , of firms that re-optimize prices within the quarter. In the firm-specific capital model,  $\gamma$  is a function of a broader set of the structural parameters. For example, the more costly it is for a firm to vary capital utilization, the steeper is its marginal cost curve and hence the smaller is  $\gamma$ . A different example is that in the firm-specific capital model, the parameter  $\gamma$  is smaller the more elastic is the firm's demand curve.<sup>9</sup> This result reflects that the more elastic is a firm's demand, the greater is the reduction in demand and output in response to a given price increase. A bigger fall in output implies a bigger fall in marginal cost which reduces a firm's incentive to raise its price.

The only way that  $\xi_p$  enters into the reduced form of the two models is via its impact on  $\gamma$ . If we parameterize the two models in terms of  $\gamma$  rather than  $\xi_p$ , they have identical implications for all aggregate quantities and prices in a standard (log-)linearized framework. This observational equivalence result implies that we can estimate the model in terms of  $\gamma$  without taking a stand on whether capital is firm-specific or homogeneous. The observational equivalence result also implies that we cannot assess the relative plausibility of the homogeneous and firm-specific capital models using macro data. However, the two models have very different implications for micro data. To assess the relative plausibility of the

<sup>2</sup> See for example the estimates in Rabanal and Rubio-Ramírez (2005) for the post 1982 era or the NBER working paper 10617 version of Eichenbaum and Fisher (2007).

<sup>3</sup> For example in calibrating their model to the micro data, Golosov and Lucas (2007) select parameters to ensure that firms change prices on average once every 1.5 quarters.

<sup>4</sup> To maintain contact with the bulk of estimated new Keynesian models we assume that firms index prices to lagged inflation. The NBER working paper 10617 version of Eichenbaum and Fisher (2007) establishes that the introduction of firm-specific capital has similar effects in models with and without indexation to lagged prices.

<sup>5</sup> See also DiCecio (2009) for a multi-sectoral general equilibrium model which allows for the same shocks that we consider. Also Edge et al. (2003) consider a general equilibrium model with two types of technology shocks. Galí, López-Salido and Vallés (2002) consider neutral technology shocks only.

<sup>6</sup> For early discussions of this idea, see Ball and Romer (1990) and Kimball (1995).

<sup>7</sup> A closely related assumption that generates an upward sloping marginal cost curve is that there are internal costs of adjusting capital.

<sup>8</sup> See Levin et al. (2008) who emphasize this point and analyze the nature of optimal policy in the non-linear versions of the two models.

<sup>9</sup> See Ball and Romer (1990) and Kimball (1995) for an early discussion of this point.

two models, we focus on the mean time between price re-optimization, and the dynamic response of the cross – firm distribution of production and prices to aggregate shocks. These implications depend on the parameters of the model, which we estimate.

We follow CEE (2005) in choosing model parameter values to minimize the differences between the dynamic response to shocks in the model and the analog objects estimated using a vector autoregressive representation of 10 post-war quarterly U.S. time series.<sup>10</sup> To compute vector autoregression (VAR) based impulse response functions, we use identification assumptions satisfied by our economic model: the only shocks that affect productivity in the long run are innovations to neutral and capital-embodied technology; the only shock that affects the price of investment goods in the long run is an innovation to capital-embodied technology<sup>11</sup>; monetary policy shocks have a contemporaneous impact on the interest rate, but they do not have a contemporaneous impact on aggregate quantities or the price of investment goods. We estimate that together these three shocks account for almost 60 percent of cyclical fluctuations in aggregate output and other aggregate quantities.

We now discuss the key properties of our estimated model. First, the model does a good job of accounting for the estimated response of the economy to both monetary policy and technology shocks. Second, according to our point estimates, households re-optimize wages on average about once a year. Third, our point estimate of  $\gamma$  is 0.014. In the homogeneous capital version of the model, this value of  $\gamma$  implies that firms change prices on average once every 9.4 quarters. But in the firm-specific capital model, this value of  $\gamma$  implies that firms change price on average once every 1.8 quarters. The reason why the models have such different implications for firms' pricing behavior is that according to our estimates, firms' demand curves are highly elastic and their marginal cost curves are very steep.

Finally, we show that the two versions of the model differ sharply in terms of their implications for the cross-sectional distribution of production. In the homogeneous capital model, a very small fraction of firms produce the bulk of the economy's output in the periods after a monetary policy shock. The implications of the firm-specific model are much less extreme. We conclude that both the homogeneous and firm-specific capital models can account for inflation inertia and the response of the economy to monetary policy and technology shocks. But only the firm-specific model can reconcile the micro-macro pricing conflict without obviously unpalatable micro implications.

It is useful to place this paper in the context of the literature. That firm-specific capital can rationalize a lower estimate of  $\xi_p$  (more frequent price re-optimization) was first demonstrated by Sbordone (1998, 2002) and further discussed by Galí et al. (2001) and Woodford (2003). In these papers, the stock of capital owned by the firm is fixed. Woodford (2005) analyzes the impact of firm-specific capital allowing for investment. As our discussion above indicates, whether firm-specific capital actually does rationalize a lower estimate of  $\xi_p$  depends critically on the other parameters characterizing firms' environments. **To the extent that capital utilization rates can easily be varied, the assumption of firm-specific capital loses its ability to rationalize low values of  $\xi_p$ . A maintained assumption of the papers just cited is that firms cannot vary capital utilization rates. So these papers leave open the question of how important firm-specific capital is once firms can vary capital utilization rates.** Similarly, the smaller is the elasticity of demand for a firm's output, the smaller is the impact of firm-specific capital on inference about  $\xi_p$ . The above cited papers condition their inference on particular assumed values for this elasticity. **The key contribution of this paper is to estimate the key parameters governing the operational importance of firm-specific capital and to assess the importance for firm-specific capital in an estimated dynamic stochastic general equilibrium model. Our key result is that the assumption of firm-specific capital does in fact rationalize relatively low values of  $\xi_p$ , thereby helping to reconcile the apparent conflicting pictures of pricing behavior painted by micro and macro data.**<sup>12</sup>

From a broader perspective, this paper belongs to a larger literature that tries to explain the mechanisms by which nominal shocks have effects on real economic activity that last longer than the frequency with which firms re-optimize prices. Perhaps the most closely related mechanisms are firm-specific labor (Woodford, 2005), sector-specific labor (Gertler and Leahy, 2008) and strategic complementarities arising from an elasticity of firm demand that is increasing in the firm's price (see for example Kimball, 1995 and Eichenbaum and Fisher, 2007). A different propagation mechanism stems from heterogeneity across sectors in the frequency of price changes (see for example Bils and Klenow, 2004; Carvalho, 2006 and Nakamura and Steinsson, 2008) and the presence of intermediate inputs (see for example Basu, 1997 and Huang, 2006).

An alternative and promising propagation mechanism arises from the assumption that firms cannot attend perfectly to all available information (see Sims, 1998, 2003). Mackowiak and Wiederholt (2009) present a model in which firms decide what variables to pay attention to, subject to a constraint on information flow. When idiosyncratic conditions are more variable or more important than aggregate conditions, firms in their model pay more attention to idiosyncratic conditions than to aggregate conditions. Their model has the important property that firms react fast and by large amounts to idiosyncratic shocks, but only slowly and by small amounts to nominal shocks. As a result nominal shocks have strong and persistent real effects. Woodford (2008) develops a generalization of the standard, full-information model of state-dependent pricing in

<sup>10</sup> The quality of our estimation strategy depends on the ability of identified VARs to generate reliable estimates of the dynamic response of economic variables to shocks. In an appendix, available upon request, we evaluate the reliability of VAR methods in our application by means of Monte Carlo simulation methods. We find that the Monte Carlo performance of our VAR based estimates of impulse response functions is very good.

<sup>11</sup> Our strategy for identifying technology shocks follows Fisher (2006).

<sup>12</sup> Since our paper was written, an interesting literature has arisen incorporating firm-specific capital into analyses of price setting. See for example Christoffel et al. (2007), de Walque et al. (2006), Eichenbaum and Fisher (2007), and Sveen and Weinke (2007a, 2007b).

which decisions about when to review a firm's existing price must be made on the basis of imprecise awareness of current market conditions. He endogenizes imperfect information using a variant of the theory of "rational inattention" proposed by Sims (1998, 2003). In related work, Mankiw and Reis (2002) and Reis (2006) stress the potential importance of slow dissemination of information to firms for generating persistent effects of nominal shocks.

The merits of these alternative propagation mechanisms is a subject of an ongoing, vigorous debate. A detailed assessment of their empirical strengths is beyond the scope of this paper. It is clear however that the debate will be settled on the field of microeconomic data. For recent reviews of how firm level data on prices bears on alternative approaches we refer the reader to Mackowiak and Smets (2008), Eichenbaum, Jaimovich and Rebelo (2008), and Klenow and Malin (2009). While we emphasize the importance of firm-specific capital in this paper, we leave open the possibility that the other propagation mechanisms discussed above may be at least as important.

The remainder of this paper is organized as follows. In Section 2 we describe our basic model economy. Section 3 describes our VAR-based estimation procedure. Section 4 presents our VAR-based impulse response functions and their properties. Sections 5 and 6 present and analyze the results of estimating our model. Section 7 discusses the implications of the homogeneous and firm-specific capital models for the cross-firm distribution of prices and production in the wake of a monetary policy shock. Section 8 concludes.

## 2. The model economy

In this section we describe the homogeneous and firm-specific capital models.

### 2.1. The homogeneous capital model

The model economy is populated by goods-producing firms, households and the government.

#### 2.1.1. Final good firms

At time  $t$ , a final consumption good,  $Y_t$ , is produced by a perfectly competitive, representative firm. The firm produces the final good by combining a continuum of intermediate goods, indexed by  $i \in [0, 1]$ , using the technology

$$Y_t = \left[ \int_0^1 y_t(i)^{\frac{1}{\lambda_f}} di \right]^{\lambda_f}, \quad (2.1)$$

where  $1 \leq \lambda_f < \infty$  and  $y_t(i)$  denotes the time  $t$  input of intermediate good  $i$ . The firm takes its output price,  $P_t$ , and its input prices,  $P_t(i)$ , as given and beyond its control. The first order necessary condition for profit maximization is:

$$\left( \frac{P_t}{P_t(i)} \right)^{\frac{\lambda_f}{\lambda_f-1}} = \frac{y_t(i)}{Y_t}. \quad (2.2)$$

Integrating (2.2) and imposing (2.1), we obtain the following relationship between the price of the final good and the price of the intermediate good:

$$P_t = \left[ \int_0^1 P_t(i)^{\frac{1}{1-\lambda_f}} di \right]^{(1-\lambda_f)}. \quad (2.3)$$

#### 2.1.2. Intermediate good firms

Intermediate good  $i \in (0, 1)$  is produced by a monopolist using the following technology:

$$y_t(i) = \begin{cases} K_t(i)^\alpha (z_t h_t(i))^{1-\alpha} - \phi z_t^* & \text{if } K_t(i)^\alpha (z_t h_t(i))^{1-\alpha} \geq \phi z_t^*, \\ 0 & \text{otherwise,} \end{cases} \quad (2.4)$$

where  $0 < \alpha < 1$ . Here,  $h_t(i)$  and  $K_t(i)$  denote time  $t$  labor and capital services used to produce the  $i$ th intermediate good. The variable,  $z_t$ , represents a time  $t$  shock to the technology for producing intermediate output. We refer to  $z_t$  as a neutral technology shock and denote its growth rate,  $z_t/z_{t-1}$ , by  $\mu_{zt}$ . The non-negative scalar,  $\phi$ , parameterizes fixed costs of production. The variable,  $z_t^*$ , is given by:

$$z_t^* = \gamma_t^{\frac{\alpha}{1-\alpha}} z_t, \quad (2.5)$$

where  $\gamma_t$  represents a time  $t$  shock to capital-embodied technology. We choose the structure of the firm's fixed cost in (2.5) to ensure that the non-stochastic steady state of the economy exhibits a balanced growth path. We denote the growth rate of  $z_t^*$  and  $\gamma_t$  by  $\mu_{z_t^*}$  and  $\mu_{\gamma_t}$  respectively, so that:

$$\mu_{z^*,t} = (\mu_{\gamma,t})^{\frac{\alpha}{1-\alpha}} \mu_{z,t}. \quad (2.6)$$

Throughout, we rule out entry into and exit from the production of intermediate good  $i$ .

Let  $\hat{\mu}_{z,t}$  denote  $(\mu_{z,t} - \mu_z)/\mu_z$ , where  $\mu_z$  is the growth rate of  $\mu_{z,t}$  in non-stochastic steady state. We define all variables with a hat in an analogous manner. The variable  $\hat{\mu}_{z,t}$  evolves according to:

$$\hat{\mu}_{z,t} = \rho_{\mu_z} \hat{\mu}_{z,t-1} + \varepsilon_{\mu_z,t} \quad (2.7)$$

where  $|\rho_{\mu_z}| < 1$  and  $\varepsilon_{\mu_z,t}$  is uncorrelated over time and with all other shocks in the model. We denote the standard deviation of  $\varepsilon_{\mu_z,t}$  by  $\sigma_{\mu_z}$ . Similarly, we assume:

$$\hat{\mu}_{\gamma,t} = \rho_{\mu_\gamma} \hat{\mu}_{\gamma,t-1} + \varepsilon_{\mu_\gamma,t}, \quad (2.8)$$

where  $\varepsilon_{\mu_\gamma,t}$  has the same properties as  $\varepsilon_{\mu_z,t}$ . We denote the standard deviation of  $\varepsilon_{\mu_\gamma,t}$  by  $\sigma_{\mu_\gamma}$ .

Intermediate good firms rent capital and labor in perfectly competitive factor markets. Profits are distributed to households at the end of each time period. Let  $P_t r_t^k$  and  $P_t w_t$  denote the nominal rental rate on capital services and the wage rate, respectively. We assume that the firm must borrow the wage bill in advance at the gross interest rate,  $R_t$ .

Firms set prices according to a variant of the mechanism spelled out in Calvo (1983). In each period, an intermediate goods firm faces a constant probability,  $1 - \xi_p$ , of being able to re-optimize its nominal price. The ability to re-optimize prices is independent across firms and time. As in CEE (2005), we assume that a firm which cannot re-optimize its price sets  $P_t(i)$  according to:

$$P_t(i) = \pi_{t-1} P_{t-1}(i). \quad (2.9)$$

Here,  $\pi_t$  denotes aggregate inflation,  $P_t/P_{t-1}$ .

An intermediate goods firm's objective function is:

$$E_t \sum_{j=0}^{\infty} \beta^j v_{t+j} [P_{t+j}(i) y_{t+j}(i) - P_{t+j}(w_{t+j} R_{t+j} h_{t+j}(i) + r_{t+j}^k K_{t+j}(i))], \quad (2.10)$$

where  $E_t$  is the expectation operator conditioned on time  $t$  information. The term,  $\beta^t v_{t+j}$ , is proportional to the state-contingent marginal value of a dollar to a household.<sup>13</sup> Also,  $\beta$  is a scalar between zero and unity. The timing of events for a firm is as follows. At the beginning of period  $t$ , the firm observes the technology shocks and sets its price,  $P_t(i)$ . Then, a shock to monetary policy is realized, as is the demand for the firm's product, (2.2). The firm then chooses productive inputs to satisfy this demand. The problem of the  $i$ th intermediate good firm is to choose prices, employment and capital services, subject to the timing and other constraints described above, to maximize (2.10).

### 2.1.3. Households

There is a continuum of households, indexed by  $j \in (0, 1)$ . The sequence of events in a period for a household is as follows. First, the technology shocks are realized. Second, the household makes its consumption and investment decisions, decides how many units of capital services to supply to rental markets, and purchases securities whose payoffs are contingent upon whether it can re-optimize its wage decision. Third, the household sets its wage rate. Fourth, the monetary policy shock is realized. Finally, the household allocates its beginning of period cash between deposits at the financial intermediary and cash to be used in consumption transactions.

Each household is a monopoly supplier of a differentiated labor service, and sets its wage subject to Calvo-style wage frictions. In general, households earn different wage rates and work different amounts. A straightforward extension of arguments in Erceg et al. (2000) and Woodford (1996) establishes that in the presence of state contingent securities, households are homogeneous with respect to consumption and asset holdings. Our notation reflects this result.

The preferences of the  $j$ th household are given by:

$$E_t^j \sum_{l=0}^{\infty} \beta^{l-t} \left[ \log(C_{t+l} - b C_{t+l-1}) - \psi_L \frac{h_{j,t+l}^2}{2} \right], \quad (2.11)$$

where  $\psi_L \geq 0$  and  $E_t^j$  is the time  $t$  expectation operator, conditional on household  $j$ 's time  $t$  information set. The variable,  $C_t$ , denotes time  $t$  consumption, and  $h_{j,t}$  denotes time  $t$  hours worked. When  $b > 0$ , (2.11) exhibits habit formation in consumption preferences.

The household's asset evolution equation is given by:

$$M_{t+1} = R_t [M_t - Q_t + (x_t - 1) M_t^a] + A_{j,t} + Q_t + W_{j,t} h_{j,t} + P_t r_t^k u_t \bar{K}_t + D_t - (1 + \eta(\mathcal{V}_t)) P_t C_t - P_t \gamma_t^{-1} (I_t + a(u_t) \bar{K}_t). \quad (2.12)$$

<sup>13</sup> The constant of proportionality is the probability of the relevant state of the world.

Here,  $M_t$ ,  $Q_t$  and  $W_{j,t}$  denote the household's beginning of period  $t$  stock of money, cash balances and time  $t$  nominal wage rate, respectively. In addition,  $\bar{K}_t$ ,  $u_t$ ,  $D_t$  and  $A_{j,t}$  denote the household's physical stock of capital, the capital utilization rate, firm profits and the net cash inflow from participating in state-contingent securities at time  $t$ , respectively. The variable  $x_t$  represents the gross growth rate of the economy-wide per capita stock of money,  $M_t^a$ . The quantity  $(x_t - 1)M_t^a$  is a lump-sum payment made to households by the monetary authority. The household deposits  $M_t - Q_t + (x_t - 1)M_t^a$  with a financial intermediary. The variable,  $R_t$ , denotes the gross interest rate.

In (2.12), the price of investment goods relative to consumption goods is given by  $\gamma_t^{-1}$  which we assume is an exogenous stochastic process. One way to rationalize this assumption is that agents transform final goods into investment goods using a linear technology with slope  $\gamma_t$ . This rationalization also underlies why we refer to  $\gamma_t$  as capital-embodied technological progress.

The variable,  $\mathcal{V}_t$ , denotes the time  $t$  velocity of the household's cash balances:

$$\mathcal{V}_t = \frac{P_t C_t}{Q_t}, \quad (2.13)$$

where  $\eta(\mathcal{V}_t)$  is increasing and convex. The function  $\eta(\mathcal{V}_t)$  captures the role of cash balances in facilitating transactions. Similar specifications have been used by a variety of authors including Sims (1994) and Schmitt-Grohe and Uribe (2004). For the quantitative analysis of our model, we require the level and the first two derivatives of the transactions function,  $\eta(\mathcal{V})$ , evaluated in steady state. We denote these by  $\eta$ ,  $\eta'$ , and  $\eta''$ , respectively. We chose values for these objects as follows. The first order condition for  $Q_t$  is:

$$R_t = 1 + \eta' \left( \frac{P_t C_t}{Q_t} \right) \left( \frac{P_t C_t}{Q_t} \right)^2.$$

Let  $\epsilon_t$  denote the interest semi-elasticity of money demand:

$$\epsilon_t \equiv - \frac{100 \times d \log(\frac{Q_t}{P_t})}{400 \times d R_t}.$$

Denote the curvature of  $\eta$  by  $\varphi$ :

$$\varphi = \frac{\eta'' \mathcal{V}}{\eta'}.$$

Then, the first order condition for  $Q_t$  implies that the interest semi-elasticity of money demand in steady state is:

$$\epsilon = \frac{1}{4} \left( \frac{1}{R-1} \right) \left( \frac{1}{2+\varphi} \right),$$

where the steady state value of  $R$  is  $\pi \mu_{z^*} / \beta$ . We parameterize  $\eta(\cdot)$  indirectly using values for  $\epsilon$ ,  $\mathcal{V}$  and  $\eta$ .

The remaining terms in (2.12) pertain to the household's capital-related income. The services of capital,  $K_t$ , are related to stock of physical capital,  $\bar{K}_t$ , by

$$K_t = u_t \bar{K}_t.$$

The term  $P_t I_t^k u_t \bar{K}_t$  represents the household's earnings from supplying capital services. The function  $a(u_t) \bar{K}_t$  denotes the cost, in investment goods, of setting the utilization rate to  $u_t$ . We assume  $a(u_t)$  is increasing and convex. These assumptions capture the idea that the more intensely the stock of capital is utilized, the higher are maintenance costs in terms of investment goods. Our log-linear approximation solution strategy requires the level and first two derivatives of  $a(\cdot)$  in steady state. We treat  $\sigma_a = a''(1)/a'(1) \geq 0$  as a parameter to be estimated and impose that  $u_t = 1$  and  $a(1) = 0$  in steady state. Although the steady state of the model does not depend on the value of  $\sigma_a$ , the dynamics do. Given our solution procedure, we do not need to specify any other features of the function  $a$ .

The household's stock of physical capital evolves according to:

$$\bar{K}_{t+1} = (1 - \delta) \bar{K}_t + \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) I_t, \quad (2.14)$$

where  $\delta$  denotes the physical rate of depreciation, and  $I_t$  denotes time  $t$  investment goods. The adjustment cost function,  $S$ , is assumed to be increasing, convex and to satisfy  $S = S' = 0$  in steady state. We treat the second derivative of  $S$  in steady state,  $S'' > 0$ , as a parameter to be estimated. Although the steady state of the model does not depend on the value of  $S''$ , the dynamics do. Given our solution procedure, we do not need to specify any other features of the function  $S$ .

### 2.1.4. The wage decision

As in Erceg et al. (2000), we assume that the  $j$ th household is a monopoly supplier of a differentiated labor service,  $h_{j,t}$ . It sells this service to a representative, competitive firm that transforms it into an aggregate labor input,  $H_t$ , using the technology:

$$H_t = \left[ \int_0^1 h_{j,t}^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}, \quad 1 \leq \lambda_w < \infty.$$

The demand curve for  $h_{j,t}$  is given by:

$$h_{j,t} = \left( \frac{W_t}{W_{j,t}} \right)^{\frac{\lambda_w}{\lambda_w - 1}} H_t. \quad (2.15)$$

Here,  $W_t$  is the aggregate wage rate, i.e., the nominal price of  $H_t$ . It is straightforward to show that  $W_t$  is related to  $W_{j,t}$  via the relationship:

$$W_t = \left[ \int_0^1 (W_{j,t})^{\frac{1}{1-\lambda_w}} dj \right]^{1-\lambda_w}. \quad (2.16)$$

The household takes  $H_t$  and  $W_t$  as given.

Households set their nominal wage according to a variant of the mechanism by which intermediate good firms set prices. In each period, a household faces a constant probability,  $1 - \xi_w$ , of being able to re-optimize its nominal wage. The ability to re-optimize is independent across households and time. If a household cannot re-optimize its wage at time  $t$ , it sets  $W_{j,t}$  according to:

$$W_{j,t} = \pi_{t-1} \mu_{z^*} W_{j,t-1}. \quad (2.17)$$

The presence of  $\mu_{z^*}$  in (2.17) implies that there are no distortions from wage dispersion along the steady state growth path.

### 2.1.5. Monetary and fiscal policy

We adopt the following specification of monetary policy:

$$\hat{x}_t = \hat{x}_{z,t} + \hat{x}_{\gamma,t} + \hat{x}_{M,t}.$$

Here  $x_t$  represents the gross growth rate of money,  $M_{t+1}/M_t$ . We assume that

$$\begin{aligned} \hat{x}_{M,t} &= \rho_{xM} \hat{x}_{M,t-1} + \varepsilon_{M,t}, \\ \hat{x}_{z,t} &= \rho_{xz} \hat{x}_{z,t-1} + c_z \varepsilon_{z,t} + c_z^p \varepsilon_{z,t-1}, \\ \hat{x}_{\gamma,t} &= \rho_{x\gamma} \hat{x}_{\gamma,t-1} + c_\gamma \varepsilon_{\gamma,t} + c_\gamma^p \varepsilon_{\gamma,t-1}. \end{aligned} \quad (2.18)$$

Here,  $\varepsilon_{M,t}$  represents a shock to monetary policy. We denote the standard deviation of  $\varepsilon_{M,t}$  by  $\sigma_M$ . The dynamic response of  $\hat{x}_{M,t}$  to  $\varepsilon_{M,t}$  is characterized by a first order autoregression, so that  $\rho_{xM}^j$  is the response of  $E_t \hat{x}_{t+j}$  to a one-unit time  $t$  monetary policy shock. The term  $\hat{x}_{z,t}$  captures the response of monetary policy to an innovation in neutral technology,  $\varepsilon_{z,t}$ . We assume that  $\hat{x}_{z,t}$  is characterized by an ARMA(1,1) process. The term,  $\hat{x}_{\gamma,t}$ , captures the response of monetary policy to an innovation in capital-embodied technology,  $\varepsilon_{\gamma,t}$ . We assume that  $\hat{x}_{\gamma,t}$  is also characterized by an ARMA(1,1) process.

In models with nominal rigidities, it is generally the case that the dynamic response functions to shocks depend heavily on the nature of monetary policy. CEE (2005) show that for a very closely related model the impulse response function to a monetary policy shock as parameterized above are very similar to the response obtained when the central bank is instead assumed to follow an explicit Taylor rule.

Finally, we assume that the government adjusts lump sum taxes to ensure that its intertemporal budget constraint holds.

### 2.1.6. Loan market clearing, final goods market clearing and equilibrium

Financial intermediaries receive  $M_t - Q_t + (x_t - 1)M_t$  from the household. Our notation reflects the equilibrium condition,  $M_t^a = M_t$ . Financial intermediaries lend all of their money to intermediate good firms, which use the funds to pay labor wages. Loan market clearing requires that:

$$W_t H_t = x_t M_t - Q_t. \quad (2.19)$$

The aggregate resource constraint is:

$$(1 + \eta(\mathcal{V}_t))C_t + \gamma_t^{-1} [I_t + a(u_t) \bar{K}_t] \leq Y_t. \quad (2.20)$$

We adopt a standard sequence-of-markets equilibrium concept, and use the aggregation approached outlined by Yun (1996). In the technical appendix to this paper, Altig et al. (ACEL henceforth, 2004), we discuss our computational strategy for approximating that equilibrium. This strategy involves taking a (log-)linear approximation about the non-stochastic steady state of the economy and using the solution methods discussed in Anderson and Moore (1985) and Christiano (2002).

## 2.2. The firm-specific capital model

In this model, firms own their own capital. The firm cannot adjust its capital stock within the period. It can only change its stock of capital over time by varying the rate of investment. In all other respects the problem of intermediate good firms is the same as before. In particular, they face the same demand curve, (2.2), production technology, (2.4)–(2.8), and Calvo-style pricing frictions, including the updating rule given by (2.9).

The technology for accumulating physical capital by intermediate good firm  $i$  is given by

$$\bar{K}_{t+1}(i) = (1 - \delta)\bar{K}_t(i) + \left(1 - S\left(\frac{I_t(i)}{I_{t-1}(i)}\right)\right)I_t(i).$$

The present discounted value of the  $i$ th intermediate good's net cash flow is given by:

$$E_t \sum_{j=0}^{\infty} \beta^j v_{t+j} \{P_{t+j}(i)y_{t+j}(i) - P_{t+j}R_{t+j}w_{t+j}(i)h_t(i) - P_{t+j}\gamma_{t+j}^{-1}[I_{t+j}(i) + a(u_{t+j}(i))\bar{K}(i)_{t+j}]\}. \quad (2.21)$$

Time  $t$  net cash flow equals sales, less labor costs (inclusive of interest charges) less the costs associated with capital utilization and capital accumulation.

The sequence of events as it pertains to the  $i$ th firm is as follows. At the beginning of period  $t$ , the firm has a given stock of physical capital,  $\bar{K}_t(i)$ . After observing the technology shocks, the firm sets its price,  $P_t(i)$ , subject to the Calvo-style frictions described above. The firm also makes its investment and capital utilization decisions,  $I_t(i)$  and  $u_t(i)$ , respectively. The time  $t$  monetary policy shock then occurs and the demand for the firm's product is realized. The firm then purchases labor to satisfy the demand for its output. Subject to these timing and other constraints, the problem of the firm is to choose prices, employment, the level of investment and utilization to maximize net discounted cash flow.

## 2.3. Implications for inflation

The equations which characterize equilibrium for the homogenous and firm-specific capital model are identical except for the equation which characterizes aggregate inflation dynamics. This equation is of the form:

$$\Delta \hat{\pi}_t = E[\beta \Delta \hat{\pi}_{t+1} + \gamma \hat{s}_t | \Omega_t], \quad (2.22)$$

where

$$\gamma = \frac{(1 - \xi_p)(1 - \beta \xi_p)}{\xi_p} \chi,$$

and  $\Delta$  is the first difference operator. The information set  $\Omega_t$  includes the current realization of the technology shocks, but not the current realization of the innovation to monetary policy. The variable  $s_t$  denotes the economy-wide average marginal cost of production, in units of the final good.

In ACEL (2004) we establish the following<sup>14</sup>:

**Proposition 1.** (i) In the homogeneous capital model,  $\chi = 1$ ; (ii) In the firm-specific capital model,  $\chi$  is a particular non-linear function of the parameters of the model.

We parameterize the firm-specific and homogeneous capital model in terms of  $\gamma$ , rather than  $\xi_p$ . Consequently, the list of parameters for the two models remains identical. Given values for these parameters, the two models are observationally equivalent with respect to aggregate prices and quantities. This means that we do not need to take a stand on which version of the model we are working with at the estimation stage of our analysis.

## 3. Econometric methodology

We employ a variant of the limited information strategy used in CEE (2005) (see also Rotemberg and Woodford, 1997). Define the ten-dimensional vector,  $Y_t$ :

<sup>14</sup> See Christiano (2004) for a discussion of the solution to firm-specific capital models in simpler settings.



$$\underbrace{Y_t}_{10 \times 1} = \begin{pmatrix} \Delta \ln(\text{relative price of investment}_t) \\ \Delta \ln(GDP_t/\text{Hours}_t) \\ \Delta \ln(GDP \text{ deflator}_t) \\ \text{Capacity Utilization}_t \\ \ln(\text{Hours}_t) \\ \ln(GDP_t/\text{Hours}_t) - \ln(W_t/P_t) \\ \ln(C_t/GDP_t) \\ \ln(I_t/GDP_t) \\ \text{Federal Funds Rate}_t \\ \ln(GDP \text{ deflator}_t) + \ln(GDP_t) - \ln(MZM_t) \end{pmatrix} = \begin{pmatrix} \underbrace{\Delta p_{It}}_{1 \times 1} \\ \underbrace{\Delta a_t}_{1 \times 1} \\ \underbrace{Y_{1t}}_{6 \times 1} \\ \underbrace{R_t}_{1 \times 1} \\ \underbrace{Y_{2t}}_{1 \times 1} \end{pmatrix}. \quad (3.1)$$

We embed our identifying assumptions as restrictions on the parameters of the following reduced form VAR:

$$\begin{aligned} Y_t &= \alpha + B(L)Y_{t-1} + u_t, \\ Eu_t u_t' &= V, \end{aligned} \quad (3.2)$$

where  $B(L)$  is a  $p$ th-ordered polynomial in the lag operator,  $L$ . The “fundamental” economic shocks,  $\varepsilon_t$ , are related to  $u_t$  as follows:

$$u_t = C\varepsilon_t, \quad E\varepsilon_t \varepsilon_t' = I, \quad (3.3)$$

where  $C$  is a square matrix and  $I$  is the identity matrix. We assume that  $\varepsilon_t$  is a martingale difference stochastic process, so that we allow for the presence of conditional heteroscedasticity.<sup>15</sup> We require  $B(L)$  and the  $i$ th column of  $C$ ,  $C_i$ , to calculate the dynamic response of  $Y_t$  to a disturbance in the  $i$ th fundamental shock,  $\varepsilon_{i,t}$ .

According to our economic model, the variables in  $Y_t$ , defined in (3.1), are stationary stochastic processes. We partition  $\varepsilon_t$  conformably with the partitioning of  $Y_t$ :

$$\varepsilon_t = \begin{pmatrix} \underbrace{\varepsilon_{\gamma,t}}_{1 \times 1} & \underbrace{\varepsilon_{z,t}}_{1 \times 1} & \underbrace{\varepsilon'_{1t}}_{1 \times 6} & \underbrace{\varepsilon_{M,t}}_{1 \times 1} & \underbrace{\varepsilon_{2t}}_{1 \times 1} \end{pmatrix}'. \quad (3.4)$$

Here,  $\varepsilon_{z,t}$  is the innovation to a neutral technology shock,  $\varepsilon_{\gamma,t}$  is the innovation in capital-embodied technology, and  $\varepsilon_{M,t}$  is the monetary policy shock.

### 3.1. Identification of impulse responses

We assume that policy makers set the interest rate so that the following rule is satisfied:

$$R_t = f(\Omega_t) + \vartheta \varepsilon_{M,t}, \quad (3.5)$$

where  $\varepsilon_{M,t}$  is the monetary policy shock and  $\vartheta > 0$  is a constant. We interpret (3.5) as a reduced form Taylor rule. To ensure identification of the monetary policy shock, we assume  $f$  is linear,  $\Omega_t$  contains  $Y_{t-1}, \dots, Y_{t-q}$  and the only date  $t$  variables in  $\Omega_t$  are  $\{\Delta a_t, \Delta p_{It}, Y_{1t}\}$ . Finally, we assume that  $\varepsilon_{M,t}$  is orthogonal to  $\Omega_t$ .

As in Fisher (2006), we assume that innovations to technology (both neutral and capital-embodied) are the only shocks which affect the level of labor productivity in the long run. In addition, we assume that capital embodied technology shocks are the only shocks that affect the price of investment goods relative to consumption goods in the long run. These assumptions are satisfied in our model.

To compute the responses of  $Y_t$  to  $\varepsilon_{\gamma,t}$ ,  $\varepsilon_{z,t}$ , and  $\varepsilon_{M,t}$ , we require estimates of the parameters in  $B(L)$ , as well as the 1st, 2nd and 9th columns of  $C$ . We obtain these estimates using a suitably modified variant of the instrumental variables strategy proposed by Shapiro and Watson (1988). See ACEL (2004) for further details.

## 4. Estimation results based on a structural vector autoregression

In this section we describe the dynamic response of the economy to monetary policy shocks, neutral technology shocks and capital embodied shocks. In addition, we discuss the quantitative contribution of these shocks to the cyclical fluctuations in aggregate economic activity. In the first subsection we describe the data used in the analysis. In the second and third subsections we discuss the impulse response functions and the importance of the shocks to aggregate fluctuations.

<sup>15</sup> Justiniano and Primiceri (2008) argue that conditional heteroscedasticity in fundamental shocks is important for explaining the ‘Great Moderation’. Similar arguments have been made by CEE (1999) and Smets and Wouters (2007).

#### 4.1. Data

With the exception of the price of investment and of monetary transactions balances, all data were taken from the FRED Database available through the Federal Reserve Bank of St. Louis.<sup>16</sup> The price of investment corresponds to the ‘total investment’ series constructed and used in Fisher (2006).<sup>17</sup> Our measure of transactions balances, *MZM*, was obtained from the Federal Reserve Bank of St. Louis’s online database. Our data are quarterly, and the sample period is 1982:1–2008:3.<sup>18</sup>

We work with the monetary aggregate, *MZM*, for the following reasons. First, *MZM* is constructed to be a measure of transactions balances, so it is a natural empirical counterpart to our model variable,  $Q_t$ . Second, our statistical procedure requires that the velocity of money is stationary. The velocity of *MZM* is reasonably characterized as being stationary. The stationarity assumption is more problematic for the velocity of aggregates like the base, *M1* and *M2*.

#### 4.2. Estimated impulse response functions

In this subsection we discuss our estimates of the dynamic response of  $Y_t$  to monetary policy and technology shocks. To obtain these estimates we set  $p$ , the number of lags in the VAR, to 4. Various indicators suggest that this value of  $p$  is large enough to adequately capture the dynamics in the data. For example, the Akaike, Hannan–Quinn and Schwartz criteria support a choice of  $q = 2, 2, 1$ , respectively.<sup>19</sup> We also compute the multivariate Portmanteau ( $Q$ ) statistic to test the null hypothesis of zero serial correlation up to lag  $n$  in the VAR disturbances. We consider  $n = 4, 6, 8, 10$ . The test statistics are, respectively,  $Q = 262, 475, 680, 880$ . Using conventional asymptotic sampling theory, these  $Q$  statistics all have a  $p$ -value very close to zero, indicating a rejection of the null hypothesis. However, we find evidence that the asymptotic sampling theory rejects the null hypothesis too often. When we simulate the  $Q$  statistic using repeated artificial data sets generated from our estimated VAR, we find that the  $p$ -values of our  $Q$  statistics are 97, 87, 90 and 95 percent, respectively. For these calculations, each artificial data set is of length equal to that of our actual sample, and is generated by bootstrap sampling from the fitted disturbances in our estimated VAR. On this basis we do not strongly reject the null hypothesis that the disturbance terms in a VAR with  $p = 4$  are serially uncorrelated.

Fig. 1 displays the response of the variables in our analysis to a one standard deviation monetary policy shock (roughly 30 basis points). In each case, there is a solid line in the center of a gray area. The gray area represents a 95 percent confidence interval, and the solid line represents the point estimates.<sup>20</sup> Except for inflation and the interest rate, all variables are expressed in percent terms. So, for example, the peak response of output is about 0.15 percent. The Federal Funds rate is expressed in units of percentage points, at an annual rate. Inflation is expressed in units of percentage points, at a quarterly rate.

Six features of Fig. 1 are worth noting. First, the effect of a policy shock on the money growth rate and the interest rate is completed within roughly one year. Other quantity variables respond over a longer period of time. Second, there is a significant liquidity effect, i.e. the interest rate and money growth move in opposite directions after a policy shock. Third, inflation responds very weakly to the policy shock. Fourth, output, consumption, investment, hours worked and capacity utilization all display hump-shaped responses. With the exception of hours worked, the peak response in these aggregates occurs roughly one year after the shock. The hump shaped response in hours worked is more drawn out, with the peak occurring after approximately two years. Fifth, velocity co-moves with the interest rate, with both initially falling in response to a monetary policy shock, and then rising. Sixth, the real wage does not respond significantly to a monetary policy shock, but after a delay the price of investment does.

Fig. 2 displays the response of the variables in our analysis to a positive, one standard deviation shock in neutral technology,  $e_{z,t}$ . By construction, the impact of this technology shock on output, labor productivity, consumption, investment and the real wage can be permanent. Because the roots of our estimated VAR are stable, the impact of a neutral technology shock on the variables whose levels appear in  $Y_t$  must be temporary. These variables are the Federal Funds rate, capacity utilization, hours worked, velocity and inflation.

According to Fig. 2 a positive, neutral technology shock leads to a persistent rise in output with a peak rise of roughly 0.35 percent over the period displayed. In addition, hours worked, investment and consumption rise in response to the

<sup>16</sup> Nominal gross output is measured by GDP, real gross output is measured by GDPC96 (real, chain-weighted GDP). Nominal investment is PCDD (household consumption of durables) plus GPDI (gross private domestic investment). Nominal consumption is measured by PCND (non-durables) plus PCESV (services) plus GCE (government expenditures). Real private domestic investment is given by GPDIC96. Real private consumption expenditures are given by PCEC96. Our *MZM* measure of money is *MZMSL*. Variables were converted into per capita terms by CNP160V, a measure of the U.S. civilian non-institutional population over age 16. A measure of the aggregate price index was obtained from the ratio of nominal to real output, GDP/GDPC96. Capacity utilization is measured by CUMFN, the manufacturing industry’s capacity index. The interest rate is measured by the federal funds rate, FEDFUNDS. Hours worked is measured by HOANBS (non-farm business hours). Hours were converted to per capita terms using our population measure. Nominal wages are measured by COMPNFB (nominal hourly non-farm business compensation). This was converted to real terms by dividing by the aggregate price index.

<sup>17</sup> We also re-estimated the VAR and the structural model using as our measures of hours and productivity, private business hours and business sector productivity, respectively. In these estimation runs, we measure consumption and output as private sector consumption and private sector output, respectively. Taking sampling uncertainty, we find that our results are robust to these alternatives data measures.

<sup>18</sup> The estimation period for the vector autoregression drops the first  $p$  quarters, to accommodate the  $p$  lags.

<sup>19</sup> See Bierens (2004) for the formulas used and for a discussion of the asymptotic properties of the lag length selection criteria.

<sup>20</sup> The confidence intervals are symmetric about our point estimates. They are obtained by adding and subtracting 1.96 times our estimate of the standard errors of the coefficients in the impulse response functions. These standard errors were computed by bootstrap simulation of the estimated model.

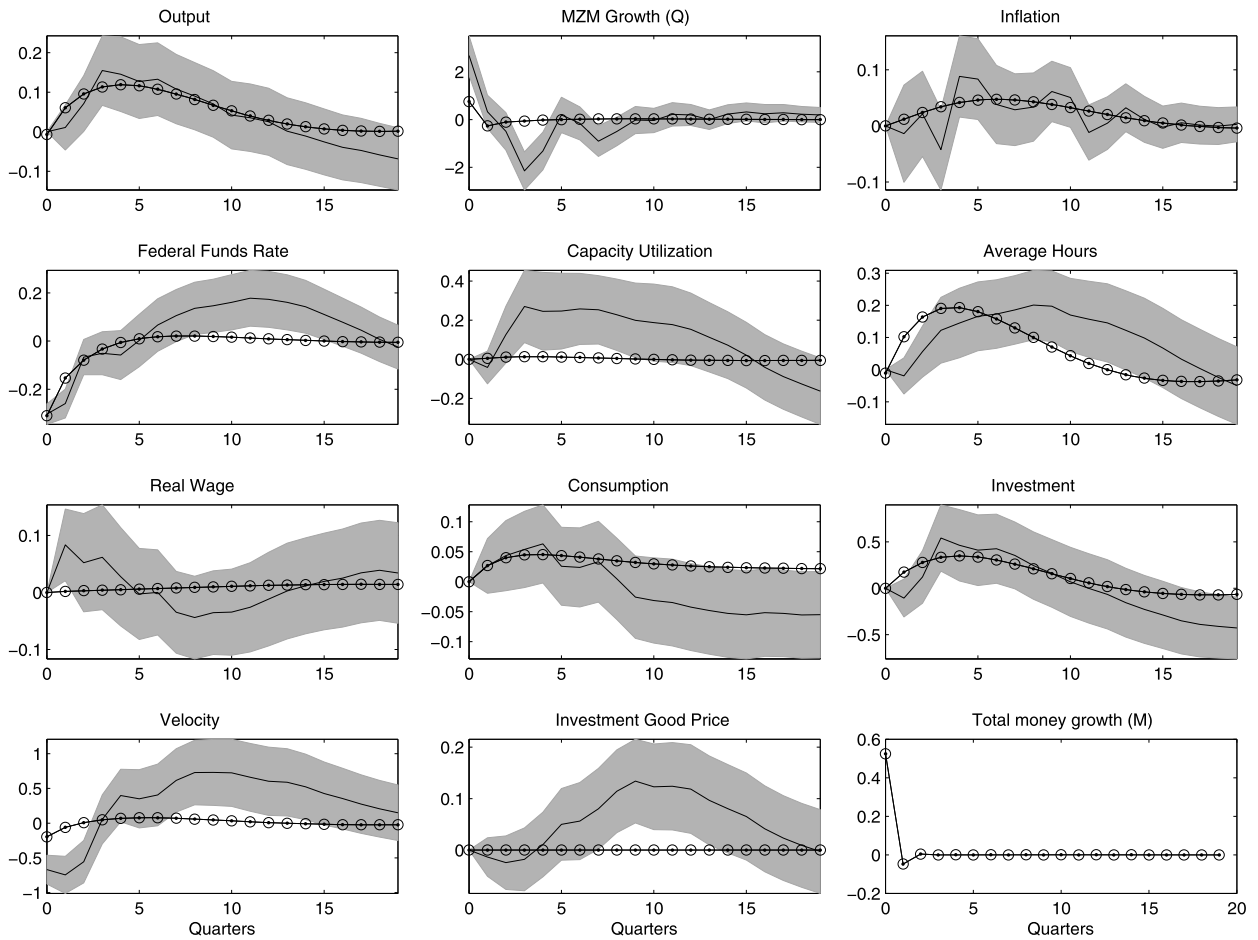


Fig. 1. Response to a monetary policy shock (—○— Model, — VAR, grey area – 95% confidence interval).

technology shock. These rises are only marginally statistically significant. Finally notice that a neutral technology shock leads to an initial sharp fall in the inflation rate.<sup>21</sup> Overall, these effects are broadly consistent with what a student of real business cycle models might expect.

Fig. 3 displays the response of the variables in our analysis to a one standard deviation positive capital-embodied technology shock,  $\varepsilon_{\gamma,t}$ . This shock leads to statistically significant rises in output, hours worked, capacity utilization, investment and the federal funds rate. At the same time, it leads to an initial fall in the price of investment of roughly 0.2 percent, followed by an ongoing significant decline. Finally, the shock also leads a marginally significant decline in real wages.

#### 4.3. The contribution of monetary policy and technology shocks to aggregate fluctuations

We now briefly discuss the contribution of monetary policy and technology shocks to cyclical fluctuations in economic activity. Table 1 summarizes the contribution of the three shocks to the variables in our analysis. We define business cycle frequencies as the components of a time series with periods of 8 to 32 quarters. The columns in Table 1 report the fraction of the variance in the cyclical frequencies accounted for by our three shocks. Each row corresponds to a different variable. Using the techniques described in Christiano and Fitzgerald (2003), we calculate the fractions as follows. Let  $f^i(\omega)$  denote the spectral density at frequency  $\omega$  of a given variable, when only shock  $i$  is active. That is, the variance of all shocks in  $\varepsilon_t$ , apart from the  $i$ th, are set to zero and the variance of the  $i$ th shock in  $\varepsilon_t$  is set to unity. Let  $f(\omega)$  denote the corresponding spectral density when the variance of each element of  $\varepsilon_t$  is set to unity. The contribution of shock  $i$  to variance in the business cycle frequencies is then defined as:

$$\frac{\int_{\omega_1}^{\omega_2} f^i(\omega) d\omega}{\int_{\omega_1}^{\omega_2} f(\omega) d\omega}, \quad \omega_1 = \frac{2\pi}{32}, \quad \omega_2 = \frac{2\pi}{8}.$$

<sup>21</sup> Alves (2004) also finds that inflation drops after a positive neutral technology shocks using data for non-U.S. G7 countries.

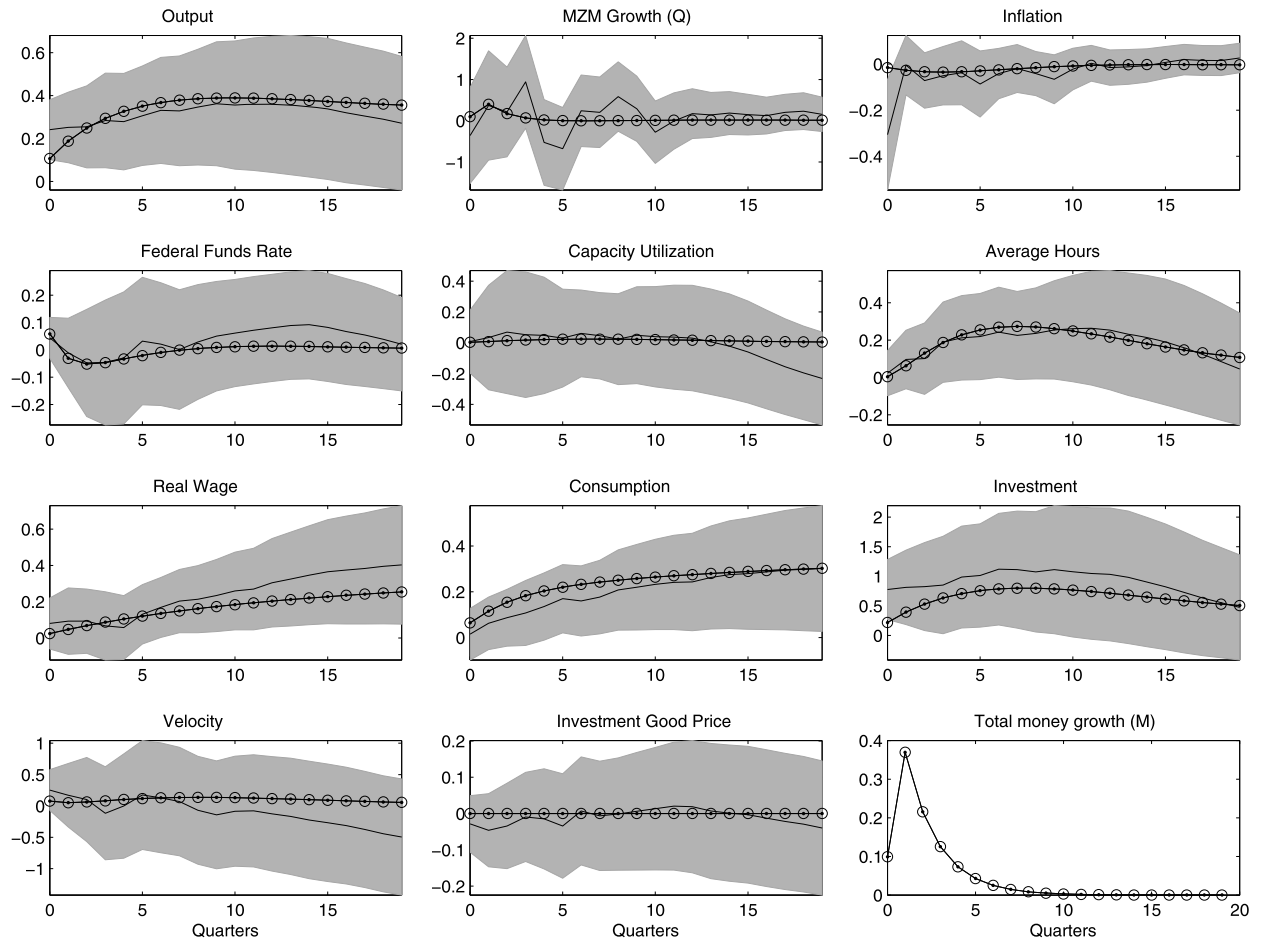


Fig. 2. Response to a neutral technology shock (—○— Model, — VAR, grey area – 95% confidence interval).

Our estimate of the spectral density is the one implied by our estimated VAR.<sup>22</sup> Numbers in parentheses are the standard errors, which we estimate by bootstrap methods. Finally, the fraction of the variance accounted for by all three shocks is just the sum of the individual fractions of the variance.

Table 1 shows that the three shocks together account for a substantial portion of the cyclical variance in the aggregate quantities. For example, they account for roughly 60 percent of the variation in aggregate output, with the capital-embodied technology shock playing the largest role. Indeed the capital-embodied technology shock is the largest contributor to the cyclical variation in all of the variables included in the VAR. Intriguingly, the capital embodied technology shock accounts for nearly 30% of the cyclical variation in the real wage, a variable whose cyclical variation is typically difficult to account for empirically.

## 5. Estimation results for the equilibrium model

In this section we discuss the estimated parameter values. In addition, we assess the ability of the estimated model to account for the impulse response functions discussed in Section 4.

### 5.1. Benchmark model parameter estimates

We partition the parameters of the model into three groups. The first group of parameters,  $\zeta_1$ , is:

<sup>22</sup> We found that the analog statistics computed using the Hodrick–Prescott filter yielded essentially the same results. We computed this as follows:

$$\frac{\int_0^\pi g(\omega) f^i(\omega) d\omega}{\int_0^\pi g(\omega) f(\omega) d\omega},$$

where  $g(\omega)$  is the frequency-domain representation of the Hodrick–Prescott filter with  $\lambda = 1600$ .

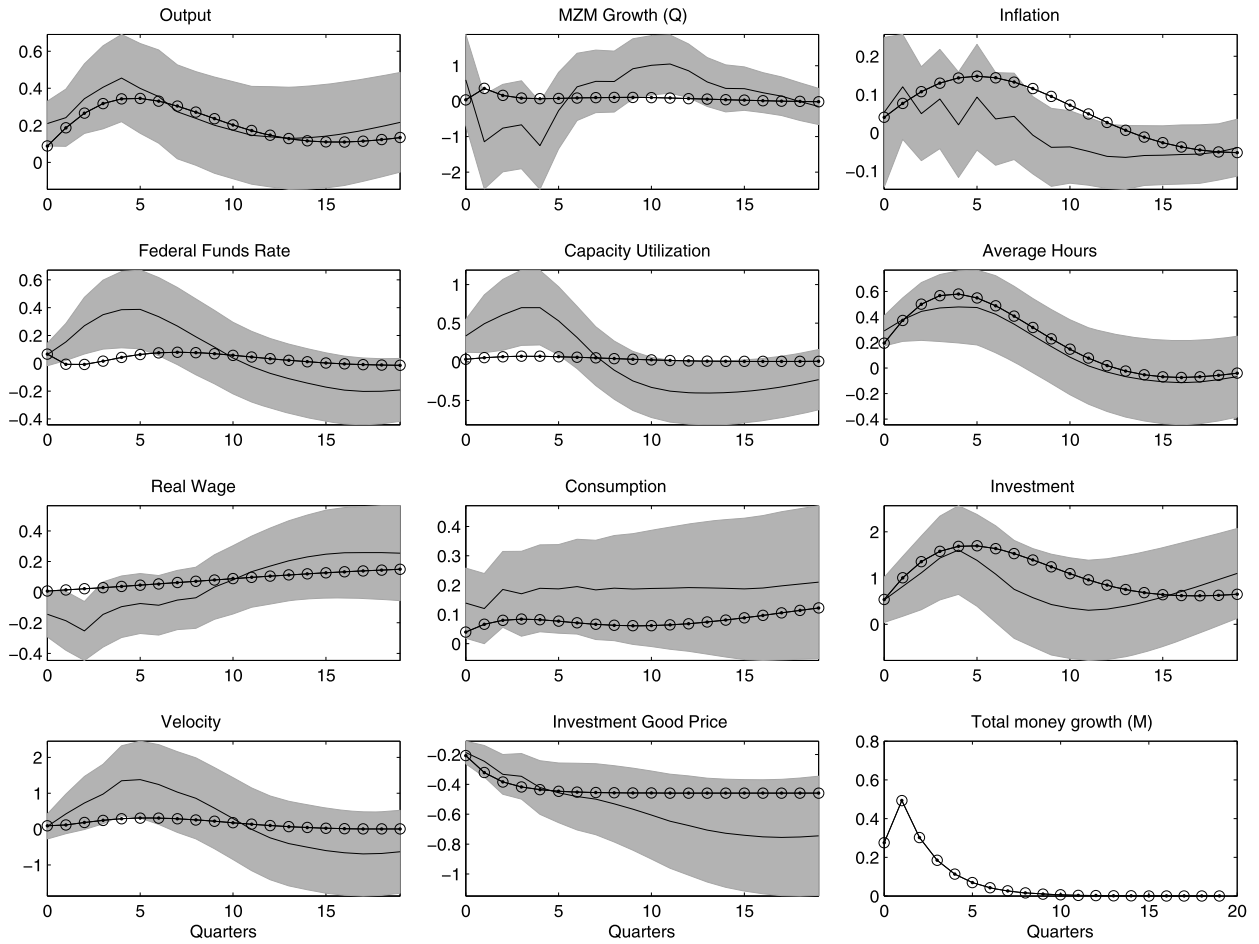


Fig. 3. Response to an embodied technology shock (—○— Model, — VAR, grey area – 95% confidence interval).

$$\zeta_1 = [\beta, \alpha, \delta, \phi, \psi_L, \lambda_w, \mu_\gamma, \mu_z, \chi, \nu, \eta].$$

The second group of parameters,  $\zeta_2$ , pertain to the ‘non-stochastic part’ of the model:

$$\zeta_2 = [\lambda_f, \xi_w, \gamma, \sigma_a, b, S'', \epsilon].$$

The third set of parameters,  $\zeta_3$ , pertain to the stochastic part of the model:

$$\zeta_3 = [\rho_{xM}, \sigma_M, \rho_{\mu_z}, \sigma_{\mu_z}, \rho_{xz}, c_z, c_z^p, \rho_{\mu_\gamma}, \sigma_{\mu_\gamma}, \rho_{x\gamma}, c_\gamma, c_\gamma^p].$$

We estimate the values of  $\zeta_2$  and  $\zeta_3$  and set the values of  $\zeta_1$  a priori. We assume  $\beta = 1.03^{-0.25}$ , which implies a steady state annualized real interest rate of 3 percent. We set  $\alpha = 0.36$ , which corresponds to a steady state share of capital income equal to roughly 36 percent.<sup>23</sup> We set  $\delta = 0.025$ , which implies an annual rate of depreciation on capital equal to 10 percent. This value of  $\delta$  is roughly equal to the estimate reported in Christiano and Eichenbaum (1992). The parameter,  $\phi$ , is set to guarantee that profits are zero in steady state. As in CEE (2005), we set the parameter,  $\lambda_w$ , to 1.05. We set the parameter  $\psi_L$  to one.

The steady state growth of real per capita GDP,  $\mu_y$ , is given by

$$\mu_y = (\mu_\gamma)^{\frac{\alpha}{1-\alpha}} \mu_z.$$

Given an estimate of  $\mu_y$  and  $\mu_\gamma$ , we use this equation to estimate  $\mu_z$ . We use data over the sample period 1959II–2001IV, the sample period in ACEL (2005), to estimate the parameters  $\mu_\gamma$  and  $\mu_x$ . If we use the sample period 1982:1–2008:3, then the implied point estimate of  $\mu_z$  is less than one, a value that seems implausible to us. It seems reasonable to extend the sample back in time because the value of  $\mu_z$  should not be affected by any change in the monetary policy regime that

<sup>23</sup> In our model, the steady state share of labor income in total output is  $1 - \alpha$ . This result reflects our assumption that profits are zero in steady state.

**Table 1**  
Decomposition of variance, business cycle frequencies.

Variable	Monetary policy shocks	Neutral technology shocks	Embodied technology shocks
Output	9 [4]	11 [11]	41 [15]
MZM growth	20 [5]	1 [7]	30 [12]
Inflation	5 [3]	12 [10]	25 [13]
Fed funds	14 [5]	2 [9]	38 [16]
Capacity util.	8 [4]	2 [9]	43 [16]
Avg. hours	9 [5]	6 [10]	45 [17]
Real wage	4 [3]	3 [10]	29 [14]
Consumption	5 [3]	8 [12]	19 [13]
Investment	9 [4]	9 [10]	33 [14]
Velocity	17 [5]	2 [8]	26 [13]
Price of inv.	11 [4]	2 [8]	41 [14]

Notes: Numbers are the fraction of variance in the business cycle frequencies accounted for by the indicated shock; number in square brackets is an estimated of the standard error (see text). All variables, except MZM growth, inflation and Fed funds, are measured in log-levels.

**Table 2**  
Estimated parameter values  $\zeta_1$ .

Model	$\lambda_f$	$\xi_w$	$\gamma$	$\sigma_a$	$b$	$S''$	$\epsilon$
Benchmark	1.01 (n.a.)	0.78 (0.08)	0.014 (0.007)	11.42 (6.86)	0.76 (0.08)	1.50 (0.83)	0.61 (0.23)
Monetary shocks only	1.01 (n.a.)	0.71 (0.16)	0.018 (0.021)	3.93 (3.45)	0.81 (0.05)	5.32 (3.30)	0.94 (0.23)
Neutral technology shocks only	1.01 (n.a.)	0.76 (0.14)	1.367e + 04 (3.7e+05)	13.39 (50.22)	0.95 (0.05)	0.01 (0.20)	2.42e + 05 (9.09e+06)
Embodied technology shocks only	1.35 (0.31)	0.96 (0.09)	0.017 (0.008)	16.70 (49.54)	0.00 (n.a.)	0.40 (0.15)	3.17 (2.41)
Low cost of varying capital util.	1.84 (0.22)	0.56 (0.05)	0.065 (0.073)	<b>0.01</b>	0.79 (0.19)	0.49 (0.65)	0.26 (0.23)
Intermediate markup	<b>1.05</b>	0.78 (0.08)	0.014 (0.007)	9.46 (5.48)	0.75 (0.08)	1.44 (0.80)	0.61 (0.24)
High markup	<b>1.20</b>	0.77 (0.09)	0.018 (0.008)	5.67 (3.36)	0.74 (0.08)	1.26 (0.72)	0.59 (0.24)

occurred in the early 1980s. For comparability with ACEL (2005) we stopped the sample period at 2001IV.<sup>24</sup> With these considerations in mind, we set the parameter  $\mu_\gamma$  to 1.0042. At an annualized rate, this value is equal to the negative of the average growth rate of the price of investment relative to the GDP deflator which fell at an annual average rate of 1.68 percent over the ACEL (2005) sample period. The average growth rate of per capita GDP in the ACEL (2005) sample period is  $\mu_y = 1.0045$ . Solving the previous equation for  $\mu_z$  yields  $\mu_z = 1.00013$ , which is the value of  $\mu_z$  we use in our analysis.<sup>25</sup> We set the average growth rate of money,  $\mu_x$ , equal to 1.017.<sup>26</sup> This value corresponds to the average quarterly growth rate of money (MZM) over the ACEL (2005) sample period.

We set the parameters  $\gamma$  and  $\eta$  to 0.45 and 0.036, respectively. The value of  $\gamma$  corresponds to the average value of  $P_t C_t / Q_t$  in the ACEL (2005) sample period, where  $Q_t$  is measured by MZM. We chose  $\eta$  so that in conjunction with the other parameter values of our model, the steady state value of  $\eta C / Y$  is 0.025. This corresponds to the percent of value-added in the finance, insurance and real estate industry (see Christiano et al., 2004).

The row labeled ‘benchmark’ in Table 2 summarizes our point estimates of the parameters in the vector  $\zeta_2$ . Standard errors are reported in parentheses. The lower bound of unity is binding on  $\lambda_f$ . So we simply set  $\lambda_f$  to 1.01 when we estimate the model. However, the estimation criterion displays very little curvature with respect to  $\lambda_f$ . When we individually test the hypotheses that  $\lambda_f$  is 1.05 or 1.20 against the null that  $\lambda_f = 1$ , we obtain a chi-square statistic equal to 0.01 and 0.2,

<sup>24</sup> Adding the years 2002–2008 makes little difference to our estimate of  $\mu_\gamma$  and  $\mu_x$ .

<sup>25</sup> For the 1982–2008 period, the investment deflator drops on average by –2.3 percent. Over the same sample period GDP per capita growth was 1.75 percent at an annualized rate, implying that  $\mu_z$  is less than one.

<sup>26</sup> The average annual growth rate of MZM over the sample period 1982–2008 and 1959–2008 is 2 and 1.7 percent, respectively.

**Table 3**Estimated parameter values  $\zeta_2$ .

$\rho_M$	$\sigma_M$	$\rho_{\mu_z}$	$\sigma_{\mu_z}$	$\rho_{xz}$	$c_z$	$c_z^p$	$\rho_{\mu_\gamma}$	$\sigma_{\mu_\gamma}$	$\rho_{x\gamma}$	$c_\gamma$	$c_\gamma^p$
Benchmark											
−0.09 (0.07)	0.13 (0.05)	0.42 (0.27)	0.17 (0.08)	0.58 (0.26)	0.14 (0.21)	0.46 (0.29)	0.55 (0.12)	0.21 (0.04)	0.61 (0.07)	0.33 (0.22)	0.39 (0.15)
Monetary shocks only											
−0.13 (0.08)	0.20 (0.05)	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Neutral technology shocks only											
n.a.	n.a.	0.65 (0.21)	0.11 (0.06)	0.65 (0.83)	−0.36 (1.04)	1.37 (2.42)	n.a.	n.a.	n.a.	n.a.	n.a.
Embodied technology shocks only											
n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	0.67 (0.12)	0.17 (0.04)	0.84 (0.11)	0.43 (0.83)	−0.11 (0.75)
Low cost of varying capital utilization											
−0.01 (0.10)	0.06 (0.05)	0.71 (0.11)	0.07 (0.04)	0.51 (0.12)	0.92 (0.68)	0.65 (0.55)	0.21 (0.26)	0.20 (0.06)	0.61 (0.11)	0.42 (0.29)	0.26 (0.16)
Intermediate markup $\lambda_f = 1.05$											
−0.09 (0.08)	0.13 (0.05)	0.40 (0.30)	0.17 (0.08)	0.58 (0.25)	0.15 (0.21)	0.45 (0.27)	0.54 (0.12)	0.21 (0.04)	0.61 (0.07)	0.34 (0.22)	0.38 (0.15)
High markup $\lambda_f = 1.20$											
−0.09 (0.08)	0.13 (0.05)	0.30 (0.45)	0.19 (0.11)	0.56 (0.23)	0.15 (0.21)	0.41 (0.24)	0.52 (0.12)	0.21 (0.04)	0.62 (0.07)	0.38 (0.22)	0.35 (0.14)

**Table 4**

Implied average time (Quarters) between reoptimization.

Model	$\frac{1}{1-\xi_p}$	
	Firm-specific capital model	Homogeneous capital model
Benchmark	1.81	9.36
Monetary shocks only	1.76	8.10
Neutral technology shocks only	$\leq 1.00^a$	$\leq 1.00^a$
Embodied technology shocks only	5.78	8.40
Low cost of varying capital util.	4.50	4.52
Intermediate markup $\lambda_f = 1.05$	3.15	9.12
High markup $\lambda_f = 1.20$	4.90	8.26

<sup>a</sup>  $\leq 1.00$  implies that prices are re-optimized each period in our quarterly model.

respectively, with associated probability values 0.0002 and 0.024, respectively. So we cannot clearly reject either hypothesis. Tables 2 and 3 report point estimates for  $\zeta_2$  and  $\zeta_3$  when we re-estimate the model setting  $\lambda_f$  to 1.05 and 1.20.

Our point estimate of  $\xi_w$  implies that wage contracts are re-optimized, on average, once every 4.5 quarters. To interpret our point estimate of  $\gamma$ , recall that in the homogeneous capital model,  $\gamma = (1 - \xi_p)(1 - \beta\xi_p)/\xi_p$ . So our point estimate of  $\gamma$  implies a value of  $\xi_p$  equal to 0.896. This implies that firms re-optimize prices roughly every 9.36 quarters (see Table 4). This value is much larger than the value used by Golosov and Lucas (2007) who calibrate their model to micro data to ensure that the firms re-optimize prices on average once every 1.5 quarters.

Table 4 shows that if we adopt the assumption that capital is firm-specific, then our estimates imply that firms re-optimize prices on average once every 1.8 quarters.<sup>27</sup> So the assumption that capital is firm-specific has a very large impact on inference about the frequency at which firms re-optimize price.

To interpret the estimated value of  $\sigma_a$ , we consider the homogeneous capital model. Linearizing the household's first order condition for capital utilization about steady state yields:

$$E \left\{ \left( \frac{1}{\sigma_a} \hat{r}_t^k - \hat{u}_t \right) \middle| \Omega_t \right\} = 0.$$

According to this expression,  $1/\sigma_a$ , equal to 0.08 of a percent, is the elasticity of capital utilization with respect to the rental rate of capital. Our estimate of  $\sigma_a$  is larger than the value estimated by CEE (2005) and indicates that it is relatively costly for firms to vary the utilization of capital.

Our point estimate of the habit parameter  $b$  is 0.76. This value is reasonably close to the point estimate of 0.66, reported in CEE (2005) and the value of 0.7 reported in Boldrin et al. (2001). The latter authors argue that the ability of standard

<sup>27</sup> This number was obtained using the algorithm discussed in ACEL (2004).

general equilibrium models to account for the equity premium and other asset market statistics is considerably enhanced by the presence of habit formation in preferences.

We now discuss our point estimate of  $S''$ . Suppose we denote by  $P_{k',t}$  the shadow price of one unit of  $\bar{k}_{t+1}$ , in terms of output. The variable  $P_{k',t}$  is what the price of installed capital would be in the homogeneous capital model if there were a market for  $\bar{k}_{t+1}$  at the beginning of period  $t$ . Proceeding as in CEE (2005), it is straightforward to show that the household's first order condition for investment implies:

$$\hat{i}_t = \hat{i}_{t-1} + \frac{1}{S''} \sum_{j=0}^{\infty} \beta^j E[\hat{P}_{k',t+j} | \Omega_t].$$

According to this expression,  $1/S''$  is the elasticity of investment with respect to a one percent temporary increase in the current price of installed capital. Our point estimate implies that this elasticity is equal to 0.66. The more persistent is the change in the price of capital, the larger is the percentage change in investment. This property holds because adjustment costs induce agents to be forward looking.

Table 3 reports the estimated values of the parameters pertaining to the stochastic part of the model. With these values, the laws of motion for the neutral and capital-embodied technology shocks are:

$$\begin{aligned} \hat{\mu}_{\gamma,t} &= \underset{(0.12)}{0.55} \hat{\mu}_{\gamma,t-1} + \varepsilon_{\mu_{\gamma},t}, & 100 \times \sigma_{\mu_{\gamma}} &= \underset{(0.04)}{0.21}, \\ \hat{\mu}_{z,t} &= \underset{(0.27)}{0.42} \hat{\mu}_{z,t-1} + \varepsilon_{\mu_z,t}, & 100 \times \sigma_{\mu_z} &= \underset{(0.08)}{0.17}. \end{aligned}$$

Numbers in parentheses are standard errors. Our estimates imply that a one-standard deviation neutral technology shock drives  $z_t$  up by 0.17 percent in the period of the shock and by 0.29 ( $= 0.17/(1 - 0.42)$ ) percent in the long run. A one-standard-deviation shock to embodied technology drives  $\gamma_t$  up by 0.21 percent immediately and by 0.47 percent in the long run. Our estimates imply that shocks to neutral technology exhibit a high degree of serial correlation, while shocks to capital-embodied technology do not.

It is interesting to compare our results for  $\hat{\mu}_{z,t}$  with the ones reported in Prescott (1986), who estimates the properties of the technology shock process using the Solow residual. He finds that the shock is roughly a random walk, and its growth rate has a standard deviation of roughly 1 percent.<sup>28</sup> By contrast, our estimates imply that the unconditional standard deviation of the growth rate of neutral technology is roughly 0.19 ( $= 0.17/\sqrt{1 - 0.42^2}$ ) percent. So we find that technology shocks are substantially less volatile but more persistent than those estimated by Prescott. In principle, these differences reflect two factors. First, from the perspective of our model, Prescott's estimate of technology confounds technology with variable capital utilization. Second, our analysis is based on different data sets and different identifying assumptions than Prescott's.

## 5.2. Impulse responses

The dotted lines in Figs. 1 through 3 display the impulse response functions of the estimated model to monetary policy, neutral technology shocks and capital-embodied shocks, respectively. Recall that the solid lines and the associated confidence intervals (the gray areas) pertain to the impulse response functions from the estimated, identified VARs.

### 5.2.1. Response to a monetary policy shock

We begin by discussing the model's performance with respect to a monetary policy shock (see Fig. 1). First, consistent with results in CEE (2005), the model does well at accounting for the dynamic response of the U.S. economy to a monetary policy shock. Most (but not all) of the model responses lie within the two-standard deviation confidence interval computed from the data. This is true even though firms in the firm-specific capital version of the model change prices on average once every 1.8 quarters.

Second, the model generates a very persistent response in output. The peak effect occurs roughly one year after the shock. The output response is positive for over three years. Third, the model accounts for the dynamic response of the interest rate to a monetary policy shock. Consistent with the data, an expansionary monetary policy shock induces a sharp decline in the interest rate which then returns to its pre-shock level within a year. The model does not account for the overshooting pattern of the interest rate in the data. The growth rate of transactions balances rises for a brief period of time after the policy shock, but then quickly reverts to its pre-shock level. But the model does not account for the overshooting pattern seen in transaction balances. Fig. 1 shows that the effects of a policy shock on aggregate economic activity persist beyond the effects on the policy variable itself, regardless of whether the policy variable is measured as the interest rate or the money supply. This property reflects the strong internal propagation mechanisms in the model.

<sup>28</sup> Prescott (1986) actually reports a standard deviation of 0.763 percent. However, he adopts a different normalization for the technology shock than we do, by placing it in front of the production function. By assumption, the technology shock multiplies labor directly in the production and is taken to a power of labor's share. The value of labor's share that Prescott uses is 0.70. When we translate Prescott's estimate into the one relevant for our normalization, we obtain  $0.763/.7 \approx 1$ .



Fourth, as in the data, the real wage remains essentially unaffected by the policy shock. Fifth, consumption, investment, and hours worked exhibit persistent, hump-shaped rises that are consistent with our VAR-based estimates. Sixth, consistent with the data, velocity falls after the expansionary policy shock. This fall reflects the rise in money demand associated with the initial fall in the interest rate. However this fall is nearly as strong as the VAR based response of velocity to a monetary policy shock. Seventh, by construction, the relative price of investment is not affected by a policy shock in the model. At least for the first two years after the policy shock, this lack of response is consistent with the response of the relative price of investment to a policy shock in the identified VAR. It is not consistent with the rise in that price in the third year after the shock. Finally, capacity utilization in the model rises by only a very small amount, and understates the estimated rise in the data.

Overall the response of the model to a monetary policy shock is quite similar to the response of the estimated model in CEE (2005). This result holds even though the models are estimated over very different sample periods. The main difference is that the size of the monetary policy shock is almost twice as large as in ACEL (2005). But conditional on a given monetary policy shock, the transmission mechanism in the two estimated models is very similar.

### 5.2.2. Response to a neutral technology shock

We now discuss the model's performance with respect to a neutral technology shock (see Fig. 2). First, the model does well at accounting for the dynamic response of the U.S. economy to a neutral technology shock. Specifically, the model accounts for the rise in aggregate output, hours worked, investment, consumption and the real wage. However, the model does not capture the extent of the fall in inflation that occurs immediately after the shock.

### 5.2.3. Response to a capital embodied technology shock

We now discuss the model's performance with respect to a capital-embodied technology shock (see Fig. 3). The model does very well in accounting for the response of the U.S. economy to this shock, except that it does not account for the rises in capacity utilization and the federal funds rate that occur after the capital-embodied technology shock. In addition money growth is high relative to the estimated response from the VAR. To see the importance of monetary policy in the transmission of capital embodied technology shocks, we compute the response of the model economy to a positive, capital embodied technology shock under the assumption that money growth remains unchanged from its steady state level. We find that output and hours worked rise by much less, while inflation falls compared to what happens when monetary policy is accommodative. We conclude that the model requires accommodative monetary policy to match the expansionary effects of a positive capital embodied technology shock.

## 6. The key features of the model

In this section we discuss the features of the data driving our estimates of the parameters determining the implications of the firm-specific and homogeneous capital models for the frequency at which firms re-optimize prices.

Our point estimate of  $\gamma$  (0.014) implies that a temporary one percent change in marginal cost results in only a 0.02 percent change in the aggregate price level.<sup>29</sup> The small value of  $\gamma$  lies at the heart of the tension between the micro and macro implications of the homogeneous capital model.

We now argue that any reasonable estimate of  $\gamma$  must be low. In Fig. 4a we plot  $\Delta\hat{\pi}_t - \beta\Delta\hat{\pi}_{t+1}$  against our measure of the log of marginal cost,  $\hat{s}_t$ .<sup>30</sup> The distribution of  $\Delta\hat{\pi}_t - \beta\Delta\hat{\pi}_{t+1}$  is at best weakly related to the magnitude of  $\hat{s}_t$ .<sup>31</sup> The relatively flat curve in Fig. 4a has a slope equal to our point estimate of  $\gamma$  (0.014). Significantly, this curve passes through the central tendency of the data. The steeper curve in Fig. 4a is drawn for a value of  $\gamma$  equal to 0.68, a value which implies that in the homogeneous capital model firms change prices roughly once every 1.8 quarters. Fig. 4a shows that raising  $\gamma$  to 0.68 leads to a drastic deterioration in fit.

Eq. (2.22) implies that the magnitude of the residuals from the lines in Fig. 4a cannot be used as a formal measure of model fit. We should focus on the size of residuals when the data are replaced by their projection onto date  $t$  information, because then (2.22) implies that least squares consistently recovers the true value of  $\gamma$ . Fig. 4b is the analog to Fig. 4a, with variables replaced by their projection onto  $F_t \equiv \{\Delta\pi_{t-s} - \beta\Delta\pi_{t+1-s}, \hat{s}_{t-s}; s = 1, 2\}$ . Figs. 4a and 4b are very similar so that our conclusions are unchanged: the data on inflation and marginal cost suggest that  $\gamma$  is small.<sup>32</sup>

The low estimated value of  $\gamma$  provides a different perspective on the inflation inertia puzzle, particularly the weak response of inflation to monetary policy shocks. Solving (2.22) forward we obtain

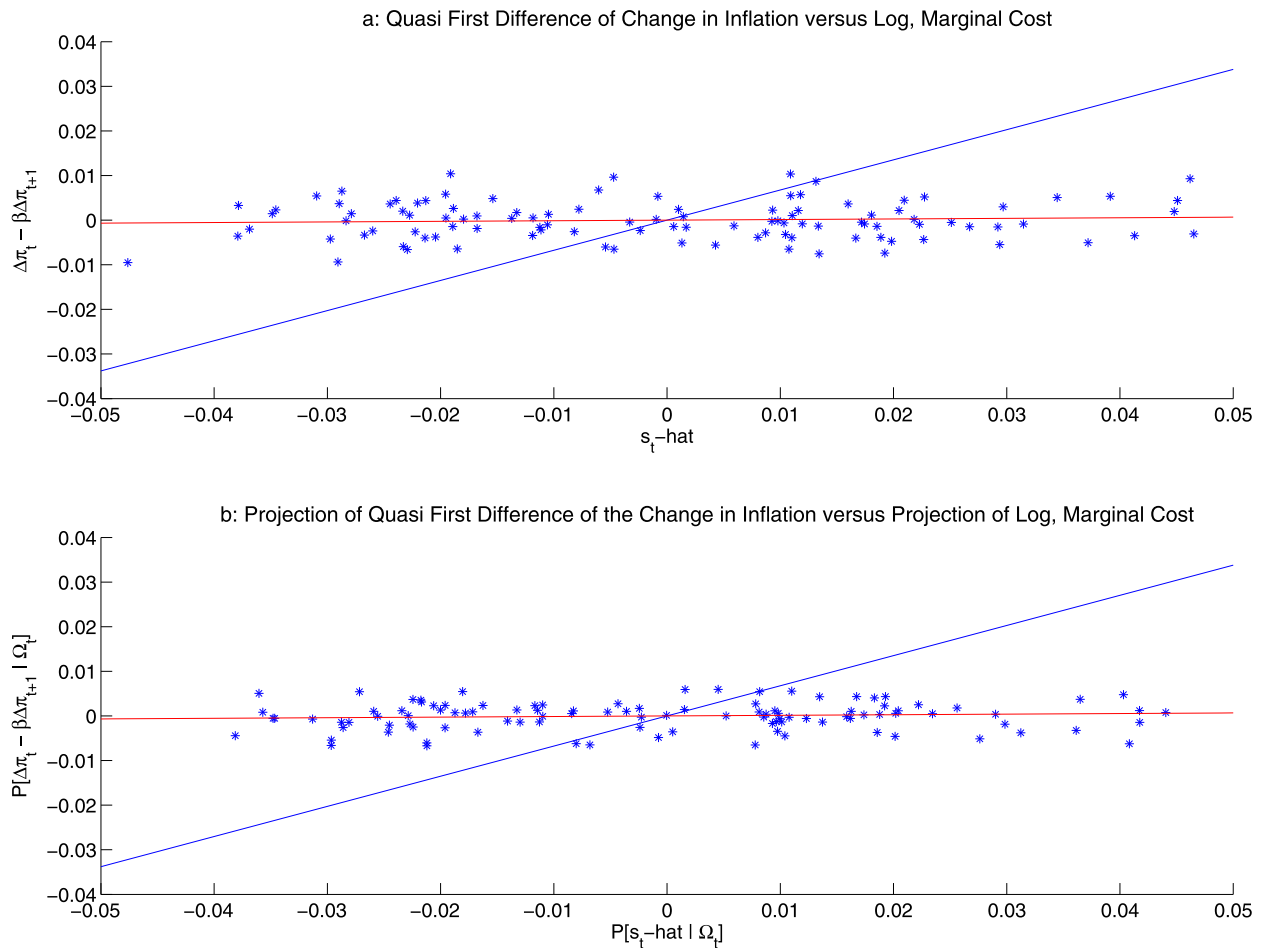
$$\Delta\hat{\pi}_t = \gamma \sum_{j=0}^{\infty} \beta^j E_t \hat{s}_{t+j}. \quad (6.1)$$

<sup>29</sup> This estimate is consistent with results in the literature. See Eichenbaum and Fisher (2007) and the references therein.

<sup>30</sup> We set  $\beta = 1.03^{-25}$ . Also, we measure marginal productivity by labor's share in GDP. In our model this is the correct measure if fixed costs are zero. This measure is approximately correct here, since our estimate of  $\phi$  is close to zero.

<sup>31</sup> Eichenbaum and Fisher (2007) argue that their estimates of  $\gamma$  are robust to alternative measures of marginal cost.

<sup>32</sup> We obtain the same results whether we work with  $\Delta\hat{\pi}_t$  or with  $\hat{\pi}_t$ .



**Fig. 4.** (a) Quasi first difference of change in inflation versus Log, marginal cost. (b) Projection of quasi first difference of the change in inflation versus projection of Log, marginal cost.

This relation makes clear why many authors incorporate features like variable capital utilization and sticky wages into their models. These features can reduce the response of expected marginal cost to shocks.<sup>33</sup> Relation (6.1) reveals another way to account for inflation inertia: assign a small value to  $\gamma$ . The evidence in Figs. 4a and 4b indicates that a small value of  $\gamma$  must be part of any successful resolution of the inflation inertia puzzle.

A low value of  $\gamma$  is clearly a problem for the homogeneous capital model. This is because the model then implies that firms re-optimize prices very infrequently, e.g., at intervals of roughly 9.5 quarters.<sup>34</sup> So to get the macro data right (i.e., a low  $\gamma$ ) we must make assumptions about the frequency at which firms re-optimize prices that seem implausible in light of the micro data. In contrast, suppose we adopt the more plausible assumption that firms re-optimize prices on average once every 1.8 quarters. Then the homogeneous capital model implies  $\gamma = 0.68$ . But this means that the model gets the macro data wrong.

In the firm-specific capital model it is possible to reconcile the low value of  $\gamma$  with a low value of  $\xi_p$ . This reflects two features of that model. The first is that not all firms set prices at the same time (i.e., ‘staggered pricing’). The second is that capital is firm-specific so that the only way a firm can adjust its capital stock is by varying its investment over time. To understand the role of these features, suppose there is an increase in the quantity of money. Flexible price firms respond by increasing their prices. Depending on how elastic demand is, these price increases cause demand to shift away from flexible price firms and towards the sticky price firms. Consequently, flexible price firms need less capital services. If firms could trade physical capital, capital would flow from flexible price firms to sticky price firms. With firm-specific capital this flow cannot occur. To the extent that flexible price firms cannot easily reduce capital utilization rates, the shadow value of capital to flexible price firms plummets. This fall acts like a decline in marginal cost and reduces the incentive of flexible price firms to raise prices in the first place. This mechanism is enhanced the more elastic is demand and the less flexible is

<sup>33</sup> See, for example, Ball and Romer (1990), CEE (2005), Dotsey and King (2009), Galí and Gertler (1999), Lindé (2005) and Smets and Wouters (2003).

<sup>34</sup> This is a straightforward implication of the homogeneous capital model discussed above, according to which  $\gamma = (1 - \xi_p)(1 - \beta\xi_p)/\xi_p$ .

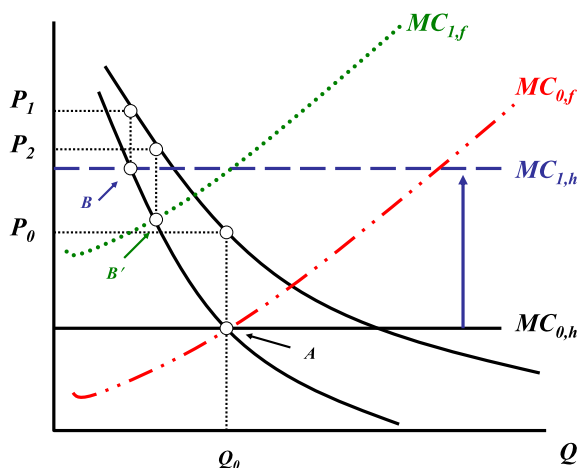


Fig. 5. Firm-specific capital and the response of price to marginal cost shocks.

capital utilization. No doubt, the assumption that capital is completely immobile between firms is unrealistic. At the same time, anything which causes a firm's marginal cost to be an increasing function of its output works in the same direction as firm-specificity of capital.<sup>35</sup>

The key parameter which governs the flexibility of capital utilization is  $\sigma_a$ . The logic in the previous paragraph suggests that for a fixed value of  $\xi_p$ , the larger is  $\sigma_a$ , the lower is  $\gamma$ . But other things equal, a lower  $\xi_p$  implies a higher  $\gamma$ . These observations suggest that for a given value of  $\gamma$ ,  $\xi_p$  is a decreasing function of  $\sigma_a$ . In fact, our point estimate of  $\sigma_a$  is large which helps explain why the value of  $\xi_p$  implied by the firm-specific model is low.

What is it about the data that leads to a large (albeit imprecise) point estimate for  $\sigma_a$ ? We recompute the impulse responses implied by our model, holding all but one of the model parameters at their estimated values. The exception is  $\sigma_a$  which we set to 0.01. The new value of  $\sigma_a$  has two major effects on the model impulse response functions. First, the responses of capital utilization to both technology shocks are stronger. The responses are so strong that, at several horizons, they lie substantially outside the corresponding empirical confidence intervals. This effect is particularly strong for a capital-embodied technology shock. Also the model has difficulty in matching the rise in output, measured net of capacity utilization costs, after a capital-embodied technology shock. Basically capacity utilization rises by such a large amount that it leads to a drag in output net of capacity utilization costs. These two effects explain why our estimation criterion settles on a high value of  $\sigma_a$ .

In Tables 2 and 3 we report the results of estimating the model subject to the constraint that  $\sigma_a$  is a small number, 0.01 (see the row labelled 'Low cost of varying capacity utilization'). Note that the point estimate rises from 0.014 to 0.065, a value that is inconsistent with the estimated value of  $\gamma$  discussed in the context of Fig. 4. Table 4 shows that consistent with our intuition, the homogeneous and firm-specific capital model now yield very similar implications for the frequency with which firms re-optimize prices, namely about once a year.

To verify our intuition about why our benchmark estimate of  $\sigma_a$  is high, we re-estimate the model including only the responses to a monetary policy shock in the criterion. We report our results in Tables 2, 3 and 4. Our point estimate of  $\sigma_a$  falls from 11.42 to 3.93. The lower value of  $\sigma_a$  allows the model to better capture the estimated rise in capital utilization that occurs after a monetary policy shock, without paying a penalty for a counterfactually large rise in capacity utilization and a fall in output after a capital-embodied technology shock. This result reconciles our findings with those reported in CEE (2005) who report a low estimated value of  $\sigma_a$  based on an estimation criterion that includes only the responses to a monetary policy shock.

To pursue our intuition about the benchmark estimate of  $\sigma_a$  we also re-estimate the model including only a capital-embodied technology shock in the estimation criterion. Tables 2, 3 and 4 show that our results are similar to the benchmark results except that our estimate of  $\sigma_a$  is higher. The higher value of  $\sigma_a$  dampens the response of capital utilization to a capital-embodied technology shock, bringing the model response closer to the VAR-based response.

Fig. 5 suggests that a high elasticity of demand also works to reduce a firm's incentive to raise price after an exogenous increase in marginal cost, i.e. a low value of  $\lambda_f$  reduces  $\gamma$ . While our estimation criterion is very insensitive to  $\lambda_f$ , it weakly prefers a very low value for this variable. To examine the role played by  $\lambda_f$ , we re-estimate the model imposing  $\lambda_f = 1.05$  and 1.20. The first of these values of  $\lambda_f$  is close to Bowman's (2003) estimate of the markup for the economy as a whole. The second value of  $\lambda_f$  is equal to the point estimate in CEE (2005). Table 2 shows that imposing different values of  $\lambda_f$  has very little impact on the estimated values of the key structural parameters of the model. Table 4 shows that the main qualitative effect of a higher value of  $\lambda_f$  is to reduce the frequency with which firms re-optimize prices in the firm-specific

<sup>35</sup> Marginal costs could be increasing because of firm-specificity of other factors of production or costs of adjusting production inputs.

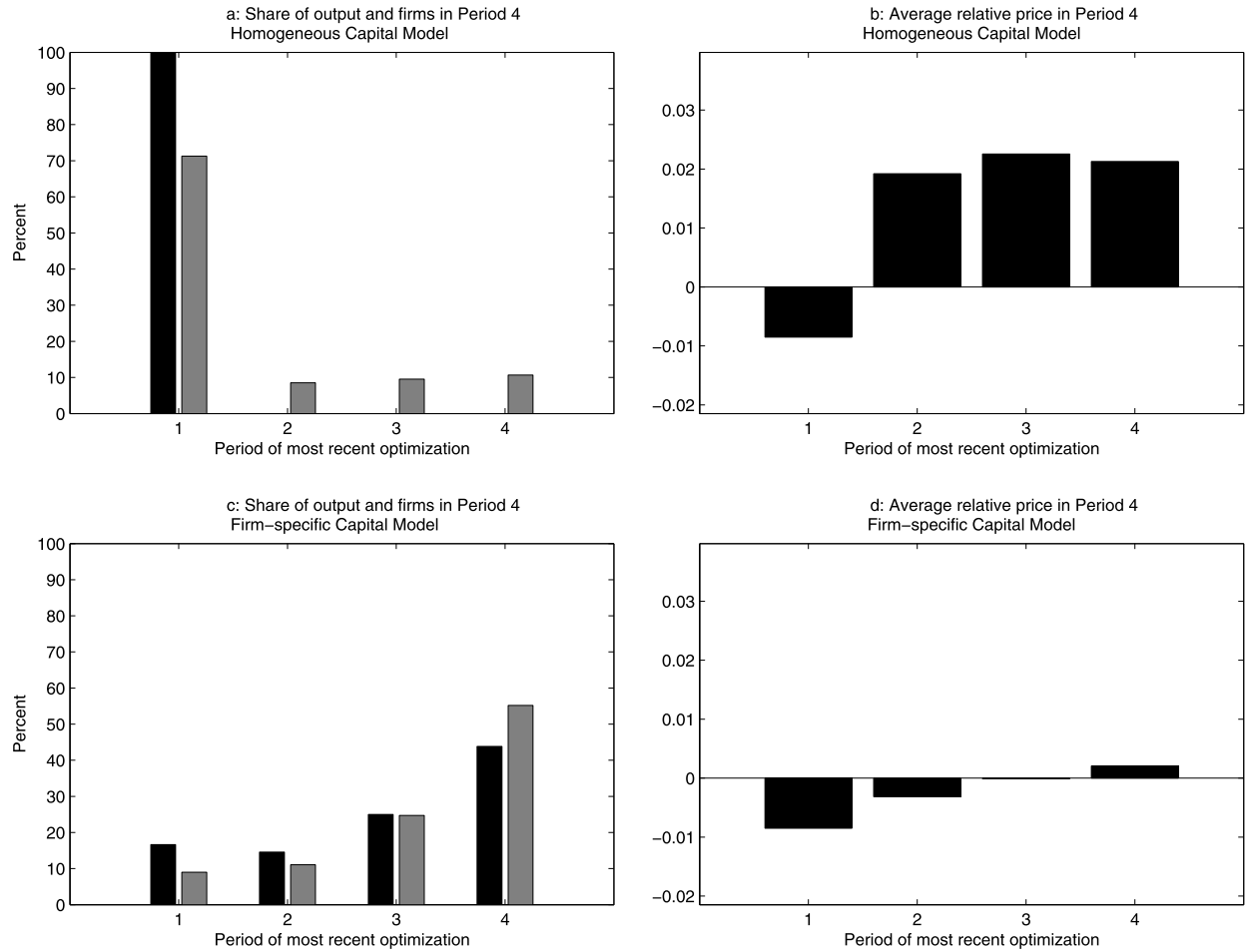


Fig. 6. Features of the distribution of output and prices across firms.

capital model. For  $\lambda_f$  equal to 1.05 and 1.20 respectively, the frequency with which firms re-optimize prices rises to once every 3.15 and 4.90 quarters respectively. We conclude that to resolve the micro-macro pricing puzzle in our framework we are compelled to take the view that  $\lambda_f$  is close to one. This last result may reflect our assumption that intermediate good firms face a constant elasticity of demand. Other specifications of demand, like the one proposed in Kimball (1995), break the link between the steady state markup and the elasticity of demand away from steady state. Incorporating changes like these may make it possible to rationalize a low  $\gamma$  with a low value of  $\xi_p$  and a higher value of  $\lambda_f$ .

## 7. Examining the microeconomic implications of the homogeneous versus firm-specific capital models

The homogeneous and firm-specific capital models imply that firms re-optimize prices on average once every 1.8 and 9.4 quarters, respectively. These results point in favor of the firm-specific capital model. We now document an even more powerful reason for preferring that model: the estimated homogeneous capital model predicts, implausibly, that a small subset of firms produce the bulk of total output after a monetary policy shock. The firm-specific capital model does not suffer from this shortcoming. However, the benchmark firm-specific capital model does have an important shortcoming. It implies that firm-level output is too volatile, relative to firm-level prices. We display a variant of the benchmark model which does not suffer from this shortcoming.

To document these findings we begin by considering the impact of a monetary policy shock on the cross-firm distribution of prices and output. We suppose that the economy is in a steady state up until period 0. In the steady state, each firm's price and quantity is the same. An expansionary monetary policy shock occurs in period 1. Given the timing convention in our model, prices and output levels are the same across firms at the end of period 1. In period 2, a fraction,  $1 - \xi_p$ , of firms re-optimize their price. The other firms update their price according to (2.9). In period 3 there are four types of firms: (i) a fraction,  $(1 - \xi_p)^2$ , of firms that re-optimize in periods 2 and 3; (ii) a fraction,  $\xi_p^2$ , of firms that do not re-optimize in periods 2 or 3; (iii) a fraction,  $(1 - \xi_p)\xi_p$ , which re-optimize in period 2 and not in period 3; and (iv) a fraction,  $\xi_p(1 - \xi_p)$ , of firms that do not re-optimize in period 2, but do re-optimize in period 3. In period  $s$  there are  $2^{s-1}$  different types of firms.

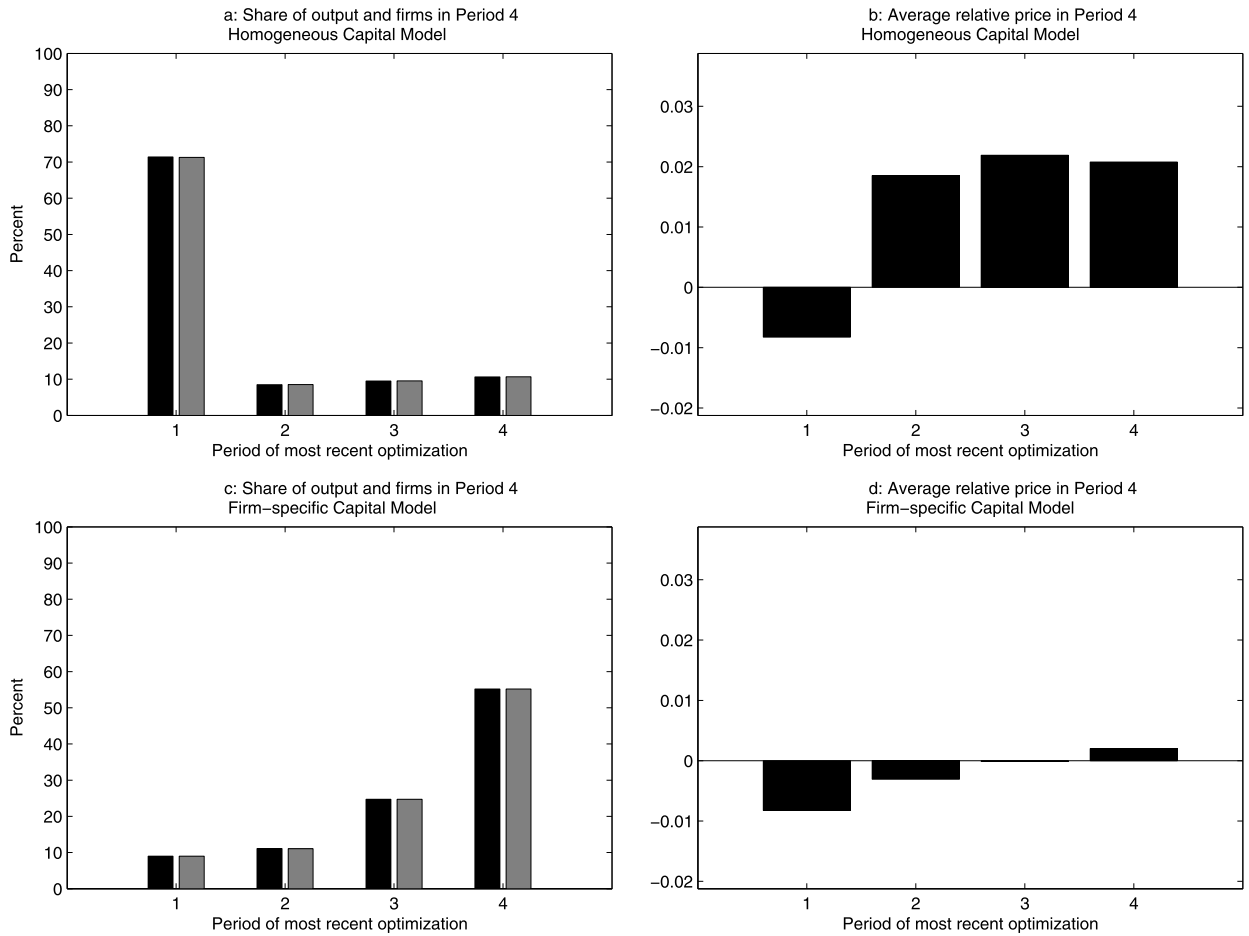


Fig. 7. Features of the distribution of output and prices across firms: lower demand elasticity.

We calculate the distribution of output and relative prices across firms in period  $s = 4$ . Figs. 6a and 6b summarize our findings for the homogeneous capital version of the model. The integers 1, 2, 3, and 4 on the horizontal axes of these figures refer to different groups of firms. The integer, 1, pertains to firms that did not re-optimize their price in periods 2, 3 and 4. The integers  $j = 2, 3$  and 4, pertain to firms which last re-optimized in period  $j$ . Fig. 6a shows the share of output (black bars) and the fraction of firms (grey bars) corresponding to the different groups of firms. For the firms in each group, Fig. 6b shows the log deviation of their price from the aggregate price.

We note several features of Figs. 6a and 6b. First, a small fraction of the firms are producing a disproportionate share of the output. Indeed, roughly 70% of the firms who did not re-optimize their prices in periods 2, 3 and 4 produce 100 percent of output. The remaining firms effectively shut down. A key factor driving this result is the high elasticity of demand for a firm's output ( $\lambda_f$  is small) in the estimated benchmark model. As we show below, we can overturn this implication by imposing a higher value of  $\lambda_f$ .

We now turn to the firm-specific capital model. Figs. 6c and 6d are the analogs to Figs. 6a and 6b. Fig. 6c shows that the dramatic degree of inequality of production associated with the homogeneous capital model no longer obtains. Still, there is some inequality in the level of production at individual firms. The average level of production by firms in a particular category corresponds to the ratio of the black bar (total production in that group) to the grey bar (number of firms in that group). In period 4, these averages are 1.8, 1.3, 1.0, and 0.8 for firms that last optimized in periods 1, 2, 3 and 4, respectively. So, the typical firm that has not been able to re-optimize its price since the monetary policy shock produces over twice as much as a firm that has not been able to re-optimize since the shock occurred. In later periods, the extent of the inequality in production is substantially mitigated.<sup>36</sup>

Fig. 7 is the analog to Fig. 6 for the estimated model when we set  $\lambda_f$  to 1.05. Comparing Figs. 6a and 7a we see that the implications of the homogeneous capital model become less extreme for the higher value of  $\lambda_f$ . It is still true that a small

<sup>36</sup> One measure of the degree of inequality in production is provided by the Gini coefficient. In periods 4, 8 and 16, these are 0.12, 0.15 and 0.26 for the firm-specific capital version of the model.

fraction of firms are producing a disproportionate share of the output. But now the fraction of output produced by the roughly 70 percent of the firms who did not re-optimize their prices in periods 2, 3 and 4 is only a bit larger than 70 percent.<sup>37</sup> However as Table 4 indicates, with  $\lambda_f$  equal to 1.05, in the homogeneous capital model firms re-optimize prices once every 9 quarters. In the firm-specific capital model firms re-optimize prices roughly once every 3 quarters. On this basis we would still prefer the firm-specific capital model.

## 8. Conclusion

We construct a dynamic general equilibrium model of cyclical fluctuations that accounts for inflation inertia even though firms re-optimize prices on average once every 1.8 quarters. To obtain this result we assume that capital is firm-specific. If we assume that capital is homogenous we can account for inflation inertia. However, this version of the model has micro implications that are implausible: firms re-optimize their prices on average once every 9.4 quarters and a monetary policy shock induces extreme dispersion in prices and output across firms. These considerations lead us to strongly prefer the firm-specific capital model.

We conclude by noting that, in this paper, we have take as given that firms re-optimize prices roughly once every two quarters. If we take the position that firms re-optimize prices on average roughly once a year, then we can reconcile the micro–macro pricing puzzle with a value of  $\lambda_f$  around roughly 1.10. As Table 4 indicates, for these values of  $\lambda_f$ , firm-specific capital still plays a critical role in generating plausible implications for the frequency with which firms re-optimize prices. At the same time higher values of  $\lambda_f$  are associated with less elastic demand curves than lower values of  $\lambda_f$ . This fact has important implications for the micro implications of the model, like the volatility of firm level output. We will pursue these implications in future research.

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<sup>37</sup> The behavior of firms when capital is firm-specific is not substantially affected by the higher value of  $\lambda_f$ .

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