A Benchmark Real Business Cycle Model

Contents:

Readings

- Kydland, F. E. and E. C. Prescott, 1982. Time to build and aggregate fluctuations. *Econometrica*, 50 (6): 1345-1370.
- Hansen, G. D., 1985. Indivisible labor and the business cycle. *Journal of Monetary Economics*, 16: 309-327.
- Cooley, T. F. and E. C. Prescott, 1995. Economic growth and business cycle. In Cooley, T. F., Frontier of business cycle research. Princeton University Press. Princeton, New Jersey, pp 1-38.
- A Toolkit for analyzing nonlinear dynamic stochastic models easily (by Uhlig Harald).

Introduction: Economic Growth and Business Cycles

☐ Kaldor's "stylized facts" of growth (as characterized by Solow (1970)):

1. Real output grows at a more or less constant rate;
2. The stock of real capital grows at a more or less constant rate greater than the rate of growth of the labour input;
3. The growth rates of real output and the stock of capital tend to be about the same;
4. The rate of profit on capital has a horizontal trend;
5. The rate of growth of output per-capita varies greatly from one country to another;
6. Economies with a high share of profits in income tend to have a high ratio of investment to output.

Introduction: Economic Growth and Business Cycles

□ Kaldor's "stylized facts" of growth (as characterized by Solow (1970)):

Remarks:

- The 2nd and 3rd of these stylized facts imply that investment-output ratio is constant;
- The 3rd and 4th imply that capital-output share is constant;
- The first four together describe an economy experiencing"balanced growth";
- The 5th and 6th stylized facts have posed more difficulty for neoclassical growth theory, and much of the modern endogenous growth literature has been concerned with these features.

Introduction: Economic Growth and Business Cycles

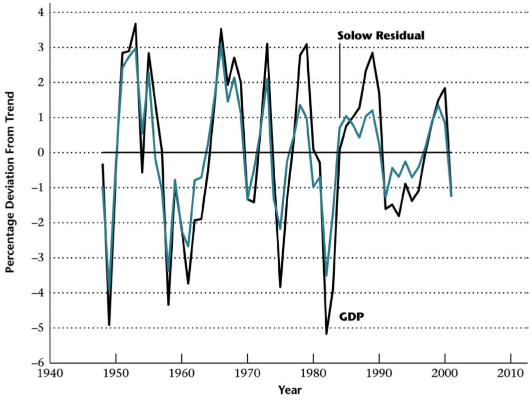
- Business cycle facts (Williamson, 2008)

Table 11.2 Data Versus Predictions of the Real Business Cycle Model with Productivity Shocks

Variable	*Data*	*Model*
Consumption	Procyclical	Procyclical
Investment	Procyclical	Procyclical
Price Level	Countercyclical	Countercyclical
Money Supply	Procyclical	—
Employment	Procyclical	Procyclical
Real Wage	Procyclical	Procyclical
Average Labor Productivity	Procyclical	Procyclical

☐ Business cycle facts (Williamson, 2008)

Figure 6.24 Percentage Deviations from Trend in Real GDP (black line)and the Solow Residual (colored line), 1948–2001



Introduction: Economic Growth and Business Cycles

☐ Modern business cycle theory (Cooley and Prescott, 1995)

- For a long time, study of short-term business cycles and study of long-term growth were divorced;
- Modern business cycle theory starts with the view that growth and fluctuations are not distinct phenomena to be studied with separate data and different analytical tools. This is due to the developments:
  1. Brock and Mirman's (1970) characterization of optimal growth in an economy with stochastic productivity shocks;
  2. The introduction of the labour-leisure choice into the basic neoclassical model.
- The artificial economies are constructed to mimic important aspects of the behavior through time of actual economies. Therefore, they are useful laboratories for studying the business cycle and for studying economic theory;
- In the rest of this session, we study Hansen's benchmark real business cycle model, in which the most common element is the neoclassical model of economic growth — the marriage of economic theory and empirical observation.

Hansen's Benchmark Real Business Cycle Model (The Environment)

Preferences:

The model economy is populated by infinitely-lived households. The preferences of households are assumed to be identical. Households maximize the expected utility over life time:

$$U(C_t, N_t) = E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\eta_c} - 1}{1 - \eta_c} - AN_t \right), \qquad 0 < \beta < 1 \qquad \eta_c > 0 \tag{1}$$

where C_t and N_t are consumption and labour respectively, β is the discount factor that households apply to future consumption, and η_c is the coefficient of relative risk aversion.

Hansen's Benchmark Real Business Cycle Model (The Environment)

Technology:

The technology is defined as a standard Cobb-Douglas production function:

$$Y_t = Z_t K_{t-1}^{\alpha} N_t^{1-\alpha} \tag{2}$$

where Z_t is total factor productivity (TFP) which is exogenously evolving according to the law of motion:

$$logZ_t = (1-\psi)logZ^{ss} + \psi logZ_{t-1} + \epsilon_t, \qquad \epsilon_t \sim i.i.d.(0, \sigma^2) \tag{3}$$

where ψ and Z^{ss} are parameters, and $0 < \psi < 1$.

Hansen's Benchmark Real Business Cycle Model (The Environment)

Technology:
As in a neoclassical growth model, capital stock depreciates at the rate δ, and households invest a fraction of income in capital stock in each period. This amount of investment forms part of productive capital in current period. Therefore the law of motion for aggregate capital stock is

$$K_t = (1-\delta)K_{t-1} + X_t, \qquad 0 < \delta < 1 \tag{4}$$

Aggregate resource constraint:

$$Y_t = C_t + X_t \tag{5}$$

Hansen's Benchmark Real Business Cycle Model (The FOCs)

The Bellman equation:

The model can be written as a Bellman equation of the form

$$V(K_{t-1}, Z_t) = \max_{C_t, X_t, K_t, N_t} \left[\frac{C_t^{1-\eta_c} - 1}{1-\eta_c} - AN_t + \beta E_t V(K_t, Z_{t+1})\right] \quad (6)$$

s.t.

$$\begin{aligned} Y_t &= C_t + X_t \\ Y_t &= Z_t K_{t-1}^{\alpha} N_t^{1-\alpha} \\ K_t &= (1-\delta)K_{t-1} + X_t \\ logZ_t &= (1-\psi)logZ^{ss} + \psi logZ_{t-1} + \epsilon_t \end{aligned}$$

Hansen's Benchmark Real Business Cycle Model (The FOCs)

The FOC w.r.t. K_t

$$1 = \beta E_t[(\frac{C_t}{C_{t+1}})^{\eta_c} R_{t+1}] \tag{7}$$

The FOC w.r.t. to N_t

$$A = C_t^{-\eta_c}(1-\alpha)\frac{Y_t}{N_t} \tag{8}$$

and

$$R_t = \alpha \frac{Y_t}{K_{t-1}} + (1-\delta) \tag{9}$$

where, R_t is the gross real return of capital, which is equal to the real return r_t plus $(1-\delta)$, i.e. $R_t = 1 + r_t - \delta$[1].

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[1] Take note, later on we use $\hat{r}_t$ represents the deviation of R_t from its steady state

Hansen's Benchmark Real Business Cycle Model (The Complete Model Economy)

$$
\begin{aligned}
Y_t &= C_t + X_t \\
Y_t &= Z_t K_{t-1}^{\alpha} N_t^{1-\alpha} \\
K_t &= (1-\delta)K_{t-1} + X_t \\
1 &= \beta E_t[(\frac{C_t}{C_{t+1}})^{\eta_c} R_{t+1}] \\
A &= C_t^{-\eta_c}(1-\alpha)\frac{Y_t}{N_t} \\
R_t &= \alpha\frac{Y_t}{K_{t-1}} + (1-\delta) \\
logZ_t &= (1-\psi)logZ^{ss} + \psi logZ_{t-1} + \epsilon_t, \qquad \epsilon_t \sim i.i.d.(0,\sigma^2)
\end{aligned}
$$

Hansen's Benchmark Real Business Cycle Model (The Steady State)

$$
\begin{aligned}
Y^{ss} &= C^{ss} + X^{ss} && (10)\\
Y^{ss} &= Z^{ss}(K^{ss})^{\alpha}(N^{ss})^{1-\alpha} && (11)\\
X^{ss} &= \delta K^{ss} && (12)\\
R^{ss} &= \frac{1}{\beta} && (13)\\
A &= \frac{1}{(C^{ss})^{\eta_c}}(1-\alpha)\frac{Y^{ss}}{N^{ss}} && (14)\\
K^{ss} &= \left(\frac{\alpha Z^{ss}}{R^{ss} - 1 + \delta}\right)^{\frac{1}{1-\alpha}} N^{ss} && (15)
\end{aligned}
$$

Hansen's Benchmark Real Business Cycle Model (The log-linearized Model[2])

$$
\begin{aligned}
\hat{y}_t &= \frac{C^{ss}}{Y^{ss}}\hat{c}_t + \frac{X^{ss}}{Y^{ss}}\hat{x}_t && (16)\\
\hat{y}_t &= \hat{z}_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{n}_t && (17)\\
\hat{k}_t &= \frac{X^{ss}}{K^{ss}}\hat{x}_t + (1-\delta)\hat{k}_{t-1} && (18)\\
0 &= E_t[\eta_c(\hat{c}_t - \hat{c}_{t+1}) + \hat{r}_{t+1}] && (19)\\
0 &= -\eta_c\hat{c}_t + \hat{y}_t - \hat{n}_t && (20)\\
R^{ss}\hat{r}_t &= \alpha\frac{Y^{ss}}{K^{ss}}(\hat{y}_t - \hat{k}_{t-1}) && (21)\\
\hat{z}_t &= \psi\hat{z}_{t-1} + \epsilon_t, \qquad \epsilon_t \sim i.i.d.(0,\sigma^2) && (22)
\end{aligned}
$$

[2] A small letter with *hat* represents the deviation from its steady state.

Calibrating a Specific Model Economy

This section follows the three-step process (Cooley and Prescott, 1995) from the general framework described in the previous section to quantitative measurements of the variables of interest — output, employment, investment, and so on.

- The first step is to restrict the model to display balanced growth. That is, in steady state, capital, consumption and investment all grow at a constant rate.
- The second step is to define the consistent measurements of the conceptual framework of the model economy and the real data.
- The parameter values of the model economy are then assigned according to the measured data during the sample period of 1970 to 2000 (RSA).

Calibrating a Specific Model Economy

- The annual aggregate capital depreciation rate δ is obtained from annual averaged values of $\frac{X}{Y}$ and $\frac{K}{Y}$. This yields an annual depreciation rate of 0.076, or a quarterly rate of 0.019.
- The standard real business cycle literature suggest that capital and labour shares of output have been approximately constant. The capital output share (α) is equal to 0.26 obtained from the steady state equation (11), whereas the labour output share ($1 - \alpha$) is 0.74.
- The discount factor β is set equal to 0.99, as in Hansen (1985), which implies an annual real interest rate of four percent in steady state.
- The coefficient of relative risk aversion η_c, is set equal to one. The parameter A, in the utility function (1), is equal to 2.6712, obtained from (14).

Calibrating a Specific Model Economy

The measurement of technology shock, also Known as Solow residual in growth accounting literature (Solow, 1957), is computed from Eq.(2) as follows:

$$\begin{aligned} logZ_t - logZ_{t-1} &= (logY_t - logY_{t-1}) - [\alpha(lnK_t - lnK_{t-1}) \\ &+ (1-\alpha)(logN_t - logN_{t-1})]. \end{aligned} \quad (23)$$

The parameter Z^{ss}, in the law of motion for TFP (3), is set equal to one. Therefore (3) becomes a first-order linear Markov process:[3]

$$logZ_t = \psi logZ_{t-1} + \epsilon_t, \qquad \epsilon_t \sim i.i.d.(0, \sigma^2) \quad (24)$$

The persistence parameter ψ is set equal to 0.95, which is consistent with the literature (Hansen, 1985). From (24) one can compute a set of innovations of technology ϵ_t. These innovations have a standard deviation of 0.0083.

[3] We can assume that quarterly variations in capital stock are approx. zero. We therefore use real GNP and quarterly hours series (see Cooley & Prescott, p.22.)

Calibrating a Specific Model Economy

As shown in the table below, all parameters of the model have now been assigned.

Parameters calibrated to the model economy

technology					*preferences*		
α	ψ	Z^{ss}	δ	σ_ϵ	β	η_c	A
0.26	0.95	1.00	0.019	0.0083	0.99	1.00	2.6712

The Dynamics of the Model

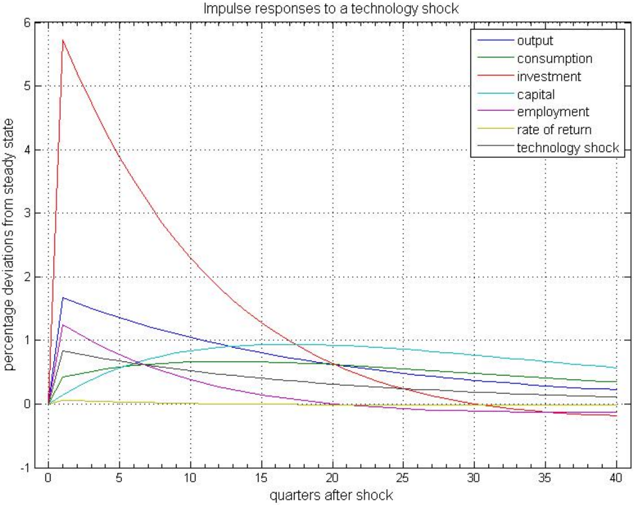

Table: Cyclical properties

Variable	standard deviation relative to output data	rbc	correlation with output data	rbc
Output (Y_t)	1.00	1.00	1.00	1.00
Consumption (C_t)	0.74	0.70	0.83	0.88
Investment (X_t)	3.10	2.51	0.90	0.90
Hours (N_t)	0.37	0.51	0.62	0.75
Productivity (Z_t)	-	0.50	-	1.00

Note: all variables, except for rates, are in log real terms and are detrended using the HP filter. Source: Cooley & Prescott, 1995: cherry-picked from Table 1.1 and 1.2.