Tutorial 3: Investment in a housing market

Jeanne Terblanche
(jeannebrink@sun.ac.za)

Set-up

Consider a continuous time model of investment in the housing market. Let H_t denote the stock of housing, I_t the investment in housing, p_t the price of a unit of housing and R_t the rent that can be earned on a unit of housing per instant.

Suppose that investment is a function of the price of housing (think of this as the supply curve of housing):

$$\begin{aligned} I_t &= I(p_t) \\ \text{with:} & \\ I'(p_t) &> 0 \end{aligned}$$

and that rent is a function of the stock of housing:

$$\begin{aligned} R_t &= R(H_t) \\ \text{with:} & \\ R'(H_t) &< 0 \end{aligned}$$

Set-up

The stock of housing depreciates at rate δ so that the evolution is given by:

$$\dot{H}_t = I\left(p_t\right) - \delta H_t$$

Lastly, assume that rental income plus capital gains must equal some exogenously fixed required rate of return r:

$$\frac{R\left(H_t\right) + \dot{p}_t}{p_t} = r$$

1.

Question 1

Question 1

1. Give an economic interpretation of the analytic assumptions made on the behaviour of investment and rent with respect to the variables they depend on.

Investment:

- Function of the price of housing.
- If price increases → higher return, thus investment increases, i.e. supply of housing increases as return increases.

$$I_t = I(p_t)$$

with:

$$I'(p_t) > 0$$

Rent:

- Function of stock of housing → evolution of housing depends on depreciation and investment.
- If housing stock increases, rent per unit of housing decreases, i.e. decreasing returns, the more houses supplied the lower the rent.

$$R_t = R(H_t)$$

with:

$$R'(H_t) < 0$$

2.

Question 2

Question 2

2. In H, p space (i.e. in a graph with the stock of housing, H_t, on the horizontal axis and the price of housing, p_t , on the vertical axis) provide a sketch of a phase diagram that describes the equilibrium of this model. It should contain the following:

a) the locus of points where $\dot{H}_t = 0$

b) the locus of points where $\dot{p}_t = 0$

c) the dynamics of H_t and p_t in each of the four quadrants defined by the two loci in (a) and (b), and

d) the saddle path that describes the equilibrium evolution to steady state.

Question 2

a) the locus of points where $\dot{H}_t = 0$

$$\dot{H}_t = I\left(p_t\right) - \delta H_t$$

$$\begin{aligned} \dot{H}_t &= 0 \\ &\Updownarrow \\ H_t &= \frac{I\left(p_t\right)}{\delta} \end{aligned}$$

Since $I'(p_t) > 0$, this locus is positively sloped. When $I(p_t) > \delta H_t$ (above the $\dot{H}_t = 0$ locus), $\dot{H}_t > 0$.

Question 2

b) the locus of points where $\dot{p}_t = 0$

$$\frac{R(H_t) + \dot{p}_t}{p_t} = r$$

$$\begin{aligned} \dot{p}_t &= 0 \\ &\Updownarrow \\ p_t &= \frac{R(H_t)}{r} \end{aligned}$$

Since $R'(H_t) < 0$, this locus is negatively sloped. When $rp_t > R(H_t)$ (to the right of the $\dot{p}_t = 0$ locus), $\dot{p}_t = rp_t - R(H_t) > 0$.

$$I'(p_t) > 0 \qquad R'(H_t) < 0$$

Question 2

Dynamics: same as in lecture

$\dot{H}_t = 0$ locus:

- Above locus: $p \uparrow \rightarrow$ $I \uparrow$ $\rightarrow$ $\dot{H}_t > 0 \rightarrow H \uparrow \rightarrow$ move right
- Below locus: $p \downarrow \rightarrow$ $I \downarrow$ $\rightarrow$ $\dot{H}_t < 0 \rightarrow H \downarrow \rightarrow$ move left

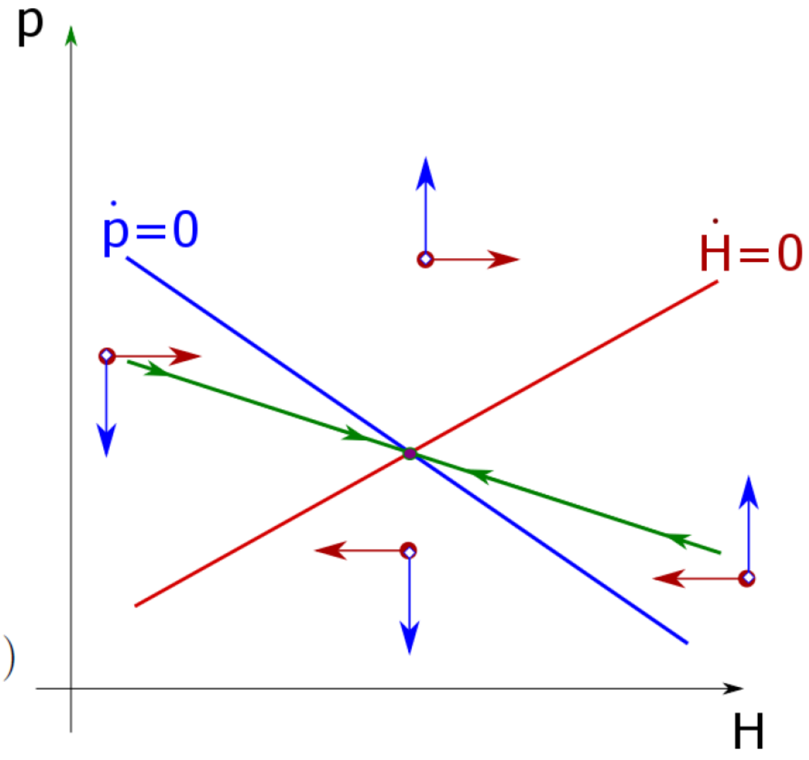
$$\dot{H}_t = I(p_t) - \delta H_t$$

$\dot{p}_t = 0$ locus:

- Right locus: $H \uparrow \rightarrow$ $R(H) \downarrow$ $\rightarrow$ $\dot{p}_t > 0 \rightarrow p \uparrow \rightarrow$ move up
- Left locus: $H \downarrow \rightarrow$ $R(H) \uparrow$ $\rightarrow$ $\dot{p}_t < 0 \rightarrow p \downarrow \rightarrow$ move down

$$\frac{R(H_t) + \dot{p}_t}{p_t} = r$$

$$\dot{p}_t = r\, p_t - R(H_t)$$



3.

Question 3

Question 3

3. Why is the $\dot{H}_t = 0$ locus not horizontal in this model? Provide an economic intuition.

In the previous model: **no depreciation**

- ∴ the locus was horizonal (constant over H)
- no depreciation → no investment was necessary to keep the capital stock constant, hence there was no adjustment cost to pay.

This model: **depreciation**

- ∴ a higher stock of housing requires a higher replacement investment to keep the stock constant, and a higher investment causes more capital adjustment costs, which requires a higher price to support.
- Hence, a higher housing stock requires a higher price to induce the level of investment that keeps the housing stock constant.

4.

Question 4

Question 4

4. Are the implied adjustment costs in this model internal or external? Explain. [Hint: $I(p_t)$ can be interpreted as a supply function. Think what the inverse supply function says about the market for houses].

The model is in very reduced form → question is just to think about the issue.

The original model focused on **external adjustment costs** - the investment depends on the price of housing.

- The price of investment is higher when investment is larger.
- ∴ the required price on capital must be higher to support faster investment, or equivalently that the cost of investment is increasing in the size of the investment.
- For example: there are some builders in the economy, to make them build houses faster, you have to pay them more, thus the costs are external to the person buying the house to rent it out.

5.

Question 5

Question 5(a)

5. Suppose the market is initially in long run equilibrium. Then, at moment t_1 it becomes known that the depreciation rate is higher than before $\delta' > \delta$, but that this higher depreciation rate will only last until some future moment t_2, after which it will return to its original level.

a) Provide a full analysis of the impact of this change on the time paths of H_t and p_t. I.e. determine which loci shift, and how the equilibrium evolves from one steady state to the next. Your answer should also include figures showing the time paths of each of the variables in question.

Change is announced at t_1 and is reversed at t_2

→ Unanticipated, temporary change (loci will shift and return to their original position)

Question 5(a)

Change is announced at t_1 and is reversed at t_2

→ Unanticipated, temporary change

Which loci are affected?

- Depreciation rate is higher between t_1 and t_2 ($\delta' > \delta$)
- $\dot{p}_t = 0$ locus unaffected
- $\dot{H}_t = 0$ locus affected
  - Higher δ → smaller RHS and therefore smaller LHS, i.e. H_t reduces.
  - Intuitively: a lower H_t for every p_t can be maintained efficiently, since the depreciation is higher.

$$\dot{p}_t = 0$$
$$\Updownarrow$$
$$p_t = \frac{R(H_t)}{r}$$

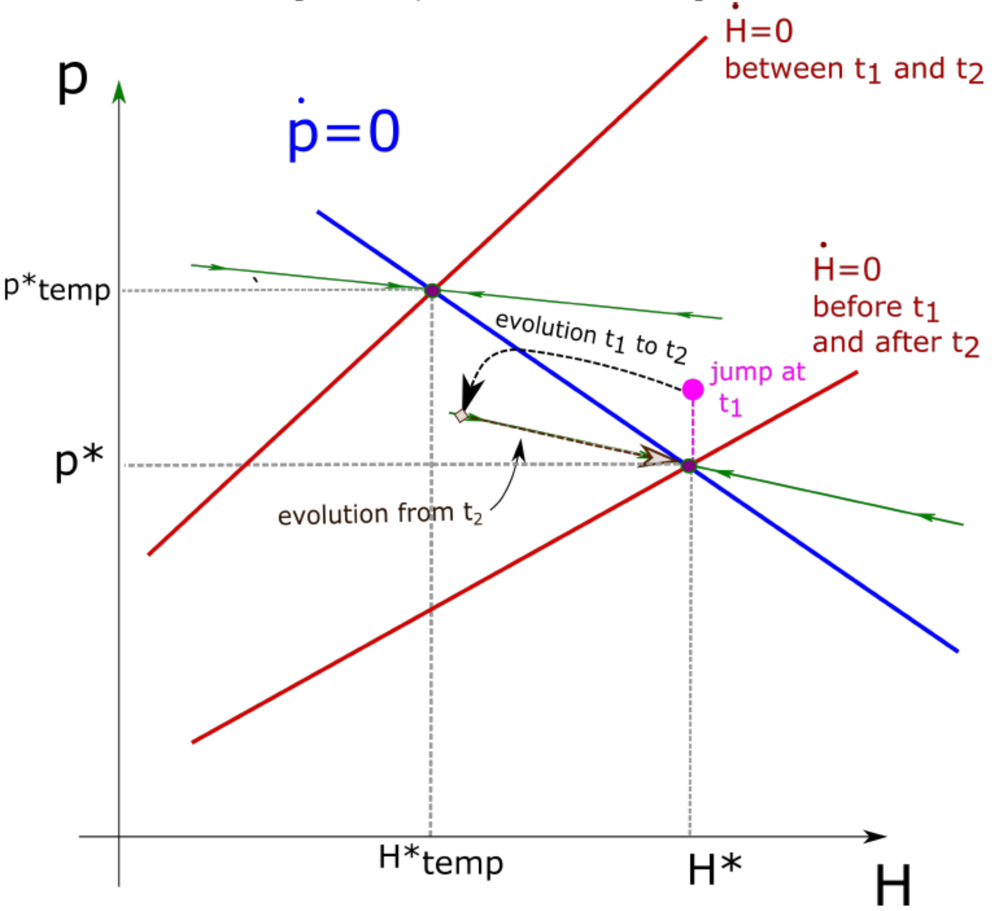
$$\dot{H}_t = 0$$
$$\Updownarrow$$
$$H_t = \frac{I(p_t)}{\delta}$$

Question 5(a)

At t_1:

- The $\dot{H}_t = 0$ locus shifts left. The $\dot{p}_t = 0$ locus is unaffected.
- This system will only stay in place until t_2 after which the system returns to its original form.
- From t_1 to t_2, there is a temporary steady state with a lower housing stock at a higher price, with associated saddle path.

Figure 2: Dynamics in Phase Diagram



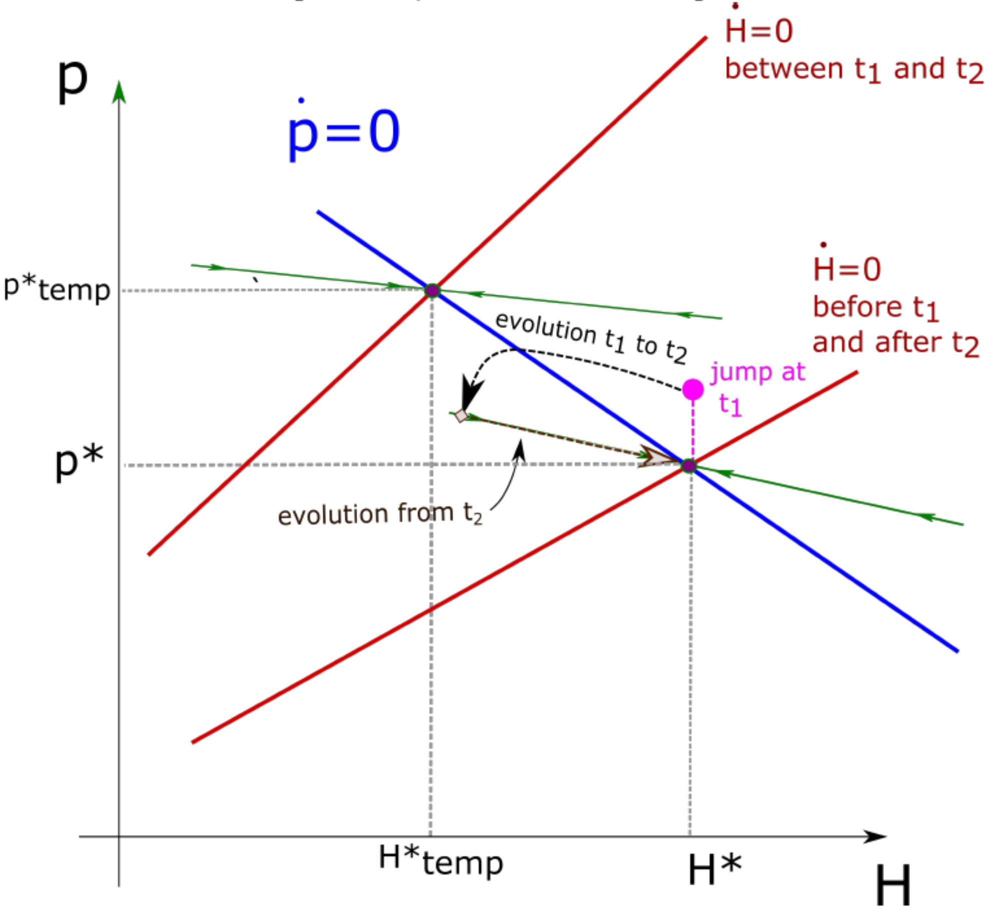
Question 5(a)

How does the economy respond?

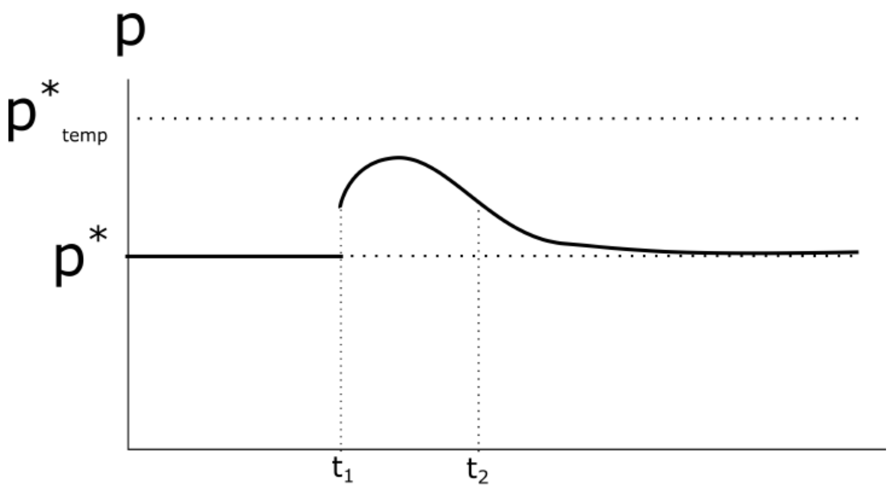
- At t_1, the price jumps up, but not all the way, to the temporary saddle path.
- The dynamics between t_1 and t_2 drive the price to rise and the housing stock to shrink until the $\dot{p}_t = 0$ locus is reached, whereafter the price begins to fall and the housing stock shrinks further.

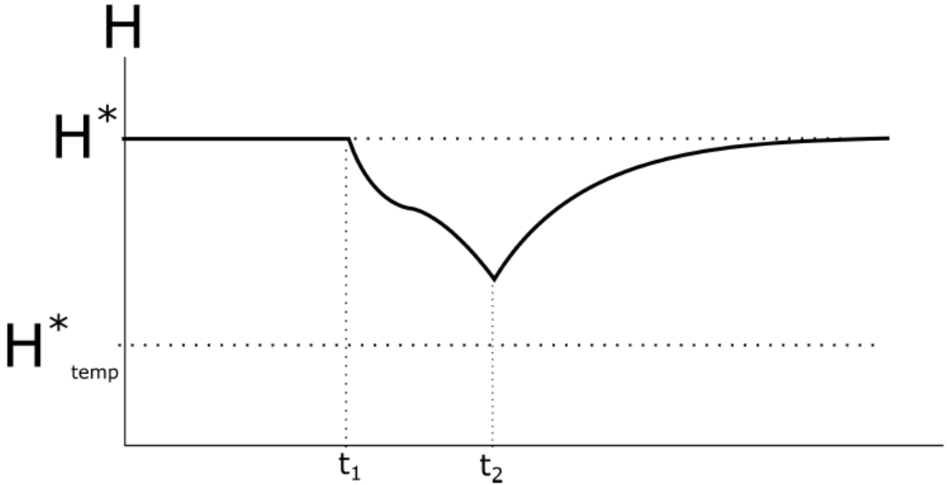
At t_2, the system is on the old saddle path that now again governs dynamics, and the economy converges to the original steady state.

Figure 2: Dynamics in Phase Diagram



Question 5(a)





Question 5(b)

b) Provide an economic story for the change in the depreciation rate. That is: what real world event would lead to this type of dynamics?

Many possible answers, e.g. a period of severe weather that damages houses faster than normal for a while.

Thank you!