



# Data revisions and DSGE models

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## ABSTRACT

The typical estimation of DSGE models requires data on a set of macroeconomic aggregates, such as output, consumption and investment, which are subject to data revisions. The conventional approach employs the time series that is currently available for these aggregates for estimation, implying that the last observations are still subject to many rounds of revisions. This paper proposes a release-based approach that uses revised data of all observations to estimate DSGE models, but the model is still helpful for real-time forecasting. This new approach accounts for data uncertainty when predicting future values of macroeconomic variables subject to revisions, thus providing policy-makers and professional forecasters with both backcasts and forecasts. Application of this new approach to a medium-sized DSGE model improves the accuracy of density forecasts, particularly the coverage of predictive intervals, of US real macro variables. The application also shows that the estimated relative importance of business cycle sources varies with data maturity.

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## 1. Introduction

The typical estimation of Dynamic Stochastic General Equilibrium (DSGE) models requires data on a set of macroeconomic aggregates, such as output, consumption and investment, which are subject to data revisions. The conventional approach uses the time series currently available for these aggregates to estimate the parameters of the model. This implies that the last observations in the time series are earlier estimates and subject to many rounds of revisions. The conventional approach has been employed to evaluate the accuracy of DSGE forecasts in real time by [Edge and Gurkaynak \(2011\)](#), [Wouters \(2012\)](#) and [Del Negro and Schorftheide \(2013\)](#). [Del Negro and Schorftheide \(2013\)](#) show that long-horizon output growth and inflation forecasts from medium-scale DSGE models are more accurate than Federal Reserve Greenbook forecasts and professional forecasters.

Though one could estimate the DSGE model using only heavily revised data by shortening the sample size to remove earlier estimates, this alternative implies that we cannot use the model as a forecasting device because the lack of information on recent observations is very damaging for the forecasting accuracy of future values. This paper proposes a release-based approach that allows the DSGE model parameters to be estimated by using revised data while retaining the usefulness of the model for real-time forecasting. The approach jointly estimates the model

parameters and the data revision processes by employing a specially designed Metropolis-in-Gibbs algorithm.

To model the data revision processes, I assume that we observe both initial releases and revised values of the macroeconomic time series of interest. My proposed method requires the augmentation of the measurement equation in a way that differs from others that are used in the literature. Data augmentation normally implies that we observe some of the endogenous variables in the model with error ([Boivin and Giannoni, 2006](#)). [Croushore and Sill \(2014\)](#) exploit the data augmentation method of [Schorftheide et al. \(2010\)](#) to measure how initial releases are explained by shocks to the revised data, assuming that the DSGE model is estimated only with the revised data. Their approach requires shortening the dataset to estimate the model only with heavily revised data in the first step, and therefore, it is not adequate for real-time forecasting. [Smets et al. \(2014\)](#) assume that the expected values of some variables are observed with measurement errors by augmenting the dataset with survey forecasts. In contrast, the release-based approach assumes that the observed endogenous variables fit the final revised values for  $t = 1, \dots, T$ . The dataset employed in the DSGE estimation includes the time series of the initial releases such that the modelling approach is able to deliver final estimates for the observations still subject to revision  $t = T - q + 2, \dots, T$ . The release-based approach assumes that the statistical agency publishes data revisions either because information on the complete effect of structural shocks was not available at the time of the initial releases or due to reduction of earlier measurement errors.

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Alternative modelling approaches modify consumers' and firms' decisions to account for real-time data availability (Coenen et al., 2005; Casares and Vazquez, forthcoming). Because, at each point in time, the last observation available is typically an initial release, these approaches match model-observed variables to initial releases, and they allow for unanticipated data revisions to have an effect on business cycle fluctuations. In this paper, I assume that agents' decisions are based on revised data, including data revision predictions for the set of observations still subject to revisions.

When forecasting economic activity, policy makers may also examine the uncertainty around recent values of output growth. The Bank of England density forecasts for UK output growth, which are published quarterly in the Inflation Report, include probabilistic assessments of past values of GDP growth still subject to revisions, that is, they include backcasts in addition to forecasts.<sup>1</sup> The release-based approach allows us to use a single model to compute predictive densities for both forecasts and backcasts. Clements and Galvão (2013a) compare backcasts and forecasts of reduced-form modelling approaches, but this is the first paper to provide similar policy-relevant predictive information based on a DSGE model.

This new approach for estimating DSGE models accounts for data uncertainty when predicting future values of macroeconomic variables subject to revisions. Empirical results with a DSGE applied to US data suggest that the new approach reduces the gap between nominal and empirical coverage rates of predictive intervals of consumption and investment growth. In general, if our macroeconomic forecasting targets are values observed after many rounds of revisions, then DSGE models estimated with the release-based approach may provide us with well-calibrated and more accurate predictive densities than the usual option of disregarding data revisions.

The release-based approach for forecasting with DSGE models provides new evidence on the nature of US data revisions and how they affect the measurement of sources of business cycle fluctuations. Earlier data revisions may be explained by the lack of complete information on the effect of structural shocks at the time of the previous release. Investment-specific shocks are a relevant source of data revisions at this early stage. Later data revisions are explained by reduction of the measurement error embedded in previous releases. As a consequence, the relative importance of a specific business cycle source may vary with the data maturity, that is, the number of quarters that a first release is available. By applying the release-based approach to the Smets and Wouters (2007) model, I find that productivity shocks explain 51% of the variance of output growth observed seven quarters after the first release, but this proportion is only 38% if computed for the first release of output growth. Future data revision shocks explain a sizeable proportion (20%–40%) of the unexpected changes in first-release estimates of real variables (output, consumption and investment), but by definition, they have no impact on fully revised data.

The approach developed in this paper can be applied to any DSGE model that can be estimated based on a linear state-space representation. Section 2 describes the new approach in contrast with the conventional approach, including a detailed description of the estimation, backcasting and forecasting methods. The release-based approach is applied to the medium-scale Smets and Wouters (2007) model, and Section 3 describes the details of this application, including descriptive statistics of data revisions of output

growth, inflation, and consumption and investment growth. Section 4 discusses full sample results including posterior estimates for alternative specifications and variance decompositions. Section 5 describes the design and the results of a real-time out-of-sample forecasting exercise, including backcasting and forecasting evaluations, and an assessment of real-time output gaps.

## 2. Forecasting with DSGE models

This section describes the conventional approach for using DSGE models for real-time forecasting, as employed by Edge and Gurkaynak (2011), Wouters (2012), Herbst and Schorftheide (2012) and Del Negro and Schorftheide (2013). Then, I demonstrate how to apply the release-based approach for real-time forecasting of DSGE models.

### 2.1. The conventional approach

Before estimation, some endogenous variables in the DSGE model are detrended based on common deterministic (Smets and Wouters, 2007) or stochastic (Del Negro and Schorftheide, 2013) trends. Then, the model is log-linearised around the deterministic steady state. Based on the log-linearised version, numerical methods are employed to solve the rational expectations model (see, e.g., Guerron-Quintana and Nason (2012) for a description of the usual techniques).

Define  $x_t$  as an  $n \times 1$  vector of the endogenous DSGE variables written as a deviation of the steady state. In practice,  $x_t$  may also include lagged variables. Define  $\theta$  as the vector of structural parameters. The solution of the DSGE model for a given vector of parameters  $\theta$  is written as

$$x_t = F(\theta)x_{t-1} + G(\theta)\eta_t \quad (1)$$

where  $\eta_t$  is a  $r \times 1$  vector of structural shocks, and thus, the matrix  $G(\theta)$  is  $n \times r$ . Note also that  $\eta_t \sim iidN(0, Q)$  and that  $Q$  is a diagonal matrix. The Eq. (1) is the state equation of the state space representation of the DSGE model.

Define  $X_t$  as the  $m \times 1$  vector of observable variables. Typically,  $m < n$  and  $m \leq r$ . Smets and Wouters (2007) medium-sized model has  $m = r = 7$ . The measurement equation is:

$$X_t = d(\theta) + H(\theta)x_t, \quad (2)$$

that is, the observable variables, such as inflation and output growth, are measured without error.

Edge and Gurkaynak (2011), Wouters (2012) and Del Negro and Schorftheide (2013) evaluate the accuracy of DSGE forecasts in real time. This means that they use only the data available at each forecast origin for estimating the vector of parameters  $\theta$ . Observables such as the output, inflation, consumption and investment are computed using national accounting data. US and UK quarterly national accounting data are initially published with a one-month delay with respect to the observational quarter. If the model is estimated at  $T + 1$ , we only have data available up to  $T$  for estimation. The measurement equation for real-time estimation is:

$$X_t^{T+1} = d(\theta) + H(\theta)x_t$$

for  $t = 1, \dots, T$ , where  $T$  is the number of observations in the initial in-sample period. Suppose that the number of quarters in the out-of-sample period is  $P$ ; a conventional real-time forecasting exercise re-estimates the model at each forecast origin from  $T + 1$  up to  $T + P$ , and the forecasts are computed using data up to  $T, \dots, T + P - 1$  at each origin.

An issue with this approach is that the model is estimated by mixing heavily revised data ( $t = 1, \dots, T - 14$ ), data subject to annual revisions ( $t = T - 13, \dots, T - 1$ ), and data subject

<sup>1</sup> For historical fan chart information, see <http://www.bankofengland.co.uk/publications/Pages/inflationreport/irprobab.aspx>.

to the initial round of revisions and annual revisions ( $t = T$ ),<sup>2</sup> while the forecasts are computed conditioned on lightly revised data ( $t = T$ ). Koenig et al. (2003) and Clements and Galvão (2013b) establish how to improve forecasts by addressing this problem in the context of distributed lag regressions and autoregressive models, respectively. Clements and Galvão (2013b) demonstrate that the conventional use of real-time data delivers estimates of autoregressive coefficients that do not converge to values that would deliver optimal forecasts in real time.

## 2.2. The release-based approach

As before, we have observations up to  $T$  from the  $T + 1$  data vintage. We could estimate the DSGE model using only the revised data by removing the last  $q$  observations of the time series that are currently available:

$$X_{t-q+1}^{T+1} = d(\theta) + H(\theta)x_{t-q+1} \text{ for } t = 1, \dots, T. \quad (3)$$

This approach is applied by Casares and Vazquez (forthcoming) and Croushore and Sill (2014). The disadvantage of this approach is that if we want to predict  $X_{T+1}, \dots, X_{T+h}$ , the fact that the observations  $X_{T-q+2}^{T+1}, \dots, X_T^{T+1}$  are not included may imply that the forecasts will be inaccurate.

The release-based approach, based on Kishor and Koenig (2012), requires the assumption that after  $q - 1$  rounds of revisions, we observe an efficient estimate of the true value. Assume that the true value  $X_t$  is observed after  $q - 1$  rounds of revisions at  $X_t^{t+q}$  for  $t = 1, \dots, T$ , with both the subscripts and superscripts varying with  $t$ . If the DSGE is estimated using the values observed after  $q - 1$  rounds of revisions, the measurement equations are:

$$X_{t-q+1}^{t+1} = d(\theta) + H(\theta)x_{t-q+1} \text{ for } t = 1, \dots, T, \quad (4)$$

and as before, the last  $q - 1$  observations have to be excluded. Note however that the number of rounds of revisions at each time period is exactly  $q - 1$  in (4) but it varies with  $t$  in (3), implying that the release-based approach recognises differences in data maturity by not mixing apples with oranges as in Kishor and Koenig (2012).

The demeaned observed revisions between first releases  $X_t^{t+1}$  and true values  $X_t^{t+q}$  are

$$rev_t^{t+q,1} = (X_t^{t+1} - X_t^{t+q}) - M_1 \text{ for } t = 1, \dots, T - q + 1.$$

This implies that we observe  $T - q + 1$  values of the full revision process to a first release at  $T + 1$ , and that the full revision process for observation  $t$  is only observed at  $t + q$  because of the statistical agency data release schedule. In general, for the  $v$ th release, the (demeaned) remaining revisions up to the true values are:

$$rev_t^{t+q+1-v,v} = (X_t^{t+v} - X_t^{t+q}) - M_v \text{ for } t = 1, \dots, T - q + v$$

and  $v = 1, \dots, q - 1$ .

At  $T + 1$  we do not observe fully revised values,  $X_t^{t+q}$ , of the observations  $T - q + 2, \dots, T$ , but we do observe earlier estimates of these observations. The release-based approach proposed in this paper employs these earlier estimates to estimate the revised value of the last  $q - 1$  observations. The approach incorporates modelling of data revisions to the DSGE estimation by assuming that both the true values of the observables  $X_t$  and the  $q - 1$  revision processes  $rev_t^1, \dots, rev_t^{q-1}$  are unobserved at  $t$ . These assumptions imply the use of filtering procedures to obtain values for  $X_t$  and  $rev_t^1, \dots, rev_t^{q-1}$  for  $t = 1, \dots, T$  when estimating the model

cast in state space.<sup>3</sup> The fact that revisions, albeit unobserved at  $t$ , are observed at  $t + q + 1 - v$  is incorporated in the smoother employed to obtain full sample estimates of the true values  $X_t$ . The release-based approach is in particularly advantageous when forecasting with DSGE models because we can use all observations available while assuming that the DSGE is estimated with only heavily revised data.

The approach augments the measurement equations (4) to include a time series of first releases, second releases, and so on, as:

$$\begin{bmatrix} X_t^{t+1} \\ X_{t-1}^{t+1} \\ \vdots \\ X_{t-q+1}^{t+1} \end{bmatrix} = \begin{bmatrix} d(\theta) + M_1 \\ d(\theta) + M_2 \\ \vdots \\ d(\theta) \end{bmatrix} + \begin{bmatrix} H(\theta) & 0_m & \cdots & 0_m & I_m & 0_m & \cdots & 0_m \\ 0_m & H(\theta) & \cdots & 0_m & 0_m & I_m & \cdots & 0_m \\ & & \ddots & & & & \ddots & \\ 0_m & 0_m & & H(\theta) & 0_m & 0_m & & 0_m \end{bmatrix} \times \begin{bmatrix} X_t \\ X_{t-1} \\ \vdots \\ X_{t-q+1} \\ rev_t^1 \\ rev_{t-1}^2 \\ \vdots \\ rev_{t-q+2}^{q-1} \end{bmatrix} \quad (5)$$

for  $t = 1, \dots, T$  and:

$$rev_t^v = (X_t^{t+v} - X_t) - M_v \text{ for } v = 1, \dots, q - 1,$$

where the  $m \times 1$  vectors  $M_v$  allow for non-zero data revisions.

Data revisions may add new information and/or reduce measurement errors, following the definitions by Mankiw and Shapiro (1986) employed by Jacobs and van Norden (2011). I consider both noise and news revisions. Noise revisions follow the classical measurement error in Sargent (1989): they are orthogonal to the true values. News revisions are correlated with the true values, and they may be caused by the statistical agency filtering the available data (Sargent, 1989). If the statistical agency filters the data before releasing it, innovations to the data revisions may be correlated with structural shocks, as in Sargent (1989). Finally, I also allow for serial correlation in the revisions as in Kishor and Koenig (2012), following the Howrey (1978) model. The data revision processes are:

$$rev_t^v = K_v rev_{t-1}^v + \xi_t^v + A_v \eta_t, \quad \xi_t^v \sim N(0, R_v) \quad (6)$$

where the serial correlation allows for noise-predictable revisions if the  $m \times m$  matrix  $K_v$  is nonzero. The own innovation term  $\xi_t^v$  allows for data revisions that are caused by a reduction of measurement errors, and we assume that the innovations are not correlated across variables, so  $R_v$  is diagonal. The last term  $A_v \eta_t$  implies that the data revisions may be caused by new information not available at the time of the current release, but included in the revised data used to compute the complete effects of the structural shocks. We can identify both types of innovations –

<sup>2</sup> This assumes three round of annual revisions published every July, as is usually the case for US National Accounts Data published by the Bureau of Economic Analysis.

<sup>3</sup> Note that  $rev_t^{t+q+1-v,v}$  is the observed revision between the  $v$ th and the  $q$ th release available up to  $t = T - q + v$ , while  $rev_t^v$  is the state variable matching the same concept.  $rev_t^v$  is unobserved at  $t$  and by using filtering procedures available up to  $t = T$ .

measurement errors and news revisions – because we assume that we eventually observe the revised or true value of each observation in the time series.<sup>4</sup> The structural innovations  $\eta_t$  drive the business cycle fluctuations in the DSGE endogenous variables as a common component because the number of shocks  $\eta_t$  is smaller than the number of endogenous variables  $x_t$ . The Eq. (6) suggests that these innovations may also drive a common component in the data revision processes of observables,  $rev_t^{(v)}$ , depending on the values in  $A_v$ .

The release-based approach implies that we need to enlarge the state vector to include revision processes. The  $n + 2(q - 1)m$  vector of state variables is

$$\alpha_t = [x'_t, \dots, x'_{t-q+1}, rev_t^{1'}, \dots, rev_{t-q+2}^{q-1'}]'$$

instead of  $\alpha_t = x_t$  in the conventional approach used in Section 2.1. The new  $mq$  vector of observables is written as

$$y_t^{t+1} = [X_t^{t+1'}, \dots, X_{t-q+1}^{t+1'}]'$$

The vector of parameters governing the data revision process is

$$\beta = [M'_1, \dots, M'_v, vec(K_1)', \dots, vec(K_v)', a_{1,1}, \dots, a_{1,m}, \dots, a_{v,1}, \dots, a_{v,m}]'$$

where  $a_{v,i}$  is the row  $i$  of matrix  $A_{(v)}$  where  $i = 1, \dots, m$ . Using the above defined vectors, the measurement equations in (5) may be written as:

$$y_t^{t+1} = D(\theta, \beta) + L(\theta, \beta)\alpha_t. \quad (7)$$

Therefore, the measurement equations do not include measurement errors to be able to incorporate the fact that we observe each data revision process at  $t + q + 1 - v$ . This approach differs from the data augmentation of Boivin and Giannoni (2006). They assume that some of the endogenous variables in the DSGE model are measured with errors using a set of observable variables. Smets et al. (2014) include measurement errors to employ survey forecasts as an approximate measure of expectations.

Measurement errors have been also employed by Ruge-Murcia (2007) and Ferroni et al. (2015) to solve stochastic singularity problems caused by there being fewer structural shocks than the number of observables in the estimation of DSGE models. Ireland (2004) suggests the inclusion of measurement errors for cases in which the DSGE is too prototypical a model to fit the observed data. In these last two cases, the measurement equations for the conventional approach are:

$$X_t = d(\theta) + H(\theta)x_t + u_t, \quad (8)$$

where the measurement errors in the  $m \times 1$  vector  $u_t$  could be serially correlated (Ireland, 2004). The release-based approach measurement equations (7) would then include a vector of measurement errors  $u_t$  in the equations for the revised data  $X_{t-q+1}^{t+1}$  as:

$$y_t^{t+1} = D(\theta, \beta) + L(\theta, \beta)\alpha_t + Bu_{t-q+1}, \quad (9)$$

where  $B = [0'_{m \times m(q-1)} I_m]'$ .<sup>5</sup> The release-based approach is able to identify both DSGE serially correlated measurement errors  $u_t$

and revisions  $rev_t^{(v)}$  due to the assumption that final values are observed at  $t + q$ , and that the set of observables is enlarged accordingly.

In the release-based approach, the  $r + (q - 1)m$  vector of state disturbances is

$$\varepsilon_t = [\eta_t', \xi_t^{1'}, \dots, \xi_{t-q+2}^{q-1'}]'$$

and therefore, the state equations are

$$\alpha_t = T(\theta, \beta)\alpha_{t-1} + R(\theta, \beta)\varepsilon_t \text{ where } \varepsilon_t \sim N(0, P), \quad (10)$$

where  $P$  is a diagonal matrix containing the variances of the structural shock innovations and of the data revision innovations along the diagonal. If the DSGE model has the same number of observables as shocks, this state-space representation implies that  $qm$  observables are driven by  $qm$  innovations. The required state matrices are

$$T(\theta, \beta) = \begin{bmatrix} F(\theta) & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ \vdots & \ddots & & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & K_{(1)} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & K_{(2)} & & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & & K_{(q-1)} \end{bmatrix}$$

and

$$R(\theta, \beta) = \begin{bmatrix} G(\theta) & 0 & 0 & \dots & 0 \\ A_{(1)} & 1 & 0 & \dots & 0 \\ A_{(2)} & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{(q-1)} & 0 & 0 & \dots & 1 \end{bmatrix}.$$

The release-based approach implies that the smoothed estimate of the true values  $X_{t|T} = E[X_t | \{y_t^{t+1}\}_{t=1}^T, \theta, \beta]$  is equal to the observed  $X_t^{t+q}$  for  $t = 1, \dots, T - q + 1$ . Note, however, that the predicted estimates  $X_{t|t-1} = E[X_t | \{y_t^{t+1}\}_{t=1}^{t-1}, \theta, \beta]$  and filtered estimates  $X_{t|t} = E[X_t | \{y_t^{t+1}\}_{t=1}^t, \theta, \beta]$  will not be equal to  $X_t^{t+q}$  for all  $t$  because filtered values are computed using  $y_t^{t+1}$  that does not include  $X_t^{t+q}$ . True values are observed at  $t + q$  for observations up to  $t = T - q + 1$  so they are incorporated when computing smoothed estimates. To compute the likelihood of an unobserved component model, we use filtered estimates  $X_{t|t}$ . This implies that if we compute the likelihood as function of  $\theta$  and  $\beta$  for the state space model in (7) and (10), the full time series of  $X_t$  is treated as an unobserved time series. If, however, our aim is measure  $X_t$  for  $t = 1, \dots, T$ , then the posterior distribution of the smoothed estimates  $p(X_t | \{y_t^{t+1}\}_{t=1}^T, \theta, \beta)$  provides the best estimate because it uses the full sample information, which implies that we are only uncertain about the last  $q - 1$  observations.

### 2.2.1. Estimation

I exploit two Bayesian Markov Chain Monte Carlo methodologies to obtain posterior distributions for the DSGE parameters  $\theta$  jointly with the parameters of the data revision processes  $\beta$ . The first method uses the Random-Walk Metropolis–Hasting (RWMH) algorithm described in Del Negro and Schorftheide (2011) for the state-space representation described by (7) and (10). As part of the RWMH approach, the prior distributions for both sets of parameters  $p(\theta, \beta)$  need to be defined, and the likelihood  $p(Y | \theta, \beta)$ , where  $Y = \{y_t^{t+1}\}_{t=1}^T$ , needs to be evaluated. The RWMH algorithm requires the numerical optimisation of the posterior kernel to obtain the variance–covariance matrix of the parameters at the posterior

<sup>4</sup> Jacobs and van Norden (2011) decompose the observed values into the true value and noise and news revisions. Their approach, similar to mine, assumes that the data revisions are a combination of both news and noise processes. However, because they assume the true value is not observed, the last revision process (say  $rev_t^{q-1}$  in our notation) can be either news or noise, but not both. The modelling choice that the true values are eventually observed solves this identification problem. A similar approach was applied by Cunningham et al. (2012), Kishor and Koenig (2012) and Croushore and Sill (2014).

<sup>5</sup> In the case of serially correlated measurement errors as in Ireland (2004), the state equations (Eq. (10)) need to be augmented to include the new set of unobserved disturbances. Note that depending on the DSGE specification, the modeller may choose fewer measurement errors than  $m$ .



mode. Because the release-based approach involves the estimation of at least  $r + 2$  parameters for each revision of order  $v$  and for each variable  $m$  in addition to the DSGE parameters (around 40 in the case of a medium-sized model), this step is highly computationally intensive and may fail in some circumstances. I present the results of the application of this approach in Sections 4 and 5 for the case that  $q = 2$  and the data revision process of only two variables are modelled.

The second method exploits the fact that conditional on a time series of true values, that is,  $X = \{X_t\}_{t=1}^T$ , we can compute posterior distributions for  $\theta$  by applying the RWMH to the state-space representation defined by (1) and (2). If we can draw  $p(\theta|\beta, X)$  and  $p(\theta|\beta, Y)$  from the conditional distributions, we can use Gibbs sampling to obtain an approximation of the joint posterior distribution of the parameters. This is also a computationally intensive MCMC algorithm, but it is less likely to fail with large  $q$  and  $m$  because the Metropolis step and the numerical optimisation of the posterior kernel are only applied to the subset  $\theta$  of the parameter space. In addition, the proposed algorithm delivers clear measures of the underlying data uncertainty on the last  $q - 1$  observations of  $X_t$ , which are drawn from a conditional distribution within the Gibbs algorithm. The algorithm is described in detail in Appendix A, which also includes a convergence analysis in comparison with the RWMH algorithm. Previously, Gibbs sampling was applied to DSGE estimation where the variance of the shocks was allowed to change over time (Justiniano and Primiceri, 2008).

The first step employs a Metropolis RW draw to obtain the conditional draw  $\theta^{(j)}|\beta^{(j-1)}, X^{(j-1)}$ . The second step draws a time series of structural shocks  $\eta_{(1)}^{(j)}, \dots, \eta_{(T)}^{(j)}$  by employing a smoothing algorithm to the state-space model defined by (1) and (2) conditional on  $\theta^{(j)}$  and  $X^{(j-1)}$ . The third step employs an independent normal-inverse gamma prior approach for the parameters in  $\beta$  such that we can use closed-form solutions to obtain the conditional draw  $\beta^{(j)}|\theta^{(j)}, X^{(j-1)}, \eta_{(1)}^{(j)}, \dots, \eta_{(T)}^{(j)}, Y$ , including draws for the variances of the data revision innovations as well. The fourth step applies a smoothing algorithm to the state space defined by (7) and (10) such that conditional on  $\beta^{(j)}, \theta^{(j)}$  and  $Y$ , we are able to obtain draws for  $X_{T-q+2}^{(j)}, \dots, X_T^{(j)}$ .

### 2.2.2. Backcasting

The algorithm provides us with a direct measure of the data uncertainty because based on a set of kept draws of  $X_{T-q+2}^{(j)}, \dots, X_T^{(j)}$  for  $j = 1, \dots, S$ , we can compute moments of the posterior distribution. For example, the posterior mean for the last observation is computed as  $\hat{X}_T = 1/S \sum_{j=1}^S X_T^{(j)}$ , which is an estimate of the fully revised values of the last observations of the observed time series. The standard deviation of the posterior distribution can be used as a measure of the data uncertainty embedded in the DSGE estimation. If, for example, we compute this value only for the last observation, we use  $std(X_T) = \sqrt{1/S \sum_{j=1}^S (X_T^{(j)} - \hat{X}_T)^2}$ . The posterior draws can also be used to compute the data uncertainty quantiles. We can call the computation of predictions for  $X_{T-q+2}^{(j)}, \dots, X_T^{(j)}$  backcasting because it predicts time periods for which initial releases are already available.

### 2.2.3. Forecasting

In addition to the backcasting computation described above, the release-based approach for the estimation of the DSGE model also provides us with forecasts of future revised values of the observables, that is,  $X_{T+1}, \dots, X_{T+h}$ , where  $h$  is the maximum forecast horizon in the quarters. To compute  $J$  draws from the predictive density of the state vector, we employ  $J$  equally spaced

draws of the saved posterior distributions of  $\theta$  and  $\beta$  in the state equation as:

$$\alpha_{T+h|T}^{(j)} = T(\theta^{(j)}, \beta^{(j)})\alpha_{T+h-1|T}^{(j)} + R(\theta^{(j)}, \beta^{(j)})\varepsilon_t \text{ where } \varepsilon_t \sim N(0, P^{(j)}).$$

Note that the draws  $\theta^{(j)}, \beta^{(j)}$  are also used to compute  $\alpha_T^{(j)}$  to condition forecasts on, which includes  $X_T^{(j)}$ . This procedure delivers the sequence of forecasts  $x_{T+1|T}^{(j)}, \dots, x_{T+h|T}^{(j)}$ . Based on the DSGE parameters draw  $\theta^{(j)}$  and a sequence of forecasts for the DSGE state variables, forecasts of future revised values of the observable variables are computed using:

$$X_{T+h|T}^{(j)} = d(\theta^{(j)}) + H(\theta^{(j)})x_{T+h|T}^{(j)}. \quad (11)$$

I evaluate forecasts computing the predictive density  $p(X_{T+h}|Y)$  as the empirical density of  $X_{T+h|T}^{(j)}$  for  $j = 1, \dots, J$ . Point forecasts are the mean of the predictive density, that is,  $\hat{X}_{T+h|T} = 1/J \sum_{j=1}^J X_{T+h|T}^{(j)}$ .

## 3. Application of the release-based approach to the Smets and Wouters model

In this section, I explain how I apply the release-based approach proposed in this paper to the Smets and Wouters (2007) (SW) model. The forecasting performance of the SW model in real time has been evaluated by Edge and Gurkaynak (2011), Herbst and Schorftheide (2012) and Del Negro and Schorftheide (2013). The equations of the log-linearised version of the SW model are described in Appendix B under the assumption of a common deterministic trend for output, consumption, investment and real wages. Similar to the approach of Smets and Wouters (2007), some of the parameters are calibrated such that the number of parameters to be estimated using the conventional approach is 36. The observation/measurement equations are:

$$\begin{bmatrix} \Delta \log(GDP_t) \\ \Delta \log(Cons_t) \\ \Delta \log(Inv_t) \\ \Delta \log(Wage_t) \\ \Delta \log(Hours_t) \\ \Delta \log(P_t) \\ FFR_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{c} \\ \bar{i} \\ \bar{w} \\ \bar{l} \\ \bar{\pi} \\ \bar{r} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ l_t \\ \pi_t \\ r_t \end{bmatrix}. \quad (12)$$

All of the observable variables are subject to revisions, except the fed fund rate  $FFR_t$  and the total population above 16 used to compute  $GDP_t$ ,  $Cons_t$ ,  $Inv_t$  and  $Hours_t$ . The GDP deflator is used to compute  $P_t$  and also to deflate the nominal observed values of  $Cons_t$ ,  $Inv_t$  and  $Wage_t$ . Table 1 provides details on how each of these variables are computed using the available observable data and the availability of quarterly vintages.

In this paper, I model data revisions of output growth, inflation, consumption and investment growth, that is, the four observables computed using national accounting data. The remaining observables are treated as the conventional approach for real-time forecasting. Because real-time data on compensation are available only for a limited period, I am not able to model revisions of real wages and prefer not to model revisions in hours.

<sup>6</sup> I am implicitly assuming that the adequate measurement equations are Eq. (7). If using instead Eq. (9), then draws from the measurement error processes  $u_t$  are also required.

**Table 1**  
Data sources.

Name	Computed with	Data/Source
$GDP_t$	Real GDP; Population above 16.	vintages from 1965Q4, Philadelphia Fed; CNP16OV, FRED/St Louis.
$Cons_t$	Personal consumption expenditure; GDP deflator; Population above 16	PCE, vintages from 1979Q4, ALFRED/St Louis; vintages from 1965Q4, Philadelphia Fed; CNP16OV, FRED/St Louis.
$Inv_t$	Fixed private investment; GDP deflator. Population above 16	FPI, vintages from 1965Q4, ALFRED/St Louis; vintages from 1965Q4, Philadelphia Fed; CNP16OV, FRED/St Louis.
$Wage_t$	Hourly compensation; GDP deflator.	COMPBNFB, vintages from 1997:Q1, ALFRED/St Louis. vintages from 1965Q4, Philadelphia Fed
$Hours_t$	Civilian employment; Average weekly hours; Population above 16.	CE16OV, vintages from 1965Q4, ALFRED/St Louis; AWHNONAG, vintages from 1970Q1, ALFRED; CNP16OV, FRED/St Louis.
$P_t$	GDP deflator.	vintages from 1965Q4, Philadelphia Fed.
$FFR_t$	Fed funds rate.	FEDFUNDS, FRED/St Louis.

Note: Dated Vintages from ALFRED were converted to quarterly data vintages by using the vintage available at the middle of the quarter to match the Philadelphia Fed real-time dataset. If source data is sampled monthly, data is converted to quarterly by averaging over quarter (before performing growth rates computation). Population and hours are converted to an index with base year in 1995.

I consider two assumptions for the number of revisions before obtaining an efficient estimate of the true values of the macroeconomic time series. With the first, I assume that the first-final (or third), that is, the value available after the initial round of monthly revisions, is an efficient estimate of the truth (as in [Kishor and Koenig \(2012\)](#) and [Garratt et al. \(2008\)](#)), implying that  $q = 2$ . Recent evidence on the impact of macro news on equity markets ([Gilbert, 2011](#); [Clements and Galvão, forthcoming](#)) suggests that surprises on the first two revisions released in the second and third month after the observational quarter have an impact on equity markets on the day of the announcement. This suggests that market participants may incorporate the impact of these revisions in their economic decisions, in agreement with our assumption that we estimate behavioural parameters using revised data. With the second, I assume that the truth is revealed two years after the observational quarter, that is,  $q = 8$ . This means that I also incorporate the first two rounds of annual revisions in additional to the initial monthly revisions. The idea that the observational quarter revisions are largely unpredictable two years after is embedded in the argument of [Croushore and Sill \(2014\)](#).

### 3.1. Characteristics of the data revisions

In this subsection, I describe the characteristics of the data revisions implied by the use of  $X_t^{t+2}$  and  $X_t^{t+8}$  as the final data in contrast to the data revisions measured using the most recent vintage as in [Casares and Vazquez \(forthcoming\)](#) and [Croushore and Sill \(2014\)](#).

[Table 2](#) presents characteristics of the data revisions of per capita output growth ( $\Delta \log(GDP)$ ), inflation ( $\Delta \log(P)$ ), per capita consumption growth ( $\Delta \log(Cons)$ ) and per capita investment growth ( $\Delta \log(Inv)$ ). The period covered is 1984Q1–2008Q3; 2008Q3 is the last observation available with the 2008Q4 vintage, which is the last vintage considered when estimating DSGE models in Sections 4 and 5. [Table 2A](#) presents results for revisions always using the first release as the initial value, but either the first-final  $X_t^{t+2}$ , the eighth release  $X_t^{t+8}$  or the 2008Q4 vintage data as the final value. [Table 2B](#) presents summary statistics for the 84–98 period with observations taken from the first releases  $X_t^{t+1}$ , the second releases  $X_t^{t+2}$ , the eighth releases  $X_t^{t+8}$  and the 2008Q4 vintage data. [Table 2C](#) presents the correlation matrix of data revisions for the four macroeconomic variables, assuming that the true value is either the second or the eighth release.

The standard deviations of the data revision processes indicate that the 7th revision has a similar size to the one measured using the 2008Q4 vintage, but the 1st revision is in general smaller, accounting for half of the revision variation of the 8th estimate. The initial revisions ( $X_t^{t+2} - X_t^{t+1}$ ) are sizeable because they are equivalent in size to the first two rounds of annual revisions ( $X_t^{t+8} - X_t^{t+2}$ ). The results of the Ljung–Box  $Q(4)$  test for a serial correlation of order 4 suggest that revisions computed using  $X_t^{t+2}$  and  $X_t^{t+8}$  as the final value are not serially correlated in general; in contrast, revisions computed using the 2008Q4 as the final value are serially correlated. Because we consider either  $X_t^{t+8}$  or  $X_t^{t+2}$  to be the true values, we expect that our results might differ from [Casares and Vazquez \(forthcoming\)](#) and [Croushore and Sill \(2014\)](#), where the last vintage is used for the true values.

Data revisions tend to increase the time series average as suggested by [Tables 2A](#) and [2B](#), but this increase is only sizeable in the case of the first revision of investment growth. Data revisions also have a large impact on the first-order serial correlation of the time series but a small impact on the unconditional variance. In general, data revisions of real variables increase the unconditional variance but decrease the unconditional variance of inflation. Correctly measuring the underlying unconditional variance may improve the coverage of interval forecasts, as suggested by [Clements \(2015\)](#).

[Table 2C](#) suggests that data revisions on the real macroeconomic variables are negatively correlated with data revisions on inflation, as expected based on the construction of these time series. The comovements of data revisions on output growth, consumption and investment are stronger when  $q = 8$ . When  $q = 2$ , data revisions on investment are mainly uncorrelated with revisions of other variables. The release-based approach can accommodate comovements in data revisions if structural shocks have a similar impact on data revisions of different variables, that is, they depend on the estimates in  $A_{(v)}$ .

### 3.2. Release-based specifications

The SW model describes business cycles fluctuations using seven shocks: spending ( $g$ ), risk-premium ( $b$ ), investment ( $i$ ), productivity ( $a$ ), price-push ( $p$ ), cost-push ( $w$ ) and monetary policy ( $r$ ). Because we are modelling data revisions of four observables, the number of coefficients to be estimated in  $A_{(v)}$  is

**Table 2**

Characteristics of data revisions on output growth, inflation, consumption and investment growth (1984Q1–2008Q3).

A: Summary statistics for data revisions							
	$\Delta \log GDP$			$\Delta \log P$			
Final:	$X_t^{t+2}$	$X_t^{t+8}$	$X_t^{08Q4}$	$X_t^{t+2}$	$X_t^{t+8}$	$X_t^{08Q4}$	
Mean	0.028	−0.002	0.065	0.024	0.057	0.026	
Stdev	0.171	0.338	0.359	0.089	0.150	0.170	
AC(1)	−0.052	0.020	−0.124	0.007	0.126	0.112	
Q(4)	0.799	8.08	25.27	1.634	2.980	10.12	
	[0.939]	[0.09]	[0.000]	[0.803]	[0.561]	[0.038]	
	$\Delta \log Cons$			$\Delta \log Inv$			
Final:	$X_t^{t+2}$	$X_t^{t+8}$	$X_t^{08Q4}$	$X_t^{t+2}$	$X_t^{t+8}$	$X_t^{08Q4}$	
Mean	−0.005	−0.009	0.041	0.106	0.025	0.010	
Stdev	0.138	0.314	0.370	0.540	0.803	0.818	
AC(1)	−0.125	0.055	0.045	0.007	0.041	0.030	
Q(4)	4.012	1.677	8.813	5.204	3.584	5.987	
	[0.404]	[0.795]	[0.066]	[0.267]	[0.465]	[0.200]	
Note: The revisions are computed with the indicated final value minus the first release.							
B: Summary statistics for releases of different maturity							
	$\Delta \log GDP$				$\Delta \log P$		
	$X_t^{t+1}$	$X_t^{t+2}$	$X_t^{t+8}$	$X_t^{08Q4}$	$X_t^{t+1}$	$X_t^{t+2}$	$X_t^{08Q4}$
Mean	0.388	0.416	0.386	0.453	0.604	0.630	0.630
Stdev	0.451	0.507	0.584	0.538	0.284	0.287	0.245
AC(1)	0.246	0.256	0.285	0.155	0.465	0.457	0.453
	$\Delta \log Cons$				$\Delta \log Inv$		
	$X_t^{t+1}$	$X_t^{t+2}$	$X_t^{t+8}$	$X_t^{08Q4}$	$X_t^{t+1}$	$X_t^{t+2}$	$X_t^{08Q4}$
Mean	0.503	0.497	0.493	0.544	0.284	0.390	0.309
Stdev	0.490	0.507	0.500	0.510	1.833	1.914	1.690
AC(1)	−0.140	−0.046	0.094	0.025	0.384	0.455	0.531
C: Correlation between revisions							
	$X_t^{t+2} - X_t^{t+1}$			$X_t^{t+8} - X_t^{t+1}$			
	$\Delta \log GDP$	$\Delta \log P$	$\Delta \log Cons$	$\Delta \log GDP$	$\Delta \log P$	$\Delta \log Cons$	
$\Delta \log GDP$	1			1			
$\Delta \log P$	−0.07	1		−0.38	1		
$\Delta \log Cons$	0.23	−0.34	1	0.59	−0.42	1	
$\Delta \log Inv$	0.14	0.01	0.01	0.33	−0.23	0.29	

28 for each  $v = 1, \dots, q - 1$ . The time series of revisions for  $v > 2$  include many zeros because the observations are not revised every quarter after the initial round of monthly revisions. As a consequence, I assume that the revision processes for  $q = 8$  are such that only  $rev_t^{(1)}$  is affected by structural shocks:

$$\begin{aligned}
 (X_t^{t+1} - X_t) &= M_{(1)} + K_{(1)}(X_{t-1}^t - X_{t-1} - M_{(1)}) \\
 &\quad + A_{(1)}\eta_t + \xi_t^{(1)}; \xi_t^{(1)} \sim N(0, R_{(1)}) \\
 (X_{t-1}^{t+1} - X_{t-1}) &= \xi_{t-1}^{(2)}; \xi_{t-1}^{(2)} \sim N(0, R_{(2)}) \\
 &\vdots \\
 (X_{t-6}^{t+1} - X_{t-6}) &= \xi_{t-6}^{(7)}; \xi_{t-6}^{(7)} \sim N(0, R_{(7)}).
 \end{aligned}$$

If  $q = 2$ , I retain the same assumption, and therefore, I estimate only the first equation of the system above.

The first specification considered is estimated using the RWMH algorithm; therefore, only data revisions of output growth and inflation are modelled, and  $q = 2$ . The number of additional parameters in this specification is 20. In the remaining two specifications, I assume no serial correlation in the data revisions, that is, all elements in  $K_{(1)}$  are zero. The removal of the serial correlation coefficient is supported by the characteristics of the data revisions described in Table 2. As a result of these assumptions, the number of additional parameters to be estimated if  $q = 8$ , the number of structural shocks is 7, and data revisions of 4 observables are modelled, is 60. If  $q = 2$ , 44 parameters are to be estimated.

In summary, I consider three vintage-based specifications. The first specification is 'MH,  $q = 2$ ', where the MH estimation method

is applied for modelling data revisions only of output growth and inflation while estimating the DSGE model and assuming that the second quarterly release is an efficient estimate of the truth. The parameters of the data revision processes of this specification are listed in Table 4. The second is the 'Gibbs-M,  $q = 2$ ' specification that models revisions of consumption and investment growth in addition to output growth and inflation. The third is the 'Gibbs-M,  $q = 8$ ' specification that extends the previous specification by considering data revision processes up to two years after the observational quarter, that is, the efficient estimate of the truth is only published eight quarters after the observational quarter.

Del Negro and Schorftheide (2013) employ a medium-sized DSGE specification assuming a common stochastic trend for consumption, investment, output and real wage instead of the common deterministic trend in Smets and Wouters (2007). Canova (2014) claims that the assumption for this trend has an impact on the estimates of the DSGE parameters. The baseline results in Sections 4 and 5 are based on the common deterministic trend model, but I will investigate the robustness of the forecasting results to the assumption of a stochastic trend.<sup>7</sup> The preliminary

<sup>7</sup> If the total factor productivity process follows a deterministic trend, this means that it exhibits features of a linear trend plus an AR(1) process. If it follows a stochastic trend, then it exhibits features of a linear trend plus a random walk (AR(1) with  $\rho = 1$ ). After calculating the first differences (or detrending), the productivity shocks follow an AR(1) process with drift in the first case and white noise with drift in the second case. Because variables such as the output, consumption and investment are detrended before the equations are log-linearised around the steady-state, some equations require small modifications. Modifications are also required for the measurement equations.

results suggest that the forecasts of output growth and inflation are in general more accurate if the common trend is deterministic.

#### 4. Full sample evaluation

In this section, I discuss results based on the estimates for the full sample. As in [Herbst and Schorftheide \(2012\)](#), I use observations since 1984, implying that the period of high inflation is not included in the estimation. In this section, I use the 2008Q4 vintage for the conventional approach, that is, the results are computed for the sample period from 1984Q1 up to 2008Q3. For the release-based approach, I use vintages from 1984Q2 up to 2008Q4.

Both estimation methods described in Section 2.2.1 require priors for the DSGE parameters, including structural shocks processes. The priors for these coefficients are the ones used in [Smets and Wouters \(2007\)](#), and I do not include the large set of calibrated coefficients suggested by [Herbst and Schorftheide \(2012\)](#); the values used are listed in [Table 3](#). The priors on the coefficients that describe the data revision processes are not restrictive. In the case of the ‘MH,  $q = 2$ ’ model, I use a normal prior for the mean revisions and autoregressive coefficients, allowing for negative serial correlation in the data revision processes. I use inverse gamma priors for the standard error of data revision innovations following the priors for the standard deviations of the structural shocks. I also assume a normal prior for the parameters measuring the impact of structural shocks on the data revision processes. The description of the complete set of priors for the data revision processes of output growth and inflation is provided in [Table 4](#). In the case of the Gibbs-M specifications, priors for the parameters of the data revision processes are described in [Appendix A](#).

##### 4.1. Conventional and release-based posteriors

In this subsection, I compare the full sample estimates of the release-based approach with the conventional one. I also discuss the estimates of the parameters governing data revision processes for different estimation methods and specifications.

[Table 3](#) presents the mean of the posterior distributions and 5% and 95% posterior quantiles for the DSGE parameters estimated with the conventional approach and three release-based approach specifications. [Table 4](#) presents posterior mean and 5% and 95% posterior quantiles for the parameters of the data revision processes of output growth and inflation. For the ‘MH,  $q = 2$ ’ specification, the values presented include all the parameters estimated, but for the other two specifications, these are only a subset of the  $\beta$  parameters. The estimates of the serial correlation parameters with the ‘MH,  $q = 2$ ’ specification support the assumption that  $K_{(1)} = 0$  in the remaining specifications.

The posterior mean estimates for the DSGE parameters obtained with the release-based approach are in general within the 90% interval estimates of the conventional approach. [Table 3](#) highlights values when this is not the case. The main impact of the vintage-based approach is observed in the autoregressive coefficients of the structural shocks in agreement with the change in the first-order serial correlation with the maturity  $q$  in [Table 2B](#). Modelling data revisions of output growth and inflation reduces the persistence of productivity shocks and increases the persistence of price shocks. If data revisions of investment are also incorporated, the variance of investment-specific shocks (only if  $q = 2$ ) increases, and the capital-share in the production function ( $\alpha$ ) decreases. Finally, the Gibbs-M specifications suggest larger consumption habit formation ( $h$ ) values than observed with the conventional approach because they include modelling of consumption data revisions.

The posterior mean and quantiles presented in [Table 4](#) allow us to compare the impact of specifications and estimation methods on the posterior distribution of the data revision parameters of output growth and inflation. The inclusion of data vintages on consumption and investment alters many of the estimates of the coefficients in  $A_{(1)}$ . In particular, the values of the output growth revisions of productivity shocks go to zero, while the coefficients for the investment-specific shocks are now significantly negative. The results for the data revisions defined as  $X_t^{t+1} - X_t$  in [Table 4](#) indicate that the 90% intervals for the parameters in the  $A_{(1)}$  matrix are generally wider with the Metropolis-in-Gibbs algorithm than with the RWMH algorithm. This implies that the Gibbs approach exploits a larger portion of the parameter space than the Metropolis approach for the  $\beta$  parameters. A comparison of both methods convergence analysis in [Appendix A.3](#) shows that indeed the inefficiency factor is normally smaller for the Gibbs algorithm.

##### 4.2. Variance decompositions

[Smets and Wouters \(2007\)](#) describe business cycles fluctuations using seven shocks: spending ( $g$ ), risk-premium ( $b$ ), investment ( $i$ ), productivity ( $a$ ), price-push ( $p$ ), cost-push ( $w$ ) and monetary policy ( $r$ ). In this subsection, I evaluate variance decompositions from innovations to each one of these shocks in addition to idiosyncratic innovations to data revisions of output growth, inflation, consumption and investment growth. These results allow us to address the issue of how the relative importance of business cycle sources changes with data maturity.

[Fig. 1](#) presents the proportion explained by each of the seven DSGE structural shocks and the future revision-specific shocks computed for the posterior mean with the release-based approach (Gibbs-M,  $q = 8$  specification). The future revision shock is specific for each variable (output growth, inflation, consumption and investment growth in panels A to D) and data maturity. [Fig. 1](#) shows the results for data maturities from the first release  $X_t^{t+1}$  up to the eighth release  $X_t^{t+8}$ , represented by bars varying from left to right. [Fig. 1](#) also includes results using the conventional approach. Recall that the data employed to estimate the DSGE model with the conventional approach are in general heavily revised, and therefore, the explained fractions may be more similar to the 8th release results, but with some weight given to the last 7 observations that are still subject to revision. The proportions presented in [Fig. 1](#) are those for responses after 40 quarters and for the observed values of each variable, which implies that these values were computed using both state (10) and measurement equations (7).

[Fig. 1](#) suggests that data uncertainty plays an important role in explaining the differences between the observed first release values and the predicted true values. The fraction values are 20% for output growth, 30% for consumption growth, 42% for investment growth and 15% for inflation. These fractions decrease with the data maturity and become zero for the 8th release based on the assumption that the true values are eventually observed. The assumption that revisions may be caused by not fully observing the effect of structural shocks at the time of the initial release implies large differences in the fraction explained by some structural shocks when the first release is observed in comparison with later releases. For example, in the case of consumption, 12% of the first-release variation is explained by investment-specific shocks, but this value drops to around 5% in later releases. In contrast, the fraction explained by productivity shocks increases from 32% in the first release to 62% in the final release.

The last bar for each shock in [Fig. 1](#) shows the proportion estimated using the conventional approach. In some cases, the conventional approach that mixes data from different maturities provides variance decomposition estimates that differ from the



**Table 3**

Priors and posteriors distributions of the DSGE parameters using the conventional and the release-based approaches.

	Priors			Conventional			MH, $q = 2$			Gibbs-M, $q = 2$			Gibbs-M, $q = 8$		
	Density	Par (1)	Par (2)	0.05	Mean	0.95	0.05	Mean	0.95	0.05	Mean	0.95	0.05	Mean	0.95
$\varphi$	normal	4.00	1.5	5.08	5.78	7.62	5.15	6.66	8.66	3.55	5.24	7.28	4.84	6.38	8.16
$\sigma_c$	normal	1.5	0.37	0.51	0.65	1.26	0.46	0.59	0.85	0.38	0.50	0.66	0.48	0.63	0.83
$h$	beta	0.70	0.10	0.60	0.69	0.75	0.66	0.74	0.80	0.71	0.78	0.85	0.76	0.77	0.78
$\xi_w$	beta	0.50	0.10	0.76	0.80	0.85	0.71	0.78	0.86	0.73	0.79	0.86	0.69	0.76	0.83
$\sigma_l$	normal	2.00	0.75	2.19	2.50	3.64	2.58	3.33	4.27	1.98	2.94	3.88	1.96	3.05	4.08
$\xi_p$	beta	0.50	0.10	0.84	0.87	0.90	0.79	0.85	0.90	0.78	0.83	0.88	0.79	0.85	0.89
$\iota_w$	beta	0.50	0.15	0.18	0.29	0.45	0.13	0.29	0.51	0.12	0.28	0.48	0.14	0.29	0.48
$\iota_p$	beta	0.50	0.15	0.16	0.41	0.62	0.14	0.27	0.40	0.16	0.31	0.45	0.19	0.37	0.53
$\psi$	beta	0.50	0.15	0.63	0.77	0.85	0.61	0.76	0.88	0.41	0.63	0.83	0.44	0.65	0.84
$\Phi$	normal	1.25	0.12	1.46	1.54	1.69	1.37	1.50	1.69	1.32	1.48	1.65	1.40	1.55	1.71
$r_\pi$	normal	1.50	0.25	1.08	1.24	1.73	1.06	1.37	1.78	1.11	1.49	1.85	1.18	1.49	1.84
$\rho$	beta	0.75	0.10	0.82	0.84	0.89	0.83	0.86	0.89	0.83	0.87	0.90	0.82	0.86	0.89
$r_y$	normal	0.12	0.05	0.15	0.22	0.24	0.12	0.18	0.26	0.10	0.17	0.24	0.13	0.18	0.25
$r_{\Delta y}$	normal	0.12	0.05	0.09	0.16	0.19	0.07	0.13	0.17	0.07	0.12	0.17	0.07	0.12	0.17
$\pi$	gam	0.62	0.10	0.56	0.65	0.71	0.52	0.62	0.71	0.50	0.61	0.71	0.57	0.65	0.72
$100(\beta^{-1} - 1)$	gam	0.25	0.10	0.15	0.33	0.39	0.15	0.28	0.41	0.15	0.27	0.42	0.14	0.27	0.40
$\bar{l}$	normal	0.00	2.00	-0.89	0.37	0.76	-1.43	0.01	1.00	-1.52	-0.44	0.52	-1.92	-0.30	1.00
$\bar{y}$	normal	0.40	0.10	0.33	0.40	0.44	0.37	0.42	0.45	0.37	0.41	0.46	0.36	0.40	0.44
$\alpha$	normal	0.30	0.05	0.13	0.16	0.19	0.11	0.14	0.19	0.06	0.09	0.13	0.07	0.11	0.15
$\rho_a$	beta	0.50	0.20	0.94	0.96	0.99	0.87	0.91	0.98	0.85	0.91	0.96	0.87	0.93	0.98
$\rho_b$	beta	0.50	0.20	0.53	0.81	0.89	0.67	0.79	0.90	0.73	0.84	0.91	0.63	0.77	0.88
$\rho_g$	beta	0.50	0.20	0.94	0.95	0.98	0.98	0.99	1.00	0.96	0.98	0.99	0.97	0.98	1.00
$\rho_i$	beta	0.50	0.20	0.54	0.72	0.79	0.43	0.58	0.76	0.16	0.36	0.54	0.46	0.61	0.74
$\rho_r$	beta	0.50	0.20	0.46	0.60	0.66	0.42	0.50	0.60	0.38	0.49	0.60	0.44	0.53	0.61
$\rho_p$	beta	0.50	0.20	0.39	0.46	0.54	0.65	0.74	0.83	0.71	0.75	0.79	0.51	0.60	0.67
$\rho_w$	beta	0.50	0.20	0.47	0.69	0.83	0.36	0.69	0.92	0.43	0.67	0.86	0.49	0.70	0.87
$\mu_p$	beta	0.50	0.20	0.23	0.42	0.52	0.63	0.69	0.76	0.62	0.68	0.74	0.32	0.49	0.61
$\mu_w$	beta	0.50	0.20	0.89	0.91	0.97	0.88	0.95	0.98	0.88	0.93	0.97	0.89	0.94	0.98
$\rho_{ga}$	beta	0.50	0.20	0.17	0.39	0.48	0.20	0.33	0.49	0.10	0.22	0.33	0.17	0.30	0.43
$\sigma_a$	invga	0.10	2.00	0.38	0.41	0.46	0.39	0.45	0.50	0.40	0.46	0.53	0.40	0.45	0.51
$\sigma_b$	invga	0.10	2.00	0.07	0.09	0.14	0.06	0.09	0.12	0.06	0.08	0.11	0.07	0.09	0.13
$\sigma_g$	invga	0.10	2.00	0.36	0.43	0.45	0.37	0.41	0.47	0.35	0.39	0.44	0.34	0.38	0.43
$\sigma_i$	invga	0.10	2.00	0.28	0.32	0.42	0.27	0.39	0.47	0.45	0.59	0.72	0.29	0.39	0.49
$\sigma_r$	invga	0.10	2.00	0.10	0.13	0.13	0.10	0.12	0.14	0.10	0.12	0.14	0.10	0.11	0.13
$\sigma_p$	invga	0.10	2.00	0.13	0.16	0.17	0.14	0.17	0.20	0.14	0.17	0.21	0.12	0.14	0.17
$\sigma_w$	invga	0.10	2.00	0.33	0.36	0.40	0.33	0.37	0.42	0.32	0.37	0.42	0.32	0.37	0.43

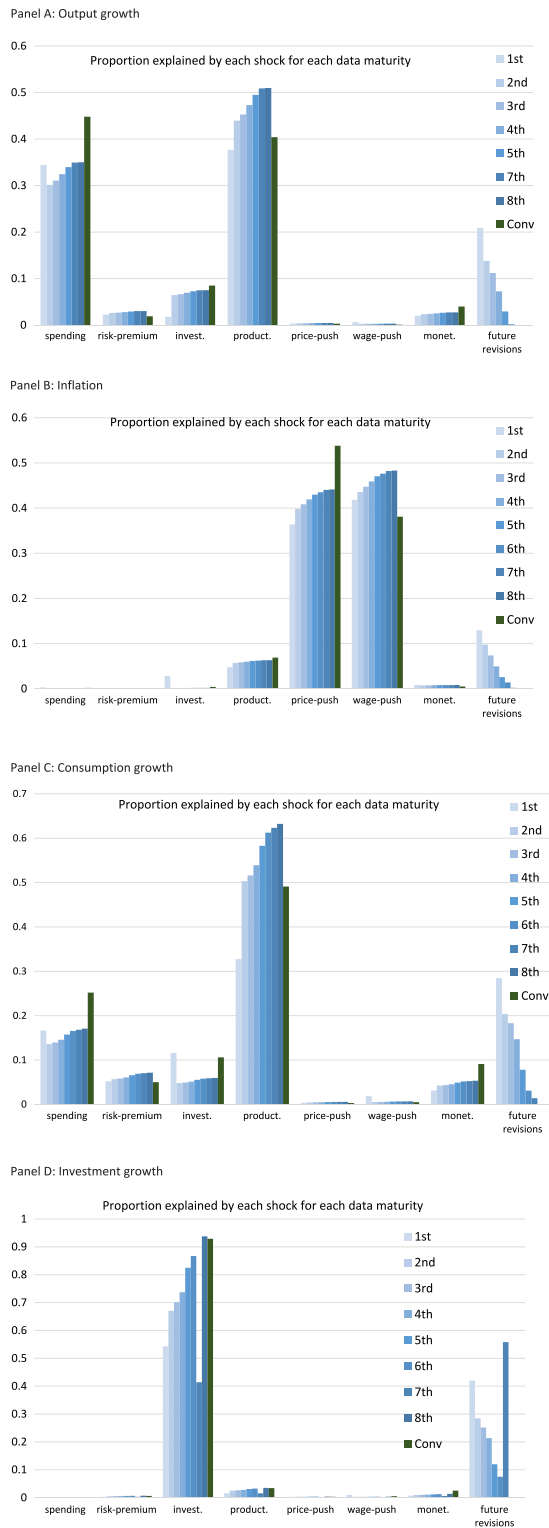
Note: fixed values:  $\lambda_w = 1.5$ ,  $\varepsilon_w = 10$ ,  $\varepsilon_p = 10$ ,  $\delta = 0.025$ ,  $g_Y = 0.18$ . See [Appendix B](#) for model equations. These results are based on observations from 1984Q1 up to 2008Q3. The three different release-based specifications are as follows. The 'MH,  $q = 2$ ' specification that employs a pure RWMH Algorithm, sets the revised value as the first-final (2nd release) and models revisions on output growth and inflation only. The 'Gibbs-M,  $q = 2$ ' employs the Metropolis in Gibbs algorithm described in the [Appendix A](#), models data revisions on output growth, consumption and investment growth, and inflation, and sets the revised value as the first final. The Gibbs-M,  $q = 8$  is as the Gibbs-M,  $q = 2$ , except that the revised value is only observed in the 8th release, that is, two years after the observation quarter.

**Table 4**

Release-based approach estimates of data revision processes parameters of output growth and inflation.

	Priors			MH, $q = 2$			Gibbs-M, $q = 2$			Gibbs-M, $q = 8$		
	Density	Para(1)	Para(2)	0.05	Mean	0.95	0.05	Mean	0.95	0.05	Mean	0.95
$M_{1,y}$	normal	0.10	0.20	-0.06	-0.03	0.00	-0.06	-0.03	0.00	-0.06	0.00	0.05
$M_{1,\pi}$	normal	0.10	0.20	-0.04	-0.02	0.00	-0.04	-0.02	0.19	-0.09	-0.06	-0.03
$k_{1,y}$	normal	0.10	0.20	-0.08	0.06	0.21						
$k_{1,\pi}$	normal	0.10	0.15	-0.10	0.06	0.19						
$\sigma_{1,y}$	invgamma	0.50	0.40	0.16	0.17	0.20	0.15	0.17	0.19	0.28	0.31	0.36
$\sigma_{1,\pi}$	invgamma	0.20	0.40	0.08	0.09	0.11	0.08	0.09	0.10	0.13	0.15	0.17
$a_{1,y,g}$	normal	-0.20	0.50	-0.12	-0.04	0.03	-0.07	0.01	0.10	-0.11	0.05	0.20
$a_{1,y,b}$	normal	-0.20	0.50	-0.57	-0.24	0.08	-0.48	-0.05	0.37	-1.10	-0.37	0.30
$a_{1,y,i}$	normal	-0.20	0.50	-0.06	0.03	0.10	-0.11	-0.05	0.01	-0.39	-0.22	-0.08
$a_{1,y,a}$	normal	-0.20	0.50	-0.23	-0.16	-0.10	-0.05	0.01	0.08	-0.20	-0.07	0.06
$a_{1,y,p}$	normal	-0.20	0.50	0.06	<b>0.26</b>	0.39	-0.35	-0.13	0.06	-0.32	0.10	0.52
$a_{1,y,w}$	normal	-0.20	0.50	-0.06	0.01	0.10	-0.14	-0.05	0.04	-0.31	-0.14	0.03
$a_{1,y,r}$	normal	-0.20	0.50	0.11	<b>0.27</b>	0.47	-0.30	-0.02	0.25	-0.21	0.29	0.78
$a_{1,\pi,g}$	normal	-0.20	0.50	-0.08	-0.04	0.00	-0.06	-0.01	0.03	-0.09	-0.02	0.05
$a_{1,\pi,b}$	normal	-0.20	0.50	-0.40	-0.16	-0.02	-0.24	-0.02	0.19	-0.25	0.07	0.39
$a_{1,\pi,i}$	normal	-0.20	0.50	-0.09	-0.03	0.02	-0.02	0.01	0.04	0.00	0.07	0.15
$a_{1,\pi,a}$	normal	-0.20	0.50	-0.02	0.01	0.06	-0.05	-0.02	0.02	-0.03	0.03	0.09
$a_{1,\pi,p}$	normal	-0.20	0.50	-0.18	-0.08	0.03	-0.08	0.02	0.13	-0.26	-0.06	0.15
$a_{1,\pi,w}$	normal	-0.20	0.50	-0.10	-0.05	-0.01	-0.06	-0.01	0.04	-0.07	0.00	0.08
$a_{1,\pi,r}$	normal	-0.20	0.50	-0.01	0.15	0.29	-0.23	-0.09	0.06	-0.32	-0.09	0.15

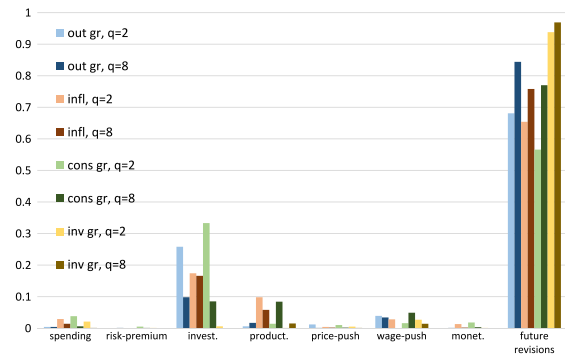
Note: See notes of [Table 3](#). The prior values in the first columns are the ones employed in the MH estimation. The priors for the Gibbs-M versions are in the [Appendix A](#). The parameters above are a subset of the parameters estimated with the Gibbs-M,  $q = 2$  and  $q = 8$  specifications since this table presents results for data revisions on output growth ( $y$ ) and inflation ( $\pi$ ). They are parameters for the revision between the first release and the final values (either  $q = 2$  or  $q = 8$ ).



**Fig. 1.** Variance decompositions computed with the release-based ( $q = 8$ ) and conventional approaches (at the posterior mean; after 40 quarters).

ones obtained with revised/true data ( $X_t$ ). In the case of output growth, for example, the conventional approach suggests that the spending shocks explain 45% of the variation in contrast with 35% with the release-based approach, while productivity shocks explain 40% with the conventional approach but 51% with the release-based approach.

In Table A1 in the Appendix C, more detailed variance decomposition results are presented for the first and last release



**Fig. 2.** Variance decompositions computed for the revisions between the first release and final values (either  $q = 2$  or  $q = 8$ ; computed at the posterior mean; after 40 quarters).

( $X_t^{t+q}$ ;  $q = 2, 8$ ) and for the conventional approach. Also included are ranges for the estimates and results for two additional release-based specifications: Gibbs-M,  $q = 2$  and MH,  $q = 2$ . These results show that if we assume that the true/revise value is observed after two quarters, then the variation of the first release explained by future revisions is small (around 2% but up to 7% for investment). Table A confirms the main result from Fig. 1 that the relative importance of different shocks in causing business cycle variation may depend on how far in the revision process we are. If our best variance decomposition estimates are obtained using revised data, we should be aware that, by estimating them using time series that include observations still subject to many rounds of revision, our inference on the relative importance of structural shocks might be mistaken.

Fig. 2 presents variance decompositions for the same set of shocks of Fig. 1 but they show the proportion of unexpected data revision variation caused by each shock. The data revision process considered is  $rev_t^{(1)} = X_t^{t+1} - X_t - M_{(1)}$ , that is, the complete revision process until the true (revised) value is observed. Fig. 2 presents variance decompositions computed at the posterior mean for Gibbs specifications with  $q = 2$  and  $q = 8$  for each one of the four variables with modelled data revision processes. The bars are ordered by variable and then specification, so we can evaluate the impact of the assumption on the final values ( $X_t^{t+2}$  or  $X_t^{t+8}$ ). It is clear that the proportion explained by future revisions, or the revision-specific shocks for each variable, is smaller if the final value is  $X_t^{t+2}$  instead of  $X_t^{t+8}$ . As a consequence, structural shock innovations are more dominant in explaining the first revision than later revisions. This is in agreement with the fact that publications of initial revisions by statistical agencies are mainly caused by the use of a more complete information set (Landefeld et al., 2008), and that the initial revisions are mainly predicted based on new information (Clements and Galvão, forthcoming).

Although the main source of unexpected revisions is a decrease in measurement errors of earlier estimates because “future revisions” shocks explain at least 70% of the variation if the truth is revealed after 8 quarters, data revisions of all variables, except investment, are also explained by investment-specific shocks. Investment-specific shocks are then the source identified using the release-based approach for the data revision comovements identified in Table 2C.

In Table A2 in the Appendix C, I present the estimated range for these variance decomposition values as well as the results for the ‘MH,  $q = 2$ ’ specification that confirms that structural shocks are more important in explaining earlier rather than later revisions.

## 5. Real-time out-of-sample evaluation

In this section, I compare the relative real-time forecasting performance of the conventional and the release-based approach

to estimate and forecast using DSGE models in real time. One of the advantages of the release-based approach is that confidence intervals for the last observation currently available can be easily computed based on the information set available for the forecaster estimating using the DSGE model. In other words, we can account for data uncertainty. As a consequence, I use a backcasting exercise to assess the empirical coverage of the data uncertainty intervals computed using the release-based specifications. I also employ this forecasting exercise to evaluate the reliability of DSGE real-time estimates of the output gap with the conventional approach and compare these estimates with the release-based approach.

### 5.1. Design of the forecasting exercise

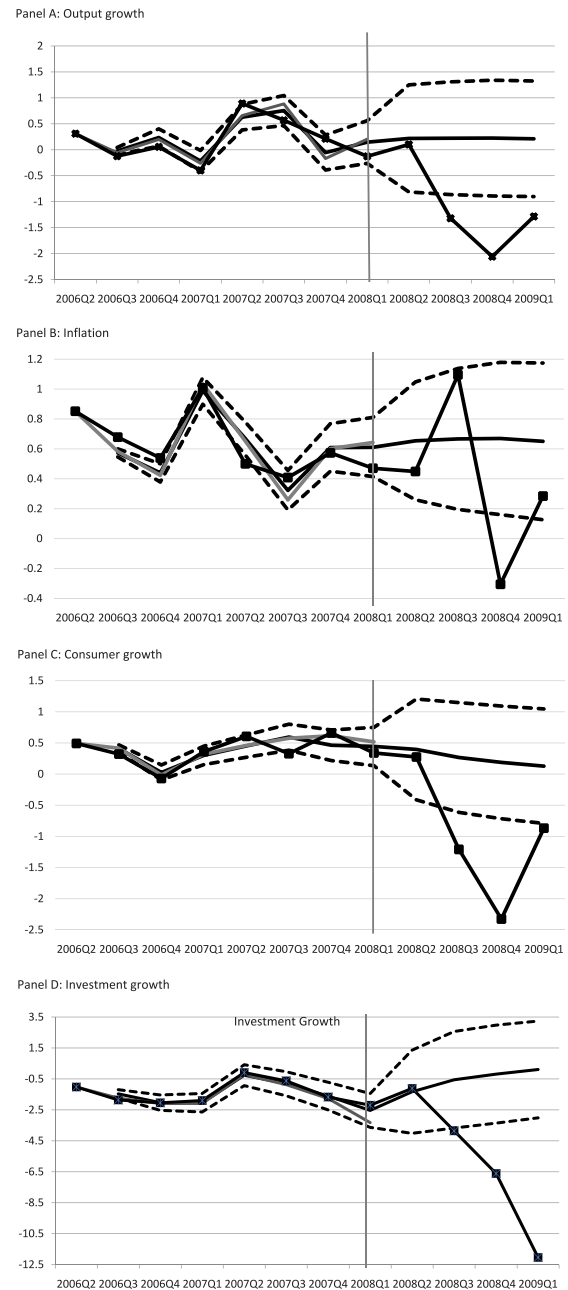
I use a real-time forecasting exercise, but instead of organising real-time vintages to match the dates of computation of the Greenbook and/or Blue Chip forecasts, as is done in [Edge and Gurkaynak \(2011\)](#), I organise the data by quarterly vintages dated at the middle of the quarter, similar to the Philadelphia Fed real-time dataset. Although this makes it more difficult to compare my results with the survey forecasts computed earlier in the quarter, it is easier to compare them with results in the literature on the impact of real-time datasets in forecasting, as surveyed by [Croushore \(2011\)](#). Details of the real-time datasets employed are provided in [Table 1](#).

The forecast accuracy is evaluated using first-final estimates as in [Edge and Gurkaynak \(2011\)](#), when the release-based specification assumes  $q = 2$  and the use of the eighth estimate if the vintage-based specification sets  $q$  to be 8. This implies that forecasts computed using the conventional approach are evaluated using both  $X_t^{t+2}$  and  $X_t^{t+8}$ . I consider 38 forecasting origins covering end-of-sample vintages from 1999Q1 up to 2008Q2 and using observations from 1984Q1. For each forecasting origin, I compute forecasts for one to four quarter horizons. For this baseline exercise, I disregard later forecasting origins because the medium-sized DSGE model is not adequate to fit central bank preferences during the recent period where the Zero Lower Bound holds. The implications for relative forecasting performance of considering this out-of-sample period are investigated in [Section 5.3](#).

Computation of forecasts using the release-based approach is performed as described in [Section 2.2.3](#). I set  $J = 2000$  using equally spaced draws from the draws kept from the posterior distribution (30,000). A similar approach is applied to compute forecasts using the conventional approach. I also use the posterior distribution of  $X_T$  (over  $S$  draws) to compute 90% intervals for the last observation and evaluate the data uncertainty.

[Fig. 3](#) provide us with examples of the application of the vintage-based approach with  $q = 8$  to compute backcasting and forecasting intervals, computed using real-time vintages up to 2008Q2, which include observations up to 2008Q1. [Fig. 3\(A, B\)](#) presents the values from 2006Q2 up to 2009Q1 for output growth and inflation, and [Fig. 3\(C, D\)](#) presents the values for consumption and investment growth. The grey line represents the last 8 observations available in the 2008Q2 vintage. The line with a square marker represents the values available 8 quarters after the observational quarter, that is,  $X_t^{t+8}$ , which are the target values. The  $X_t^{t+8}$  time series is only equal to the equivalent time series from the 2008Q2 vintage at the 2006Q2 observation. The black line represents our mean/point forecasts for this period. The dashed lines represent the 90% confidence bands. These forecasts are based on the ‘Gibbs-M,  $q = 8$ ’ specification (see [Table 4](#)).

As expected, the intervals are wider for the last observation available (2008Q1) than for earlier dates. The interval widths show that the forecasting uncertainty is larger than the data uncertainty. Note also that the model does not perform well for observations from 2008Q3, as reported by [Del Negro and Schorftheide \(2013\)](#) on

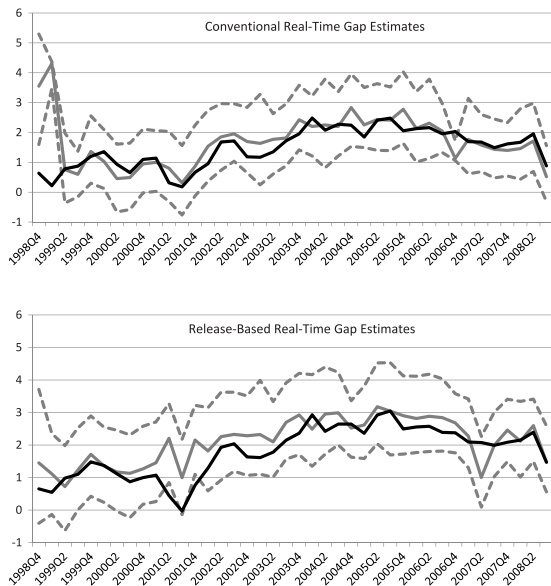


**Fig. 3.** Backcasts and forecasts with the release-based approach ( $q = 8$ ) at the 2008Q2 forecast origin with 90% intervals. (Grey line: values from the 2008Q2 vintage; line with square markers: final values—8 quarters after the observational quarter.)

DSGE models not being able to provide accurate forecasts during the 2008–2009 crisis.

The ability to use the same model to provide uncertainty assessments for both backcasts and forecasts is one of the main contributions of the release-based approach proposed in this paper. [Fig. 3](#) provides us with an example of these uncertainty measurements for a specific forecasting origin. A formal assessment of the modelling approach considering both data and forecasting uncertainty is presented below.

I evaluate the coverage of the predictive intervals and the calibration of both backcasts and forecasts. I compute the empirical coverages using nominal coverages of 70% and 90%. [Clements \(2015\)](#) argues that accounting for data revisions may have a large impact on the prediction interval coverages of autoregressive models.



**Fig. 4.** Real-time gap estimates with 90% intervals: conventional and release-based (Gibbs-M,  $q = 8$ ) approaches (black line: final estimates (the posterior mean) computed with each approach).

If the predictive densities approximate the true data density well, the probability integral transforms (PITs) should be uniform, implying that the density is well-calibrated. We use the test proposed by Berkowitz (2001) to assess the uniformity while imposing no restriction on the serial correlation of PITs over time, as in Clements (2004). This implies that we can evaluate the calibration of density forecasts at all forecast and backcast horizons because we expect some serial correlation in the PITs if we are not dealing with a one-step-ahead prediction.

The relative forecasting performance of a model is measured using the squared error loss function for point forecasts and log scores for density forecasts. The advantage of using log scores is that if model A has a larger log score than model B, this means that model A is closer to the true density using the Kullback–Leibler information criteria (Mitchell and Wallis, 2011). I compute log-scores  $\log p_{j,h,t}$  for  $t = 1, \dots, P$  for model  $j$  based on the predictive density for horizon  $h$  using numerical methods.<sup>8</sup> I test for equal forecasting accuracy using the difference in log scores between the model  $i$  under the null hypothesis and the model  $j$  under the alternative as

$$d_{h,t} = \log p_{i,h,t} - \log p_{j,h,t},$$

and therefore, the null hypothesis is rejected in favour of the model under the alternative if the mean of  $d_{h,t}$  is significantly smaller than zero. I use a  $t$ -statistic that employs the Newey–West estimator to obtain  $\text{var}(d_{h,t})$  and asymptotic normality to obtain critical values, as in Diebold and Mariano (1995), Giacomini and White (2006) and Amisano and Giacomini (2007). A similar test statistic is also used when comparing the mean squared forecasting errors so that we can evaluate whether the alternative model is a more accurate point forecaster than the model under the null hypothesis.

## 5.2. Backcasting evaluation

The first empirical question that I address using the forecasting exercise described above is whether the release-based approach

is able to provide a good measure of data uncertainty for the last observation available at each vintage in the out-of-sample period, that is,  $\{X_t\}_{t=T}^{t=T+P-1}$ . I use two release-based specifications in this exercise: Gibbs-M,  $q = 2$  and  $q = 8$ . Note that each of these specifications is evaluated using their assumed actual value ( $X_t^{t+2}$  and  $X_t^{t+8}$ ). Empirical coverages computed for the 38 forecasting origins considered and  $p$ -values for the Berkowitz test are presented in Table 5 for the four variables that we model the data revision processes for.

The performance of the specification with  $q = 2$  is not good because the null hypothesis of uniformity is rejected for all variables and the predictive intervals are too narrow in comparison with the nominal values. The specification with  $q = 8$  performs better, and we find that the backcasting densities for inflation and investment growth are well calibrated and that their empirical coverages are near the nominal value. Even with  $q = 8$ , the predictive intervals heavily undercover the actual realisations, particularly when predicting consumption growth.

This disappointing performance of the release-based approach in providing accurate backcasts for the last observation of each vintage maybe be related to the fact that we are not using any additional information for forecasting these data revisions. A successful predictive model for data revisions should normally incorporate additional information (Cunningham et al., 2012; Clements and Galvão, forthcoming). This paper does not aim to provide us with an outstanding data revision forecasting model, and therefore, this issue is not pursued further. However, the release-based approach when estimated using the Gibbs-M algorithm is sufficiently flexible that we can incorporate additional observables on the data revision regression equations. I leave this task for future research.

## 5.3. Forecasting evaluation

The second empirical question addressed by this empirical exercise is the relative performance of the release-based approach in comparison with the conventional approach in real-time forecasting. The statistics presented in Table 6 to evaluate the forecasting performance of both approaches are similar to those in Table 5, that is, they include coverage measures and  $p$ -values for the Berkowitz test. Results are presented for two forecasting horizons:  $h = 1$  and  $h = 4$ . In addition to the four variables of Table 5, I include results for the fed fund rate. This last variable is not subject to revision, and therefore, the conventional approach results for both actual values,  $X_t^{t+2}$  and  $X_t^{t+8}$ , are exactly the same. For the other variables, the change in the target values affects the coverage and calibration of conventional density forecasts.

The results in Table 6 suggest that the release-based approach addresses issues of undercoverage of the conventional approach. The predictive intervals using the conventional approach are too narrow for the  $X_t^{t+8}$  values of consumption and investment growth at both horizons. Coverage rates with the release-based approach for  $q = 8$  are closer to the nominal values, and there is evidence that the predictive density is calibrated well for all variables except the fed rate. The DSGE model is not able to provide good forecasts of the fed fund rates at both horizons (confirming the results of Herbst and Schorftheide (2012) and Del Negro and Schorftheide (2013)) using either approaches to deal with real-time data.

One could argue that the conventional approach provides density forecasts that are in general well calibrated if the first-final ( $q = 2$ ) is taken as the actual, as in Edge and Gurkaynak (2011). However, the usual argument, as presented in Del Negro and Schorftheide (2013), is that how we define the actual does not matter for the DSGE forecasting performance. The results in Table 6 suggest that the actual does matter when evaluating interval coverage and density calibration. It also shows that if we

<sup>8</sup> In a first step, I use a non-parametric Kernel estimator with Gaussian weights and bandwidth computed using cross-validation to estimate the predictive density over a grid of 1000 values between  $-15\%$  and  $+15\%$ . Then, in a second step, I use the smoothed predictive density to obtain the log score at the realisation value.



**Table 5**

Coverage and calibration of backcasts for the last observation with the release-based approach.

	Coverage: 70%		Coverage: 90%		Berkowitz Test—uniformity	
	$q = 2$	$q = 8$	$q = 2$	$q = 8$	$q = 2$	$q = 8$
Output gr.	22.5%	62.5%	45%	82.5%	0.006	0.000
Inflation	52.5%	70%	72.5%	82.5%	0.003	0.213
Consumption gr.	50%	52.5%	57.5%	72.5%	0.000	0.001
Investment gr.	35%	72.5%	55%	90%	0.000	0.534

Note: Computed for forecasting origins from 1999Q1 up to 2008Q1. The entries employ the Gibbs-M release-based specifications for  $q = 2$  (2nd quarterly release is the final value) and  $q = 8$  (release published two years after the observational quarter is the final value). Estimation was carried out with expanding windows over the out-of-sample period. Entries for Berkowitz test are  $p$ -values.

**Table 6**

Coverage and calibration of one and four-quarter ahead forecasts with the conventional (conv) and release-based (RB) approaches.

	Coverage 70%				Coverage 90%				Berkowitz Test—uniformity			
	$q = 2$		$q = 8$		$q = 2$		$q = 8$		$q = 2$		$q = 8$	
	Conv.	RB	Conv.	RB	Conv.	RB	Conv.	RB	Conv.	RB	Conv.	RB
$h = 1$												
Output gr.	82.5%	77.5%	65%	77.5%	92.5%	<b>90%</b>	<b>90%</b>	<b>90%</b>	0.067	0.189	0.187	0.680
Inflation	65%	75%	62.5%	<b>70%</b>	77.5%	85%	82.5%	87.5%	0.226	0.173	0.804	0.991
Consumption gr.	60%	65%	55%	72.5%	87.5%	82.5%	82.5%	<b>90%</b>	0.193	0.013	0.087	0.228
Investment gr.	72.5%	80%	47.5%	82.5%	97.5%	92.5%	67.5%	92.5%	0.170	0.055	0.002	0.039
Fed rate	67.5%	72.5%	67.5%	65%	82.5%	82.5%	82.5%	85%	0.034	0.039	0.034	0.010
$h = 4$												
Output gr.	<b>70%</b>	67.5%	<b>70%</b>	72.5%	85%	85%	87.5%	82.5%	0.974	0.664	0.652	0.493
Inflation	57.5%	75%	75%	72.5%	82.5%	<b>90%</b>	85%	<b>90%</b>	0.871	0.008	0.926	0.582
Consumption gr.	67.5%	67.5%	60%	<b>70%</b>	77.5%	82.5%	85%	77.5%	0.141	0.078	0.059	0.388
Investment gr.	62.5%	<b>70%</b>	52.5%	77.5%	87.5%	87.5%	82.5%	87.5%	0.170	0.346	0.192	0.513
Fed rate	40%	42.5%	40%	42.5%	52.5%	60%	52.5%	57.5%	0.000	0.000	0.000	0.000

Note: See notes of Table 5.

aim to predict revised data, then density forecasts with the release-based approach may provide better coverage and calibration.

The statistics in Table 6 are not adequate for evaluating if the vintage-based approach is relatively more accurate than the conventional approach. As a consequence, in Table 7, I present  $t$ -statistics and  $p$ -values of the test of equal accuracy using both MSFEs and log scores as the loss function. Results are presented for the two release-based specifications of Table 6 and the same five variables, but for  $h = 1, 2, 4$ . The negative  $t$ -statistics suggest that the release-based approach is more accurate than the conventional approach.

The specification that assumes the first-final estimate is the revised value ( $q = 2$ ) has a forecasting performance that is in general similar to that of the conventional approach, though it is worse in some cases. However, the specification with  $q = 8$  is able to significantly improve forecasts of consumption and investment growth at the first two horizons, presenting reductions of RMSFEs of around 30%. The caveat here is that fed fund rate forecasts are deteriorated at  $h = 1$ . One-step-ahead forecasts of output growth are also largely improved with  $q = 8$ , but not significantly at a 10% level. In agreement with previous results, the 'Gibbs-M,  $q = 8$ ' specification may perform better than the conventional approach in forecasting.

Table 8 presents additional forecasting results to investigate the robustness of the baseline results discussed above and presented in Table 7. Table 8 presents  $t$ -statistics on the equal accuracy test of log scores (because the tests deliver qualitatively similar results based on log scores and MSFEs in Table 7) for the three most popular variables in US macroeconomic forecasting (output growth, inflation and the fed rate). The additional four release-based specifications considered are 'Gibbs-M,  $q = 2$ ' and 'Gibbs-M,  $q = 8$ ' specifications, but modelling data revisions of only output growth and inflation, and 'MH,  $q = 2$ ' specifications with a common deterministic trend (as evaluated in Section 4) and a common stochastic trend.

Significant gains/losses in density forecasting performances by using the release-based approach instead of the conventional one for real-time forecasting may depend on the model specification because by modelling additional data revision processes, we increase the model complexity, which may not always be beneficial for forecasting. The specification with a common stochastic trend suggests one-step-ahead improvements for all three variables, although this is only statistically significant for the interest rate. The MH,  $q = 2$  specification shows significant improvements in predicting one-step-ahead output growth. The removal of data revisions of consumption growth and investment from the information set improves the forecasts of the fed fund rates.

Finally, I compare the specifications in Table 7 for forecasting origins from 2008Q3 up to 2013Q3 (using as actuals data released up to the 2016Q2 vintage if  $q = 8$ ) to check if the relative performance of the release-based approach sustains during this most recent period, which is in general associated with a zero lower bound in the policy rate. The results in Table 9 suggest that, as in the earlier period, the release-based approach improves forecasts of output, consumption and investment growth when predicting heavily revised data ( $q = 8$ ) and it is equivalent to the conventional approach if predicting the first final ( $q = 2$ ). During the most recent period, the relative performance of the release-based approach has improved when forecasting inflation, but the approach performs badly when predicting the fed rate.

In summary, I find evidence that the release-based approach can improve real-time forecasts at short horizons. By accounting for the uncertainty arising from future data revisions, the release-based approach improves the predictive density forecasts, particularly the interval coverages, of real macro variables.

#### 5.4. Real-time output gap estimates

Orphanides and van Norden (2002) argue that real-time estimates of the output gap are unreliable, and Watson (2007)

**Table 7**

Comparing release-based and conventional approaches in forecasting: equal accuracy tests based on MSFEs and logscores.

	$q = 2$			$q = 8$		
	RMSFE ratio	MSFE test	Logscore test	RMSFE ratio	MSFE test	Logscore test
Out. gr., $h = 1$	1.07	1.190 [0.883]	0.90 [0.815]	0.88	−0.937 [0.174]	−1.13 [0.129]
$h = 2$	1.09	1.406 [0.920]	1.35 [0.912]	1.00	0.065 [0.526]	−0.222 [0.412]
$h = 4$	1.02	1.067 [0.858]	−0.568 [0.284]	1.08	0.517 [0.697]	0.971 [0.834]
Inflation, $h = 1$	1.03	0.659 [0.745]	0.186 [0.574]	1.07	0.320 [0.626]	0.29 [0.614]
$h = 2$	1.08	1.218 [0.888]	1.51 [0.935]	1.04	0.183 [0.573]	0.57 [0.718]
$h = 4$	1.10	1.102 [0.865]	<b>1.79</b> <b>[0.964]</b>	1.22	0.954 [0.829]	1.231 [0.891]
Cons. gr., $h = 1$	1.02	0.268 [0.606]	0.273 [0.608]	0.76	− <b>1.668</b> <b>[0.048]</b>	− <b>1.506</b> <b>[0.066]</b>
$h = 2$	1.04	0.605 [0.727]	1.005 [0.843]	0.81	− <b>1.287</b> <b>[0.099]</b>	−1.031 [0.151]
$h = 4$	1.02	0.316 [0.624]	−1.064 [0.144]	1.04	0.194 [0.577]	−0.043 [0.483]
Inv. gr., $h = 1$	1.07	0.738 [0.77]	2.182 [0.985]	0.58	− <b>3.07</b> <b>[0.001]</b>	−2.893 [0.002]
$h = 2$	1.11	1.236 [0.892]	2.212 [0.987]	0.67	− <b>2.088</b> <b>[0.019]</b>	−1.988 [0.023]
$h = 4$	1.08	1.255 [0.895]	0.894 [0.816]	1.27	0.684 [0.753]	0.287 [0.613]
Fed rate, $h = 1$	1.02	0.454 [0.675]	−0.25 [0.400]	1.04	<b>1.680</b> <b>[0.954]</b>	<b>1.514</b> <b>[0.935]</b>
$h = 2$	0.98	−0.581 [0.281]	−0.51 [0.306]	1.01	0.895 [0.815]	−0.291 [0.386]
$h = 4$	0.98	−0.780 [0.218]	−1.18 [0.120]	1.02	1.024 [0.847]	−0.089 [0.464]

Notes: The RMSFE ratio column is the ratio between the release-based and the conventional RMSFEs. The following two columns are  $t$ -statistics of the Diebold and Mariano test of equal accuracy with the loss function indicated. The values in brackets are  $p$ -values. The forecasting model under the null is the DSGE model estimated with the conventional approach and the models under the alternative are the release-based Gibbs-M specifications with  $q = 2$  and  $q = 8$ . The  $t$ -statistics are based on 38 observations for forecasting origins from 1999Q1 up to 2008Q2 (vintage dates).

**Table 8**

Comparing release-based alternative specifications and conventional approaches in forecasting: equal accuracy tests based on logscores.

	Output growth			Inflation			Fed rate		
	$h = 1$	$h = 2$	$h = 4$	$h = 1$	$h = 2$	$h = 4$	$h = 1$	$h = 2$	$h = 4$
Gibbs-M, $q = 2$ ; only $y$ and $p$	0.41 [0.660]	0.866 [0.807]	− <b>1.32</b> <b>[0.093]</b>	−0.249 [0.401]	0.44 [0.671]	<b>1.37</b> <b>[0.916]</b>	− <b>1.82</b> <b>[0.034]</b>	0.82 [0.795]	−1.16 [0.121]
Gibbs-M, $q = 8$ , only $y$ and $p$	−0.895 [0.185]	0.519 [0.698]	0.811 [0.791]	0.350 [0.636]	0.68 [0.754]	<b>1.293</b> <b>[0.902]</b>	−0.367 [0.357]	−1.258 [0.104]	−0.925 [0.178]
MH, $q = 2$	− <b>2.15</b> <b>[0.016]</b>	−0.377 [0.353]	−1.014 [0.155]	−0.583 [0.280]	−0.284 [0.389]	<b>1.424</b> <b>[0.922]</b>	1.096 [0.864]	1.984 [0.970]	−0.309 [0.379]
MH, $q = 2$ Stoch trend	−1.19 [0.116]	0.712 [0.762]	0.837 [0.799]	−1.223 [0.111]	0.639 [0.739]	0.708 [0.760]	− <b>1.586</b> <b>[0.056]</b>	−0.196 [0.422]	−0.741 [0.229]

Notes: See notes of Table 7. These entries are  $t$ -statistics of for the test of equal accuracy for the difference in log scores. Negative values means that the release-based approach indicated in the first column is more accurate than the conventional approach. Values in brackets are  $p$ -values. The release-based alternative specifications considered in this Table are described in details in Section 5.3.

shows that this is mainly related to the two-sidedness of typical filters employed in the output gap computation. Based on the estimated coefficients and a set of observables, we can employ the Smets and Wouters (2007) model to compute a time series of the output gap. The output gap in the SW model is the difference between the current log (output) and the log(output) that would hold if there were no frictions. This measure of the output gap is one of the systematic components in the Taylor rule. In this subsection, I compare conventional and release-based measures of the output gap computed in real time based on their ability to replicate final estimates, which are the values computed using the 2008Q4 vintage in the conventional case and vintages up to 2008Q4 for the release-based specification.

At each forecast origin, I save estimates of the output gap and 90% intervals for the last observation ( $t = T, \dots, T + P - 1$ ) so

that I can plot a time series of real-time estimates<sup>9</sup>. The real-time gap estimates are computed for the conventional approach and the ‘Gibbs-M,  $q = 8$ ’ release-based approach, which was chosen based on its performance in the previous exercises.<sup>10</sup>

<sup>9</sup> These are real-time estimates because I only use data available up to the specific date to estimate the DSGE and compute the output gap with increasing larger windows of data.

<sup>10</sup> The conventional approach estimates are computed by using 5000 equally spaced draws from the saved posterior distribution of the DSGE parameters estimated using the RWMH algorithm. For each parameter draw, I use a smoother, similar to the one described in the Appendix A, to obtain estimates for the state variables, which are required for the output gap computation. Then, the empirical distribution of the last observation of the output gap is used to compute the gap

**Table 9**

Comparing release-based and conventional approaches in forecasting over the 2008–2013 period: equal accuracy tests based on MSFEs and logscores.

	$q = 2$			$q = 8$		
	RMSFE ratio	MSFE test	Logscore test	RMSFE ratio	MSFE test	Logscore test
Out. gr., $h = 1$	1.01	0.143 [0.557]	−0.20 [0.421]	0.63	<b>−1.589</b> <b>[0.056]</b>	<b>−1.405</b> <b>[0.080]</b>
$h = 2$	1.10	0.910 [0.819]	0.246 [0.597]	0.65	−1.180 [0.119]	−0.993 [0.161]
$h = 4$	1.00	−0.014 [0.495]	−0.613 [0.270]	0.38	−1.133 [0.129]	−1.159 [0.123]
Inflation, $h = 1$	0.96	−0.532 [0.297]	<b>−1.286</b> <b>[0.099]</b>	0.98	−0.055 [0.478]	−0.888 [0.187]
$h = 2$	0.87	−1.103 [0.135]	−0.389 [0.349]	0.80	−0.877 [0.190]	−1.168 [0.122]
$h = 4$	0.78	<b>−1.396</b> <b>[0.081]</b>	−0.693 [0.244]	0.75	−0.892 [0.186]	−0.793 [0.214]
Cons. gr., $h = 1$	1.07	0.590 [0.723]	1.038 [0.850]	0.72	−0.937 [0.174]	−0.961 [0.168]
$h = 2$	1.28	1.063 [0.856]	−1.107 [0.134]	0.68	−1.269 [0.102]	−0.258 [0.398]
$h = 4$	1.14	0.607 [0.728]	0.792 [0.786]	0.44	−1.247 [0.106]	−1.283 [0.100]
Inv. gr., $h = 1$	0.98	−0.271 [0.393]	−1.043 [0.149]	0.66	−1.187 [0.118]	−0.290 [0.386]
$h = 2$	0.99	−0.267 [0.395]	0.363 [0.642]	0.73	−0.688 [0.252]	−0.983 [0.163]
$h = 4$	1.05	0.563 [0.713]	−0.223 [0.412]	0.41	−1.230 [0.109]	<b>−1.344</b> <b>[0.090]</b>
Fed rate, $h = 1$	0.99	−0.277 [0.391]	1.185 [0.882]	1.06	1.198 [0.885]	<b>1.803</b> <b>[0.964]</b>
$h = 2$	0.92	−1.150 [0.125]	−0.644 [0.260]	1.08	<b>1.610</b> <b>[0.946]</b>	<b>1.847</b> <b>[0.968]</b>
$h = 4$	0.92	−0.898 [0.185]	−0.805 [0.211]	1.12	<b>1.550</b> <b>[0.939]</b>	<b>1.892</b> <b>[0.971]</b>

Notes: The RMSFE ratio column is the ratio between the release-based and the conventional RMSFEs. The following two columns are  $t$ -statistics of the Diebold and Mariano test of equal accuracy with the loss function indicated. The values in brackets are  $p$ -values. The forecasting model under the null is the DSGE model estimated with the conventional approach and the models under the alternative are the release-based Gibbs-M specifications with  $q = 2$  and  $q = 8$ . The  $t$ -statistics are based on 21 observations for forecasting origins from 2008Q3 up to 2013Q3 (vintage dates).

Fig. 4 presents the time series of the output gap estimates and 90% intervals obtained with the conventional approach in the upper plot and the release-based approach in the lower plot. Each plot also includes the estimate (posterior mean) obtained using the full sample (2008Q4 vintage), that is, the final value. The real-time output gap measures obtained with the DSGE model are remarkably reliable even if we employ the conventional approach. There are issues of unreliability for the 1998Q4 and 1999Q1 observations, but in general, there is a good match between the real-time and final measures. The release-based approach performs better in the sense that no large failure is noted. More importantly, if we measure the average width of the 90% interval over these 40 quarters for both approaches, we find that the width computed using the conventional approach is 2.2%, which is smaller than the 2.4% width computed using the release-based approach. These results are in agreement with the ones in Table 6 based on interval coverage rates: the conventional approach may underestimate the uncertainty around estimates obtained using the DSGE model by disregarding the impact of future data revisions.

## 6. Conclusions

Sargent (1989) argued that the behaviour of the statistical agency that provides data on output, inflation and other macroeco-

nommic variables should be taken into account when fitting a DSGE model to data. This paper proposes an approach for joint estimation of DSGE parameters and data revision processes. The release-based approach allows the statistical agency to revise data to reduce initial measurement errors and add new information to the initially released estimates. Households, firms and the government make their decisions using revised data. Because these entities and the econometrician have only the initial releases of the last observations, they use past data revision processes to compute estimates of the last observations.

The application of the release-based approach to the Smets and Wouters (2007) model suggests that initial releases differ from the final values because data revisions reduce initial measurement errors and the best estimates at the time of the initial release are not able to incorporate the effects of structural shocks, particularly investment-specific structural shocks. In addition, the release-based approach improves the real-time accuracy of predictive densities by bringing coverage rates closer to the nominal values when predicting heavily revised data. The improvements in forecasting performance are explained by considering the data uncertainty in the observations still subject to revision.

I also provide evidence that future data revisions are an important source of unexplained variation in initial releases of real macroeconomic variables (output, consumption and investment) and inflation. As the process of releasing revised data progresses, the size of revision shocks decreases and the correlation between unexpected data revisions and structural shocks may change. This implies that the estimated relative importance of business cycle sources varies with data maturity.

Future research should investigate the possible impact of data revisions on the measurement and identification of news and

estimate (mean) and the 90% interval (5% and 95% quantiles). For the release-based approach, I exploit the fact that at each Gibbs iteration, the algorithm requires the computation of a draw of the state vector, which allows us to compute the output gap. After removing 20% burn-in draws, I use the Gibbs posterior draws to compute the gap estimate (posterior mean) and the 90% interval for the last observation.

noise shocks as drivers of business cycles as defined and argued by Blanchard et al. (2013).

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## Appendix A. Metropolis-in-Gibbs algorithm for vintage-based estimation of DSGE models

### A.1. Priors and initialisation

The first step of the Gibbs sampling described below requires us to have values for  $\Sigma_\theta$ , which is the variance-covariance matrix of the parameters of the DSGE model at the posterior mode, computed based on the Hessian at the posterior mode. We compute the posterior mode of  $\theta$  using the state equation in (1) and the following measurement equations:

$$X_{t-q+1}^{t+1} = d(\theta) + H(\theta)x_{t-q+1} \text{ for } t = 1, \dots, T,$$

that is, only observations up to  $T - q + 1$  are employed in the computation of  $\hat{\Sigma}_\theta$  to initialise the required Metropolis step.<sup>11</sup> The priors on the DSGE parameters  $p(\theta)$  for the applications described in this paper are set as in Smets and Wouters (2007).

The Gibbs sampling algorithm also requires priors on the parameters of the data revision process  $\beta$ . The algorithm below exploits draws for  $\beta_{i,v}$  for  $i = 1, \dots, m$  and  $v = 1, \dots, q - 1$ , that is, I draw the parameters for each round of the data revision process and observed variable separately. Assuming that the structural shocks  $\eta_t$  and  $X_t$  are observed, I can write a regression for the  $v$  remaining revision rounds for each variable in the vector  $X_t$  as:

$$(X_{i,t}^{t+v} - X_{i,t}) = M_{i,v} + K_{i,v}(X_{i,t-1}^{t+v-1} - X_{i,t-1}) + A_{i,v}\eta_t + \xi_{i,t}^{(v)} \\ \text{for } i = 1, \dots, m \text{ and } v = 1, \dots, q - 1$$

$$z_{v,i,t} = w'_{v,i,t}\beta_{i,v} + \xi_{i,t}^{(v)}; w_{v,i,t} = (1, z_{v,i,t-1}, \eta_t)'$$

where  $\beta_{i,v}$  and  $w_{v,i,t}$  are  $k \times 1$  vectors with  $k = r + 2$  and  $r$  is the number of DSGE structural shocks innovations.

I define normal/inverse gamma independent priors for each revision regression. The priors on  $\beta_{i,v}$  are  $N(\underline{\beta}_{i,v}, \underline{V}_{i,v})$  where  $\underline{\beta}_{i,v} = \mathbf{0}_{k \times 1}$  and the prior variance diagonal elements are:

$$V_{i,v,j} = \varphi s_{v,i}^2 (W'_{j,v,i} W_{j,v,i})^{-1} \text{ for } j = 1, \dots, k$$

where  $W_{v,i}$  is a  $N \times k$  matrix with all observations of the row vector  $w'_{v,i,t}$ . I set  $\varphi = 25^2$  so that the prior variance is data dependent, but its degree of tightness is controlled by  $\varphi$ , which is set to a high value, that is, it is a loose prior. The implicit prior on the  $\text{var}(\xi_{i,t}^{(v)}) = \sigma_{v,i}^2$  is:

$$\sigma_{v,i}^2 \sim IG(s_{v,i}^2, \underline{v}).$$

I compute the prior for the variance as:

$$\underline{s}_{v,i}^2 = \frac{1}{N} \sum_{j=1}^N (z_{v,i,t} - w'_{v,i,t} \hat{\beta}_{i,v})^2$$

<sup>11</sup> Note that if the DSGE model in the conventional approach includes measurement errors, then  $X_{t-q+1}^{t+1} = d(\theta) + H(\theta)x_{t-q+1} + u_{t-q+1}$  should be used instead, and the parameters describing the measurement errors are drawn within the Metropolis step in the algorithm below.

where  $\hat{\beta}_{i,v}$  is the OLS estimate assuming that  $X_t = X_t^{t+q}$ , that is, using observations only up to  $T - q + 1$ , including the smoothed values of  $\eta_t$ . The scale is set as  $\underline{v} = 0.005$ .

Initial values for  $X_{T-q+1}^{(0)}, \dots, X_T^{(0)}$  are obtained by applying the state smoothing recursion described in Durbin and Koopman (2012, 4.4.4) to the state equations (10) with  $\theta$  set to the posterior mode values computed as described above and  $\beta$  set as the OLS estimates using observations up to  $T - q + 1$  and  $\eta_t$  smoothed using the disturbance smoothing recursion in Durbin and Koopman (2012, 4.5.3).

### A.2. The Metropolis-in-Gibbs Algorithm:

1. Conditional on  $X_{T-q+1}^{(j-1)}, \dots, X_T^{(j-1)}$ , a draw of the DSGE parameters  $\theta^{(j)}$  is obtained using a Metropolis step. A random walk candidate draw is:

$$\varrho = \theta^{j-1} + \varpi \text{ where } \varpi \sim N(0, c^2 \hat{\Sigma}_\theta),$$

where  $c$  is set such that the acceptance rates are around 30%. The candidate draw is accepted such that  $\theta^j = \varrho$  with probability:

$$\alpha(\varrho|\theta^{j-1}) = \min \left\{ 1, \frac{p(X^{(j-1)}|\varrho)p(\varrho)}{p(X^{(j-1)}|\theta^{(j-1)})p(\theta^{(j-1)})} \right\},$$

where  $p(X^{(j-1)}|\varrho)$  is the likelihood function computed at  $\varrho$  using data on  $X_t$  up to  $T$  with the last  $T - q + 1$  observations from the previous draw.

2. Conditional on  $\theta^{(j)}$  and  $X_{T-q+1}^{(j-1)}, \dots, X_T^{(j-1)}$ , we obtain a draw of  $\eta_1^{(j)}, \dots, \eta_T^{(j)}$  using a smoother. Recall that  $\eta_t \sim N(0, Q)$ ; therefore, we obtain draws for the DSGE innovation shocks as:

$$\eta_t^{(j)} \sim N(\hat{\eta}_{t|T}, \hat{Q}_{t|T}) \text{ for } t = 1, \dots, T,$$

where  $\hat{\eta}_{t|T} = E[\eta_t|X, \theta^{(j)}]$  and  $\hat{Q}_{t|T} = \text{var}[\eta_t|X, \theta^{(j)}]$  are computed using the disturbance smoothing recursion of Section 4.5.3 in Durbin and Koopman (2012).

3. Conditional on  $X_{T-q+1}^{(j-1)}, \dots, X_T^{(j-1)}$  and  $\eta_1^{(j)}, \dots, \eta_T^{(j)}$ , draws are obtained for the data revision parameters  $\beta_{i,v}$  and  $\sigma_{i,v}^2$  for  $i = 1, \dots, m$  and  $v = 1, \dots, q$  using normal and inverse gamma and closed-form solutions as in Koop (2003, ch. 4). The conditional draws are

$$\beta_{i,v}^{(j)}|\sigma_{i,v}^2, X, \eta_1^{(j)}, \dots, \eta_T^{(j)} \sim N(\bar{\beta}_{i,v}, \bar{V}_{i,v})$$

$$\sigma_{i,v}^{2(j)}|\beta_{i,v}, X, \eta_1^{(j)}, \dots, \eta_T^{(j)} \sim IG(\bar{s}_{v,i}^2, \bar{v})$$

where

$$\bar{V}_{i,v} = \underline{V}_{i,v}^{-1} + \left( (\sigma_{i,v}^2)^{-1} (W'_{v,i} W_{v,i}) \right)^{-1}$$

$$\bar{\beta}_{i,v} = \underline{\beta}_{i,v} (\underline{V}_{i,v}^{-1} \underline{\beta}_{i,v} + (\sigma_{i,v}^2)^{-1} (W'_{v,i} Z_{v,i}))^{-1}$$

$$\bar{s}_{v,i}^2 = \sum_{j=1}^T (z_{v,i,t} - w'_{v,i,t} \hat{\beta}_{i,v})^2 + \underline{v} s_{v,i}^2$$

$$\bar{v} = T + \underline{v}.$$

4. Conditional on DSGE and data revision process parameter draws, we use the state-space representation in (7) and (10) to draw  $X_{T-q+1}^{(j)}, \dots, X_T^{(j)}$ . Recall that the full vector of state disturbances is  $\varepsilon_t = [\eta_t', \xi_t^{1'}, \dots, \xi_t^{q-1'}]'$  and the variances of structural shocks innovations are part of  $\theta^{(j)}$  and that variances of the data revision innovations are  $\sigma_{i,v}^{2(j)}$  in the previous step. We obtain smoothed draws of the state vector  $\alpha_t$  by first obtaining smoothed draws of  $\varepsilon_t$  as

$$\varepsilon_t^{(j)} \sim N(\hat{\varepsilon}_{t|T}, \hat{P}_{t|T}) \text{ for } t = 1, \dots, T,$$



**Table 10**  
Convergence analysis.

	Metropolis-in-Gibbs				RWMH			
	$\theta$		$\beta$		$\theta$		$\beta$	
	InEff	PSR	InEff	PSR	InEff	PSR	InEff	PSR
Mean	1067	1.15	436	1.06	2122	1.26	2029	1.27
Median	277	1.01	70	1.01	1822	1.15	1822	1.15
10% quantile	95	1.00	17	1.00	566	1.03	412	1.03
90% quantile	3344	1.44	670	1.07	3881	1.53	4063	1.71

Notes: These values are computed for the DSGE model described in Section 3 and assuming that only data revisions of output growth and inflation are modelled,  $q = 2$  and no serial correlation in the revisions.  $\theta$  is the vector 36 DSGE parameters and  $\beta$  is the vector of 18 data revision parameters. InEff is the inefficiency factor computed with autocorrelation lag length of 15. PSR is the potential scale reduction that measures convergence by comparing within-chain and between-chain variance of the draws. Results are based on four chains of 20,000 draws with different starting values where the first 4000 draws are removed before the computation of the statistics in the table.

where  $\hat{\varepsilon}_{t|T}$  and  $\hat{P}_{t|T}$  are computed with the disturbance smoothing recursion in Durbin and Koopman (2012, 4.5.3). Then, we can obtain smoothed draws for the full state vector as:

$$\alpha_{t|T}^{(j)} = T(\theta^{(j)}, \beta^{(j)})\alpha_{t-1|T}^{(j)} + R(\theta^{(j)}, \beta^{(j)})\varepsilon_{t-1|T}^{(j)}$$

for  $t = T - q + 2, \dots, T$

which include draws for  $X_{T-q+2|T}^{(j)}, \dots, X_{T|T}^{(j)}$  because they are state variables. The advantage of this algorithm as suggested by Durbin and Koopman (2012, 4.9.3) is that we draw from a multivariate normal of dimension  $r + (q - 1)m$  instead of dimension  $n + 2(q - 1)m$ .

### A.3. Convergence analysis

This subsection provides analysis of the convergence performance of the Metropolis-in-Gibbs algorithm for estimating the DSGE model of Section 3 with the release-based approach in comparison with the RWMH algorithm. Both algorithms are applied for a specification of the model in Section 3 that assumes that data revisions are modelled only on output growth and inflation,  $q = 2$ , and there is no serial correlation in the revisions ( $K_{(1)} = 0$ ). This specification has 36  $\theta$  (DSGE) parameters and 18  $\beta$  (data revisions) parameters, and was chosen because for this reasonably limited number of parameters both algorithms can be easily applied, while for a large number of parameters (such as the ‘Gibbs-M,  $q = 8$ ’ specification in Sections 3–5), only the Metropolis-in-Gibbs is recommended. For both algorithms, I computed 20,000 draws from four chains with different initial values randomised around posterior mode values. Then I remove the first 4000 draws of each chain for initialisation. The scale parameter  $c$  is set within each algorithm to obtain the candidate draws for the DSGE model such as the acceptance rate is around 30%, as suggested by Herbst and Schorftheide (2016) to minimise the inefficiency factor.

Table 10 presents results for the inefficiency factor and the potential scale reduction factor assuming that the model is estimated using the full sample (as in Section 4.1). Average inefficiency factors are around 2000 for the RWMH algorithm, but there are 1000 for DSGE parameters and 400 for data revision parameters with the Gibbs-M algorithm. This means we need fewer draws for convergence with the Metropolis-in-Gibbs algorithm. These inefficiency values are compatible with the ones in Herbst and Schorftheide (2016) in particular if considering the large number of parameters estimated. The potential scale reduction (PSR) factor in Table 10 compares the convergence across chains. Convergence means values near 1. Average PSR values suggest that the Metropolis-in-Gibbs algorithm has converged, while the RWMH algorithm needs a few more replications. In Sections 4 and 5, we present results for ‘MH,  $q = 2$ ’ specifications using a chain of 70,000 replications to take this issue into account. One could potentially improve the performance of the RWMH algorithm following suggestions by Haario et al. (1999)

and alternatives in Herbst and Schorftheide (2016), but since convergence results support the Metropolis-in-Gibbs algorithm, additional algorithm improvements are left for future research.

### Appendix B. Smets and Wouters (2007) Model

In this appendix, I describe the log-linearised Smets and Wouters (2007) model. All endogenous variables present log-deviations from the steady state.

The endogenous variables are the following: output  $y_t$ ; consumption  $c_t$ ; labour, hours worked  $l_t$ ; nominal interest rate  $r_t$ ; inflation  $\pi_t$ ; real wage  $w_t$ ; wage markup  $\mu^w$ ; price markup  $\mu^p$ ; investment  $i_t$ ; value of capital stock  $q_t$ ; capital installed  $k_t$ ; capital services used in production  $k_t^s$ ; rental rate of capital  $r_t^k$ ; and capital utilisation costs  $z_t$ . The seven shocks are the following: total factor productivity  $\varepsilon_t^a$ ; investment-specific technology  $\varepsilon_t^i$ ; risk premium  $\varepsilon_t^b$ ; exogenous spending  $\varepsilon_t^g$ ; price-push  $\varepsilon_t^p$ ; cost-push  $\varepsilon_t^w$ ; and monetary policy  $\varepsilon_t^r$ .

1. Aggregate resource constraint:  $y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g$ .
2. From the consumption Euler equation:  $c_t = c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1}) - c_3 (r_t - E_t r_{t+1} + \varepsilon_t^b)$ .
3. From the investment Euler equation:  $i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon_t^i$ .
4. Arbitrage equation for the value of capital:  $q_t = q_1 E_t q_{t+1} + (1 - q_1) E_t r_{t+1}^k - (r_t - E_t r_{t+1} + \varepsilon_t^b)$ .
5. Production function  $y_t = \phi_b (\alpha k_t^s + (1 - \alpha) l_t + \varepsilon_t^a)$ .
6. Capital used:  $k_t^s = k_{t-1} + z_t$ .
7. Capital utilisation costs:  $z_t = z_1 r_t^k$ .
8. Dynamics of capital accumulation:  $k_t = k_1 k_{t-1} + (1 - k_1) i_t + k_2 \varepsilon_t^i$ .
9. Firms’ markup:  $\mu_t^p = \alpha (k_t^s - l_t) + \varepsilon_t^a - w_t$ .
10. Phillips Curve:  $\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \varepsilon_t^p$ .
11. Solution for rental-rate of capital:  $r_t^k = -(k_t - l_t) + w_t$ .
12. Workers’ markup:  $\mu_t^w = w_t - \left[ \sigma_l l_t + \frac{1}{1-\lambda/\gamma} (c_t - \lambda/\gamma c_{t-1}) \right]$ .
13. Wage dynamics:  $w_t = w_1 w_{t-1} + (1 - w_1) (E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w$ .
14. Monetary Policy rule:  $r_t = \rho r_{t-1} + (1 - \rho) \{ r_\pi \pi_t + r_y (y_t - y_t^p) \} + r_{\Delta y} [(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \varepsilon_t^r$ .

In order to link the parameters of the above equations with the structural parameters in Table 2, please refer to Smets and Wouters (2007).

The equations for the shocks are

1. exogenous spending:  $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a$
2. risk premium:  $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$
3. investment:  $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$
4. productivity:  $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$
5. price-push:  $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p$
6. cost-push:  $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w$
7. monetary policy:  $\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r$ .

## Appendix C. Additional results

Supplementary material related to this article can be found online at <http://dx.doi.org/10.1016/j.jeconom.2016.09.006>.

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