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EMPIRICAL APPROACHES TO SOVEREIGN DEBT DEFAULT AND MONETARY-FISCAL INTERACTIONS‡

Estimating Sovereign Default Risk†

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Since 2009, mounting fears over a eurozone sovereign debt crisis have heightened risks of sovereign debt restructuring and outright default. The objective of this article is to better understand the probability of (partial) default in eurozone countries.

While there is a growing literature on sovereign default risk premia for developed countries, a divide between empirical and theoretical studies remains. Using rational expectation models, theoretical studies show that the government's willingness/ability to service its debt depends on the underlying macroeconomic fundamentals and therefore is country specific; see Bi (2012), among many others. Moreover, economic agents' beliefs about the future states of the economy are important for the formation of default probabilities. Many empirical studies, nevertheless, use panel regressions that cannot account for country heterogeneity, or use historical fiscal responses to construct "debt limits" that are backward looking in nature.

This article aims to bridge this gap by using Bayesian methods to estimate a standard real business cycle model that allows for sovereign default. We estimate the model using post-EMU data for Italy and Greece and evaluate each country's historical probability of sovereign default. Although we find that Greece historically had a lower probability of default for a given level of debt, our estimates suggest that the Italian government is more willing to service its debt than the Greek government.

I. Model

Following Bi (forthcoming), our model is a closed economy with linear production technology, whereby output depends on the level of productivity (A_t) and the labor supply (n_t). Household consumption (c_t) and government purchases (g_t) satisfy the aggregate resource constraint,

$$c_t + g_t = A_t n_t, \tag{1}$$

where A_t follows an $AR(1)$ process,

$$A_t - A = \rho^A (A_{t-1} - A) + \varepsilon_t^A \tag{2}$$

with $\varepsilon_t^A \sim \mathcal{N}(0, \sigma_A^2)$.

The government finances lump-sum transfers to households (z_t) and unproductive purchases by levying a tax (τ_t) on labor income and issuing one-period bonds (b_t). Let q_t be the price of the bond in units of consumption at time t. For each unit of the bond, the government promises to pay the household one unit of consumption in the next period. The bond contract is, however, not enforceable. At each period, a stochastic fiscal limit (b_t^*) is drawn from its distribution $\mathcal{B}^*$. If the debt surpasses the fiscal limit, then it partially defaults. The default scheme can be summarized as,

$$\Delta_t = \begin{cases} 0 & \text{if } b_{t-1} < b_t^* \\ \delta & \text{if } b_{t-1} \geq b_t^* \end{cases}.$$

We specify the cumulative density function of the fiscal limit distribution as a logistical

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function with parameters η_1 and η_2 dictating its shape:

$$(3) \quad P(b_{t-1} \geq b_t^*) = \frac{\exp(\eta_1 + \eta_2 b_{t-1})}{1 + \exp(\eta_1 + \eta_2 b_{t-1})}$$

The government's budget constraint is given by

$$(4) \qquad \tau_t A_t n_t + b_t q_t = b_t^d + g_t + z_t$$

with $b_t^d = (1 - \Delta_t) b_{t-1}$. The tax rate and government spending evolve according to the following rules,

$$(5) \qquad \tau_t = u_t^\tau + \gamma^\tau (b_t^d - b)$$

$$(6) \qquad g_t = u_t^g + \gamma^g (b_t^d - b),$$

where $u_t^\tau = (1 - \rho^\tau)\tau + \rho^\tau \tau_{t-1} + \varepsilon_t^\tau$ and $u_t^g = (1 - \rho^g) g + \rho^g g_{t-1} + \varepsilon_t^g$ with $\varepsilon_t^\tau \sim \mathcal{N}(0, \sigma_\tau^2)$ and $\varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2)$ are $AR(1)$ components. The nondistortionary transfers are modeled as a residual in the government budget constraint, exogenously determined by an AR(1) process,

$$(7) \qquad z_t = (1 - \rho^z) z + \rho^z z_{t-1} + \varepsilon_t^z$$

with $\varepsilon_t^z \sim \mathcal{N}(0, \sigma_z^2)$.

With access to the sovereign bond market, a representative household chooses consumption (c_t), labor supply (n_t), and bond purchases (b_t) by solving

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t)$$

$$s.t. \;\; A_t n_t (1 - \tau_t) + z_t - c_t = b_t q_t - b_t^d.$$

The bond price reflects the household's expectation about the probability and magnitude of sovereign default in the next period. The optimal solution to the household's maximization problem must also satisfy the following transversality condition,

$$\lim_{j \to \infty} E_t \beta^{j+1} \frac{U_c(t+j+1)}{U_c(t)} b_{t+j+1}^d = 0.$$

We use the monotone mapping method to solve the decision rule of the bond price in terms of the state vector. At time t, the state vector is $\psi_t = (b_t^d, c_{t-1}, A_t, u_t^g, z_t, u_t^\tau)$, and the decision rule of the bond price can be solved as $q_t = q(\psi_t)$. We assume the utility function is $U(c_t, n_t) = \log(c_t - h\bar{c}_{t-1}) + \phi \log(1 - n_t)$. The online Appendix discusses the solution procedure in detail.

II. Estimation

The model is estimated for two countries: Italy (1999:II–2010:III) and Greece (2001:II–2010:III). The start dates represent the quarter following each country's official adoption of the euro. Five observables are used for the estimation, including real output, government spending, tax revenue, government debt, and a 10-year real interest rate. Data sources include the OECD, BIS, Consensus Economics, and the Survey of Professional Forecasts. See the online Appendix for a detailed description of the data.

A. *Methodology*

We estimate the model using Bayesian methods. This section briefly outlines the estimation procedure, and we refer the interested reader to the online Appendix for more details. The equilibrium system is written in the nonlinear state-space form

$$(8) \qquad \mathbf{x}_t = f(\mathbf{x}_{t-1}, \varepsilon_t, \theta)$$

$$(9) \qquad \mathbf{v}_t = \mathbf{A}\mathbf{x}_t + \mathbf{e}_t,$$

where observables $\mathbf{v}_t$ are linked with model variables $\mathbf{x}_t$ via the matrix $\mathbf{A}$, θ denotes model parameters, and $\mathbf{e}_t$ is a vector of measurement errors distributed $\mathcal{N}(0, \boldsymbol{\Sigma})$. We assume $\boldsymbol{\Sigma}$ is a diagonal matrix and calibrate the standard deviation of each measurement error to be 20 percent of the standard deviation of the corresponding observable variable.

We use a particle filter to approximate the likelihood function, as in Fernández-Villaverde and Rubio-Ramírez (2007). We combine the likelihood $\mathcal{L}(\theta \mid \mathbf{v}^T)$ with a prior density $p(\theta)$ to obtain the posterior kernel, which is proportional to the posterior density. We assume that parameters are independent a priori. However, we discard any prior draws that do not deliver a unique rational expectation equilibrium, as the analysis is restricted to the determinacy parameter subspace. We construct the posterior distribution of the parameters using the random walk Metropolis-Hastings algorithm. In each estimation, we sample 38,000 draws from the posterior distribution and discard the first

15,000 draws.[1] The sample is thinned by every 25 draws, and the likelihood is computed using 40,000 particles.

B. *Prior Distributions*

We impose dogmatic priors over some parameters. The discount rate is 0.99, so that the deterministic net interest rate is 1 percent. We calibrate the household's leisure preference parameter ϕ such that a household spends 25 percent of its time working at the steady state. The deterministic debt to GDP ratio, government spending to GDP ratio, and tax rate are calibrated to the mean values of our data samples. For Italy: $\overline{g}/\overline{y} = 0.1966$, $\overline{b}/\overline{y} = 1.19 \times 4$, and $\tau = 0.4148$. For Greece: $\overline{g}/\overline{y} = 0.1795$, $\overline{b}/\overline{y} = 1.14 \times 4$, and $\tau = 0.3387$.

The priors for the remaining parameters are listed in Table 1. For the responses of government spending and taxes to debt, we form priors for the long run responses in terms of percentage deviations from steady state, that is,

$$\gamma^{g,L} \equiv \frac{\gamma^g \overline{b}}{\overline{g}(1-\rho^g)}, \quad \gamma^{\tau,L} \equiv \frac{\gamma^\tau \overline{b}}{\overline{\tau}(1-\rho^\tau)}.$$

These values are more comparable to estimates in the literature.

For the standard deviations of shocks, we form priors for the standard deviations relative to the relevant steady-state variables: $\sigma_{k,p} \equiv \sigma_k/\overline{J}$ for $J = \{A, g, \tau, z\}$ and $k = \{a, g, \tau, z\}$.

We estimate one parameter from the fiscal limit distribution. Given the distribution specification in equation (3), the parameters η_1 and η_2 can be uniquely determined by two points on the distribution, $(\tilde{b}^*, \tilde{p}^*)$ and $(\hat{b}^*, \hat{p}^*)$:

$$\eta_2 = \frac{1}{\tilde{b}^* - \hat{b}^*} \log\left(\frac{\tilde{p}^*}{\hat{p}^*} \frac{1-\hat{p}^*}{1-\tilde{p}^*}\right)$$

$$\eta_1 = \log \frac{\tilde{p}^*}{1-\tilde{p}^*} - \eta_2 \tilde{b}^*.$$

[1] We use Fortran 90 code compiled in Intel Visual Fortran for the estimation. We use the computer server system at the Bank of Canada, which uses Xeon CPU X7460 at 2.66GHz and has 23 processors with 64G RAM. One evaluation using the particle filter takes 35 seconds. These computational constraints limit the number of draws from the Metropolis-Hastings algorithm.

TABLE 1—PRIORS

	Function	Italy	Greece
h	Beta	0.5 (0.2)	0.5 (0.2)
$\tilde{b}^*$	Uniform	1.6 (0.013)	1.6 (0.013)
$\gamma^{\tau,L}$	Gamma	1.1 (0.3)	1.1 (0.3)
$\gamma^{g,L}$	Gamma	0.4 (0.2)	1.1 (0.3)
ρ^a	Beta	0.8 (0.1)	0.8 (0.1)
ρ^g	Beta	0.8 (0.1)	0.8 (0.1)
ρ^τ	Beta	0.8 (0.1)	0.8 (0.1)
ρ^z	Beta	0.3 (0.1)	0.8 (0.1)
$\sigma_{a,p}$	Gamma	0.005 (0.003)	0.005 (0.003)
$\sigma_{g,p}$	Gamma	0.005 (0.003)	0.005 (0.003)
$\sigma_{\tau,p}$	Gamma	0.005 (0.003)	0.005 (0.003)
$\sigma_{z,p}$	Gamma	0.2 (0.1)	0.2 (0.1)

Note: The distribution and mean are listed, as well as the standard deviation in parentheses.

Since $(\tilde{b}^*, \tilde{p}^*)$ and $(\hat{b}^*, \hat{p}^*)$ provide a more intuitive description about the fiscal limit distribution than η_1 and η_2, we can fix $\tilde{p}^*$ and $\hat{p}^*$ at certain levels and estimate the corresponding $\tilde{b}^*$ and $\hat{b}^*$, instead of estimating η_1 and η_2 directly. We choose $\tilde{p}^* = 0.3$ and $\hat{p}^* = 0.999$. Unfortunately, given that defaults are never observed in our data, the data is uninformative about the upper bound of the distribution. Therefore, we estimate $\tilde{b}^*$ and fix the difference between $\tilde{b}^*$ and $\hat{b}^*$ to be 40 percent of steady-state output, which is chosen to capture the observation that once risk premia begin to rise, they do so rapidly. Given the lack of guidance for the parameter $\tilde{b}^*$, we adopt a diffuse uniform prior over the interval 1.4 to 1.8.

To our knowledge, this article is the first attempt to estimate a DSGE model of sovereign default. Thus, prior to estimating the model with real data, we performed estimations with simulated data (see the online Appendix). The results revealed that we cannot jointly identify the

TABLE 2—ITALY ESTIMATES

	Prior		Posterior: $\delta^A = 0.3788$		Posterior: $\delta^A = 0.2$		Posterior: $\delta^A = 0.0947$	
	mean	[5, 95]	median	[5, 95]	median	[5, 95]	median	[5, 95]
h	0.5	[0.17, 0.83]	0.14	[0.06, 0.21]	0.11	[0.03, 0.24]	0.11	[0.02, 0.26]
$\tilde{b}^*$	1.6	[1.42, 1.78]	1.52	[1.46, 1.60]	1.47	[1.43, 1.51]	1.60	[1.44, 1.78]
$\gamma^{\tau,L}$	1.1	[0.64, 1.67]	0.53	[0.45, 0.66]	0.56	[0.44, 0.68]	0.56	[0.28, 0.70]
$\gamma^{g,L}$	0.4	[0.12, 0.82]	0.30	[0.16, 0.56]	0.51	[0.26, 0.75]	0.54	[0.25, 0.80]
ρ^a	0.8	[0.61, 0.94]	0.96	[0.95, 0.97]	0.96	[0.95, 0.97]	0.96	[0.94, 0.98]
ρ^g	0.8	[0.61, 0.94]	0.84	[0.72, 0.87]	0.85	[0.78, 0.90]	0.86	[0.77, 0.91]
ρ^z	0.3	[0.15, 0.48]	0.49	[0.38, 0.67]	0.47	[0.30, 0.62]	0.50	[0.32, 0.66]
ρ^τ	0.8	[0.61, 0.94]	0.84	[0.81, 0.85]	0.84	[0.82, 0.87]	0.84	[0.81, 0.88]
$\sigma_{a,p}$	0.005	[0.001, 0.01]	0.010	[0.009, 0.012]	0.01	[0.009, 0.012]	0.01	[0.009, 0.012]
$\sigma_{g,p}$	0.005	[0.001, 0.01]	0.006	[0.005, 0.007]	0.006	[0.005, 0.008]	0.006	[0.005, 0.008]
$\sigma_{z,p}$	0.2	[0.07, 0.39]	0.13	[0.10, 0.17]	0.14	[0.10, 0.18]	0.13	[0.10, 0.18]
$\sigma_{\tau,p}$	0.005	[0.001, 0.01]	0.007	[0.006, 0.008]	0.007	[0.006, 0.009]	0.008	[0.006, 0.009]

rate of partial default δ and the fiscal limit parameter $\tilde{b}^*$ when defaults are not observed in the data. Given this limitation, we estimate our model for three different calibrations of δ: 0.0978, 0.05, and 0.0245. These calibrations imply annualized rates of default (δ^A) of 37.88 percent, 20 percent, and 9.78 percent, respectively. The range covers the actual default rates of emerging market economies over the period 1998 to 2005, as documented by Bi (2011).

III. Estimation Results

A. *Posterior Estimates*

Tables 2 and 3 compare the medians and 90 percent credible intervals of the posterior distributions estimated from the three specifications for each country.

For the Italian estimates (Table 2), several observations are noticeable when comparing across the δ calibrations. The estimates of $\tilde{b}^*$ suggest that if agents expect a 37.88 percent annualized default rate, there is a 30 percent probability of default when the current debt–to–annualized steady-state GDP ratio is between 1.46–1.60. In contrast, if agents expect a 20 percent default rate, the debt to GDP ratio associated with a 30 percent default probability ranges from 1.43–1.51. The lower $\tilde{b}^*$ estimates for a lower δ^A calibration are consistent with theory. The model tries to match the risk premium in the data through the values of δ^A and $\tilde{b}^*$. When δ^A is higher, agents expect to lose more of the face value of debt following a default. Thus, households demand a higher interest rate to compensate for this risk. For the given risk premium implied by the data, therefore, a higher $\tilde{b}^*$ is needed to offset a higher δ^A value.

With a low rate of default $(\delta^A = 0.0947)$, $\tilde{b}^*$ is not identified from Italian data, as the 90 percent posterior credible interval mirrors the prior. The correlation between Italian government debt and the ten-year interest rate is only 0.01, suggesting any risk premium for Italian debt is low over our sample and not easily identifiable. Thus, it appears that with a low default probability, which causes the model to resemble a no default, approximately linear model, the resulting loss in nonlinearity in the model makes the data uninformative about $\tilde{b}^*$.

Turning to the Greek estimates (Table 3), we see that the data are informative about $\tilde{b}^*$ for all δ^A calibrations.[2] The estimates of $\tilde{b}^*$ suggest that if agents expect a 37.88 percent annualized default rate, there is a 30 percent probability of default when the debt-to-GDP ratio is between 1.58–1.78. In contrast, if agents expect a 9.47 percent rate of default, the debt-to-GDP ratio

[2] Our estimates suggest the presence of multiple modes for Greece. These results may reflect the difficulty in estimating an unconditional fiscal limit with our data, as a recent shift in the fiscal limit in Greece seems apparent. A companion paper, Bi and Traum (2011), addresses this issue by estimating a state-dependent fiscal limit.

Table 3—Greece Estimates

	Prior		Posterior: $= \delta^A$ 0.3788		Posterior: $= \delta^A$ 0.2		Posterior: $= \delta^A$ 0.0947	
	mean	[5, 95]	median	[5, 95]	median	[5, 95]	median	[5, 95]
h	0.5	[0.17, 0.83]	0.13	[0.06, 0.25]	0.18	[0.11, 0.39]	0.08	[0.04, 0.18]
$\tilde{b}^*$	1.6	[1.42, 1.78]	1.67	[1.58, 1.78]	1.69	[1.58, 1.79]	1.45	[1.40, 1.57]
$\gamma^{\tau,L}$	1.1	[0.64, 1.67]	0.82	[0.54, 1.09]	0.73	[0.48, 1.00]	1.14	[0.94, 1.48]
$\gamma^{g,L}$	1.1	[0.64, 1.67]	1.73	[0.87, 2.97]	1.47	[1.12, 1.80]	1.51	[1.08, 1.78]
ρ^a	0.8	[0.61, 0.94]	0.91	[0.89, 0.92]	0.91	[0.89, 0.93]	0.90	[0.88, 0.91]
ρ^g	0.8	[0.61, 0.94]	0.88	[0.82, 0.96]	0.87	[0.83, 0.93]	0.87	[0.85, 0.96]
ρ^z	0.8	[0.61, 0.94]	0.74	[0.62, 0.86]	0.77	[0.66, 0.88]	0.85	[0.76, 0.92]
ρ^τ	0.8	[0.61, 0.94]	0.81	[0.75, 0.85]	0.79	[0.74, 0.83]	0.83	[0.81, 0.88]
$\sigma_{a,p}$	0.005	[0.001, 0.01]	0.010	[0.009, 0.013]	0.012	[0.011, 0.014]	0.01	[0.009, 0.012]
$\sigma_{g,p}$	0.005	[0.001, 0.01]	0.021	[0.017, 0.027]	0.024	[0.020, 0.027]	0.02	[0.019, 0.025]
$\sigma_{z,p}$	0.2	[0.07, 0.39]	0.29	[0.24, 0.36]	0.29	[0.23, 0.35]	0.30	[0.23, 0.36]
$\sigma_{\tau,p}$	0.005	[0.001, 0.01]	0.010	[0.008, 0.013]	0.010	[0.008, 0.012]	0.01	[0.008, 0.013]

associated with a 30 percent probability of default ranges from 1.40–1.57. As mentioned above, the lower $\tilde{b}^*$ estimates for a lower δ^A calibration are consistent with theory.

Interestingly, the $\tilde{b}^*$ estimates for Greece are virtually identical for the high and mid range default rate calibrations. Holding $\tilde{b}^*$ constant, a higher default rate implies a larger risk premium in the model. In order to avoid this, $\gamma^{g,L}$, the response of government spending to debt, adjusts. In the high δ^A calibration, the posterior for $\gamma^{g,L}$ has more values concentrated at higher levels than the posterior for the mid δ^A calibration. Ceteris paribus, a larger $\gamma^{g,L}$ implies a stronger response of government spending to debt, which lowers the risk premium. Thus, the two estimated specifications still imply similar risk premia overall.

Looking across countries, we find that the debt level associated with a 30 percent probability of default is higher in Greece than in Italy. This is due to the fact that over the sample period, Greece is estimated to adjust taxes and expenditures more systematically with fluctuations in its debt (that is, $\gamma^{g,L}$ and $\gamma^{\tau,L}$ are higher in Greece).

B. *Laffer Curve and Fiscal Limit*

In this section, we use the structural estimates to further explore how the market perceives the political willingness/ability to service its debt in Italy and Greece.

The proportional tax on labor income distorts a household's behavior as it lowers the after-tax wage and may induce households to work less. A higher tax rate can raise tax revenue when the existing rate is low, but it can reduce tax revenue when the existing rate is high, producing a Laffer curve. Laffer curves are usually dynamic as the shape depends on the state of the economy. In our model, for given levels of productivity and government purchases (A_t, g_t), the government can collect the maximum level of tax revenue, denoted as $T^{\max}(A_t, g_t)$, at the peak of the dynamic Laffer curve, denoted as $\tau^{\max}(A_t, g_t)$. The maximum level of debt that the government can possibly pay back is the sum of the discounted maximum fiscal surplus $(s_t^{\max})$ in all future periods.

$$\mathcal{B}^{\max} = E\sum_{t=0}^{\infty} \beta^{t+1} \frac{U_c^{\max}(A_{t+1}, g_{t+1})}{U_c^{\max}(A_0, g_0)} s_t^{\max},$$

where $s_t^{\max} = (T^{\max}(A_t, g_t) - g_t - z_t)$. $\mathcal{B}^{\max}$ is obtained, however, under the assumption that the government is willing to raise the tax at the peak of the Laffer curve, while angry protesters on Athens' streets illustrate the powerful political obstacles to raising tax rates in reality. A reduced-form representation of the political economy perspective is to discount the fiscal surplus not only by a pure rate of time preference, but also by an additional political factor (β^{pol}).

TABLE 4—MODEL-IMPLIED $\tilde{b}^{\max}$ AND ESTIMATED $\tilde{b}^{*}$

	Italy $= \delta^A$ 0.3788		Italy $= \delta^A$ 0.0947		Greece $= \delta^A$ 0.3788		Greece $= \delta^A$ 0.0947	
	mean	[5, 95]	median	[5, 95]	median	[5, 95]	median	[5, 95]
h	2.45	[2.38, 2.49]	2.47	[2.24, 2.51]	3.32	[3.15, 3.36]	3.26	[3.07, 3.35]
$\tilde{b}^{*}$	1.52	[1.46, 1.6]	1.6	[1.44, 1.78]	1.67	[1.58, 1.78]	1.45	[1.40, 1.57]
$\gamma^{\tau,L}$	0.62	[0.59, 0.67]	0.65	[0.58, 0.73]	0.5	[0.48, 0.54]	0.45	[0.42, 0.48]

$$\mathcal{B}^{*} = E\sum_{t=0}^{\infty}\beta^{t+1}\beta^{pol}\,\frac{U_c^{\max}(A_{t+1}, g_{t+1})}{U_c^{\max}(A_0, g_0)}\,s_t^{\max}.$$

Given a particular set of parameter draws (θ_i), we can compute the model-implied distribution, $\mathcal{B}_i^{\max}$, and the corresponding $\tilde{b}_i^{\max}$, at which the default probability is 0.3. $\tilde{b}_i^{*}$ is the corresponding draw for the debt threshold from our estimates. Thus, the ratio between the estimated $\tilde{b}_i^{*}$ and the model-implied $\tilde{b}_i^{\max}$ gives the political factor β_i^{pol}. The online Appendix discusses the procedure in detail.

The top row in Table 4 shows the median and the 90 percent credible intervals for $\tilde{b}^{\max}$ for Italy and Greece for various δ^A calibrations. The model-implied debt threshold is higher in Greece than in Italy, regardless of the δ^A calibrations. This implies that, were both Italian and Greek governments willing to tax at the peak of their Laffer curves, the latter would be able to service a higher level of debt; we assume the average levels of government spending and transfers in the future are the same as the historical levels, which are higher in Italy than in Greece.

For comparison, the second row in Table 4 lists the median and the 90 percent credible confidence intervals for the estimated $\tilde{b}^{*}$ in both countries. Interestingly, the implied political factor β^{pol}, calculated as the ratio between the estimated $\tilde{b}^{*}$ and the model-implied $\tilde{b}^{\max}$, is lower in Greece than in Italy. The median is 0.62–0.65 for Italy, but 0.45–0.5 for Greece. One interpretation is that the market perceives the Italian government is willing to raise taxes to the peak of the Laffer curve with a probability of 62 percent to 65 percent, while the Greek government with a mere 45 percent to 50 percent. In other words, the political willingness to service debt is 12 percent to 20 percent higher in Italy than in Greece.

IV. Discussion

In this article, we show how to estimate a structural model of sovereign default. Although our nonlinear model allows interactions among fiscal policy instruments and the fiscal limit, it is only a first step toward understanding and estimating default probabilities for developed countries. A companion paper, Bi and Traum (2011), extends the analysis to a state-dependent fiscal limit and offers a broader comparison across eurozone countries. To understand fully the complexities associated with default risk, several other features are worthy of modeling attention, including the interaction of monetary and fiscal policies; the interaction of the financial sector and the government; and open economy issues including foreign holdings of debt and contagion risks.

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