Growth Theory

Session 2: The Solow-Swan Model

ECO5021F: Macroeconomics
University of Cape Town

Readings

Required

- Romer, D. (2019). Advanced Macroeconomics. Chapter 1.

Recommended

- Econtalk: Romer on Growth (2007); Spence on Growth (2010)
- Mankiw, G., D. Romer, and D.N. Weil. (1992) “A Contribution to the Empirics of Economic Growth,” Quarterly Journal of Economics, vol.107, p.407-437
- Solow, R.M. (1994). “Perspectives on Growth Theory,” Journal of Economic Perspectives, vol. 8, no. 1, pp. 45-54.

Contents:

The Harrod-Domar model

Assumptions in this model

- A given technology exhibiting fixed factor proportions (K/L).
  - i.e.,: Leontief/Perfect Complements
- The national income equation: $Y_t = C_t + S_t$.
- For equilibrium: we require that $I_t = S_t$
- The evolution of capital stock is given by $K_{t+1} = (1-\delta)K_t + I_t$, where δ is the depreciation rate of capital.
- The capital output ratio is fixed: $(K/Y = \Delta K/\Delta Y = \upsilon)$.
- Making a few substitutions, we can now rewrite our capital accumulation equation as:

$$\begin{aligned} \upsilon Y_{t+1} &= (1-\delta)\upsilon Y_t + sY_t \\ Y_{t+1} - Y_t &= (s/\upsilon - \delta)Y_t \\ \frac{Y_{t+1} - Y_t}{Y_t} &= (s/\upsilon - \delta) \end{aligned}$$

The Harrod-Domar model

Fundamental equation & implications

$$\frac{Y_{t+1} - Y_t}{Y_t} = s \cdot \frac{1}{v} - \delta \tag{1}$$

$\Rightarrow$ The growth rate of GDP is primarily determined by the savings ratio (s) and capital output ratio (v).

- From (1) it is clear that HD growth theory “sanctioned the overriding importance of capital accumulation in the quest for enhanced growth.” (Shaw, 1992)
- Conclusion: growth proportional to savings

The Harrod-Domar model

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- From (1) it is clear that HD growth theory "sanctioned the overriding importance of capital accumulation in the quest for enhanced growth." (Shaw, 1992)
- Conclusion: growth proportional to savings
- Central development problem was simply to increase resources devoted to investment
- Implications: since budgetary surpluses could substitute for domestic savings, fiscal policy became identified as the primary growth instrument. Government had a role to play.
- Empirically: massive failure (think Soviet Union)

The Harrod-Domar model

Why the epic failure?

- In this model K/Y and K/L is fixed.
- Solow (1956) major critique of HD: "even for the long run the economic system is at best balanced on a knife-edge of equilibrium growth."[1]
- For equilibrium K and Y, as well as K and L, must always grow at the same rate.

Key:

- no trade-off between K and L;
- no diminishing returns to K and L

[1] If the magnitudes of the key parameters (s, K/Y, n) were to change $\rightarrow$ either growing unemployment or prolonged inflation. (n is the population (labour) growth rate.)

The Solow-Swan model

Basic idea

Goal: Develop a simple framework for the proximate causes and the mechanics of economic growth and cross-country income differences

In short: Endogenizes the capital/labour (K/L) ratio in a world with an exogenous savings rate, productivity growth rate. No micro foundations

Solow-Swan model tries to emphasize the fact that output is related in some systematic way to inputs in the production process

The Solow-Swan model

Basic idea

- Model has two underlying principles
  1. Goods and labour markets clear
  2. Diminishing return to capital
- Solow used this model to answer questions related to
  1. The dynamics of growth
  2. The long-term relationship between growth and savings, population growth and technological progress
  3. Convergence in income growth rates

The Solow-Swan model

Assumptions: Households

- Closed economy, with unique final good.
- Economy is inhabited by a large number of households, and for now households *will not* be optimizing.
  - This is the key difference between the Solow and other neoclassical growth models.
- Households save a constant exogenous fraction $s \in (0, 1)$ of their disposable income.

The Solow-Swan model

Assumptions: Firm

- The aggregate production function is a function of **capital, labour and technology**

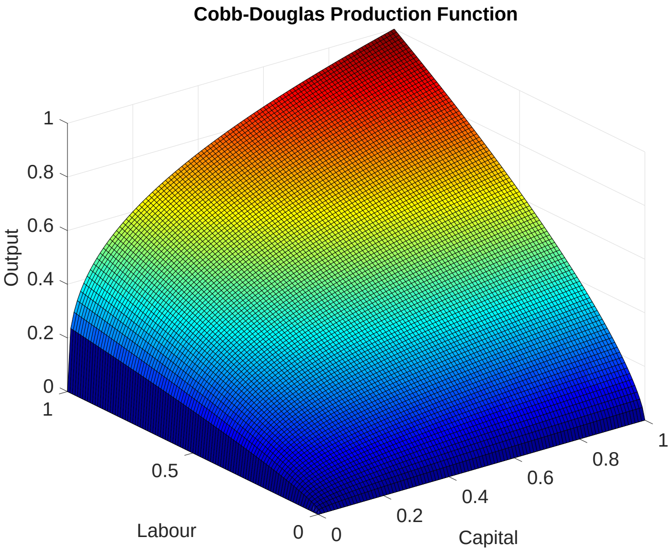
$$Y = F(K, L, A)$$

- Major assumption is that technology is **free** (publicly available, non-excludable, non-rival)
- Assume that all firms have access to the same **production function**. This aggregate production function for the unique final good is

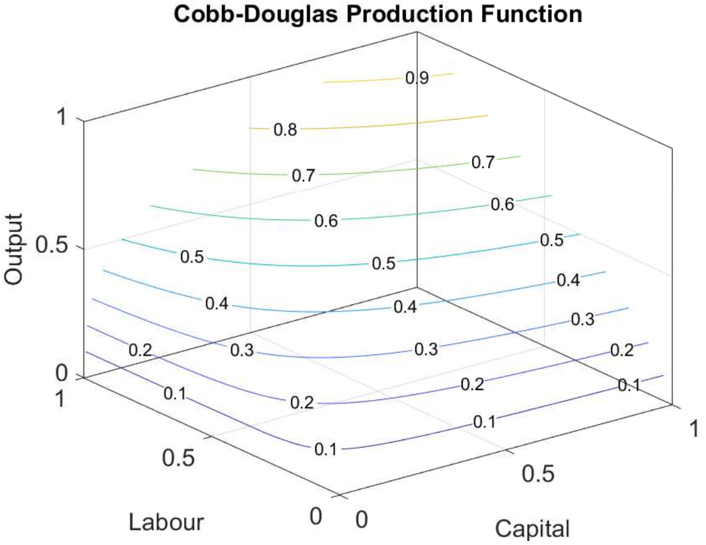
$$Y = K^{\alpha}(AL)^{1-\alpha}$$

- Referred to as a **Cobb-Douglas production function** with *labour augmenting technology*

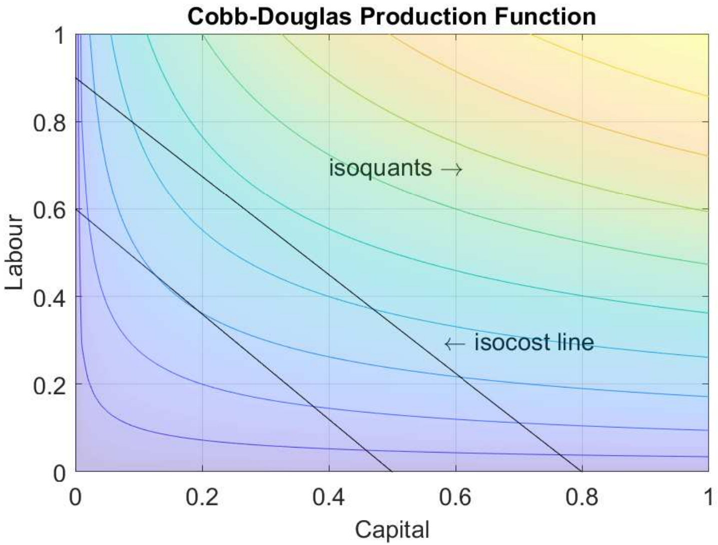
The Solow-Swan model



The Solow-Swan model



The Solow-Swan model



The Solow-Swan model

Assumptions: Firm

- With the assumption of constant returns to scale we can write the equation in its intensive form

$$y = f(k) = (K/AL)^\alpha = k^\alpha$$

- Some assumptions concerning the intensive form are:[2]

$$\text{Concave} = \begin{cases} f(0) = (0)^\alpha & = 0 \\ f'(k) = \alpha k^{\alpha-1} & > 0 \\ f''(k) = \alpha k(\alpha-1)^{\alpha-2} & < 0 \end{cases}$$

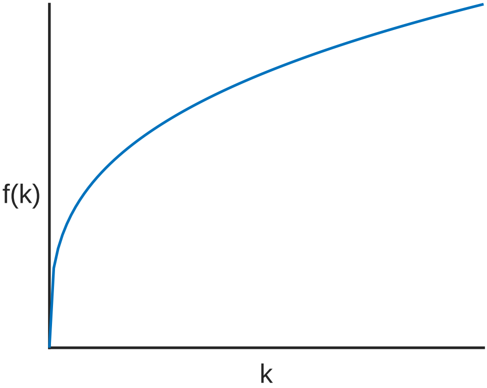
$$\text{Inada} = \begin{cases} \lim\limits_{k\to\infty} = f'(k) & = 0 \\ \lim\limits_{k\to 0} = f'(k) & = \infty \end{cases}$$

[2] $\partial y/\partial k = \partial f(k)/\partial k = f'(k) = \alpha(y/k)$

The Solow-Swan model

Assumptions: Firm

- Intensive form of this production function can be drawn in two dimensional space



Concavity (strict)? Inada conditions?

The Solow-Swan model

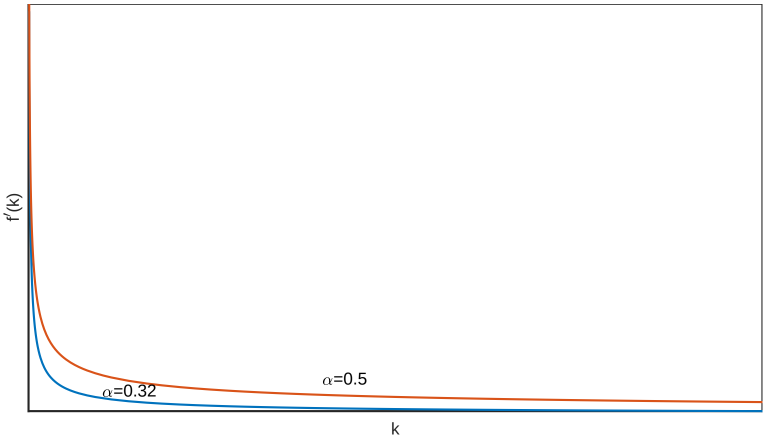
Assumptions: Firm

- The first three assumptions relate to the shape of the function
- These assumptions tell us the function is concave:

  i.e. increasing ($f'(\cdot) > 0$) at a decreasing rate ($f''(\cdot) < 0$)
- The final two assumptions are known as the **Inada conditions** and are required for stability of the model
- Inada conditions state the marginal productivity $f'(\cdot)$ is large for small values of capital per effective labour and *vice versa*

The Solow-Swan model

Figure: A graph of the effective capital derivative: $f'(k)$.



Gives an idea of the limits ...

The Solow-Swan model

Discrete v. Continuous

- Most of us are familiar with discrete time growth rates, namely:

$$g = \frac{Y_{t+1} - Y_t}{Y_t} = \frac{\Delta Y_{t+1}}{Y_t}$$

- One can represent the Solow model with discrete time dynamics, but Romer uses the continuous version.
- In discrete time, if output grows by, e.g., 4% then we would have that $Y_{t+1} = (1+g)Y_t = (1.04)Y_t$
- Applying this formula year after year we would have:

$$\begin{array}{rl} \text{two periods:} & Y_{t+2} = (1+g)Y_{t+1} = (1+g)^2 Y_t \\ & \vdots \\ n \text{ periods:} & Y_{t+n} = (1+g)Y_{t+(n-1)} = (1+g)^n Y_t \end{array}$$

The Solow-Swan model

Discrete v. Continuous

- If we think about variables as continuous functions of time we can use the methods of calculus and **differential equations**
- Our notation will be slightly different for continuous time ...
  - $\Rightarrow$ we use $Y(t)$ instead of Y_t
- The (instantaneous) change of Y per unit of time at moment t is a *time derivative*:

$$\frac{dY(t)}{dt} = \dot{Y}(t) \tag{2}$$

- Measures the amount of change in a variable as time passes
- If you look at the previous slide, it is similar to the discrete time first difference equation $\rightarrow \Delta Y_{t+1} = Y_{t+1} - Y_t$
- Provides the *amount* of growth in Y, but *not the rate* of growth
- To establish a *growth rate* we must divide by the level:

$$\begin{aligned} \text{discrete growth rate} \qquad & g_Y = \Delta Y_{t+1}/Y_t \\ \text{continuous growth rate} \qquad & g_Y = \dot{Y}(t)/Y_t \end{aligned}$$

The Solow-Swan model

Discrete v. Continuous

Given the initial value of Y at time 0 and a generic rate g, how large will Y be at some time t in future:

- In discrete time: $Y_t = (1+g)^t Y_0$
- Continuous time version $\rightarrow$ *continuous* compounding
- Gives the corresponding formula:

$$Y(t) = e^{gt} \cdot Y(0) \tag{3}$$

The Solow-Swan model

Discrete v. Continuous

Given the initial value of Y at time 0 and a generic rate g, how large will Y be at some time t in future:

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- How long will it take a country to double in size?

The Solow-Swan model

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  Rule of 70: $70/g = t$
  $\Rightarrow 70/2\% = 35yrs$ or $70/4\% = 17.5yrs$

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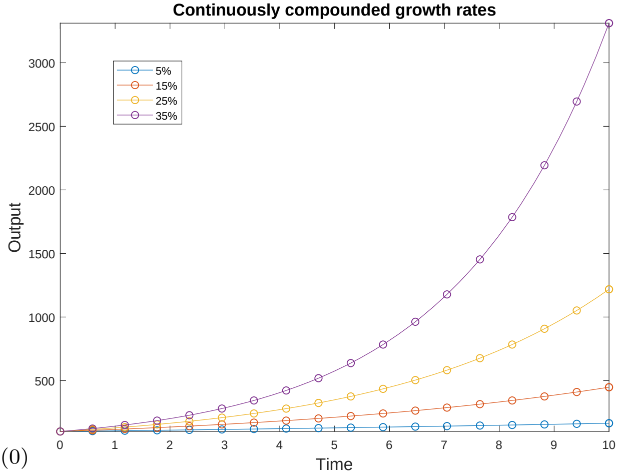
  Rule of 70: $70/g = t$
  $\Rightarrow 70/2\% = 35yrs$ or $70/4\% = 17.5yrs$
  Take natural logarithm (inverse of e) of Eq. (3):

$$\begin{aligned} lnY(t) &= [lnY(0)] + gt \\ ln(Y(t)/Y(0)) &= gt \\ ln(2) &= gt \Rightarrow ln(2)/g = t \Rightarrow 69.3/(100 * g) = t \end{aligned}$$

The Solow-Swan model

Discrete v. Continuous



$$Y(t) = e^{gt} \cdot Y(0)$$

More generally, e^{gt} shows the effects of continuously compounding growth (or interest) over the period $[0, t]$.

Analogously, the present value of 1 unit of output at t is e^{-gt} (see Romer pp.13-14)

The Solow-Swan model

The evolution of the inputs of production

- In the textbook Romer assumes that the labour force (L) and stock of “knowledge” or “effectiveness of labour” (A) both grow at constant rates:

$$\dot{L}(t) = nL(t),$$
$$\dot{A}(t) = gA(t),$$

where n and g are exogenous parameters and where a dot over a variable denotes a derivative wrt time: $\dot{X}(t) = dX(t)/dt$

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- The growth rate of a variable is equal to the time derivative of the natural log of the variable:

$$\frac{d\ln L(t)}{dt} = \underbrace{\frac{d\ln L(t)}{dL(t)} \cdot \frac{dL(t)}{dt}}_{\text{chain rule}} = \frac{1}{L(t)} \cdot \dot{L}(t)$$

- The same calculations are true for technology, just replace L with A, and n with g.

The Solow-Swan model

The evolution of the inputs of production

- Output is divided between consumption and investment

$$Y(t) = C(t) + I(t),$$

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- In the closed economy we have that savings equals investment

$$S(t) = I(t) = sY(t),$$

  where the fraction of output devoted to investment (s) is exogenous and constant.

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- In the closed economy we have that savings equals investment

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where the fraction of output devoted to investment (s) is exogenous and constant.

- The equation of motion (or evolution) of the capital stock can be described by the following equation:

$$\dot{K}(t) = sY(t) - \delta K(t),$$

where existing capital depreciates at rate δ.

The Dynamics of the Model

The dynamics of k

- How do we determine the behaviour of the economy we have just described?
- The evolution of two of the three inputs into production (L and A) is exogenous ...
- Because the economy may be growing over time we focus on the capital stock per unit of effective labour: $k = K/AL$.
- We can use the chain rule (for partial differentiation) to derive the evolution of k:[3]

$$\dot{k}(t) = sf(k(t)) - (n + g + \delta)k(t). \tag{4}$$

[3] Recall: output per unit of effective labour $y = f(k)$ and the fraction of output that is saved is s.

The Dynamics of the Model

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Where does the $n + g$ come from? . . . Derivation for your tutorial!

[3]Recall: output per unit of effective labour $y = f(k)$ and the fraction of output that is saved is s.

The Dynamics of the Model

The fundamental equation

$$\dot{k}(t) = sf(k(t)) - (n + g + \delta)k(t)$$

Eq. (4) states that the *rate of change* of the the capital stock per unit of effective labour is the difference between actual investment, $sf(k)$, and *break-even* investment, $(n + g + \delta)k$.

The Dynamics of the Model

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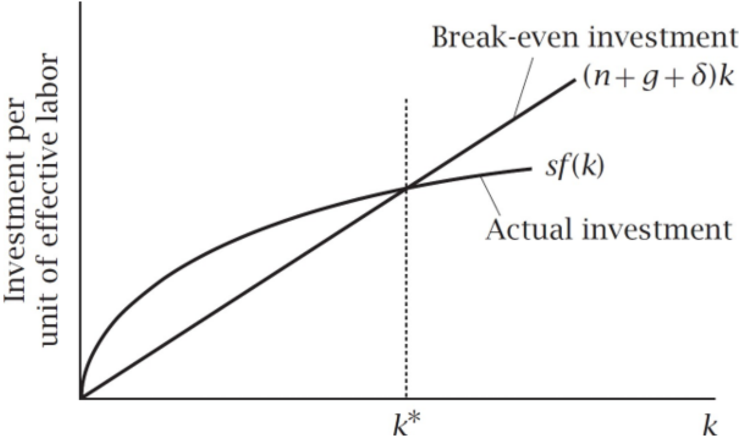
We can plot the two terms of this equation for $\dot{k}$ as functions of k . . .

The Dynamics of the Model

Phase diagram



Recall the Inada conditions . . . $f'(k) \to \infty$ as $k \to 0$ and $f'(k) \to 0$ as $k \to \infty$ and $f''(k) < 0$.

The Dynamics of the Model

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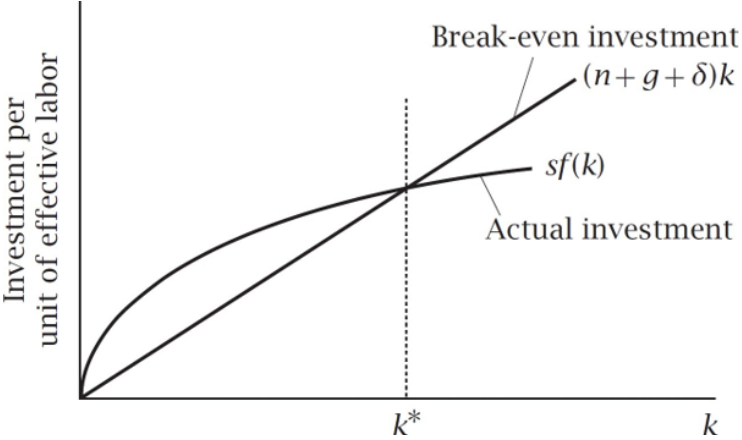

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Hint: slopes of the lines are $sf'(k)$ and $(n + g + \delta)$.

The Dynamics of the Model

Phase diagram

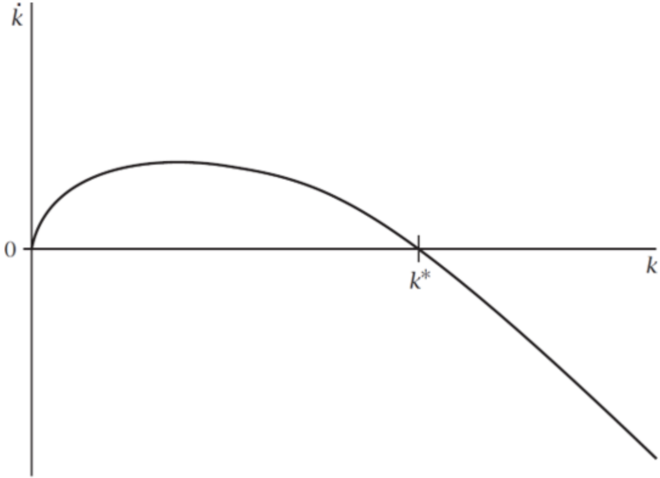


FIGURE 1.3 The phase diagram for k in the Solow model

Regardless of where k starts, it converges to k^* and remains there ...

The Dynamics of the Model

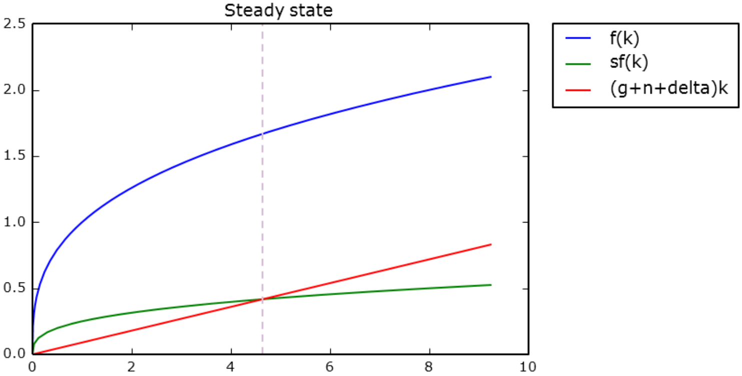
The Balanced Growth Path (stable equilibrium)

- This "steady state" does not mean that the economy is stagnating.
- Instead the economy has reached the *balanced growth path*: each variable of the model is growing at a constant rate. Characteristics of the BGP follow as:

1. Variables expressed per unit of effective labour (k^*, y^*, c^*) remain unchanged
2. By assumption labour and knowledge (or technological progress) are growing at n and g.
3. Given that capital stock is $K = kAL$ and we have constant returns to scale:

$$\begin{aligned} \dot{K}/K;\ \ \dot{A}L/AL;\ \ \dot{Y}/Y &= n + g \\ K/L;\ \ Y/L &= g \end{aligned}$$

The Steady State



Given that $\dot{k}(t) = 0$ on the *balance growth path*, we can solve for the steady-state levels of capital and output per unit of effective labour

$$k^* = \left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}}$$

$$y^* = \left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

The Dynamics of the Model

Consumption in the Steady State

Introducing households into the model raises the issue of welfare from consumption (instead of output)

- Consumption: $C = Y - S = Y - I$ in the closed economy
- In consumption per unit of effective labour terms:

$$c = f(k) - sf(k)$$

- On the BGP: actual investment $sf(k^*)$, equals break-even investment, $(n + g + \delta)k^*$
- Therefore, c^* becomes:

$$c^* = f(k^*) - (n + g + \delta)k^* \tag{5}$$

The Dynamics of the Model

Consumption in the Steady State

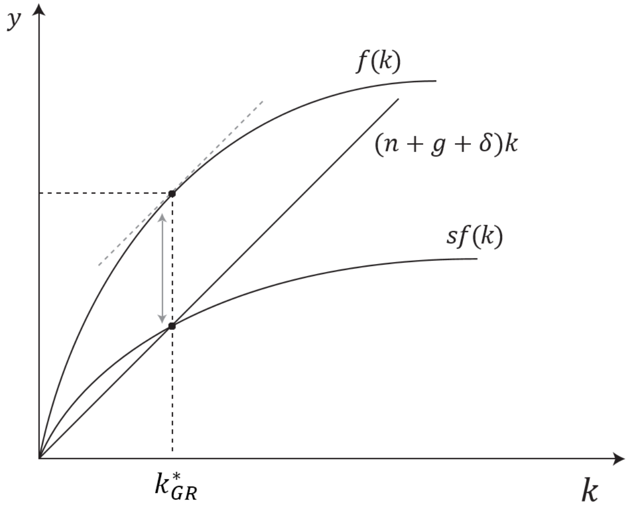
Note: re-writing the fundamental equation in terms of consumption delivers some interesting results:

$$c = f(k) - sf(k)$$

- The capital-effective labour ratio has two opposing effects on consumption per unit of effective labour
  1. A rise in k raises income, $f(k)$; and thereby per capita consumption
  2. However, a rise in k raises the amount of investment required to maintain the capital stock per effective labour, and so lowers the per capita consumption

The Dynamics of the Model

Golden Rule Level of Capital Stock



From (5): $\frac{\partial c^*}{\partial k^*} = f'(k^*) - (n+g+\delta) = 0 \Rightarrow k^*_{GR}$

- This diagram tells us that you cannot raise consumption indefinitely by raising the capital-effective labour ratio
- There is going to be a **golden rule level of capital stock** which *maximises* consumption per effective unit of output
- In this model there is *no reason* to expect that the balanced growth path $=$ the golden rule level of capital stock
- The reason is that the *rate of savings in this model is exogenous*; which means there is no guarantee that the optimum amount of savings is done

The Dynamics of the Model

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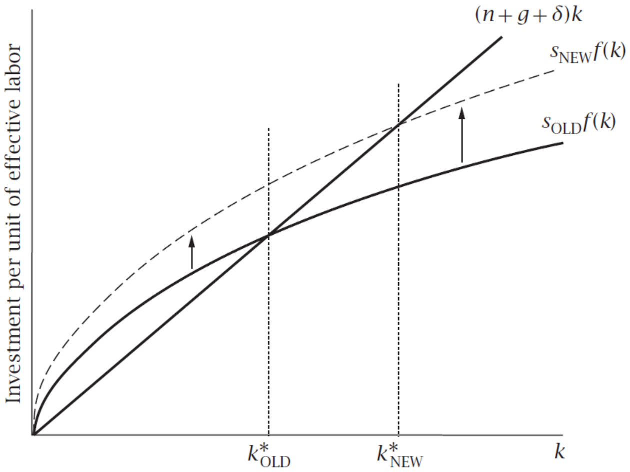
…back to this in a moment …

The Impact of a Change in the Savings Rate

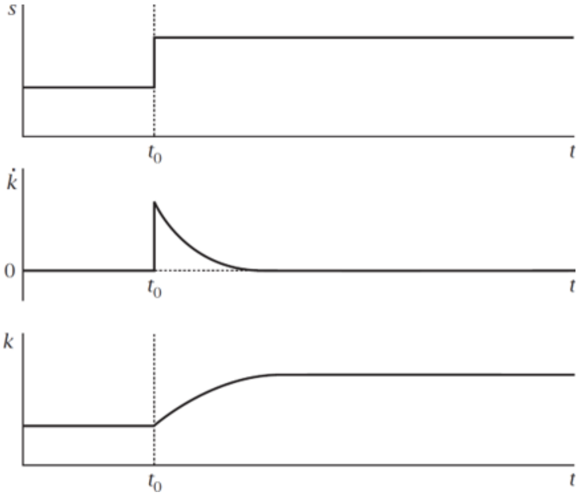
The impact on output



- A savings rate change does NOT affect long run growth in Y/L
- Has a *level effect* but not a *growth effect*

The Impact of a Change in the Savings Rate

The effects of an increase in the saving rate



The Impact of a Change in the Savings Rate

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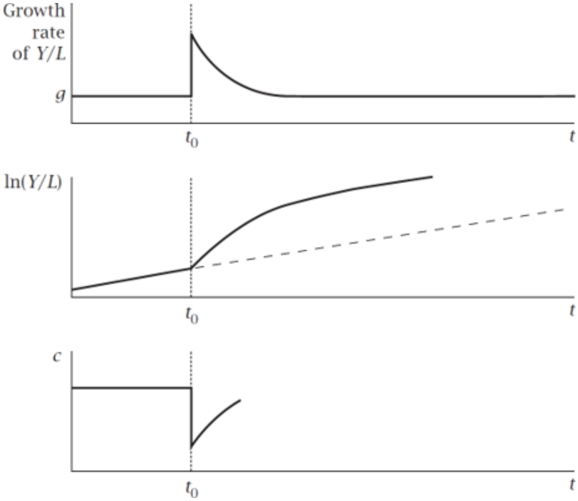


What happens to consumption?

The Impact of a Change in the Savings Rate

The impact on consumption

- Recall: a change in the savings rate has *level effect* but not a *growth effect*

$$c^* = f(k^*) - (n+g+\delta)k^*$$

Take the partial derivative of steady-state consumption wrt savings:

$$\frac{\partial c^*}{\partial s} = [f'(k^*) - (n+g+\delta)]\frac{\partial k^*}{\partial s}\ . \tag{6}$$

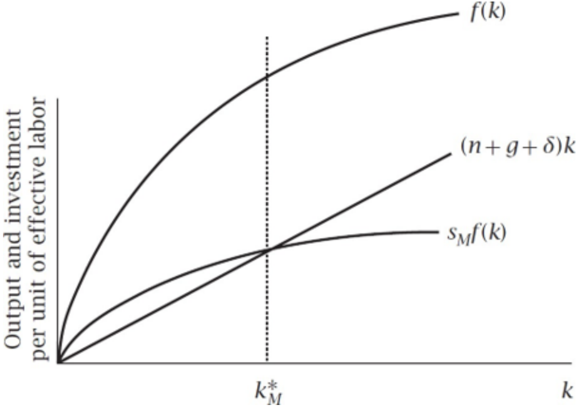
And recall from (5):

$$\frac{\partial c^*}{\partial k^*} = f'(k^*) - (n+g+\delta) = 0 \Rightarrow k^*_{GR}\ \ldots$$

The Impact of a Change in the Savings Rate

Case 1: the impact on consumption

Case 1: $f'(k^*) = (n+g+\delta)$

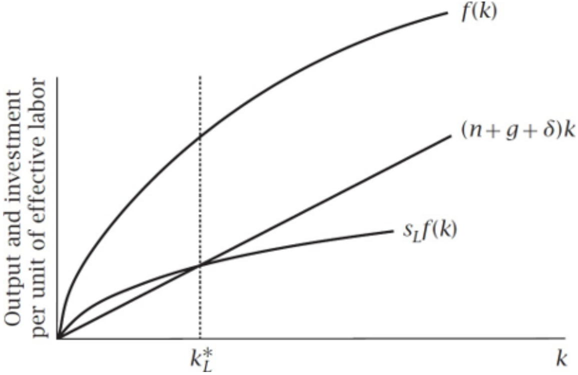


A marginal change in s has no effect on consumption in the long-run and consumption is at its maximum possible level. $\frac{\partial c^*}{\partial k^*} = f'(k^*) - (n+g+\delta) = 0 \Rightarrow k_M^* = k_{GR}^*$

The Impact of a Change in the Savings Rate

Case 2: the impact on consumption

Case 2: $f'(k^*) > (n+g+\delta)$, s low, k^* low

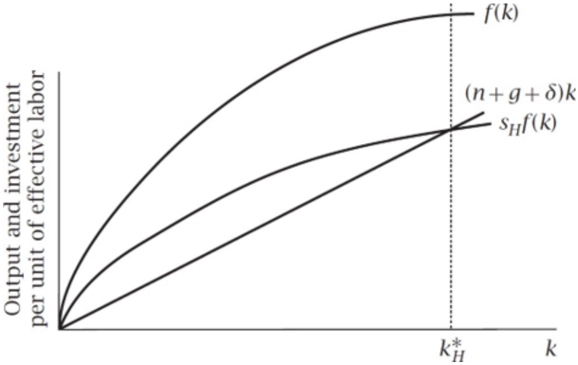


- $\uparrow s$ raises consumption in the long-run $\Rightarrow$ implies a higher standard of living in the long run
- i.e., if the steady state level of capital k^* does not exceed the golden rule capital stock: $k^*_L < k^*_M$.

The Impact of a Change in the Savings Rate

Case 3: the impact on consumption

Case 3: $f'(k^*) < (n+g+\delta)$, s high, k^* high



- $\uparrow s$ lowers consumption even when the economy has reached its new balanced growth path.

Quantitative Implications

The effect on output in the long run (see Romer pp.23-26)

$$\begin{aligned} \frac{\partial y^*}{\partial s} &= f'(k^*)\frac{\partial k^*}{\partial s} \\ \frac{\partial k^*}{\partial s} &= \frac{f(k^*)}{(n+g+\delta) - sf'(k^*)} \end{aligned}$$

Substituting in yields:

$$\frac{\partial y^*}{\partial s} = \frac{f(k^*)f'(k^*)}{(n+g+\delta) - sf'(k^*)}$$

which we can convert into an elasticity by $\times\ s/y^*$:

$$\frac{s}{y^*}\frac{\partial y^*}{\partial s} = \frac{\alpha_K(k^*)}{1-\alpha_K(k^*)},$$

where $\alpha_K(k^*) = k^* f'(k^*)/f(k^*)$ is the elasticity of output wrt capital at $k = k^*$. [We use the BGP condition: $sf(k^*) = (n+g+\delta)k^*$.]

In most countries, $\alpha = 0.33 \rightarrow$ elasticity of y^* wrt s is 0.5. A small value implies a low impact of savings on output.

Quantitative Implications

The speed of convergence

- Not only interested in eventual effect of some change, we also want to know how fast those effects occur (pp. 25).
- Again, we can use approximations around the long-run equilibrium.
- Our goal is to determine how rapidly k approaches k^*
- Since $\dot{k}$ is determined by k (Eq.4):

$$\dot{k}(t) \approx -\lambda[k(t) - k^*]$$

  where $\lambda = -\partial\dot{k}(k)/\partial k|_{k=k^*}$. When $k = k^*$, $\dot{k} = 0$.
- Since $\dot{k}$ is positive when k is slightly below k^* and negative when it is slightly above, $\partial\dot{k}(k)/\partial k|_{k=k^*}$ is negative (i.e., λ is positive).
- The growth rate of $k(t) - k^*$ is approximately constant and equal to $-\lambda$:

$$k(t) \approx k^* + e^{-\lambda t}[k(0) - k^*] \ ,$$

  $\lambda = [1 - \alpha_K(k^*)](n + g + \delta) \approx 4\%$ or about a 17-year *half-life* (see p.27)

Central questions in Growth Theory

Technological progress (productivity growth)

- Ultimately, the shift in production technology is the only factor of the model that can bring about change in long term output growth to the model
- i.e., only differences in the effectiveness of labour can account for the vast differences in wealth across time and space
  - Productivity raises steady-state output in two ways:
    1. Directly, by increasing output for a given k
    2. Indirectly, by raising the steady state k^*
  - Variations in the accumulation of capital do not account for significant differences:
    1. Directly, k differs by factor of $X^{1/\alpha}$
    2. Indirectly, differences in rate of return on capital, $f'(k) - \delta$.
- Problem with Solow model is that it does not try and understand technological advancement (black box)
- Romer, Chapters 3 and 4 address this.

Central questions in Growth Theory

Poverty Traps

What about population growth?

- Many countries consider high population growth as development problem and try to reduce it with policy measures (e.g. China's "one-child family" policy)
- However, many counter-arguments to this logic (small population may reduce chance of technological advance)
  - **Econtalk:** *Spence on Growth* & *Easterly on Growth*
- There are two types of poverty traps: technologically-induced poverty traps and demographically-induced poverty traps. Both cases involve the inclusion of a non-linearity into the system.[4]
  - endogenous population growth ($n = f(y) = n(k)$)
  - non-linearities in the production function ($f''(k) > 0 \ \& < 0$)
- Tutorial application...

[4] See Snowdon (2008) *The Solow Model, Poverty Traps and the Foreign Aid debate*

Empirical applications

Growth accounting

How much of growth is due to changes in specific factors of production?

- From the production function, $Y(t) = F(K(t), A(t)L(t))$ we can derive an expression for the growth rate of output per worker (p. 30)

$$
\begin{aligned}
\frac{\dot{Y}(t)}{Y(t)} &= \alpha_K \frac{\dot{K}(t)}{K(t)} + \alpha_L \frac{\dot{L}(t)}{L(t)} + R(t) \\
\frac{\dot{Y}(t)}{Y(t)} - \frac{\dot{L}(t)}{L(t)} &= \alpha_K \left[\frac{\dot{K}(t)}{K(t)} - \frac{\dot{L}(t)}{L(t)} \right] + R(t) \, , \qquad (7)
\end{aligned}
$$

  where $R(t)$ is called the *Solow residual*

- i.e., decompose the growth of output per worker into the contribution of growth of capital per worker and the Solow residual.

Growth accounting: South Africa

Table 6 Sources of output growth in South Africa: 1985 – 2004

Human Capital Treatment	Period	Output growth	Capital contribution	Labour contribution	Total factor productivity
No provision for human capital	1985-1994	0.8	0.45	0.63	-0.28
	1995-2004	3.1	0.62	0.62	1.86
Human capital based on average years of schooling	1985-1994	0.8	0.45	1.11	-0.76
	1995-2004	3.1	0.62	0.88	1.60
Human capital represented by 3 skills levels	1985-1994	0.8	0.45	1.49	-1.14
	1995-2004	3.1	0.62	0.95	1.53

Table 7 Recent studies on the sources of output growth in South Africa

Study	Period	Output growth	Capital contribution	Labour contribution	Total factor productivity
Arora (2005)	1980-1994	1.2	0.8	0.7	-0.4
	1995-2003	2.9	0.7	0.9	1.3
Fedderke (2002)	1970s	3.21	2.54	1.17	-0.49
	1980s	2.20	1.24	0.62	0.34
	1990s	0.94	0.44	-0.58	1.07

Source: Du Plessis and Smit (2007)

Growth accounting: South Africa

Table 8 Sources of output growth in South Africa: Sectoral: 1985-2004

	Output Growth		Capital contribution		Labour contribution		Total factor productivity	
	1985-1994	1995-2004	1985-1994	1995-2004	1985-1994	1995-2004	1985-1994	1995-2004
Primary sector	0.47	0.31	0.51	0.32	-0.43	-0.75	0.39	0.74
- Agriculture, forestry and fishing	3.89	0.44	-1.38	-0.18	-0.04	-0.35	2.55	0.61
- Mining and quarrying	-0.58	0.26	1.45	0.50	-1.07	-1.37	-0.96	1.13
Secondary sector	-0.03	2.73	0.21	0.31	-0.50	-1.22	0.26	3.64
- Manufacturing	-0.1	2.78	0.49	0.7	-0.47	-0.67	-0.12	2.75
- Electricity, gas and water	3.95	1.61	-0.29	-0.86	-1.55	-0.92	5.79	3.39
- Construction (contractors)	-2.64	3.48	-0.83	1.65	-0.36	-3.44	-1.45	-1.61
Tertiary sector	1.41	3.79	0.54	0.72	0.24	0.97	0.64	2.10
- Wholesale and retail trade, catering and accommodation	-0.11	4.3	0.48	1.07	0.27	1.18	-0.86	2.05
- Transport, storage and communication	1.58	6.85	0.09	0.97	2.8	-1.6	4.29	7.48
- Financial intermediation, insurance, real estate and business services	1.77	5.16	0.76	0.76	4.11	3.26	-3.10	1.14

Data source: Quantec

Source: Du Plessis and Smit (2007)

Empirical applications

Convergence & Savings-Investment

Major empirical implication of the model is the phenomenon of convergence. Do poor countries tend to grow faster than rich countries?

Solow model predicts:

- If countries converge to their balanced growth paths $\Rightarrow$ expect poor countries to "catch-up"
- at higher k^*, $f'(k*) - \delta$ is lower in rich countries $\therefore$ capital should flow from rich to poor countries
- lags in the diffusion of knowledge

pp. 32-35: See discussion of Baumol (1986) and the follow-up by De Long (1988)

Savings-Investment correlation and the Feldstein-Horioka paradox:

- High $S - I$ correlation likely not due to barriers to capital mobility; rather
- underlying forces affecting both S and I