A general equilibrium model of imperfect competition and price setting

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This model is entirely static, and focusses exclusively on the new element we introduce: a real role for relative price differences. As such we us a very simplified production side: no capital, linear in labour with a perfectly competitive labour market and no productivity shocks.

Adding capital would add an accumulation/growth component. For now we want to focus only on the variation around that trend, so we might as well ignore the trend entirely (for now). We also have already studied the optimal investment in capital in a model where there are interesting interactions with labour supply, so we abstract from that here. We study this simplified model as the pricing side will form the benchmark period-by-period outcome in a fully dynamic economy when prices are prefectly flexible. In such an economy there will be no real impacts of nominal shocks. The only thing that will determine outcomes is real demand levels.

To allow real effects of prices, we need to assume an imperfection: firms have pricing power. This will inevitibly lead to the equilibrium being socially inefficient. This is not a result of main import however.

Or goal with this model is to have a tractable mathematical description of a situation in which relative prices have real effects. Heuristically: in an efficient market we expect that the products in highest demand (due to the relative preference of consumers) to have the highest relative price, so that the most resources are devoted to the production of the most welfare beneficial product. If there are no barriers to price adjustment, and all price information is common knowledge, the market equilibrium set of relative prices will be efficient in the *relative* allocation of productive resources, even though imperfect competition in the goods market means that the *aggregate* level of productive resources employed is not efficient.

We will use a very simple mathemetical analogy for this "different preferences across goods": we will assume there exists a variety of distinct "consumption" goods (indexed along a continuum), and that consumers prefer some combination of all of them. The composition of the aggregate consumption bundle will be governed by a single parameter. The fact that consumers want some combination, means that the goods are not perfect substitutes. And once we assume there is a single firm producing each of these types goods, each has a relative monopoly in the production of its specific good and hence pricing power. Not infinite power, of course: analogously to the simple monopoly in micro: relative price that is "too high" in some sense will drive consumers to substitute away from a good and will not yield maximal profits.

1 Production Technology and Labour markets:

There exists a continuum of consumption goods, indexed by $i \in [0, 1]$, each produced by a single firm with monopoly rights to that good, so that firm i can set the price for good i.

- Denote the consumption of good i by C_i and the price paid for good i, P_i
- Firm i produces only good i using only labour, which is supplied by the representative household to each firm (so each household supplies some amount of labour to each firm).
- Production function:

$$Y_i = L_i$$

  - Firm i hires labour quantity L_i on a perfectly competitive labour market

$$Y_i = L_i$$

- No government, trade or capital, so:

$$\begin{aligned} Y_i &= C_i \\ \therefore Y &= C \end{aligned}$$

2 The Representative Household:

- Households gain utility out of consuming the full continuum of consumption goods $\{C_i\}_{i \in [0,1]}$ and disutility of supplying labour $\{L_i\}_{i \in [0,1]}$ the full continuum of firms in the following way:

$$\begin{aligned} U &= C - \frac{L^\gamma}{\gamma} \\ \text{where} & \\ C &= \left[\int_0^1 C_i^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}} \\ L &= \int_0^1 L_i di \\ \gamma, \eta &> 1 \\ L &\in [0, 1] \end{aligned}$$

- Since we are in a static case, consumption smoothing is not relevant, so the aggregate utility of consumption function is linear as a simplification (it will become non-linear again in the next model).
- Similarly, we are not interested in the production side of this economy, so we assume households are indifferent to the distribution of their labour across the different firms.
- The disutility of labour function is strictly convex as before, so that the marginal onerousness of labour increases in labour
- The preferences across types in the continuum is represented by a standard Constant Elasticity of Substitution utility function, although now the substitution is across types, not aggregate consumption in different periods.
    * the elasticity of substitution across types is: η
    * $\lim_{\eta\to\infty} \frac{\eta-1}{\eta} = \lim_{\eta\to\infty} \frac{\eta}{\eta-1} = 1$, so the utility function is linear, or all goods are perfect substitutes. This means that only the lowest price good will be consumed, which in turn implies that all relative prices are always equal to 1.
    * $\lim_{\eta\to 1} \frac{\eta-1}{\eta} = 0$ and $\lim_{\eta\to 1} \frac{\eta}{\eta-1} = \infty$, and one can show that $\lim_{\eta\to 1} \left[\int_0^1 C_i^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}} = \min_{i\in[0,1]} \{C_i\}$, means or all goods are perfect complements. The optimal choice will always be to consume equal amounts of all goods, independent of relative prices, thus we only consider cases where $\eta > 1$.
    * Note: when building intuition or using the rules of differentiation, $\left[\int_0^1 C_i^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}$ behaves almost exactly like $\left[\sum_i C_i^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$ or even $\left[C_1^{\frac{\eta-1}{\eta}} + C_2^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$.

- Budget Constraint:
    - Households receive
        * W per unit of labour supplied,
        * R in total as profits from the firms, and
        * spend all of their income on the continuum of consumption goods, spending P_iC_i on each good i, so that their budget constraint is given by:

$$R + WL = \int_0^1 P_i C_i di$$

        * Since different types of labour are perfect substitutes in both utility and production functions, all relative wages are equal to 1 so we just consider the aggregate wage W

- Note: By assumption utility is additively separable between consumption (of any type) and labour. By the linear nature of the budget constraint wage income and consumption choices are also additively separable

  - This means we can treat the "Consumption choice given disposable income" and "Labour supply choice to obtain disposable income" entirely separately

Whatever the outcome of the labour decision, there will be some amount of money $S = R + WL$ available to be spent on consumption goods. What we want to determine is how much is spent on each type of good.

- Utility function over C_i:

$$\left[\int_0^1 C_i^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}$$

- Budget Constraint: On each good i, the consumer spends P_iC_i and she spends all of her budget, so the budget constraint is:

$$\int_0^1 P_iC_i di = S$$

- Lagrangian:

$$\mathcal{L} = \left[\int_0^1 C_i^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}} + \lambda\left[S - \int_0^1 P_iC_i di\right]$$

- FOC with respect to *one* of the goods C_j:

  - We have to use the chain rule for the utility function: Let the inner function be $F = \int_0^1 C_i^{\frac{\eta-1}{\eta}} di$ then the outer function becomes $F^{\frac{\eta}{\eta-1}}$. We want $\frac{\partial F^{\frac{\eta}{\eta-1}}}{\partial F} \cdot \frac{\partial F}{\partial C_j}$

- $\frac{\partial \mathcal{L}}{\partial C_j} = 0$:

$$\begin{aligned}
\frac{\eta}{\eta-1}\left[\int_0^1 C_i^{\frac{\eta-1}{\eta}} di\right]^{\frac{1}{\eta-1}} \frac{\eta-1}{\eta} C_j^{\frac{-1}{\eta}} &= \lambda P_j \\
C_j &= \underbrace{\lambda^{-\eta}\left[\int_0^1 C_i^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}}_{\text{not function of } j} P_j^{-\eta} \\
C_j &= AP_j^{-\eta} \\
\text{equivalently: } C_i &= AP_j^{-\eta}
\end{aligned}$$

- As usual, for a demand function we want it in the form:

$$\begin{aligned}
C_j &= f\left(\{\text{money available}\}, P_j, \{\text{all other prices}\}\right) \\
C_j &= f\left(S, P_j, \{P_i\}_{i \in [0,1]}\right)
\end{aligned}$$

- Plug $C_j = AP_j^{-\eta} \forall j$ into budget constraint and solve for A:

$$\begin{aligned}
\int_0^1 P_i A P_i^{-\eta} di &= S \\
A \int_0^1 P_i^{1-\eta} di &= S \\
A &= \frac{S}{\int_0^1 P_i^{1-\eta} di} \\
\therefore C_j &= \frac{S}{\int_0^1 P_i^{1-\eta} di} P_j^{-\eta}
\end{aligned}$$

- Plug this back into the utility function to find C as a function of S and P:

$$
\begin{aligned}
C &= \left[\int_0^1 \left(\frac{S}{\int_0^1 P_i^{1-\eta} di} P_j^{-\eta}\right)^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}} \\
&= \left[\int_0^1 \underbrace{\left(\frac{S}{\int_0^1 P_i^{1-\eta} di}\right)^{\frac{\eta-1}{\eta}}}_{\text{not function of } j} \left(P_j^{-\eta}\right)^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}} \\
&= \left[\left(\frac{S}{\int_0^1 P_i^{1-\eta} di}\right)^{\frac{\eta-1}{\eta}} \int_0^1 \left(P_j^{-\eta}\right)^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}} \\
&= \left[\left(\frac{S}{\int_0^1 P_i^{1-\eta} di}\right)^{\frac{\eta-1}{\eta}} \int_0^1 P_j^{1-\eta} dj\right]^{\frac{\eta}{\eta-1}} \\
&= \left[S^{\frac{\eta-1}{\eta}} \left(\int_0^1 P_i^{1-\eta} di\right)^{-\left(\frac{\eta-1}{\eta}\right)} \left(\int_0^1 P_j^{1-\eta} dj\right)\right]^{\frac{\eta}{\eta-1}} \\
&= \frac{S}{\left[\int_0^1 P_i^{1-\eta} di\right]} \left[\int_0^1 P_j^{1-\eta} dj\right]^{\frac{\eta}{\eta-1}}
\end{aligned}
$$

note: $\int_0^1 P_i^{1-\eta} di = \int_0^1 P_j^{1-\eta} dj$

$$
C = \frac{S}{\left[\int_0^1 P_i^{1-\eta} di\right]^{\frac{1}{1-\eta}}}
$$

- Now we **define** aggregate price-index P (for this economy) to be the cost of buying one unit of the composite good C *when the consumer optimally allocates her expenditure over the different available goods given her utility function*:

$$
P = \left[\int_0^1 P_i^{1-\eta} di\right]^{\frac{1}{1-\eta}}
$$

  - This means that *by definition*, "total expenditure" is equal to total expendable resources, or

$$
C = \frac{S}{\left[\int_0^1 P_i^{1-\eta} di\right]^{\frac{1}{1-\eta}}} = \frac{S}{P}
$$

$C = \frac{S}{P}$

    * Do not try to make this be "fundamentally intuitive" - it is a matter of definition that is

extraordinarily convenient in large models

- Put differently: in this model the "correct" way of aggregating prices for the purposes of calculating inflation (which we will do later, in dynamic models) is not some weighted linear average of prices, but the non-linearly weighted average defined above.
  - If this feels like an "unacceptable degree of cheating", ask yourself if you could rigorously defend the idea that the "appropriate" view of "aggregate price" should necessarily be a linear average.
  - Even the CPI is a specific weighted average, chain weighted over time

- Finally, we can now rearrange our individual good demand to get a beautifully simple demand function that depends only on the good's price relative to the aggregate price and aggregate demand:

$$\begin{aligned}
P &= \left[\int_0^1 P_i^{1-\eta} di\right]^{\frac{1}{1-\eta}} \\
P^{1-\eta} &= \left[\int_0^1 P_i^{1-\eta} di\right] \\
\text{so:} & \\
C_j &= \frac{S}{\int_0^1 P_i^{1-\eta} di} P_j^{-\eta} \\
&= \frac{S}{P^{1-\eta}} P_j^{-\eta} \\
&= \frac{S}{P} \frac{P_j^{-\eta}}{P^{-\eta}} \\
&= \left(\frac{P_j}{P}\right)^{-\eta} \frac{S}{P} \\
C_j &= \left(\frac{P_j}{P}\right)^{-\eta} C
\end{aligned}$$

- In equilibrium: $C_i = Y_i (= L_i)$:

$$Y_i = \left(\frac{P_i}{P}\right)^{-\eta} Y$$

- We already know from above that $C = \frac{WL+R}{P}$ so we can simply plug this into the utility function and do unconstrained optimization to find the labour supply function:

$$\max_L \left\{ \frac{WL+R}{P} - \frac{L^\gamma}{\gamma} \right\}$$

- FOC and rearranging:

$$\begin{aligned}\frac{W}{P} - L^{\gamma-1} &= 0\\ L &= \left(\frac{W}{P}\right)^{\frac{1}{\gamma-1}}\end{aligned}$$

3 Firms

- Firms are assumed to be profit maximizers
- They are perfectly informed about the structure of consumer demand we derived above, and can perfectly observe all prices.
- Given that the demand is a known function of the *relative* price of firm i's goods, *relative* price $\frac{P_i}{P}$ to maximize profit.

  – Note that relative prices are real concepts.

- Real Profits of firm i are real revenues minus real costs:

$$\frac{R_i}{P} = \frac{P_i}{P}Y_i - \frac{W}{P}L_i$$

- Given the simple production function: $Y_i = L_i$ and consumer demand $Y_i = \left(\frac{P_i}{P}\right)^{-\eta} Y$:

$$\begin{aligned}\frac{R_i}{P} &= \frac{P_i}{P}Y_i - \frac{W}{P}L_i\\ &= \frac{P_i}{P}Y_i - \frac{W}{P}Y_i\\ &= \frac{P_i}{P}\left(\frac{P_i}{P}\right)^{-\eta} Y - \frac{W}{P}\left(\frac{P_i}{P}\right)^{-\eta} Y\\ \frac{R_i}{P} &= \left(\frac{P_i}{P}\right)^{1-\eta} Y - \frac{W}{P}\left(\frac{P_i}{P}\right)^{-\eta} Y\end{aligned}$$

- FOC w.r.t $\left(\frac{P_i}{P}\right)$:

$$(1-\eta)\left(\frac{P_i}{P}\right)^{-\eta} Y + \eta\frac{W}{P}\left(\frac{P_i}{P}\right)^{-\eta-1} Y = 0$$

- Rearranging:

$$\frac{P_i}{P} = \frac{\eta}{\eta-1}\frac{W}{P}$$

- I.e. firms charge a markup over marginal costs, where the size of the markup is a function of the elasticity of demand η

4 Equilibrium

- In equilibrium:

$$L = C = Y$$

- From the labour supply curve:

$$\begin{aligned} L &= \left(\frac{W}{P}\right)^{\frac{1}{\gamma-1}} \\ \frac{W}{P} &= Y^{\gamma-1} \end{aligned}$$

- Since all firms face the same elasticity of demand and same real wage, they will charge the same relative price expressed in terms of output:

$$\frac{P_i^*}{P} = \frac{\eta}{\eta-1} Y^{\gamma-1}$$

- When all firms charge the same price $P_i^* = P^*$, the aggregate price is also equal to this price by definition of the price aggregator, hence we get the result that if prices are fully flexible, the real output is determined by the features of the economy alone and is not a function of aggregate prices:

$$Y^* = \left(\frac{\eta-1}{\eta}\right)^{\frac{1}{\gamma-1}} < 1$$

- The real wage:

$$\begin{aligned} \frac{W^*}{P} &= \left(\frac{\eta-1}{\eta}\right) < 1 \\ L^* &= \left(\frac{\eta-1}{\eta}\right)^{\frac{1}{\gamma-1}} < 1 \end{aligned}$$

- To close the model, we need to assume something about aggregate nominal demand. We take the simplest possible option and assume it is entirely determined by money supply:

$$Y = \frac{M}{P}$$

- This means the equilibrium price is:

$$P^* = \frac{M}{\left(\frac{\eta-1}{\eta}\right)^{\frac{1}{\gamma-1}}}$$

5 Implications:

- due to market power, production is below socially optimal amount:
- To show this we use ask whether a "benevolent Social Planner" that takes over all decisions has only one problem: maximize utility of all (so the representative household):

$$C - \frac{L^{\gamma}}{\gamma}$$

  – Subject to the aggregate Constraint: $C = L$

$$\max_{L} L - \frac{L^{\gamma}}{\gamma}$$

$$\begin{aligned} &\text{FOC:} \\ L^{*\gamma-1} &= 1 \\ L^{*} &= 1 \end{aligned}$$

- This is because firms have pricing power in the consumption goods market while the consumer-workers are price takers, while in the perfectly competitive labour market, both firms and consumer-workers are wage takers. Thus the equilibrium real wage is less than 1, inducing a smaller than full use of resources.
- It should be emphasized that the *relative* allocation of productive resources is efficient, it is just the scale that is inefficient.
- If we allow for a government, we can design a profit tax whose revenues are used to subsidize wages that would regain full efficiency, but that is not of central interest for our purposes.