Dynamic Stochastic General Equilibrium Models of Fluctuations

Session 8: Introduction to DSGE

ECO5021F: Advanced Macroeconomics
University of Cape Town

Readings

Required

- **Chapter 7.1−7.4, 7.6−7.7** (Introduction to DSGE and Calvo pricing)
- **Chapter 7.8 to 7.10** (Canonical DSGE)

Recommended

- Interview with Greg Mankiw (in Snowdown and Vane)

Romer's "building blocks"

- **Romer** uses a common framework to present different dynamic models with nominal imperfections
- State vs. time dependent models [Discuss]
- This is a **discrete time model**
- Firms are **imperfectly competitive** (have some market power)
- Production uses only labour as an input

$$C_{it} = Y_{it} = L_{it}$$

- Closed economy with no government

Romer's "building blocks"

- **Households**: Maximise utility, take as given the path of real wages (perfectly competitive) and the real interest rate
- **Firms**: Maximise the present value of their profits, subject to some constraint on the ability to set prices
- **Central Bank**: Set the path for the real interest rate

Households

- We will combine the two components of the consumption problem from the previous lectures
- Intertemporal IS curve and labour supply curve
- Demand for individual goods

- In aggregate, we assume that,

$$\sum_{t=0}^{\infty} \beta^t \left[U(C_t) - V(L_t) \right], \quad 0 < \beta < 1$$

which gives $$\sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\theta}}{1-\theta} - B \frac{L_t^{\gamma}}{\gamma} \right], \quad \gamma > 1,\ \theta > 0,\ B > 0$$

where $$C = \left[\int_0^1 C_i^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \quad \eta > 1$$

Households

- As we found in previous lectures, this will yield a
    1. Labour supply curve
    2. Intertemporal IS equation
- In equilibrium, the increase in household income from dL extra labour must equal the disutility of that increase

$$U'(C_t)\frac{W_t}{P_t} = V'(L_t)$$
$$\frac{W_t}{P_t} = \frac{BL_t^{\gamma-1}}{C_t^{-\theta}}$$

- Because $Y_t = L_t = C_t$, we can substitute for L_t and C_t

$$\frac{W_t}{P_t} = BY_t^{\theta+\gamma-1} \tag{1}$$

- For the intertemporal IS equation we have,

$$\ln Y_t = -\frac{1}{\theta}r_t - \frac{1}{\theta}\ln\beta + \ln Y_{t+1} \tag{2}$$

Firm: Dynamic price setting

- As previously, the demand for each type of good in any period t is given by,

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} \cdot Y_t$$

- Real profit is specified as,

$$\begin{aligned}\frac{R_{it}}{P_t} &= \frac{P_{it}}{P_t} Y_{it} - \frac{W_t}{P_t} L_{it} \\ &= \left(\frac{P_{it}}{P_t}\right)^{1-\eta} Y_t - \frac{W_t}{P_t}\left(\frac{P_{it}}{P_t}\right)^{-\eta} Y_t \\ &= Y_t P_t^{\eta-1}(P_{it}^{1-\eta} - W_t P_{it}^{-\eta})\end{aligned}$$

Firm: Dynamic price setting

- This next part is a bit tricky, so let us elaborate on the explanation by **Romer**
- Consider the problem of a firm setting it's price that may **remain in place** for a number of periods
- We can *normalize* the date of the decision to "period 0"
- Firm faces the problem of setting a single price to **maximise profits** over some horizon

- **Important**: Firm's profit accrues to households $\rightarrow$ values profits according to utility they provide

Detour: Diamond model

- Remember the Euler equation from the **Diamond model** (p.79), solving for $1 + r_{t+1}$

$$1 + r_{t+1} = (1+\rho)\frac{C_{2,t+1}^{\theta}}{C_{1,t}^{\theta}} = (1+\rho)\frac{C_{1,t}^{-\theta}}{C_{2,t+1}^{-\theta}} = (1+\rho)\frac{U'(C_{1,t})}{U'(C_{2,t+1})}$$

- The RHS of the equation $\rightarrow$ definition of the utility function
- In the **Diamond model**, individuals only live for two periods, so the comparison is t and $t+1$

- The owners of firms in our Keynesian model are longer lived, so consumption tradeoffs at **more distant points in time** need to be considered.

Firm: Dynamic price setting

- If we were to generalise the result from the **Diamond model** to reflect the tradeoff between consumption at time zero and time t, we would get,

$$1 + \bar{r}_t = \overbrace{\prod_{s=0}^{t} (1 + r_s)}^{\text{Cumulative return}} = (1 + \rho)^t \frac{U'(C_0)}{U'(C_t)}$$

$$\frac{1}{1 + \bar{r}_t} = \left[\frac{1}{(1 + \rho)} \right]^t \frac{U'(C_t)}{U'(C_0)} = \beta^t \frac{U'(C_t)}{U'(C_0)} = \lambda_t$$

- **Romer** defines the $\beta^t U'(C_t)/U'(C_0)$ as the marginal utility of the representative household's consumption in period t relative to period 0
- $\lambda_t \rightarrow$ discount factor that translates period t profits to period 0 units
- This discount factor λ_t is defined this way because firms ultimately belong to households
- Normally, on the **firm** side, the real interest rate is the "market" discount factor that governs the transfer of wealth across time
- **In equilibrium** $\rightarrow$ market and subjective discount factors should equal

Firm: Dynamic price setting

- Another *new* component that is introduced is $q_t \rightarrow$ probability that a price P_i set today (period 0) has not been changed t periods later
- This probability depends on the firm's future decisions about whether or not to change price
- We are essentially going to look at several **alternative models** for q_t (could be state or time dependent)
- For now, it is only going to be a probability **parameter**, leaving determination unspecified

Firm: Dynamic price setting

- Problem facing the firm is to set it’s price to maximise,

$$\max_{P_i} \sum_{t=0}^{\infty} \mathbb{E}_0 \left[q_t \lambda_t \frac{R_i}{P_t} \right]$$

- Substituting for the definition of real profit,

$$\max_{P_i} \sum_{t=0}^{\infty} \mathbb{E}_0 \left[q_t \lambda_t Y_t P_t^{\eta-1} (P_i^{1-\eta} - W_t P_i^{-\eta}) \right]$$

- We will now add some elements to simplify the problem

Firm: Dynamic price setting

- Recall (previous lecture) that in the perfectly flexible price setting scenario the problem is static (in each period the optimal price only relies on variables from that period)
- Optimal price P_t^* is markup over marginal cost,

$$\frac{P_t^*}{P_t} = \frac{\eta}{\eta - 1}\frac{W_t}{P_t}$$
$$P_t^* = \frac{\eta}{\eta - 1}W_t$$
$$W_t = \frac{\eta - 1}{\eta}P_t^*$$

- Substitute this value for W_t into the equation from the previous slide,

$$\max_{P_i} \sum_{t=0}^{\infty} \mathbb{E}_0 \left[q_t \lambda_t Y_t P_t^{\eta-1} (P_i^{1-\eta} - \frac{\eta - 1}{\eta} P_t^* P_i^{-\eta}) \right]$$

- It is not obvious how we are going to solve this, so we have to make some simplifying assumptions to get to an approximation

Approximation of the solution

- Two assumptions are made by **Romer**, to simplify things:
  1. Inflation is low; economy always close to flexible price equilibrium
  2. β is close to one (λ_t is close to constant in steady state)
- First assumption is quite common in complicated non-linear models $\rightarrow$ approximations are often only valid close to the point around the steady state
- The first step is going to be to define variables in terms of **log deviation from steady state**
- This normalisation means that the log-steady state values are zero, so we only consider the logs of variables

Approximation of the solution

- Firms problem can be written as,

$$\max_{p_i} \sum_{t=0}^{\infty} \mathbb{E}_0 \left[q_t \lambda_t Y_t P_t^{\eta-1} F(p_i, p_t^*) \right]$$

- where p_i and p_t^* denote the logs of P_i and P_t^*
- since we made the assumption that the economy stays near it's flexible price equilibrium,
  1. eliminate $\lambda_t Y_t P_t^{\eta-1} \rightarrow$ variation negligible relative to variation in q_t and p_t^*
  2. $F(p_i, p_t^*)$ can be well approximated around the flexible price equilibrium $p_i = p_t^*$
- Period t profits are maximised where $p_i = p_t^*$

Approximation of the solution

- Using Taylor expansion, we can write:

$$F(p_i, p_t^*) \approx F(p_t^*, p_t^*) + \frac{\partial F(p_i, p_t^*)}{\partial p_i}(p_i - p_t^*) + \frac{1}{2}\frac{\partial^2 F(p_i, p_t^*)}{\partial^2 p_i}(p_i - p_t^*)^2$$

- We know that $\frac{\partial F(p_i, p_t^*)}{\partial p_i} = 0$ and $\frac{\partial^2 F(p_i, p_t^*)}{\partial^2 p_i} < 0 \rightarrow$ profits are maximised
- Taking this into account we can write the approximation as,

$$F(p_i, p_t^*) \approx F(p_t^*, p_t^*) - K(p_i - p_t^*)^2, \quad K > 0$$

- In order to maximise profits, we need to minimize,

$$\min_{p_i} \sum_{t=0}^{\infty} q_t (p_i - p_t^*)^2$$

- We drop the expectations operator $\mathbb{E}_0$ for now $\rightarrow$ add it back later

Approximation of the solution

- Minimisation problem is now straightforward,

$$
\begin{aligned}
A &= \min_{p_i} \sum_{t=0}^{\infty} \left[q_t (p_i - p_t^*)^2 \right] \\
&= \min_{p_i} \sum_{t=0}^{\infty} \left[q_t (p_i^2 - 2 p_i p_t^* + {p_t^*}^2) \right] \\
&= \min_{p_i} \left[\sum_{t=0}^{\infty} q_t p_i^2 - 2 \sum_{t=0}^{\infty} q_t p_i p_t^* + \sum_{t=0}^{\infty} q_t {p_t^*}^2 \right] \\
&= \min_{p_i} \left[p_i^2 \sum_{t=0}^{\infty} q_t - 2 p_i \sum_{t=0}^{\infty} q_t p_t^* + \sum_{t=0}^{\infty} q_t {p_t^*}^2 \right]
\end{aligned}
$$

$$
\left(\frac{\partial A}{\partial p_i} \right) \qquad 2 p_i \sum_{t=0}^{\infty} q_t = 2 \sum_{t=0}^{\infty} q_t p_t^*
$$

$$
p_i = \sum_{t=0}^{\infty} \omega_t p_t^*
$$

Firm: Dynamic price setting

$$p_i = \sum_{t=0}^{\infty} \omega_t p_t^*$$

- where $\omega_t \equiv q_t / \sum_{\tau=0}^{\infty} q_\tau$
- Important to know what p_i and p_t^* represent
  - p_i is the **price that the firm sets in period 0** (knowing it will be in place now, and probably some future period)
  - p_t^* is the price that would be ideal for firm to set in period t if it were to set prices independently each period
- Firms would set $p_i = p_t^*$ in every period if there is no cost of price adjustment
- The firm should set a price equal to a **weighted average** of the ideal prices in each future period
- It is weighted proportional to the probability that the current (period 0) price will still be in effect during that future period

Firm: Dynamic price setting

$$p_i = \sum_{t=0}^{\infty} \omega_t p_t^*$$

- Let us use an **example** to illustrate how this works
- Suppose **it is known** that the newly set price will be in effect for two periods $\rightarrow$ optimal price for the firm to set is (**unweighted**) average between the desired price in the first period and the desired price in the second period
- If the new price will be in effect for the first period and there is a 50% chance it will be in effect the second period (but not any longer) $\rightarrow$ firm sets price at **weighted average** of the two ideal prices, with 2/3 weight to the first period and 1/3 weight to the second

Firm: Dynamic price setting

- Finally, in the case of uncertainty, we will reintroduce out expectations operator $\mathbb{E}_0$,

$$p_i = \sum_{t=0}^{\infty} \omega_t \mathbb{E}(p_t^*)$$

- Finally, let us combine some of our equations (in logarithms) and input them into this function

$$\begin{aligned}
w_t &= p_t + b + (\theta + \gamma - 1)y_t \\
p_t^* &= \ln(\frac{\eta}{\eta - 1}) + w_t \\
y_t &= m_t - p_t \\
\therefore \quad p_t^* &= \ln(\frac{\eta}{\eta - 1}) + p_t + b + (\theta + \gamma - 1)(m_t - p_t) \\
p_t^* &= \phi m_t + (1 - \phi)p_t \\
\therefore \quad p_i &= \sum_{t=0}^{\infty} \omega_t \mathbb{E}[\phi m_t + (1 - \phi)p_t]
\end{aligned}$$

- We have assumed that $\ln(\frac{\eta}{\eta-1}) + b = 0$ and $\phi = (\theta + \gamma - 1) > 0$

Predetermined prices: Fischer model

- Prices are set in an **asynchronous** way for two periods
- One half of prices are set by "**Group A**", who presets its price at the end of period $t-1$ for period t and $t+1$
- This means that A's prices are fixed for two periods
- "**Group B**" presets at the end of period $t-2$ its price for $t-1$ and t
- In any period, half of prices are ones set in the previous period (p_t^1), and half are set two periods ago (p_t^2), average price is then,

$$p_t = \frac{1}{2}(p_t^1 + p_t^2)$$

- Our assumptions imply that p_t^1 equals the expectation as of period $t-1$ of p_{it}^* and p_t^2 equals the expectation as of period $t-2$ of p_{it}^*:

$$p_t^1 = \mathbb{E}_{t-1}[\phi m_t + (1-\phi)p_t] = \phi \mathbb{E}_{t-1} m_t + (1-\phi)\frac{1}{2}(p_t^1 + p_t^2)$$
$$p_t^2 = \mathbb{E}_{t-2}[\phi m_t + (1-\phi)p_t] = \phi \mathbb{E}_{t-2} m_t + (1-\phi)\frac{1}{2}(\mathbb{E}_{t-2} p_t^1 + p_t^2)$$

Predetermined prices: Fischer model

- **Goal:** Find out how price level and output evolve over time, given the behaviour of m
- Begin by solving for p_t^1, which yields,

$$p_t^1 = \frac{2\phi}{1+\phi}\mathbb{E}_{t-1}m_t + \frac{1-\phi}{1+\phi}p_t^2$$

- Take expectation as of $t-2$ on both sides (law of iterated projections),

$$\mathbb{E}_{t-2}\, p_t^1 = \frac{2\phi}{1+\phi}\mathbb{E}_{t-2}m_t + \frac{1-\phi}{1+\phi}p_t^2$$

- Now we can substitute this into the equation for p_t^2, which gives us,

$$p_t^2 = \phi\mathbb{E}_{t-2}m_t + (1-\phi)\frac{1}{2}\left(\frac{2\phi}{1+\phi}\mathbb{E}_{t-2}m_t + \frac{1-\phi}{1+\phi}p_t^2 + p_t^2\right)$$

- Solving this expression then gives us,

$$p_t^2 = \mathbb{E}_{t-2}m_t$$

Predetermined prices: Fischer model

- Substitute this value for p_t^2 into the equation for p_t^1 and simplify to get,

$$p_t^1 = \mathbb{E}_{t-2}m_t + \frac{2\phi}{1+\phi}\left(\mathbb{E}_{t-1}m_t - \mathbb{E}_{t-2}m_t\right)$$

- Now substitute into $p_t = (p_t^1 + p_t^2)/2$ and $y_t = m_t - p_t$, to get:

$$\begin{aligned} p_t &= \mathbb{E}_{t-2}m_t + \frac{\phi}{1+\phi}\left(\mathbb{E}_{t-1}m_t - \mathbb{E}_{t-2}m_t\right) \\ y_t &= \frac{1}{1+\phi}\left(\mathbb{E}_{t-1}m_t - \mathbb{E}_{t-2}m_t\right) + \left(m_t - \mathbb{E}_{t-1}m_t\right) \end{aligned}$$

- There are a few lessons to be learned from these equations
    1. **Unanticipated AD shifts** have real effects $(m_t - \mathbb{E}_{t-1}m_t) \rightarrow$ price setters don't know m_t when they set prices
    2. AD shifts that become **anticipated after the first prices are set** affect output $\rightarrow$ proportion of information about m_t that arrives between $t-2$ and $t-1$ is passed into output, the rest into prices
    3. Output does not depend on $\mathbb{E}_{t-2}m_t \rightarrow$ once all price setters have had chance to respond, **demand shocks no longer affect output**

Predetermined prices: Fischer model

- Some information about the **real rigidity** parameter ϕ
- $\phi \rightarrow$ measures responsiveness of desired real prices to changes in aggregate real output
- When ϕ is small $\rightarrow$ greater real rigidity
  - Large real rigidity means price-setters are reluctant to allow variations in their relative prices (**prices sticky**)
  - **Real effects** from money shocks can be large
- When $\phi > 1$ price-setters are willing to make large price changes
  - **Real effects** from money shocks are small

Calvo model and NK Phillips Curve

- Most influential of the staggered price setting models
- Each period a firm has probability α of getting to change it's price (follows a *Poisson process*)
  - Implication is that probability that a firm has to change it's price is **the same each period** (regardless of what happened before)
- The probability of having price fixed until time j is $q_j = (1-\alpha)^j \rightarrow$ since $(1-\alpha)$ share of the population do not adjust for j periods in a row
- **Important** for the result are the following conditions:
  - Number of firms is large, so by the law of large numbers $\rightarrow$ each period exactly a fraction α of firms set their prices anew
  - Firms are chosen at **random**

Calvo model and NK Phillips Curve

- Average price in any period t is:

$$p_t = \alpha x_t + (1 - \alpha) p_{t-1}$$

- where p is the average price and x is the price set by firms who can change their price
- Average price charged by firms that do not change their levels **equals** average price charged by all firms in the previous period
- Subtract p_{t-1} from both sides $\rightarrow$ gives an expression for **inflation**

$$p_t - p_{t-1} = \pi_t = \alpha(x_t - p_{t-1})$$

- In what follows we will look for a value of x_t $\rightarrow$ average price set by those who can set price (**reset price**)

Calvo model and NK Phillips Curve

- We begin by looking at the expression for the rule we derived before,

$$p_i = \sum_{t=0}^{\infty} \frac{q_t}{\sum_{\tau=0}^{\infty} q_\tau} p_t^*$$

- One of the assumptions made before was that $\beta \approx 1 \rightarrow$ introduce β and minimise the firm's loss function to get,

$$x_t = \sum_{j=0}^{\infty} \frac{\beta^j q_j}{\sum_{k=0}^{\infty} \beta^k q_k} \mathbb{E}_t p_{t+j}^*$$

- $q_j \rightarrow$ probability that the price will still be in effect in period $t+j$
- $\sum_{k=0}^{\infty} \beta^k q_k \rightarrow$ expected number of periods that price will still be in effect
- **Poisson** assumption from Calvo implies that $q_j = (1-\alpha)^j$
- Using the geometric sum formula we get,

$$\sum_{k=0}^{\infty} \beta^k q_k = \sum_{k=0}^{\infty} [\beta(1-\alpha)]^k = \frac{1}{1-\beta(1-\alpha)}$$

Calvo model and NK Phillips Curve

- This simplification helps us to rewrite the equation as,

$$x_t = [1 - \beta(1-\alpha)] \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j \mathbb{E}_t p^*_{t+j}$$

- This equation states that the optimal solution is for the firm to set its price equal to a weighted average of prices it would have expected to set in future (if there were **no price rigidities**)
- Firm is not able to change price each period $\rightarrow$ tries to keep close to **optimal frictionless price**
- Through some further manipulation and the use of $\pi_t = \alpha(x_t - p_{t-1})$ we will find the **NK Phillips curve**
- **Important:** x_t is not the general price level, it is the price that each individual firm sets $\rightarrow$ that is why we need to use our equation for the general price level $p_t = \alpha x_t + (1-\alpha)p_{t-1}$

Calvo model and NK Phillips Curve

$$x_t = [1-\beta(1-\alpha)]\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j\mathbb{E}_t p^*_{t+j} \tag{3}$$

$$\begin{aligned} x_t = {} & [1-\beta(1-\alpha)]\beta(1-\alpha)\mathbb{E}_t p^*_t + \\ & \beta(1-\alpha)[1-\beta(1-\alpha)]\left[\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j\mathbb{E}_t p^*_{t+1+j}\right] \end{aligned} \tag{4}$$

$$x_t = [1-\beta(1-\alpha)]p^*_t + \beta(1-\alpha)\mathbb{E}_t x_{t+1} \tag{5}$$

- Last equation uses the fact that p^*_t is known at time t and shifts forward equation (3) one period to get x_{t+1}
- Subtract $x_t - p_t$ from both sides gives us,

$$\begin{aligned} (x_t - p_{t-1}) - (p_t - p_{t-1}) = \\ & [1-\beta(1-\alpha)](p^*_t - p_t) + \beta(1-\alpha)(\mathbb{E}_t x_{t+1} - p_t) \end{aligned}$$

Calvo model and NK Phillips Curve

$$(x_t - p_{t-1}) - (p_t - p_{t-1}) = \\ [1 - \beta(1-\alpha)](p_t^* - p_t) + \beta(1-\alpha)(\mathbb{E}_t x_{t+1} - p_t)$$

- From our aggregate price equation, we have that $x_t - p_{t-1} = \pi_t/\alpha$ and $\mathbb{E}_t x_{t+1} - p_t = E_t \pi_{t+1}/\alpha$.
- In addition, $p_t - p_{t-1} = \pi_t$ and $p_t^* - p_t = \phi y_t$ (from Ch. 6, p. 273, (6.60))
- Substituting these values into the equation above gives us the **NK Phillips curve**,

$$(\pi_t/\alpha) - (\pi_t) = [1 - \beta(1-\alpha)]\phi y_t + \beta(1-\alpha)(E_t \pi_{t+1}/\alpha)$$

$$\pi_t = \frac{\alpha}{1-\alpha}[1 - \beta(1-\alpha)]\phi y_t + \beta \mathbb{E}_t \pi_{t+1}$$

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \pi_{t+1}, \qquad \kappa \equiv \frac{\alpha[1-(1-\alpha)\beta]\phi}{1-\alpha}$$

Example: 3-Equation New Keynesian DSGE model

- The basic New Keynesian model with Calvo price setting simplifies to a system of three equations
    1. **New Keynesian Phillips curve** $\rightarrow$ relating inflation to output gap (supply side)
    2. **Dynamic IS equation** $\rightarrow$ linking evolution of AD (and the output gap) to the nominal interest rate (demand side)
    3. **Monetary policy rule** $\rightarrow$ rule for setting the nominal interest rate (normally a Taylor rule)

- Represented below is the system of equations,

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \pi_{t+1} + u_t^s \qquad \text{NK Phillips curve}$$

$$y_t = -\frac{1}{\theta}(i_t - \mathbb{E}_t \pi_{t+1}) + \mathbb{E}_t y_{t+1} + u_t^d \qquad \text{Dynamic IS equation}$$

$$i_t = \phi_y y_t + \phi_\pi \pi_t + u_t^m \qquad \text{Monetary policy rule}$$

Inflation inertia

$$\mathbb{E}_t\pi_{t+1} - \pi_t = \underbrace{\frac{1-\beta}{\beta}}_{\text{small}\approx 0.01} \pi_t - \frac{\kappa}{\beta}\underbrace{(y_t - \bar{y}_t)}_{\approx \text{ real MC}}$$

- There are some criticisms of this version of the Phillips curve, **in the model** we have that
  - Disinflation can be achieved **costlessly** (anti-inertia)
    - Inflation inertia $\rightarrow$ inflation is costly to reduce (difficult to do quickly)
    - No tradeoff between inflation and output gap stabilisation
  - Anticipated disinflation could **generate a boom**!
  - Inflation is purely forward looking $\rightarrow$ past inflation is irrelevant
  - Inflation leads the output gap measure
- **Mankiw** (2001) states that the NKPC *cannot even come close to explaining the dynamic effect of monetary policy on inflation and unemployment*

Inflation leading the output gap?

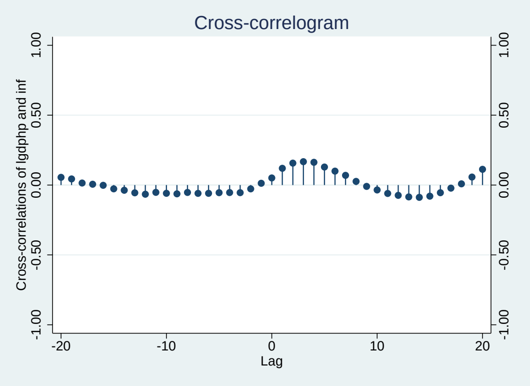


Figure: The output gap and inflation

The Figure shows a cross-correlogram between the output gap (estimated with a **two-side HP filter**) and inflation in South Africa. The cross-correlogram shows a mild effect of the output gap on subsequent inflation, but no evidence of inflation leading the output gap as would be required by an NKPC with this output gap. (Burger and du Plessis, 2006)

Inflation inertia

$$\mathbb{E}_t \pi_{t+1} - \pi_t = \underbrace{\frac{1-\beta}{\beta}}_{\text{small}\approx 0.01} \pi_t - \frac{\kappa}{\beta} \underbrace{(y_t - \bar{y}_t)}_{\approx \text{ real MC}}$$

- Two shortcomings were revealed through empirical analysis
  1. **Fuhrer and Moore** (1995)$\rightarrow$ output gap tends to lead inflation
     - Sluggishness adjustment of prices observed in data
     - Hump shape response to monetary policy (**not jump variable**)
  2. **Ball** (1994) $\rightarrow$ disinflation is costly, associated with recession
- However, there is still some hope!
- **Christiano, Eichenbaum and Evans** (2001) show that persistent effects are possible
  - They achieve this through introduction of habit formation, variable capital utilisation and investment adjustment costs
- It might also be that our measure of the output gap is poor, rather use marginal cost?

Perhaps labour share (MC) works better

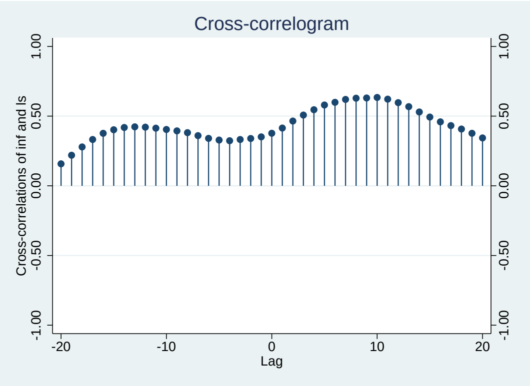


Figure: Inflation and the labour income share

The cross-correlogram between the labour income share and inflation in South Africa shows that, empirically, inflation anticipates movements in the labour income share as required by the alternative specification of the NKPC. (Burger and du Plessis, 2006)

Hybrid Phillips Curve

- **Fuhrer** (1997) nests the traditional expectations augmented Phillips curve (accelerationist) and the NKPC
- The model is also derived from microfoundations, and is given by,

$$\pi_t = \kappa(y_t - \bar{y}_t) + \gamma_f \mathbb{E}_t \pi_{t+1} + \gamma_b \pi_{t-1}$$

$$\text{where} \quad \lambda = \frac{(1-\omega)[1-\beta\theta][1-\theta]}{\theta}$$

$$\gamma_f = \frac{\beta\theta}{\phi}$$

$$\gamma_b = \frac{\omega}{\phi}$$

$$\phi = \theta + \omega[1 - \theta(1-\beta)]$$

- Allows for a proportion of backward looking price setters ω
- Sometimes in the literature you will see $\theta = (1-\alpha)$

Hybrid Phillips Curve

- **Gali and Gertler** (1999) take a formal econometric approach and try to estimate this equation, substituting output with marginal cost (and adding a disturbance term)

$$\pi_t = \lambda S_t + \gamma_f \mathbb{E}_t \pi_{t+1} + \gamma_b \pi_{t-1} + e_t$$

- Link between output and marginal cost $\rightarrow$ when output is above normal, marginal cost is high and relative prices are increased
- They use labour share of income S_t as proxy for marginal cost
- However, **Rudd and Whelan** (2005) provide evidence against the usage of S_t as a measure of marginal cost

Alternatives to the NKPC

- Hybrid NKPC (includes price indexing) $\rightarrow$ **Christiano, Eichenbaum and Evans**
  - Not a single price, but rather past inflation indexed price path
- Sticky information Phillips curve $\rightarrow$ **Mankiw and Reis** (2002)
  - Rather than assume firms cannot adjust their prices, assume they infrequently update their information
  - Nominal rigidity $\rightarrow$ changes are based on **information not price**
  - Model predicts **inflation inertia** and no boon for announced disinflation
- *Generalized* New-Keynesian Phillips Curve $\rightarrow$ **Ascari and Sbordone** (2014)
  - positive trend inflation: $\pi > 0$ in steady state
  - standard NK models assume zero trend (steady state) inflation, such that: $\pi = 0 \rightarrow i = r$ in steady state.
  - implications: determinacy and stability region for monetary policy shrinks (requires more **hawkish** policy); generates endogenous **persistence** and real economic (**welfare**) costs via price dispersion; interacts with price indexation (**adaptive** expectations) and the elasticity of substitution (**markups**)

South African Evidence

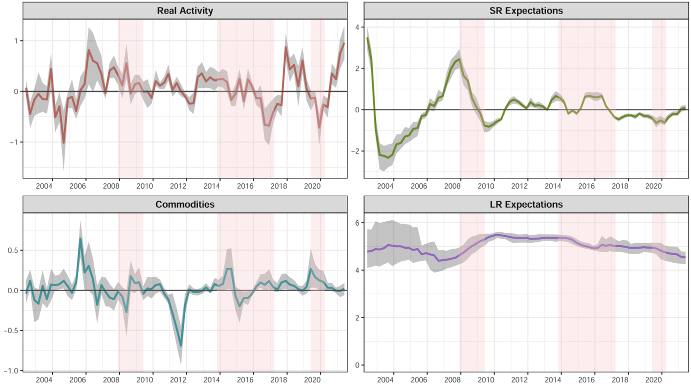
- **Burger and du Plessis** (2006) look at several versions of the NK Phillips curve
  - NKPC with labour income share merits consideration in the study of SA inflation dynamics
  - Parameter estimates are sensitive to model specification and sample periods, and often subject to weak instrument problems
  - Find that prices are fixed on average, for the sample period 1985Q1−2003Q4, between 3.5 and 4 quarters.
- **Reid and du Rand** (2015) estimate a Sticky Information Phillips Curve á la Mankiw and Reis
  - Find that the marginal cost calculation approach is not suited to the South African case
  - Prices are fixed for around 1.3 quarters
- **Fedderke and Liu** (2018) look at a range of inflation models: Phillips curve, New Keynesian Phillips curve, monetarist and structural models of inflation
  - the single most robust covariate of inflation is unit labour cost, decomposed into the nominal wage (strong positive relationship) and real labour productivity (weak negative relationship).
  - inflationary pressure: output gap does not statistically robust; money growth and govt. expenditure robust.

South African Evidence

- **Viegi and Dadam** (2020) estimate a New Keynesian Wage Phillips Curve – wage formation mechanism very insensitive to overall macroeconomic conditions
- **Botha, Kuhn, and Steenkamp** (2020) consider an augmented Phillips curve specification that accounts for various drivers of inflation and test different measures of economic slack in forecasting inflation and GDP outcomes
  - show that a Phillips curve relationship continues to exist in South Africa.
  - the slack measure that performs best is one that includes labour market indicators.
  - While the output gap-inflation channel continues to operate, the contributions to inflationary pressures of factors such as past inflation and inflation expectations have been much more important over recent years
- **du Rand, Hollander, and van Lill** (2023) estimate the Phillips curve relationship using a novel deep learning technique.
  - multiple measures of economic slack/tightness and inflation expectations
  - long-run inflation expectations are the dominant determinant of inflation, with these expectations anchored around 5% historically but declining since the 2008 global financial crisis.
  - renewed empirical support for the relevance of the Phillips curve: short-run expectations and real economic activity are significant

Hemisphere Neural Network Phillips Curve

Figure 2: Contributions of hemispheres to inflation estimate



Note: contributions of real activity, short-run expectations, commodities, and long-run expectations to inflation over time. Dashed lines represent the beginning of the out-of-sample period. Recessions (business cycle turning points as retrieved from the SARB) are depicted in pink. The grey shading represents the 68% credible region.

Source: authors' elaboration.

Figure: du Rand, Hollander, and van Lill (2023)

The Canonical NK model

The canonical three-equation new Keynesian model of Clarida, Galí, and Gertler (2000)

- The price-adjustment equation is the new Keynesian Phillips curve of Section 7.4
    - strong microeconomic foundations
    - comparative simplicity
- The aggregate demand equation of the model is the new Keynesian IS curve of Sections 6.1 and 7.1.
- The final equation describes monetary policy
    - we assume the central bank follows *a forward looking interest-rate rule*, adjusting the interest rate in response to changes in expected future inflation and output.

The Canonical NK model

The three core equations are:

$$
\begin{aligned}
y_t &= E_t[y_{t+1}] - \frac{1}{\theta} r_t + u_t^{IS}, \quad \theta > 0, & (6)\\
\pi_t &= \beta E_t[\pi_{t+1}] + \kappa y_t + u_t^{\pi}, \quad 0 < \beta < 1, \quad \kappa > 0, & (7)\\
r_t &= \phi_\pi E_t[\pi_{t+1}] + \phi_y E_t[y_{t+1}] + u_t^{MP}, \quad \phi_\pi > 0, \quad \phi_y \geq 0. & (8)
\end{aligned}
$$

Equation (6) is the new Keynesian IS curve, (7) is the new Keynesian Phillips curve, and (8) is the forward-looking interest-rate rule. The shocks follow independent AR-1 processes:

$$
\begin{aligned}
u_t^{IS} &= \rho_{IS} u_{t-1}^{IS} + e_t^{IS}, \quad -1 < \rho_{IS} < 1, & (9)\\
u_t^{\pi} &= \rho_\pi u_{t-1}^{\pi} + e_t^{\pi}, \quad -1 < \rho_\pi < 1, & (10)\\
u_t^{MP} &= \rho_{MP} u_{t-1}^{MP} + e_t^{MP}, \quad -1 < \rho_{MP} < 1, & (11)
\end{aligned}
$$

where e^{IS}, e^{π}, and e^{MP} are white-noise disturbances that are uncorrelated with one another.

The Canonical NK model

Next session we will do a practical application of the NK model. You can use any variation to compare to this baseline, for example:

- NKPC with indexation (simply replace the forward-looking NK equation with the equation (7.76) on p. 344, Section 7.7.
- Alternative MP rules
- Habit formation, Sticky wages, Credit market (for these extensions you'll use pre-existing code)

Discussion:

- The model's implications for the costs of disinflation (e.g., SARB moving from 4.5% to 3%)