Investment in a Housing Market

Honours Macroeconomics Problem Set

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This problem is adapted from:

> Poterba, J.M., 1984. "Tax subsidies to owner-occupied housing: an asset-market approach" *The Quarterly Journal of Economics*, 99(4), pp.729-752.

Consider a continuous time model of investment in the housing market. Let H_t denote the stock of housing, I_t the investment in housing, p_t the price of a unit of housing and R_t the rent that can be earned on a unit of housing per instant.

Suppose that investment is a function of the price of housing (think of this as the supply curve of housing):

$$\begin{aligned} I_t &= I(p_t) \\ \text{with:} & \\ I'(p_t) &> 0 \end{aligned}$$

and that rent is a function of the stock of housing:

$$\begin{aligned} R_t &= R(H_t) \\ \text{with:} & \\ R'(H_t) &< 0 \end{aligned}$$

The stock of housing depreciates at rate δ so that the evolution is given by:

$$\dot{H}_t = I(p_t) - \delta H_t$$

Lastly, assume that rental income plus capital gains must equal some exogenously fixed required rate of return r:

$$\frac{R(H_t) + \dot{p}_t}{p_t} = r$$

1. Give an economic interpretation of the analytic assumptions made on the behaviour of investment and rent with respect to the variables they depend on.

2. In H, p space (I.e.: in a graph with the stock of housing, H_t, on the horizontal axis and the price of housing, p_t, on the vertical axis) provide a sketch graph of phase diagram that describes the equilibrium of this model. It should contain the following:

   (a) the locus of points where $\dot{H}_t = 0$,

   (b) the locus of points where $\dot{p}_t = 0$,

   (c) the dynamics of H_t and p_t in each of the four quadrants defined by the two loci in (a) and (b), and

   (d) the saddle path that describes the equilibrium evolution to steady state.

3. Why is the $\dot{H}_t = 0$ locus not horizontal in this model? Provide an economic intuition.

4. Are the implied adjustment costs in this model internal or external? Explain. (Hint: $I(p_t)$ can be interpreted as a supply function. Think what the *inverse* supply function says about the market for houses)

5. Suppose the market is initially in long run equilibrium. Then, at moment t_1 it becomes that depreciation rate is higher than before $\delta' > \delta$, but that this higher depreciation rate will only last until some future moment t_2, after which it will return to its original level.

   (a) Provide a full analysis of the impact of this change on the time paths of H_t and p_t. I.e. determine which loci shift, and how the equilibrium evolves from one steady state to the next. Your answer should also include figures showing the time paths of each of the variables in question.

   (b) Provide an economic story for the change in the depreciation rate. That is: what real world event would lead to this type of dynamics?