Consumption

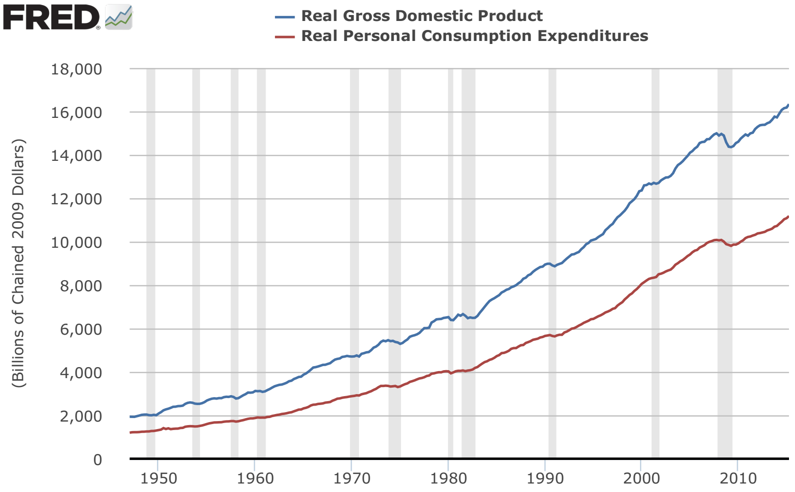
Macroeconomics

What will we do today?

- Which chapters?
  - **Chapter 8** (Consumption) – Required
  - Podcast with Vernon Smith (in Econtalk) – Required
- Theories of consumption

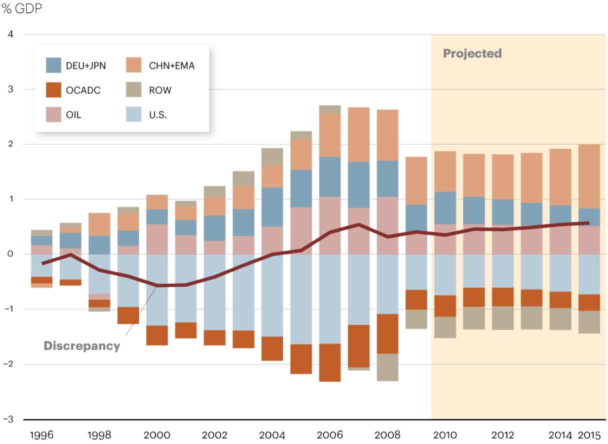
Reasons for studying consumption

- "...the sole end and object of all economic activity" (**JMK**)
- Consumption perhaps related to **happiness**...
- Significant proportion of **AD** $\rightarrow$ will determine how monetary and fiscal policy affects output
- Decision to **save** (consume) is a decision to accumulate capital
  - Important for **current and future expenditure**
  - Influenced by, and impacts on, financial market developments
- Impact on the balance of payments (**BoP**) $\rightarrow$ has been important part of "international imbalances"
  - National accounts $\rightarrow S - I = (G_c - T) + (X + TR - M)$
  - The situation where some countries have more assets than others (long term **deficits** in advanced economies)



Shaded areas indicate US recessions - 2015 research.stlouisfed.org

Figure 1
Global imbalances



Notes: CHN+EMA: China, Hong Kong SAR, Indonesia, Korea, Malaysia, Philippines, Singapore, Taiwan Province of China, and Thailand; DEU+JPN: Germany and Japan; OCADC: Bulgaria, Croatia, Czech Republic, Estonia, Greece, Hungary, Ireland, Latvia, Lithuania, Poland, Portugal, Romania, Slovak Republic, Slovenia, Spain, Turkey, and United Kingdom; OIL: Oil exporters; ROW: rest of the world; U.S.: United States.

Source: IMF October 2010 World Economic Outlook report

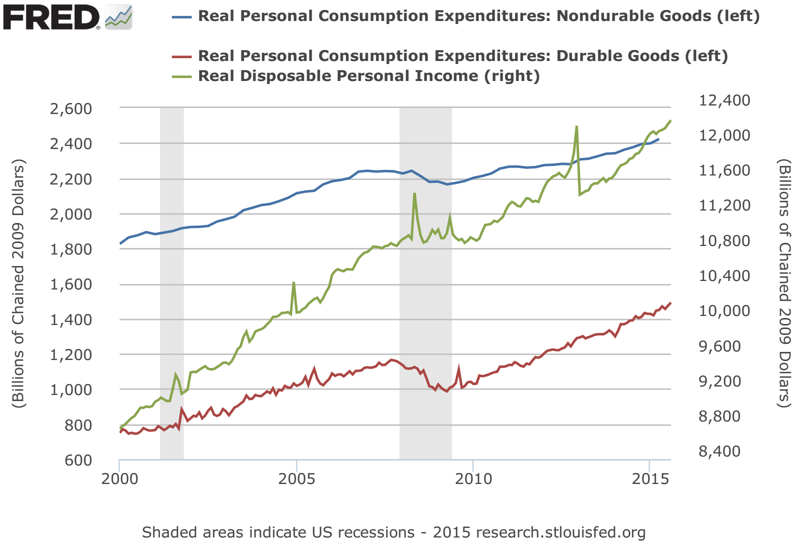
African context

- Many households live close to the subsistence in a **risky** environment $\rightarrow$ **idiosyncratic shocks** lead to high income variability and persistent poverty
- **Shallow credit markets** means limited access to services for savings, insurance and money transfers
- They need strategies to cope with **risk**
  - Agricultural diversification
  - Consumption smoothing $\rightarrow$ self insurance through **precautionary savings** (i.e. smoothing over time) or **risk sharing** (i.e. smoothing across households)

- Recent focus on **microfinancing** to promote financial inclusion

Relevant stylised facts

- Time series of non-durable consumption is **more smooth** than either durable consumption or disposable income
  - Non-durable goods not going to drive **business cycle movements**
- Cohort data in developed countries have a “**hump**” in consumption and income across the life cycle
  - **Hump** is less pronounced *per equivalent adult*
- Important differences between **time series** and **cross section** evidence on the relationship between consumption and income



FRED
Real Personal Consumption Expenditures: Nondurable Goods (left)
Real Personal Consumption Expenditures: Durable Goods (left)
Real Disposable Personal Income (right)
(Billions of Chained 2009 Dollars)
2,600
2,400
2,200
2,000
1,800
1,600
1,400
1,200
1,000
800
600
12,400
12,000
11,600
11,200
10,800
10,400
10,000
9,600
9,200
8,800
8,400
(Billions of Chained 2009 Dollars)
2000
2005
2010
2015
Shaded areas indicate US recessions - 2015 research.stlouisfed.org

Figure 4.1: Total Expenditure

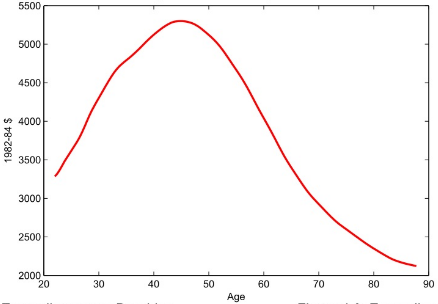


Figure 4.2: Expenditures non Durables

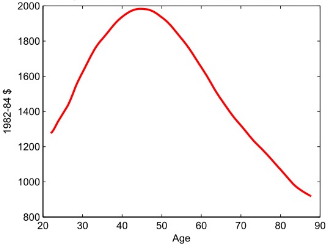


Figure 4.3: Expenditures Durables

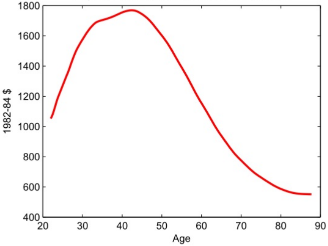


Figure 4.4: Total Expenditure, Adult Equivalent

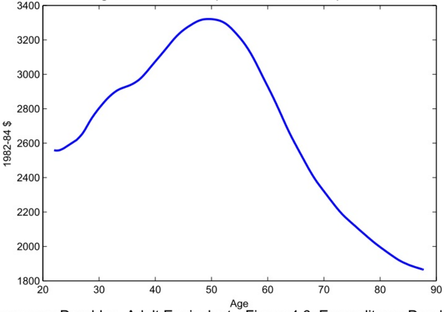


Figure 4.5: Expenditures non Durables, Adult Equivalent

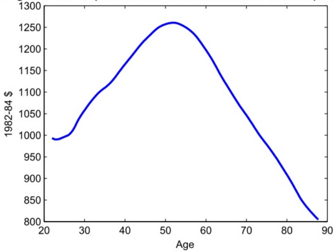
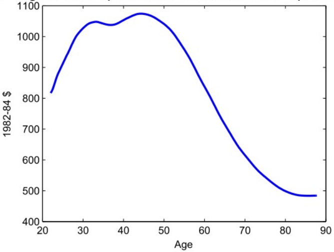
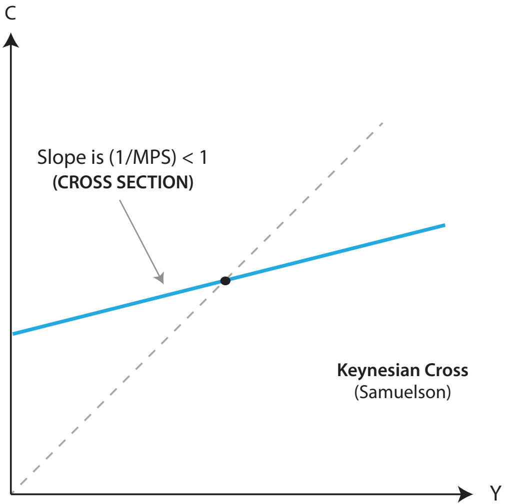
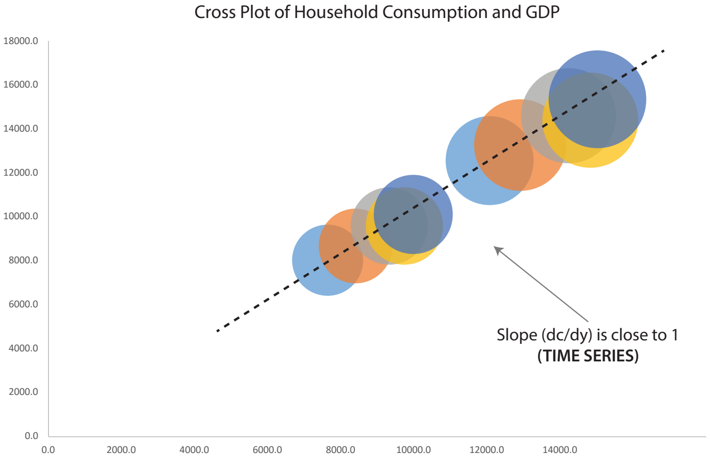


Figure 4.6: Expenditures Durables, Adult Equivalent




C
Slope is (1/MPS) < 1
(CROSS SECTION)
Keynesian Cross
(Samuelson)
Y



Cross Plot of Household Consumption and GDP
18000.0
16000.0
14000.0
12000.0
10000.0
8000.0
6000.0
4000.0
2000.0
0.0
0.0
2000.0
4000.0
6000.0
8000.0
10000.0
12000.0
14000.0
Slope (dc/dy) is close to 1
(TIME SERIES)

Old theories of consumption (**Keynes**)

- **Keynes** was the first to formalise a consumption function
- Consumption depends on **income** and *subjective needs and the psychological propensities and habits of the individuals composing it*

- **Keynes** in the *General Theory*, states that

  The fundamental psychological law, upon which we are entitled to depend with great confidence both a priori from our knowledge of human nature and from the detailed facts of the experience, is that men are disposed, as a rule and on average, to increase their consumption as their income increases, but not by as much as their income...

Old theories of consumption (**Keynes**)

- Based on the Keynesian consumption function the **Absolute Income Hypothesis** (AIH) gives the following relation

$$C_t = a + bY_t^d, \quad a > 0,\, 1 > b > 0$$

- where a is autonomous consumption, b is the marginal propensity to consume and Y_t^d is after tax (disposable) income
- Several **problems** with this model $\rightarrow$ empirical and theoretical

Empirical problems

- $\partial C/\partial Y \rightarrow$ contender for title of "most estimated coefficient"

- Early empirical work found problems with the **Keynesian consumption function**
- Cross sectional estimates were not the problem $\rightarrow$ time series estimates showed a largely **proportional relationship** between consumption and income
- **Kuznets paradox** $\rightarrow$ percentage of Y_t^d that is consumed is remarkably constant in the LR
- In order to better understand this paradox, consider the average propensity to consume (APC), defined as,

$$APC \equiv C/Y = \frac{a + bY}{Y}$$
$$APC = \frac{a}{Y} + b$$

- Implies that with higher levels of income APC will be lower, but Kuznets finds that $a = 0$ and $MPC = APC$

Empirical problems

- Time series implications:
  - $Y_t \to \infty \Rightarrow$ APC declines to negligible amount $\Rightarrow$ Savings $\to \infty$
  - Implies rich countries should have very high savings rates $\to$ **reduce consumption as they grow**
  - Kuznets found savings rate stable over time, even though income increased significantly
- Cross section implications:
  - Across households $\to$ implies households with higher levels income should save proportionally **smaller portion of income**
  - Cross section evidence by Brumberg and Modigliani (1954) suggest that the *opposite is true*

Empirical problems

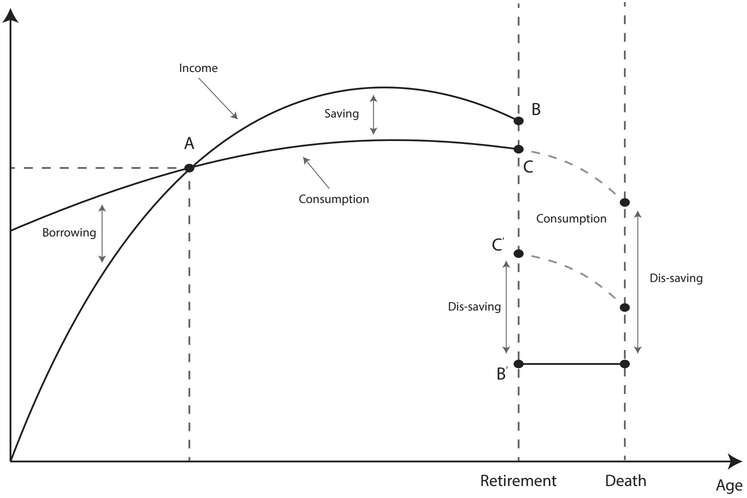
- In light of the Kuznets paradox a movement was started toward better understanding consumption $\rightarrow$ first attempt was the **relative-income hypothesis**
  - Consumption also depends on income relative to the past and relative to other households

- **Friedman** showed that Kuznets paradox can be explained by distinguishing between **permanent** and **transitory** income
- Over time the variation in permanent income dominates variation in transitory income
  - Implies that the slope of the consumption function will be **proportional to income**
- When variation in transitory income dominates (as in the cross section) then the slope will be as **JMK** predicted

- We will get back to this, and do the empirical specification, after discussing **Friedman's PIH model** in more detail

Life Cycle Hypothesis

- Modigliani's key insight $\rightarrow$ household's use savings (and dis-saving / borrowing) to smooth consumption over the **life-cycle**
- Savings treated as **future consumption**
- This perspective exposes some common fallacies about savings
  - Keeping up with the future Joneses
  - The poor save proportionally less
- Let us take a look at the graphical presentation of this theory


Income
Saving
B
A
C
Consumption
Borrowing
Consumption
C′
Dis-saving
Dis-saving
B′
Retirement
Death
Age

Modern consumption theory (PIH)

- The **Permanent Income Hypothesis** (PIH) was developed in 1957 by **Friedman** in response to difficulties explaining long run consumption patterns

- We start with a simple model, where households with lifetime of T periods have the following utility function

$$U = \sum_{t=1}^{T} u(C_t), \quad u'(\bullet) > 0, \quad u''(\bullet) < 0$$

- Discount and interest rates are assumed to be zero
- With initial wealth A_0 and lifetime earnings Y_t, the lifetime BC is,

$$\sum_{t=1}^{T} C_t \leq A_0 + \sum_{t=1}^{T} Y_t$$

Modern consumption theory (PIH)

- The Lagrangian for this problem is,

$$\mathcal{L} = \sum_{t=1}^{T} u(C_t) + \lambda \left(A_0 + \sum_{t=1}^{T} Y_t - \sum_{t=1}^{T} C_t \right)$$

- Expanding the Lagrangian we have,

$$\begin{aligned}\mathcal{L} =& [u(C_1) + u(C_2) + \ldots + u(C_T)] + \\ & \lambda \left(A_0 + Y_1 + Y_2 + \ldots Y_T - (C_1 + C_2 + \ldots + C_T) \right)\end{aligned}$$

- The first order condition for this problem is,

$$\begin{aligned}\left(\frac{\partial \mathcal{L}}{\partial C_1}\right) &\qquad u'(C_1) = \lambda \\ \left(\frac{\partial \mathcal{L}}{\partial C_t}\right) &\qquad u'(C_t) = \lambda\end{aligned}$$

- Holds for all $t \in \{1, T\} \rightarrow$ consumption constant for all periods
- $C_t = C_1 = C_2 = \ldots = C_T$ and $\sum_{t=1}^{T} C_t = T \times C_t$

Modern consumption theory (PIH)

- Substituting this result into the budget constraint yields,

$$\sum_{t=1}^{T} C_t = A_0 + \sum_{t=1}^{T} Y_t$$

$$T \times C_t = A_0 + \sum_{t=1}^{T} Y_t$$

$$C_t = \underbrace{\frac{1}{T}\left[A_0 + \sum_{t=1}^{T} Y_t\right]}_{\text{Permanent income}} \quad \forall\, t$$

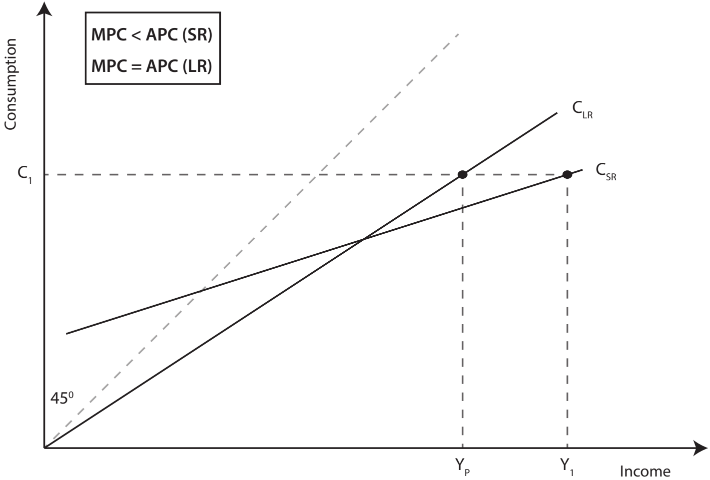
- The **PIH** states that lifetime resources are **evenly spread** over the household's lifetime
- Key insight $\rightarrow$ consumption is determined by **permanent income**
- Windfalls are spread over the household's lifetime, and temporary tax cuts will have little impact on consumption

Modern consumption theory (PIH)

- We can also use this model to analyse **savings behaviour**

$$
\begin{aligned}
S_t &= Y_t - C_t \\
\therefore \quad S_t &= \underbrace{Y_t - \frac{1}{T}\left[A_0 + \sum_{t=1}^{T} Y_t\right]}_{\text{Transitory income}}
\end{aligned}
$$

- Difference between current income Y_t and permanent income is **transitory income** $\rightarrow Y_t - Y_t^P = Y_t^T$
- Savings is high when income is high **relative to the average** $\rightarrow$ transitory income is high
- **Important:** Saving is used to smooth the path of consumption!


MPC < APC (SR)
MPC = APC (LR)
Consumption
C_1
C_{LR}
C_{SR}
45^0
Y_P
Y_1
Income

Modern consumption theory (PIH)

- **Friedman** used regression analysis to find support for his theory
- By construction for individual i, we have that $C_{it} = Y_i^P$ and $Y_{it} = Y_{it}^T + Y_i^P$
- Since transitory income Y_{it}^T reflects departures from Y_i^P, on average it will equal zero
- In an independently drawn sample, it is expected that,

$$\sum_{i=0}^{N} Y_{it}^T \approx 0$$

- In a regression of the Keynesian consumption function we have,

$$C_i = a + bY_i + e_i$$

- The OLS estimators are $\hat{b} = \frac{\mathrm{Cov(Y,C)}}{\mathrm{Var}(Y)}$ and $\hat{a} = \bar{C} - \hat{b}\bar{Y}$

Modern consumption theory (PIH)

- If the permanent income hypothesis is accurate we have that,

$$\hat{b} = \frac{\text{Cov}(Y^P + Y^T, Y^P)}{\text{Var}(Y^P + Y^T)} = \frac{\text{Var}(Y^P)}{\text{Var}(Y^P) + \text{Var}(Y^T)}$$

- In cross section $\rightarrow$ we expect $\text{Var}(Y^T) \gg 0$, which means that $0 < \hat{b} < 1$
- In time series $\rightarrow$ we expect $\text{Var}(Y^T) \approx 0$, which means that $\hat{b} \approx 1$
  - This matches the Kuznets paradox $\rightarrow \hat{a} = (1 - \hat{b})\bar{Y}^P \approx 0$

- Matches pattern found empirically $\rightarrow$ not strong enough evidence
- **Robert Hall** (1978) attempts to fix this with rational expectations model

Modern consumption theory (PIH)

- Some useful things about using $\mathbb{E}$ to define moments of a series

$\mathbb{E}(Y) \rightarrow$ Expected value of Y (mean)

$\mathbb{E}\left[Y-\mathbb{E}(Y)\right]^2 = \mathrm{Var}(Y) \rightarrow$ Variance of Y

$\mathbb{E}\left[(Y-\mathbb{E}(Y))(X-\mathbb{E}(X))\right] = \mathrm{Cov}(Y,X) \rightarrow$ Covariance of Y and X

- In our example above we know that $\mathbb{E}(Y^T) = 0$ and $\mathbb{E}(Y^T Y^P) = 0$

$$\hat{b} = \frac{\mathbb{E}\left[(Y-\mathbb{E}(Y))(C-\mathbb{E}(C))\right]}{\mathbb{E}\left[Y-\mathbb{E}(Y)\right]^2}$$

$$\hat{b} = \frac{\mathbb{E}\left[(Y^P+Y^T-\mathbb{E}(Y^P))(Y^P-\mathbb{E}(Y^P))\right]}{\mathbb{E}\left[Y^P+Y^T-\mathbb{E}(Y^P)\right]^2}$$

- To simplify this, we can expand the numerator,

$$\mathbb{E}\left[(Y^P Y^P + Y^T Y^P - \mathbb{E}(Y^P)Y^P) - (Y^P\mathbb{E}(Y^P) - Y^T\mathbb{E}(Y^P) + \mathbb{E}(Y^P)\mathbb{E}(Y^P))\right]$$

$$\mathbb{E}\left[(Y^P Y^P - \mathbb{E}(Y^P)Y^P) - (Y^P\mathbb{E}(Y^P) - \mathbb{E}(Y^P)\mathbb{E}(Y^P))\right]$$

$$\mathbb{E}\left[(Y^P - \mathbb{E}(Y^P))(Y^P - \mathbb{E}(Y^P))\right] = \mathrm{Var}(Y^P)$$

Uncertainty and random Walks

- Let's use the same model as before, however, now the future income stream is **uncertain**
- **Quadratic** instantaneous utility function, giving lifetime utility as,

$$\mathbb{E}(U) = \mathbb{E}\left[\sum_{t=1}^{T}\left(C_t - \frac{a}{2}C_t^2\right)\right], \quad a > 0$$

- subject to the lifetime budget constraint,

$$\sum_{t=1}^{T} C_t \leq A_0 + \sum_{t=1}^{T} Y_t$$

- The Lagrangian setup then is,

$$\mathcal{L} = \mathbb{E}\left[\sum_{t=1}^{T}\left(C_t - \frac{a}{2}C_t^2\right)\right] + \lambda\left(A_0 + \sum_{t=1}^{T} Y_t - \sum_{t=1}^{T} C_t\right)$$

Uncertainty and random Walks

- First order condition with respect to first-period consumption is,

$$\left(\frac{\partial \mathcal{L}}{\partial C_1}\right) \qquad \mathbb{E}_1(1 - aC_1) = \lambda$$

- First-period consumption is known at $t = 1 \rightarrow (1 - aC_1) = \lambda$
- At time $t = 1$, HH choice of consumption for any period t is,

$$\left(\frac{\partial \mathcal{L}}{\partial C_t}\right) \qquad \mathbb{E}_1(1 - aC_t) = \lambda$$

- Combining these expressions we have the choice of first period consumption,

$$(1 - aC_1) = \mathbb{E}_1(1 - aC_t)$$
$$C_1 = \mathbb{E}_1(C_t)$$

Uncertainty and random Walks

- Plugging $C_1 = \mathbb{E}_1(C_t)$ this into budget constraint yields,

$$\sum_{t=1}^{T} \mathbb{E}_1[C_t] = A_0 + \sum_{t=1}^{T} \mathbb{E}_1[Y_t]$$

$$\sum_{t=1}^{T} C_1 = A_0 + \sum_{t=1}^{T} \mathbb{E}_1[Y_t]$$

$$T \times C_1 = A_0 + \sum_{t=1}^{T} \mathbb{E}_1[Y_t]$$

$$C_1 = \frac{1}{T}\left[A_0 + \sum_{t=1}^{T} \mathbb{E}_1[Y_t]\right]$$

Uncertainty and random Walks

- Recall that we had from the FOC for consumption today, relative to some future period $t \rightarrow C_1 = \mathbb{E}_1(C_t)$
- Condition holds for each consumption value chosen at $t = 1$
- Then consumption today C_1 is equal to the expected consumption next period C_2

$$\begin{aligned} C_1 &= \mathbb{E}_1(C_2) \\ \therefore \quad C_{t-1} &= \mathbb{E}_{t-1}(C_t) \end{aligned}$$

- Suppose that e_2 is the error between forecasted and actual C_2, this means that $\rightarrow e_2 = C_2 - \mathbb{E}_1(C_2)$
- More generally, we can let the error be e_t, which would yield,

$$\begin{aligned} e_t &= C_t - \mathbb{E}_{t-1}(C_t) \\ C_t &= \mathbb{E}_{t-1}(C_t) + e_t \end{aligned}$$

- Combine with $C_{t-1} = \mathbb{E}_{t-1}[C_t]$, to get $C_t = C_{t-1} + e_t$

Uncertainty and random Walks

$$C_t = C_{t-1} + e_t$$
$$C_t - C_{t-1} = e_t$$
$$\Delta C_t = e_t$$

- Consumption must be a Martingale $\rightarrow$ **random walk** if e_t is i.i.d
- Random walk is an AR(1) process where the autoregressive parameter is one $\rightarrow$ integrated of order 1, I(1) process

- Why does this imply changes in consumption are **unpredictable**?
- If consumption is expected to rise in future, household will respond by consuming more today $\rightarrow$ in order to **smooth out this increase** over his lifetime

Uncertainty and random Walks

- **Romer** gives a detailed account of what would happen if we solve for consumption in period 2, with the end result being,

$$C_2 = C_1 + \frac{1}{T-1}\left(\sum_{t=2}^{T}\mathbb{E}_2[Y_t] - \sum_{t=2}^{T}\mathbb{E}_1[Y_t]\right)$$

- In parenthesis we have the difference between the forecasted lifetime income between periods 1 and 2
- Consumption in period 2 will differ from consumption in period 1 only if there was a shock that caused lifetime income to deviate from the forecast in period 1

- This expression is consistent with **certainty equivalence**
  - Implies that individuals consume the same amount they would if future income were certain to be equal to their means
- In other words, $C_1 = C_2$ if $\sum_{t=2}^{T}\mathbb{E}_2[Y_t] = \sum_{t=2}^{T}\mathbb{E}_1[Y_t]$

Uncertainty and random Walks: Empirical application

- Several papers tried to use the specification of Hall (1978) to construct empirical tests of the **PIH**

- **Hall** (1978) found the following empirical results
  1. Consumption follows a **random walk process** $\rightarrow$ from estimate of marginal utility on it's past value $C_t = \beta C_{t-1} + \varepsilon_t$, finds $\beta = 1$
  2. Consumption cannot be predicted based on past values of consumption $\rightarrow$ **F-test** on coefficients of lags beyond the first

- General implications $\rightarrow$ the data supports the **PIH**
- Faced with an unexpected decline in income, consumption declines by the amount of the fall in permanent income
- No reason to expect that consumption would **rebound**

- Remember Ramsey model $\rightarrow \dot{C}/C$ depends on $\{r, k, \rho, \theta\}$
  - Effect of income on consumption was through its effect on the level $\rightarrow$ **NOT the growth rate**
  - Level of consumption path only depends on **PV of lifetime income** $\rightarrow$ not on when income is received

Uncertainty and random Walks: Empirical application

- There have been many tests of the random walk hypothesis (**Romer** Section 8.3)
- **Hall** predicted that consumption would only adjust to the extent that permanent income adjusts over the business cycle
- However, aggregate data suggests that there is *excess sensitivity* for consumption
  - **Marjorie Flavin** (1993) $\rightarrow$ changes in income seemed to cause consumption to change by more than predicted
  - Tests with household level data (cross section) by **Shea** (1995) $\rightarrow$ also found large *excess sensitivity*
- Excess sensitivity does not hold for large expected movements

Interest rate and saving

- The potential impact of higher (lower) interest rates on saving is an **important policy topic**
- Household with **CRRA** utility and non-zero discount rate ρ

$$U = \sum_{t=1}^{T} \left(\frac{1}{1+\rho} \right)^t \cdot \frac{C_t^{1-\theta}}{1-\theta}$$

- Following the budget constraint when the interest rate is r gives,

$$\sum_{t=1}^{T} \left(\frac{1}{1+r} \right)^t C_t \leq A_0 + \sum_{t=1}^{T} \left(\frac{1}{1+r} \right)^t Y_t$$

Interest rate and saving

- We can set the problem up as a Lagrangian,

$$\mathcal{L} = \sum_{t=1}^{T} \beta^t \frac{C_t^{1-\theta}}{1-\theta} + \lambda \left(A_0 + \sum_{t=1}^{T} \left(\frac{1}{1+r} \right)^t Y_t - \sum_{t=1}^{T} \left(\frac{1}{1+r} \right)^t C_t \right)$$

- The FOC with respect to the consumption choice in period t is

$$\left(\frac{\partial \mathcal{L}}{\partial C_t} \right) \qquad \beta^t C_t^{-\theta} = \lambda \frac{C_t}{(1+r)^t}$$

$$\lambda = \beta^t (1+r)^t C_t^{-\theta}$$

- FOC holds for each period, so for period $t+1$

$$\left(\frac{\partial \mathcal{L}}{\partial C_{t+1}} \right) \qquad \lambda = \beta^{t+1} (1+r)^{t+1} C_{t+1}^{-\theta}$$

Interest rate and saving

- Combining these we obtain the **Euler equation**

$$\beta^t(1+r)^t C_t^{-\theta} = \beta^{t+1}(1+r)^{t+1} C_{t+1}^{-\theta}$$
$$\frac{C_t^{-\theta}}{C_{t+1}^{-\theta}} = \beta(1+r)$$
$$\frac{C_{t+1}}{C_t} = [\beta(1+r)]^{\frac{1}{\theta}}$$

- Consumption will follow a **random walk** if $[\beta(1+r)]^{\frac{1}{\theta}} = 1$
- Interest rate affects the relative value of consumption today versus tomorrow in the utility function
  - As interest rate rises household saves more $\rightarrow$ consumption tomorrow will rise relative to consumption today
  - We can see households seek to equate the discounted present value of marginal utility of consumption $\rightarrow$ not consumption itself

Interest rate and saving

- Alternatively, we can use calculus of variations
- The marginal utility of consumption in period t is $C_t^{-\theta}/(1+\rho)^t$

$$\left(\frac{1}{1+\rho}\right)^t C_t^{-\theta} = (1+r)\left(\frac{1}{1+\rho}\right)^{t+1} C_{t+1}^{-\theta}$$

$$\therefore \quad \frac{C_t^{-\theta}}{C_{t+1}^{-\theta}} = \frac{(1+r)}{(1+\rho)}$$

$$\therefore \quad \frac{C_{t+1}}{C_t} = \left(\frac{1+r}{1+\rho}\right)^{\frac{1}{\theta}}$$

- Which is the same result as before
- Does not imply a predictable bath for consumption if interest and discount rates are dissimilar
- Substitution effect observed is outweighed by income effect

Responding to anomalies

- The evidence of excess sensitivity have stimulated new theoretical ideas
  - Precautionary saving
  - Liquidity constraint
  - Departure from rationality (especially impatience)
- All three seem necessary to explain the anomaly

Econometric problem

- **Trygve Haavelmo** $\rightarrow$ used consumption example to point out inconsistency bias that can occur in **OLS** estimation of MPC
- Consider the following **simultaneous equation model**. From the national accounting identity we have that

$$Y_t = C_t + I_t$$

- An estimable form for the Keynesian consumption function is,

$$C_t = a + bY_t + u_t$$

- where a and b are coefficients and u_t is a disturbance term
- Keynesian consumption function assumes that Y_t is an **exogenous variable**, however, according to the national accounts identity it is an **endogenous variable**
- This means we have **simultaneity bias**, with $\mathrm{Cov}(u_t, Y_t) \neq 0 \rightarrow$ our estimate of b will be biased (quick proof follows)
- Once the "Haavelmo problem" was accounted for, corrected estimates of MPC turned out to be **significantly lower**

Simultaneity Bias

- Substitute the value for C_t from the consumption function into the national accounts equation to get,

$$\begin{aligned} Y_t &= \beta_0 + \beta_1 Y_t + u_t + I_t \\ Y_t(1-\beta_1) &= \beta_0 + u_t + I_t \\ Y_t &= \frac{\beta_0}{1-\beta_1} + \frac{I_t}{1-\beta_1} + \frac{u_t}{1-\beta_1} \\ \mathbb{E}(Y_t) &= \frac{\beta_0}{1-\beta_1} + \frac{I_t}{1-\beta_1} \end{aligned}$$

- Combining the last two equations gives us,

$$Y_t - \mathbb{E}(Y_t) = \frac{u_t}{1-\beta_1}$$

- From this, we can look at $\mathrm{Cov}(u_t, Y_t) = \mathbb{E}[Y_t - \mathbb{E}(Y_t)][u_t - \mathbb{E}(u_t)]$,

$$\begin{aligned} &\mathbb{E}[Y_t - \mathbb{E}(Y_t)]u_t, \quad \text{where } \mathbb{E}(u_t) = 0 \\ &\mathbb{E}\left[\frac{u_t}{1-\beta_1}\right]u_t \Rightarrow \left[\frac{1}{1-\beta_1}\right]\mathbb{E}(u_t^2) \Rightarrow \left[\frac{\sigma^2}{1-\beta_1}\right] \neq 0 \end{aligned}$$

Simultaneity Bias

- Next we prove the biasedness of OLS estimator $\hat{\beta}_1$,

$$\begin{aligned}
\text{plim } \hat{\beta}_{1,OLS} &= \frac{\text{Cov}(C_t, Y_t)}{\text{Var}(Y_t)} \\
&= \frac{\text{Cov}(a + bY_t + u_t, Y_t)}{\text{Var}(Y_t)} \\
&= \frac{b\text{Var}(Y_t) + \text{Cov}(u_t, Y_t)}{\text{Var}(Y_t)} \\
&= b + \frac{\text{Cov}(u_t, Y_t)}{\text{Var}(Y_t)} \\
&\neq b
\end{aligned}$$

Simultaneity Bias: IV

- The solution $\rightarrow$ use I_t as a instrumental variable for Y_t
- Investment can only be used as an instrument if $\text{Cov}(I_t, u_t) = 0$ and $\text{Cov}(I_t, Y_t) > 0$
- We use **two-stage least squares** (2SLS) to resolve endogeneity
  - In the first stage $\rightarrow$ regression of endogenous regressor Y_t on the instrument I_t, to obtain a fitted value $\hat{Y}_t$
  - In the second stage $\rightarrow$ dependent variable of interest C_t is regressed on the fitted value $\hat{Y}_t$