# The Significance Level as a Long-Run Error Rate

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**Significance level**

Reject $H_0$ if $P - value \leq \alpha$

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When doing hypothesis testing..

# MISTAKES WILL SOMETIMES BE MADE

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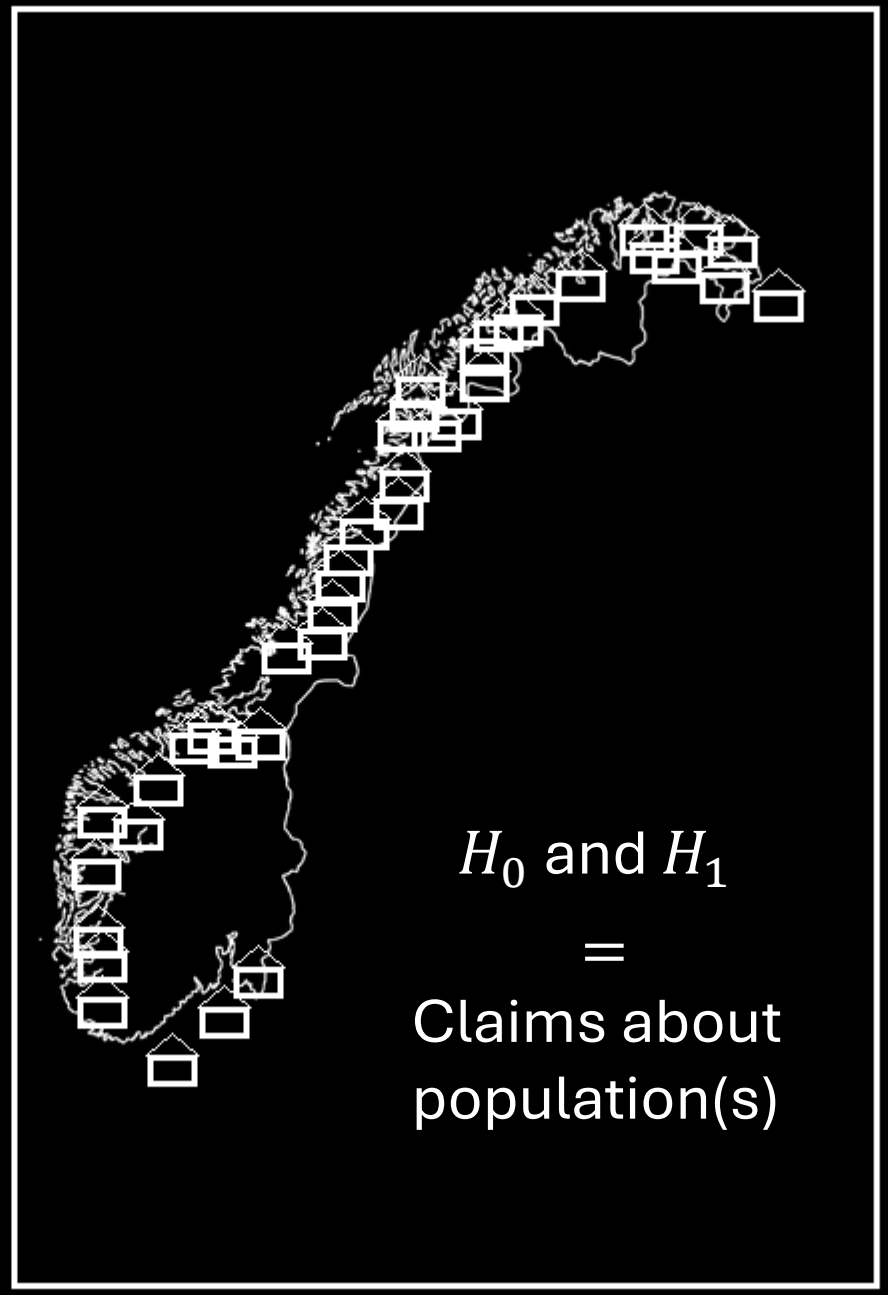
Population

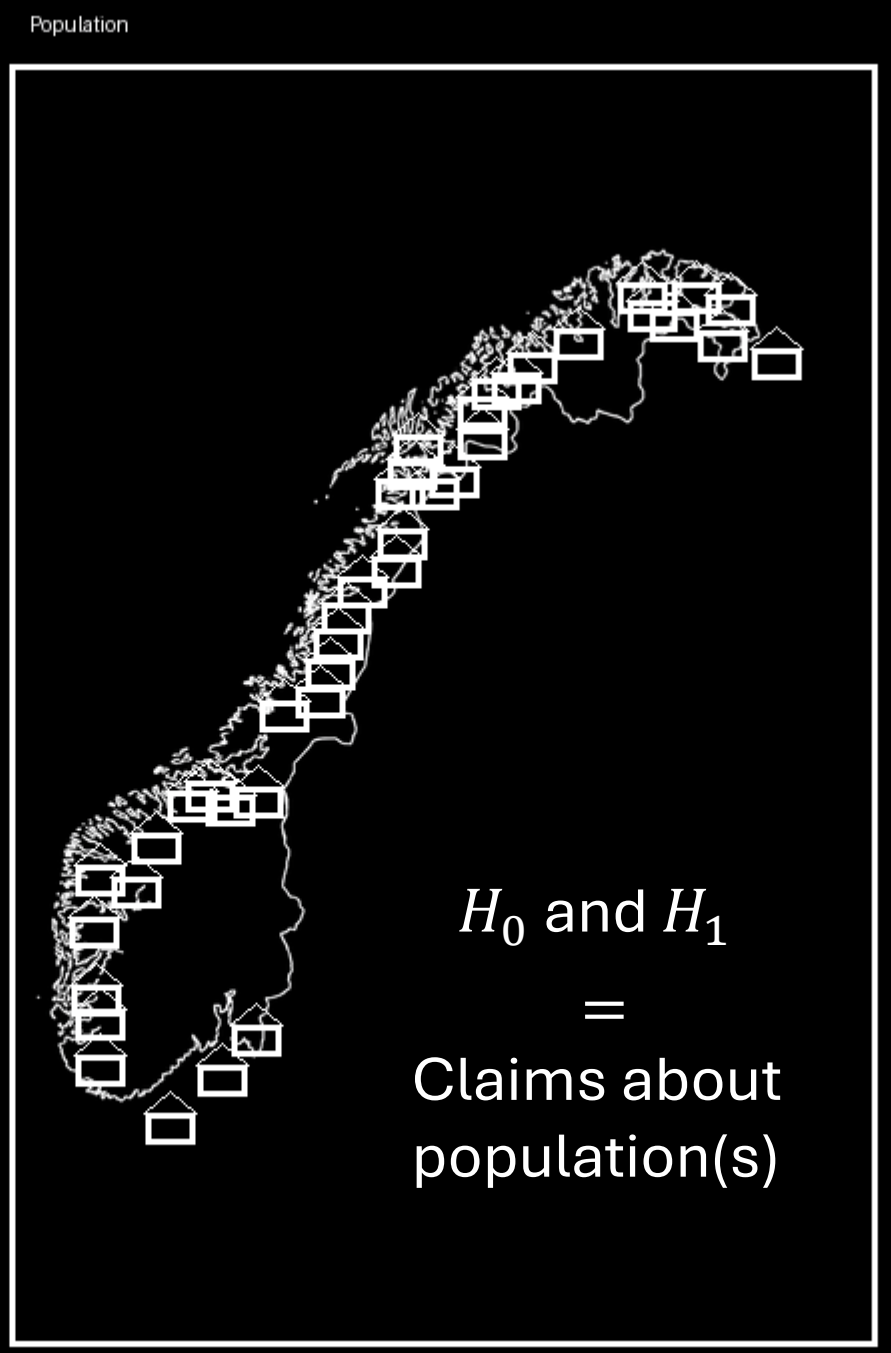


$H_0$ and $H_1$

=

Claims about population(s)

Our conclusions about $H_0$ and $H_1$ are based on:



J. Neyman and Pearson 1933:

“…Without hoping to know whether each separate hypothesis is true or false, we may search for rules to govern our behaviour with regard to them, in following which we insure that, in the long run of experience, we shall not often be wrong”

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States of reality:

Possible decisions

| | $H_0$ is true | $H_0$ is false |
|---|---|---|
| Reject $H_0$ | Type I Error | Correct decision |
| Retain $H_0$ | Correct decision | Type II Error |

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Decision rule:

Reject $H_0$ if $P - value \leq \alpha$

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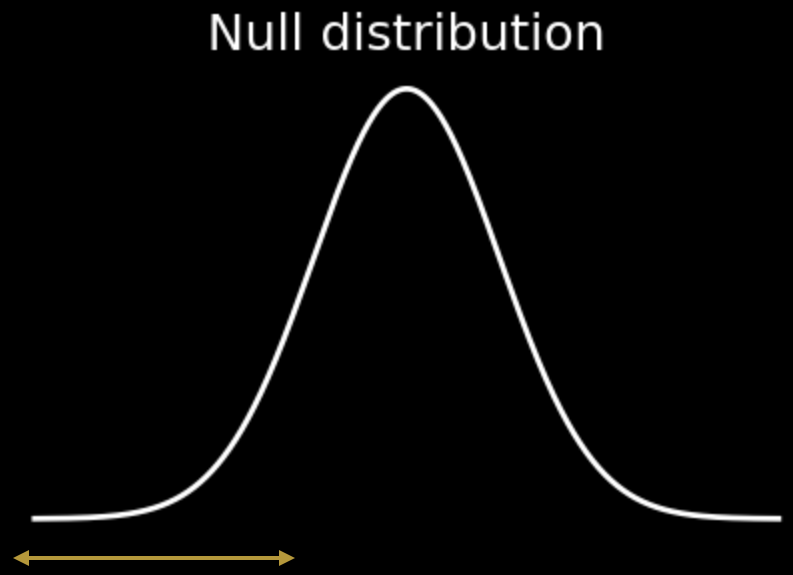
Decision rule:

Reject $H_0$ if $P - value \leq \alpha$

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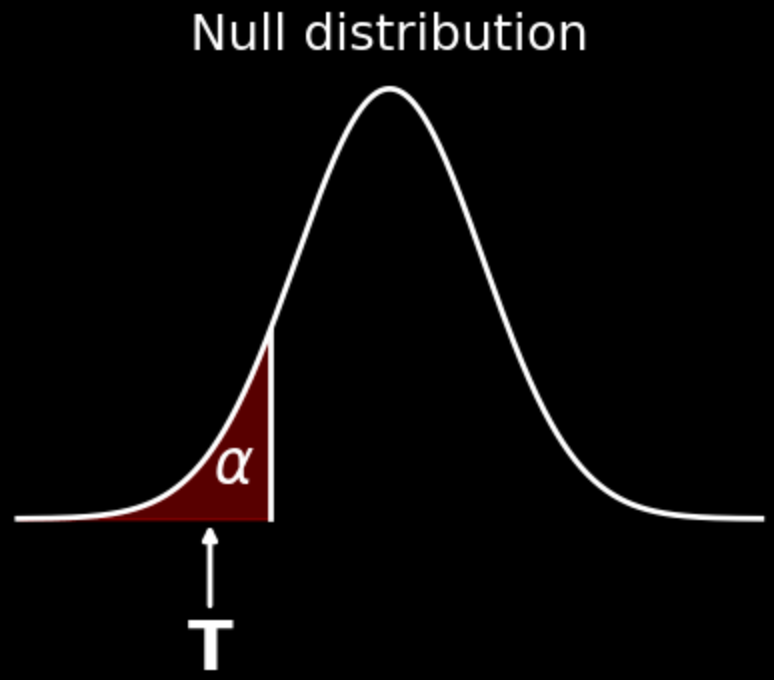
Decision rule:

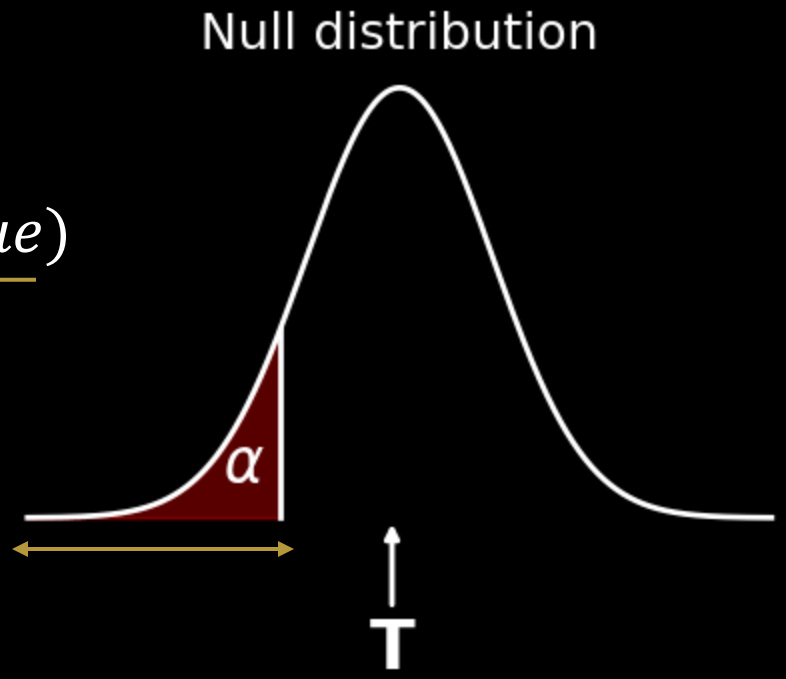
Reject $H_0$ if $P - value \leq \alpha$



$P(Type\ I\ Error) = P(Reject\ H_0 | H_0\ true)$

Decision rule:

Reject $H_0$ if $P - value \leq \alpha$




If $H_0$ is true:

1. Reject $H_0$ = Type I Error
2. T follows the Null distribution

$\Rightarrow P(Type\ I\ Error) = \alpha$

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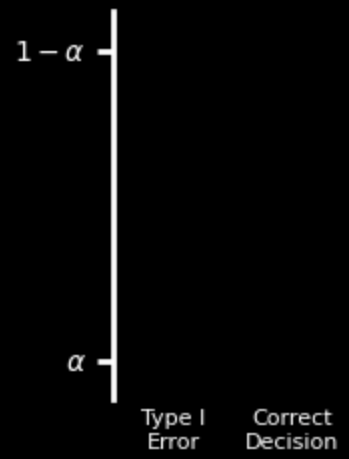


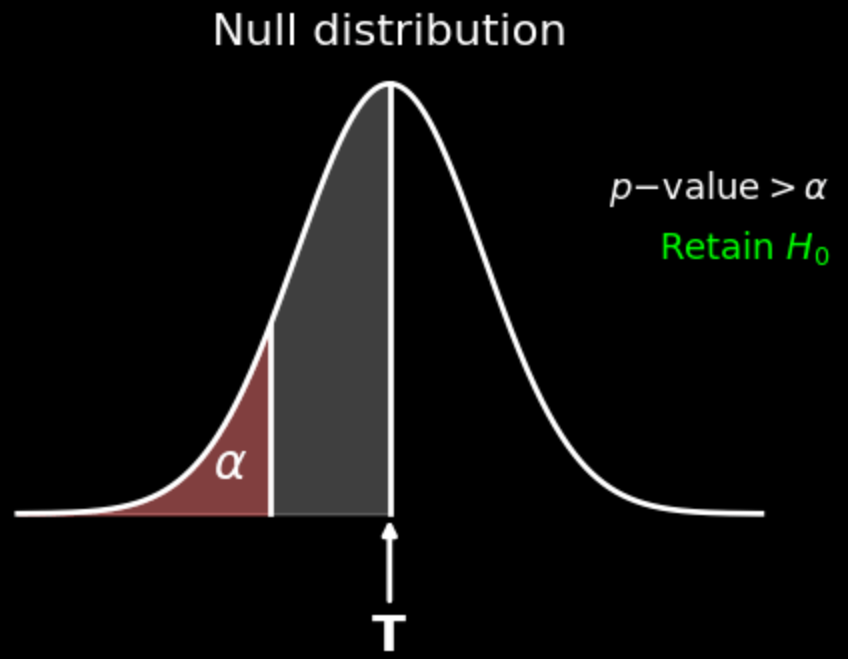


$$\Rightarrow P(Type\ I\ Error) = \alpha$$

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**Hypotheses:**

$H_0$ : The batch of airbags is **defective**.

$H_A$ : The batch of airbags is **safe.**

**Action:** If we reject $H_0$ the batch is approved for installation.

Type I error: Approving a batch of defective airbags → **Risk of injury or death**.

Very low $\alpha$ → Ensures defective airbags rarely pass quality checks.

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**Hypotheses:**

$H_0$ : The batch of airbags is **defective**.

$H_A$ : The batch of airbags is **safe.**

**Common misconception:**

• "A proportion α of all approved batches are defective."

**The significance level is a conditional error rate:**

• α is the proportion of *defective batches that we mistakenly approve* in the long run.

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**1000 A/B tests with hypotheses:**

$H_0 : p_{new} = p_{current}$

$H_A : \ p_{new} > p_{current}$

$$1000 \times 0.05 = 50 \text{ Significant results}$$

«Multiple testing problem»

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Why not set $\alpha = 0,$ so that $P(Type\ I\ Error) = 0$?

| | $H_0$ is true | $H_0$ is false |
|---|---|---|
| Reject $H_0$ | Type I Error | Correct decision |
| Retain $H_0$ | Correct decision | Type II Error |

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# Neyman–Pearson framework:

- *We cannot avoid mistakes.*
- *We can aim to control how often we make them in the long run.*

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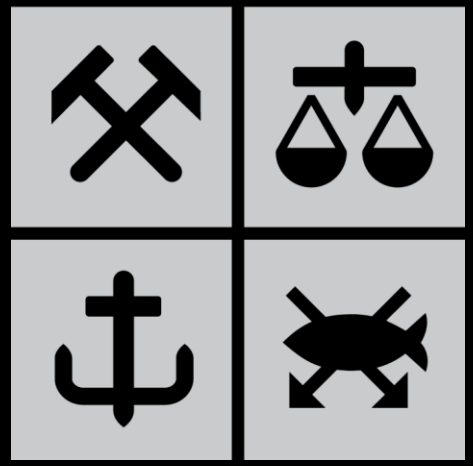

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