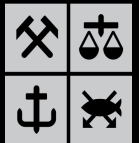


LEAST SQUARES ESTIMATORS

NHH
TECH3



$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

$$Q(\beta_0, \beta_1) = \sum_{i=1}^n \varepsilon_i^2 = \sum_{i=1}^n (y_i - \beta_0 - \beta_1 x_i)^2$$

$$\frac{\partial Q}{\partial \beta_0} = 0 \quad \text{and} \quad \frac{\partial Q}{\partial \beta_1} = 0$$

$$\begin{aligned} \frac{\partial Q}{\partial \beta_0} &= \frac{\partial}{\partial \beta_0} \sum_{i=1}^n (y_i - \beta_0 - \beta_1 x_i)^2 \\ &= \sum_{i=1}^n 2(y_i - \beta_0 - \beta_1 x_i) \cdot (-1) \end{aligned}$$

$$= \sum_{i=1}^n 2(y_i - \beta_0 - \beta_1 x_i) \cdot (-1)$$

$$= -\cancel{2} \sum y_i + \cancel{2} \beta_0 \cdot n + \cancel{2} \beta_1 \sum x_i = 0$$

$$-n \bar{y} + n \beta_0 + \beta_1 n \bar{x} = 0$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$\frac{\partial Q}{\partial \beta_1} = \frac{\partial}{\partial \beta_1} \left(\sum_{i=1}^n (y_i - \beta_0 - \beta_1 x_i)^2 \right)$$

$$= \sum_{i=1}^n 2(y_i - \beta_0 - \beta_1 x_i) \cdot (-x_i)$$

$$= -\cancel{2} \sum_{i=1}^n x_i y_i + \cancel{2} \beta_0 \sum x_i + \cancel{2} \beta_1 \sum x_i^2 = 0$$

$$= \sum_{i=1}^n 2(y_i - \beta_0 - \beta_1 x_i) \cdot (-x_i)$$

$$= -2 \sum_{i=1}^n x_i y_i + 2\beta_0 \sum_{i=1}^n x_i + 2\beta_1 \sum_{i=1}^n x_i^2 = 0$$

$$\hat{\beta}_1 \sum_{i=1}^n x_i^2 = \sum_{i=1}^n x_i y_i - \underbrace{\hat{\beta}_0 \sum_{i=1}^n x_i}$$

$$\hat{\beta}_0 \sum_{i=1}^n x_i = \hat{\beta}_0 n \bar{x}$$

$$= (\bar{y} - \hat{\beta}_1 \bar{x}) n \bar{x}$$

$$= n \bar{y} \bar{x} - n \hat{\beta}_1 \bar{x}^2$$

$$\hat{\beta}_1 \sum_{i=1}^n x_i^2 = \sum_{i=1}^n x_i y_i - (n \bar{x} \bar{y} - n \hat{\beta}_1 \bar{x}^2)$$

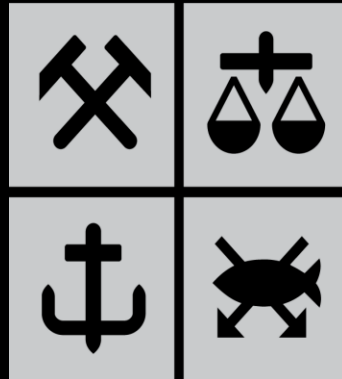
$$\hat{\beta}_1 \sum_{i=1}^n x_i^2 - \hat{\beta}_1 n \bar{x}^2 = \sum_{i=1}^n x_i y_i - n \bar{x} \bar{y}$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n x_i y_i - n \bar{x} \bar{y}}{\sum_{i=1}^n x_i^2 - n \bar{x}^2}$$

$$\hat{\beta}_1 = \frac{\hat{\text{Cov}}(x, y)}{s_x^2}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

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Sondre Hølleland
Geir Drage Berentsen