

THE BINOMIAL DISTRIBUTION



The number of **successes** in n independent
Bernoulli trials has a binomial distribution

If X is the number of successes in n trials, each with success probability p , then

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

for $x \in \{0, 1, 2, \dots, n\}$

Scenario:

A company knows from past campaigns that 10% of customers click a marketing email.

They send an email to $n = 10$ customers.

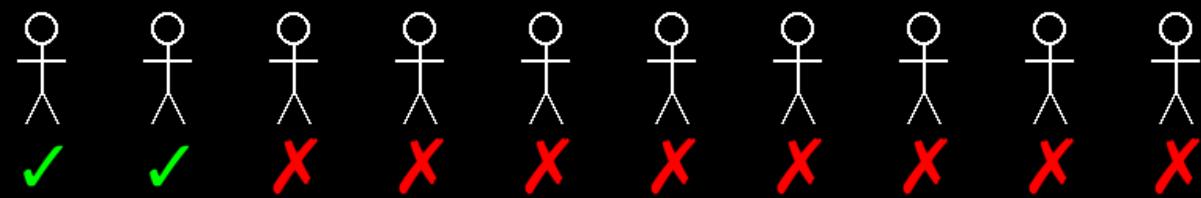
Question:

What is the probability that exactly 2 customers click?

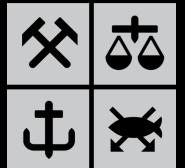
$$P(\text{ } \begin{array}{cccccccccc} \text{♀} & \text{♀} & \text{♀} & \text{♂} & \text{♂} & \text{♀} & \text{♂} & \text{♀} & \text{♂} & \text{♂} \\ \checkmark & \checkmark & \times \end{array})$$

$$= 0.1 \cdot 0.1 \cdot 0.9 \cdot 0.9$$

$$= 0.1^2 0.9^8$$



NHH
TECH3



$$P\left(\begin{array}{cccccccccc} \text{boy} & \text{boy} \\ \checkmark & \times & \checkmark & \times \end{array}\right)$$

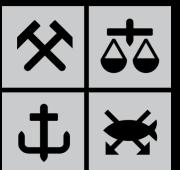
$$= 0.1 \cdot 0.9 \cdot 0.1 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9$$

$$= 0.1^2 0.9^8$$

45



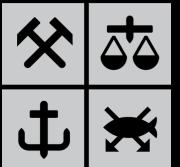
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$$P(\text{Two clicks}) = P\left(\bigcup_{\text{Set 1}} \bigcup_{\text{Set 2}} \right)$$

The diagram illustrates the probability of two clicks. It shows three sets of binary strings of length 10. Set 1 contains strings where the first 5 bits are 'X' and the last 5 are 'X'. Set 2 contains strings where the first 5 bits are 'X' and the last 5 are 'V'. Set 3 contains strings where the first 5 bits are 'V' and the last 5 are 'X'. The sets overlap, with some strings containing both 'X' and 'V' in the first 5 positions. The union of these sets represents all possible outcomes where at least one click occurs.

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$$P(\text{Two clicks}) = \overbrace{0.1^2 0.9^8 + 0.1^2 0.9^8 + \dots + 0.1^2 0.9^8}^{45}$$
$$= 45 \times 0.1^2 0.9^8 \approx 0.19$$

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

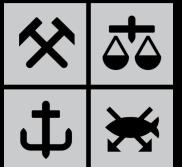
where X follows a binomial distribution with $n = 10$ trials, and success probability $p = 0.1$.

$$X \sim Bin(10, 0.1)$$



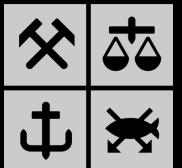
$$P(X = 2) = \binom{10}{2} 0.1^2 0.9^8$$

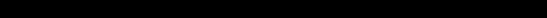
«10 choose 2»



$$P(X = 2) = \binom{10}{2} 0.1^2 0.9^8$$

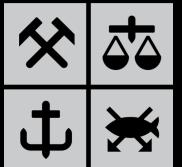
«10 choose 2»



 «10 choose 2» $\binom{10}{2} = \frac{10!}{2!(10-2)!} = 45$

$$P(X = 2) = \binom{10}{2} 0.1^2 0.9^8 = 45 \cdot 0.1^2 0.9^8 \approx 0.19$$

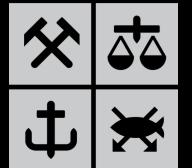
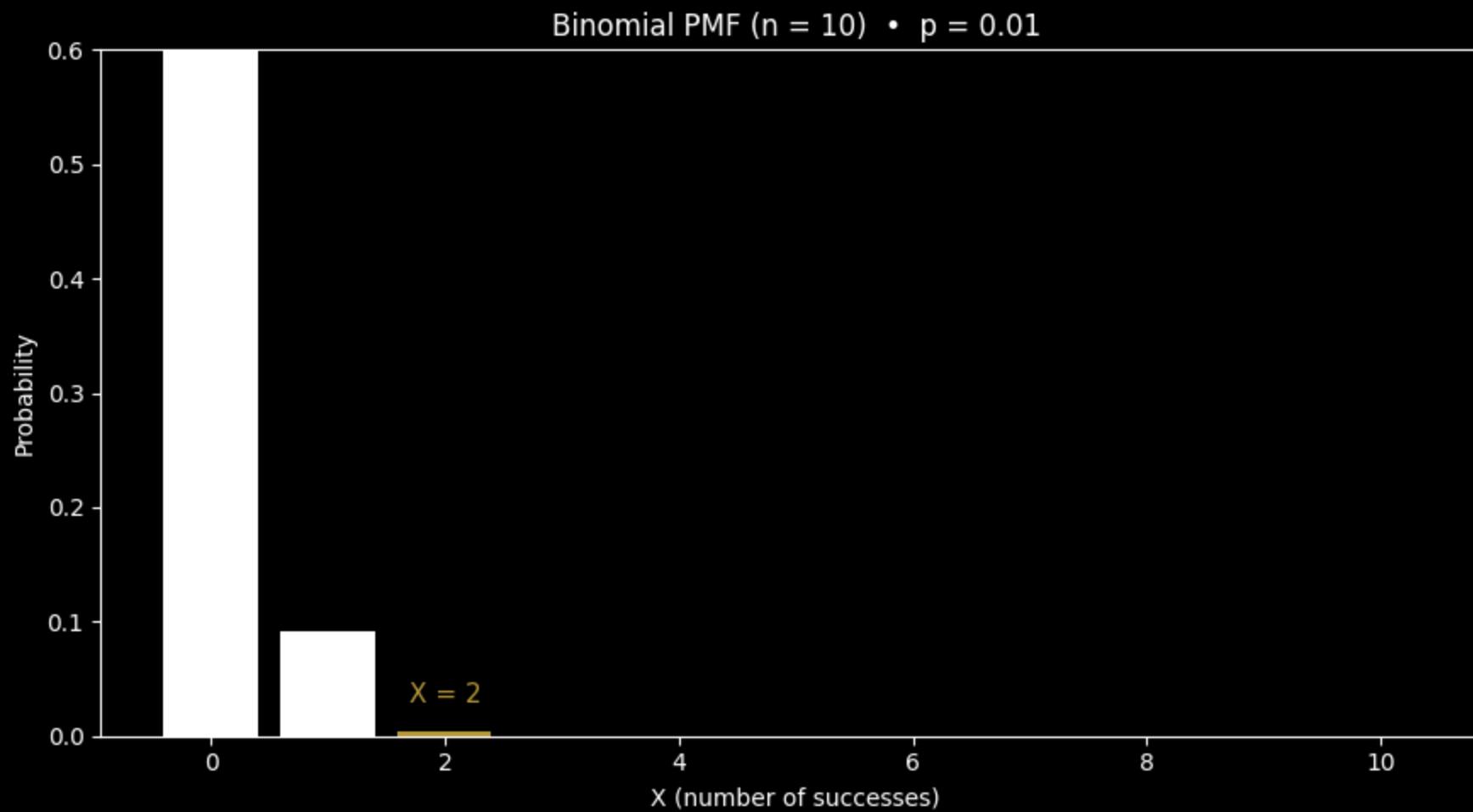
$$P(\textcolor{brown}{\checkmark}, \textcolor{red}{X}, \textcolor{green}{\checkmark}, \textcolor{red}{X}, \textcolor{brown}{\checkmark}, \textcolor{red}{X}, \textcolor{brown}{\checkmark}, \textcolor{red}{X}, \textcolor{brown}{\checkmark}, \textcolor{red}{X})$$



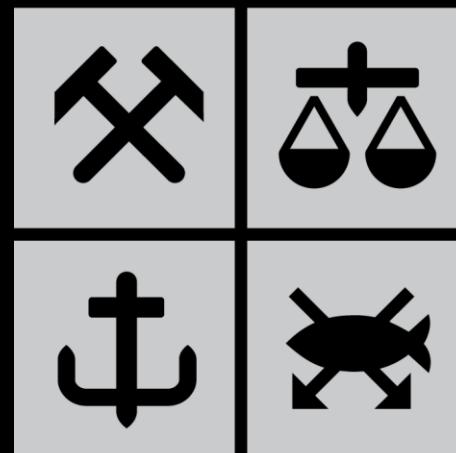
Number of ways to choose x successes in n trials

Probability of any specific sequence with x successes

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$$



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Sondre Hølleland
Geir Drage Berentsen