Week #8
Revenue of single Item auctions

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Four cute results

- Simple versus optimal mechanisms
  - Hartline-Roughgarden EC-09
- Auction versus Negotiation
  - Bulow-Klemperer AER-96
- Digital good auctions
  - Optimal competitive auctions
  - Chen-Gravin-Lu, STOC-14
- Look-ahead auctions
  - Ronen EC-01

Quick review: Myerson auction

- One item
- N buyers
- Each with valuation v_i, independently from F_i
- Auction rule:
  - Compute *virtual value* based on reported v_i and F_i
    - $v_i-(1-F_i(v_i))/f_i(v_i)$
  - Delete all the *negative virtual values*
  - Run second price auction on the nonnegative, if any
  - Charge winner *lowest possible bid* for him to win
  - Key Lemma: expected revenue = expected virtual value

Example: the iid, regular case

- One item
- N buyers
- Each with valuation v_i, independently from F
  - Every buyer has the same distribution
- Auction rule:
  - Compute *virtual value* based on reported v_i and F
    - $v_i-(1-F(v_i))/f(v_i)$
    - Regular: virtual value increasing in v_i
  - Delete all the *negative virtual values*
    - *Impose a reserve price:* $v_i-(1-F(v_i))/f(v_i)=0$
  - Run second price auction on the nonnegative, if any
    - Virtual value and real value induce the same ranking
  - Charge winner *lowest possible bid* for him to win
- Equivalent to second price auction with a reserve

Myerson auction in the non-iid case

- Problems:
  - Not efficient, not intuitive
    - I am the highest bidder, why didn't I win?
  - Need too much information
    - The seller knows the value distribution of each bidder
  - Hard to implement in practice
    - Revenue not high as expected
      - Assumptions hard to fulfil: iid, regular, distribution known
      - Long term vs short term

Simple versus optimal auctions

- [Hartline, Roughgarden, EC-2009]
- Design simple auction that approximates optimal revenue
- Second price auction?
  - Arbitrarily far from optimal
- Need a reserve price!
  - Hmmm….

Monopoly reserve

- Imagine buyer i is the single bidder in the market, what is the seller-optimal price?
  - Monopoly reserve r*: $r^*-(1-F_i(r^*))/f_i(r^*)=0$
  - In other words, the value whose virtual value is 0

Second price with r* is a 2-approximation of Myerson

- Theorem: For MHR distributions, second price auction with r* is a 2-approximation of Myerson
  - MHR: $h(z)=f_i(z)/(1-F_i(z))$ weakly increasing
  - MHR → regular
  - Fact: for any auction,
    - expected revenue = expected virtual value
  - Proof on Board

What if we don't know the distributions?

- Single Item, bidders iid?
  - Bulow-Klemperer Theorem (Next Slide)
- Infinitely many copies of the same item (IE: digital copy of a song), no prior information
  - Digital Goods Auctions (After)

Bulow-Klemperer Theorem

- Motivation:
  - In the real world, bidders are usually i.i.d.
    - Have you ever seen an auction that sets a different price for different people?
  - In the real world, learning distributions is hard

Bulow-Klemperer Theorem

- Let 2nd(F,n) denote the expected revenue of running the second price auction with n bidders sampled independently from F.
- Let OPT(F,n) denote the expected revenue of running Myerson's auction with n bidders sampled independently from F.
- Theorem (Bulow-Klemperer): If F is regular,

  $$\text{2nd}(F,n+1) \geq \text{OPT}(F,n)$$

- Big Picture: With i.i.d. bidders, rather than learn the distributions, try to attract more bidders.

Bulow-Klemperer Theorem

- Proof of Theorem consequence of two simple claims.
- Let COPT(F,n) denote the maximum attainable expected revenue by an auction that always awards the item to someone.
- 1) $COPT(F,n+1) \geq OPT(F,n)$
- 2) $2nd(F,n+1) = COPT(F,n+1)$

Bulow-Klemperer Theorem

- 1) COPT(F,n+1) ≥ OPT(F,n)
  - Proof: Run Myerson's auction on the first n bidders. If Myerson doesn't award the item, give it to bidder n+1 for free. Revenue is exactly OPT(F,n), item is always given away.

Bulow-Klemperer Theorem

- 2) 2nd(F,n+1) = COPT(F,n+1)
  - Proof: We know that expected revenue is exactly expected virtual welfare. If F is regular, then the highest bidder has the highest virtual value. So 2nd always chooses the highest virtual value, thus maximizing revenue among all the auctions that always give the item away.
  - The only difference between 2nd and Myerson is that Myerson sometimes withholds the good from sale if all virtual values are negative.

Digital Good Auctions

- Infinitely many copies of a single item
  - a song
  - Use a posted price
- Want a worst-case guarantee
  - Optimal Revenue is wrong benchmark: impossible to attain (highest bid could be 1, 10, 10000000…)
- $F^{(2)}$ benchmark: $\max_{p \mid (\#\text{bidders with } v > p) > 1} \{p^*(\#\text{bidders with } v > p)\}$
  - In English: Allowed to choose any price where at least two bidders are willing to purchase.
  - Why $F^{(2)}$? Anything stronger unattainable in worst-case. Simple approximation to $F^{(2)}$ possible (next slide)

Digital Good Auctions

- Random Sample Auction: For each bidder, flip a coin. Denote by T the set of bidders with tails, and H the set of bidders with heads.
  - Find the optimal price for bidders in T, p(T) and the optimal price for bidders in H, p(H). Charge price p(T) to bidders in H, and p(H) to bidders in T.
  - Truthful? Bidders are offered a price that they can't control. Dominant strategy to tell the truth.
  - Optimal? Theorem: Random Sample Auction has expected revenue equal to at least $F^{(2)}/15$. There are instances where the expected revenue is at most $F^{(2)}/4$.

What if the values are correlated?

- Why independence is not a realistic assumption?
- Solution: 1-lookahead auctions (Ronen, EC-01)
  - Rank all bids in increasing order
  - Reject all bids except the highest bid
  - Compute the conditional distribution of highest value
  - Design an optimal one-buyer auction for this dist.
- Theorem: 1-lookahead auction is truthful, IR and a 2-approximation of the optimal revenue.

Proof.

- IR is obvious
- Truthfulness
  - For the highest bidder, truthful since he is in a posted price auction
    - A single buyer auction is truthful iff it is a posted price
  - For others, if win, pay at least the second highest, better off losing by reporting truthfully
- 2-approximation
  - On board