人工智能

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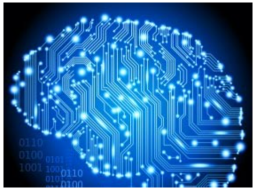
Knowledge 3



一阶谓词逻辑

First-order Logic: syntax and semantics

First-order logic

- Whereas propositional logic assumes world contains facts, first-order logic(like natural language) assumes the world contains

- **Objects:** people, house, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries…..
- **Relations:** red, round, bogus, prime, multistoried…,
  brother of, bigger than, inside, part of, has color, occurred after,
  owns, comes between,…
- **Functions:** father of, best friend, third inning of, one more than, end of…

Syntax of FOL: Basic elements

Constants	*KingJohn, 2, UCB,…*
Predicates	Brother, >,…
Functions	*Sqrt, LeftLegOf,…*
Variables	x, y, a, b,…
Connectives	$\wedge \quad \vee \quad \neg \quad \Rightarrow \quad \Leftrightarrow$
Equality	=
Quantifiers	$\forall \quad \exists$

Atomic sentences

Atomic sentence = $predicate(term_1, \dots, term_n)$
or $term_1 = term_2$

Term = $function(term_1, \dots, term_n)$
or *constant* or *variable*

E.g., $Brother(KingJohn, RichardTheLionheart)$
$> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))$

Complex sentences

Complex sentences are made from atomic sentences using connectives

$$\neg \boldsymbol{S},\ \ \boldsymbol{S_1} \wedge \boldsymbol{S_2},\ \ \boldsymbol{S_1} \vee \boldsymbol{S_2},\ \ \boldsymbol{S_1} \Rightarrow \boldsymbol{S_2},\ \ \boldsymbol{S_1} \Leftrightarrow \boldsymbol{S_2}$$

E.g., $Slibing(KingJohn, Richard) \Rightarrow Slibing(Richard, KingJohn)$

$> (1{,}2) \vee \leq (1{,}2)$

$> (1{,}2) \wedge \neg > (1{,}2)$

Truth in first-order logic

- Sentences are true with respect to a model and an interpretation
- Model contains ≥ 1 objects(domain elements) and relations among them

指示物

- Interpretation specifies referents for

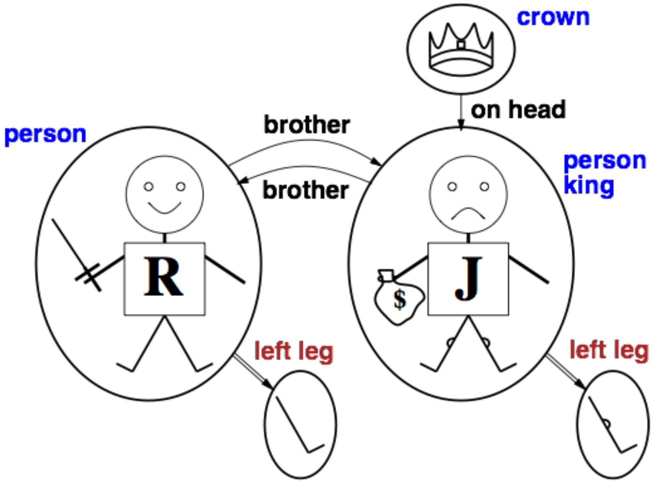
  constant symbols ⟶ objects

  predicate symbols ⟶ relations

  function symbols ⟶ functional relations

- An atomic sentence $predicate(term_1, \dots, term_n)$ is true iff the objects referred to by $term_1, \dots, term_n$ are in the relation referred to by $predicate$

Models for FOL: Example



- Consider the interpretation in which
- $Richard$ ⟶ Richard the Lionheart
- $John$ ⟶ the evil King John
- $Brother$ ⟶ the brotherhood relation

- Under this interpretation, $Brother(Richard, John)$ is true just in case Richard the Lionheart and the evil King John are in the brotherhood relation in the model

Models for FOL: Lots!

- Entailment in propositional logic can be computed by enumerating models
- We can enumerate the FOL models for a given KB vocabulary:
- For each number of domain elements *n* from 1 to ∞

  For each k-ary predicate P_k in the vocabulary

    For each possible k-ary relation on *n* objects

      For each constant symbol *C* in the vocabulary

        For each choice of referent for *C* from *n* objects …

- Computing entailment by enumerating FOL models is not easy!

Universal quantification

- $\forall < variables >< sentence >$
- Everyone at Berkeley is smart:

  $\forall x \;\; At(x, Berkeley) \;\Rightarrow Smart(x)$
- $\forall x \;\; P$ is true in a model *m* iff *P* is true with *x* being each possible object in the model
- **Roughly** speaking, equivalent to the conjunction of instantiations of P

$$
\begin{aligned}
&\boldsymbol{\big(At(KingJohn, Berkeley) \Rightarrow Smart(KingJohn)\big)} \\
\wedge &\boldsymbol{\big(At(Richard, Berkeley) \Rightarrow Smart(Richard)\big)} \\
\wedge &\boldsymbol{(At(Berkeley, Berkeley) \Rightarrow Smart(Berkeley))} \\
\wedge &\boldsymbol{\ldots\ldots}
\end{aligned}
$$

A common mistake to avoid

- Typically, $\Rightarrow$ is the main connective with $\forall$
- Common mistake: using $\land$ as the main connective with $\forall$

$$\forall x \quad At(x, Berkeley) \land Smart(x)$$

- Means "Everyone is at Berkeley and everyone is smart"

Existential quantification

- $\exists < variables >< sentence >$
- Someone at Berkeley is smart:

  $\exists x \;\; At(x, Berkeley) \land Smart(x)$

- $\exists x \;\; P$ is true in a model *m* iff *P* is true with *x* being some possible object in the model
- **Roughly** speaking, equivalent to the disjunction of instantiations of P

$$
\begin{aligned}
&\left(\boldsymbol{At(KingJohn, Berkeley) \land Smart(KingJohn)}\right) \\
\lor &\left(\boldsymbol{At(Richard, Berkeley) \land Smart(Richard)}\right) \\
\lor &\left(\boldsymbol{At(Berkeley, Berkeley) \land Smart(Berkeley)}\right) \\
\lor &\; \boldsymbol{\ldots..}
\end{aligned}
$$

Another common mistake to avoid

- Typically, $\wedge$ is the main connective with $\exists$
- Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

$$\exists x \quad At(x, Stanford) \Rightarrow Smart(x)$$

- Is true if there is anyone who is not at Stanford!

Properties of quantifiers

- $\forall x\ \forall y$ is the same as $\forall y\ \forall x$ (why??)
- $\exists x\ \exists y$ is the same as $\exists y\ \exists x$ (why??)
- $\exists x\ \forall y$ is not the same as $\forall y\ \exists x$

- $\exists x\ \forall y\ Loves(x, y)$
  "There is a person who loves everyone in the world"

- $\forall y\ \exists x\ Loves(x, y)$
  "Everyone in the world is loved by at least one person"

- Quantifier duality: each can be expressed using the other

$$\forall x\ Likes(x, IceCream) \qquad \neg\exists x\ \neg Likes(x, IceCream)$$
$$\exists x\ Likes(x, Broccoli) \qquad \neg\forall x\ \neg Likes(x, Broccoli)$$

Fun with sentences

- Brothers are siblings
- $\forall x, y \;\; Brother(x,y) \;\Rightarrow Sibling(x,y)$.
- "Sibling" is symmetric
- $\forall x, y \;\; Sibling(x,y) \;\Leftrightarrow Sibling(y,x)$.
- One's mother is one's female parent
- $\forall x, y \;\; Mother(x,y) \;\Leftrightarrow \big(Female(x) \land Parent(x,y)\big)$.
- A first cousin is a child of a parent's sibling
- $\forall x, y \;\; Firstcousin(x,y) \;\Leftrightarrow \exists p, ps \; parent(p,x) \land Sibling(ps,p) \land Parent(ps,y)$.

Fun with sentences

- 不到长城非好汉。

- **到了长城就是好汉。**

Equality

- $term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object
- E.g., $1 = 2$ and $\forall x \ \times\left(Sqrt(x), Sqrt(x)\right) = x$ are satisfiable

  $2 = 2$ is valid
- E.g., definition of (full) $Sibling$ in terms of $Parent$:

  $$\forall x, y \ \ Sibling(x, y) \Leftrightarrow [\neg(x = y) \wedge \exists m, f \ \neg(m = f) \wedge Parent(m, x) \wedge Parent(f, x) \wedge Parent(m, y) \wedge Parent(f, y)]$$

Back to the wumpus world again

- Define adjacency:

$$\forall x, y, a, b\ Adjacent([x,y],[a,b]) \Leftrightarrow$$

$$\big(x = a \wedge (y = b - 1 \vee y = b + 1)\big) \vee \big(y = b \wedge (x = a - 1 \vee x = a + 1)\big).$$

- Location predicator, x is at square s at time t:

$$\forall t\ At(WUMPUS, [2,2], t).$$

$$\forall x, s_1, s_2, t\ \ At(x, s_1, t) \wedge At(x, s_2, t) \Rightarrow s_1 = s_2$$

- Define property for squares:

$$\forall s, t\ At(AGENT, s, t) \wedge Breeze(t) \Rightarrow Breezy(s).$$

$$\forall s, t\ At(PIT, s, t) \Rightarrow Pit(s).$$

$$\forall s, t\ At(WUMPUS, s, t) \Rightarrow Wumpus(s).$$

- Rules of the Wumpus world can be defined.

$$\forall s\ Breezy(s) \Leftrightarrow \exists r\ \ Adjacent(r, s) \wedge Pit(r)$$

$$\forall t\ HaveArrow(t+1) \Leftrightarrow \big(HaveArrow(t) \wedge \neg Action(shoot, t)\big).$$

Short Summary

- First-order logic:
  - Objects and relations are semantic primitives
  - Syntax: constants, functions, predicates, equality, quantifiers
- Increased expressive power: sufficient to define Wumpus world

研究形式逻辑的目的是什么？

- 知识表示
  - 将一组知识形式化为符号
- 知识推理
  - 通过形式推演，自动推出结论
    - 可靠
    - 完备

一个可靠的[illegible]

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Homework

8.15 Explain what is wrong with the following proposed definition of the set membership predicate $\in$:

$$\forall x, s \;\; x \in \{x|s\}$$
$$\forall x, s \;\; x \in s \Rightarrow \forall y \;\; x \in \{y|s\} .$$

8.20 Arithmetic assertions can be written in first-order logic with the predicate symbol $<$, the function symbols $+$ and $\times$, and the constant symbols 0 and 1. Additional predicates can also be defined with biconditionals.

a. Represent the property "x is an even number."

b. Represent the property "x is prime."

c. Goldbach's conjecture is the conjecture (unproven as yet) that every even number is equal to the sum of two primes. Represent this conjecture as a logical sentence.