# Zoom算法的若干种情况

# 要满足的条件

- $\alpha_{low}$和$\alpha_{high}$组成的区间内存在满足强Wolfe条件的步长
- $\alpha_{low}$是所有生成的满足充分下降条件的步长中，函数值最小的
- $\emptyset'(\alpha_{low})(\alpha_{high} - \alpha_{low}) < 0$

- $\emptyset(\alpha_{low}) < \emptyset(\alpha_{high})$
- $\alpha_j$在$\alpha_{low}$和$\alpha_{high}$组成的区间内

# 情况1

**repeat**

Interpolate (using quadratic, cubic, or bisection) to find
    a trial step length $\alpha_j$ between $\alpha_{lo}$ and $\alpha_{hi}$;
Evaluate $\phi(\alpha_j)$;
**if** $\phi(\alpha_j) > \phi(0) + c_1\alpha_j\phi'(0)$ or $\phi(\alpha_j) \geq \phi(\alpha_{lo})$
    ❌                                              ✔
    $\alpha_{hi} \leftarrow \alpha_j$;
**else**
    Evaluate $\phi'(\alpha_j)$;
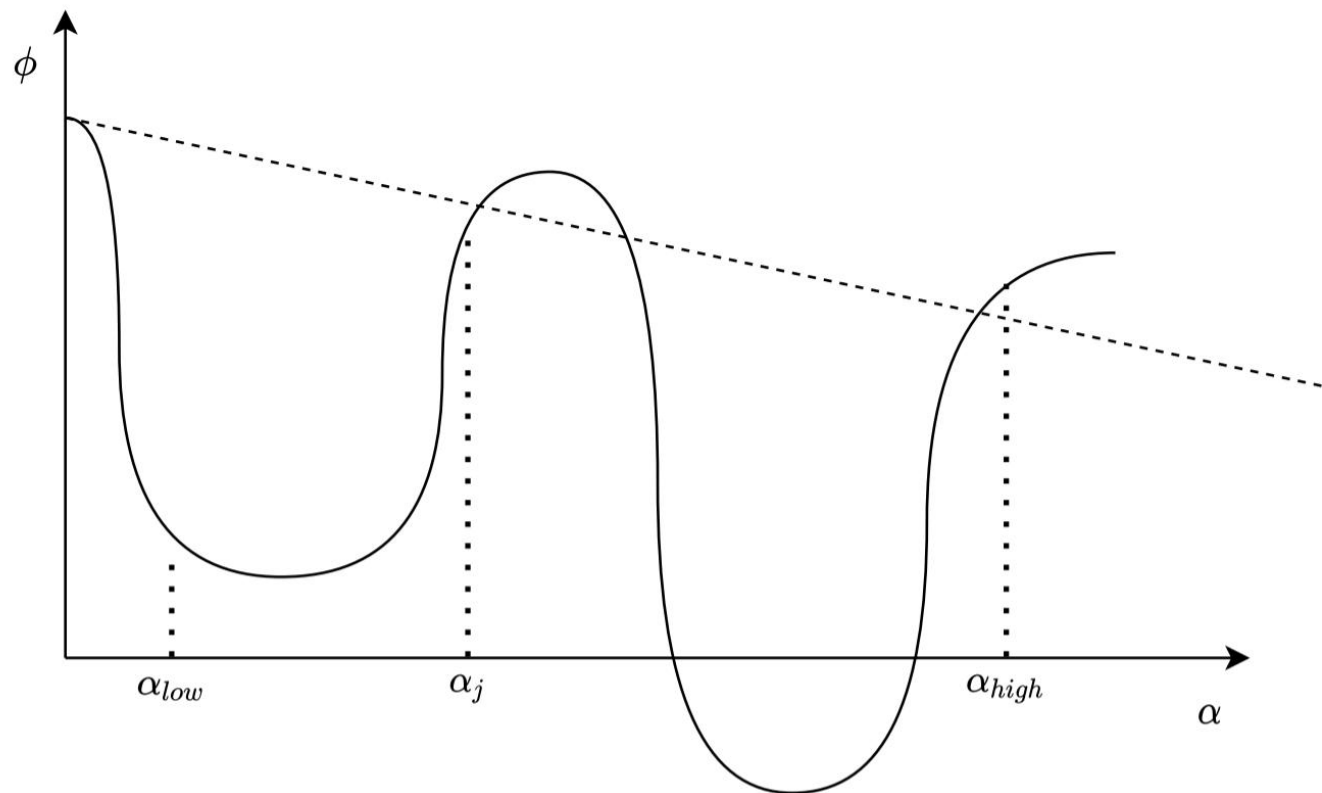    **if** $|\phi'(\alpha_j)| \leq -c_2\phi'(0)$
        Set $\alpha_* \leftarrow \alpha_j$ and **stop**;
    **if** $\phi'(\alpha_j)(\alpha_{hi} - \alpha_{lo}) \geq 0$
        $\alpha_{hi} \leftarrow \alpha_{lo}$;
    $\alpha_{lo} \leftarrow \alpha_j$;
**end (repeat)**



- 搜索区间变为$[\alpha_{low}, \alpha_j]$

# 情况2

repeat

    Interpolate (using quadratic, cubic, or bisection) to find

        a trial step length $\alpha_j$ between $\alpha_{lo}$ and $\alpha_{hi}$;

    Evaluate $\phi(\alpha_j)$;

    **if** $\phi(\alpha_j) > \phi(0) + c_1\alpha_j\phi'(0)$ or $\phi(\alpha_j) \geq \phi(\alpha_{lo})$

        $\alpha_{hi} \leftarrow \alpha_j$;

    **else**

        Evaluate $\phi'(\alpha_j)$;

        **if** $|\phi'(\alpha_j)| \leq -c_2\phi'(0)$
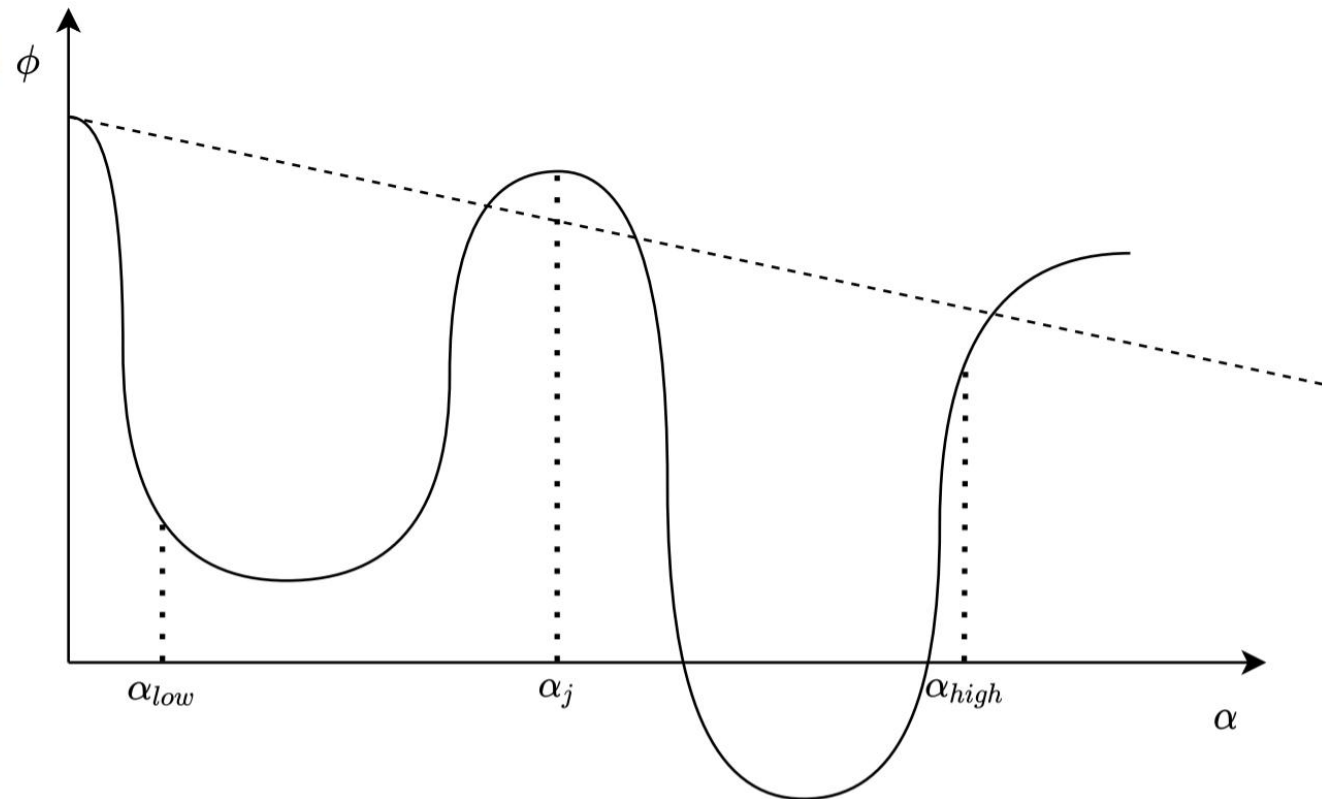
            Set $\alpha_* \leftarrow \alpha_j$ and **stop**;

        **if** $\phi'(\alpha_j)(\alpha_{hi} - \alpha_{lo}) \geq 0$

            $\alpha_{hi} \leftarrow \alpha_{lo}$;

        $\alpha_{lo} \leftarrow \alpha_j$;

end (repeat)



- 搜索区间变为$[\alpha_{low}, \alpha_j]$

# 情况3

**repeat**

   Interpolate (using quadratic, cubic, or bisection) to find
       a trial step length $\alpha_j$ between $\alpha_{lo}$ and $\alpha_{hi}$;

   Evaluate $\phi(\alpha_j)$;

   **if** $\phi(\alpha_j) > \phi(0) + c_1\alpha_j\phi'(0)$ or $\phi(\alpha_j) \geq \phi(\alpha_{lo})$

      $\alpha_{hi} \leftarrow \alpha_j$; ✔      ✘

   **else**

      Evaluate $\phi'(\alpha_j)$;

      **if** $|\phi'(\alpha_j)| \leq -c_2\phi'(0)$
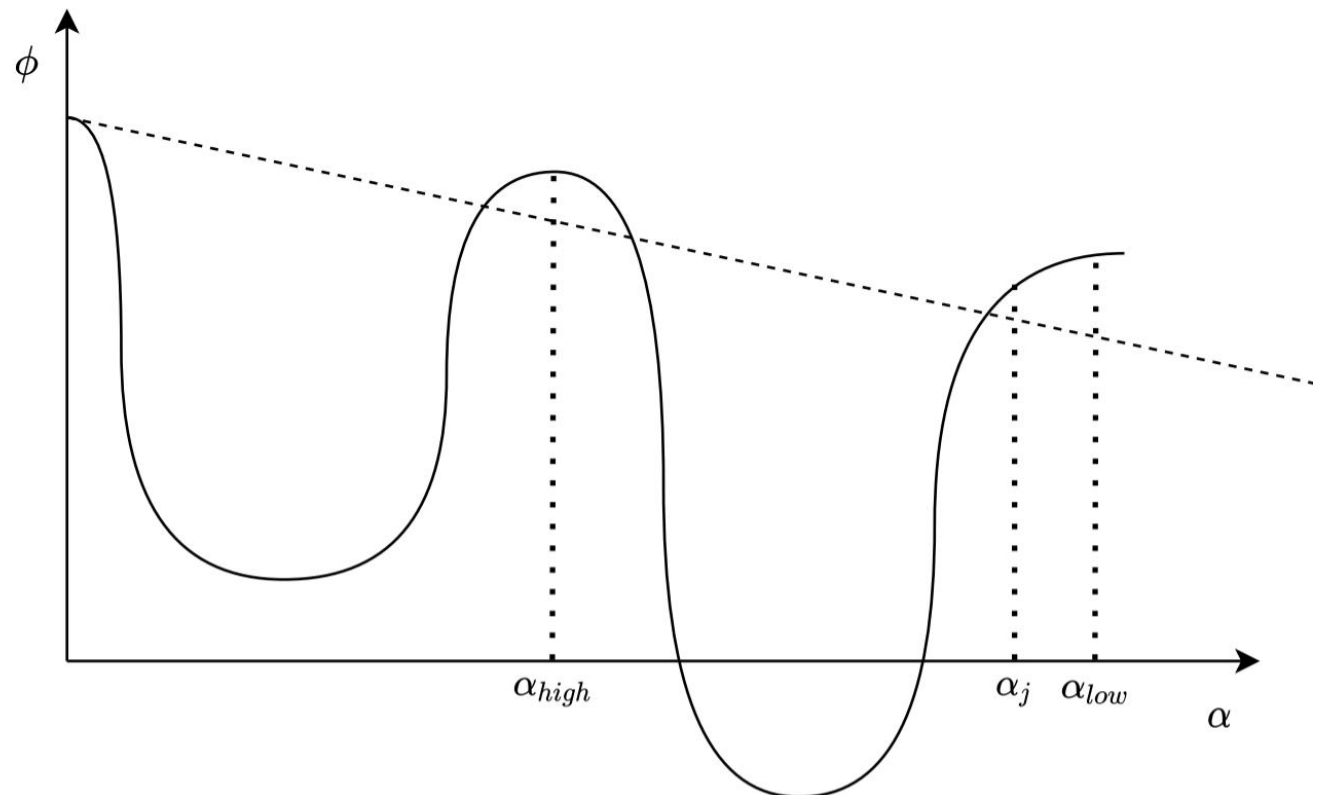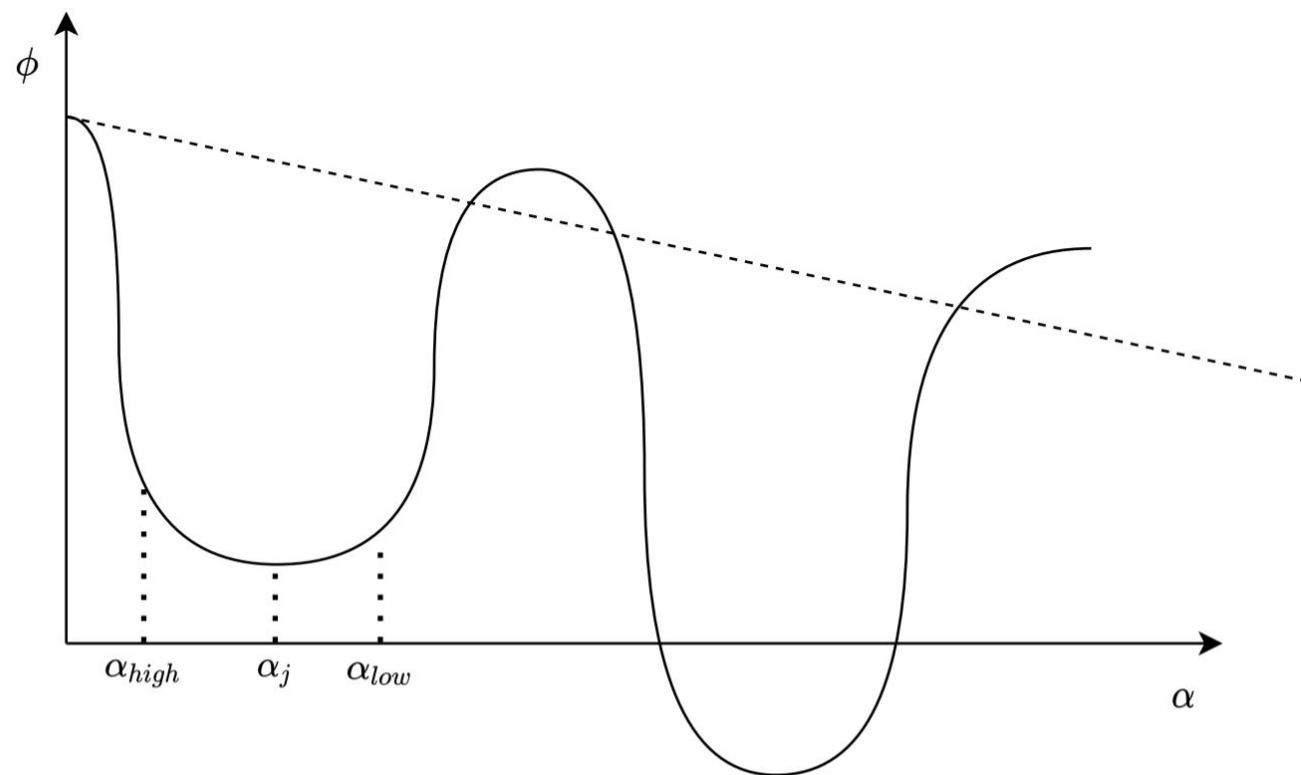
         Set $\alpha_* \leftarrow \alpha_j$ and **stop**;

      **if** $\phi'(\alpha_j)(\alpha_{hi} - \alpha_{lo}) \geq 0$

         $\alpha_{hi} \leftarrow \alpha_{lo}$;

      $\alpha_{lo} \leftarrow \alpha_j$;

**end (repeat)**



- 与 $\alpha_{low}$ 满足充分下降条件矛盾

# 情况4

**repeat**

    Interpolate (using quadratic, cubic, or bisection) to find
        a trial step length $\alpha_j$ between $\alpha_{\text{lo}}$ and $\alpha_{\text{hi}}$;

    Evaluate $\phi(\alpha_j)$;

    **if** $\phi(\alpha_j) > \phi(0) + c_1\alpha_j\phi'(0)$ or $\phi(\alpha_j) \geq \phi(\alpha_{\text{lo}})$

        $\alpha_{\text{hi}} \leftarrow \alpha_j$; ❌ ❌

    **else**

        Evaluate $\phi'(\alpha_j)$;

        **if** $|\phi'(\alpha_j)| \leq -c_2\phi'(0)$

            Set $\alpha_* \leftarrow \alpha_j$ and **stop**;

        **if** $\phi'(\alpha_j)(\alpha_{\text{hi}} - \alpha_{\text{lo}}) \geq 0$

            $\alpha_{\text{hi}} \leftarrow \alpha_{\text{lo}}$;

    $\alpha_{\text{lo}} \leftarrow \alpha_j$;

**end (repeat)**



- 进入下一个判定

# 情况4-1



**repeat**

    Interpolate (using quadratic, cubic, or bisection) to find
        a trial step length $\alpha_j$ between $\alpha_{lo}$ and $\alpha_{hi}$;

    Evaluate $\phi(\alpha_j)$;

    **if** $\phi(\alpha_j) > \phi(0) + c_1\alpha_j\phi'(0)$ or $\phi(\alpha_j) \geq \phi(\alpha_{lo})$ ❌    ❌

        $\alpha_{hi} \leftarrow \alpha_j$;

    **else**

        Evaluate $\phi'(\alpha_j)$;

        **if** $|\phi'(\alpha_j)| \leq -c_2\phi'(0)$ ✔
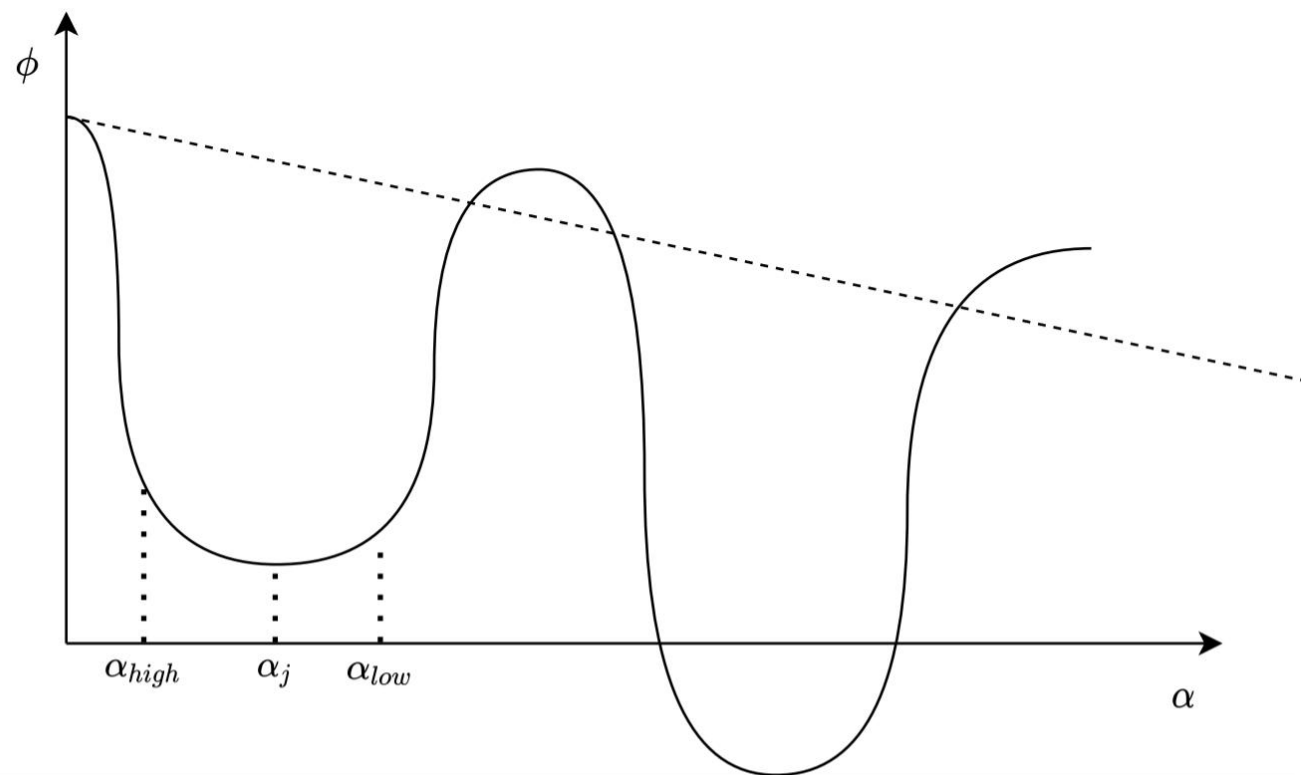
            Set $\alpha_* \leftarrow \alpha_j$ and **stop**;

        **if** $\phi'(\alpha_j)(\alpha_{hi} - \alpha_{lo}) \geq 0$

            $\alpha_{hi} \leftarrow \alpha_{lo}$;

    $\alpha_{lo} \leftarrow \alpha_j$;

**end (repeat)**

- $\alpha_* = \alpha_j$，算法终止

# 情况4-2

repeat

Interpolate (using quadratic, cubic, or bisection) to find
a trial step length $\alpha_j$ between $\alpha_{lo}$ and $\alpha_{hi}$;
Evaluate $\phi(\alpha_j)$;
**if** $\phi(\alpha_j) > \phi(0) + c_1\alpha_j\phi'(0)$ or $\phi(\alpha_j) \geq \phi(\alpha_{lo})$
    $\alpha_{hi} \leftarrow \alpha_j$; ❌      ❌
**else**
    Evaluate $\phi'(\alpha_j)$;
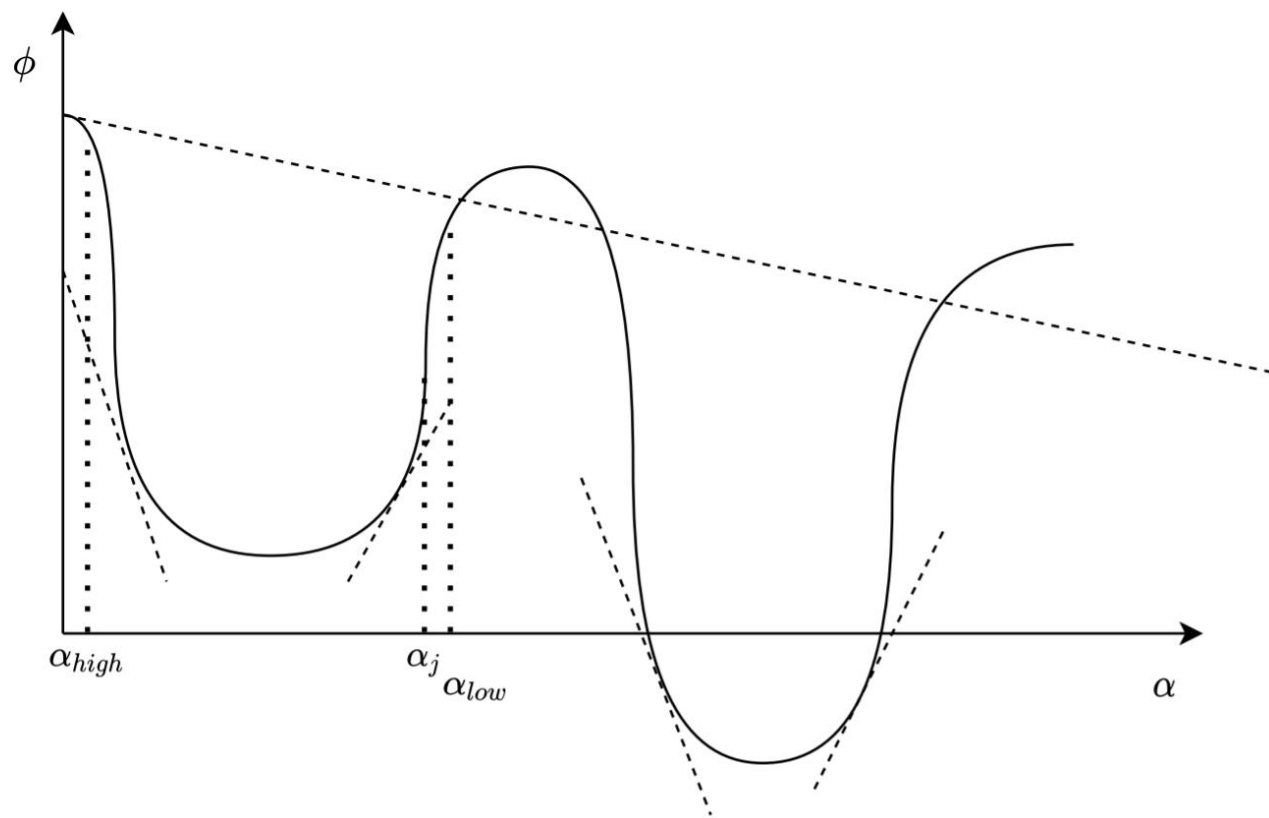    **if** $|\phi'(\alpha_j)| \leq -c_2\phi'(0)$ ❌
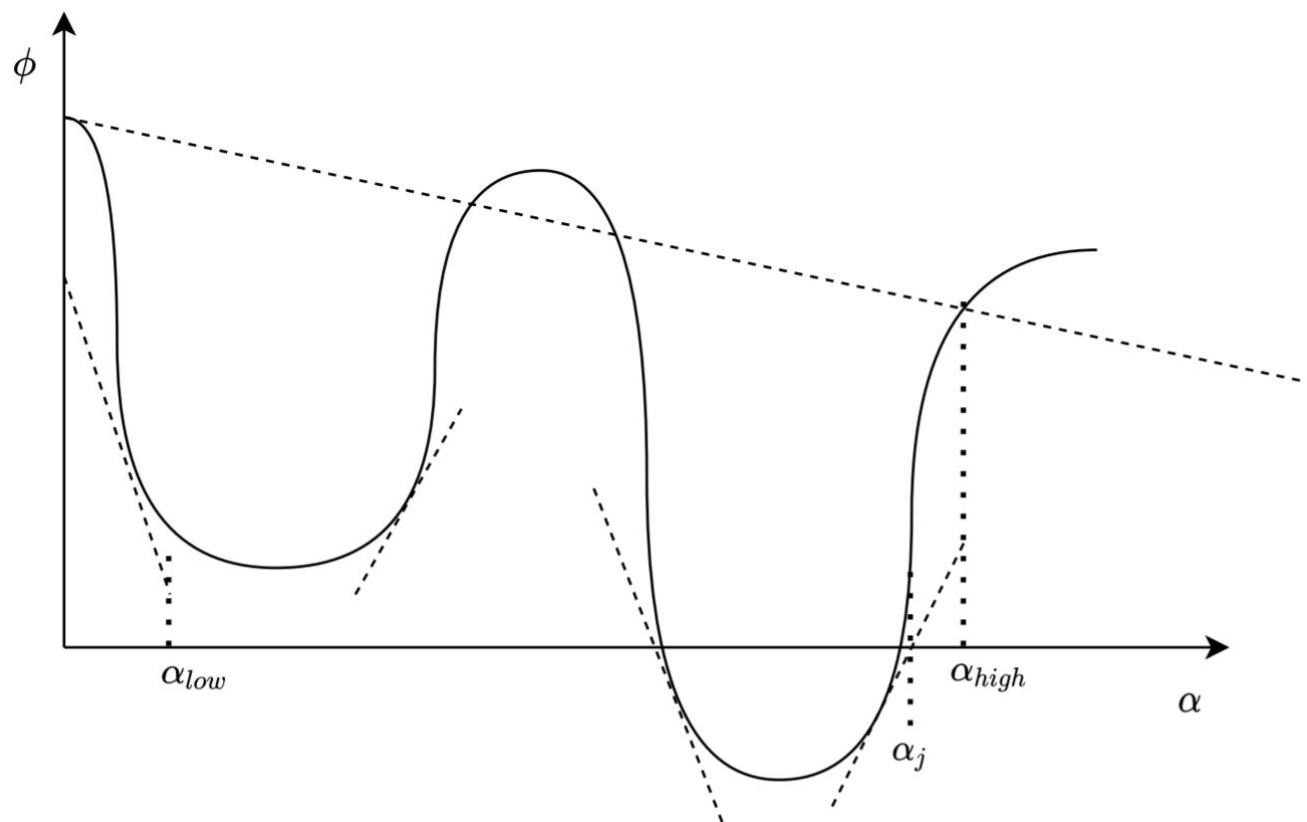        Set $\alpha_* \leftarrow \alpha_j$ and **stop**;
    **if** $\phi'(\alpha_j)(\alpha_{hi} - \alpha_{lo}) \geq 0$
        $\alpha_{hi} \leftarrow \alpha_{lo}$;
    $\alpha_{lo} \leftarrow \alpha_j$;

end (repeat)



- 进入下一个判定

# 情况4-2-1

**repeat**

    Interpolate (using quadratic, cubic, or bisection) to find
        a trial step length $\alpha_j$ between $\alpha_{\text{lo}}$ and $\alpha_{\text{hi}}$;

    Evaluate $\phi(\alpha_j)$;

    **if** $\phi(\alpha_j) > \phi(0) + c_1\alpha_j\phi'(0)$ or $\phi(\alpha_j) \geq \phi(\alpha_{\text{lo}})$

        $\alpha_{\text{hi}} \leftarrow \alpha_j$; ❌ ❌

    **else**

        Evaluate $\phi'(\alpha_j)$;

        **if** $|\phi'(\alpha_j)| \leq -c_2\phi'(0)$ ❌

            Set $\alpha_* \leftarrow \alpha_j$ and **stop**;

        **if** $\phi'(\alpha_j)(\alpha_{\text{hi}} - \alpha_{\text{lo}}) \geq 0$ ✔

            $\alpha_{\text{hi}} \leftarrow \alpha_{\text{lo}}$;

    $\alpha_{\text{lo}} \leftarrow \alpha_j$;

**end (repeat)**



- 搜索区间变为 $[\alpha_{low}, \alpha_j]$

# 情况4-2-2

**repeat**
    Interpolate (using quadratic, cubic, or bisection) to find
        a trial step length $\alpha_j$ between $\alpha_{lo}$ and $\alpha_{hi}$;
    Evaluate $\phi(\alpha_j)$;
    **if** $\phi(\alpha_j) > \phi(0) + c_1\alpha_j\phi'(0)$ ❌ or $\phi(\alpha_j) \geq \phi(\alpha_{lo})$ ❌
        $\alpha_{hi} \leftarrow \alpha_j$;
    **else**
        Evaluate $\phi'(\alpha_j)$;
        **if** $|\phi'(\alpha_j)| \leq -c_2\phi'(0)$ ❌
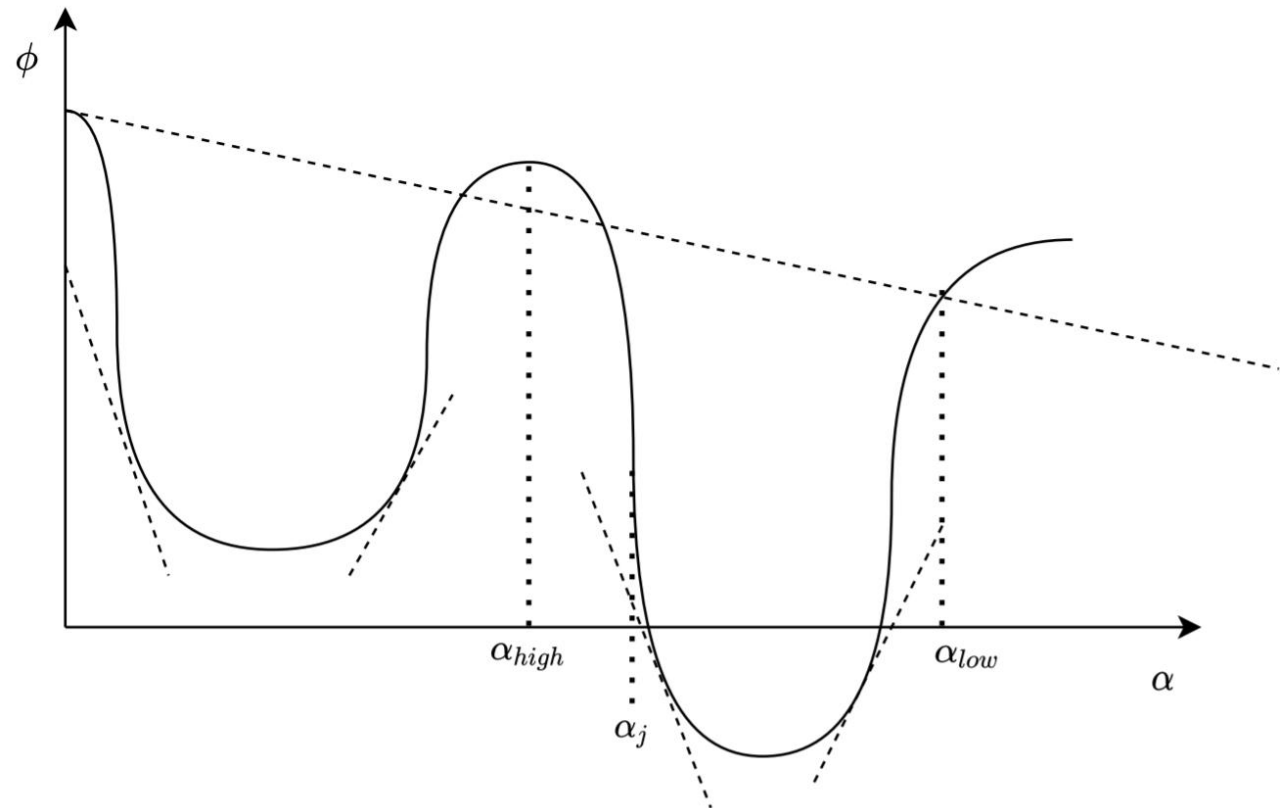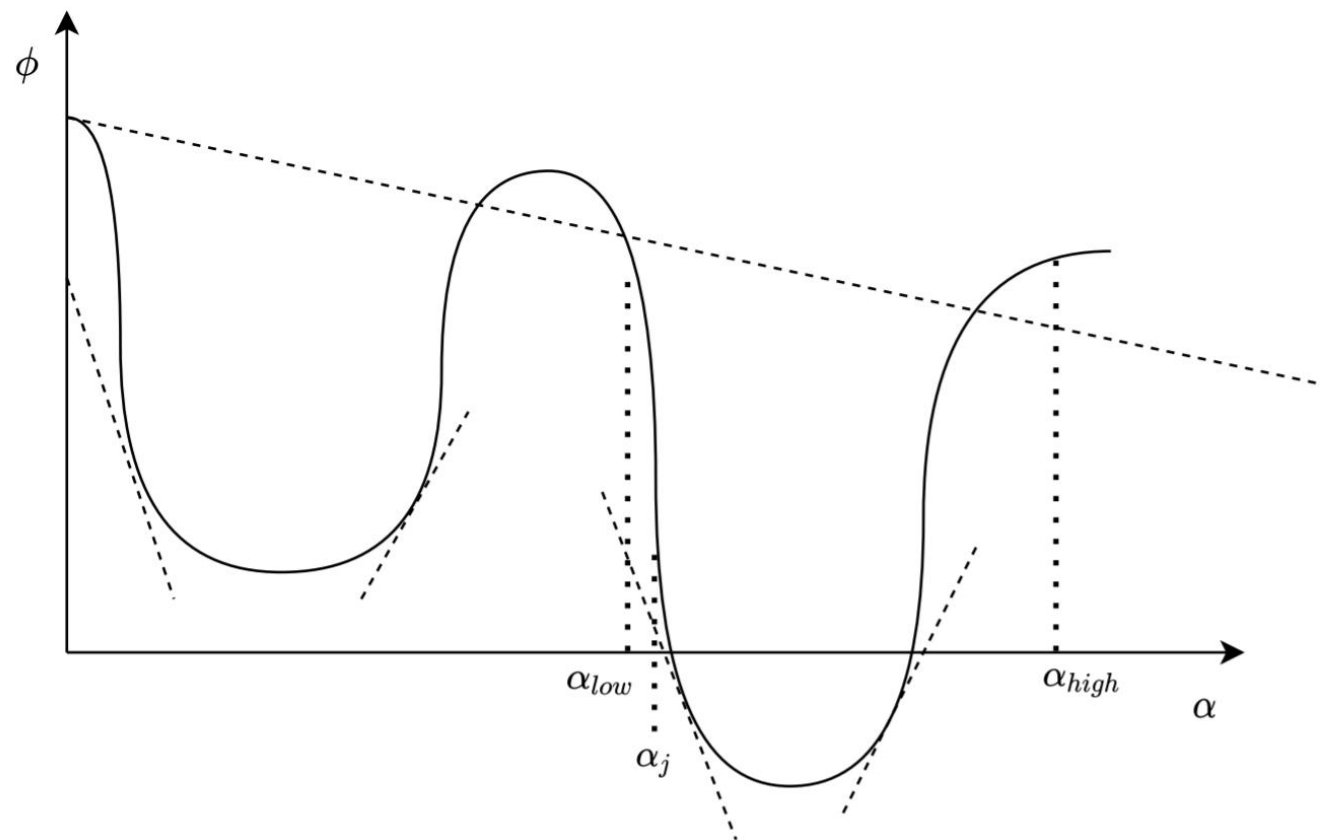            Set $\alpha_* \leftarrow \alpha_j$ and **stop**;
        **if** $\phi'(\alpha_j)(\alpha_{hi} - \alpha_{lo}) \geq 0$ ✔
            $\alpha_{hi} \leftarrow \alpha_{lo}$;
        $\alpha_{lo} \leftarrow \alpha_j$;
**end (repeat)**



- 搜索区间变为 $[\alpha_{low}, \alpha_j]$

# 情况4-2-3

**repeat**
    Interpolate (using quadratic, cubic, or bisection) to find
        a trial step length $\alpha_j$ between $\alpha_{\mathrm{lo}}$ and $\alpha_{\mathrm{hi}}$;
    Evaluate $\phi(\alpha_j)$;
    **if** $\phi(\alpha_j) > \phi(0) + c_1\alpha_j\phi'(0)$ or $\phi(\alpha_j) \geq \phi(\alpha_{\mathrm{lo}})$
        $\alpha_{\mathrm{hi}} \leftarrow \alpha_j$; ❌        ❌
    **else**
        Evaluate $\phi'(\alpha_j)$;
        **if** $|\phi'(\alpha_j)| \leq -c_2\phi'(0)$ ❌
            Set $\alpha_* \leftarrow \alpha_j$ and **stop**;
        **if** $\phi'(\alpha_j)(\alpha_{\mathrm{hi}} - \alpha_{\mathrm{lo}}) \geq 0$ ❌
            $\alpha_{\mathrm{hi}} \leftarrow \alpha_{\mathrm{lo}}$;
        $\alpha_{\mathrm{lo}} \leftarrow \alpha_j$;
**end (repeat)**



- 搜索区间变为$[\alpha_j, \alpha_{high}]$

# 情况4-2-4

repeat

    Interpolate (using quadratic, cubic, or bisection) to find
        a trial step length $\alpha_j$ between $\alpha_{lo}$ and $\alpha_{hi}$;

    Evaluate $\phi(\alpha_j)$;

    **if** $\phi(\alpha_j) > \phi(0) + c_1\alpha_j\phi'(0)$ or $\phi(\alpha_j) \geq \phi(\alpha_{lo})$ ❌    ❌

        $\alpha_{hi} \leftarrow \alpha_j$;

    **else**

        Evaluate $\phi'(\alpha_j)$;

        **if** $|\phi'(\alpha_j)| \leq -c_2\phi'(0)$ ❌
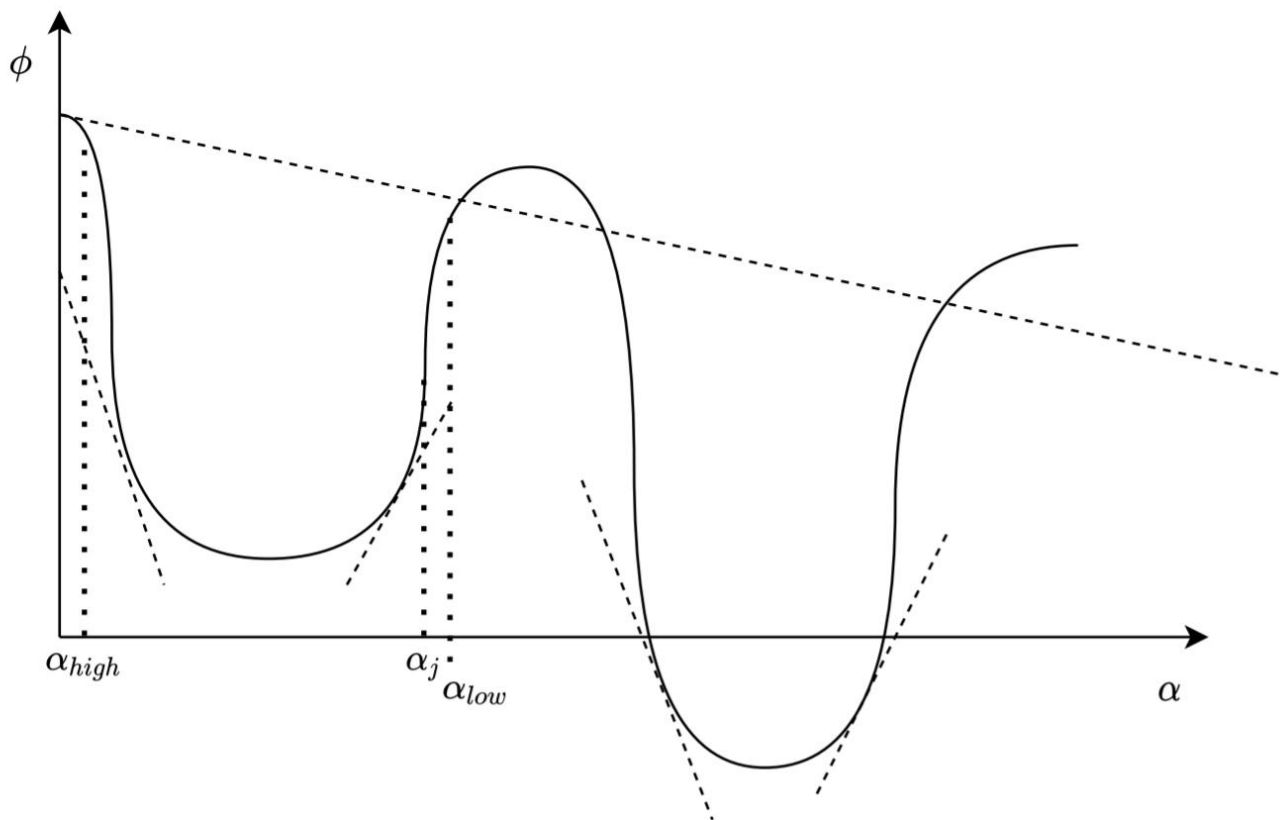
            Set $\alpha_* \leftarrow \alpha_j$ and **stop**;

        **if** $\phi'(\alpha_j)(\alpha_{hi} - \alpha_{lo}) \geq 0$ ❌

            $\alpha_{hi} \leftarrow \alpha_{lo}$;

    $\alpha_{lo} \leftarrow \alpha_j$;

end (repeat)



- 搜索区间变为 $[\alpha_{high}, \alpha_j]$