Week #8
An Introduction to Auction Design

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Outline of this lecture

- Outline
  - Examples of auctions
  - Single item auctions
  - Revenue equivalence and optimal auction design
  - Simple versus optimal auctions
  - Ad auctions and sponsored search

The oldest auction?

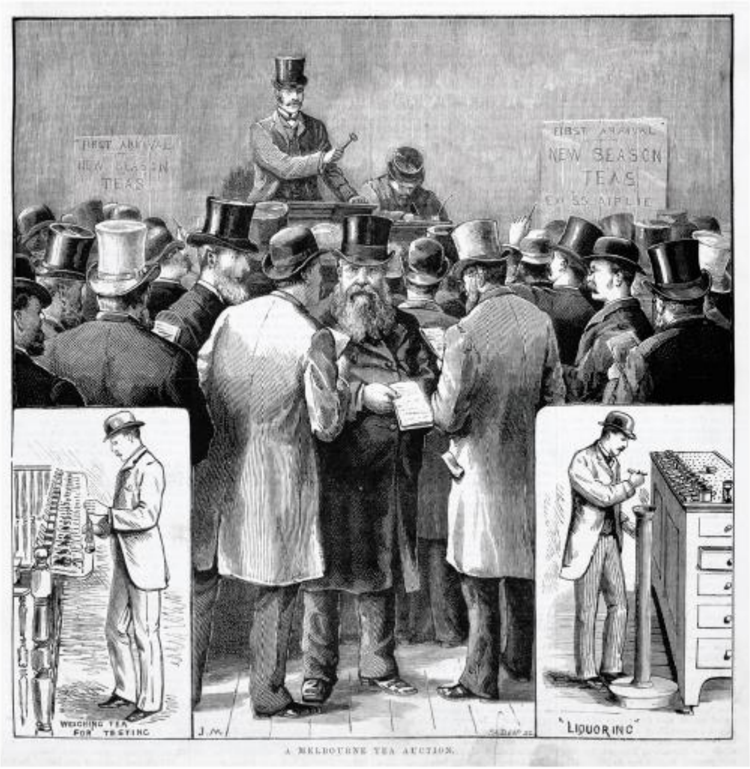
The Babylonian marriage auction (500 BC)

Edwin Long, 1875, The Babylonian marriage market

Facts of Babylonian marriage auction

- Two groups
  - Group 1: Men pay women for marriage
  - Group 2: Women pay men for marriage
- Order of auction
  - In descending order of *beauty*
    - Beauty is in the eye of the auctioneer
- Outside option
  - Illegal to get married via any other means
- Get-along phase
  - In case of mutual rejection --- refund and re-auction

Melbourne Tea Auction



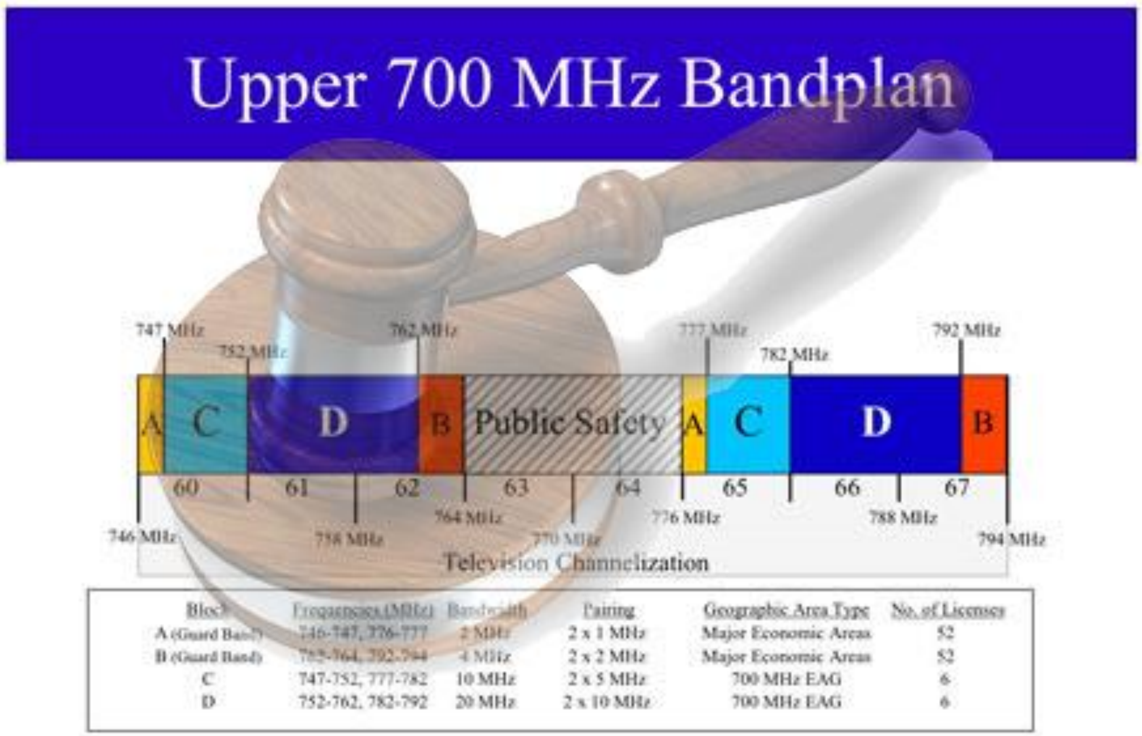
Tea Auction, Melbourne, Australia, 1885.

Dutch flower auction

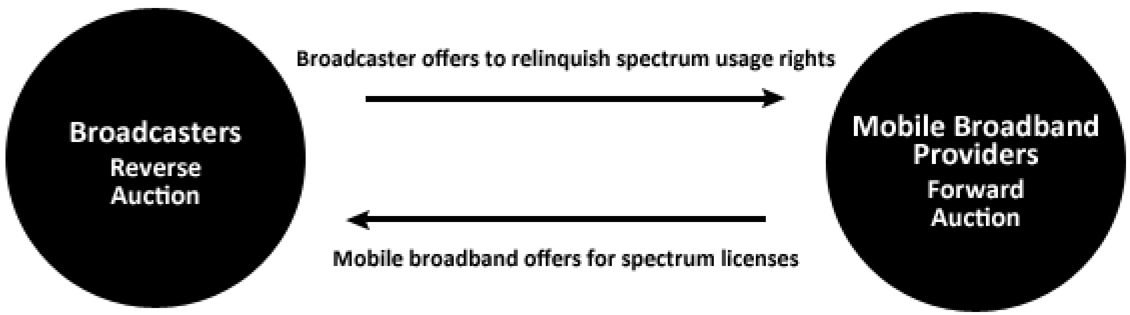
Japan Bluefin Tuna Auction

- 34usd (Spain) vs 46 usd (Japan)
- Record price: 1.7 M USD, 220 KG
- Fish market = 43 football courts
    - Largest seafood market

FCC wireless spectrum auction (Paul Milgrom)



FCC incentive auction
Broadcast → Mobile (Paul Milgrom)

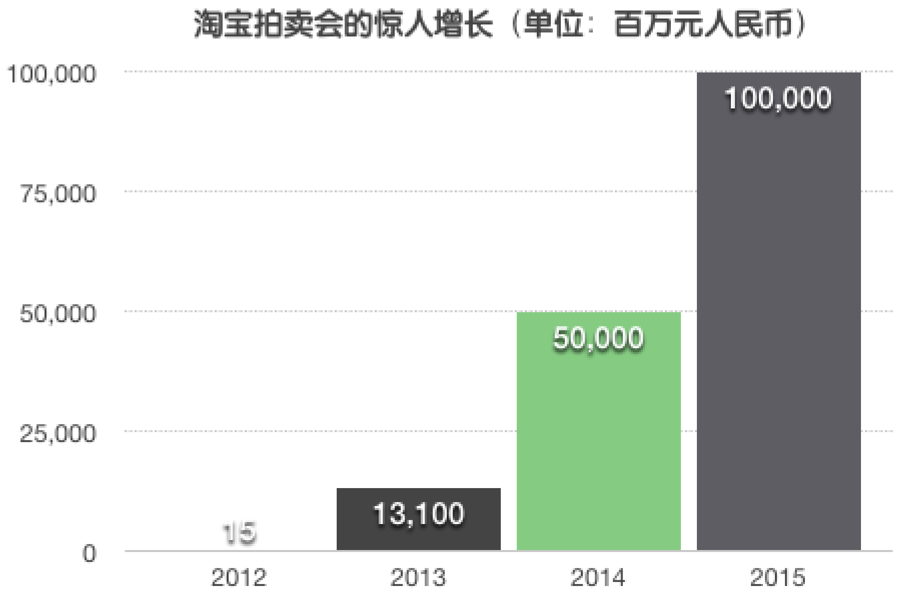


A common feature

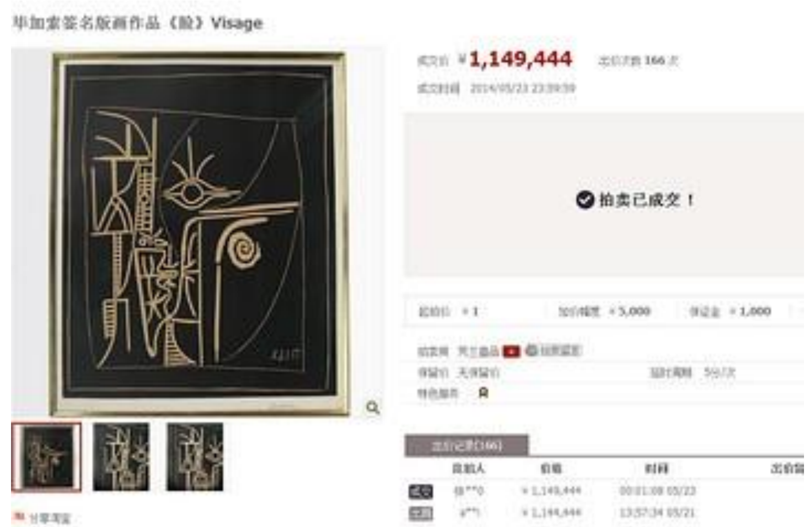
- Value decreases over time
- Market thin
- E.g., Flower, Tuna
- Use auction as a means to
  - *aggregate demand*
  - *increase competition*

Taobao (eBay) auctions

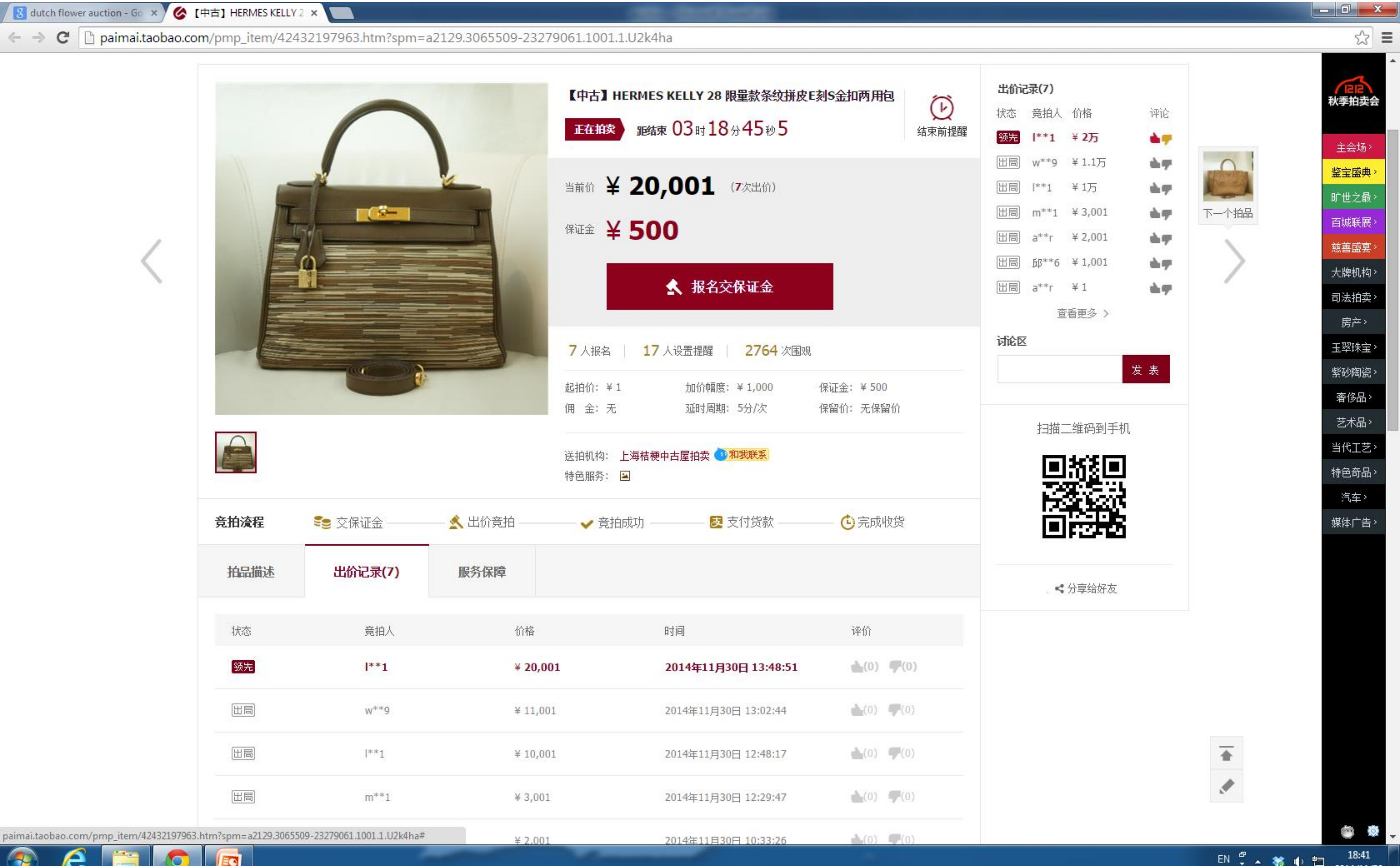
- Jewelry, Art, and Antique auctions
  - http://paimai.taobao.com/
- Juristic auctions
  - http://sf.taobao.com/
- Car plates auctions



Example of Taobao auctions: Auction of a Picasso painting

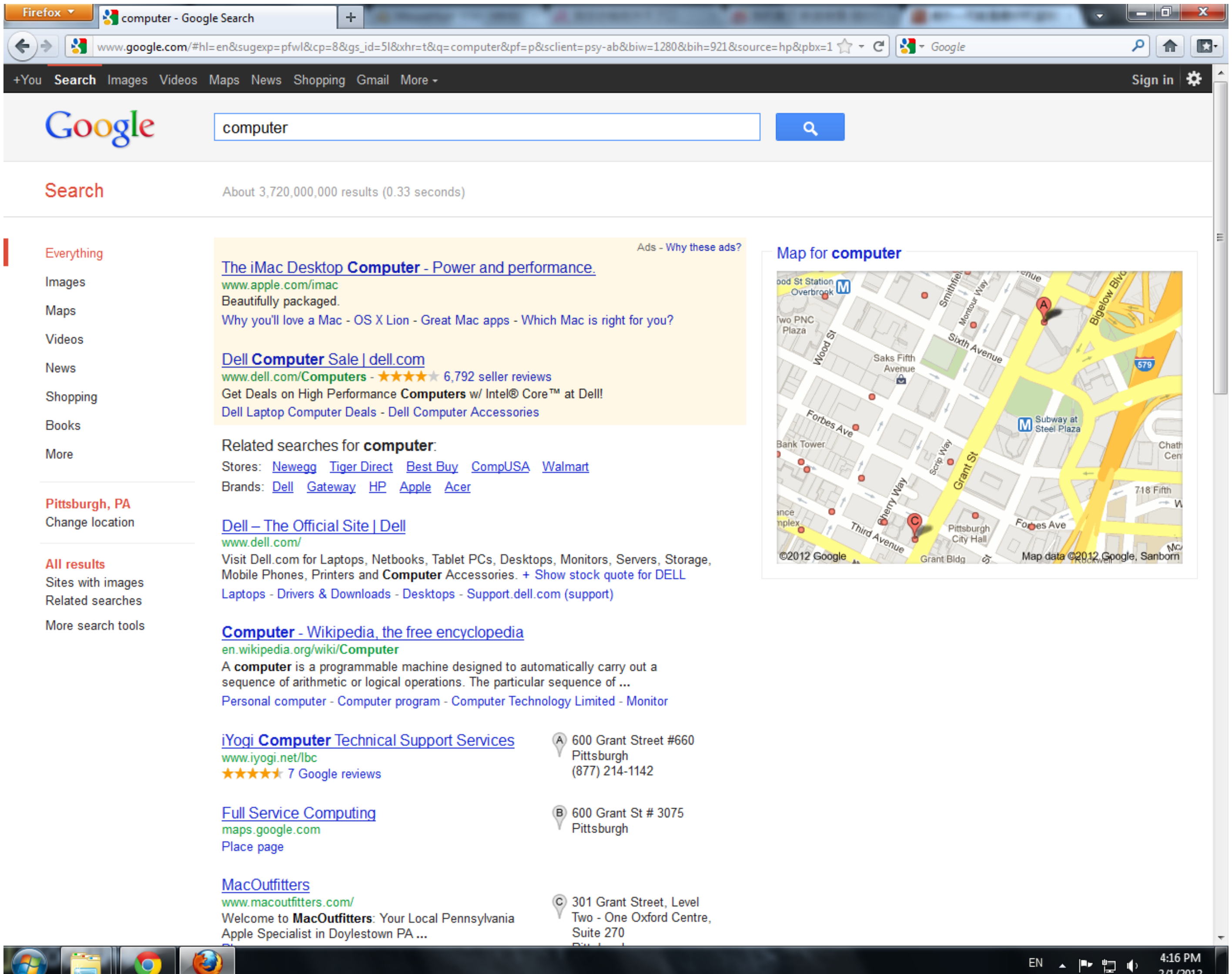


Example of Taobao auctions: Auction of a 2nd-hand Hermes bag



Example of Taobao auctions: Auction of a corrupted real estate

Sponsored search: selling advertisements



Many others

- Spectrum auctions
- Car plate auctions
- Wheat auctions
- Dairy auctions

Auctions

- What is auction ?
  - Methods for allocating resources
  - Participants: one seller, multiple bidders
  - Agreement between seller and bidders
    - Who gets the goods?
    - How much does everyone pay?

  - Reverse (procurement) auction: one buyer, multiple sellers
  - Auction = quasi-linear mechanism

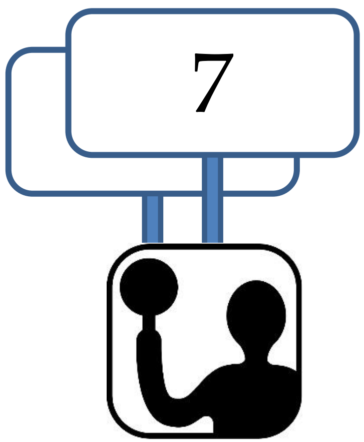
Single-item auctions
[Shoham and Leyton-Brown, Chapter 10, Multiagent Systems, 2009]

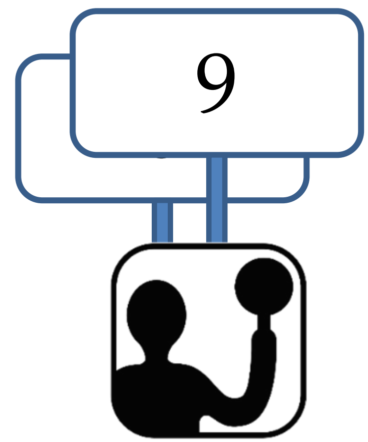
Different single-item auction formats

- English auction
- Dutch auction
- First-price auction
- Second-price auction

English auction

Payment = 9





Dutch auction

Payment = 10

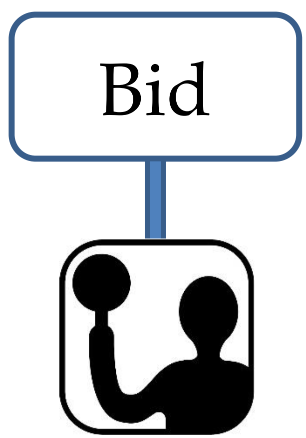
=

First price auction

Payment = 10

Second price auction

Payment = 7



Analysis

Independent private value (IPV) setting

- Each buyer i has a private valuation v_i towards the item
  - Maximum amount of money i is willing to pay
  - v_i is sometimes called the buyer's type
- v_i is drawn from a publicly known distribution F_i
- Only buyer i knows the realized value of v_i
- Others, including the seller, know F_i
- Valuations are drawn independently among buyers
- Technical assumptions on CDF F_i
  - $[0, v_{max}]$
  - Continuous density function, full support

Bayesian game induced by the auction

- Definition. Strategy of buyer i
  - For any realization of v_i, specifies a bid b_i
  - Formally: $s_i(v_i) = b_i$
- Strategy profile: $s(v) = \left(s_1(v_1), \dots, s_n(v_n)\right)$
  - $s_{-i}(v_{-i}) = \left(s_1(v_1), \dots, s_{i-1}(v_{i-1}), s_{i+1}(v_{i+1}), \dots, s_n(v_n)\right)$

Dominant strategy equilibrium (DSE)

- Definition (Dominant Strategy Equilibrium).

  Strategy profile $s^* = (s_1^*, \ldots, s_n^*)$ is a dominant strategy equilibrium (DSE) in a sealed bid auction if, for all agents i,

$$u_i(s_i^*(v_i), b_{-i}) \geq u_i(b_i, b_{-i})$$

  for all values v_i, bids b_i and bids b_{-i} from others.

Bayes Nash equilibrium (BNE)

- Definition (Bayes Nash equilibrium).

  Strategy profile $s^* = (s_1^*, \dots, s_n^*)$ is a Bayes Nash equilibrium (BNE) in a sealed bid auction if, for all agents i, and all values v_i,

  $$\mathbb{E}_{v_{-i}}\left[u_i\left(s_i^*(v_i), s_{-i}^*(v_{-i})\right)\right] \geq \mathbb{E}_{v_{-i}}\left[u_i\left(b_i, s_{-i}^*(v_{-i})\right)\right]$$

  for all bids b_i.

Analysis of second price auction

Truthfulness

- Theorem: Truthfulness ($s_i(v_i)=v_i$) is a dominant strategy in second-price auction
- Proof?

Second-Price proof

Theorem

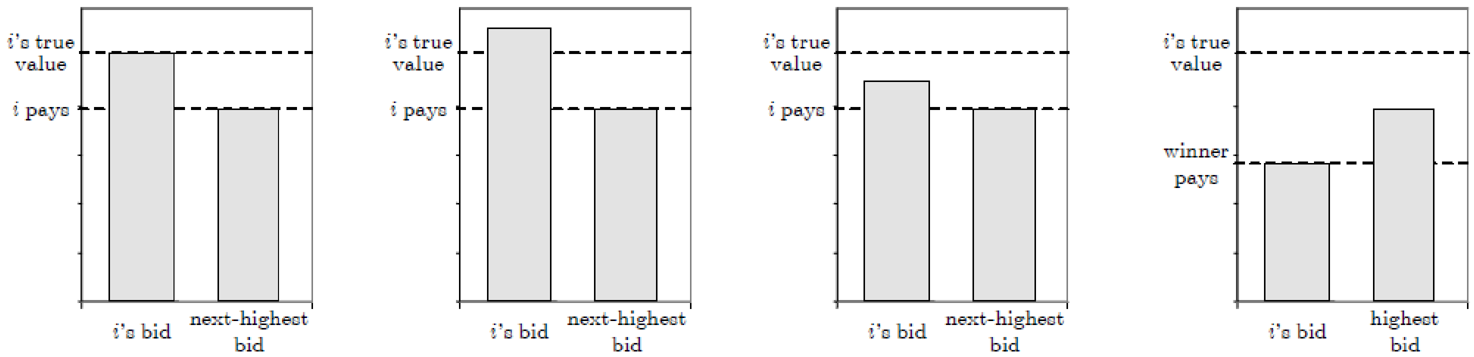
Truth-telling is a dominant strategy in a second-price auction.

Proof.

Assume that the other bidders bid in some arbitrary way. We must show that i's best response is always to bid truthfully. We'll break the proof into two cases:

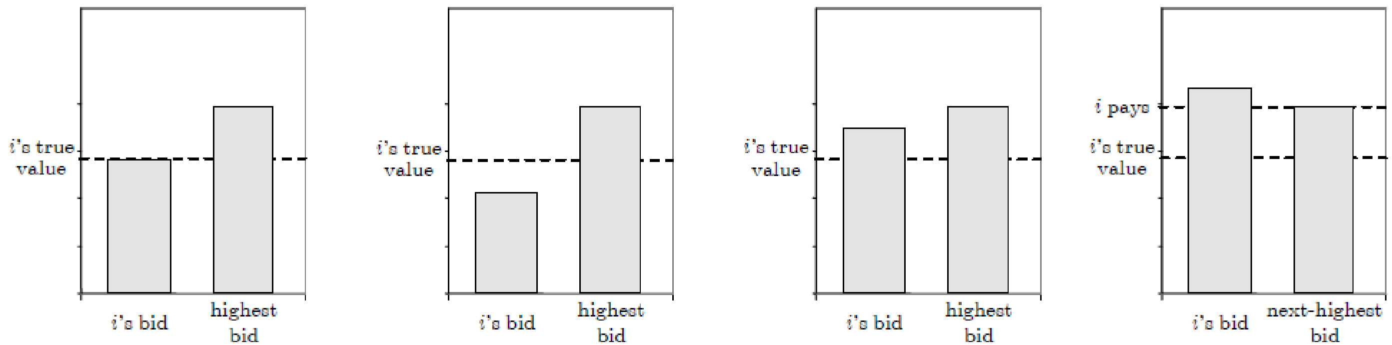
1. Bidding honestly, i would win the auction
2. Bidding honestly, i would lose the auction

Case 1



- Bidding honestly, i is the winner
- If i bids higher, he will still win and still pay the same amount
- If i bids lower, he will either still win and still pay the same amount. . . or lose and get utility of zero.

Case 2



- Bidding honestly, i is not the winner
- If i bids lower, he will still lose and still pay nothing
- If i bids higher, he will either still lose and still pay nothing. . . or win and pay more than his valuation.

Analysis of first price auction

Equivalence of Dutch and 1st price auctions

- Theorem: First price and Dutch auctions are strategically equivalent
- Proof. What do you need to calculate before a Dutch auction?

Analysis

Theorem

In a first-price auction with two risk-neutral bidders whose valuations are drawn independently and uniformly at random from $[0,1]$*,* $(\frac{1}{2}v_1, \frac{1}{2}v_2)$ *is a Bayes-Nash equilibrium strategy profile.*

Proof.

Assume that bidder 2 bids $\frac{1}{2}v_2$, and bidder 1 bids s_1. From the fact that v_2 was drawn from a uniform distribution, all values of v_2 between 0 and 1 are equally likely. Bidder 1's expected utility is

$$E[u_1] = \int_0^1 u_1 dv_2. \tag{1}$$

Note that the integral in Equation (1) can be broken up into two smaller integrals that differ on whether or not player 1 wins the auction.

$$E[u_1] = \int_0^{2s_1} u_1 dv_2 + \int_{2s_1}^1 u_1 dv_2$$

Analysis

Theorem

In a first-price auction with two risk-neutral bidders whose valuations are drawn independently and uniformly at random from $[0, 1]$, $(\frac{1}{2}v_1, \frac{1}{2}v_2)$ is a Bayes-Nash equilibrium strategy profile.

Proof (continued).

We can now substitute in values for u_1. In the first case, because 2 bids $\frac{1}{2}v_2$, 1 wins when $v_2 < 2s_1$, and gains utility $v_1 - s_1$. In the second case 1 loses and gains utility 0. Observe that we can ignore the case where the agents have the same valuation, because this occurs with probability zero.

$$\begin{aligned} E[u_1] &= \int_0^{2s_1} (v_1 - s_1) dv_2 + \int_{2s_1}^{1} (0) dv_2 \\ &= (v_1 - s_1) v_2 \Big|_0^{2s_1} \\ &= 2v_1 s_1 - 2s_1^2 \end{aligned} \tag{2}$$

Analysis

Theorem

In a first-price auction with two risk-neutral bidders whose valuations are drawn independently and uniformly at random from $[0,1]$, $(\frac{1}{2}v_1, \frac{1}{2}v_2)$ *is a Bayes-Nash equilibrium strategy profile.*

Proof (continued).

We can find bidder 1's best response to bidder 2's strategy by taking the derivative of Equation (2) and setting it equal to zero:

$$\begin{aligned} \frac{\partial}{\partial s_1}(2v_1 s_1 - 2s_1^2) &= 0 \\ 2v_1 - 4s_1 &= 0 \\ s_1 &= \frac{1}{2}v_1 \end{aligned}$$

Thus when player 2 is bidding half her valuation, player 1's best strategy is to bid half his valuation. The calculation of the optimal bid for player 2 is analogous, given the symmetry of the game and the equilibrium.

More than two bidders

- Very narrow result: two bidders, uniform valuations.
- Still, first-price auctions are not incentive compatible
  - hence, unsurprisingly, not equivalent to second-price auctions

Theorem

In a first-price sealed bid auction with n risk-neutral agents whose valuations are independently drawn from a uniform distribution on the same bounded interval of the real numbers, the unique symmetric equilibrium is given by the strategy profile $\left(\frac{n-1}{n}v_1, \ldots, \frac{n-1}{n}v_n\right)$.

- proven using a similar argument, but more involved calculus
- a broader problem: that proof only showed how to *verify* an equilibrium strategy.
  - How do we identify one in the first place?

Solving BNE of first price auction?

- Two bidders, symmetric uniform
  - Vickrey. 1950s.
- Two bidders, asymmetric uniform (2010)
  - [Kaplan and Zamir, 2010]
- Two bidders, general case
  - Open
- Two bidder, sequential case
  - Optimal commitments in first-price auction
  - [Tang, Wang, Zhang. EC-16]

Which one yields higher revenue?

Facts

- Order statistic of n iid variables
  - $v_{(1)}$ is the highest value
  - $v_{(k)}$ is the *k*-th highest value
- Order statistic of n iid uniform samples:

$$\mathbb{E}_v \left[v_{(k)} \mid \text{n samples IID } \sim \ U(0,1) \right] = \frac{n+1-k}{n+1}$$

- Two buyers, uniform [0,1], revenue comparison
  - 1st price: $\frac{1}{2} \times v_{(1)} = 1/2 \times 2/3 = 1/3$
  - 2nd price: $v_{(2)} = 1/3$

Facts

- n buyers, uniform[0, 1]
  - 1^{st} price: n-1/n$\times$ $v_{(1)}$ = (n-1/n) $\times$(n/n+1)=n-1/n+1
  - 2^{nd} price: $v_{(n)}$ = n-1/n+1
- Exercise: n buyers, iid drawn from F on [0,1]
  - Fact: 1^{st} price auction: $s_i(v_i)= E(v_{(1)} \mid v_{(1)} < v_i)$
  - $v_{(1)}$ now denotes first-order statistic of n-1 values
  - Show revenue equivalence
- Question:
  - How general can this equivalence hold?

Revenue equivalence theorem

- Theorem: Auctions that, in equilibrium,
  - result in the same allocation rule,
  - assign 0 utility to bidders in their lowest types,

  yield the same expected revenue.
- Proof via Myerson's Lemma.
- Lemma: $p_i(v_i)=u_i'(v_i)$ (Myerson 1981)
  - By the definition of BNE.
- Proof of the theorem:
  - $payment(v_i)$

  $=v_i p_i(v_i) - u_i(v_i)$

  $= v_i p_i(v_i) - \int_0^{v_i} p_i(v_i)dv_i - u_i(0)$

  $Revenue = \sum_1^n E\ payment(vi)$ QED

Facts

- Fact: first and second price auctions share the same allocation rule p_i in BNE, when bidders' valuations are iid.
  - First price auction: symmetric increasing BNE
  - The item is allocated to the bidder with the highest valuation in equilibrium

- Question: are they optimal in revenue?

2^{nd} price auction sometimes fails

- Example?

Claude and Paloma, Picasso, $ 28M by Jianlin Wang (2015, NY)

- 1 strong buyer + several very weak buyers

Revenue optimal auctions (1,1)

- *Example*: 1 item, 1 buyer
- Seller: one item for sale
  - $q=1$ if sold, $q=0$ if reserve;
- Buyer: valuation x from Uniform [0,1]
  - Exact value known to buyer, dist. known to seller
  - Buyer utility: $x \times q - t$
- First and second price auction end up with 0 revenue
- Auction = set a price p (in the 1-item case)
- Buyer's decision: *buy, if* $x > p$
- Problem: *max* $p(1\text{-}p)$,
  - Solution: $p = 0.5$
- Easy for any distribution
  - If x is known, *revenue* $= x$

So…what is optimal?

- Question: what is the set of all possible auctions?
  - Does it help to add sequentiality (Dutch, English)?
  - Does it help to add a richer bidding language?
- Revelation principle: No!
  - It is without loss of generality to focus on *direct mechanisms*
  - A subset of *single-round auction*, where
    - Everyone reports a bid
    - Report truthfully is an equilibrium strategy
  - In other words, for any indirect mechanism, there is a truthful direct mechanism that can do the same
    - Problem: the direct mechanism may be difficult to find
    - Problem: the direct mechanism may have a weird form

Revenue optimal auction (Myerson)

- *Example*: 1 item, n buyers
- Seller: one item for sale
- Buyers: x_i IID from Uniform [0,1]
- Optimal auction:
  - Second price auction with *reserve* 0.5
    - Think of 0.5 as the seller's bid, a competitive bid for strong buyer

Optimal auction (Myerson 1981, Nobel prize 2007)

- For asymmetric distributions (regular case)
- For buyer *i*, $x_i \sim F_i, f_i$
- Procedure:
  - Each buyer *i* reports a bid v_i
  - Seller computes virtual bids $v_i - (1-F_i(v_i))/f_i(v_i)$
    - Delete all negative virtual bids (Myerson reserve: seller's virtual bid=0)
    - Higher price at the cost of no transaction
  - Run 2nd price auction on the remaining *virtual bids*
  - Winner pays *threshold price*
- Interpretation:
  - Second price auction on virtual bids
  - Not necessarily sell to the highest bidder
    - First-degree price discrimination
    - Boost weak bidders, increase competition
  - Not necessarily sell the item at all
    - Reserve pricing

Applying Myerson auction

- Apply to all *single-parameter* settings
  - *Example*: Seller has k units of identical items
    - Such as sponsored search setting (in a few slides)
    - Feasibility constraints: first slot k_1 units, second slot k_2 units
- Procedure
  - Report bids
  - Rank by virtual bids
  - Threshold payments (payment identity)
- Special case
  - Buyers have the iid *regular* distribution
  - GSP allocation (rank by bids) with Myerson reserve
- In general
  - GSP is different in both allocation and payment

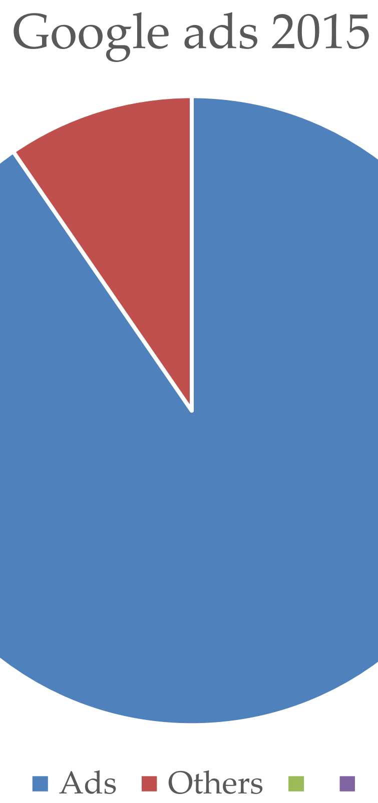
Difficulties to apply Myerson directly

- Don't rank by bids
  - "Not fair!" -- Buyers may complain
- Price discrimination
  - Different prices for one unit of impression
  - Different reserves
- Repeated auction vs. single-round auction
  - Myerson may charge too much
  - Drive advertisers away
  - Hurt ecosystem (objective: α *revenue* + β *welfare* + γ*clicks*) [EC-14, Key et al]
- Prior dependence
  - Rely on value distribution, could be inaccurate and hard to infer [EC-15]
- Complex (irrational) bidding behaviors

What can be done?

- Incorporate insights from Myerson into GSP framework
  - Reserve pricing
    - Constant
    - Bidder specific
    - Randomized
    - Dynamic
  - Price discrimination
    - Boost weak buyers
      - Quality score
      - Squashing, anchoring
    - Increase competition
  - Make *good* use of bidding data
    - Understand different levels of reserves
    - Understand the same reserve for different keywords
    - Understand the effect of dynamic reserve
      - E.g., what happens when we change reserve from 0.6 to 0.7?

Sponsored search auctions


Google ads 2015
Ads
Others

Statistics

- Google advertising revenue:
  - 2011: $36,531M
  - 2012: $43,686M
  - …
  - 2015: $67,390M
- Hal Varian:
  - "What most people don't realize is that all that money comes pennies at a time"
- References:
  - Varian 2008: Position auctions
  - Edelman et. al 2007: Internet advertising and generalized second price auctions

Model

- K positions $k = 1, \ldots, K$
- N bidders $i=1,\ldots, N$
- Bidder i values position k at V= v_i*x_k
  - v_i is the value of a click to bidder i
  - x_k is the probability of a click at this slot

- Efficient allocation is assortative
  - v and x follow the same order

GSP Auction Rules

- Each agent *i* submits bid b_i
- Positions assigned in order of bids
- Agent i's price per click is bid of agent in the next slot down.
- Let b^k denote *k*th highest value and v^k value.
- Payoff of *k*th highest bidder:

$$v^k \cdot x_k - b^{k+1} \cdot x_k = (v^k - b^{k+1}) \cdot x_k$$

Example

- Two positions: receive 200 and 100 clicks
- Advertisers 1,2,3 have per-click values \$10, \$4, \$2.

- GSP auction
  - One eqm: truthful bids of \$10, \$4, \$2.
  - Revenue is 200*\$4 + 100*\$2 = \$1000.

Example, continued…

- Consider VCG auction
  - Dominant to bid true value.
  - Advertiser 2 pays \$200 (displaces 3) for 100 clicks, or \$2 per click.
  - Advertiser 1 pays \$600 (displaces 3 *and* 2) for 200 clicks, or \$3 per click.
  - Revenue of \$800 is lower than GSP…

Facts

- Theorem: If bidders were to bid the same amount under VCG and GSP, each bidder's payment would be $\text{payment}_{gsp} \geq \text{payment}_{vcg}$
- Proof. Induction on positions. QED

Truthful bidding?

- Not a dominant strategy to bid “truthfully”
  - Two positions, with 200 and 100 clicks.
  - Consider bidder with value 10
  - Faces competing bids of 4 and 8.
    - Bidding 10 wins top slot, pay 8: profit 200 • 2 = 400.
    - Bidding 5 wins next slot, pay 4: profit 100 • 6 = 600.
  - If competing bids are 6 and 8, better to bid 10…

GSP equilibrium Analysis

- Full information Nash equilibrium
  - NE means no gain from changing positions

- A Nash eqm is a profile of bids $b^1, \ldots, b^K$ such that

$$(v^k - b^{k+1}) \cdot x_k \geq (v^k - b^{m+1}) \cdot x_m \quad \text{for } m>k$$

$$(v^k - b^{k+1}) \cdot x_k \geq (v^k - b^{m}) \cdot x_m \quad \text{for } m<k$$

- Lots of Nash equilibria, including some that are inefficient (try to show this).

Locally Envy-Free

- *Definition:* An equilibrium is *locally envy-free* if no player can improve his payoff by exchanging bids with the player ranked one position above him.
  - Motivation: "squeezing" – if an equilibrium is not LEF, there might be an incentive to squeeze.
  - Add the constraint for all k

$$(v^k - b^{k+1}) \cdot x_k \geq (v^k - b^k) \cdot x_{k-1}$$

Stable Assignments

- Treat positions as players. Coalition value from a position-bidder pair is $v_i x_k$, and price of position is p_k
  - Payoff to agent is $(v_i - p_k) x_k$
  - Payoff to position is $p_k x_k$
- All stable assignments are efficient (assortative), and the relevant blocks are bidders looking to move up or down one position. (think about this).
- Prices that support a stable allocation satisfy:

$$(v_k - p_k) \bullet x_k \geq (v_k - p_{k-1}) \bullet x_{k-1}$$
$$(v_k - p_k) \bullet x_k \geq (v_k - p_{k+1}) \bullet x_{k+1}$$

Equivalence Result

- ***Theorem:***
  - Outcome of any locally envy-free equilibrium is a stable assignment.
  - Provided that |N|>|K|, any stable assignment is an outcome of a locally envy-free equilibrium.

Revenue and Prices

- ***Theorem***
  - There exists a bidder-optimal stable assignment (equivalently, GSP equilibrium) and a seller-optimal one.
  - The bidder optimal stable assignment is payoff-equivalent to the VCG outcome.

- ***Corollary:*** any locally envy free GSP equilibrium generates at least as much revenue as VCG.

Remainder of the lecture

- Discussions of recent papers
- Lessons learned

GSP with monopoly reserve

Simple vs. Optimal Mechanisms [Hartline & Roughgarden, EC-09]

- Motivation:
  - Myerson does not rank by bids
  - Buyers may complain
- Question:
  - What if we insist on ranking by bids?
  - How good is 2^{nd} price auction with Myerson reserve?
    - Known: for IID, this is exactly Myerson auction
- Theorem: For any *MHR* distributions, 2^{nd} price auction with Myerson reserve is a *2-approximation* of optimal revenue
  - Even in the worst case, better than half
  - 2^{nd} price auction is *simple* and approximately *optimal*

Simple vs. optimal mechanisms [Hartline&Roughgarden, EC-09]

- Still not fair?
  - Different reserves for different buyers
  - Buyers may complain
- What if we insist on *anonymous reserve*?
  - Taobao and Ebay: one reserve for all buyers
- Theorem. For any regular distribution, 2nd price auction with *some* anonymous reserve is at least a *4-approximation* of Myerson auction

Lessons learnt

- Ranking
  - Fairest (most efficient): deterministic by bid
  - Optimal: deterministic by virtual value
    - Tradeoff: mixed ranking [Shen&Tang, AAMAS-2017]
    - Higher bid has *a better chance* to be ranked higher
    - Higher virtual value has *a better chance* to be ranked higher
    - Myerson reserve, anonymous reserve
    - Analyze worst case guarantee, test actual performance
    - Baidu field experiments

Related academic work

- [Alaei, et. al. FOCS-2015]
  - Optimal auction vs. Anonymous pricing
  - 2nd price auction with *some* anonymous reserve is an *e-approximation* of Myerson auction, (still not tight)
  - Sequential posted pricing is an *e*-approximation
- [Tang & Wang, EC-14]
  - Simple and optimal auctions
- [Li and Yao, PNAS-13], [Yao, SODA15], [Yao, EC17]
  - Simple versus optimal auctions for multiple items
- [Ostrovsky & Schwartz, EC-09]
  - Reserve prices in ad auctions: a field experiments
  - Large-scale field experiments on Yahoo ads

Yahoo experiments on reserve prices [Ostrovsky & Schwartz, EC-09]

- Motivation
  - Optimal reserve prices observed in practice are lower than the theoretically predicted optima
  - Bulow-Klemperer Theorem: for iid regular distributions, 2nd price auction with n+1 bidders yields more revenue than Myerson auction with n bidders
    - Instead of optimizing reserves, try to recruit one more bidder!
    - Proof: Compare to the optimal auction that always sells the item for n+1
  - Infinite iid bidders: 2nd price = Myerson
  - Rumor: In practice, reserve price may not be as important as theory predicts to be?
- Methodology
  - A field experiment on Yahoo GSP auctions
    - Comparing to the fixed 10 cents (US) in the old design
  - to verify/refute the above rumor

Yahoo experiments on reserve prices [Ostrovsky & Schwartz, EC-09]

- Summary of findings
  - Reserve pricing plays a substantial role in revenue
  - Especially effective for keywords with
    - High search volumes, or,
    - High Myerson reserves, or
    - Relatively small number of advertisers (thin market)
- Summary of experimentation design
  - 460000 keywords
  - Valuation fits to a log-normal distribution for each keyword
  - Reserve =α (myerson reserve) + (1-α)*(10 cents)
    - α is uniformly from 0.4, 0.5, 0.6, conservation reasons
    - fact: midpoint of 10 cent and Myerson reserve is almost optimal
  - A/B test: 95% treatment group, 5% control group

Yahoo experiments on reserve prices [Ostrovsky & Schwartz, EC-09]

- More details
  - Downside (or upside?)
    - Reduce the number of ads per page by 0.91 (almost one fewer ad per page)
    - Better user experience
  - Upside: two measurements
    - Without isolating search volume impact
      - Increase average revenue by 8% - 13% per keyword
      - No difference for α=0.4, 0.5, 0.6
    - Isolate search volume impact
      - Increase average revenue by 3% per keyword

Lessons learnt

- All of the above!
- Treat Yahoo experiments seriously!
- Follow-ups: reinforcement mechanism design
  - [IJCAI17, Tang], in a few slides
  - Automated adjustment of reserve prices
  - Compared to the manual approach by Yahoo!

Different ranking rules

Optimizing tradeoffs among stakeholders in ad auctions [EC14, Key et al, AAMAS17, Shen&Tang]

- Objective: *max* α*revenue* + β*welfare* + γ*clicks*
  - Show it is a special case of single-parameter setting
    - Can use Myerson paradigm to derive theoretical optimum
  - Implementation via GSP
    - A rank score with Myerson reserve
  - Extend to the case
    - *max* α*revenue*+β*welfare*
      - *Subject to clicks* > *c*
  - Extend to the case
    - Number of ads per page is constrained
  - Experimentation over Bing/Baidu data

GSP with Squashing [Lahaie & Pennock, EC07]

- Rank bidders by $b_i e_i^q$
  - e_i is the probability of being clicked on if noticed by a user (aka. *advertiser effect* or *quality score*)
  - q is a parameter, thus a class of auctions
- Empirical evaluation based on data from Yahoo!
  - Findings
  - Setting $q < 1$ increases revenue
  - Tuning q results in more significant revenue gains than setting reserve prices

GSP with anchoring [Leyton-Brown and Thompson, EC-13]

- Anchoring GSP
  - Bidders face a uniform reserve price r
  - Those whose bids exceed r are then ranked by $(b_i-r)e_i$

- Findings
  - Anchoring is optimal for some simple distributions
  - Anchoring performs well for other distributions

Price of prior dependence

Price of prior dependence How to evaluate Myerson auction? [Tang & Zeng, EC18]

- Motivation
  - In practice: advertisers bid lower than their true value
  - Even in a truthful auction!
- Interpretation
  - Advertisers realize that their past bids might potentially be used against themselves in the future
  - Conservative and shade their bids
- Model:
  - Consider a stationary state
  - Advertisers carefully report value distributions
  - Seller uses reported distributions to set Myerson parameters
  - Question: under equilibrium, revenue?

Findings

- Theorem. The revenue of Myerson auction in the reported setting is equivalent to the revenue of first price auction in the standard setting
  - Myerson is not robust against smart buyers
  - Two U[0,1] report U[1/4,1/2] in equilibrium, i.e., under-report
  - Advertisers can remove the effect of *prior-dependent* reserve
- Theorem. The revenue of 2^{nd} auction in the reported setting is the same as the standard setting
  - Second price auction is prior free (hold for any prior-free)
- Theorem. Revenue of 2^{nd} price auction with α*Myerson reserve in the reported setting is less than or equal to 2^{nd} price auction in the standard setting
  - Prior dependent
  - Similar to the current design

Lessons learned

- Caution:
  - Design shall be robust wrt. priors
    - Incentivize buyers to reveal valuation distribution
    - Or, does not heavily rely on prior distribution
    - Or, rely on prior in an indirect way that it is difficult to manipulate
  - Vanilla GSP and VCG are prior free
- Note
  - Not all advertisers are smart --- the design should reflect and exploit such irrationality

Reinforcement mechanism design
Dynamic reserve pricing in ad auctions
[Shen et. al., AAAI2020, joint work with Baidu ads]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贴吧
锤子科技 坚果 Pro 上市百天 限时特惠
坚果 Pro 上市100天,锤子科技官方商城,优惠岂止百元.仅限8月17日-8月18日.漂亮依然,实力依然.
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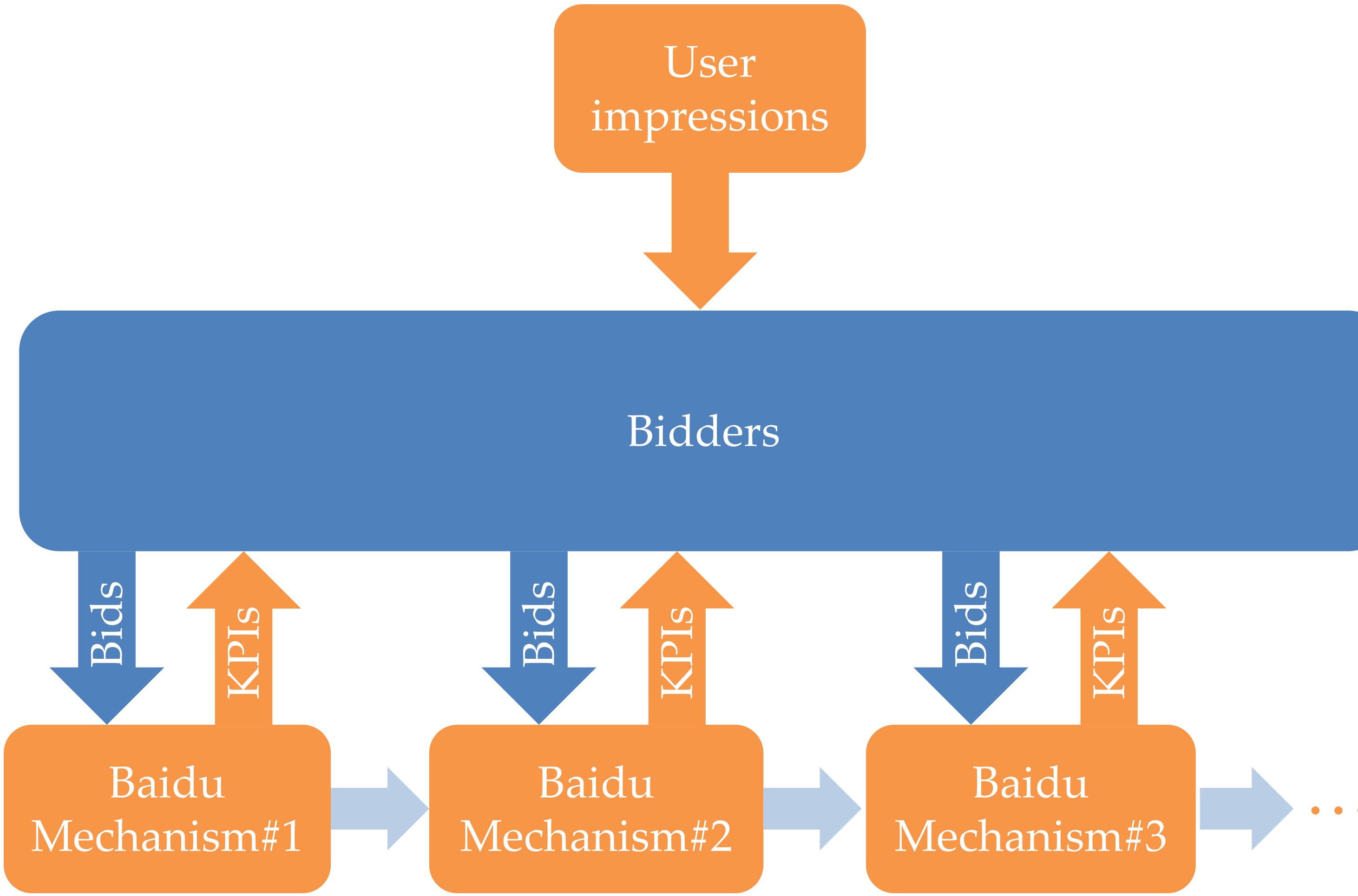
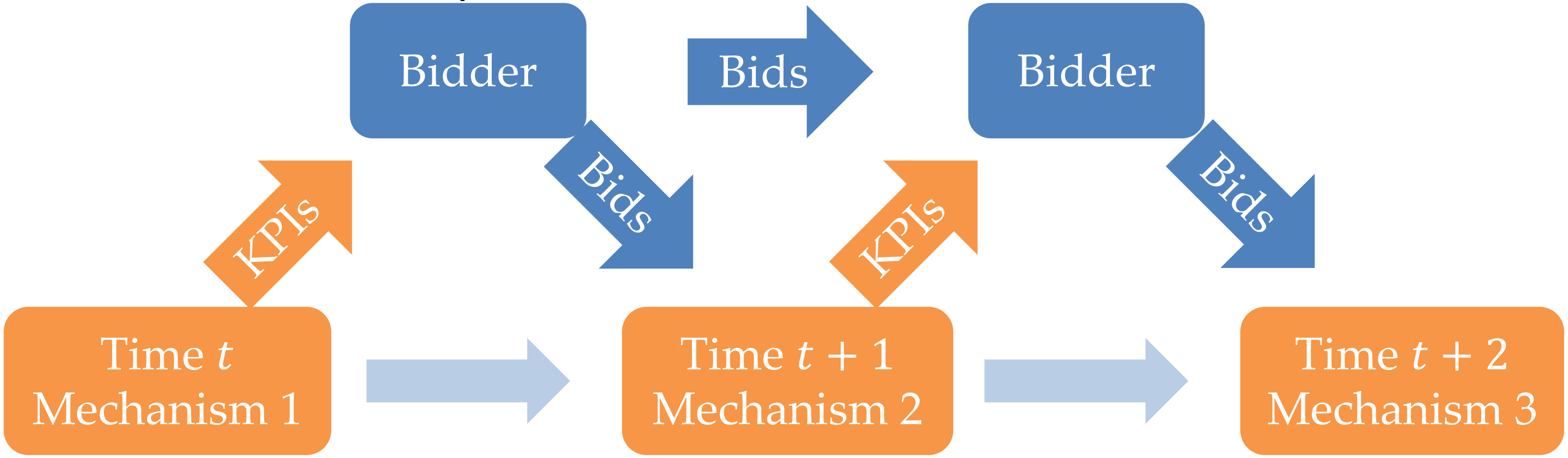
Dynamic GSP auctions



Markov bidder model

A bidder's bid distribution at time $t + 1$ depends only on:

- His/her bid distribution at time t
- The KPIs he/she receives at time t



Reinforcement mechanism design

The dynamic mechanism design problem is a Markov decision process (MDP):

- Joint bid distribution (and history) as state;
- Reserve prices as action;
- State transition defined by the *bidder model*;
- Maximize long-term discounted revenue

Implementations

- MDPs are hard to solve
  - Dynamic environment
  - High dimensional state space and action space.
- Reasonable simplifications:
  - Consider top bidders for each keyword
  - Search a small neighborhood of current prices
- Use heuristics:
  - Monte–Carlo Tree Search algorithm

Bidder model

- Bidder model

  Represented as a RNN (LSTM)

  Input:

  - KPIs containing stats of several consecutive days
  - Temporal features

  Output:

  - Bid *distribution* for the next time step

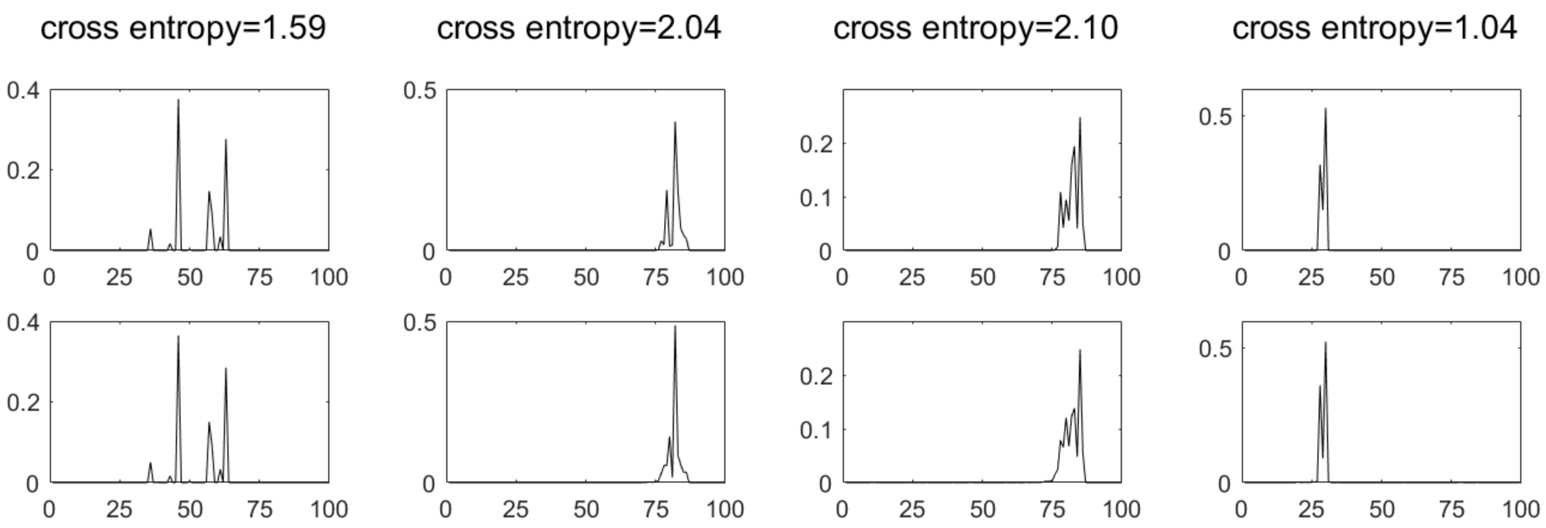
  Discretize bid distributions to 100 buckets

  Minimize cross-entropy

Results on 9-month Baidu dataset (400 keywords, 6TB)

- Advertiser model

  Average cross-entropy among all test instances: 1.67



Performance comparison

- MCTS algorithm
- Compare 5 policies:

  BAIDU: Baidu current reserve prices

  STATIC_50: always use 50 cents

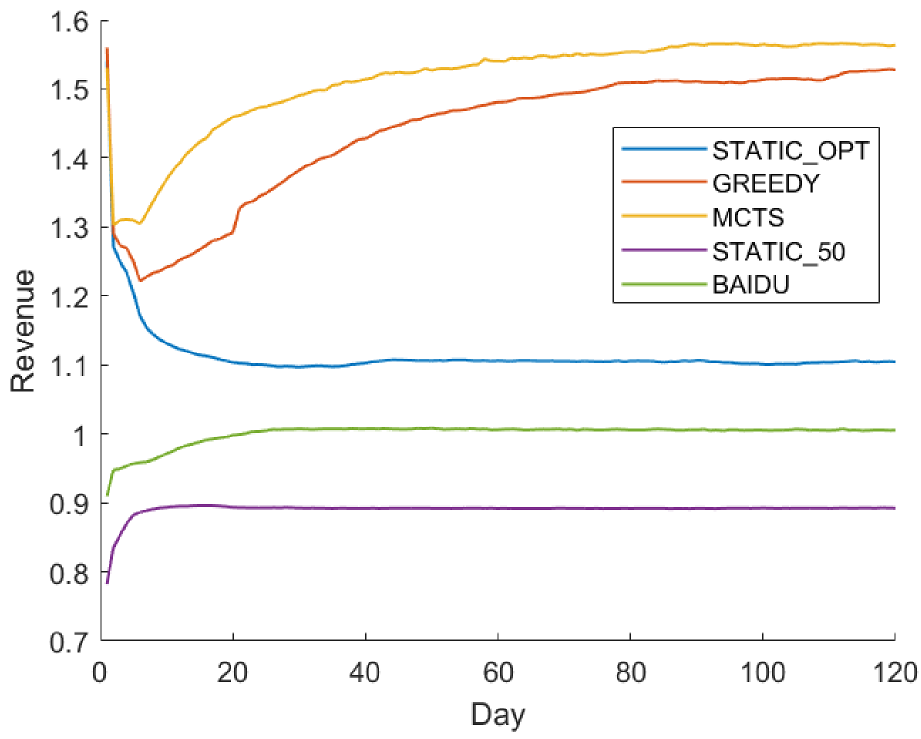
  STATIC_OPT: revenue optimal static reserve based on historical bid distribution

  GREEDY: randomly choose a bidder, greedily change his reserve price

  MCTS: our framework implemented by MCTS

Results

- Revenue comparison by simulation



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