

Economics and Computation

Week #11: Auction: applications

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Two applications of auctions

- eBay (Taobao, Amazon, JD)
- Google ads (Bing, Yahoo!, Facebook)

eBay

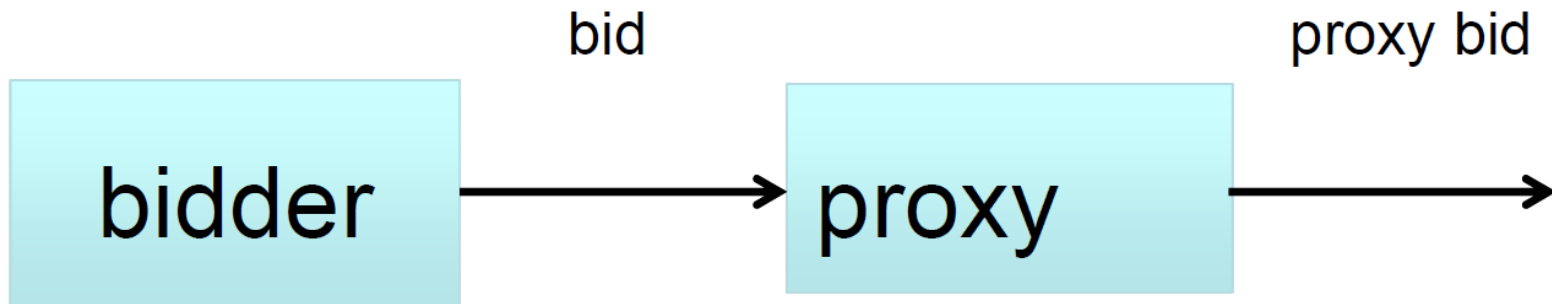
eBay

- Launched as AuctionWeb in 1995
- By 2012, total gross merchandise volume on eBay marketplace \$67b each year
- Evolution from beanie babies to sales channel for retailers

Gross transaction volume comparison (year of 2015)

- Alibaba = Taobao+Tmall = 261+165=\$426B
- Amazon = \$ 88B
- eBay = \$83B
- Walmart = \$ 471B
- 2016: Alibaba surpasses Walmart

Proxy bidding



Rules and notations

- $b^{(1)}$, $b^{(2)}$: first, second highest bids placed with proxy agents
 - Must bid at least starting price
 - Current winner: highest bid
 - $\text{price} = \min(b^{(2)} + \text{incr}, b^{(1)})$ (starting price if only one bid)
 - $\text{ask-price} = \text{price} + \text{incr}$
 - secret reserve: say “reserve not met”
- Taobao:
Two bidders
meet secret
reserve!

Example

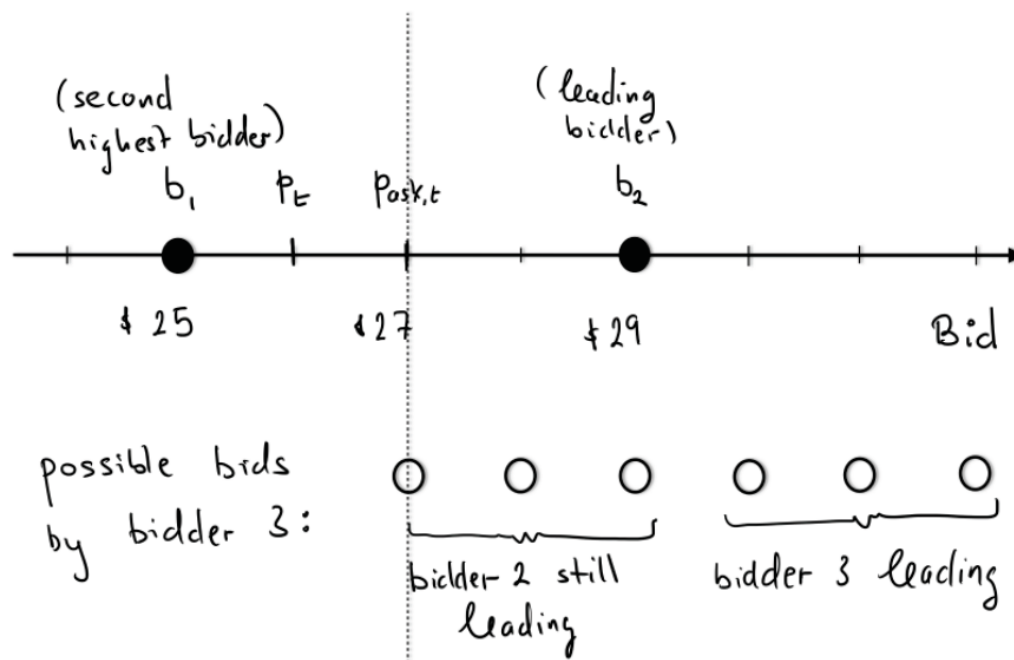
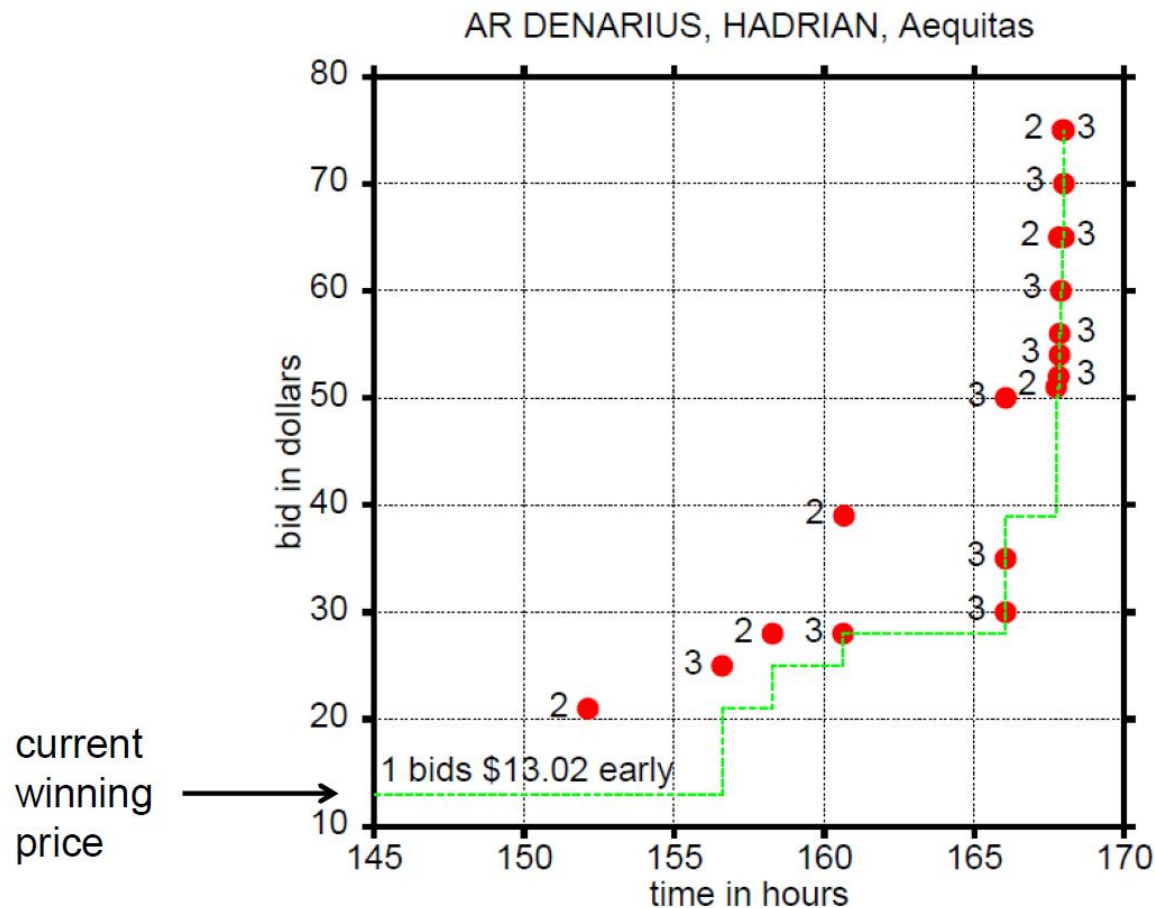


Figure 7.4: An eBay auction with bid increment \$1 and current bids $b_1 = \$25$ and $b_2 = \$29$. Bidder 2 the leading bidder with price \$26. The ask price is \$27. Bidder 3 can bid \$27 or higher, but only a bid of \$30 or higher will make it the leading bidder.

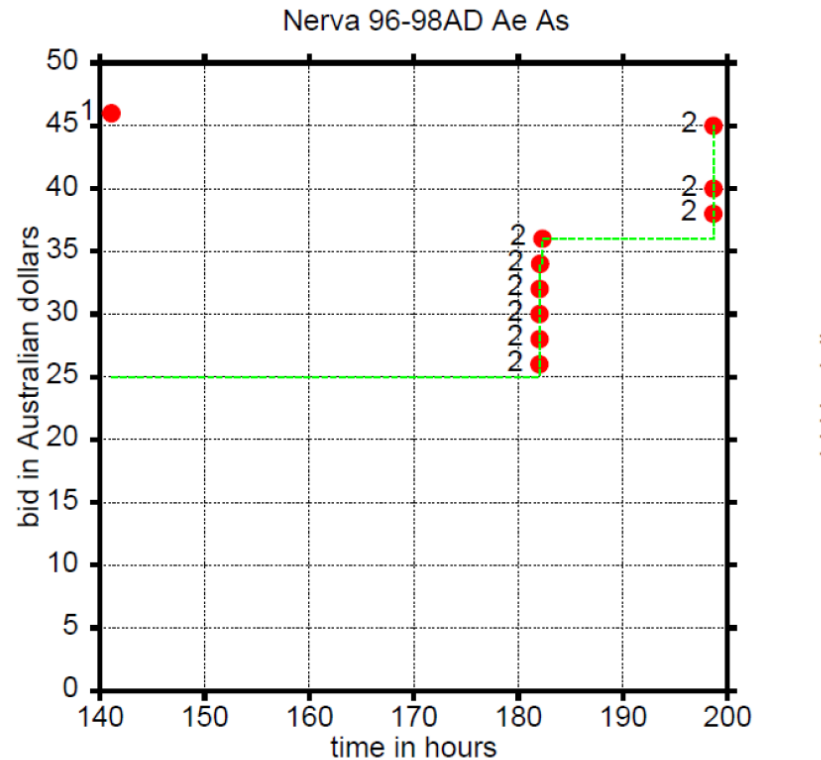
Recommendation by eBay

- Simplicity: “enter maximum then sit back and watch”

A bidding war towards the end

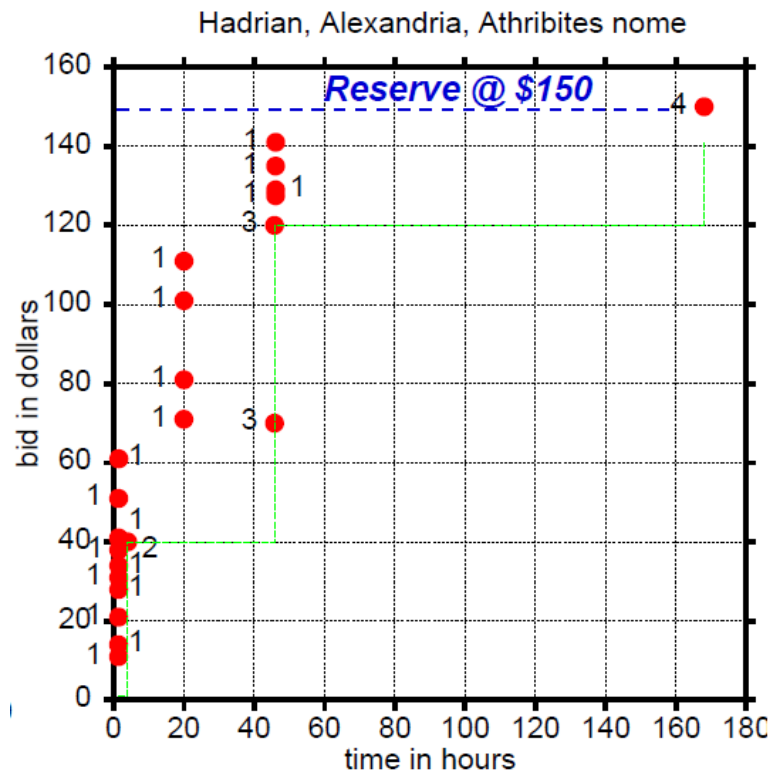


Potential downside of early bidding



c: Potential downside of early bidding. (Note:

Searching for secret reserves



d: Searching for a secret reserve price.

eBay design choice

- Simplicity: “enter maximum then sit back and watch”

vs. SPSB:

- Transparency
- Coordinated bidding across auctions
- Non private value environments
- Entertainment value

Hard vs soft closing rules

- eBay: hard closing rule
 - can use tools such as eSnipe to submit late bids
- Amazon auctions: soft closing rule
 - automatically extend for 10 mins if bid placed in last 10 mins
 - no time at which a buyer can bid at which other buyers cannot respond

Taobao auction closing rules

- 1) 司法拍卖出价延时的基本规则

在设置了出价延时的拍品竞拍结束的前2分钟（以系统接受竞价的时间显示为准），如果有竞买人出价竞拍，那么该次拍卖时间在此次出价的自动延时的基础上自动延时5分钟，循环往复直到没有竞买人出价竞拍时，拍卖结束。

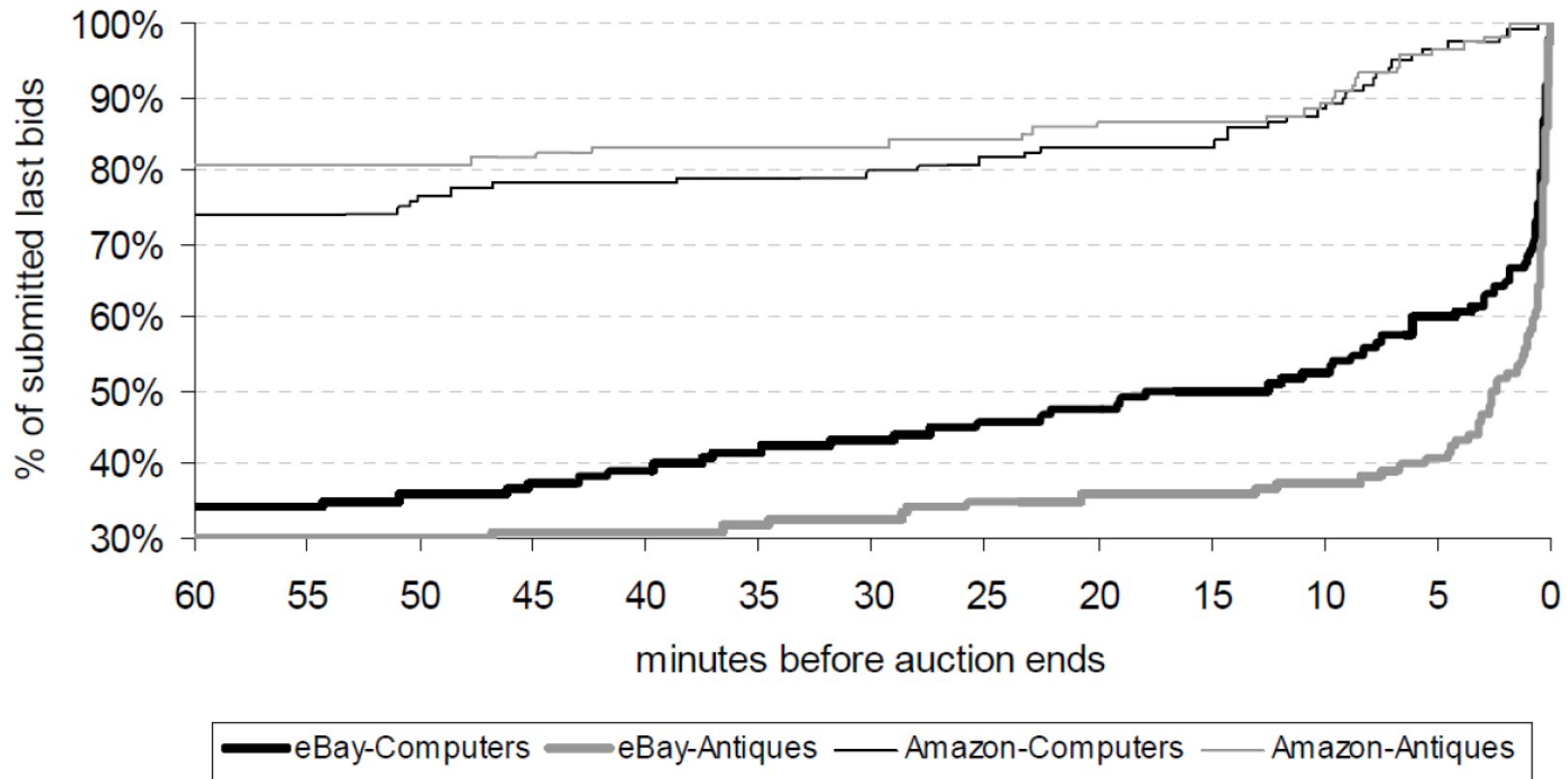
- 2) 司法拍卖出价延时举例说明

比如：假设某件拍品的拍卖结束时间是8月8日22点整，如果在8月8日21点58分15秒，有竞买人出价，那么系统将拍卖结束时间自动延长至8月8日22点03分15秒；如果在22点03分有竞买人出价，那么系统将自动延时到22点08分。

- 3) 司法拍卖出价延时的效果

出价延时给拍卖用户带来的便利包括以下几点：1.将给到竞买人更加充分的参与竞拍的机会。2.避免因网络或者电脑原因导致的延迟而错失出价机会。

Distribution of time remaining for final bids



Time remaining: before current scheduled close time
Final bid = the last bid submitted by any bidder

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Hard vs Soft closing rule

- Pros and Cons
 - Soft closing=Early bidding: shared information promotes common value
 - Hard closing=late bidding: allow seller to choose a closing time that attracts large demand (cf. 11.11)

A “Seller Experiment”

- Listings form a seller experiment if same seller, same item (category, title, sub-title)
- Free shipping, no secret reserve, no BIN
- One posted price transaction
- **Listings with different starting prices**
- 19,000+ experiments; 490,000+ listings in 2009. Large variation on starting price.

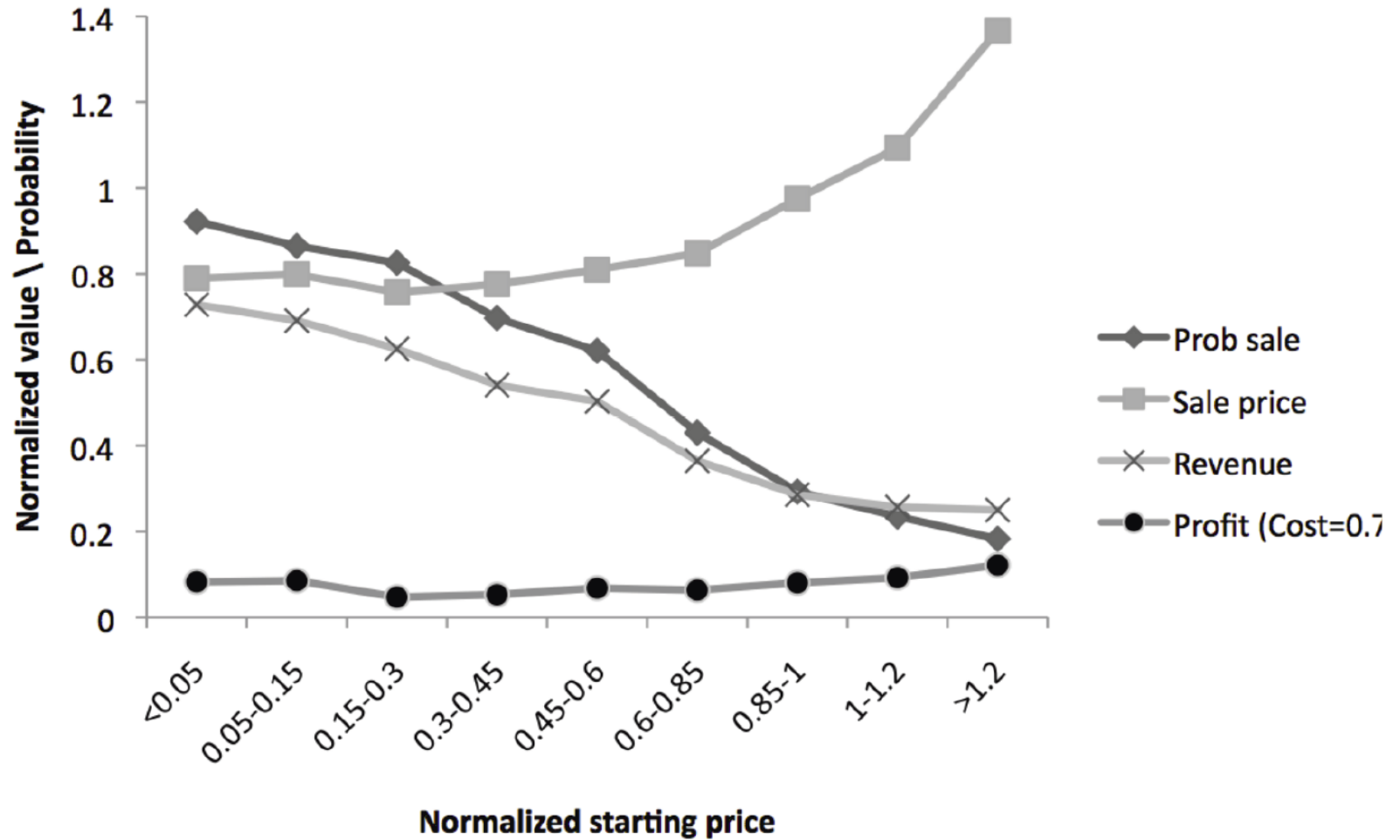
Reference value

- Posted price transaction gives a reference value for the item
- Normalize starting price and sale price on items by this reference value
- Can pool data across multiple seller experiments (= multiple items)

Effect of starting price

- Some earlier studies found
 - lower starting price, better odds of sale
 - lower starting price, higher price conditional on sale... “bidding frenzy” story
- But small field experiments.

Results



Additional findings

- BIN prices above 1.0 tend to increase seller revenue (Why?)
- Timing/duration does not matter much (just don't close late at night!)

Aside: Trend to Fixed Prices?

(Einav et al. 2012)

- 90% transaction volume was auctions in 2003; today less than 36%
- Not category specific
- Auction price (conditioned on sale) 5% less than fixed price in '03; 17% less in '09
- Shift in demand to posted price listings

Aside: Trend to Fixed Prices?

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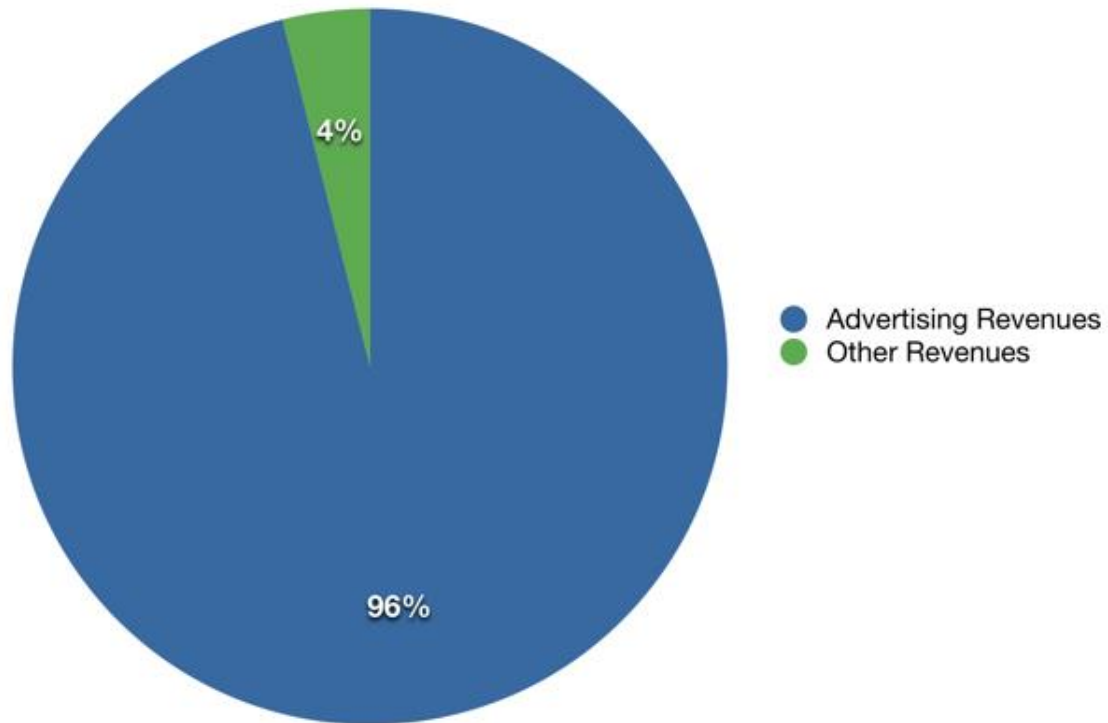
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- **Possible reasons:**
 - competition for time, other things to do online!
 - smart phones; preference for efficiency

Reputation system design

- Readings:
 - Reputation and feedback systems in online platform markets. AER-2016.
 - The limits of reputation in platform markets: an empirical analysis and field experiments.

Google: sponsored search auctions

Googles Revenues 1Q 2012



Statistics

- Google advertising revenue:
 - 2011: \$36,531M
 - 2012: \$43,686M
 - 2013 first 3 quarters: \$ 36,600 M
- Hal Varian: Google Chief Economist
 - “What most people don’t realize is that all that money comes pennies at a time”
- References:
 - Varian 2008: Position auctions
 - Edelman et. al 2007: Internet advertising and generalized second price auctions







Keyword Auctions

- Advertiser submit bids for keywords
 - Offer a dollar payment *per click*.
 - Alternatives: price per impression, or per conversion.
- Separate auction for every query
 - Positions awarded in order of bid (more on this later).
 - Advertisers pay bid of the advertiser in the position below.
 - “Generalized second price” auction format.
- Some important features
 - Multiple positions, but advertisers submit only a single bid -
- “simplification” (cf Milgrom’s lecture).
 - Search is highly targeted, and transaction oriented.

Brief History of Sponsored Search Auctions



- Pre-1994: advertising sold on a per-impression basis, traditional direct sales to advertisers.
- 1994: Overture (then GoTo) allows advertisers to bid for keywords, offering some amount *per click*. Advertisers pay their bids.
- Late 1990s: Yahoo! and MSN adopt Overture, but mechanism proves unstable - advertisers constantly change bids to avoid paying more than necessary.
- 2002: Google modifies keyword auction to have advertisers pay minimum amount necessary to maintain their position (i.e. GSP)- followed by Yahoo! and MSN.



Example

- Two positions: receive 200 and 100 clicks
- Advertisers 1,2,3 have per-click values \$10, \$4, \$2.
- Overture auction
 - Advertiser 2 has to bid \$2.01 to get second slot
 - Advertiser 1 wants to bid \$2.02.
 - But then advertiser 2 wants to top this, and so on.
- GSP auction
 - One eqm: truthful bids of \$10, \$4, \$2.
 - Revenue is $200 \times \$4 + 100 \times \$2 = \$1000$.

VCG for keywords auction

- Same allocation rule as GSP
- Everyone pays the damages he causes by his participation



Example, continued...

- Consider VCG auction
 - Dominant to bid true value.
 - Advertiser 2 pays \$200 (displaces 3) for 100 clicks, or \$2 per click.
 - Advertiser 1 pays \$600 (displaces 3 *and* 2) for 200 clicks, or \$3 per click.
 - Revenue of \$800 is lower than GSP...

Model

- K positions $k= 1, \dots, K$
- N bidders $i=1,\dots, N$
- Bidder i values position k at $V= v_i x_k$
 - v_i is the value of a click to bidder i
 - x_k is the probability of a click at this slot
- Efficient allocation:
 - v and x follow the same order



GSP Auction Rules

- Each agent i submits bid b_i
- Positions assigned in order of bids
- Agent i 's price per click is bid of agent in the next slot down.
- Let b^k denote k th highest value and v^k value.
- Payoff of k th highest bidder:

$$v^k \cdot x_k - b^{k+1} \cdot x_k = (v^k - b^{k+1}) \cdot x_k$$

Observations of GSP

- **Theorem.** If bidders were to bid the same amount under VCG and GSP, then each bidder's payment would be $\text{pay}_{\text{gsp}} \geq \text{pay}_{\text{vcg}}$
- Proof. Induction on positions. QED



Truthful bidding?

- Not a dominant strategy to bid “truthfully”
 - Two positions, with 200 and 100 clicks.
 - Consider bidder with value 10
 - Faces competing bids of 4 and 8.
 - Bidding 10 wins top slot, pay 8: profit $200 \cdot 2 = 400$.
 - Bidding 5 wins next slot, pay 4: profit $100 \cdot 6 = 600$.
 - If competing bids are 6 and 8, better to bid 10...



GSP equilibrium Analysis

- Full information Nash equilibrium
 - NE means no gain from changing positions
- A Nash eqm is a profile of bids b^1, \dots, b^K such that
$$(v^k - b^{k+1}) \cdot x_k \geq (v^k - b^{m+1}) \cdot x_m \quad \text{for } m > k$$
$$(v^k - b^{k+1}) \cdot x_k \geq (v^k - b^m) \cdot x_m \quad \text{for } m < k$$
- Lots of Nash equilibria, including some that are inefficient (try to show this).



Locally Envy-Free

- *Definition:* An equilibrium is *locally envy-free* if no player can improve his payoff by exchanging bids with the player ranked one position above him.
 - Motivation: “squeezing” – if an equilibrium is not LEF, there might be an incentive to squeeze.
 - Add the constraint for all k

$$(v^k - b^{k+1}) \cdot x_k \geq (v^k - b^k) \cdot x_{k-1}$$



Stable Assignments

- Treat positions as players. Coalition value from a position-bidder pair is $v_i x_k$, and price of position is p_k
 - Payoff to agent is $(v_i - p_k) x_k$
 - Payoff to position is $p_k x_k$
- All stable assignments are efficient (assortative), and the relevant blocks are bidders looking to move up or down one position. (think about this).
- Prices that support a stable allocation satisfy:

$$(v_k - p_k) \cdot x_k \geq (v_k - p_{k-1}) \cdot x_{k-1}$$

$$(v_k - p_k) \cdot x_k \geq (v_k - p_{k+1}) \cdot x_{k+1}$$



Equivalence Result

- **Theorem:**
 - Outcome of any locally envy-free equilibrium is a stable assignment.
 - Provided that $|N| > |K|$, any stable assignment is an outcome of a locally envy-free equilibrium.



Revenue and Prices

- **Theorem**

- There exists a bidder-optimal stable assignment (equivalently, GSP equilibrium) and a seller-optimal one.
- The bidder optimal stable assignment is payoff-equivalent to the VCG outcome.

- **Corollary:** any locally envy free GSP equilibrium generates at least as much revenue as VCG.



Example of LEF Equilibria

- Three positions with 300, 200, 100 clicks
- Four bidders with values \$3, \$2, \$1, \$1
- Efficient assignment is assortative
- Supporting prices
 - Bidder 3 pays \$100 for slot 3, $p_3 = 1$.
 - Bidder 2 pays \$200-300 for slot 2, $p_2 \in [1, 3/2]$.
 - Bidder 1 pays \$400-600 for slot 3, $p_3 \in [4/3, 2]$.
- Try solving for bids that generate these prices.
- Relationship between VCG and LEF eqm
 - VCG payments are \$100, \$200, \$400, revenue \$700.
 - LEF payments range from \$700 up to \$1000.



Structure of Clearing Prices

- Supporting prices satisfy

$$(v_k - p_k) \cdot x_k \geq (v_k - p_{k-1}) \cdot x_{k-1}$$

$$(v_k - p_k) \cdot x_k \geq (v_k - p_{k+1}) \cdot x_{k+1}$$

- Re-arranging we get

$$p_{k-1}x_{k-1} \geq p_k x_k + v_k(x_{k-1} - x_k)$$

$$p_{k-1}x_{k-1} \leq p_k x_k + v_{k-1}(x_{k-1} - x_k)$$

- This gives a simple *recursive* way to find the highest and lowest equilibrium payments.



Features of Equilibrium

- Allocation is efficient (assortative)
- Increasing price of marginal clicks
 - Varian points out this is testable.
 - Implies bidders are click-constrained!
 - Pricing should be linear if bidders satiated...
- Bids “reveal” bounds on bidder values.
 - Apparently not so easy to invert in practice.
 - Actual bidding is surprisingly unstable...



Ascending auction

- Incomplete information about values
- Price rises from zero, advertisers can drop out at any time, fixing their bid.
- ***Theorem (Edelman et al.).***
 - There is a unique perfect equilibrium in which an advertiser with value v_i drops at
$$p_i(n, h, v_i) = (x_n / x_{n-1})(v_i - b_{n+1})$$
 - The equilibrium outcome is the same as VCG
 - The equilibrium is an *ex post* equilibrium.



Optimal auction design

- Suppose each bidder i draws values from F_i
 - Define marginal revenue: $MR_i(v_i) = v_i - (1 - F_i)/f_i$
 - Seller has total quantity $x = x_1 + \dots + x_K$.
- Optimal auction problem:
 - Choose allocation of clicks z_1, \dots, z_N to maximize $\sum_i MR_i(v_i) z_i$ subject to the allocation being feasible.
 - Solution: assign slots in order of marginal revenue, so long as it is positive.
- Optimal reserve prices: if the environment is symmetric, optimal to run a position auction with reserve price r^* that satisfies $MR(r^*) = 0$.
 - Of course, need to know distribution of per-click values...



Further issues

- Each query is a separate game
 - Advertisers really have portfolio of bids & broad match...
 - Ignores budget constraints, diminishing returns, etc
 - Hard to think about eg. competing platforms
- Model doesn't allow for much uncertainty
 - Click rates, effectiveness of advertising are known.
 - Seems to be a lot of experimentation in practice. Why?
- Many aspects of search not captured
 - How do people decide whether/what to click?
 - Is there an interaction with “organic” search?
- “Non-search” internet advertising
 - Google uses same auction to place ads on non-query web pages (AdSense).
 - Other companies use related mechanisms to match ads and eyeballs, and sometimes quite different approaches.

Acknowledgement

- eBay Slides adapted from David Parkes@Harvard
- Google slides adapted from Jonathan Levin@Stanford