MAT-215 Problem Set 4

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Problem 1

We have the set $\{x \in \mathbb{R} : x^2 - 3x - 4 > 0\}$. We can write this set equivalently as

$$\{x \in \mathbb{R} : x < -1\} \cup \{x \in \mathbb{R} : x > 4\}.$$

We can do this because the parabola $x^2 - 3x - 4 = (x-4)(x+1)$ is only greater than 0 when $x < -1$ or when $x > 4$.

Problem 2

We have the set $A = \{6n : n \in \mathbb{N}\} \cap \{10n : n \in \mathbb{N}\}$.

a. The set $\{6n : n \in \mathbb{N}\}$ has elements $\{0, 6, 12, 18, 24, 30, 36, ...\}$. The set $\{10n : n \in \mathbb{N}\}$ has elements $\{0, 10, 20, 30, 40, 50, 60, ...\}$. We can see that the only elements that occur in both these sets are $\{0, 30, 60, ...\}$.

b. My conjecture about the parametric description of A is

$$A = \{30n : n \in \mathbb{N}\}$$

Problem 3

We define $A \triangle B = \{x : x \text{ is an element of exactly one of } A \text{ or } B\}$

a. $\{1, 3, 8, 9\} \triangle \{2, 3, 4, 7, 8\} = \{1, 2, 4, 7, 9\}$

b. I claim that $A \triangle B = (A \cup B) \setminus (A \cap B)$. Consider the example in part a. We have $A \cup B = \{1, 2, 3, 4, 7, 8, 9\}$. We have $A \cap B = \{3, 8\}$. Thus,

$$(A \cup B) \setminus (A \cap B) = \{x : x \in A \cup B \text{ and } x \notin A \cap B\} = \{1, 2, 4, 7, 9\}$$

c. We let $A = \{2n : n \in \mathbb{N}\}$ and $B = \{3n : n \in \mathbb{N}\}$. Listing out elements A, we have $\{0, 2, 4, 6, 8, 10, 12, 14, ...\}$. Listing out elements of B, we have $\{0, 3, 6, 9, 12, 15, 18, 21, ...\}$. We can see that

$$A \triangle B = \{2, 3, 4, 8, 9, 10, 14, 15, 16, 20, 21, 22, 26, ...\}$$

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d. Consider the set $C = \{3 + 6n : n \in \mathbb{N}\} = \{3, 9, 15, 21, ...\}$ Notice that $A \triangle B \setminus C = \{2, 4, 8, 10, 14, 16, 20, 22, ...\}$. Now consider the set $D = \{2 + 6n : n \in \mathbb{N}\} = \{2, 8, 14, 20, 26, ...\}$. Notice that $A \triangle B \setminus C \setminus D = \{4, 10, 16, 22, 28, \}$. We define this final set as $E = \{4 + 6n : n \in \mathbb{N}\}$. Thus, $C \cup D \cup E = A \triangle B$.

Problem 4

Using the domain $\mathbb{R}$, we get that

$$\text{range}(f) = \{x \in \mathbb{R} : x > 0\}$$

since the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = e^x$ returns outputs greater than 0 for all inputs in *reals*.

Problem 5

a. In order to compute $(g \circ f)(6)$, we first determine $f(6)$. The number of positive divisors of 6 is 4. Then, we compute $g(4)$. The number of primes less than 4 is 2. Thus, $(g \circ f)(6) = 2$.

In order to compute $(g \circ f)(36)$, we first determine the number of positive divisors of 36. We find that there are 9 (more specifically, $1, 36, 6, 2, 18, 3, 12, 4, 9$). Next, we compute $g(9)$. There are 4 primes less than 9, so $(g \circ f)(36) = 4$.

b. In order to find an example of $n \in \mathbb{N}^+$ with $(g \circ f)(n) = 3$, we start by finding an example of $m \in \mathbb{N}^+$ with $g(m) = 3$. There are only two candidates for m, 5 and 6, as they both have only 3 primes less than or equal to m. Next, we need $n \in \mathbb{N}^+$ with $f(n) = 5$ or $f(n) = 6$. We see that $n = 12$ has 6 positive divisors, so an example of n such that $(g \circ f)(n) = 3$ is $n = 12$.

Problem 6

We define the function $f : \mathbb{Q} \to \mathbb{Q}$ with $f(a) = 5a - 3$.

a. $f(2) = 5(2) - 3 = 10 - 3 = 7$

b. $f(-10) = 5(-10) - 3 = -50 - 3 = -53$

c. $f\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - 3 = 4 - 3 = 1$

d. Let $b \in \mathbb{Q}$ be arbitrary. Notice that

$$\begin{aligned} f\left(\frac{b+3}{5}\right) &= 5\left(\frac{b+3}{5}\right) - 3 \\ &= (b+3) - 3 \\ &= b \end{aligned}$$

Thus, we have found an example of $a \in \mathbb{Q}$ such that $f(a) = b$. Therefore, $\mathbb{Q} \subseteq \text{range}(f)$.