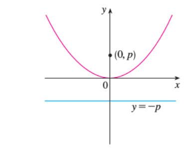
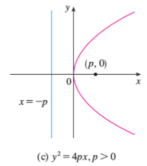
MATH 153 Ch. 10 & 12 Formula Sheet

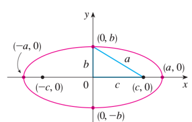
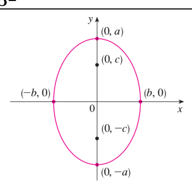
Conic Sections

For unshifted standard forms of conic sections, let $h=0, k=0$.

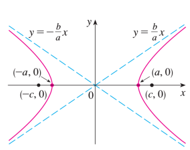
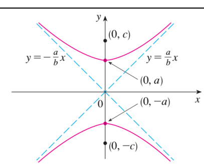
Parabolas

Graph		
Equation	$(x-h)^2=4p(y-k)$ $y=(x-h)^2+k$	$(y-k)^2=4p(x-h)$ $x=(y-k)^2+h$
Directrix	$y=k-p$	$x=h-p$
Foci	$(h,k+p)$	$(h+p,k)$
Vertices	(h,k)	(h,k)

Ellipses $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

Graph	 $a \geq b > 0$	 $b \geq a > 0$
Center	(h,k)	(h,k)
Foci	$(h \pm c, k)$ $(c^2=a^2-b^2)$	$(h, k \pm c)$ $(c^2=b^2-a^2)$
Vertices	$(h \pm a, k)$	$(h, k \pm b)$

Hyperbolas

Graph		
Equation	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{b^2}-\frac{(x-h)^2}{a^2}=1$
Center	(h,k)	(h,k)
Foci ($c^2=a^2+b^2$)	$(h \pm c, k)$	$(h, k \pm c)$
Vertices	$(h \pm a, k)$	$(h, k \pm b)$
Asympt.	$y=k \pm \frac{b}{a}(x-h)$	$y=k \pm \frac{b}{a}(x-h)$

Parametric Equations

General Form of Parametric Equations

$$x=f(t), \quad y=g(t), \quad a \leq t \leq b$$

Parametric Equations of an Ellipse

$$x=a\cos(\omega)t, \quad y=b\sin(\omega)t$$

Traces an ellipse exactly once in a counter-clockwise direction starting at the point $(a,0)$ in the range $0 \leq t \leq 2\pi$

Parametric Derivatives

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \text{where } \frac{dx}{dt} \neq 0$$

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

Parametric Tangents

Tangent Line: $y=F(a)+m(x-a)$ where $\left.\frac{d}{dx}\right|_{t=a}=F'(a)$

Horizontal Tangents: $\frac{dy}{dt}=0$, where $\frac{dx}{dt} \neq 0$

Vertical Tangents: $\frac{dx}{dt}=0$, where $\frac{dy}{dt} \neq 0$

Parametric Areas Under Curves

$$A=\int_\alpha^\beta g(t)f'(t)\ \mathrm{dt}, \quad a=f(\alpha), b=f(\beta)$$

$$A=\int_\beta^\alpha g(t)f'(t)\ \mathrm{dt}, \quad b=f(\alpha), a=f(\beta)$$

where α and β are values of t, a and b are values of x, y.

Parametric Arc Length

$$L=\int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\ \mathrm{dt}$$

The calculated arc length may not be proportional to the number of times the parametric curve has been traced.

Parametric Surface Areas

$$S=\int 2\pi y\ \mathrm{ds} \quad \text{rotation about x-axis}$$

$$S=\int 2\pi x\ \mathrm{ds} \quad \text{rotation about y-axis}$$

$$\mathrm{ds}=\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\ \mathrm{dt} \quad \text{if } x=f(t), y=g(t), \alpha \leq t \leq \beta$$

Polar Coordinates

Alt. Representations of Polar Coordinates

$$(r, \theta+2\pi n)$$

$$(-r, \theta+(2\pi+1)n)$$

Polar to Cartesian Formula

$$x=r\cos(\theta)$$

$$y=r\sin(\theta)$$

Cartesian to Polar Formula

$$r^2=x^2+y^2; \quad r=\sqrt{x+y}$$

$$\theta=\tan^{-1}\left(\frac{y}{x}\right)$$

Basic Polar Graphs

Symmetry Tests

Symmetric about the polar axis or x-axis	$(r,-\theta)$
Symmetric about the pole/origin	$(-r,\theta)$
Symmetric about $\theta=\pi/2$ or y-axis	$(-r,-\theta)$

Lines

Polar	Cartesian
$\theta=\beta$	$y=(\tan\beta)x$
$r\cos\theta=a$	$x=a$
$r\sin\theta=b$	$y=b$

Circles

Polar	Cartesian
$r=a$	$x^2+y^2=a^2$
$r=2a\cos\theta$	$(x-a)^2+y^2=(\lvert a\rvert)$
$r=2b\sin\theta$	$x^2+(y-b)^2=(\lvert b\rvert)$

Polar Tangents

$$\frac{dy}{dx}=\frac{\frac{dr}{d\theta}\sin\theta+r\cos\theta}{\frac{dr}{d\theta}\cos\theta-r\sin\theta}$$

Polar Areas Under Curves

$$A=\int_\alpha^\beta \frac{1}{2}r^2 d\theta$$

Polar Arc Length

$$L=\int ds=\int \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}d\theta$$

Polar Conic Sections

Horizontal Conic: $r=\frac{ed}{1 \pm e\cos\theta}, \quad x \pm d$

Vertical Conic: $r=\frac{ed}{1 \pm e\sin\theta}, \quad y \pm d$

The conic is an ellipse if $e<1$, a parabola if $e=1$ or a hyperbola if $e>1$.

3D Coordinate Space

Distance Between Two Points in 3D

$$d(P_1, P_2) = \sqrt{(x_2 - x_2)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Eq. of Sphere with center (h, k, l) and radius r:

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$

Vectors

Magnitude of a Vector: $\quad |\vec{a}|| = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}$

Direction of a Vector: $\theta = \arctan\left(\frac{\vec{a}_x}{\vec{a}_y}\right)$

Unit Vector: $\quad \vec{u} = \frac{1}{|\vec{a}|}\vec{a}$

Net Angle Between Two Vectors

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

Direction Cosines

$$\frac{1}{|a|}\vec{a} = \langle \cos\alpha, \cos\beta, \cos\gamma \rangle$$

Scalar projection of $\vec{b}$ onto $\vec{a}$:

$$\text{comp}_{\vec{a}}\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$

Vector projection of $\vec{b}$ onto $\vec{a}$:

$$\text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}^2|}\vec{a}$$

Dot Product

Given two vectors $\vec{a} = \langle a_1, a_2, \ldots, a_n \rangle$ and $\vec{b} = \langle b_1, b_2, \ldots, b_n \rangle$, the dot product $\vec{a} \cdot \vec{b}$ is:

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$$

$$\vec{a} \cdot \vec{b} = |a||b| \cos\theta$$

Cross Product

Given two vectors $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$ in $\mathbf{R^3}$, the cross product $\vec{a} \times \vec{b}$ is:

$$\vec{a} \times \vec{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$

$$|\vec{a} \times \vec{b}| = |a||b| \sin\theta$$

Other Formulas

Exponent Properties

$$a^m a^n = a^{n+m}$$
$$(a^n)^m = a^{nm}$$
$$\frac{a^n}{a^m} = a^{n-m}$$
$$(ab)^n = a^n b^n$$
$$\left(\frac{a^n}{b^m}\right) = \frac{a^{nk}}{b^{mk}}$$
$$\frac{1}{a^{-n}} = a^n$$

Logarithm Properties

$$\log_b(M \cdot N) = \log_b(M) + \log_b(N)$$
$$\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$$
$$\log_b(M^k) = k \cdot \log_b(M)$$

Trignometric Identities

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \frac{1}{\cos^2 x}$$
$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$
$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$
$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\begin{aligned} \cos(2x) &= \cos^2(x) - \sin^2(x) \\ &= 2\cos^2(x) - 1 \\ &= 1 - 2\sin^2(x) \end{aligned}$$

Trigonometry Reference

Common Values of Sine and Cosine

θ	Cosine	Sine
0	1	0
$\pi/6$	$\sqrt{3}/2$	$1/2$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$	$1/2$	$\sqrt{3}/2$
$\pi/2$	0	1
π	-1	0
$3\pi/2$	0	-1