```
clear all
close all
%Part 1 Observation
% Case I
m = 10;
n = 10;

independent_columns_counter = 0;
dependent_columns_counter = 0;
for i = 1:5
    A = rand(m, n);
    if rank(A) == n
        independent_columns_counter = independent_columns_counter + 1;
    else
        dependent_columns_counter = dependent_columns_counter + 1;
    end
end
disp(['Case I - Linearly independent: ',
num2str(independent_columns_counter)]);
disp(['Case I - Linearly dependent: ', num2str(dependent_columns_counter)]);


% Case II
m = 10;
n = 9;

independent_columns_counter = 0; %reinitializes the counter for case II
dependent_columns_counter = 0;
for i = 1:5
    A = rand(m, n);
    if rank(A) == n
        independent_columns_counter = independent_columns_counter + 1;
    else
        dependent_columns_counter = dependent_columns_counter + 1;
    end
end
disp(['Case II - Linearly independent: ',
num2str(independent_columns_counter)]);
disp(['Case II - Linearly dependent: ', num2str(dependent_columns_counter)]);


% Case III
m = 10;
n = 11;

independent_columns_counter = 0;
dependent_columns_counter = 0;
for i = 1:5
    A = rand(m, n);
    if rank(A) == n
        independent_columns_counter = independent_columns_counter + 1;
    else
```

```
        dependent_columns_counter = dependent_columns_counter + 1;
    end
end
disp(['Case III - Linearly independent: ',
num2str(independent_columns_counter)]);
disp(['Case III - Linearly dependent: ',
num2str(dependent_columns_counter)]);


%Question 1

%Case I.1. Generate a 12x12 matrix

m = 12;
n = 12;
A = rand(m, n);
b = rand(m,1);

augmented_matrix1 = [A,b]
case_I= rref(augmented_matrix1)

%unique solution bc every row of the coeff matrix corresponds to a constant
%matrix b

%Case II.1. Generate a 15x13 matrix
m = 15;
n = 13;
A = rand(m, n);
b = rand(m,1);

augmented_matrix2 = [A,b];
case_II = rref(augmented_matrix2)
% no solution as the second last row has the augmented matrix at 0 = 1
% which is not possible

%Case III.1. Generate a 15x16 matrix
m = 15;
n = 16;
A = rand(m, n);
b = rand(m,1);

augmented_matrix3 = [A,b];
case_III = rref(augmented_matrix3)
%the last column of the coeff matrix is a free variable. it will create a
%general solution with parameters --> infinitely many solutions


%Question 2. Exceptions
%Case I.2. Generate a 3x3 matrix with a 3x1 constant matrix b
a1 = [3 2 -5]';
a2 = [1 1 -2]';
a3 = [5 3 -8]';
b = [4 1 6]'; % column vector of constants (right hand side values of an
augmented matrix)
```

```
case1_augmentedmatrix = [a1 a2 a3 b] %augmented matrix Ax = b
case1_exception_reduced = rref(case1_augmentedmatrix)
% the system is inconsistent as the last row of the matrix is 0 = 1 which
% is not true


%Case II.2. Generate a 4x3 matrix with a 4x1 constant matrix
x1 = [ 1  4 7 10]';
x2 = [ 2 5 8 11]';
x3 = [3 6 9 12 ]';
x= [7 18 29 40]';

case2_augmentedmatrix = [x1 x2 x3 x]

case2_reduced= rref(case2_augmentedmatrix)


%CASE III.2
a1 = [ 1 5 9]';
a2 = [ 2 6 10]';
a3 = [ 3 7 11]';
a4 = [4 8 12]';
b = [10 26 41]';

case3_augmentedmatrix = [a1 a2 a3 a4 b]

case3_reduced = rref(case3_augmentedmatrix)
%the system is inconsistent

Case I - Linearly independent: 5
Case I - Linearly dependent: 0
Case II - Linearly independent: 5
Case II - Linearly dependent: 0
Case III - Linearly independent: 0
Case III - Linearly dependent: 5

augmented_matrix1 =

  Columns 1 through 7

    0.0894    0.3634    0.8866    0.2300    0.7320    0.8756    0.9373
    0.4241    0.7710    0.4401    0.5514    0.5689    0.7462    0.7697
    0.0510    0.6687    0.4387    0.5903    0.3170    0.2665    0.3936
    0.2014    0.1402    0.6839    0.8129    0.3267    0.3634    0.6377
    0.1444    0.0966    0.1116    0.8159    0.3972    0.7745    0.5645
    0.4606    0.9741    0.5538    0.2150    0.1013    0.8502    0.3518
    0.8012    0.1483    0.9701    0.7021    0.2712    0.6682    0.2149
    0.1117    0.3561    0.6033    0.8802    0.0306    0.2766    0.9961
    0.3835    0.1702    0.6539    0.2652    0.6619    0.2148    0.7046
    0.8431    0.7734    0.6686    0.8423    0.9065    0.5359    0.5253
    0.4336    0.7536    0.4341    0.8050    0.3757    0.6382    0.4140
    0.9913    0.2008    0.5100    0.3658    0.2578    0.0400    0.2096
```

```
  Columns 8 through 13

    0.8053    0.2658    0.3372    0.8574    0.9992    0.1069
    0.4037    0.0082    0.8050    0.9427    0.8247    0.3279
    0.1029    0.7279    0.5262    0.9913    0.5769    0.6567
    0.8733    0.8097    0.0976    0.8105    0.4926    0.8768
    0.6159    0.3766    0.4545    0.2480    0.2337    0.7621
    0.6239    0.3197    0.9618    0.2663    0.1537    0.8119
    0.2970    0.8933    0.4473    0.1055    0.7182    0.4783
    0.6840    0.4677    0.1586    0.2897    0.6006    0.2848
    0.4248    0.9221    0.5832    0.1613    0.5142    0.4535
    0.7309    0.6723    0.9546    0.7347    0.8416    0.2873
    0.6329    0.1247    0.8472    0.4427    0.2837    0.3574
    0.9148    0.4556    0.0863    0.7415    0.5010    0.6131


case_I =

  Columns 1 through 7

    1.0000         0         0         0         0         0         0
         0    1.0000         0         0         0         0         0
         0         0    1.0000         0         0         0         0
         0         0         0    1.0000         0         0         0
         0         0         0         0    1.0000         0         0
         0         0         0         0         0    1.0000         0
         0         0         0         0         0         0    1.0000
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0

  Columns 8 through 13

         0         0         0         0         0    0.7898
         0         0         0         0         0   -0.0380
         0         0         0         0         0   -0.2946
         0         0         0         0         0   -0.1343
         0         0         0         0         0   -0.3235
         0         0         0         0         0    0.8493
         0         0         0         0         0    0.8103
    1.0000         0         0         0         0   -0.3799
         0    1.0000         0         0         0    0.8943
         0         0    1.0000         0         0   -0.3572
         0         0         0    1.0000         0    0.8229
         0         0         0         0    1.0000   -1.4424


case_II =

  Columns 1 through 13

     1     0     0     0     0     0     0     0     0     0     0     0
```

```
0
     0     1     0     0     0     0     0     0     0     0     0     0
0
     0     0     1     0     0     0     0     0     0     0     0     0
0
     0     0     0     1     0     0     0     0     0     0     0     0
0
     0     0     0     0     1     0     0     0     0     0     0     0
0
     0     0     0     0     0     1     0     0     0     0     0     0
0
     0     0     0     0     0     0     1     0     0     0     0     0
0
     0     0     0     0     0     0     0     1     0     0     0     0
0
     0     0     0     0     0     0     0     0     1     0     0     0
0
     0     0     0     0     0     0     0     0     0     1     0     0
0
     0     0     0     0     0     0     0     0     0     0     1     0
0
     0     0     0     0     0     0     0     0     0     0     0     0
1
     0     0     0     0     0     0     0     0     0     0     0     0
0
     0     0     0     0     0     0     0     0     0     0     0     0
0

  Column 14

     0
     0
     0
     0
     0
     0
     0
     0
     0
     0
     0
     0
     1
     0


case_III =

  Columns 1 through 7

    1.0000         0         0         0         0         0         0
```

```
         0    1.0000         0         0         0         0         0
         0         0    1.0000         0         0         0         0
         0         0         0    1.0000         0         0         0
         0         0         0         0    1.0000         0         0
         0         0         0         0         0    1.0000         0
         0         0         0         0         0         0    1.0000
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0

  Columns 8 through 14

         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
    1.0000         0         0         0         0         0         0
         0    1.0000         0         0         0         0         0
         0         0    1.0000         0         0         0         0
         0         0         0    1.0000         0         0         0
         0         0         0         0    1.0000         0         0
         0         0         0         0         0    1.0000         0
         0         0         0         0         0         0    1.0000
         0         0         0         0         0         0         0

  Columns 15 through 17

         0    0.7653   -3.7122
         0    0.6557    0.9599
         0   -0.3500    0.5941
         0    0.9941   -0.8489
         0   -0.7780   -0.0185
         0   -0.4855   -1.9131
         0    0.5934    0.6742
         0    1.2334    2.2304
         0    0.4714   -3.6066
         0   -1.2190    0.2362
         0   -0.7235    2.3561
         0   -0.5425    0.8485
         0    0.2450   -0.2409
         0   -0.4532    0.7092
    1.0000    0.3075    1.9763


case1_augmentedmatrix =
```

```
     3     1     5     4
     2     1     3     1
    -5    -2    -8     6


case1_exception_reduced =

     1     0     2     0
     0     1    -1     0
     0     0     0     1


case2_augmentedmatrix =

     1     2     3     7
     4     5     6    18
     7     8     9    29
    10    11    12    40


case2_reduced =

    1.0000         0   -1.0000    0.3333
         0    1.0000    2.0000    3.3333
         0         0         0         0
         0         0         0         0


case3_augmentedmatrix =

     1     2     3     4    10
     5     6     7     8    26
     9    10    11    12    41


case3_reduced =

     1     0    -1    -2     0
     0     1     2     3     0
     0     0     0     0     1
```

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