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Journal of Econometrics 131 (2006) 151–177

JOURNAL OF Econometrics

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Breaks and persistency: macroeconomic causes of stock market volatility

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Available online 25 February 2005

Abstract

In the paper we study the relationship between macroeconomic and stock market volatility, using S&P500 data for the period 1970–2001. We find evidence of a twofold linkage between stock market and macroeconomic volatility. Firstly, the break process in the volatility of stock returns is associated with the break process in the volatility of the Federal funds rate and M1 growth. Secondly, two common long memory factors, mainly associated with output and inflation volatility, drive the break-free volatility series. While stock market volatility also affects macroeconomic volatility, the causality direction is stronger from macroeconomic to stock market volatility.
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JEL classification: C32; F30; G10

Keywords: Stock market volatility; Macroeconomic volatility; Long memory; Fractional cointegration; Structural change

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doi:10.1016/j.jeconom.2005.01.007

1. Introduction

Why does stock market volatility change over time? This questions was asked by Schwert (1989) at the end of the 1980s. His goal was to explain the time-varying stock return volatility by means of the time-varying volatility of macroeconomic and financial variables. The basic conclusion of the paper was that "the amplitude of the fluctuations in aggregate stock volatility is difficult to explain using simple models of stock valuation". Schwert (1989) also found mixed results with respect to the direction of causality between return volatility and the volatility of macroeconomic and financial variables. He found that: (a) inflation volatility predicts stock volatility but only for the sub-period 1953–1987 and stock volatility does not predict inflation volatility, (b) money growth volatility predicts stock volatility in various sub-samples and stock volatility predicts money growth volatility from 1920 to 1952, (c) industrial production volatility weakly explains the volatility of stock returns, while stock volatility helps to predict industrial production volatility in two sub-samples. Overall his results point to a positive linkage between macroeconomic volatility and stock market volatility, with the direction of causality being stronger from the stock market to the macroeconomic variables. Moreover, the level of macroeconomic volatility explains less than half of the volatility of stock returns. In some periods the ratio is even lower: in 1929–1939 the volatility of macroeconomic variables increased but not by a factor of three as in the case of stock return volatility. Finally, he found evidence that stock market uncertainty is higher during recessions than expansions.[1]

A weakness in Schwert (1989) is that it does not accurately model the persistence properties of volatility and it ignores the potential downward bias affecting the estimates, due to the use of noisy volatility proxies. In fact, since Schwert's study there have been many advances in the theoretical and empirical understanding of econometric models for time-varying volatility. Many studies have focused on the causes of persistence of volatility of asset returns, pointing to the presence of structural change, long memory, or both. For instance, Hamilton and Susmel (1994) have found that the conditional variance process of the US stock market can be described by a switching regime model with three persistent states. The interpretation of the authors is that the high volatility state was triggered by general business downturn. These findings have largely been confirmed by So et al. (1998) and Hamilton and Lin (1996), while Kim and Kim (1996) have suggested that the switch to the high volatility state may be due to an increased volatility in the fad component of the returns, rather than to an increase in the volatility of fundamentals. Evidence of switching regimes in the conditional variance process have been also found for some European countries by Morana and Beltratti (2002).

The alternative explanation of long range dependence has been also proposed to account for persistence of the conditional variance process (see for instance Ding et al., 1993; Baillie et al., 1996; Bollerslev and Mikkelsen, 1996; Andersen and Bollerslev, 1997), with long memory being the consequence, for instance, of the cross-sectional aggregation of a large number of volatility components or news

[1]See also Campbell et al. (2001) and Whitelaw (1994).

information arrival processes with different degrees of persistence (Granger, 1980; Andersen and Bollerslev, 1997). While some recent contributions have cast doubts on the hypothesis that long memory is a real feature of the data generating process of the volatility of financial returns (Granger and Hyung, 2004; Mikosch and Starica, 1998), other authors have suggested that both long memory and structural change characterize the structure of financial returns volatility (Morana and Beltratti, 2004; Lobato and Savin, 1998).

In this paper, we provide further evidence on the economic causes of volatility persistence for stock market returns. In particular, we study the relationship between S&P500 returns volatility and the volatility of some macroeconomic factors over the period 1970–2001. Improving on Schwert (1989), we take into account recent evidence about the stochastic process followed by volatility, and allow for both long memory and structural breaks. This allows us to study the relations among breaks in the series and among break-free series. Moreover, in order to account for the presence of observational noise, we extend the nonlinear log periodogram estimator of Sun and Phillips (2003) to the multivariate case and develop a semiparametric noise filtering approach for perturbed long memory processes. Finally, and contrary to what previously done in the literature, we follow a structural approach. We exploit the long-run properties of the volatility processes investigated and identify the cointegration space and the sources of persistent volatility dynamics.

We believe there are merits in the proposed analysis since we provide evidence of short- and long-term linkages, which could not be determined or disentangled by using a single component approach. An econometric analysis which does not account for multiple components of volatility cannot disentangle meaningful relations among the volatility series in our sample.[2] However, as it will be shown in the paper, accurately modelling the persistence properties of the series and accounting for observational noise allows to draw much different conclusions concerning the linkages between macroeconomic and stock market volatility. Particularly important for justifying our approach, we show that the relations between volatility of the variables change depending on the specific component under investigation.

Our main results are the following. First, we find evidence that the process describing volatility of the US market is characterized by both long memory and structural change. We confirm an important result obtained by Campbell et al. (2001), that is that the post 1995 period has not witnessed an increase in overall market volatility, but we qualify it because we show that the length of time spent in the high volatility regime is unusual compared to the past. Second, the break process in stock market volatility can be related to the break process in the volatility of macroeconomic factors, Federal funds rate and M1 growth in particular, with the

[2] In the sample period used in this paper, 1970–2001, linkages between macroeconomic and stock market volatility are even harder to find than in Schwert's sample. The p-values of standard Granger causality tests from stock return volatility are: 0.14 for output volatility, 0.23 for inflation volatility, 0.77 for the Federal funds rate volatility, and 0.16 for money growth volatility. The p-values of standard Granger causality tests from the volatility of macroeconomic variables to stock returns volatility are, respectively, 0.57, 0.82, 0.41, and 0.78.

causality direction of this linkage being stronger from macroeconomic volatility to stock market volatility. Third, fractional cointegration analysis, carried out on the break-free log variance processes, points to the existence of three cointegrating vectors, linking output growth, money growth, stock market, the Federal funds rate, and inflation volatility. Fourth, on the basis of the variance decomposition analysis, we find that the two common long memory factors driving the five processes are largely explained by output and inflation volatility. Fifth, our decomposition suggests that a 1% stock market volatility increase is determined by a 0.85% increase in the nonpersistent component and a 0.15% in the persistent component. Sixth, the findings point to the existence of causality linkages which are stronger from macroeconomic volatility to stock market volatility than the other way around: macroeconomic volatility contributes to both persistent and nonpersistent stock market volatility fluctuations, albeit the bulk of stock market volatility fluctuations are largely associated with idiosyncratic "financial" shocks; stock market volatility exercises only a limited influence on macroeconomic volatility.

The rest of the paper is organized as follows. In Section 2 we introduce the econometric methodology, and in Section 3 we investigate the time series properties of the data and the relationship between macroeconomic and stock market volatility. Finally, in Section 4 we conclude.

2. Econometric methodology

2.1. The common long memory factor model

We consider the following common long memory factor model

$$\tilde{\mathbf{x}}_t = \mathbf{\Theta}\boldsymbol{\mu}_t + \mathbf{b}_t + \mathbf{u}_t,$$

$$\Delta^d \boldsymbol{\mu}_t = \boldsymbol{\varepsilon}_t, \tag{1}$$

where $\tilde{\mathbf{x}}_t$ is a $p \times 1$ vector of observations on log variances of the processes which are assumed to be fractionally cointegrated,[3] $\mathbf{b}_t$ is a $p \times 1$ vector of break processes, $\mathbf{\Theta}$ is the $p \times k$ factor loading matrix with $k<p$, $\boldsymbol{\mu}_t$ is a $k \times 1$ vector of observations on the long memory factors ($I(d)$ $0<d<0.5$), $\boldsymbol{\varepsilon}_t \sim i.i.d.(\mathbf{0}, \mathbf{\Sigma}_\varepsilon)$ $\mathbf{\Sigma}_\varepsilon = \mathbf{I}_k$ with dimension $k \times 1$, $\mathbf{u}_t$ is a $p \times 1$ vector of observations on the weakly dependent components ($I(0)$),[4] with $\mathbf{\Phi}(L)\mathbf{u}_t = \mathbf{\Omega}(L)\mathbf{v}_t$, all the roots of the polynomial matrices in the lag operator $\mathbf{\Phi}(L)$ and $\mathbf{\Omega}(L)$ are outside the unit circle, $\mathbf{\Phi}(0) = \mathbf{\Omega}(0) = \mathbf{I}_p$, and $\mathbf{v}_t \sim i.i.d.(\mathbf{0}, \mathbf{\Sigma}_u)$ with dimension $p \times 1$. When the loading matrix $\mathbf{\Theta}$ has reduced rank $k<p$, then there

[3]In terms of our implementation of the approach the vector process $\mathbf{x}_t$ corresponds to the vector of log variance processes for stock market returns, industrial production growth, money growth, Federal funds rate, inflation.

[4]The approach is also valid when $\mathbf{u}_t \sim I(b)$, $b>0$, $0<b<d$.

exist k common long memory factors. This implies the existence of $p-k$ fractional cointegration relationships relating the p log variance processes. We will return later to the possibility of considering log variances as observed variables.

This specification is general and allows volatility persistence to be related to three different sources, two stochastic (one represented by the long memory component and one by the weakly dependent stationary process), and the other deterministic (the break process). The model builds on existing evidence that the time series process of log variance is frequently characterized by both long memory and structural breaks, as shown in the papers mentioned in the introduction. Both these components may be common across volatility processes, pointing to different types of long-run relationships linking the series analyzed.

The model has been estimated in two steps. In the first step the existence of a break process in the various log variance processes is investigated. To this aim a Markov switching mean model has been employed, as suggested by Timmerman (2001), Morana and Beltratti (2004) and Morana (2002). By applying an argument presented in Ang and Bekaert (1998), consistent estimation of the break process can be obtained by the Markov switching model, using the two-step approach, if the omitted variables are not regime dependent.[5] A test for a break process can then be performed by means of an augmented Engle and Kozicki (1993) feature test (see Morana, 2002).[6] Then, break-free series can be obtained by subtracting the estimated break process from the actual data ($\mathbf{x}_t = \tilde{\mathbf{x}}_t - \hat{\mathbf{b}}_t$), and semiparametric methods can be employed to assess the presence of long memory in the data and the existence of stochastic long-run linkages relating the volatility processes.The approach of Morana (2004a) has then been employed to estimate the fractional cointegration relationships and the common long memory factor structure (see Appendix A). This two-step methodology is standard in the literature when frequency domain tools are employed to estimate the memory parameter (see for instance Granger and Hyung, 2004).

A persistent–nonpersistent decomposition (P–NP decomposition) of the observed variables can be performed through the decomposition of Kasa (1992), which can be written as

$$
\begin{aligned}
\mathbf{x}_t &= \boldsymbol{\Theta}\boldsymbol{\mu}_t + \mathbf{u}_t,\\
\boldsymbol{\mu}_t &= (\boldsymbol{\Theta}'\boldsymbol{\Theta})^{-1}\boldsymbol{\Theta}'\mathbf{x}_t,\\
\mathbf{u}_t &= \boldsymbol{\beta}(\boldsymbol{\beta}'\boldsymbol{\beta})^{-1}\boldsymbol{\beta}'\mathbf{x}_t,
\end{aligned} \tag{2}
$$

[5]Only for output growth and inflation volatility our modelling approach requires a third step, which consists of (linearly) filtering out the observational noise component.

[6]A neglected break process may lead to the spurious detection of long memory, while removing a spurious break from a long memory process may induce antipersistence in the level of the series (see for instance Granger and Hyung, 2004; Diebold and Inoue, 2001). The augmented Engle and Kozicki feature test amounts to checking the statistical significance of a candidate break process in an ARFIMA model. By controlling for both long memory and structural change, this test is expected to provide reliable results concerning the causes of persistence. See Morana (2002) also for a discussion of alternative methodologies.

where $\mathbf{\Theta}(\mathbf{\Theta}'\mathbf{\Theta})^{-1}\mathbf{\Theta}'\mathbf{x}_t$ is the persistent (long memory component) and $\boldsymbol{\beta}(\boldsymbol{\beta}'\boldsymbol{\beta})^{-1}\boldsymbol{\beta}'\mathbf{x}_t$ is the nonpersistent (I(0)) component or the less persistent I(b) component $b>0$, $d-b>0$, when $\mathbf{u}_t \sim I(b)$.[7]

The proportion of variance of the long memory factors explained by each process can be determined through the Choleski decomposition of the variance of each factor components, determined from the actual processes $\mathbf{x}_t$ through the weights $(\mathbf{\Theta}'\mathbf{\Theta})^{-1}\mathbf{\Theta}'$. Similarly, the proportion of variance of the persistent (nonpersistent) components explained by each process can be determined through the Choleski decomposition of the variance of the composing elements, determined from the actual processes $\mathbf{x}_t$ through the weights $\mathbf{\Theta}(\mathbf{\Theta}'\mathbf{\Theta})^{-1}\mathbf{\Theta}'$ $(\boldsymbol{\beta}(\boldsymbol{\beta}'\boldsymbol{\beta})^{-1}\boldsymbol{\beta}')$. The Choleski decomposition allows to account for the presence of correlation between the composing elements. Since in this latter case the outcome of the decomposition may depend on the ordering of the composing elements, upper and lower bounds should be constructed to assess the sensitivity of the results. A similar approach has been proposed by Baillie et al. (2002).

3. Empirical results

3.1. Descriptive analysis

The data used in the paper have been obtained by Datastream (daily data for the S&P500 index) and by FRED (monthly figures for industrial production and the consumer price index; weekly figures for the Federal funds rate, M1). The sample period is 1970:1–2001:9. Realized variance processes (Andersen et al., 2001) for interest rates, stock market returns, and money growth, have been obtained by summing the squared daily and weekly conditional mean innovations over the corresponding months.[8] For the rate of growth of industrial production and the inflation rate the squared monthly innovations have been employed. Andersen et al. (2001) have shown that the realized variance estimator is a consistent estimator of integrated variance in the frequency of sampling, yielding virtually noise-free volatility estimates. However, in our case we do not have high frequency data to compute realized variance processes. We have time series observed at different frequencies, i.e. daily, weekly and monthly, and this is unavoidable given our objective which is to link financial volatility and macroeconomic variables volatility. For the monthly series our realized variance is simply the square of the innovations, and we do not expect to be able to observe integrated variance without measurement error. Therefore, we have developed a methodology to compute a noise-free estimate of the observed (log) variance, which is discussed in detail in Appendix B. The noise

[7]The $\mathbf{u}_t$ vector is $I(b)$ when the cointegrating residuals are I(b) or when the largest order of fractional integration of the cointegrating residuals is I(b). Note in fact that the $\mathbf{u}_t$ vector is computed as a linear combination of the cointegrating residuals.

[8]The conditional mean innovations have been obtained by fitting time series models to the various series. For reason of space we do not report the results of the analysis, which are available from the authors upon request.

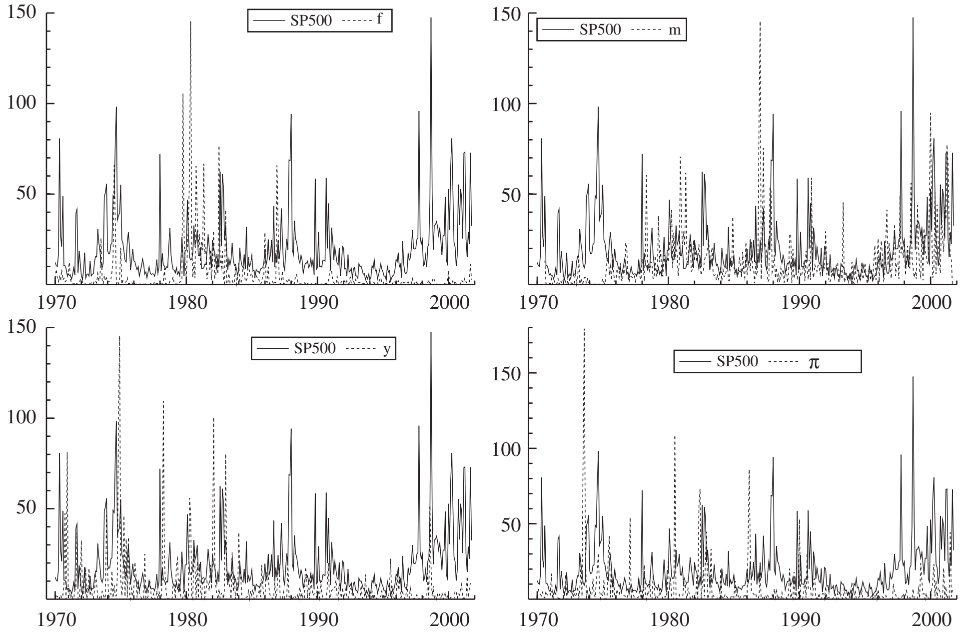

Fig. 1. Realized variance processes (SP500: Standard and Poors 500 Index returns; m: M1 growth rate; y: industrial production growth rate; π: inflation rate; f: Federal funds rate). The volatility processes, apart from the inflation series, have been rescaled to match the range of the volatility of the S&P 500 returns.

filtering procedure has been applied to the break-free log variance series, since the approach is suited to handle purely nondeterministic processes.

In Fig. 1 we report the estimated conditional variance series. To help the readability of the graphs, outlying observations were removed from the data and the variance processes for the Federal funds rate, nominal money growth, and industrial production were rescaled to match the range of variation of the stock volatility series.[9] Coherent with what found by Schwert (1989), stock volatility tends to show a higher unconditional mean and standard deviation than the volatility process of the macroeconomic factors. We also find that the most volatile macroeconomic factor is inflation, followed by M1 growth, the Federal funds rate, and industrial production.[10]

Fig. 1 reveals a very interesting pattern on the comovement between stock market volatility and macroeconomic volatility. The figure shows that different macroeconomic factors seem to be important to explain various stock returns volatility periods. For instance, the Federal funds rate volatility is high together with the

[9]The outlying observations are October 1987 for the S&P500 returns volatility and January and February 1970, and September 2001 from M1 growth volatility.

[10]The figures for the variance series are as follows. Unconditional mean: 21.12, 0.53, 6.09, 0.63, 1.01; unconditional standard deviations: 44.33, 1.16, 15.26, 1.70, 6.05. Figures are for the S&P500 returns, industrial production growth, inflation, the Federal funds rate and the M1 rate of growth, respectively.

volatility of stock returns following the two oil price shocks. The same result holds for inflation and output growth. Interestingly, the relationship between M1 and stock volatility seems to be more stable than for the other macroeconomic factors, in particular for the last part of the sample, characterized by large volatility in both variables. Overall, these pictures seem to clarify why it may be hard to explain stock returns volatility on the basis of a fixed-weight linear combination of the volatility of macroeconomic variables, as is done by the linear regression framework with fixed coefficients.

3.2. Structural change analysis

As a preliminary step in the identification of the sources of volatility persistence, we have tested for structural breaks and long memory, using the Markov switching model to obtain a candidate break process. We have then evaluated the long memory properties of the break-free processes. Since the log variance series have an unconditional distribution closer to Gaussian than the variance series, the statistical analysis is performed on the log transformed processes.

In Fig. 2 we compare the estimated break process with the one which would have been obtained from the smoothed probabilities estimated by the Markov switching

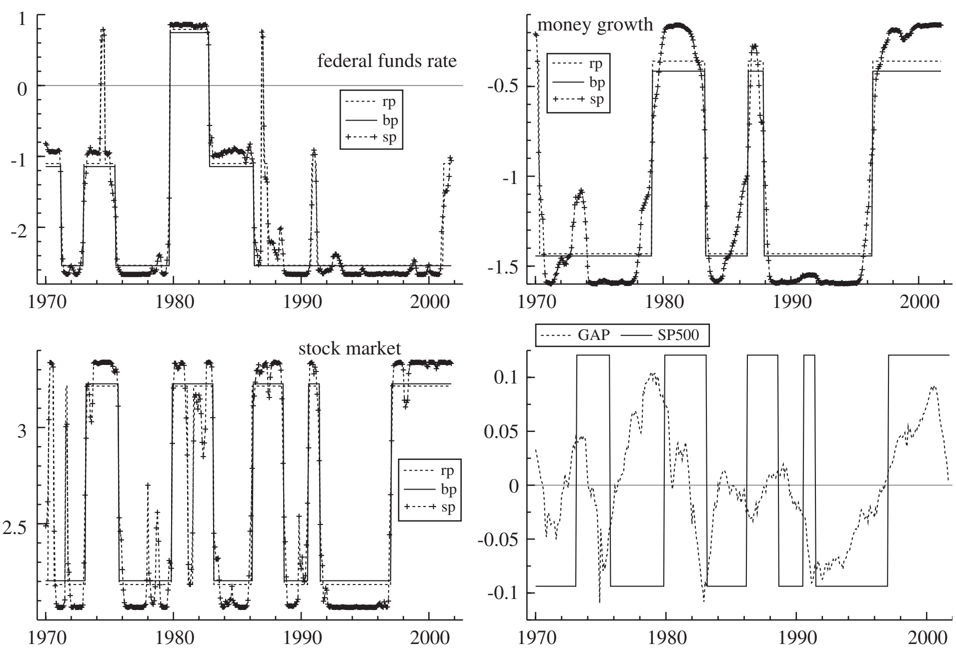

Fig. 2. Estimated break processes from the Markov switching model for log variance series: actual smoothed probabilities (*sp*), rounded smoothed probabilities (*rp*), adjusted rounded probabilities (estimated break process, *bp*). The last plot contrasts the estimated stock market break process (S&P 500) with the output gap (GAP).

model and by rounding them, in order to exactly partition the observations across regimes. In order to model infrequent changes only, we decided not to estimate a break point if the new regime lasted for less than a year. As shown in Fig. 2, this assumption is not restrictive. In fact, the estimated break process is very close to the one which would have been obtained by using the rounded smoothed probabilities (without neglecting the switches lasting less than one year), with the advantage of avoiding switching across regimes with very short duration, which cannot be interpreted as structural breaks.

Interestingly, we were unable to find any significant break points in the inflation and industrial production volatility processes only.[11] On the basis of the BIC criterion three regimes have been selected for the Federal funds rate log variance process, while two regimes have been selected for the stock market returns and M1 growth log variance processes. Although no break points have been detected in output growth volatility by the Markov switching model, a break point in 1984:5 has been imposed, coherent with the graphical inspection of the actual data, the analysis of the long memory properties of the series, and the literature on the "Great Moderation" (see Stock and Watson, 2002). The estimated break points for the Federal funds rate, M1 growth and stock market returns are six, five and nine, respectively.[12] As shown in the figure, often there is a close relation between the dating of the break points in the variance of the macroeconomic factors and in the variance of stock returns. For instance, the increase in stocks volatility following the oil shocks is matched by an increase in the volatility of the Federal funds rate. In addition, the increase in volatility at the end of the 1980s and 1990s is matched by an increase in the volatility of M1 growth. Moreover, the breaks in stock returns volatility of August 1990 and July 1991 can also be related to the volatility of the Federal funds rate. Since the switch to the higher volatility regime for this latter variable lasted for less than a year, the break points were not estimated. However, they can be seen in Fig. 2.[13]

Finally, stock market volatility and industrial production volatility regimes do not seem to be related, even though there is a clear visual association between the output gap and the regimes of stock return volatility.[14] Stock return volatility is high in cases of an increasing output gap in the middle of the 1970s, at the beginning of the 1980s, at

[11]The augmented Engle and Kozicki test suggests that the break processes are not spurious, since the break process is statistically significant and the null for the one sided test that the estimated fractional differencing parameter is greater or equal to zero cannot be rejected for all the processes. For reasons of space we do not report additional details concerning the estimated break process, which can however be found in the preprint version of this paper.

[12]This does not imply that there is no relation among volatility of stocks, inflation and output. There can be, and in fact there is, as discussed later, a relation among the break-free series.

[13]Also the switch to the high volatility regime for the stock market volatility process occurred in August 1990 lasted for less than one year. However, the estimated smoothed probabilities for July 1990 are very close to the cut off threshold (0.5) for both the low and high volatility regimes (0.58 and 0.42), so that we decided to estimate the break point. On the other hand, the switch in the Federal funds and short-term rates lasted just for 5 and 6 months, respectively.

[14]The output gap has not been considered in the econometric work. It has been included in the figure simply to illustrate the relation between stock market volatility and the business cycle.

the beginning of the 1990s. As already noted in the literature, see the references mentioned in the Introduction, there is a close association between stock market volatility and business cycle dynamics, with stocks tending to become more volatile during recessions. Only in the middle of the 1980s and of the 1990s there was a simultaneous occurrence of a decreasing output gap and a high stock return volatility. From the plot it can be noted that the dating of the stock market volatility break process is very close to the NBER business cycle turning points.[15] This pattern has been stable during the 1970s, the 1980s and the beginning of the 1990s, while stock market volatility at the end of the 1990s behaved differently, since the switch to the high volatility regime took place when the US economy was still in a phase of expansion.

In summary it is striking that the five episodes of high stock market volatility which we find in the sample are all associated with episodes of high volatility of one or more macroeconomic variables. Even if the analysis is performed in terms of the original probabilities and not of the rounded probabilities, the result remains true for 5 episodes out of 6. The only one which is not explained by macroeconomic variables is the episode occurring at the end of the 1972, which lasted for a very short period of time. More precisely, the 1973–1975, the 1979–1983 and the 1990–1991 regimes (two oil shocks and a small economic recession not clearly associated with any exogenous shocks) are associated with high interest rate volatility and a declining level of economic activity. The 1986–1988 and the 1997–present episodes (the final part of an economic downturn and the last part of the longest economic expansion, the latter including the Russian and the LTCM crises) are associated with high volatility of money supply and a changing pattern of economic activity, which was going down and then up in 1986–1988 and was going up and then down in 1997–present. It could be that monetary policy was simply ineffective or not used widely during the first oil shock, due to the particular nature of the shock, while monetary policy caused Fed funds volatility in 1979–1983 and 1990–1991 and money growth volatility in 1986–1988 and 1997–present.

Clarida et al. (2000) have suggested that monetary policy has tended to be accommodative in the 1960s and 1970s, while since 1979 has tended to be more preemptive and aggressive towards controlling inflation, with real and nominal short-term interest rates increasing to curb expected inflationary developments in the economy. This may explain a smoother development in the Federal funds rate and its reduced volatility in the more recent period, and, conversely why money growth has tended to be more volatile. In particular, the period 1986–1988 encompasses the stock market crash of October 1987, while the period 1990–1991 may be associated with the oil price shock determined by the First Gulf War. The observed volatility movements are consistent with the reaction of monetary policy to the shocks, with interest rates being affected by the expected developments in inflation only in the latter period.

[15]The dating of the NBER business cycle turning points is as follows. Contractions: January 1970–November 1970; December 1973–March 1975; February 1980–July 1980; August 1981–November 1982; August 1990–March 1991. Expansions: December 1970–November 1973; April 1975–January 1980; August 1980–July 1981; December 1982–July 1990; April 1991–February 2001.

They are also coherent with the assessment of the "Great Moderation" provided by Stock and Watson (2002). The time varying standard deviations computed by these latter authors for the inflation and the 3-month Treasury bills rate point to a smooth decline in inflation volatility and to more abrupt changes in short-term rates volatility, with both series being relative quiescent in the last part of the sample. Only for the volatility of the short-term rate the evidence would point to a structural break process. However, according to Stock and Watson (2002) improved monetary policy would have been only one element contributing to the "Great Moderation", unusually quiescent macroeconomic shocks being mostly responsible for the bulk of the observed dynamics. Our analysis provides additional evidence for stock market returns volatility and money growth volatility. It is noteworthy that these variables show an increase in volatility since the mid-1990s. Regardless of the stability of most macroeconomic variables of the "Great Moderation" period, it is possible to see that money growth volatility has recently been high together with stock market volatility.

Moreover, what is extraordinary about the recent period is the long time spent in the high volatility regime. The stock market is in the high volatility regime since February 1997. It has spent almost five years in a high volatility regime. So there is something new about stock market volatility, which a simple model with a deterministic trend, like the one used by Campbell et al. (2001) cannot detect.

To measure the degree of association between stock market and macroeconomic volatility, Probit models were fitted on the rounded estimated smoothed probabilities.[16] In Table 1 we report some statistics on the predictive ability of the estimated Probit models for M1 growth and the Federal funds rate volatility, selecting a success cut off value equal to 0.5. As shown in the table, the predictive ability of the M1 growth volatility break processes is superior to that of the Federal funds rate. M1 growth volatility accurately predicts the stock market high volatility state 87% of the times and the stock market low volatility state 63% of the times. The percentage gain of moving from a constant probability model, i.e. using a constant as a regressor, to a switching model where the transition across states for stock market volatility is explained by M1 growth volatility is about 47%. Figures for the Federal funds rate are 52%, 74%, and 19%. The stock market volatility break process also has predictive ability for both the M1 growth and the Federal funds rate break processes, although causality seems to be stronger in the opposite direction. In fact, the figures for the percentages of correct predictions for the high and low volatility states are 62% and 66% for the Federal funds rate and 80% and 75% for the M1 growth, pointing to an increase in the ability to predict the high volatility state, but to a reduction in the ability of predicting the low volatility state. However, the percentage gain of moving from a constant probability model to a switching model where regime switching is explained by stock market volatility falls

[16]The rounded probabilities yield a dichotomous process, which enable the use of Probit models. In the estimated models the probabilities that the stock market volatility is in the low state is regressed on the probabilities that the macroeconomic factor volatility is in the same state. Then estimation is repeated inverting the role of the dependent and independent variables.

Table 1
Prediction evaluation

Panel A: M1 growth volatility

M1 → SM	*Dep* = 0	*Dep* = 1	Total	SM → M1	*Dep* = 0	*Dep* = 1	Total
Total	170	212	382		134	248	382
Correct	107	185	292		107	185	292
% Correct	62.94	87.26	76.44		79.85	74.60	76.44
% Incorrect	37.06	12.74	23.56		20.15	25.40	23.56
Total gain	62.94	−12.74	20.94		79.85	−25.40	11.52
% Gain	62.94	—	47.06		79.85	—	32.84

Panel B: Federal funds rate volatility

F → SM	*Dep* = 0	*Dep* = 1	Total	SM → F	*Dep* = 0	*Dep* = 1	Total
Total	170	212	382		143	239	382
Correct	88	157	245		88	157	245
% Correct	51.76	74.06	64.14		61.54	65.69	61.54
% Incorrect	48.24	25.94	35.86		38.46	34.31	35.86
Total gain	51.76	−25.94	8.64		61.54	−34.31	1.57
% Gain	51.76	—	19.41		61.54	—	4.20

The table reports statistics on the predictive ability of the M1 growth volatility (Panel A) and Federal funds rate volatility (Panel B) break processes for the stock market volatility break process (first three columns: M1 → SM; F → SM), and the predictive ability of the stock market volatility break process for M1 growth volatility (Panel A) and Federal funds rate volatility (Panel B) break processes (last three columns: SM → M1; SM → F). The success cut off has been set to 0.5. Total is the total number of zero ($Dep = 0$) and one ($Dep = 1$) entries in the dependent variables; Correct is the number of accurate predictions for the zero ($Dep = 0$) and one ($Dep = 1$) entries; % Correct and % Incorrect denote the percentages of accurate and inaccurate predictions, respectively; Total Gain denotes the difference between the percentages of observation correctly predicted by the model and a constant probability model, % Gain denotes Total Gain as a percentage of the numbers of predictive failures in the constant probability model.

to 4% and 32%, supporting a model where the direction of causality runs from macroeconomic volatility to stock market volatility.[17]

3.3. Long memory properties of break-free series

As a preliminary step to fractional cointegration analysis, the long memory properties of the break-free volatility processes need to be assessed. In order to control for the possible presence of noise we have employed the nonlinear log periodogram estimator proposed by Sun and Phillips (2003). While this estimator is

[17]It should be noted that over the sample considered there is evidence of instability in the structural relationship linking macroeconomic and stock market volatility, with money growth or interest rates volatility being more important in given historical periods. On the basis of this results we have not carried out a formal test for the existence of a common break process between stock market volatility and each macroeconomic factor.

less efficient than the standard log periodogram estimator (Geweke and Porter-Hudak, 1983; Robinson, 1995), in the case of perturbed long memory processes it has the advantage of not being affected by the downward bias which affects the other semiparametric estimators available in the literature.[18] In the paper we have also extended the estimator of Sun and Phillips (2003) to the multivariate case, in order to test for the equality of the fractional differencing operators for the various log variance processes and obtain a constrained estimate (see Appendix B).

In Table 2 we report the results of the long memory analysis for the break-free log variance series, including the tests for the presence of noise. First, there is evidence of noise for the inflation and output log variance processes, since for this two latter processes the estimated (inverse) long-run signal to noise ratio is significantly greater than zero (the point estimate is close to one in both cases). This result will be taken into account later when performing fractional cointegration analysis.

Second, the estimates provided by the nonlinear log periodogram estimator in the stable region (Taqqu and Teverovsky, 1998) are in the range 0.15–0.25 for the various processes, pointing to the presence of a moderate degree of long memory in the break-free log variance processes. Third, in the table we also report the results of the bivariate tests for the equality of the fractional differencing parameter for the various log variance processes, computed from the multivariate nonlinear log periodogram estimator, and the estimated restricted fractional differencing parameter in correspondence of the selected bandwidth. As is shown in the table, the null of equality of the fractional differencing parameters is never rejected, while the constrained estimates range in the interval 0.16–0.25. Imposing the restriction that all the processes show the same degree of long memory leads to a point estimate of the fractional differencing parameter equal to 0.22 (0.13). Overall, it can be concluded that there is evidence of a moderate degree of long memory once structural change is allowed for, which seems to be common for the series analyzed. These latter findings invite fractional cointegration analysis.[19]

3.4. Fractional cointegration analysis

Our information set is composed of five log variance processes characterized by the same degree of long memory. As for standard cointegration analysis, the number of cointegration relationships and the common long memory factors sum to the number of variables in the information set. In order to control for the observational noise detected in the output and inflation log variance processes, the noise filtering approach described in Appendix B has been implemented. We point to the preprint version of this paper for details about the performance of the proposed semiparametric approach to noise filtering. In short, Monte Carlo simulation

[18]The small sample properties of the nonlinear log periodogram estimator have been evaluated by means of a Monte Carlo exercise. The results suggest that, for sample sizes as the one employed in the paper, the fractional differencing parameter is unbiasedly estimated. Details are available upon request to the authors (see also Sun and Phillips, 2003).

[19]See the preprint version of this paper for additional results concerning the semiparametric estimation of the fractional differencing parameter.

Table 2
Semiparametric estimates, univariate and multivariate estimates

U	y_{bf}	π	f_{bf}	m_{bf}	sm_{bf}
d_{NLP}	0.20	0.25	0.16	0.15	0.23
	(0.09)	(0.09)	(0.15)	(0.52)	(0.14)
	[140]	[130]	[155]	[35]	[120]
β_{NLP}	0.99	1.03	0.00	0.00	0.00
	(0.30)	(0.28)	(1.08)	(5.88)	(0.81)

BM	π	f_{bf}	m_{bf}	sm_{bf}
y_{bf}	0.24	0.16	0.18	0.22
	(0.11)	(0.11)	(0.33)	(0.10)
	[0.58]	[0.60]	[0.80]	[0.64]
	[90]*	[135]*	[30]*	[110]*
π		0.18	0.19	0.25
		(0.15)	(0.12)	(0.08)
		[0.64]	[0.75]	[0.69]
		[105]*	[110]*	[125]*
f_{bf}			0.16	0.21
			(0.15)	(0.11)
			[0.91]	[0.86]
			[160]*	[100]*
m_{bf}				0.22
				(0.10)
				[0.81]
				[110]*
joint	0.22			
	(0.13)			
	[80]*			

The table reports the fractional differencing parameter (d) estimated from the univariate (U), bivariate and multivariate (BM) nonlinear log periodogram estimator, with standard errors in brackets, the p-value of the test for the equality of the fractional differencing parameter for each pair of variables in squared brackets, and the selected bandwidth in stared square brackets. "*joint*" denotes the constrained estimate from the multivariate nonlinear log periodogram estimator. β denotes the estimated inverse long-run signal to noise ratio. The series analyzed are the break-free log-variance processes for the Federal funds rate (f_{bf}), the S&P500 returns (sm_{bf}), the inflation rate (π), the rate of growth of industrial production (y_{bf}) and the rate of growth of M1 (m_{bf}).

strongly support the semiparametric filter which yields a similar performance to the standard parametric Wiener–Kolmogorov filter. According to Monte Carlo results, an optimal bandwidth equal to 80 ordinates has been selected for the computation of the optimal contemporaneous trimmed filter.[20]

[20]See the preprint version of the paper for additional details.

Table 3

Panel A: Fractional cointegration tests

$h=3$	*eig*	*pv*		1%	5%	10%
ev_1	0.000	0.000	$rank=1$	0.000	0.000	0.000
ev_2	0.001	0.001	$rank=2$	0.000	0.000	0.000
ev_3	0.042	0.069	$rank=3$	0.138	0.118	0.107
ev_4	0.164	0.271	$rank=4$	0.646	0.557	0.509
ev_5	0.400	0.659				

Panel B: Estimated eigenvectors

	$\boldsymbol{\beta}_1$	$\boldsymbol{\beta}_2$	$\boldsymbol{\beta}_3$	$\boldsymbol{\Theta}_1$	$\boldsymbol{\Theta}_2$
y_{bf}	0	0	1	0.706	0
	(–)	(–)	(–)	(0.012)	(–)
π	−0.233	−0.179	0	0.216	1.121
	(0.009)	(0.013)	(–)	(0.019)	(0.011)
f_{bf}	−0.681	−0.802	−0.753	0.557	0
	(0.015)	(0.024)	(0.024)	(0.020)	(–)
m_{bf}	1	0	0	0.485	0.200
	(–)	(–)	(–)	(0.017)	(0.013)
sm_{bf}	0	1	0	0.429	0.261
	(–)	(–)	(–)	(0.011)	(0.009)

Panel A reports the Robinson and Yajima (2002) cointegrating rank test for the full set of variables. *eig* denotes the estimated eigenvalues (ev_i, $i=1,\ldots,5$), *pv* the proportion of explained variance, $rank=i$, $i=1,\ldots,4$, denotes the corresponding test (at significance levels 1%, 5%, 10%), and h is the number of periodogram ordinates used in the computation of the test. Panel B reports the identified cointegrating vectors (first three columns) and the identified factor loading matrix (last two columns). Standard errors have been computed using the jack-knife. The series analyzed are the break-free log-variance processes for the Federal funds rate (f_{bf}), the S&P500 returns (sm_{bf}), the inflation rate (π), the rate of growth of industrial production (y_{bf}) and the rate of growth of M1 (m_{bf}).

In Table 3 Panel A we report the results of the Robinson and Yajima (2002) fractional cointegrating rank test, computed setting the bandwidth equal to three periodogram ordinates. As shown in the table, the number of cointegration relationships is equal to two when the threshold $0.1 \times r/n$ is employed. Yet the bulk of the variability of the series appears to be explained by the two largest eigenvalues, accounting together for a proportion of variance of about 93%, pointing to three cointegration relationships and two common persistent factors. Since the selection of the threshold in the Robinson and Yajima (2002) test is arbitrary, we have therefore selected three cointegrating vectors.[21]

[21]This is also in the light of the Monte Carlo results of Morana (2004a), which point to downward bias in the estimated nonzero eigenvalues.

The identified cointegrating vectors (Table 3) suggest that stock market volatility and money growth volatility may be related to the Federal funds rate and inflation volatility, while a bivariate relationship relates the Federal funds rate volatility to output volatility.[22] Although for the stock market and money growth volatility processes the squared multiple coherence is lower than the expected unitary value[23] (0.74 and 0.81, respectively), the exclusion restrictions only lead to a marginal reduction in the proportion of explained power at the zero frequency (0.70 and 0.74, respectively), supporting the identifying constraints. The fact that a similar structure has been found for stock market and money growth volatility is also coherent with the relatively high squared simple coherence at the zero frequency for this two latter series (about 0.65). Also the squared simple coherence at the zero frequency for output and the Federal funds rate volatility is lower than the expected unitary value (about 0.60). However, also for this latter series the reduction in the multiple squared coherence due to the exclusion of the inflation rate log variance process from the cointegrating vector, conditional to the exclusion of the stock market and money growth log variance processes, is negligible (about 0.01). The persistence analysis of the cointegrating errors also supports the identifying restrictions imposed on the cointegration space, since the estimated fractional differencing parameters are significantly lower than the common estimated value 0.22.[24]

3.4.1. *Common factors analysis*

The factor model has been identified by imposing the required additional restriction that output volatility is affected only by the first factor. Given the structure of the cointegration space, this implies that the second factor does not affect the Federal funds rate volatility process. Support for the identifying restrictions imposed on the cointegration space and the factor loading matrix is provided by the analysis of the persistent factors. In fact, the correlation coefficient between the factors obtained from the unconstrained and constrained models is about 0.99 and 0.98 for the first and second factors, respectively.

Table 4, Panel A shows that the interpretation of the factors is clear-cut.[25] The variance of the first factor is largely explained by output growth volatility (50% and 60% are, respectively, the lower and upper bound described at the end of the second section), with the Federal funds rate (the bounds are 20–32%), money growth

[22]The small sample properties of the frequency domain principle components estimator have been evaluated by means of a Monte Carlo exercise. The results suggest that the estimator has good properties also with moderate sample sizes, as the one employed in the paper. Details are available upon request to the authors (see also Morana, 2004a).

[23]See Morana (2004b).

[24]The estimates tend to vary depending on the estimator employed and the selected bandwidth, being in the range -0.028 to 0.11 in correspondence of the optimally selected bandwidths. See the preprint version of the paper for details.

[25]The decompositions reported in Table 4, Panels A, B, and C has also been performed using the innovations as in Baillie et al. (2002), rather than the actual processes. The results are numerically close and are available upon request from the authors.

Table 4

Panel A: Factors, proportion of explained variance

	y_{bf}	π	f_{bf}	m_{bf}	sm_{bf}
$\mu_{1,\mathrm{U}}$	0.589	0.031	0.318	0.177	0.075
$\mu_{1,\mathrm{L}}$	0.492	0.020	0.201	0.087	0.034
$\mu_{2,\mathrm{U}}$	0.071	0.922	0.053	0.030	0.002
$\mu_{2,\mathrm{L}}$	0.052	0.865	0.018	0.003	0.000

Panel B: Persistent components, proportion of explained variance

	y_{bf}	π	f_{bf}	m_{bf}	sm_{bf}
$p_{y,\mathrm{U}}$	0.586	0.029	0.318	0.177	0.075
$p_{y,\mathrm{L}}$	0.492	0.020	0.201	0.087	0.034
$p_{\pi,\mathrm{U}}$	0.016	0.967	0.010	0.060	0.008
$p_{\pi,\mathrm{L}}$	0.007	0.911	0.001	0.020	0.002
$p_{f,\mathrm{U}}$	0.586	0.029	0.318	0.177	0.075
$p_{f,\mathrm{L}}$	0.492	0.020	0.201	0.087	0.034
$p_{m,\mathrm{U}}$	0.416	0.218	0.208	0.271	0.093
$p_{m,\mathrm{L}}$	0.342	0.198	0.136	0.128	0.042
$p_{sm,\mathrm{U}}$	0.504	0.088	0.260	0.263	0.095
$p_{sm,\mathrm{L}}$	0.419	0.065	0.171	0.126	0.044

Panel C: Nonpersistent components, proportion of explained variance

	y_{bf}	π	f_{bf}	m_{bf}	sm_{bf}
$u_{y,\mathrm{U}}$	0.666	0.022	0.255	0.103	0.047
$u_{y,\mathrm{L}}$	0.575	0.015	0.184	0.090	0.037
$u_{\pi,\mathrm{U}}$	0.259	0.093	0.106	0.635	0.086
$u_{\pi,\mathrm{L}}$	0.197	0.037	0.036	0.464	0.071
$u_{f,\mathrm{U}}$	0.264	0.002	0.768	0.047	0.018
$u_{f,\mathrm{L}}$	0.212	0.001	0.665	0.007	0.008
$u_{m,\mathrm{U}}$	0.078	0.043	0.031	0.939	0.011
$u_{m,\mathrm{L}}$	0.046	0.011	0.001	0.841	0.002
$u_{sm,\mathrm{U}}$	0.286	0.047	0.134	0.144	0.556
$u_{sm,\mathrm{L}}$	0.232	0.034	0.080	0.051	0.454

Panels A, B and C report the contribution of each variable (upper (U) and lower bounds (L)) to the explanation of the variance of the persistent factors (μ_1, μ_2), the variance of the persistent components (p_i, $i = y, \pi, f, m, sm$), and the variance of the nonpersistent components (u_i, $i = y, \pi, f, m, sm$), respectively. The series analyzed are the break-free log-variance processes for the Federal funds rate (f_{bf}), the S&P500 returns (sm_{bf}), the inflation rate (π), the rate of growth of industrial production (y_{bf}) and the rate of growth of M1 (m_{bf}).

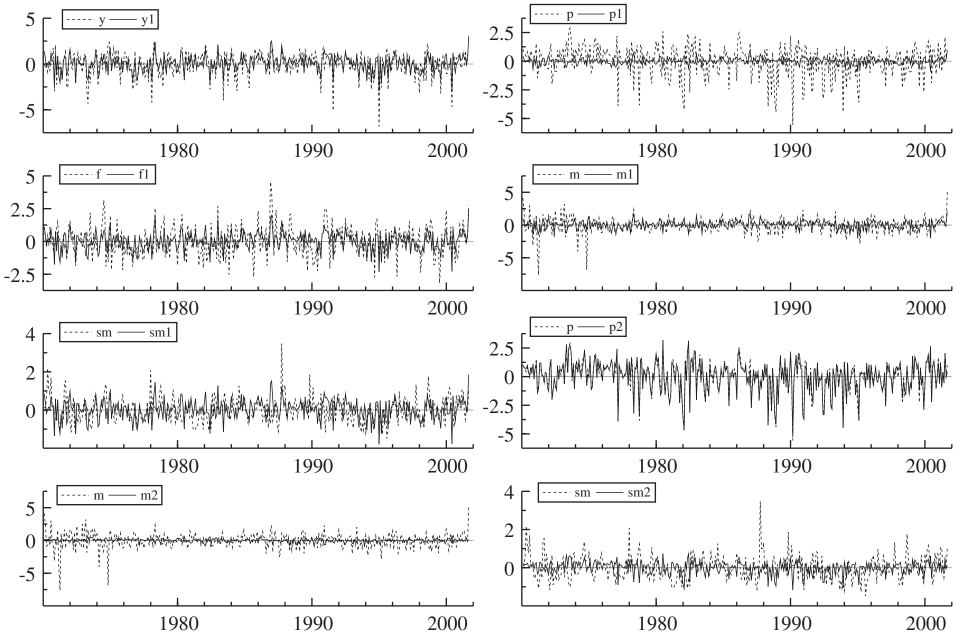

Fig. 3. Historical decomposition for break-free log variance series: persistent factors (1, 2) (output (y), inflation (p), Federal funds rate (f), money growth (m), stock market (sm)).

(9–18%) and stock market volatility (3–8%) yielding a smaller contribution. According to the historical decomposition reported in Fig. 3, this factor is the driving force of the persistent volatility components for stock market returns and the core macroeconomic variables, apart from inflation.

On the other hand, the variance of the second factor is largely explained by inflation volatility (86–93%). While the first persistent shock exercises a positive impact on all the variables, the second persistent shock does not affect output and the Federal funds rate volatility (by construction), leading to an increase in money growth and stock market volatility which is smaller in magnitude than the one determined by the first factor. According to the historical decomposition, the second persistent factor largely explains inflation volatility, showing also a nonnegligible contribution to the explanation of stock market volatility.

From Table 4, Panel B, it is possible to note that, coherent with the contribution of each variable to the factors, the persistent volatility dynamics may be associated with the own variables for inflation and output growth, with output growth volatility yielding an important contribution to the explanation of the variance of the persistent volatility dynamics of the Federal funds rate (50–59%), money growth (34–42%) and stock market returns (42–50%), and inflation providing a nonnegligible contribution to the explanation of persistent money growth volatility (20–22%). The Federal funds rate and money growth volatility also contribute to the explanation of persistent output growth (20–32%; 9–18%) and stock market (17–26%; 13–26%) volatility, in addition to provide a nonnegligible contribution to the explanation of the own (20–30%; 13–27%) and cross (14–21%; 9–18%) dynamics. On the other hand, the contribution of stock market volatility to the explanation of the persistent macroeconomic volatility dynamics, apart from inflation, is smaller in magnitude (3–9%).[26]

By computing the total differential for each log variance process from the factor model it is possible to disentangle the relative importance of the nonpersistent and persistent volatility dynamics. For the generic process x_i, we have $\mathrm{d}x_{i,t} = a_i\,\mathrm{d}x_{i,t} + (1-a_i)\,\mathrm{d}x_{i,t}$, where a_i is the corresponding entry in the matrix $\boldsymbol{\Theta}(\boldsymbol{\Theta}'\boldsymbol{\Theta})^{-1}\boldsymbol{\Theta}'$ and $(1-a_i)$ in the matrix $\boldsymbol{\beta}(\boldsymbol{\beta}'\boldsymbol{\beta})^{-1}\boldsymbol{\beta}'$. The results suggest that a 1% stock market volatility increase is determined by a 0.85% increase in the nonpersistent component and a 0.15% increase in the persistent component. For output, inflation, the Federal funds rate, and money growth the figures are 0.56 and 0.44, 0.04 and 0.96, 0.72 and 0.28, 0.82 and 0.18, for the nonpersistent and persistent components, respectively. Therefore, the findings suggest that the contribution of the nonpersistent dynamics to the overall volatility dynamics is not negligible. The latter may be interpreted in terms of disequilibrium dynamics, since they are estimated as linear combinations of the cointegrating errors.

From Table 4, Panel C it can be noted that the nonpersistent dynamics are largely idiosyncratic for output growth (58–67%), the Federal funds rate (67–77%), money growth (84–94%) and stock market (45–56%) volatility. Output growth volatility seems to matter also for inflation, the Federal funds rate and stock market volatility (20–29%), while the Federal funds rate volatility seems to matter also for output growth volatility (19–26%), and money growth volatility for inflation volatility (46–64%). The contribution to the explanation of the stock market nonpersistent dynamics of these two latter processes is smaller (5–14%). Finally, stock market volatility yields a limited contribution to the explanation of the nonpersistent macroeconomic volatility dynamics, in particular for inflation (7–9%) and output volatility (4–5%).

4. Conclusions

Why does stock market volatility change over time? We have tried to answer this question with up-to-date econometric models. The linkages between macroeconomic and stock market volatility are hard to measure without proper econometric models, and in fact are virtually absent from our sample, neglecting structural breaks and long memory dynamics. However, an accurate modelling of the statistical properties of the series allows several links to emerge. Discrete changes in monetary policy, affecting the volatilities of interest rates and money growth, seem to be the best

[26]Our results are coherent with evidence provided in Engle and Rodrigues (1989), Flavin and Wickens (2003) and Sadorsky (2003), where M1 growth volatility, inflation volatility, oil price volatility and term premium volatility have been found to significantly affect stock market volatility.

candidate to account for breaks in the volatility of stock returns and therefore to explain the level and discrete jumps in volatility. From this perspective the prolonged period of high stock market volatility, going from 1997 to the end of 2001, peculiarly occurring during a phase of an economic expansion, can be associated with an increase in money growth volatility.

After accounting for the structural breaks, there remain interesting relations among the long memory series. The fractional cointegration analysis points to three long-run relationships linking stock market, money growth, inflation, the Federal funds rate, and output growth volatility, with money growth and stock market volatility being related to the Federal funds rate and inflation volatility, and the Federal funds rate volatility being related to output growth volatility. The two common long memory factors driving the five processes are mainly associated with output growth and inflation volatility. Our decomposition suggests that a 1% stock market volatility increase is determined by a 0.85% increase in the nonpersistent component and a 0.15% in the persistent component. We find that output growth volatility contributes to both persistent and nonpersistent stock market volatility, albeit the latter can be mainly associated with an idiosyncratic financial shock.

Stock market volatility also plays a role, yet limited, in the explanation of macroeconomic volatility dynamics, particularly output and inflation volatility. We partially confirm Schwert's original findings. There seem to be bidirectional links between volatilities in the stock market and in macroeconomic variables, although we find stronger evidence of causality running from macroeconomic to stock market volatility than the other way around.

It is noticeable that we are able to obtain these results even in a sample period characterized by weak overall relations among volatility of the variables; we attribute this achievement to the use of a multi-component econometric model which acknowledges structural breaks and estimates dynamics with different persistence characteristics. We also believe there are economic reasons for considering different components. While volatility of the output of the economy, that we may interpret as uncertainty about growth, moves slowly over time, uncertainty about policy variables may well shift suddenly, reflecting expectations of innovations on the part of policy-makers. Stock markets incorporate such uncertainty by means of the relation between expectations of fundamentals and stock pricing. It is therefore plausible that volatility of the stock market can be separated into different components and that such components are differently affected by the volatility of macroeconomic and policy variables themselves characterized by breaks, more persistent and less persistent dynamics.

Acknowledgements

The authors are grateful to the editor F.X. Diebold and three anonymous referees for constructive comments. C. Morana gratefully acknowledges funding from the "Ricerca d'Eccellenza" project financed by Piedmont Region no. 21302CARPSE.

Appendix A

Let us assume the following common long memory factor model

$$\mathbf{x}_t = \mathbf{\Theta}\boldsymbol{\mu}_t + \mathbf{u}_t,$$
$$\Delta^d \boldsymbol{\mu}_t = \boldsymbol{\varepsilon}_t, \tag{3}$$

where $\mathbf{x}_t$ is a $p \times 1$ vector of observations on the p fractionally cointegrated processes, $\mathbf{\Theta}$ is the $p \times k$ factor loading matrix with $k<p$, $\boldsymbol{\mu}_t$ is a $k \times 1$ vector of unobserved long memory factors ($I(d)$ $0<d<0.5$), $\boldsymbol{\varepsilon}_t \sim i.i.d.(\mathbf{0}, \mathbf{\Sigma}_\varepsilon)$ with dimension $k \times 1$ and $\mathbf{\Sigma}_\varepsilon = \mathbf{I}_k$, $\mathbf{u}_t$ is a $p \times 1$ vector of unobserved weakly dependent components ($I(0)$), with $\mathbf{\Phi}(L)\mathbf{u}_t = \mathbf{\Omega}(L)\mathbf{v}_t$, all the roots of the polynomial matrices in the lag operator $\mathbf{\Phi}(L)$ and $\mathbf{\Omega}(L)$ are outside the unit circle, $\mathbf{\Phi}(0) = \mathbf{\Omega}(0) = \mathbf{I}_p$, and $\mathbf{v}_t \sim i.i.d.(\mathbf{0}, \mathbf{\Sigma}_v)$ with dimension $p \times 1$.

Following Morana (2004a), applying fractional differencing yields

$$\Delta^d \mathbf{x}_t = \mathbf{\Theta}\boldsymbol{\varepsilon}_t + \Delta^d \mathbf{u}_t \tag{4}$$

and the associated spectral matrix

$$\mathbf{f}(\omega) = \mathbf{\Theta}\mathbf{f}_\varepsilon(\omega)\mathbf{\Theta}' + \mathbf{\Theta}\mathbf{f}_{\varepsilon,\Delta^d u'}(\omega) + \mathbf{f}_{\Delta^d u,\varepsilon'}(\omega)\mathbf{\Theta}' + \mathbf{f}_{\Delta^d u}(\omega), \tag{5}$$

where the $\mathbf{f}_i(\omega)$ matrices contain the spectral and cross spectral functions for the given vectors, evaluated at the frequency $\omega_i = 2\pi i/T$ and T is the sample size. Evaluation at the zero frequency ($\omega_0 = 0$) yields

$$\mathbf{f}(0) = \frac{1}{2\pi}\mathbf{\Theta}\mathbf{\Theta}', \tag{6}$$

since $\mathbf{f}_{\varepsilon,\Delta^d u'}(0) = \mathbf{0}, \mathbf{f}_{\Delta^d u,\varepsilon'}(0) = \mathbf{0}, \mathbf{f}_{\Delta^d u}(0) = \mathbf{0}$.[27]

Since $\mathbf{\Theta}\mathbf{\Theta}'$ is of reduced rank $k<p$, also $\mathbf{f}(0)$ will be of reduced rank equal to k. Moreover, from $\mathbf{f}(0) = (1/2\pi)\mathbf{\Theta}\mathbf{\Theta}'$ it follows that the matrix $\mathbf{\Theta}\mathbf{\Theta}'$ is identified, while the identification of $\mathbf{\Theta}$, given the assumption of orthogonality of the factors, requires the imposition of $k(k-1)/2$ equality restrictions on $\mathbf{\Theta}$ (see Anderson, 1984).

From the symmetry property, it follows that the spectral matrix can be factorized as

$$2\pi\mathbf{f}(0) = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}', \tag{7}$$

where $\mathbf{\Lambda}$ is the $k \times k$ diagonal matrix of (real) nonzero eigenvalues of $2\pi\mathbf{f}(0)$ ordered in descending order and the matrix $\mathbf{Q}$ is the $p \times k$ matrix of the associated orthogonal eigenvectors. By writing $\mathbf{Q}^* = \mathbf{Q}\mathbf{\Lambda}^{1/2}$, we then have

$$2\pi\mathbf{f}(0) = \mathbf{Q}^*\mathbf{Q}^{*\prime}. \tag{8}$$

[27] $\mathbf{f}_{\Delta^d u}(0) = \mathbf{0}$ follows from the fact that the $\mathbf{u}_t$ vector is I(0), so that applying the fractional differencing filter leads to an overdifferenced vector process with null spectral matrix at the zero frequency. $\mathbf{f}_{\varepsilon,\Delta^d u'}(0) = \mathbf{0}$, $\mathbf{f}_{\Delta^d u,\varepsilon'}(0) = \mathbf{0}$ follow from the above argument and the $\boldsymbol{\varepsilon}_t$ having a finite spectrum at the zero frequency. Moreover, the *i.i.d.* assumption implies $\mathbf{f}_\varepsilon(\omega) = (1/2\pi)\mathbf{\Sigma}_\varepsilon$ at all frequencies. Since $\mathbf{\Sigma}_\varepsilon$ is orthonormal we then have $\mathbf{f}_\varepsilon(\omega) = (1/2\pi)\mathbf{I}_k$. See properties 1–3 in Section 3 in Morana (2004a). Note that the same results hold for the case in which the $\mathbf{u}$ vector is $I(b)$ $b>0$, $d-b>0$, since $\Delta^d\mathbf{u} \sim \mathrm{I(b-d)}$.

The matrix $\hat{\mathbf{Q}}^*$, obtained from the largest eigenvalues of $2\pi\hat{\mathbf{f}}(0)$ and the associated eigenvectors, is therefore our estimator of the factor loading matrix $\mathbf{\Theta}$.

As performed, estimation will enforce orthogonality of the factors. Estimation subject to additional $k(k-1)/2$ zero identification restrictions can be carried out as follows (Warne, 1993). Let us write the factor loading matrix as

$$\mathbf{Q}^* = \mathbf{Q}_0^* \boldsymbol{\rho}, \tag{9}$$

where $\boldsymbol{\rho}$ is a $k \times k$ matrix, $\mathbf{Q}_0^* = \mathbf{Q}^*(\mathbf{Q}_{k,k}^*)^{-1}$, and $\mathbf{Q}_{k,k}^*$ is the square matrix of order k composed of the first k rows and columns of the matrix $\mathbf{Q}^*$. Hence, the upper square submatrix of order k of $\mathbf{Q}_0^*$ is the identity matrix. From the relationship $2\pi\mathbf{f}(0) = \mathbf{Q}^*\mathbf{Q}^{*\prime}$ and $\mathbf{Q}^* = \mathbf{Q}_0^*\boldsymbol{\rho}$ we then have

$$\boldsymbol{\rho}\boldsymbol{\rho}' = (\mathbf{Q}_0'^{*}\mathbf{Q}_0^*)^{-1}\mathbf{Q}_0^*(2\pi\mathbf{f}(0))\mathbf{Q}_0^*(\mathbf{Q}_0'^{*}\mathbf{Q}_0^*)^{-1}. \tag{10}$$

The matrix $\boldsymbol{\rho}\boldsymbol{\rho}'$ is positive definite and symmetric, containing $k(k+1)/2$ distinct parameters which can be estimated through its Choleski decomposition, leading to a lower triangular $\boldsymbol{\rho}$. Hence, after estimation of $\mathbf{Q}^*$ and $\mathbf{Q}_0^*$, $\hat{\boldsymbol{\rho}}$ can be obtained from the Choleski decomposition of $(\hat{\mathbf{Q}}_0'^{*}\hat{\mathbf{Q}}_0^*)^{-1}\hat{\mathbf{Q}}_0^*(2\pi\hat{\mathbf{f}}(0))\hat{\mathbf{Q}}_0^*(\hat{\mathbf{Q}}_0'^{*}\hat{\mathbf{Q}}_0^*)^{-1}$, and the estimated factor loading matrix can then be written as

$$\hat{\mathbf{Q}}^* = \hat{\mathbf{Q}}_0^*\hat{\boldsymbol{\rho}}. \tag{11}$$

Following the above described procedure $k(k-1)/2$ zero restrictions will be imposed on the elements i,j, $i = 1,\ldots,k$, $j = i+1,\ldots,k$ of the matrix $\hat{\mathbf{Q}}^*$. The factor loading matrix can also be rotated to add further interpretability to the results.

A.1. Estimation of the cointegration space

Given the orthogonality property of the eigenvectors, it follows that

$$\mathbf{Q}_{1,\ldots,k}'\mathbf{Q}_{k+1,\ldots,p} = \underset{k\times(p-k)}{\mathbf{0}}, \tag{12}$$

where $\mathbf{Q}_{1,\ldots,k}$ and $\mathbf{Q}_{k+1,\ldots,p}$ denote the submatrices composed of the k eigenvectors associated with the first k largest roots, and the last $r = p-k$ eigenvectors associated with the zero roots, respectively. Hence $\mathbf{Q}_{k+1,\ldots,p}$ is a right null space basis of the factor loading matrix, which is the definition of the cointegration space, since the cointegration relationships are the linear combinations of the variables which remove the persistent ($I(d)$) component from them. We can write therefore $\boldsymbol{\beta} = \mathbf{Q}_{k+1,\ldots,p}$, where $\boldsymbol{\beta}$ denote the $p \times r$ cointegration matrix, obtaining

$$\boldsymbol{\beta}'\mathbf{Q}^* = \boldsymbol{\beta}'\mathbf{\Theta} = \underset{r\times k}{\mathbf{0}}.$$

The matrix $\hat{\mathbf{Q}}_{k+1,\ldots,p}$, obtained from the eigenvectors associated to the smallest eigenvalues of $2\pi\hat{\mathbf{f}}(0)$, is therefore our estimator of the cointegration space. See Morana (2004a) for additional results and asymptotic properties.

Appendix B

B.1. A semiparametric approach to noise filtering

Following Sun and Phillips (2003), consider the perturbed long memory univariate process:

$$x_t = \mu_t + u_t,$$

$$\Delta^d \mu_t = \varepsilon_t, \tag{13}$$

$0<d<0.5$, $\varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$ and $u_t \sim NID(0, \sigma_u^2)$. Its spectrum can then be written as

$$f_x(\omega_i) = \left(2\sin\frac{\omega_i}{2}\right)^{-2d} f^*(\omega_i), \tag{14}$$

where $\omega_i = 2\pi i/T$ denotes the frequency in radians and T is the sample size. By taking logs:

$$\ln f_x(\omega_i) = -2d \ln \omega_i + \ln f^*(\omega_i) - 2d \ln\left(2\omega_i^{-1}\sin\left(\frac{\omega_i}{2}\right)\right). \tag{15}$$

By writing

$$\ln f^*(\omega_i) = \ln f_\varepsilon(\omega_i) + \frac{f_u(\omega_0)}{f_\varepsilon(\omega_0)}\omega_i^{2d} + O(\omega_i^{4d})$$

and replacing $f_x(\omega_i)$ with the periodogram $I_x(\omega_i)$, one has then the nonlinear log periodogram regression

$$\ln I_x(\omega_i) = \alpha - 2d \ln \omega_i + \beta\omega_i^{2d} + w_x(\omega_i), \quad \omega_j \to 0^+, \tag{16}$$

where $w_x(\omega_i)$ is a disturbance term, and α, β, and d are the intercept, the long-run (inverse) signal to noise ratio, and the fractional differencing parameter, respectively. In particular, $\alpha = \ln f_\varepsilon(\omega_0) - c$ ($c = 0.577216\ldots$ is the Euler constant), $\beta = f_u(\omega_0)/f_\varepsilon(\omega_0)$, where $f_\varepsilon(\omega_0)$ and $f_u(\omega_0)$ are the spectral matrix at the zero frequency of the signal and noise components of the process x, respectively, and $w_x(\omega_i) = \ln f^*(\omega_i) - \ln f_\varepsilon(\omega_0) - \beta\omega_i^{2d} - 2d[\ln(2\sin\omega_i/2) - \ln \omega_i]$. The estimator proposed by Sun and Phillips (2003) is the minimizer of the averaged squared errors, requiring the minimization of the objective function

$$Q(d,\beta) = \frac{1}{m}\sum_{l=1}^{m} w_{\omega_l}^2, \tag{17}$$

$$w_{\omega_l} = \left(\ln I_{\omega_l} - \frac{1}{m}\sum_{k=1}^{m}\ln I_{\omega_k}\right) + 2d\left(\ln \omega_l - \frac{1}{m}\sum_{k=1}^{m}\ln \omega_k\right) - \beta\left(\omega_l^{2d} - \frac{1}{m}\sum_{k=1}^{m}\omega_k^{2d}\right).$$

Sun and Phillips (2003) have proved the consistency and asymptotic normality of the estimator. From the nonlinear log periodogram regression a semiparametric noise filtering approach can be easily implemented. By writing the noise corrected log periodogram for the generic jth process as $\ln I_j^c(\omega_i) = \ln I_j(\omega_i) - \hat{\beta}_j \omega_i^{2d_j}$, it is possible to recover an estimate of the periodogram for the unperturbed long memory process as $I_j^c(\omega_i) = \exp(\ln I_j^c(\omega_i))$. Similarly to the Wiener–Kolmogorov approach, two sided time domain weights to filter the long memory signal from the observed process can be computed from the inverse Fourier transform of the (semiparametric) transfer function

$$h_\mu(\omega_i) = \frac{I_j^c(\omega_i)}{I_j(\omega_i)}. \tag{18}$$

A modified version of the semiparametric filter can be computed by trimming the lowest frequency, with bandwidth determined through Monte Carlo simulation. The modified semiparametric filter is computed as follows:

$$\ln I_j^c(\omega_i) = \ln I_j(\omega_i) - \hat{\gamma}_j \omega_i^{2d_i},$$

$$\hat{\gamma}_j = 0 \quad 0 < \omega_i \leqslant \frac{2\pi m^*}{T},$$

$$\hat{\gamma}_j = \hat{\beta}_j \quad \omega_i > \frac{2\pi m^*}{T},$$

i.e. by not filtering out the lowest frequencies in the computation of the term $\ln I_i^c(\omega)$.

B.2. The multivariate nonlinear log periodogram estimator

When p perturbed long memory processes are available, a multivariate generalization of the estimator of Sun and Phillips (2003) can be implemented in a seemingly unrelated nonlinear log periodogram framework, similarly to the extension provided by Robinson (1995) for the linear log periodogram estimator. We would then have

$$\begin{aligned} \ln I_1(\omega_i) &= \alpha_1 - 2d_1 \ln \omega_i + \beta_1 \omega_i^{2d_1} + w(\omega_i)_1, \\ &\vdots \\ \ln I_p(\omega_i) &= \alpha_p - 2d_p \ln \omega_i + \beta_p \omega_i^{2d_p} + w(\omega_i)_p, \end{aligned} \tag{19}$$

The multivariate model can be estimated by means of a GLS approach, where the objective function to be minimized, concentrated with respect to the intercept vector, can be written as

$$Q(\mathbf{d}, \boldsymbol{\beta}) = \sum_{i=1}^{p} \sum_{j=1}^{p} \sigma^{ij} \mathbf{w}_i' \mathbf{w}_j, \tag{20}$$

where $\mathbf{w}_s$, $s = i,j = 1, \ldots, p$ is a $m \times 1$ vector of residuals, with generic element

$$\mathbf{w}_{s,\omega_l} = \left(\ln I_{s,\omega_l} - \frac{1}{m}\sum_{k=1}^{m} \ln I_{s,\omega_k}\right) + 2d_s\left(\ln \omega_l - \frac{1}{m}\sum_{k=1}^{m} \ln \omega_k\right) - \beta_s\left(\omega_l^{2d_s} - \frac{1}{m}\sum_{k=1}^{m} \omega_k^{2d_s}\right),$$

where m denotes the bandwidth employed for estimation. Finally, σ^{ij} denotes the i,j elements of the contemporaneous variance covariance matrix $\mathbf{\Sigma}$, i.e. $\sigma^{ij} = E[w_{i,\omega_l} w'_{j,\omega_l}]$. Since the $\mathbf{\Sigma}$ matrix is not known, a two-step procedure can be followed to obtain efficient estimates of the parameters. In the first step univariate estimation is performed on each equation separately by means of the estimator proposed by Sun and Phillips (2003), obtaining an estimate of the residuals vectors $\hat{\mathbf{w}}_s$, $s = 1, \ldots, p$, which can be employed to compute a consistent estimate of the elements σ^{ij}, $i,j = 1, \ldots, p$ as $\hat{\sigma}^{ij} = \hat{\mathbf{w}}_i'\hat{\mathbf{w}}_j/m$. This yields the feasible GLS estimator, which requires the minimization of the function

$$Q(\mathbf{d}, \boldsymbol{\beta}) = \sum_{i=1}^{p}\sum_{j=1}^{p} \hat{\sigma}^{ij}\mathbf{w}_i'\mathbf{w}_j. \tag{21}$$

Asymptotic standard errors can be computed as the square root of the diagonal elements of the matrix

$$AsyVar[\hat{\mathbf{d}}, \hat{\boldsymbol{\beta}}] = \left[\sum_{i=1}^{p}\sum_{j=1}^{p} \hat{\sigma}^{ij}\mathbf{h}_i(\mathbf{d}, \boldsymbol{\beta})'\mathbf{h}_j(\mathbf{d}, \boldsymbol{\beta})\right]^{-1}, \tag{22}$$

where $\mathbf{h}_s(\mathbf{d}, \boldsymbol{\beta})$, $s = i,j = 1, \ldots, p$ is an $m \times 2p$ matrix of pseudoregressors obtained as the derivatives of the function $\mathbf{Z}_s(\mathbf{d}, \boldsymbol{\beta})$ in the compact formulation of the model in deviations from the mean

$$\ln \tilde{\mathbf{I}}_s = \tilde{\mathbf{Z}}_s(\mathbf{d}, \boldsymbol{\beta}) + \mathbf{w}_s. \tag{23}$$

Since only the parameter d_s and β_s enter in the generic sth equation, the matrix $\mathbf{h}_s$ will contain $2p - 2$ zero columns, corresponding to the omitted parameters. We then have

$$\mathbf{h}_{s,\omega_l,d_s} = -2\left[(1 - \beta_s(\omega_l^{d_s})^2)\ln \omega_l - \frac{1}{m}\sum_{k=1}^{m}(1 - \beta_s(\omega_k^{d_s})^2)\ln \omega_k\right], \tag{24}$$

$$\mathbf{h}_{s,\omega_l,\beta_s} = \omega_l^{2d_s} - \frac{1}{m}\sum_{k=1}^{m} \omega_k^{2d_s} \tag{25}$$

with $\mathbf{h}_{s,\omega_l,d_s}$ denoting the generic element (frequency ω_l) of the pseudoregressor vector obtained by differentiating the $\mathbf{Z}_s(\mathbf{d}, \boldsymbol{\beta})$ function with respect to d_s, and $\mathbf{h}_{s,\omega_l,\beta_s}$ the generic element (frequency ω_l) of the pseudoregressor vector obtained by differentiating the $\mathbf{Z}_s(\mathbf{d}, \boldsymbol{\beta})$ function with respect to β_s.

Linear restrictions can be easily tested in our framework. Of particular interest are restrictions which involve the equality of the fractional differencing parameter for two or more processes. Assuming the following ordering for the vector of parameters $(\mathbf{d}', \boldsymbol{\beta}')$, with $\mathbf{d} = (d_1, \ldots, d_p)'$ and $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_p)'$, let us consider the case of homogeneous restrictions

$$\mathrm{H}_0 : \mathbf{Rd} = \mathbf{0}, \tag{26}$$

where $\mathbf{R}$ is an $h \times p$ matrix of rank equal to $h < p$. Following Robinson (1995), the test statistic is

$$\hat{\mathbf{d}}'\mathbf{R}' \left[(\mathbf{R}, \mathbf{0}) \left[\sum_{i=1}^{p} \sum_{j=1}^{p} \hat{\sigma}^{ij} \mathbf{h}_i(\mathbf{d}, \boldsymbol{\beta})' \mathbf{h}_j(\mathbf{d}, \boldsymbol{\beta}) \right]^{-1} (\mathbf{R}, \mathbf{0})' \right] \mathbf{R}\hat{\mathbf{d}} \sim \chi^2_{(h)}, \tag{27}$$

where $\mathbf{0}$ is a null matrix with dimension $h \times p$.

If the hypothesis under the null cannot be rejected, the restricted model can be estimated with gains in terms of efficiency. For instance, a model with equal fractional differencing parameter for the various processes can be easily estimated from the constrained model

$$\begin{aligned} \ln I_1(\omega_i) &= \alpha_1 - 2d \ln \omega_i + \beta_1 \omega_i^{2d} + w(\omega_i)_1, \\ &\vdots \\ \ln I_p(\omega_i) &= \alpha_p - 2d \ln \omega_i + \beta_p \omega_i^{2d} + w(\omega_i)_p \end{aligned} \tag{28}$$

and minimizing the function in (21).

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