COMP 680 Problem Set 01

1. [20 pts] Suppose our experimental equipment consists of 1 green 6-sided die, and 1 red 6-sided die.

1.1. [10 pts] Experiment 1 consists of rolling both dice, and observing at the color and top face on the left, and observing the color and top face on the right.

Example 1: R2, G2 is possible outcome

Example 2: G2. R2 is a possible outcome (distinct from example 1)

Enumerate the sample space for this experiment.

S = { (R1,G1), (R1,G2), (R1,G3), (R1,G4), (R1, G5), (R1, G6),

(R2,G1), (R2,G2), (R2,G3), (R2,G4), (R2, G5), (R2, G6),

(R3,G1), (R3,G2), (R3,G3), (R3,G4), (R3, G5), (R3, G6),

(R4,G1), (R4,G2), (R4,G3), (R4,G4), (R4, G5), (R4, G6),

(R5,G1), (R5,G2), (R5,G3), (R5,G4), (R5, G5), (R5, G6),

(R6,G1), (R6,G2), (R6,G3), (R6,G4), (R6, G5), (R6, G6) }

1.2. [10 pts] Experiment 2 consists of rolling both dice and summing 2 top faces.

Enumerate the sample space for this experiment

S= {2,3,4,5,6,7,8,9,10,11,12}

2. [20 pts] You are preparing a quarterfinal bracket for a single-elimination tournament.

A,B,C,D are the 4 teams in the quarterfinal. Enumerate all of the possible tournament matchups, including the winner. The order in a matchup does not matter.

Round 1: {A,B}, {C,D}

Round 2: {A,C}

Winner : {A}

Round 1: {A,B}, {C,D}

Round 2: {A,C}

Winner : {C}

Round 1: {A,B}, {C,D}

Round 2: {A,D}

Winner : {A}

Round 1: {A,B}, {C,D}

Round 2: {A,D}

Winner : {D}

Round 1: {A,B}, {C,D}

Round 2: {B,C}

Winner : {B}

Round 1: {A,B}, {C,D}

Round 2: {B,C}

Winner : {C}

Round 1: {A,B}, {C,D}

Round 2: {B,D}

Winner : {B}

Round 1: {A,B}, {C,D}

Round 2: {B,D}

Winner : {D}

Round 1: {A,C}, {B,D}

Round 2: {A,B}

Winner : {A}

Round 1: {A,C}, {B,D}

Round 2: {A,B}

Winner : {B}

Round 1: {A,C}, {B,D}

Round 2: {A,D}

Winner : {A}

Round 1: {A,C}, {B,D}

Round 2: {A,D}

Winner : {D}

Round 1: {A,C}, {B,D}

Round 2: {C,B}

Winner : {C}

Round 1: {A,C}, {B,D}

Round 2: {C,B}

Winner : {B}

Round 1: {A,C}, {B,D}

Round 2: {C,D}

Winner : {C}

Round 1: {A,C}, {B,D}

Round 2: {C,D}

Winner : {D}

Round 1: {A,D}, {B,C}

Round 2: {A,B}

Winner : {A}

Round 1: {A,D}, {B,C}

Round 2: {A,B}

Winner : {B}

Round 1: {A,D}, {B,C}

Round 2: {A,C}

Winner : {A}

Round 1: {A,D}, {B,C}

Round 2: {A,C}

Winner : {C}

Round 1: {A,D}, {B,C}

Round 2: {D,B}

Winner : {D}

Round 1: {A,D}, {B,C}

Round 2: {D,B}

Winner : {B}

Round 1: {A,D}, {B,C}

Round 2: {D,C}

Winner : {D}

Round 1: {A,D}, {B,C}

Round 2: {D,C}

Winner : {C}

3. [20 pts] Let S= {a, b, c, d} be the sample space for an experiment.

3.1. [10 pts] Suppose the {a} is in the Sigma Algebra for the sample space. Is {b} necessarily in the Sigma Algebra?

Σ = {S, ∅, {a}, {b,c,d} } so {b} is not necessarily in the Sigma Set

3.2. [10 pts] Suppose {a} and {b} are in the Sigma Algebra. Is the {c} necessarily in the Sigma Algebra?

Σ = {S, ∅, {a}, {b}, {a,b}, {c,d}, {a,c,d}, {b,c,d} } so {c} is not necessarily in the Sigma Set.

4. [20 pts] Let (S, Σ, P) be a probability tuple with sample space S, sigma algebra Σ and probability function P. Also, let $A, B \in \Sigma$, where $A \subseteq B$.

Show that $P(A) \leq P(B)$.

$B = A \cup (B \cap A^c)$	Because $A \subseteq B$
$P(A) + P(B \cap A^c) = P(B)$	Let $\varepsilon = P(B \cap A^c)$ then $0 \leq \varepsilon \leq 1$

$P(A) + \varepsilon = P(B)$

$P(A) \leq P(B)$

5. [20 pts] Let (S, Σ, P) be a probability tuple with sample space S, sigma algebra Σ and probability function P. Also, let $E1, E2 \in \Sigma$

Show that $P(E1 \cap E2) \geq P(E1) + P(E2) - 1$

Hint: You could start with the Inclusion/Exclusion formula for 2 sets:

$P(E1 \cup E2) = P(E1) + P(E2) - P(E1 \cap E2)$

$0 \leq P(E1 \cup E2) \leq 1$	Because max value of $P(E1 \cup E2)$ is $P(S)$ and $E1, E2 \in \Sigma$
$P(E1) + P(E2) - P(E1 \cap E2) \leq 1$	Because Inclusion/Exclusion formula for 2 sets

$P(E1 \cap E2) \geq P(E1) + P(E2) - 1$