COMP 680 Problem Set 07

Several problems in this section will require linear algebra. The *`scipy.linalg`* module supplies all the fundamental routines needed for this module. In addition, some problems may be easier to solve if you use distributions from the *`scipy.stats`* package.

1. [16 pts] A small computer lab has 2 terminals. The number of students working in this lab is recorded at the end of every hour. A computer assistant notices the following pattern:

    - If there are 0 or 1 students in a lab, then the number of students in 1 hour has a 50-50% chance to increase by 1 or remain unchanged.

    - If there are 2 students in a lab, then the number of students in 1 hour has a 50-50% chance to decrease by 1 or remain unchanged.

    1.1. [5 pts] Write the transition probability matrix for this Markov chain.

    1.2. [5 pts] Is this a regular Markov chain? Justify your answer.

    1.3. [6 pts] Suppose there is nobody in the lab at 7 am. What is the probability of nobody working in the lab at 10 am?

2. [16 pts] A computer system can operate in two different modes. Every hour, it remains in the same mode or switches to a different mode according to the transition probability matrix

$$\begin{pmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{pmatrix}$$

    2.1. [6 pts] Compute the 2-step transition probability matrix.

    2.2. [10 pts] If the system is in Mode I at 5:30 pm, what is the probability that it will be in Mode I at 8:30 pm on the same day?

3. [16 pts] Markov chains find direct applications in genetics. Here is an example. An offspring of a black dog is black with probability 0.6 and brown with probability 0.4. An offspring of a brown dog is black with probability 0.2 and brown with probability 0.8.

   3.1. [8 pts] Write the transition probability matrix of this Markov chain.

   3.2. [8 pts] Rex is a brown dog. Compute the probability that his grandchild is black.

4. [16 pts] A Markov chain has the transition probability matrix

$$\begin{pmatrix} 0.3 & \dots & 0 \\ 0 & 0 & \dots \\ 1 & \dots & \dots \end{pmatrix}$$

   4.1. [4 pts] Fill in the blanks.

   4.2. [6 pts] Show that this is a regular Markov chain.

   4.3. [6 pts] Compute the steady-state probabilities.

5. [18 pts] Customers of an internet service provider connect to the internet at the average rate of 12 new connections per minute. Connections are modeled by a Binomial counting process.

   5.1. [9 pts] What frame length Δ gives the probability 0.15 of an arrival during any given frame?

   5.2. [9 pts] With this value of Δ, compute the expectation and standard deviation for the number of seconds between two consecutive connections.

6. [18 pts] On the average, Mr. Z drinks and drives once in 4 years. He knows that

   - Every time when he drinks and drives, he is caught by police.
   - According to the laws of his state, the third time when he is caught drinking and driving results in the loss of his driver's license.
   - Poisson process is the correct model for such "rare events" as drinking and driving.

   What is the probability that Mr. Z will keep his driver's license for at least 10 years?