Holly Do - COMP 614

# Problem Set 02

## 1. [15 pts] Recall from Problem Set 01 that

$P(E1 \cap E2) \geq P(E1) + P(E2) - 1$

## 1.1 [10 pts] Use that fact to show that $P(E1 \cap E2 \cap E3) \geq P(E1) + P(E2) + P(E3) - 2$

$P(E1 \cap E2 \cap E3) \geq P(E1 \cap E2) + P(E3) - 1$ because $P(E1 \cap E2 \cap E3) = P((E1 \cap E2) \cap E3)$

$P(E1 \cap E2 \cap E3) \geq P(E1) + P(E2) -1 + P(E3) - 1$ because $P(E1 \cap E2) \geq P(E1) + P(E2) - 1$

$P(E1 \cap E2 \cap E3) \geq P(E1) + P(E2) + P(E3) - 2$

## 1.2 [5 pts] Using both previous facts, show that $P (E1 \cap E2 \cap E3 \cap E4 \geq P(E1) + P(E2) + P(E3) + P (E4) - 3$

$P (E1 \cap E2 \cap E3 \cap E4 ) \geq P (E1 \cap E2 \cap E3) + P (E4) - 1$ because $P (E1 \cap E2 \cap E3 \cap E4 ) = P((E1 \cap E2 \cap E3) \cap E4 )$

$P (E1 \cap E2 \cap E3 \cap E4 ) \geq P(E1) + P(E2) + P(E3) - 2 + P (E4) - 1$ because of 1.1

$P (E1 \cap E2 \cap E3 \cap E4 ) \geq P(E1) + P(E2) + P(E3) + P (E4) - 3$

## 2. [10 pts] Professor Zorro Mostel teaches 3 courses: Advanced Swordsmanship, Advanced Horsemanship, and Beginning Calligraphy. His enrollment in Advanced Swordsmanship is 40. The enrollment in Advanced Horsemanship is 21. His enrollment in Beginning Calligraphy is 15 students. There are 12 students taking both Advanced courses. There are 10 students taking both Advanced Swordsmanship and Beginning Calligraphy. There are 6 students taking Advanced Horsemanship and Beginning Calligraphy. 3 brave students are taking all 3 courses. How many distinct students does Zorro teach?

Let set A be the number of students enrolled in Advanced Swordsmanship

Let set B be the number of students enrolled in Advanced Horsemanship

Let set C be the number of students enrolled in Beginning Calligraphy

Approach # 1:

If we take sum from each class's number of enrollment, the number of students who are counted more than once:

$2 *(A \cap B \cap C) + ((A \cap B)- (A \cap B \cap C)) + ((B \cap C)- (A \cap B \cap C)) + ((A \cap C)- (A \cap B \cap C)) = 3*2 +3+7+9 = 25$

Distinct students Zorro teaches is: (40+21+15) –25 = 51 distinct students

Approach # 2:

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

=40+21+15-12-10-6+3= 51 distinct students

**3.** [10 pts] Leroy Narkon has 9 different books in English 7 different books in Klingon, and 13 different books in Czech. How many ways can Leroy pick an ordered triple of books such that each of the 3 books is in a different language?

We choose one book from each section, that would be C(9,1) * C(7,1) * C(13,1) choices.

Each of these combinations, we can switch the order of three books around, which is 3*2*1 combinations.

So the total of ordered triple books such that each of the 3 books is in a different language:

C(3,1) * C(2,1) * C(9,1) * C(7,1) * C(13,1) = 3 * 2 * 9 * 7 * 13 = 4914 ordered triples

**4.** [10 pts] 2 6-sided die are rolled. How many outcomes have both odd numbers on the 2 dice?

Because first die's odd outcomes are 1,3,5 and second die's odd outcomes are 1,3,5 so the number of outcomes have both odd numbers on the 2 dice is 3*3 = 9 outcomes

**5.** [10 pts] Let S = {♣,♦,♥,♠} be a set of symbols. We are interested in symbol sequences.

**5.1** [2 pt] How many 3-symbol sequences without repetition are there that must contain the symbol ♦?

First, we put the symbol ♦ in one of the 3 spaces, that results in C(3,1) choices.

Then for the other two slots, since without repetitions, the number of combinations for these two slots is 3*2.

Total number of 3-symbol sequences without repetition are there that must contain the symbol ♦ : C(3,1) * 3 * 2 = 3*3*2 = 18 sequences

**5.2** [1 pt] How many 3-symbol sequences are there when no symbol is repeated?

Because no symbol is repeated, the first slot has four symbol options, the second has three options, and the third has two options. Thus, the number of 3-symbol sequences when no symbol is repeated is:

P(4,3) = 4*3*2 = 24 sequences

**5.3** [1 pt] How many 2-symbol sequences are there when symbol repetition is allowed?

If repetition is allowed, the first slot has four symbol options, the second has four options. Thus, the number of 2-symbol sequences when symbol repetition is allowed:

4*4 = 16 sequences

**5.4** [2 pts] How many 4-symbol sequences with repetition are there that must contain at least 1 ♣ symbol?

At least 1 ♣ symbol meaning one, two, three, or four times.

The number of 4-symbol sequences with repetition that must contain at least 1 ♣ symbol is the total number of 4-symbol sequences with repetition minus the number of sequences that has zero ♣ symbol with repetition:

(4*4*4*4) - (3*3*3*3) = 175 sequences

## 5.5 [4 pts] Consider 2 different types of sequence:

Type 1: sequences with repetition that have at least 2 ♥ symbols

Type 2: sequences have exactly 3 ♠ symbols. The remaining symbols in the sequence are not repeated

The set J of sequences of interest are either Type 1 sequences of length 4, or type 2 sequences of length 5. How many sequences are in set J?

**Type 1: sequences with repetition that have at least 2 ♥ symbols**

Approach # 1:

At least 2 ♥ symbols meaning two, three, or four symbols.

Number of sequences that have exactly one ♥ symbols: 3*3*3* C (4,1) = 108

Number of sequences that have zero ♥ symbols: 3*3*3*3 = 81

Number of sequences of type 1: sequences with repetition that have at least 2 ♥ symbols is the total number of sequences with repetition minus Number of sequences that have zero or exactly one ♥ symbols: (4*4*4*4) - 108 - 81 = 256 – 108 - 81 = 67 sequences

Approach # 2:

Number of sequences that have two ♥ symbols: 3*3* C (4,2) = 6 *3*3 = 54

Number of sequences that have three ♥ symbols: 3* C (4,3) = 4*3 =12

Number of sequences that have four ♥ symbols: C (4,4) = 1

Number of sequences of type 1: sequences with repetition that have at least 2 ♥ symbols is 54+12+1 = 67 sequences

**Type 2: sequences have exactly 3 ♠ symbols. The remaining symbols in the sequence are not repeated, length 5**

Approach # 1:

Sequences of length 5 that have exactly 3 ♠ symbols: C(5,3) = (5*4)/2 = 10

For the two slots left over in the 5-length sequence, the number of ways is 3*2 = 6

Number of sequences of type 2: sequences have exactly 3 ♠ symbols. The remaining symbols in the sequence are not repeated, length 5: 10*6 = 60 sequences

Approach # 2:

Placing first non-spade among the 3 spades, we have 4 slots or 4 choices.

After that, placing the second non-spade among the 5 slots or 5 choices.

So, we have 4*5 ways to place 2 non-spades among 3 spades.

Because the positions imply the order, to calculate the ways of selecting 2 non-spade suits from 3 non-spade suits, we take C(3,2) = 3

So, total of combinations for type 2 is 4*5*3 = 60 sequences

**Set J has 67 + 60 = 127 sequences**

## 6. [10 pts] There are 6 people at a dinner party. They will all be seated around a circular table. There are no assigned seats. Any seating arrangement that has the same left and right neighbors for the participants is considered equivalent

## 6.1. [4 pts] How many distinct seating arrangements are there for this dinner party?

In a non-circular arrangement, there are 6! Permutations. Because this is a circular arrangement and any seating arrangement that has the same left and right neighbors for the participants is considered equivalent. The first person has 6 choices which are equivalent to each other. So, we divide by 6 to eliminate the repetitions. Thus, we have:

5! Arrangements

## 6.2. [6 pts] In general, for N people, how many distinct seating arrangements are there?

Using the same logic explained above, for N people, the number of distinct seating arrangements are:

(N-1)! Arrangements

## 7. [10 pts] Show the inclusion/exclusion for N = 3. That is, show that

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$- P(A \cap B) - P(A \cap C) - P(B \cap C)$

$+ P(A \cap B \cap C)$

Note: You may use the inclusion/exclusion principle for N = 2.

$P(E1 \cup E2) = P(E1) + P(E2) - P(E1 \cap E2)$

$P(A \cup B \cup C)$

$= P(A \cup B) + P(C) - P((A \cup B) \cap C)$ because $P(A \cup B \cup C) = P((A \cup B) \cup C)$

$= P(A)+P(B)+P(C)-P(A \cap B) - P((C \cap B) \cup P(C \cap A))$ because distributive law of sets

$= P(A)+P(B)+P(C) - P(A \cap B) - ((P(C \cap B) + P(C \cap A) - P(C \cap B \cap C \cap A))$

$= P(A)+P(B)+P(C) - P(A \cap B) - (P(C \cap B) - P(C \cap A) + P(C \cap B \cap C \cap A)$

$= P(A)+P(B)+P(C)-P(A \cap B)-(P(C \cap B) - P(C \cap A) + P(C \cap B \cap A)$ because $C \cap C = C$

$= P(A)+P(B)+P(C)-P(A \cap B)-(P(B \cap C) - P(A \cap C) + P(A \cap B \cap C))$ because $A \cap C = C \cap A$

8. [15 pts] For the inclusion/exclusion principle for N = 4,

$P(E1 \cup E2 \cup E3 \cup E4)$

$= P((E1 \cup E2 \cup E3) \cup E4) = P((E1 \cup E2 \cup E3) + P(E4) - P((E1 \cup E2 \cup E3) \cap E4)$

$= P(E1)+P(E2)+P(E3)-P(E1 \cap E2)-(P(E2 \cap E3) - P(E3 \cap E1) + P(E4) + P(E1 \cap E2 \cap E3)$

$- P((E1 \cup E2 \cup E3) \cap E4)$

$= P(E1)+P(E2)+P(E3)-P(E1 \cap E2)-(P(E2 \cap E3) - P(E3 \cap E1) + P(E4) + P(E1 \cap E2 \cap E3)$

$- P((E1 \cap E4) \cup (E2 \cap E4) \cup (E4 \cap E3))$

$= P(E1)+P(E2)+P(E3)-P(E1 \cap E2)-(P(E2 \cap E3) - P(E3 \cap E1) + P(E4) + P(E1 \cap E2 \cap E3) - (P(E1\cap E4)-P(E2 \cap E4)-P(E4 \cap E3)+P(E1\cap E2\cap E4)+P(E2\cap E4\cap E3) + P(E1\cap E4\cap E3) - P(E1\cap E2\cap E3\cap E4))$

[5 pts] How many singleton terms (e.g.) P(Ei) are in the I/E formula? Because there are 4 sets so:

4 singleton terms

[5 pts] How many P(Ei ∩ Ej) are in the I/E formula? Because you can choose 2 sets out of 4 and intersect them so:

C(4,2) = 4*3/2 = 6 terms

[5 pts] How many P(Ei ∩ Ej ∩ Ek) are in the I/E formula? Because you can choose 3 sets out of 4 and intersect them so:

C(4,3) = 4 terms

# 9. [10 pts] Suppose that a website allows its users to pick their own usernames for accounts, but imposes some restrictions. The 1st character must be an upper-case letter in the English alphabet. The 2nd through 6th characters can be letters (both upper-case and lower-case allowed) in the English alphabet or decimal digits (0–9). The 7th position must be ‘@’ or ‘.’. The 8th through 12th positions allows lower- case English letters, ‘*’, ‘%’, and ‘#’. The 13th position must be a digit (1-5).

How many users can the website accept registrations from?

Upper-case letters in the English alphabet: 26

Lower-case letters in the English alphabet: 26

Upper-case and lower-case letters in the English alphabet: 52

Decimal digits (0–9): 10

Letters (both upper-case and lower-case allowed) in the English alphabet or decimal digits (0–9) : 62

7th position must be ‘@’ or ‘.’ : 2

Lower- case English letters, ‘*’, ‘%’, and ‘#’ :26+3=29

Digit (1-5) : 5

The total number of passwords/users that get accepted is 26*62*62*62*62*62*2*

29*29*29*29*29*5