COMP 680 Problem Set 04

You may want to use the Python statistics framework to solve these problems. If you do use Python, please show the relevant parts of your code.

1. [11 pts] Lifetime of a certain hardware is a continuous random variable with

Density Function:

f(x)= k - x/50 for 0<x<10 years

f(x)= 0 for all other x

1.1. [4 pts] Find k

Integral of f(x)= k - x/50 with x from 0 to 10 is 1 because of definition of pdf.

$\int_0^{10} k - \left(\frac{x}{(50)}\right) dx$ = k*x – ((x^2/2)*1/50) = 1

k*x – (x^2/100) = 1 with x from 0 to 10

10k – (10^2/100) - 0 = 1

10k = 1+1 = 2

k = 2/10 = 0.2

1.2. [3 pts] What is the probability of a failure within the first 5 years?

```
from scipy.integrate import quad
result, error = quad(lambda x: 0.2 - x/50, 0, 5)
print(result)
```

$\int_0^{5} k - \left(\frac{x}{(50)}\right) dx$ = 0.75 using python as a calculator.

1.3. [4 pts] What is the expectation of the lifetime?

E(X) is integral of x* f(x) dx with x from 0 to 10 with f(x) is the probability of x.

```
from scipy.integrate import quad
result, error = quad(lambda x: x*(0.2 - x/50), 0, 10)
```

```
print(result)
```

$\int_0^{10} x\left(0.2 - \left(\frac{x}{(50)}\right)\right) dx$ = 3.3333333333333334

2. [11 pts] The time it takes a printer to finish printing a job is an Exponential random variable with expectation of 12 seconds. You send a job to the printer at 10:00 am, and it appears to be third in line. What is the probability that your job will be finished before 10:01?

E(X) = 12 so λ = 1/12 and θ = 1/ λ = 12

X = X1+ X2+X3 so k = 3

Time a = 10:00, b = 10:01 so (b-a) = 1 min = 60 seconds

The distribution is Gamma because it the sum of single exponential trials.

```
k = 3
scale = 12
result = stats.gamma.cdf(60, a=k, scale=scale)
print(result)
```

P((X1+ X2+X3) <= 60) = 0.8753479805169189

3. [12 pts] Two computer specialists are completing work orders. The 1st specialist receives 60% of all orders. Each order takes her Exponential amount of time with parameter λ1 = 3 per hour. The 2nd specialist receives the remaining 40% of orders. Each order takes him Exponential amount of time with parameter λ2 = 2 per hour. A certain order was submitted 30 minutes ago, and it is still not ready. What is the probability that the first specialist is working on it?

P(S1) = 60%

P(S2) = 40%

λ1 = 3 per hour so θ = 1/3

λ2 = 2 per hour so θ = 1/2

t = 30 minutes = 0.5 hr

P(S2 | T > 0.5 hr) = (P(T>0.5 | S2) * P(S2)) / P(T>0.5)

P(T>0.5 | S2) = 1 - cdf(Exp,0.5,.5)

```
result = stats.expon.cdf(0.5, loc=0, scale=0.5)
print (result)
```

cdf(Exp,0.5,.5) = 0.6321205588285577

P(T>0.5 | S2) = 1 - cdf(Exp,2,.5) = 0.36787944117144233

P(T > .5 | S1) = 1 - cdf(Exp,3,.5)

```
result = stats.expon.cdf(0.5, loc=0, scale=1/3)
print (result)
```

cdf(Exp,3,.5) = 0.7768698398515702

P(T > .5 | S1) = 1 - cdf(Exp,3,.5) = 0.22313016

P(T>0.5) = P(T > .5 | S1) P(S1) + P(T > .5 | S2) P(S2) = (0.22313016 * 0.6) + (0.36787944 * 0.4) = 0.133878+ 0.147= 0.281

According to Bayes Rules:

P(S1 | T > 0.5 hr) = (P(T>0.5 | S1) * P(S1)) / P(T>0.5) = 0.133878096 / 0.281 = 0.4764

4. [11 pts] The time X it takes to reboot a certain system has Gamma distribution

with E(X) = 20 min and Std(X) = 10 min.

4.1. [6 pts] Compute parameters of this distribution.

E(X) = 20 min = k*θ so θ = 20/k

Std(X) = 10 min so Var(X) = k*(θ^2) = k * (400/k^2) = 400/k = 100 so k = 4

θ = 20/k = 20/4 = 5

So, this Gamma distribution has k = 4 and θ = 5

4.2. [5 pts] What is the probability that it takes less than 15 minutes to reboot this system?

```
k = 4
scale = 5
result = stats.gamma.cdf(15, a=k, scale=scale)
print(result)
```

0.35276811121776874

5. [11 pts] For a Standard Normal random variable Z, compute

Standard normal distribution meaning E(X) = 0 and Std(X) = 1

5.1. [1 pt] $P(Z \geq 0.99)$

= 1 - 0.8389129404891691 = 0.16108706

5.2. [1 pt] $P(Z \leq -0.99)$

```
k = 0
scale = 1
result = stats.norm.cdf(-0.99, loc=k, scale=scale)
print (result)
```

= 0.1610870595108309

5.3. [1 pt] $P(Z < 0.99)$

```
from scipy import stats
k = 0
scale = 1
result = stats.norm.cdf(0.99, loc=k, scale=scale)
print (result)
```

= 0.8389129404891691

5.4. [2 pts] $P(|Z| > 0.99)$

= 0.16108706 + 0.16108706 = 0.32217412

5.5. [1 pt] P(Z < 10.0)

```
k = 0
scale = 1
result = stats.norm.cdf(10, loc=k, scale=scale)
print (result)
```

1

5.6. [1 pt] P(Z > 10.0)

1-1 = 0

5.7. [4 pts] With probability 0.9, variable Z is less than what?

```
k = 0
scale = 1
result = stats.norm.cdf(1.285, loc=k, scale=scale)
print (result)
```

0.90000

So Z <= 1.285 then probability 0.9

6. [11 pts] The Cretin 3.5 program is divided into 3 blocks that are being compiled on 3 different computers (in parallel). Each block takes an Exponential distributed amount of time to compile, 5 minutes on the average. The time required to compile 1 block is independent of the time to compile the other blocks. The program is completed when all the blocks are compiled.

1. [5 pts] Compute the probability that the parallel compile of Cretin 3.5 will take 7 mins or less.

2. [6 pts] Compute the probability that the parallel compile of Cretin 3.5 will take more than 10 minutes

Because these three blocks are independent and parallel, the complete time is the maximum time any of the three finishes.

E(X) = 5 mins for each block = 1/ λ = θ so λ = 1/5

```
scale = 5
result = stats.expon.cdf(7, scale=scale)
print (result)
```

P(t <= 7 mins) = 0.7534030360583935

```
scale = 5
result = stats.expon.cdf(10, scale=scale)
print (result)
```

P(t < 10 mins) = 0.8646647167633873

P(t >= 10 mins) =1 - 0.8646647167633873 = 0.1353353

7. [11 pts] For some electronic component, the time until failure has Gamma distribution with parameters α = 2 and λ = 2 years−1. {read as 2 per year). Compute the probability that the component fails within the first 6 months.

Gamma:

α = 2 = k

λ = 2 per year so θ = 1/ λ = 0.5

t = 0.5 year = 6 months

We need to find P(T <= 0.5)

```
import scipy

k = 2
scale = 1 / 2

result = stats.gamma.cdf(0.5, a=k, scale=scale)
print(result)
```

P(T <= 0.5) = 0.2642411176571153

8. [11 pts] Prove the memoryless property of Geometric distribution. That is, if X has Geometric distribution with parameter p, show that P(X ≥ t + x | X ≥ t) = P(X > x), ∀x ≥ 0

Assume that:

$P(X \geq t + x \mid X \geq t) = P(X > x), \forall x \geq 0$

And

$P(G \geq T) = q^{T-1}$

G : geometric distributed random variable

Q = 1-p probability of failure

We have:

$P(G \geq t + x \mid G \geq t) = P(G \geq (t + x) \cap G \geq t) / P(G \geq t)$

$= P(G \geq t + x) / P(G \geq t)$

$= q^{t+x-1} / q^{t-1}$

$= q^{x}$

$= P(G \geq (x + 1))$

$= P(G > x)$

9. [11 pts] There is an assembly line at the ACME corporation that takes on average 36 mins to produce 1 Roadrunner Trap. The standard deviation of that assembly line is 18 mins. On the assumption that the entire assembly line time is Gamma distributed, and that each individual stage is Exponentially distributed.

E(X) = 36 mins

Std(X) = 18 mins

9.1. [6 pts] Compute the number of stages in the assembly line.(Meaning k =?)

$E(X) = k*\theta = 36$ so $\theta = 36/k$

Std(X) = 18

$Var(X) = k*(\theta^2) = 18x18 = 324 = k * ((36*36)/(k^2))$

$324 = 1296/k$ so $k = 4$ and $\theta = 36/k = 36/4 = 9$

So $k = 4$ and $\theta = 9$

9.2. [5 pts] Compute the rate of each assembly line stage. Rate of each assembly line is also the rate of Gamma:

$\lambda = 1/\ \theta = 1/9$