Multi-Item Auction Based Mechanism for Mobile Data Offloading: A Robust Optimization Approach

Dongqing Liu, Abdelhakim Hafid, Lyes Khoukhi

***Abstract*—The opportunistic utilization of access devices to offload mobile data from cellular network has been considered as a promising approach to cope with the explosive growth of cellular traffic. To foster this opportunistic utilization, we consider a mobile data offloading market where mobile network operator (MNO) can sell bandwidth made available by the access points (APs) to increase MNO's profit. We formulate the offloading problem as a multi-item auction and study MNO's profit maximization problem. We discuss the conditions to (i) offload the maximum amount of data traffic, (ii) foster the participation of mobile subscribers (MSs) (individual rationality), (iii) prevent market manipulation (incentive compatibility) and (iv) preserve budget feasibility of MSs. Then, we propose a robust optimization based method to implement multi-item auction mechanism. We further propose two iterative algorithms that efficiently solve the offloading problem. The simulation results show the efficiency and robustness of our proposed methods for cellular data offloading.**

***Index Terms*—Multi-item auction, mobile data offloading, heterogeneous networks, robust optimization.**

I. Introduction

The rapid growth of mobile data traffic raises big challenges to cellular network. Global mobile data traffic grew 63 percent and reached 7.2 exabytes per month in 2016, which is 18-fold over the past 5 years [1]. The huge amount of mobile data traffic exceeds the capacity of cellular network and reduces the network quality [2]. To address such challenges, one simple solution is to increase the capacity of cellular network, which is inefficient and expensive due to the corresponding expensive investments in radio access networks and the core infrastructure. One promising solution, namely mobile data offloading, is to offload cellular traffic to other kinds of networks, e.g. WiFi APs and femtocells; this can solve the cellular traffic overload problem [3, 4].

Although mobile data offloading can significantly reduce cellular traffic, the task of developing a comprehensive and reliable mobile data offloading system remains challenging. A key challenge is how to achieve an efficient data offloading coordination among multiple mobile devices. By opportunistic utilization of lower cost APs, MSs will have better wireless access service with lower cost. In contrast, MNOs who have deployed these APs want to maximize the revenue by selling bandwidth. Thus, how to effectively allocate this bandwidth to mobile devices effectively becomes a key problem to be solved.

Given the limited bandwidth of APs deployed in a mobile data offloading market, when demands of mobile devices exceed supply, MNO needs to allocate the bandwidth to mobile devices and decide the price for allocated bandwidth in order to achieve the highest revenue. Auction mechanism is considered as an economically efficient approach towards the allocation of APs' bandwidth, and assigns bandwidth to MSs who value it the most [5–11]. In a real-world data offloading market, the bidding prices of MSs are private information unavailable for MNO. However, MNO may use historical information to identify the numerical characteristics of the bidding prices. Consequently, it is natural to consider how to model the bidding prices based on historical information. We use uncertainty set to model the possibility of bidding prices. MNO assumes that all bidding prices belong to the uncertainty set derived from historical information. Then, MNO makes an offloading decision based on the uncertainty set instead of some fixed bidding prices.

In this paper, we focus on designing an efficient auction mechanism for allocating APs' bandwidth among multiple MSs; this is considered as a multi-item auction problem. MNO which owns the network infrastructure acts as the auctioneer and sells bandwidth to mobile devices through an auction. We formulated the auction problem based on robust optimization which models the desirable properties (budget feasibility, incentive compatibility, and individual rationality) of optimal auctions enabling the auctioneer to use historical data or prior knowledge of valuations. The uncertainty of item valuations is modeled as an uncertainty set, which is constructed based on limit theorems of probability theory. The optimal auction mechanism with reservation price has the structure of a Vickrey-Clarke-Groves (VCG) mechanism [12].

The main contributions of this paper can be summarized as follows:

- We characterize the interaction among MNO and MSs in a multi-item auction aiming at maximizing the MNO's revenue and the amount of offloaded traffic from MSs. Our proposed multi-item auction calculates reservation prices based on the uncertainty set and the MSs' budgets; this can prevent market manipulation. Our proposed auction is implemented by robust optimization. Instead of requiring the full knowledge of MSs' valuations, robust optimization uses few information of MSs' valuations to calculate the optimal solution.

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- Since the optimal multi-item auction problem is difficult to solve, we further propose two greedy auctions that can solve the offloading market problem in polynomial time, while preserving the properties of budget feasibility, incentive compatibility, and individual rationality. These two greedy auctions outperforms each other in different network scenarios.
- We perform numerical analysis and comparative evaluation of the proposed optimal and greedy auctions, considering variant network scenarios. We further illustrate that the proposed offloading mechanisms can improve cellular data offloading performance and has higher robustness compared to Myerson auction.

The rest of the paper is organized as follows. Section II presents related work. Section III presents the system model. Section IV formulates the multi-item auction as a robust optimization problem. Section V and Section VI propose the optimal and greedy auction mechanisms, to solve the offloading market problem, respectively. Section VII illustrates and analyzes the numerical results. Section VIII concludes the paper.

II. Related work

A. Mobile Data Offloading

To cope with the growth of cellular traffic, some previous contributions have studied efficient data offloading methods from the perceptive of data offloading decision making. Jung et al. [13] proposed a WiFi based offloading model to maximize per-user throughput by collecting network information, e.g., the number of MSs and their data demands. Cheung et al. [14] proposed a Markov decision process based network selection algorithm for delay-tolerant applications under the setting of a single MS. Barbarossa et al. [15] proposed a centralized scheduling algorithm to jointly optimize the communication and computation resource allocations among multiple users with latency requirements. Kang et al. [16] studied the offloading problem from MNO's perspective and proposed a usage-based charging model to maximize MNO's revenues. Wu et al. [17] studied optimal resource allocation for data offloading via dual-connectivity, while taking into account the trade-off between optimal bandwidth allocation for base stations and optimal power allocation for MSs.

Other contributions have investigated data offloading problems based on auction theory or game theory. Chen et al. [18] studied the scenario where multiple users can access the same wireless base station, and designed a decentralized offloading mechanism that ensures the scalability of the proposed mechanism with the number of MSs. Zhou et al. [19] proposed a reverse auction based incentive framework for cellular traffic offloading, and provided a prediction model for WiFi data offloading potential. Cheng et al. [20] took into consideration users' mobility information and proposed an auction based offloading mechanism to maximize MSs' social welfare and improve MNO's revenues. Lee et al. [21] proposed a two-stage sequential game to model the interaction between MNO and MSs, and demonstrated, via simulations, that WiFi offloading is economically beneficial for both MNO and MSs. Paris et al. [22] proposed a reverse auction based offloading algorithm leasing WiFi APs, owned by third parties, to allocate bandwidth to multiple MSs. However, all these works assume that all players are rational and will take the truthful bidding. Different from existing contributions, we consider to implement worst case optimality as long as the bid values belonging to the uncertainty set constructed by historical bidding information.

B. Multi-Item Auction

Most existing studies on multi-item mechanisms aim to maximize the MNO's revenue or incentivize the participation of MSs. Zhao et al. [23] proposed an online auction method to maximize the value of services in mobile crowdsourcing (MCS), and to incentivize the participation of MSs in MCS applications. Gan et al. [24] proposed a reverse auction method to incentivize the participation of MSs in MCS applications. Wang et al. [25] designed a truthful, individual rational, budget feasible and quality-aware algorithm for task allocation in MCS. However, these works only considered the budget feasibility of MNO. This is because that, in MCS, MSs consume their own resources such as computational resources and computing power to help MNO solve a complex problem. MNO needs to pay MSs in return. In contrast, in our model, MSs request the bandwidth resources of MNO. Thus, we need to consider the budget feasibility of all MSs, which is more complex than MCS.

Other works consider the budget constraints of MSs. Bhattacharya et al. [26] proposed an approximation algorithm to solve the multi-item auction problem. Wang et al. [27] studied distributed truthful auction mechanism for task allocation in mobile cloud computing (MCC). They proposed an auction model considering computational efficiency, individual rationality, truthfulness guarantee of the bidders, and budget balance. Jin et al. [28] investigated the resource sharing problem for cloudlets in MCC. They proposed an incentive mechanism to charge MSs and reward cloudlets. Although these works considered the budget constraints of MSs, they didn't use the historical bidding information. In this paper, we design an optimal multi-item auction mechanism based on the historical bidding information, while taking into consideration MSs' budget constraints.

Compared with the above mechanisms, the auction problem designed in this paper is rather challenging, and has the following differences: (1) we take full advantage of historical bidding information and prevent abnormal auction to destroy the multi-item auction; (2) we consider the worst case optimization problem; thus, our proposed method has strong robustness compared to other optimal auction mechanisms; and (3) our optimal auction considers reservation prices that are functions of the uncertainty set and the budgets, thus can potentially protect the MNO's revenue.

III. System Model

In this section, we present the economic definitions and network model that are considered in our multi-item auction mechanism; the objective of this mechanism is to implement efficient mobile data offloading. A scenario of data offloading among multiple APs and MSs is shown in Figure 1, where MSs, in the coverage area of APs, engage in an auction to acquire bandwidth (in WiFi network). We first model

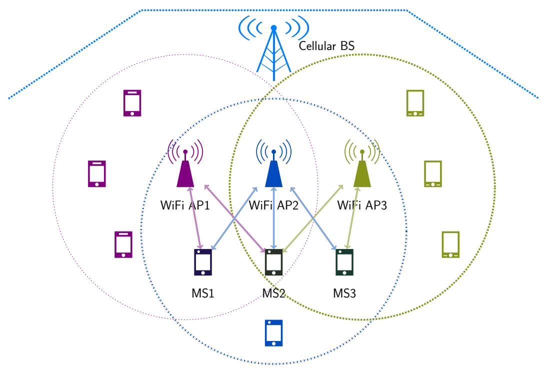


Fig. 1. An illustration of data offloading auction model. WiFi APs are managed by a single MNO that provides network access to MSs (e.g., MS1). The network capacity of WiFi APs (e.g., AP1) is allocated to MSs for data traffic offloading. In this scenario, MS1, MS2 and MS3 bid for bandwidth (i.e., AP1, AP2 and AP3) with different valuations. Considering the coverage area of each AP, MS2 can bid for three APs, while MS1 and MS3 can bid for two APs. MNO who is the auctioneer allocates different APs' bandwidth to MSs. The winning MSs can use bandwidth determined by MNO.

the uncertainty of MNO's beliefs on MS's valuations using uncertainty set. Then, we introduce the general economical definitions for multi-item auction.

Let $\mathcal{N}$ denote the set of MSs, and $\mathcal{M}$ denote the set of APs owned by MNO, where $|\mathcal{N}| = n$ and $|\mathcal{M}| = m$. MS i has a private valuation for the unit bandwidth usage associated with AP j, denoted by v_{ij} which is unknown to MNO. Let $\boldsymbol{v} = \{v_{ij} | i \in \mathcal{N}, j \in \mathcal{M}\}$ denote the private valuation matrix. Thus, for AP j, $\boldsymbol{v}_j = (v_{1j}, \ldots, v_{nj})$ denotes the column vector of private valuation matrix $\mathcal{P}$. Moreover, MS i is budget constrained and the available budget is denoted by $B_i, i \in \mathcal{N}$, while AP j is bandwidth constrained and the available bandwidth is denoted by $C_j, j \in \mathcal{M}$. In this paper, we consider that the valuation information is private (only known to MS) and budget information is public (known to MNO). [1]

For AP j, since the private valuations of MSs are hidden from MNO, we model MNO's beliefs on the valuations of n MSs using uncertainty set $\mathcal{U}_j$, where the valuation vector $\boldsymbol{v}_j \in \mathcal{U}_j$. MNO's belief on valuations for all APs is denoted as $\mathcal{U} = \{\mathcal{U}_j\}_{j \in \mathcal{M}}$.

We assume that the valuations for AP j are independent and identically distributed, as well as the expectation and deviation of AP j are μ_j and δ_j respectively. Based on the central limit theory, the distribution of

$$\frac{\sum_{i=1}^{n} v_{ij} - n \cdot \mu_j}{\sqrt{n} \cdot \delta_j}$$

is approximately a standard normal distribution when $n \to \infty$.

[1]The budget information can be extended to private situation by uncertainty set with extra computaional complexity.

TABLE I
NOTATION USED IN THE PAPER

$\mathcal{N}$	Set of Mobile Subscribers
$\mathcal{M}$	Set of Access Points
$\mathcal{U}$	Uncertainty set of $\boldsymbol{v}$
$\boldsymbol{B} = \{B_i\}_{i \in \mathcal{N}}$	MS budget constraints
$\boldsymbol{D} = \{D_i\}_{i \in \mathcal{N}}$	MS bandwidth demand
$\boldsymbol{C} = \{C_i\}_{i \in \mathcal{M}}$	AP bandwidth constraints
$\boldsymbol{v} = \{v_{ij}\}_{i \in \mathcal{N}, j \in \mathcal{M}}$	Bid matrix
$\boldsymbol{v}_k = \{v_{kj}\}_{j \in \mathcal{M}}$	Bid vector of MS k
$\boldsymbol{v}_{-k} = \{v_{ij}\}_{i \in \mathcal{N} \setminus \{k\}, j \in \mathcal{M}}$	Bid vector except for MS k
$\boldsymbol{z} = \{z_{ij}\}_{i \in \mathcal{N}, j \in \mathcal{M}}$	Worst case bid vector
$\boldsymbol{x}^* = \{x^*_{ij}\}_{i \in \mathcal{N}, j \in \mathcal{M}}$	Nominal allocation in worst case
$\boldsymbol{r}^* = \{r^*_{ij}\}_{i \in \mathcal{N}, j \in \mathcal{M}}$	Reservation prices in worst case
$\boldsymbol{y}^{\boldsymbol{v}} = \{y^{\boldsymbol{v}}_{ij}\}_{i \in \mathcal{N}, j \in \mathcal{M}}$	Adapted allocation
$\boldsymbol{y}^{\boldsymbol{v}_{-k}} = \{y^{\boldsymbol{v}_{-k}}_{ij}\}_{i \in \mathcal{N} \setminus \{k\}, j \in \mathcal{M}}$	Adapted allocation without MS k
$\boldsymbol{a}^{\boldsymbol{v}} = \{a^{\boldsymbol{v}}_{ij}\}_{i \in \mathcal{N}, j \in \mathcal{M}}$	Real allocation
$\boldsymbol{p}^{\boldsymbol{v}} = \{p^{\boldsymbol{v}}_i\}_{i \in \mathcal{N}}$	Real payments

Thus, the uncertainty set $\mathcal{U}_j$ can be constructed as follows.

$$\mathcal{U}_j = \left\{ \; (v_{1j}, \ldots, v_{nj}) \; \middle| \; -\Gamma \leq \frac{\sum_{i=1}^{n} v_{ij} - n \cdot \mu_j}{\sqrt{n} \cdot \delta_j} \leq \Gamma \; \right\}, \tag{1}$$

where $\underline{F}_j$ and $\overline{F}_j$ are the lower bound and upper bound of the competition function $f_i(k)$, respectively. Γ is a parameter that controls the conservativeness of the historical valuations. For example, under the central limit theorem, the probability that $(\hat{v}_{1j}, \cdots, \hat{v}_{nj})$ belongs to

$$-\Gamma \leq \frac{\sum_{i=1}^{n} v_{ij} - n \cdot \mu_j}{\sqrt{n} \cdot \delta_j} \leq \Gamma$$

can be calculated by

$$\mathbb{P}\big((\hat{v}_{1j}, \cdots, \hat{v}_{nj}) \in \mathcal{U}_j\big) = 2\Phi(\Gamma) - 1, \tag{2}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal. If we set Γ to 1,2 and 3, then $\mathbb{P}((\hat{v}_{1j}, \cdots, \hat{v}_{nj}) \in \mathcal{U}_j)$ is 0.683, 0.955 and 0.997, respectively. A smaller Γ makes MNO consider only those valuations with higher probability. A larger Γ makes MNO consider a larger range of valuations, which increases the accuracy of auction at the cost of computational complexity. Thus, MNO needs to choose a proper Γ to balance the accuracy and computational complexity of the auction.

IV. Problem Statement

In this section, we formulate the multi-item auction based data offloading problem as a robust optimization problem. Our objective is to maximize the total revenue of MNO for all valuations in the uncertainty set $\mathcal{U}$. We first introduce the decision variables that represent the allocation rule and the payment rule. Then, we define the properties that the allocation and payment rules should satisfy in order to implement an efficient auction. The notations used in this paper are described in Table I.

A. Allocation and Payment Rules

The decision variable $\boldsymbol{x^v} = \{x_{ij}^{\boldsymbol{v}}\}_{i\in\mathcal{N},j\in\mathcal{M}}$ describes APs' bandwidth allocation among multiple MSs based on the valuation matrix $\boldsymbol{v}$, that is, if the valuation matrix is $\boldsymbol{v}$, MNO will allocate $x_{ij}^{\boldsymbol{v}}$ bandwidth of AP j to MS i. If MS i is not in the coverage area of AP j, then $x_{ij}^{\boldsymbol{v}} = 0$. The decision variable $\boldsymbol{p^v} = \{p_i^{\boldsymbol{v}}\}_{i\in\mathcal{N}}$ denotes the payment of MSs according to current valuation matrix $\boldsymbol{v}$, where $p_i^{\boldsymbol{v}}$ is the total payment of MS i for using the bandwidth of APs. Thus, $p_i^{\boldsymbol{v}} \geq 0$.

Given the allocation variable $\boldsymbol{x^v}$ and payment variable $\boldsymbol{p^v}$, we can derive the utility (i.e., the difference of total valuation and payment) of MS i as follows,

$$U_i^{\boldsymbol{v}} = \sum_{j\in\mathcal{M}} v_{ij}\cdot x_{ij}^{\boldsymbol{v}} - p_i^{\boldsymbol{v}}, \quad i\in\mathcal{N}, \quad \boldsymbol{v}\in\mathcal{U}. \tag{3}$$

The allocation and payment variables should satisfy the following properties in order to implement an efficient multi-item auction.

- Individual Rationality (IR). This property ensures non-negative utilities (i.e., the payment of MS should be less than his obtained valuation) for MSs who bid truthfully. Formally,
$$p_i^{\boldsymbol{v}} \leq \sum_{j\in\mathcal{M}} v_{ij}\cdot x_{ij}^{\boldsymbol{v}}, \forall i\in\mathcal{N}, \forall\boldsymbol{v}\in\mathcal{U}. \tag{4}$$
- Budget Feasibility (BF). This property ensures the payment of each MS is within his budget constraint. Formally,
$$p_i^{\boldsymbol{v}} \leq B_i, \forall i\in\mathcal{N}, \forall\boldsymbol{v}\in\mathcal{U}, \tag{5}$$
where B_i is the limited budget of MS i.
- Incentive Compatibility (IC). This property ensures that MS cannot improve his utility by bidding untruthfully. Thus, the utility of MS under truthful bidding is higher than untruthful biddings; this allows avoiding market manipulation by MSs. Formally,
$$\begin{aligned} &U_i^{(\boldsymbol{v}_i,\boldsymbol{v}_{-i})} \geq U_i^{(\boldsymbol{u}_i,\boldsymbol{v}_{-i})}, \forall i\in\mathcal{N},\\ &\forall(\boldsymbol{v}_i,\boldsymbol{v}_{-i})\in\mathcal{U}, \forall(\boldsymbol{u}_i,\boldsymbol{v}_{-i})\in\mathcal{U}, \end{aligned} \tag{6}$$
where $\boldsymbol{v}_i = \{v_{ij}\}_{j\in\mathcal{M}}$ is the truthful valuation of MS i and $\boldsymbol{u}_i = \{u_{ij}\}_{j\in\mathcal{M}}$ is a possible valuation of MS i. $\boldsymbol{v}_{-i} = \{v_{kj}\}_{k\in\mathcal{N}\setminus\{i\},j\in\mathcal{M}}$ denotes the valuation matrix obtained by omitting the valuations from MS i. By substituting Eq. (3) into Eq. (6), we have
$$\begin{aligned} &\sum_{j\in\mathcal{M}} v_{ij}\cdot x_{ij}^{(\boldsymbol{v}_i,\boldsymbol{v}_{-i})} - p_i^{(\boldsymbol{v}_i,\boldsymbol{v}_{-i})}\\ &\qquad \geq \sum_{j\in\mathcal{M}} v_{ij}\cdot x_{ij}^{(\boldsymbol{u}_i,\boldsymbol{v}_{-i})} - p_i^{(\boldsymbol{u}_i,\boldsymbol{v}_{-i})},\\ &\qquad \forall i\in\mathcal{N}, \forall(\boldsymbol{v}_i,\boldsymbol{v}_{-i})\in\mathcal{U}, \forall(\boldsymbol{u}_i,\boldsymbol{v}_{-i})\in\mathcal{U}, \end{aligned} \tag{7}$$
With some mathematical manipulation of Eq. (7), we obtain the following equation.
$$\begin{aligned} &\sum_{j\in\mathcal{M}} v_{ij}\cdot\left(x_{ij}^{(\boldsymbol{u}_i,\boldsymbol{v}_{-i})} - x_{ij}^{(\boldsymbol{v}_i,\boldsymbol{v}_{-i})}\right)\\ &\qquad + p_i^{(\boldsymbol{u}_i,\boldsymbol{v}_{-i})} - p_i^{(\boldsymbol{v}_i,\boldsymbol{v}_{-i})} \geq 0,\\ &\qquad \forall i\in\mathcal{N}, \forall(\boldsymbol{v}_i,\boldsymbol{v}_{-i})\in\mathcal{U}, \forall(\boldsymbol{u}_i,\boldsymbol{v}_{-i})\in\mathcal{U}. \end{aligned} \tag{8}$$

B. Optimal auction problem

The optimal auction design problem, based on the above property constraints, is formulated as a robust optimization problem, with the objective to maximize the revenue of MNO for all the valuations in set $\mathcal{U}$. Since MNO's beliefs on MSs' valuations are modeled as an uncertainty set, we focus on maximizing the worst case revenue. The network constraints, including APs' bandwidth constraints and MSs' demand constraints, are also formulated in the optimization problem.

$$\max_{\boldsymbol{x^v},\boldsymbol{p^v}} \quad W \tag{9a}$$

$$s.t. \quad W - \sum_{i\in\mathcal{N}} p_i^{\boldsymbol{v}} \leq 0, \forall\boldsymbol{v}\in\mathcal{U} \tag{9b}$$

$$p_i^{\boldsymbol{v}} \leq \sum_{j\in\mathcal{M}} v_{ij}\cdot x_{ij}^{\boldsymbol{v}}, \forall i\in\mathcal{N}, \forall\boldsymbol{v}\in\mathcal{U} \tag{9c}$$

$$p_i^{\boldsymbol{v}} \leq B_i, \forall i\in\mathcal{N}, \forall\boldsymbol{v}\in\mathcal{U} \tag{9d}$$

$$\begin{aligned} &\sum_{j\in\mathcal{M}} v_{ij}\cdot\left(x_{ij}^{(\boldsymbol{u}_i,\boldsymbol{v}_{-i})} - x_{ij}^{(\boldsymbol{v}_i,\boldsymbol{v}_{-i})}\right)\\ &+ p_i^{(\boldsymbol{u}_i,\boldsymbol{v}_{-i})} - p_i^{(\boldsymbol{v}_i,\boldsymbol{v}_{-i})} \geq 0, \forall i\in\mathcal{N},\\ &\forall(\boldsymbol{v}_i,\boldsymbol{v}_{-i})\in\mathcal{U}, \quad \forall(\boldsymbol{u}_i,\boldsymbol{v}_{-i})\in\mathcal{U} \end{aligned} \tag{9e}$$

$$\sum_{i\in\mathcal{N}} x_{ij}^{\boldsymbol{v}} \leq C_j, \forall j\in\mathcal{M}, \forall\boldsymbol{v}\in\mathcal{U} \tag{9f}$$

$$\sum_{j\in\mathcal{M}} x_{ij}^{\boldsymbol{v}} \leq D_i, \forall i\in\mathcal{N}, \forall\boldsymbol{v}\in\mathcal{U} \tag{9g}$$

$$x_{ij}^{\boldsymbol{v}} \geq 0, \forall i\in\mathcal{N}, \forall j\in\mathcal{M}, \forall\boldsymbol{v}\in\mathcal{U} \tag{9h}$$

$$p_i^{\boldsymbol{v}} \geq 0, \forall i\in\mathcal{N}, \forall\boldsymbol{v}\in\mathcal{U}. \tag{9i}$$

Constraint (9b) ensures the maximization of worst case revenue considering all the possible valuations in the uncertainty set $\mathcal{U}$. Constraints (9c), (9d) and (9e) correspond to IR, BF and IC properties, respectively. Constraint (9f) ensures that the bandwidth allocation should not exceed the available bandwidth of an AP. Constraint (9g) guarantees that each MS cannot obtain over-demanding bandwidth. Note that the demand D_i varies over time due to the stochastic nature of MS traffic. We consider a quasi-static network scenario [29], and analyze the auction mechanism in a data offloading period (e.g., ten seconds), during which D_i remains unchanged for all $i\in\mathcal{N}$. Finally, Constraint (9i) prevents negative allocation and payment for MSs. Note that $\boldsymbol{v}\in\mathcal{U}$ is defined as $\{\boldsymbol{v} = (\boldsymbol{v}_1,\cdots,\boldsymbol{v}_m)|\boldsymbol{v}_j\in\mathcal{U}_j, \forall j\in\mathcal{M}\}$. $\boldsymbol{v}_j\in\mathcal{U}_j$ is short for the following two constraints derived from Eq. (1).

$$\begin{aligned} &\sum_{i=1}^{n} v_{ij} - n\cdot\mu_j \leq \Gamma\sqrt{n}\cdot\delta_j, \forall j\in\mathcal{M},\\ &\sum_{i=1}^{n} v_{ij} - n\cdot\mu_j \geq -\Gamma\sqrt{n}\cdot\delta_j, \forall j\in\mathcal{M}. \end{aligned}$$

For simplicity, we use $\boldsymbol{v}\in\mathcal{U}$ to stand for the above constraints in the rest of the paper.

V. Optimal auction mechanism

In this section, we propose the optimal auction mechanism to solve the optimization problem (9) in order to determine an optimal allocation and payment rules. That is, how APs' bandwidth is shared among multiple MSs, and how much MSs are charged for using allocated bandwidth. Our optimal auction mechanism illustrated in Algorithm 1, takes as input the uncertainty set $\mathcal{U}$, MS budget vector $\boldsymbol{B}$, AP constraint vector $\boldsymbol{C}$, MS demand vector $\boldsymbol{D}$ and bid matrix $\boldsymbol{v}$, and calculates as output the real allocation matrix $\boldsymbol{a^v}$ and the payment vector $\boldsymbol{p^v}$. We will refer to Algorithm 1 as Optimal Algorithm in the rest of paper. We first introduce the details of Optimal Algorithm. Then, we show the auction properties of Optimal Algorithm.

Algorithm 1 Optimal offloading auction mechanism

Input: $\mathcal{U}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}, \boldsymbol{v}, \mathcal{M}, \mathcal{N}$
Output: $\boldsymbol{a^v}$, $\boldsymbol{p^v}$

1: $(\boldsymbol{z}, \boldsymbol{x}^*) \leftarrow$ solving problem (10)
2: $(\boldsymbol{\xi}^*, \boldsymbol{\eta}^*, \boldsymbol{\lambda}^*, \boldsymbol{\theta}^*) \leftarrow$ solving problem (13)
3: **for** $i \in \mathcal{N}$ **do**
4:     **for** $j \in \mathcal{M}$ **do**
5:         $r_{ij}^* = \xi_j^* + (\eta_i^* + \lambda_i^* + \theta_i^*) \cdot z_{ij}$
6:     **end for**
7: **end for**
8: $\boldsymbol{y^v} \leftarrow$ solving problem (14)
9: **for** $k \in \mathcal{N}$ **do**
10:     $\boldsymbol{y}^{\boldsymbol{v}_{-k}} \leftarrow$ solving problem (15)
11: **end for**
12: **for** $i \in \mathcal{N}$ **do**
13:     **for** $j \in \mathcal{M}$ **do**
14:         $a_{ij}^{\boldsymbol{v}} = x_{ij}^* + y_{ij}^{\boldsymbol{v}}$
15:     **end for**
16: **end for**
17: **for** $k \in \mathcal{N}$ **do**
18:     Calculate $p_k^{\boldsymbol{v}}$ using Eq. (16)
19: **end for**

Fig. 2 shows the relationship among different optimization problems in Optimal Algorithm. Optimal Algorithm consists of two phases, the phase of nominal allocation (Steps $1-7$, left column of Fig. 2) and the phase of final allocation (Steps $8-19$, right column of Fig 2). The aim of nominal allocation is to calculate the reservation price $\boldsymbol{r}^* = \{r_{ij}\}_{i\in\mathcal{N}, j\in\mathcal{M}}$ and the nominal allocation $\boldsymbol{x}^* = \{x_{ij}\}_{i\in\mathcal{N}, j\in\mathcal{M}}$. MS i has to bid at least r_{ij} in order to use the bandwidth provided by AP j. $\boldsymbol{x}^*$ represents the best allocation in worst case scenario, which is part of the final allocation calculated in the phase of final allocation. Reservation price is obtained by calculating problems (10) and (13) sequentially. Final allocation calculates the real allocation $\boldsymbol{a^v}$ and final payment $\boldsymbol{p^v}$ based on a specific bid matrix $\boldsymbol{v}$. The final allocation $\boldsymbol{a^v} = \boldsymbol{x}^* + \boldsymbol{y^v}$, where $\boldsymbol{y^v} = \{y_{ij}^{\boldsymbol{v}}\}_{i\in\mathcal{N}, j\in\mathcal{M}}$, called adapted allocation, denotes the best allocation for a specific bid matrix $\boldsymbol{v}$. Final allocation is based on the results of optimization problems (14) and (15). Problems (14) and (15) can be calculated independently, since their inputs don't rely the results of each other.

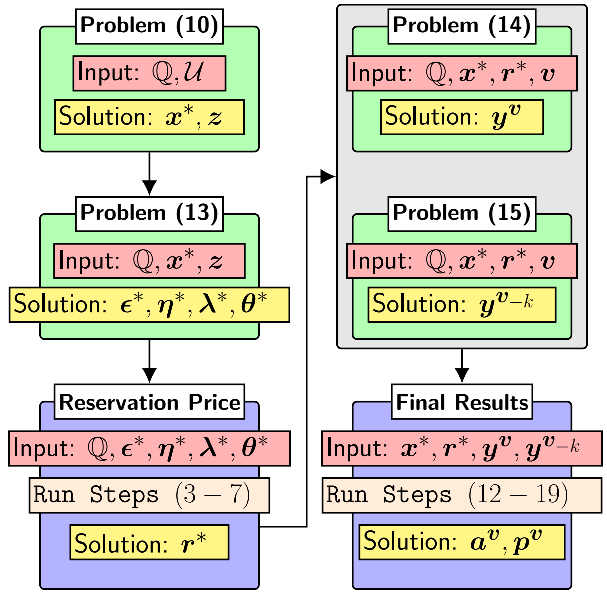


Fig. 2. Flow chart of the proposed Optimal Algorithm. The left column denotes the nominal allocation phase, while the right column denotes the final allocation phase. Note that set $\mathbb{Q} = \{\boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}, \mathcal{M}, \mathcal{N}\}$ contains the information of MSs' budgets and demands, as well as the capacity constraints of APs.

A. Phase of Nominal Allocation

In the phase of nominal allocation, Step 1 calculates the worst case bid matrix $\boldsymbol{z}$ and reservation price $\boldsymbol{r}^*$ by solving the bilinear optimization problem (10), where the constraints (10b), (10c) and (10d) are derived from constraints (9d), (9f) and (9g), respectively. Constraint (10e) that captures the IC and IR properties of problem (9) is used to calculate the worst case bid matrix $\boldsymbol{z}$, under which the obtained payoff $\sum_{j\in\mathcal{M}} x_{ij} \cdot z_{ij}$ for MS i is minimum. The nominal allocation $\boldsymbol{x}^*$ is a preallocation that corresponds to the worst case bid matrix $\boldsymbol{z}$.

$$\max_{\boldsymbol{x},\boldsymbol{v}} \quad \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{M}} x_{ij} v_{ij} \tag{10a}$$

$$s.t. \quad \sum_{j\in\mathcal{M}} x_{ij} \cdot v_{ij} \le B_i, \forall i \in \mathcal{N}, \tag{10b}$$

$$\sum_{i\in\mathcal{N}} x_{ij} \le C_i, \forall j \in \mathcal{M}, \tag{10c}$$

$$\sum_{j\in\mathcal{M}} x_{ij} \le D_j, \forall i \in \mathcal{N}, \tag{10d}$$

$$\sum_{j\in\mathcal{M}} x_{ij} \cdot v_{ij} \le \sum_{j\in\mathcal{M}} x_{ij} \cdot u_{ij}, \forall \boldsymbol{u} \in \mathcal{U}, \forall i \in \mathcal{N}, \tag{10e}$$

$$\boldsymbol{x} \ge 0, \ \boldsymbol{v} \in \mathcal{U}. \tag{10f}$$

In order to obtain the reservation price $\boldsymbol{r}^*$, We first simplify the problem (10) as a linear programming problem with decision variable $\boldsymbol{x}$ by: 1) replacing variable $\boldsymbol{v}$ with constant $\boldsymbol{z}$ (obtained in Step 1); 2) replacing Constraint (10e) with Eq. (11).

$$\sum_{j\in\mathcal{M}} x_{ij}\cdot v_{ij} \le \sum_{j\in\mathcal{M}} x^*_{ij}\overline{u}^i_j, \quad \forall i\in\mathcal{N}, \tag{11}$$

where

$$\overline{\boldsymbol{u}}^i = arg\min_{\boldsymbol{u}\in\mathcal{U}} \sum_{j\in\mathcal{M}} x^*_{ij}\cdot u_{ij}, \quad \forall i\in\mathcal{N}. \tag{12}$$

Then, we can obtain the dual problem of simplified problem (10) as follows.

$$\min_{\boldsymbol{\xi},\boldsymbol{\eta},\boldsymbol{\lambda},\boldsymbol{\theta}} \quad \sum_{j\in\mathcal{M}}\xi_j C_i + \sum_{i\in\mathcal{N}}\Big(\eta_i B_i + \lambda_i D_i + \theta_i \sum_{j\in\mathcal{M}} x^*_{ij}\overline{u}^i_j\Big) \tag{13a}$$

$$s.t. \quad \xi_j + z_{ij}(\eta_i+\lambda_i+\theta_i) \ge z_{ij}, \forall i\in\mathcal{N}, j\in\mathcal{M}, \tag{13b}$$

$$\xi_j, \eta_i, \lambda_i, \theta_i \ge 0, \forall i\in\mathcal{N}, j\in\mathcal{M}. \tag{13c}$$

The decision variables $\boldsymbol{\xi}^* = \{\xi^*_j\}_{j\in\mathcal{M}}$, $\boldsymbol{\eta}^* = \{\eta^*_i\}_{i\in\mathcal{N}}$, $\boldsymbol{\lambda}^* = \{\lambda^*_i\}_{i\in\mathcal{N}}$ and $\boldsymbol{\theta}^* = \{\theta^*_i\}_{i\in\mathcal{N}}$ correspond to the constraints (10c), (10b), (10d) and (11), respectively. Step 2 calculates the solution of dual problem (13) used to obtain the reservation price $\boldsymbol{r}^*$ in Steps $3-7$, where r^*_{ij} represents the minimum price that MS i should bid in order to use bandwidth of AP j.

B. Phase of Final Allocation

In the phase of final allocation, We first calculates the adapted allocation $\boldsymbol{y}^{\boldsymbol{v}}$ based on bid matrix $\boldsymbol{v}$ in Step 8. The adapted allocation $\boldsymbol{y}^{\boldsymbol{v}}$ is obtained by solving the linear problem (14). The objective function (Eq. (14a)) of this problem maximizes the social welfare (i.e., the total valuations of all MSs) taking into consideration the reservation price $\boldsymbol{r}^*$. Thus, Constraints (14b), (14c) and (14d) are adjusted by considering the impact of nominal allocation $\boldsymbol{x}^*$ and reservation price $\boldsymbol{r}^*$ obtained from the phase of nominal allocation.

$$\max_{\boldsymbol{y}^{\boldsymbol{v}}} \quad \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{M}} y^{\boldsymbol{v}}_{ij}\cdot(v_{ij}-r^*_{ij}) \tag{14a}$$

$$s.t. \quad \sum_{i\in\mathcal{N}} y^{\boldsymbol{v}}_{ij} \le C_i - \sum_{i\in\mathcal{N}} x^*_{ij}, \quad \forall j\in\mathcal{M}, \tag{14b}$$

$$\sum_{j\in\mathcal{M}} y^{\boldsymbol{v}}_{ij}\cdot v_{ij} \le B_i - \sum_{j\in\mathcal{M}} x^*_{ij}\cdot r^*_{ij}, \quad \forall i\in\mathcal{N}, \tag{14c}$$

$$\sum_{j\in\mathcal{M}} y^{\boldsymbol{v}}_{ij} \le D_j - \sum_{j\in\mathcal{M}} x^*_{ij}, \quad \forall i\in\mathcal{N}. \tag{14d}$$

Then we calculate the adapted allocation $\boldsymbol{y}^{\boldsymbol{v}_{-k}}$ without considering the auction participation of MS k in Steps $9-11$. $\boldsymbol{y}^{\boldsymbol{v}_{-k}}$ is used to calculate the final payment of MS k and is obtained by solving the linear problem (15), which is a reduced version of problem (14) by deleting the bidder k from the set of bidders.

$$\max_{\boldsymbol{y}^{\boldsymbol{v}_{-k}}} \quad \sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y^{\boldsymbol{v}_{-k}}_{ij}\cdot(v_{ij}-r^*_{ij}) \tag{15a}$$

$$s.t. \quad \sum_{i\in\mathcal{N}\setminus\{k\}} y^{\boldsymbol{v}_{-k}}_{ij} \le C_i - \sum_{i\in\mathcal{N}\setminus\{k\}} x^*_{ij}, \quad \forall j\in\mathcal{M}, \tag{15b}$$

$$\sum_{j\in\mathcal{M}} y^{\boldsymbol{v}_{-k}}_{ij}\cdot v_{ij} \le B_i - \sum_{j\in\mathcal{M}} x^*_{ij}r^*_{ij}, \quad \forall i\in\mathcal{N}\setminus\{k\}, \tag{15c}$$

$$\sum_{j\in\mathcal{M}} y^{\boldsymbol{v}_{-k}}_{ij} \le D_j - \sum_{j\in\mathcal{M}} x^*_{ij}, \quad \forall i\in\mathcal{N}\setminus\{k\}. \tag{15d}$$

With $\boldsymbol{x}^*$ and $\boldsymbol{r}^*$ obtained in the phase of nominal allocation, as well as $\boldsymbol{y}^{\boldsymbol{v}}$ and $\boldsymbol{y}^{\boldsymbol{v}_{-k}}$ obtained in this phase, we can calculate the final allocation $\boldsymbol{a}^{\boldsymbol{v}}$ and the final payment $\boldsymbol{p}^{\boldsymbol{v}}$ for all $k\in\mathcal{N}$. Steps $12-16$ calculate the final allocation $\boldsymbol{a}^{\boldsymbol{v}}$ that is the sum of nominal allocation $\boldsymbol{x}^*$ and adapted allocation $\boldsymbol{y}^{\boldsymbol{v}}$. Steps $17-19$ calculate the final payment $\boldsymbol{p}^{\boldsymbol{v}}$ using Eq. (16), where $p^{\boldsymbol{v}}_k$ consists of the payment of using $\boldsymbol{a}^{\boldsymbol{v}}_k$ bandwidth and the difference between the optimal value of the objective function obtained with and without the participation of k. This payment scheme guarantees the IR property of Optimal Algorithm. Furthermore, we show that Optimal Algorithm can implement an efficient auction according to Theorem 1.

$$\begin{aligned} p^{\boldsymbol{v}}_k =& \sum_{j\in\mathcal{M}} y^{\boldsymbol{v}}_{kj}\cdot r^*_{kj} + \sum_{j\in\mathcal{M}} x^*_{kj}\cdot r^*_{kj} \\ &+ \sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y^{\boldsymbol{v}_{-k}}_{ij}(v_{ij}-r^*_{ij}) - \sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y^{\boldsymbol{v}}_{ij}(v_{ij}-r^*_{ij}), \\ &\forall k\in\mathcal{N}. \end{aligned} \tag{16}$$

Theorem 1. *The proposed auction mechanism illustrated in Optimal Algorithm has the properties of incentive compatibility, budget feasibility, individual rationality and worst case optimality.*

The proof of Theorem 1 is illustrated in Appendix A.

C. Design Rational

We discuss the relationship of our proposed mechanism and VCG mechanism as follows.

(a) The allocation rule has a structure similar to that of VCG mechanism, where the bandwidth is allocated to a set of MUs in order to maximize a social welfare function. In Optimal Algorithm, the social welfare function is defined in Eq. (14a), which is parameterized by the reservation price $\boldsymbol{r^*}$.
(b) The payment rule, as defined in Eq. (16), is also similar to that of VCG mechanism. Each MU is charged with the opportunity cost, which is defined as the lowest amount that MU has to bid in order to win the allocation.
(c) Unlike VCG mechanism, we calculate the reservation price $\boldsymbol{r^*}$ in the worst case. Thus, r_{ij} is defined as the lowest price that MNO would be willing to accept for allocating the corresponding bandwidth from AP j to

MS i. The reservation price is a threshold price; the bids less than the reservation price will not be accepted. The reservation price can accelerate the auction process, since the set of prices that are lower than the reservation price can be discarded.

(d) Unlike VCG mechanism, we focus on the case where the payments of MUs provided by the optimal mechanism do not exceed their budget constraints. Standard mechanisms, such as VCG mechanism and its variants, are not applicable here [30, 31].

In summary, the well-known VCG mechanism is a dominant strategy mechanism, which can achieve ex-post incentive compatibility (truth-telling is a dominant strategy for every player in the game). However, VCG mechanism cannot implement the budget feasibility of the auction, which costs extra payment from MUs and decreases their payoffs. Thus, it cannot be properly used in the problem that we are solving in this paper. Compared with VCG mechanism, our proposed optimal mechanism is an incentive efficient mechanism that can maximize the expected total payoff of all MUs. Additionally, it achieves the budget feasibility of MUs. There is no extra cost paid in the auction when applying our optimal mechanism while VCG mechanism can not.

D. Solving Optimal Algorithm

Solving Optimal Algorithm needs to calculate one bilinear optimization problem (10) and three linear optimization problems (13), (14) and (15). The linear problems can be solved using simplex method [32]. The bilinear problem, which is the computation intensive step in the proposed mechanism, is NP-hard [33]. However, we can solve problem (10) in polynomial time to achieve global ϵ-optimal solution. This is based on the observation that both inner and outer optimization problems of problem (10) are linear optimization problems. Thus, fixing the inner optimal solution, there always exists an extreme point solution to the outer problem and vice versa. We can use Bender decomposition algorithm [34] to solve problem (10) by simply enumerating all the extreme points. Please refer to [34] for details.

VI. Greedy Auction Mechanism

In this section, we turn to the concept of two-sided matching [35] to solve the data offloading problem in polynomial time. In our two-sided matching scenario, one matching partners are MSs and another matching partners are APs. Note that each AP can be matched to multiple MSs. We propose two greedy auction mechanisms: 1) *MatchingAP* scheme, i.e., it is AP which selects MSs that it will provide network connection to; 2) *MatchingMS* scheme, i.e., it is MS which selects appropriate AP for network connection. Then, we show that these two algorithms satisfy the properties of individual rationality and incentive capability.

A. MatchingAP Scheme

The greedy algorithm for MatchingAP scheme, illustrated in Algorithm 2, is composed of two phases, namely, allocation phase and payment phase. The allocation phase aims to select MSs for each AP that can offload mobile data traffic. The payment phase calculates the price paid by each winner by considering the maximum bid from un-winning MSs. This payment scheme is widely used in second price auction to derive a truthful bidding [36].

Algorithm 2 Greedy MatchingAP Scheme

Input: $\boldsymbol{b}, \boldsymbol{d}, \mathcal{M}, \mathcal{N}, \boldsymbol{C}$
Output: $\boldsymbol{a}, \boldsymbol{p}$
1: $\boldsymbol{M} \leftarrow Sort(j \in \mathcal{M}, \frac{C_j}{|\mathcal{N}_j|}, \text{“}non-decreasing''})$
2: $\boldsymbol{N} \leftarrow \mathcal{N}$
3: **while** $\boldsymbol{M} \neq \varnothing \wedge \boldsymbol{N} \neq \varnothing$ **do**
4: $\quad j \leftarrow Next(\boldsymbol{M})$, $\boldsymbol{M} \leftarrow \boldsymbol{M} \setminus \{j\}$
5: $\quad \boldsymbol{N}_j \leftarrow Sort(i \in \mathcal{N}_j, b_i, \text{“}non-decreasing''})$
6: $\quad$ **while** $\sum_{i \in \mathcal{N}_j} a_{ij} \leq C_j \wedge \boldsymbol{N}_j \neq \varnothing$ **do**
7: $\quad\quad i \in Next(\boldsymbol{N}_j)$
8: $\quad\quad$ **if** $\sum_{j \in \mathcal{M}} a_{ij} = 0 \wedge d_i + \sum_{i \in \mathcal{N}_j} a_{ij} \leq C_j$ **then**
9: $\quad\quad\quad a_{ij} \leftarrow d_i$
10: $\quad\quad\quad \boldsymbol{N} \leftarrow \boldsymbol{N} \setminus \{i\}$
11: $\quad\quad$ **end if**
12: $\quad$ **end while**
13: **end while**
14: **for all** $j \in \mathcal{M}$ **do**
15: $\quad p_k \leftarrow \max_{\{i \in \mathcal{N}_j | a_{ij} = 0\}} b_i$
16: $\quad$ **for all** $i \in \mathcal{N}_j \wedge a_{ij} = d_i$ **do**
17: $\quad\quad p_i \leftarrow p_j \cdot d_i$
18: $\quad$ **end for**
19: **end for**

In Algorithm 2, Step 1 defines the allocation order for the set of APs. The sorted list $\boldsymbol{M}$ is obtained by sorting all APs participating in the auction in a non-decreasing order of bandwidth per number of covered MSs (i.e., the potential bidders for each AP). The allocation phase (Steps $3-13$) considers APs starting from the first AP in $\boldsymbol{M}$. In MatchingAP scheme, each AP can select MSs under its radio coverage area as potential bidders. Since one AP may have multiple bidders, we define an allocation rule for each AP, which states that the bidder who bids higher value has a higher probability to be served, as shown in Step 5, where MSs under the coverage of AP j are sorted in a non-decreasing order according to the bids submitted by MSs. The bandwidth allocation phase continues until AP j has allocated all its bandwidth or it has no more MSs to be considered (Step 6). For each MS, if it is not allocated to other APs (i.e., served by other APs) and the network demand does not exceed the bandwidth of AP j, it will be allocated to AP j (Steps $8-9$). The payment phase (Steps $14-19$) defines the price paid by each winning MS as the maximum bid value of the set of un-winning MSs. The final payment of MS i is calculated by the market clearing price p_k (obtained in Step 15) and the network demand D_i (Step 17).

B. MatchingMS Scheme

In the following, we present the greedy algorithm illustrated in Algorithm 3 for MatchingMS scheme; it has the same

Algorithm 3 Greedy MatchingMS Scheme

Input: $\boldsymbol{b}, \boldsymbol{d}, \mathcal{M}, \mathcal{N}, \boldsymbol{C}$

Output: $\boldsymbol{a}, \boldsymbol{p}$

1: $\boldsymbol{N} \leftarrow Sort(i \in \mathcal{N}, \max_{j\in\mathcal{M}} b_{ij}, \text{“}non-decreasing''\text{)}$
2: $\boldsymbol{M} \leftarrow \mathcal{M}$
3: **while** $\boldsymbol{M} \neq \varnothing \wedge \boldsymbol{N} \neq \varnothing$ **do**
4: $\quad i \leftarrow Next(\boldsymbol{N})$, $\boldsymbol{N} \leftarrow \boldsymbol{N} \setminus \{i\}$
5: $\quad \boldsymbol{M}_i \leftarrow Sort(j \in \mathcal{M}_i, C_j, \text{“}non-decreasing''\text{)}$
6: $\quad$ **while** $\sum_{j\in\mathcal{M}} a_{ij} < d_i \wedge \boldsymbol{M}_i \neq \varnothing$ **do**
7: $\quad\quad j \in Next(\boldsymbol{M}_i)$
8: $\quad\quad$ **if** $d_i + \sum_{i\in\mathcal{N}_j} a_{ij} \leq C_j$ **then**
9: $\quad\quad\quad a_{ij} \leftarrow d_i$
10: $\quad\quad\quad \boldsymbol{M} \leftarrow \boldsymbol{M} \setminus \{j\}$
11: $\quad\quad$ **end if**
12: $\quad$ **end while**
13: **end while**
14: **for all** $j \in \mathcal{M}$ **do**
15: $\quad p_k \leftarrow \max_{\{i\in\mathcal{N}_j | a_{ij}=0\}} b_i$
16: $\quad$ **for all** $i \in \mathcal{N}_j \wedge a_{ij} = d_i$ **do**
17: $\quad\quad p_i \leftarrow p_j \cdot d_i$
18: $\quad$ **end for**
19: **end for**

algorithm structure as MatchingAP scheme. It also includes allocation and payment phases. Particularly, MatchingMS scheme has same payment rule as MatchingAP scheme.

In Algorithm 3, Step 1 sorts the set of MSs by the maximum bid in a non-decreasing order. Since we aim to maximize the revenue of MNO, MSs are considered according to the allocation order obtained in $\boldsymbol{N}$. The allocation phase (Steps $3-13$) terminates until all MSs or APs are considered. In the inner loop, MS selects one AP that can provide network connection to it. APs that cover MS i are sorted in the list $\boldsymbol{M}_i$ according to bandwidth (Step 5). The network selection phase continues until MS i has selected one AP or it has no more APs to consider (Step 6). For each AP, if it has enough bandwidth to satisfy the demand of MS, it will be selected by MS (Steps $8-9$). The payment phase (Steps $14-19$) is the same as that in Algorithm 2.

These two algorithms satisfy the properties of individual rationality and incentive capability, since they adopted the similar auction structure used in [22]. The budget feasibility is satisfied by the fact that the payment of each MS will not be greater than its bid, i.e., if MS i selects bid $b_i \leq \frac{B_i}{d_i}$, then its final payment satisfies $p_i \leq b_i$.

We next analyze the time complexity of MatchingAP and MatchingMS. We consider the time complexity of MatchingAP in three parts.

- (Step 1) In MatchingAP algorithm, the construction of an AP preference list is the first step. Since there are m APs, with an efficient sorting algorithm, we can get the AP preference list in time of $\mathcal{O}(m\log(m))$.
- (Steps $3-13$) We first consider the outer while loop of the algorithm. This loop will terminate when the set of APs or the set of MSs becomes empty. Thus, the maximum number of loops is $max\{m,n\}$. In step 5, we construct an MS preference list for each AP with the complexity of $\mathcal{O}(n\log(n))$. Then, we consider the inner while loop from step 6 to step 12. It is obvious that the maximum number of loops is n, which is the total attempts made by an AP. Indeed, assume that every summation has time complexity $\mathcal{O}(1)$. The total complexity of inner loop is $\mathcal{O}(n)$. Since step 5 has higher complexity than that of inner loop, the total complexity of outer loop is $\mathcal{O}(n\log(n)max\{m,n\})$.
- (Steps $14-19$) It is easy to see that the time complexity of these steps is $\mathcal{O}(mn)$.

Finally, the total complexity of the MatchingAP is $\mathcal{O}\big(max\{m\log(m),(n\log(n)max\{m,n\}),mn\}\big)$. In general cases, the number of MSs is larger than that of APs, i.e., $n > m$. Thus, the time complexity of MatchingAP is $\mathcal{O}(n^2\log(n))$, mainly due to steps $3-13$. Following the similar analysis, we can obtain that the time complexity of MatchingMS is $\mathcal{O}\big(max\{n\log(n),(m\log(m)max\{m,n\}),mn\}\big)$. In general cases, the time complexity of MatchingMS is $\mathcal{O}\big((mn\log(m))\}\big)$.

VII. Numerical Results

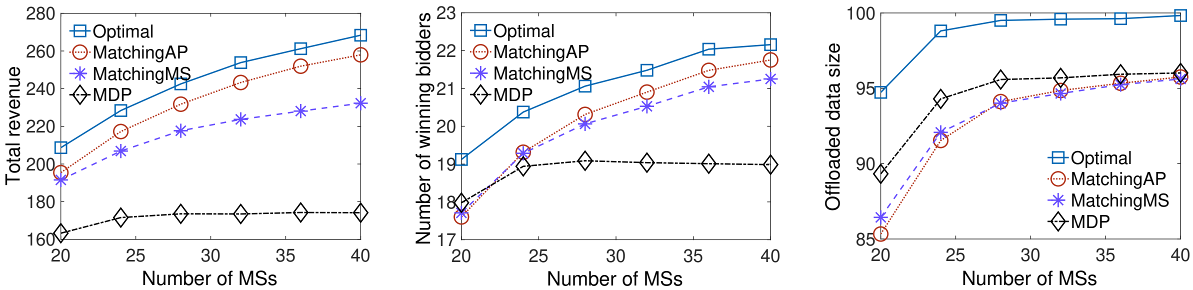
In this section, we evaluate the performance of the proposed auction mechanism for selling APs' bandwidth to MSs in proximity. More specifically, we aim to evaluate the impact of AP density (the number of APs), budget constraint and uncertainty set of valuation on the performance of the proposed mechanisms in order to implement an effective mobile data offloading marketplace. We first introduce the parameter settings, then we illustrate and discuss the numerical results achieved by the proposed offloading schemes.

We compare our proposed schemes, namely optimal scheme (Optimal Algorithm) and two greedy schemes (i.e., MatchingAP scheme and MatchingMS scheme), with the work in [37, 38], denoted as MDP scheme, since this work aims to maximize the amounts of offloaded data based on Markov Decision Process. The following performance metrics are considered in the evaluation.

- Total revenue: The total payoff of MNO.
- Offloaded traffic: The amount of traffic that can be offloaded.
- Winning MSs: The number of MSs that win the auction.

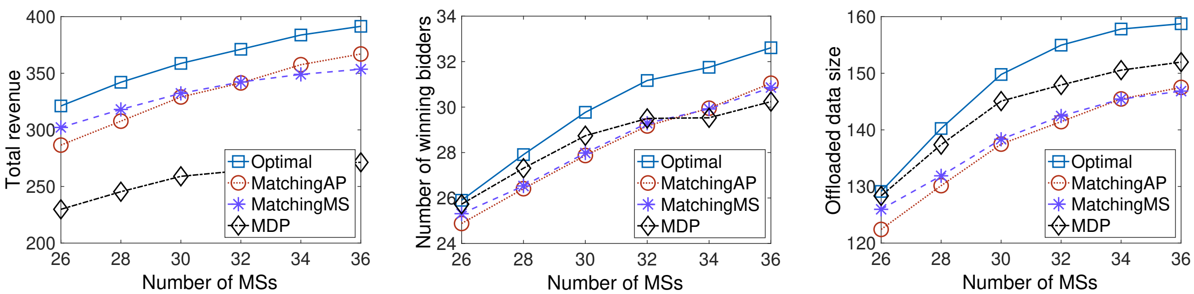
A. Simulation Setup

In our evaluation, we consider a measurement-based model [29], where there is an MNO represented by a macrocell BS. The number of APs and MSs, located in the coverage of BS, are chosen uniformly from the intervals $[2, 20]$ and $[10, 60]$, respectively. Unless stated otherwise, we use the information from [19, 29, 39] to set the parameters' values. Each MS submits a bid drawn from a normal distribution with mean value equal to $2\$/Mb$ and derivation equal to $1\$/Mb$. The maximum bandwidth of each AP is in the range of $[5Mbps, 40Mbps]$, while the traffic demand of each MS is in the range of $[2Mbps, 10Mbps]$. The budget of MS is selected from the range of $[10\$, 20\$]$. We use the historical bidding information, i.e., μ_j and δ_j, to construct the uncertainty set U_j, $\forall j \in \mathcal{M}$. We assume that μ_j and δ_j are drawn randomly from



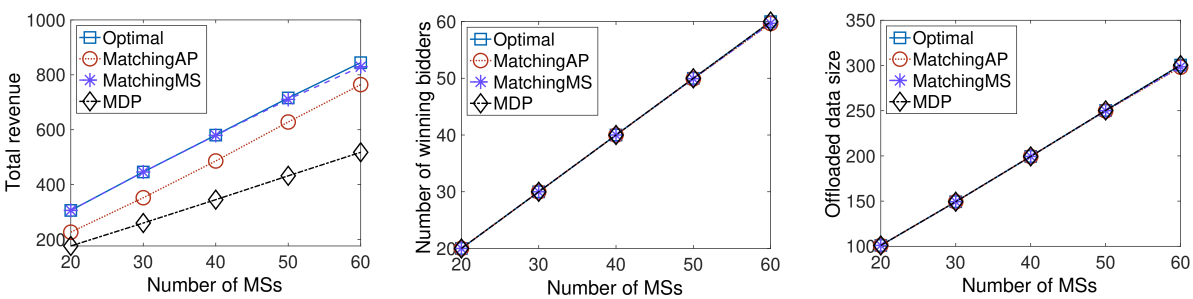
(a) Revenue (b) Number of Winners (c) Offloaded Traffic

Fig. 3. Performance comparison with low AP density ($m = 5$).



(a) Revenue (b) Number of Winners (c) Offloaded Traffic

Fig. 4. Performance comparison with medium AP density ($m = 10$).



(a) Revenue (b) Number of Winners (c) Offloaded Traffic

Fig. 5. Performance comparison with high AP density ($m = 20$).

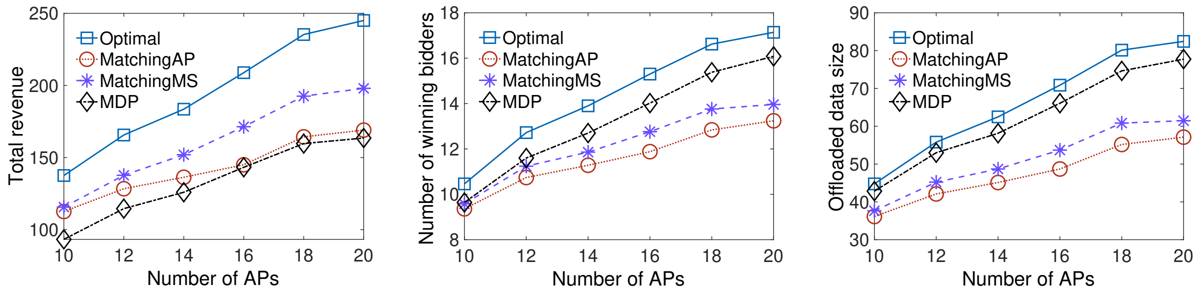
the intervals $[1\$, 3\$]$ and $[1\$, 2\$]$, respectively. We consider the scenario with high conservativeness of historical valuations by setting Γ to 1. To compare the performance of two greedy schemes, we define I_m^n as $\frac{n}{m}$, which is the ratio between the number of MSs and the number of APs. I_m^n measures the competition among MSs. Larger value of I_m^n implies higher competition among MSs.

B. Impact of AP Density in Homogeneous Networks

In order to evaluate the impact of AP density on the performance of our proposed mechanisms, we consider three levels of AP density, i.e., m is equal to 5, 10, and 20, respectively. Each AP's bandwidth is set to $30Mbps$. The simulation results are shown in Figs. 3, 4 and 5.

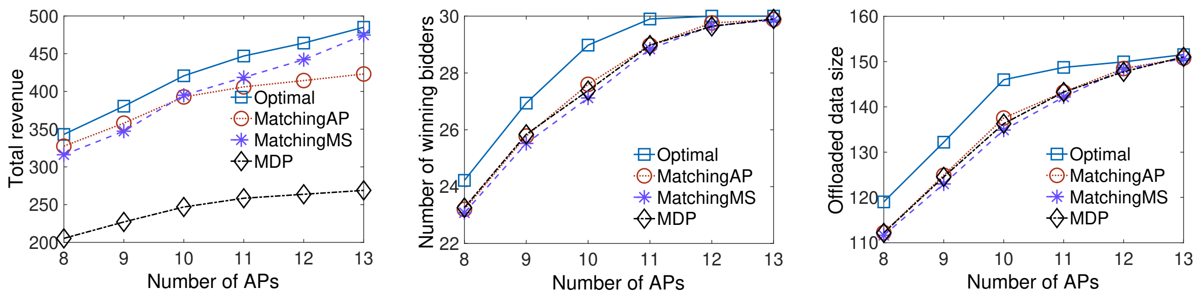
We first evaluate MNO's revenue for three levels of AP density, as shown in Figs. 3(a), 4(a) and 5(a), respectively. We observe that MNO's revenue increases with the number of MSs. The larger number of MSs, the higher competition MSs may have, and consequently MNO can choose MSs with higher bid values. Moreover, optimal scheme outperforms two greedy schemes and MDP scheme in all scenarios. We further observe that MatchingAP outperforms MatchingMS in low AP density scenario (see in Fig. 3(a)), while MatchingMS outperforms MatchingAP in high AP density scenario ((see in Fig. 5(a))). This is because MatchingAP can take advantage of the competition among MSs to obtain higher revenue. This observation can be further validated by Fig. 4(a), where MatchingAP achieves higher revenue than MatchingMS only when $n > 32$. Note that $m = 10$ in Fig. 4(a). Thus, we can obtain a threshold ratio when MatchingAP outperforms MatchingMS; that is $I_m^{n*} = 3.2$ in our settings. Since $I_m^n \geq 4 > I_m^{n*}$ in Fig. 3(a), MatchingAP achieves higher revenue than MatchingMS. In Fig. 5(a), MatchingAP achieves lower revenue than MatchingMS due to $I_m^n \leq 3 < I_m^{n*}$.

We then evaluate the number of winning bidders of dif-



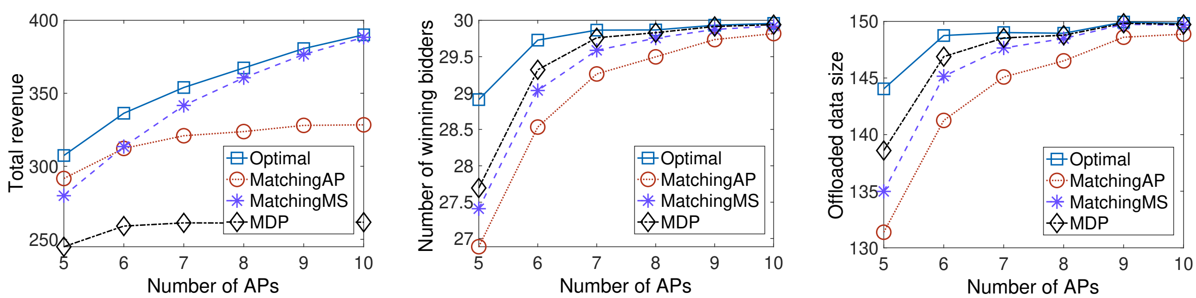
(a) Revenue (b) Number of Winners (c) Offloaded Traffic

Fig. 6. Performance comparison with low capacity ($C_j = 5$ Mbps)



(a) Revenue (b) Number of Winners (c) Offloaded Traffic

Fig. 7. Performance comparison with medium capacity ($C_j = 25$ Mbps)



(a) Revenue (b) Number of Winners (c) Offloaded Traffic

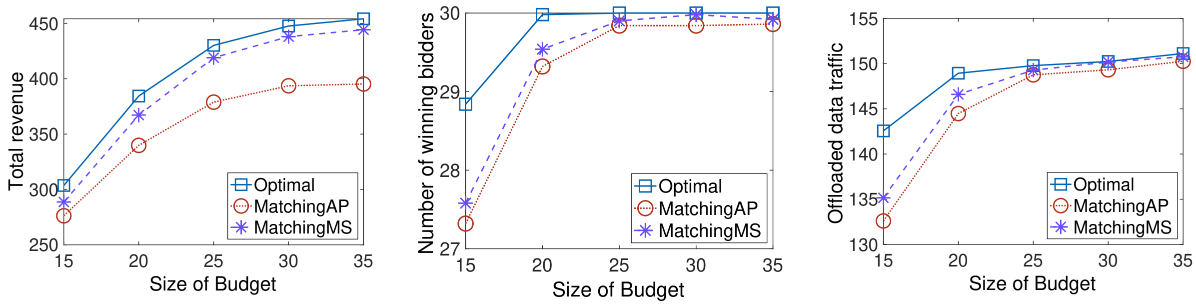
Fig. 8. Performance comparison with high capacity ($C_j = 40$ Mbps)

ferent schemes. Figs. 3(b), 4(b) and 5(b) show that optimal scheme has the largest number of winning MSs; this indicates that optimal scheme can implement better fairness allocation among multiple MSs. Figure 5(b) illustrates that the number of winning MSs for all schemes increase linearly with the number of MSs. This is because high AP density implies enough bandwidth for traffic demand from MSs. However, it is not the same case for low AP density and medium AP density, where the total APs' bandwidth is not sufficient to support a large number of MSs. By comparing Figs. 5(a) and 5(b), we observe that MatchingMS achieves higher revenue than MatchingAP, even when two greedy schemes have the same number of winning MSs. This observation implies that two greedy schemes allocate bandwidth to different sets of MSs and MatchingMS can select the set of MSs with higher bid values in high AP density scenario.

We finally investigate how AP density affects the data offloading performance. We plot the offloaded traffic versus the number of MSs in Figs. 3(c), 4(c) and 5(c). We see that optimal scheme achieves the highest size of offloaded traffic and MDP scheme outperforms two greedy schemes, since MDP scheme aims to maximize the size of offloaded traffic. However, as illustrated in Figs. 3, 4 and 5, we observe that two greedy schemes achieve higher revenue than MDP scheme, even if MDP scheme can offload more data traffic. This is due to that MDP scheme does not take advantage of the competition of MSs to obtain revenue. Fig. 5(c) shows that all the schemes achieve the same size of offloaded traffic, since all traffic demands of MSs are satisfied (see Fig. 5(b)).

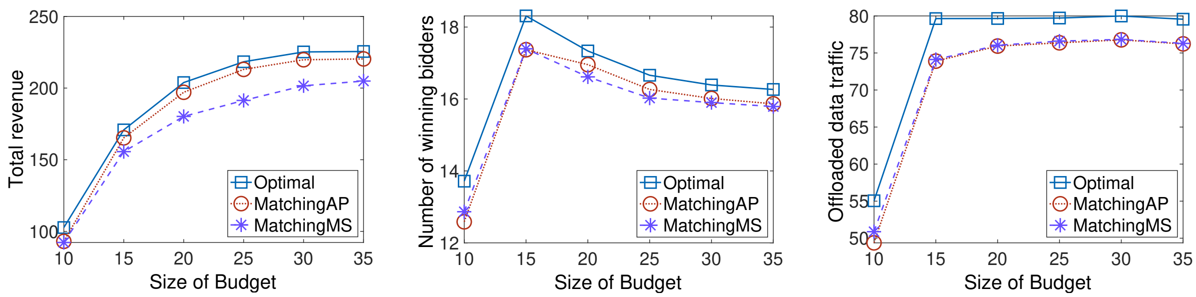
C. Impact of AP Bandwidth in Heterogeneous Networks

In order to evaluate the effect of AP bandwidth on the performance of our proposed schemes, we consider three levels of bandwidth C, namely low, medium and high, corresponding



(a) Revenue (b) Number of Winners (c) Offloaded Traffic

Fig. 9. Performance comparison with budget constraint ($n = 30, m = 10$)



(a) Revenue (b) Number of Winners (c) Offloaded Traffic

Fig. 10. Performance comparison with budget constraint ($n = 30, m = 5$).

to 5, 25, and $40Mbps$, respectively. The number of MSs varies in the range of $[30, 40]$ and the demand of an MS varies in the range of $[3Mbps, 10Mbps]$. The simulation results are shown in Figs. 6, 7 and 8.

Figs. 6(a), 7(a) and 8(a) show the variation of revenue with the number of APs. We observe that the revenues of all schemes increase with the number of APs. As more APs participate in the auction, MNO has more bandwidth provided to MSs, leading to higher revenue. We find that optimal scheme outperforms the other schemes in all scenarios. MatchingMS achieves higher revenue than MatchingAP when AP bandwidth is low, as shown in Figure 6(a). Note that with low bandwidth 5 Mbps, each AP can serve one MS at most, since the minimum demand of MS is 3 Mbps. In this scenario, the final payment of winning MS is the same as its bid value, since the only bidder is the winning MS itself. According to the sorting rule of MatchingMS (see Algorithm 3), MS with a higher bid value has higher chance of winning the auction, resulting in a higher revenue.

However, the situation changes when AP bandwidth increases to 25 Mbps, as shown in Figure 7(a), where one AP can serve multiple MSs. In this scenario, MatchingAP achieves higher revenue than MatchingMS when $m < 10$. This is because MatchingAP selects AP based on its average bandwidth for each MS; larger AP bandwidth can serve more MSs and lead to higher competition among MSs, achieving higher revenue. While MatchingMS simply decides winning MSs based on bid values, without considering the introduction of more competition among MSs. Particularly, in the high bandwidth scenario, as shown in Fig. 8(a) where AP bandwidth is 40 Mbps, MatchingMS achieves higher revenue than MatchingAP when $m > 6$. This is because the benefit of competition among MSs is decreased with sufficient bandwidth provided by a large number of APs.

Figs. 6(b), 7(b) and 8(b) show that the number of winning MSs increase with the the number of APs, since large number of APs increases the potential of satisfying the demand of MSs. We observe that the optimal scheme has the highest number of winning MSs. The curves, as shown in Figs. 6(c), 7(c) and 8(c), follow similar trends as Figs. 6(b), 7(b) and 8(b), respectively, due to the fact that the offloaded traffic increases with the number of winning MSs.

We summarize that the optimal scheme outperforms all other schemes in all scenarios. MatchingMS outperforms MatchingAP in the following two scenarios:

- High AP density: In this case, choosing MS with higher value generates higher revenue, since its demand can always be satisfied;
- Low AP bandwidth: This leads to a special case of data offloading, where one AP is connected to at most one MS at a time.

D. Impact of Budget Constraint

We evaluate the effect of budget constraint on the performance of our proposed schemes. We consider two scenarios based on whether the aggregate bandwidth of APs can satisfy the bandwidth demands of MSs or not. Fig. 9 shows the result when the aggregate bandwidth of APs is sufficient, i.e., $m = 10$, while Fig. 10 shows the result when the aggregate bandwidth of APs cannot satisfy all the demands from MSs,

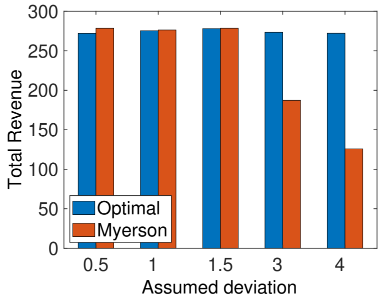


Fig. 11. Robustness comparison with different deviation

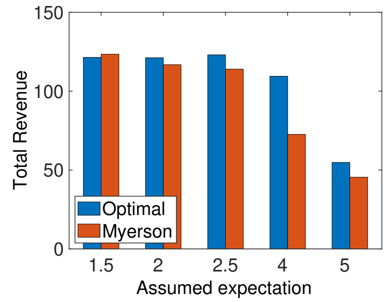


Fig. 12. Robustness comparison with different expectation

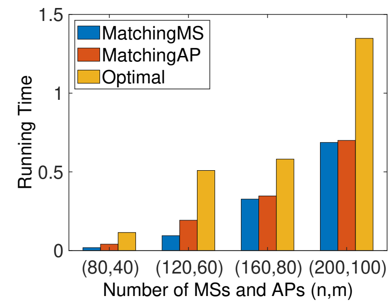


Fig. 13. Scalability comparison among different schemes

i.e., $m = 5$. We observe that the optimal method can obtain the highest revenue in all cases, as shown in Figs. 9(a) and 10(a). MatchingMS outperforms MatchingAP when the aggregate bandwidth of APs is sufficient, while MatchingAP outperforms MatchingMS when the aggregate bandwidth of APs is small.

In Fig. 9, we further observe that the total revenue increases with the value of budget. When $B_i \geq 25$, all MSs win the auction (see Fig. 9(b)) and bandwidth demands are satisfied (see Fig. 9(c)). Thus, the number of winning bidders and the offloaded traffic cannot increase with the value of budget when $B_i \geq 25$. However, the total revenue still increases when $B_i \geq 25$ (see Fig. 9(a)), since higher budget indicates higher valuation from MSs.

Fig. 10 shows the scenario where the total bandwidth demands of MSs is larger than the aggregate bandwidth of APs. As shown in Fig. 10(c), when $B_i \geq 15$, the offloaded traffic cannot increase the value of budget. This implies that all bandwidth of APs have been allocated. We observe that, when $B_i \geq 15$, the increase of budget leads to higher revenue (see Fig. 10(a)) and smaller number of winning bidders (see Fig. 10(b)). It is because higher budget increases the winning probability of MSs who have higher valuations and larger bandwidth demands. Thus, the total revenue increases while the number of winning MSs decreases when $B_i \geq 15$.

E. Robustness and Scalability Analysis

Now we illustrate the robustness and scalability of the proposed optimal offloading method. In order to show the robustness of the proposed method, we consider the scenario where the assumed distributions of MSs' valuations differ from the practical distributions, i.e., MNO's belief on the value of μ_j and δ_j is different from the realized value of μ_j^* and δ_j^*. We compare optimal scheme with Myerson auction [40] that is an optimal auction with reservation price. Myerson auction calculates the reservation price by solving the following equation.

$$1 - F_j(v_j) = v_j * f_j(v_j), \tag{17}$$

where $F_j(.)$ and $f_j(.)$ are the cumulative distribution function and probability density function, respectively, of the probability distribution that the valuation v_j is sampled from. Note that our method calculates the reservation price by solving the bilinear programming problem (10). Thus, the reservation prices obtained by Myerson auction are different from that calculated by our proposed method in most cases.

We consider a simple scenario where valuation v_j follows the normal distribution with parameters $\mu_j^* = 3$ and $\delta_j^* = 2$, for all $j \in \mathcal{M}$, where μ_j^* and δ_j^* are the practical expectation and deviation of the normal distribution, respectively. The number of APs is 10 and the number of MSs is 30.

We first investigate the revenue achieved by MNO when the assumed deviation δ_j is different from the practical deviation δ_j^*. To evaluate the impact of different deviations, we choose $\delta_j \in \{0.5, 1, 1.5, 3, 4\}$. Fig. 11 shows the total revenue obtained by Myerson auction and our optimal scheme. The larger value of δ_j, the lower revenue that the Myerson auction can obtain. For example, when $\delta_j = 4$, optimal scheme outperforms Myerson auction by 56%. This is because the reservation price used in Myerson auction depends on the assumed distribution. Thus, a misspecified (e.g., non-realistic) distribution reduces the performance of Myerson auction. Furthermore, our optimal scheme can achieve better performance due to its insensitivity to the assumed distribution.

We further evaluate how the assumed expectation μ_j affects the total revenue when using the Myerson auction and our optimal scheme. Fig. 12 shows the total revenue obtained by Myerson auction and optimal scheme, when the value of μ_j is chosen from $\{1.5, 2, 2.5, 4, 5\}$. We observe that both methods achieve good performance when $\mu_j < \mu_j^*$. However, the situation changes when $\mu_j > \mu_j^*$, e.g., $\mu_j = 5$, where both methods achieve lower revenue due to the misspecification of μ_j.

We conclude that both Myerson auction and optimal scheme are sensitive to the misspecification of μ_j. Furthermore, Myerson auction is sensitive to the misspecification of δ_j, especially when $\delta_j > \delta_j^*$, while optimal scheme is insensitive to the misspecification of δ_j. Thus, optimal scheme has stronger robustness than Myerson auction when the deviation of normal distribution is misspecified.

Lastly, we evaluate the running time of the proposed schemes on an Intel (R) Core(TM) i7-2620M CPU 2.70GHz processor with RAM of 16.00 GB and 64-bit Linux operating system. We measure the running time (seconds) of different schemes with different numbers of APs and MSs. In Fig. 13, we observe that MatchingAP achieves the lowest running time in all cases. The running time of optimal scheme increases faster than the two other schemes with the number of APs and MSs. Note that when the number of APs is 100 and the number of MSs is 200, the running time of optimal scheme is

1.34 seconds, which is a reasonable value, since the auction is executed every ten seconds.

VIII. Conclusion

This paper proposed a new trading marketplace where MNO can sell bandwidth made available by its own APs to offload data traffic of MSs. The offloading problem was formulated as a multi-item auction based robust optimization approach to guarantee individual rationality, incentive capability and budget feasibility for realistic scenarios in which only part of the valuation information of MSs is known to MNO. In order to solve efficiently (i.e., in polynomial time) the offloading problem for large-scale network scenarios, we also proposed two greedy algorithms. Numerical results show that the proposed schemes capture well the economical and networking essence of the problem, thus representing a promising solution to implement a trading marketplace for next-generation access networks composed of heterogeneous systems.

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Appendix A
Proof of the properties of the proposed auction mechanism

In this appendix, we present the proof that our proposed auction mechanism has the following properties in sequence, i.e., incentive compatibility (see Lemma 2), budget feasibility (see Lemma 3), individual rationality (see Lemma 4) and worst case optimality (see Lemma 5).

Lemma 1. *If $\boldsymbol{z}$ and $\boldsymbol{x}^*$ are the optimal solution of problem* (10)*, then $\boldsymbol{z}$ and $\boldsymbol{x}^*$ satisfy the following conditions:*

$$\sum_{j\in\mathcal{M}} x^*_{ij}\cdot z_{ij} \leq B_i, \quad \forall i\in\mathcal{N}, \tag{18}$$

$$\sum_{i\in\mathcal{N}} x^*_{ij} \leq C_j, \quad \forall j\in\mathcal{M}, \tag{19}$$

$$\sum_{j\in\mathcal{M}} x^*_{ij} \geq D_i, \quad \forall i\in\mathcal{N}, \tag{20}$$

$$\sum_{j\in\mathcal{M}} x^*_{ij}\cdot z_{ij} \leq \sum_{j\in\mathcal{M}} x^*_{ij}\cdot u_{ij}, \quad \forall \boldsymbol{u}\in\mathcal{U},\ \forall i\in\mathcal{N}. \tag{21}$$

$$\sum_{k\in\mathcal{N}} p^{\boldsymbol{z}}_k = \sum_{k\in\mathcal{N}}\sum_{j\in\mathcal{M}} x^*_{kj}\cdot z^*_{kj} \tag{22}$$

Proof. Lemma 1 can be proved by considering a reduced version of problem (10), where we set $\boldsymbol{v}=\boldsymbol{z}$. Thus, the original bilinear optimization problem (10) is reduced to a new linear optimization problem, since the only variable is $\boldsymbol{x}$. The relations (18), (19), (20) and (21) that $\boldsymbol{z}$ and $\boldsymbol{x}^*$ satisfy are derived directly from the constraints (10b), (10c), (10d) and (10e), respectively. Eq. (22) is derived from the objective function of problem (10). ☐

Lemma 2. *The proposed auction mechanism with final allocation matrix $\boldsymbol{a}^v$ and payment vector $\boldsymbol{p}^v$, satisfies the property of incentive compatibility. That is $U_k^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})} \geq U_k^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}$, which means that MS k gets higher utility with truthful bidding $\boldsymbol{v}_k$.*

Proof. We assume that the private valuation for MS k is $\boldsymbol{v}_k\in\mathbb{R}^m$, and the private valuation for the rest $(n-1)$ MSs is $\boldsymbol{v}_{-k}\in\mathbb{R}^{(n-1)}\times\mathbb{R}^m$. Now if MS k chooses to bid with valuation $\boldsymbol{u}_k\in\mathbb{R}^m$ instead of $\boldsymbol{v}_k$; using Eq. (3), where the utility $U_k^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}$ is the difference of payoff and payment, we obtain the utility of MS k as follows:

$$U_k^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})} = \sum_{j\in\mathcal{M}} a_{kj}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}\cdot v_{kj} - p_k^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}. \tag{23}$$

With the fact that $a^{\boldsymbol{v}}_{ij} = x^*_{ij} + y^{\boldsymbol{v}}_{ij}$ (Step 14 in Optimal Algorithm) and Eq. (16), Eq. (24) can be rewritten as

$$\begin{aligned} U_k^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})} =& \sum_{j\in\mathcal{M}} y_{kj}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}\cdot v_{kj} + \sum_{j\in\mathcal{M}} x^*_{kj}\cdot v_{kj} \\ &- \sum_{j\in\mathcal{M}} y_{kj}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}\cdot r^*_{kj} - \sum_{j\in\mathcal{M}} x^*_{kj}\cdot r^*_{kj} \\ &- \sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}}(v_{ij}-r^*_{ij}) \\ &+ \sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}(v_{ij}-r^*_{ij}). \end{aligned} \tag{24}$$

By substituting the following identical equation

$$\begin{aligned} &\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}(v_{ij}-r^*_{ij}) \equiv \sum_{j\in\mathcal{M}} y_{kj}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}\cdot v_{kj} - \\ &\sum_{j\in\mathcal{M}} y_{kj}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}\cdot r^*_{kj} + \sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}(v_{ij}-r^*_{ij}), \end{aligned} \tag{25}$$

into Eq. (24) and some mathematical manipulations, we have

$$\begin{aligned} U_k^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})} =& \sum_{j\in\mathcal{M}} x^*_{kj}\cdot v_{kj} - \sum_{j\in\mathcal{M}} x^*_{kj}\cdot r^*_{kj} \\ &- \sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}}(v_{ij}-r^*_{ij}) \\ &+ \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}(v_{ij}-r^*_{ij}). \end{aligned} \tag{26}$$

Similarly, we get the utility $U_k^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})}$ when MS k bid truthfully as follows:

$$\begin{aligned} U_k^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})} =& \sum_{j\in\mathcal{M}} x^*_{kj}\cdot v_{kj} - \sum_{j\in\mathcal{M}} x^*_{kj}\cdot r^*_{kj} \\ &- \sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}}(v_{ij}-r^*_{ij}) \\ &+ \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})}(v_{ij}-r^*_{ij}). \end{aligned} \tag{27}$$

By subtracting Eq. (26) from Eq. (27), we have

$$\begin{aligned} U_k^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})} - U_k^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})} =& \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})}(v_{ij}-r^*_{ij}) \\ &- \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}(v_{ij}-r^*_{ij}). \end{aligned} \tag{28}$$

Note that $y_{ij}^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})}$ is the optimal solution of problem (14), while $y_{ij}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}$ is a feasible solution of problem (14). Thus, we obtain

$$\sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})}(v_{ij}-r^*_{ij}) \geq \sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}(v_{ij}-r^*_{ij}), \tag{29}$$

which demonstrates that $U_k^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})} \geq U_k^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}$, due to Eq. (28). ☐

Lemma 3. *The proposed auction mechanism with final allocation matrix $\boldsymbol{a}^{\boldsymbol{v}}$ and payment vector $\boldsymbol{p}^{\boldsymbol{v}}$, satisfies the property of budget feasibility. That is $p^{\boldsymbol{v}}_k \leq B_k$, which implies that the payment of MS k is smaller than its budget.*

Proof. We first construct an allocation matrix $\tilde{\boldsymbol{y}}^{\boldsymbol{v}}\in\mathbb{R}^{n\times m}$ based on $\boldsymbol{y}^{\boldsymbol{v}_{-k}}\in\mathbb{R}^{(n-1)\times m}$, where

$$\tilde{y}^{\boldsymbol{v}}_{ij} = \begin{cases} y^{\boldsymbol{v}_{-k}}_{ij}, & \forall i\in\mathcal{N}\setminus\{k\},\ \forall j\in\mathcal{M}, \\ 0, & i=k,\ \forall j\in\mathcal{M}. \end{cases} \tag{30}$$

Thus, we can obtain the following identical equation:

$$\sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y^{\boldsymbol{v}_{-k}}_{ij}(v_{ij}-r^*_{ij}) \equiv \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{M}} \tilde{y}^{\boldsymbol{v}}_{ij}(v_{ij}-r^*_{ij}). \tag{31}$$

Note that $\tilde{\boldsymbol{y}}^{\boldsymbol{v}}$ is a feasible solution to problem (15). That is, $\tilde{\boldsymbol{y}}^{\boldsymbol{v}}$ satisfies all the constraints of problem (15). From Eq. (15b), we obtain

$$\sum_{i\in\mathcal{N}} \tilde{y}^{\boldsymbol{v}}_{ij} \leq C_i - \sum_{i\in\mathcal{N}} x^*_{ij}, \quad \forall j\in\mathcal{M}. \tag{32}$$

From Eq. (15c), we obtain that $\forall i\in\mathcal{N}\setminus\{k\}$,

$$\sum_{j\in\mathcal{M}} \tilde{y}^{\boldsymbol{v}}_{ij}\cdot v_{ij} \leq B_i - \sum_{j\in\mathcal{M}} x^*_{ij}\cdot r^*_{ij}. \tag{33}$$

Note that

$$\sum_{j\in\mathcal{M}} \tilde{y}^{\boldsymbol{v}}_{kj}\cdot v_{kj} = 0 \leq B_i - \sum_{j\in\mathcal{M}} x^*_{ij}\cdot r^*_{ij}. \tag{34}$$

By combine Eqs. (33) and (34), we obtain

$$\sum_{j\in\mathcal{M}} \tilde{y}^{\boldsymbol{v}}_{ij}\cdot v_{ij} \leq B_i - \sum_{j\in\mathcal{M}} x^*_{ij}\cdot r^*_{ij}, \quad \forall i\in\mathcal{N}. \tag{35}$$

Similarly, we can obtain

$$\sum_{j\in\mathcal{M}} \tilde{y}_{ij}^{\boldsymbol{v}} \le D_j - \sum_{j\in\mathcal{M}} x_{ij}^*, \quad \forall i \in \mathcal{N}. \tag{36}$$

From Eqs. (32), (35) and (36), we show that $\tilde{\boldsymbol{y}}^{\boldsymbol{v}}$ is a feasible solution to problem (14), since it satisfies the constraints (14b), (14c) and (14d). Note that $\boldsymbol{y}^{\boldsymbol{v}}$ is the optimal solution to problem (14), Thus, we obtain

$$\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{M}} \tilde{y}_{ij}^{\boldsymbol{v}}(v_{ij} - r_{ij}^*) \le \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}}(v_{ij} - r_{ij}^*). \tag{37}$$

By substituting Eq. (25) into Eq. (16), we have

$$\begin{aligned} p_k^{\boldsymbol{v}} = & \sum_{j\in\mathcal{M}} y_{kj}^{\boldsymbol{v}} \cdot v_{kj} + \sum_{j\in\mathcal{M}} x_{kj}^* \cdot r_{kj}^* \\ & + \sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}}(v_{ij} - r_{ij}^*) \\ & - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}}(v_{ij} - r_{ij}^*). \end{aligned} \tag{38}$$

By substituting Eq. (31) into Eq. (38), we have

$$\begin{aligned} p_k^{\boldsymbol{v}} = & \sum_{j\in\mathcal{M}} y_{kj}^{\boldsymbol{v}} \cdot v_{kj} + \sum_{j\in\mathcal{M}} x_{kj}^* \cdot r_{kj}^* \\ & + \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{M}} \tilde{y}_{ij}^{\boldsymbol{v}}(v_{ij} - r_{ij}^*) \\ & - \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}}(v_{ij} - r_{ij}^*). \end{aligned} \tag{39}$$

Due to Eq. (37), we have

$$p_k^{\boldsymbol{v}} \le \sum_{j\in\mathcal{M}} y_{kj}^{\boldsymbol{v}} \cdot v_{kj} + \sum_{j\in\mathcal{M}} x_{kj}^* \cdot r_{kj}^*. \tag{40}$$

From Eq. (14c), we obtain that $p_k^{\boldsymbol{v}} \le B_k$. □

Lemma 4. *The proposed auction mechanism with final allocation matrix $\boldsymbol{a}^{\boldsymbol{v}}$ and payment vector $\boldsymbol{p}^{\boldsymbol{v}}$, satisfy the property of individual rationality. That is, if MS k bids truthfully with valuation vector $\boldsymbol{v}_k$, it will get a nonnegative utility $U_k^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})} \ge 0$.*

Proof. By substituting Eq. (31) into Eq. (37), we obtain

$$\sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}}(v_{ij} - r_{ij}^*) \le \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}}(v_{ij} - r_{ij}^*). \tag{41}$$

From Eqs. (27) and (41), we obtain

$$U_k^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})} \ge \sum_{j\in\mathcal{M}} x_{kj}^* \cdot v_{kj} - \sum_{j\in\mathcal{M}} x_{kj}^* \cdot r_{kj}^*. \tag{42}$$

From Lemma 1, we get

$$\sum_{j\in\mathcal{M}} x_{kj}^* \cdot v_{kj} \ge \sum_{j\in\mathcal{M}} x_{kj}^* \cdot z_{kj} = \sum_{j\in\mathcal{M}} x_{kj}^* \cdot r_{kj}^*. \tag{43}$$

This implies that $U_k^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})} \ge 0$. □

Lemma 5. *The proposed auction mechanism with final allocation matrix $\boldsymbol{a}^{\boldsymbol{z}}$ and payment vector $\boldsymbol{p}^{\boldsymbol{z}}$, satisfies the property of worst case optimality. That is $\sum_{k=1}^{n} \boldsymbol{p}_k^{\boldsymbol{z}} \le \sum_{k=1}^{n} \boldsymbol{p}_k^{\boldsymbol{v}}$, which means that the revenue of MNO under valuation matrix $\boldsymbol{z}$ (obtained by Optimal Algorithm) is smaller than that under $\boldsymbol{v} \in \mathcal{U}$.*

Proof. We first construct an allocation matrix $\tilde{\boldsymbol{y}}^{\boldsymbol{v}} \in \mathbb{R}^{(n-1)\times m}$ based on $\boldsymbol{y}^{\boldsymbol{v}} \in \mathbb{R}^{n\times m}$, where

$$\tilde{y}_{ij}^{\boldsymbol{v}_{-k}} = y_{ij}^{\boldsymbol{v}}, \quad \forall i \in \mathcal{N}\setminus\{k\}, \forall j \in \mathcal{M}. \tag{44}$$

Thus, we have the identical equation

$$\sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}}(v_{ij} - r_{ij}^*) \equiv \sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} \tilde{y}_{ij}^{\boldsymbol{v}_{-k}}(v_{ij} - r_{ij}^*). \tag{45}$$

In the following, we show that $\tilde{\boldsymbol{y}}^{\boldsymbol{v}}$ is a feasible solution to problem (14). From Eq. (14b), we obtain that $\forall j \in \mathcal{M}$,

$$\sum_{i\in\mathcal{N}\setminus\{k\}} \tilde{y}_{ij}^{\boldsymbol{v}_{-k}} = \sum_{i\in\mathcal{N}\setminus\{k\}} y_{ij}^{\boldsymbol{v}} \le C_i - \sum_{i\in\mathcal{N}\setminus\{k\}} x_{ij}^*. \tag{46}$$

From Eq. (14c), we obtain that $\forall i \in \mathcal{N}\setminus\{k\}$,

$$\sum_{j\in\mathcal{M}} \tilde{y}_{ij}^{\boldsymbol{v}_{-k}} \cdot v_{ij} = \sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}} \cdot v_{ij} \le B_i - \sum_{j\in\mathcal{M}} x_{ij}^* \cdot r_{ij}^*. \tag{47}$$

From Eq. (14d), we obtain that $\forall i \in \mathcal{N}\setminus\{k\}$,

$$\sum_{j\in\mathcal{M}} \tilde{y}_{ij}^{\boldsymbol{v}_{-k}} = \sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}} \le D_j - \sum_{j\in\mathcal{M}} x_{ij}^*. \tag{48}$$

From Eqs. (46), (47) and (48), we show that $\tilde{\boldsymbol{y}}^{\boldsymbol{v}}$ is a feasible solution to problem (15), since it satisfies constraints (15b), (15c) and (15d). Notice that $\boldsymbol{y}^{\boldsymbol{v}_{-k}}$ is the optimal solution to problem (15). Thus, we have

$$\sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} \tilde{y}_{ij}^{\boldsymbol{v}_{-k}}(v_{ij} - r_{ij}^*) \le \sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}}(v_{ij} - r_{ij}^*). \tag{49}$$

From Eqs. (45) and (49), we derive

$$\sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}}(v_{ij} - r_{ij}^*) \le \sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}}(v_{ij} - r_{ij}^*). \tag{50}$$

From Eqs. (16) and (50), we derive

$$p_k^{\boldsymbol{v}} \ge \sum_{j\in\mathcal{M}} y_{kj}^{\boldsymbol{v}} \cdot r_{kj}^* + \sum_{j\in\mathcal{M}} x_{kj}^* \cdot r_{kj}^*. \tag{51}$$

From Lemma 1, we know that

$$\sum_{k\in\mathcal{N}} p_k^{\boldsymbol{z}} = \sum_{k\in\mathcal{N}}\sum_{j\in\mathcal{M}} x_{kj}^* \cdot r_{kj}^*. \tag{52}$$

Finally, we get that $\sum_{k\in\mathcal{N}} p_k^{\boldsymbol{z}} \le \sum_{k\in\mathcal{N}} p_k^{\boldsymbol{v}}$. □

With Lemmas $2-5$, we complete the proof of Theorem 1.