Multi-Item Auction Based Mechanism for Mobile Data Offloading With Budget Constrained Users

Dongqing Liu, Abdelhakim Hafid, Lyes Khoukhi

Abstract

The opportunistic utilization of third party WiFi access devices to offload data traffic from cellular network has been considered as a promising approach to reduce cellular traffic. To foster this opportunistic utilization, we propose a new and open market approach where a mobile operator can sell bandwidth made available by the access points to increase the network capacity. We formulate the offloading problem as a multi-item auction considering the most general case of partial cellular traffic to be offloaded. We discuss the conditions to (i) offload the maximum amount of data traffic according to the capacity made available by access devices, (ii) foster the participation of access point owners (individual rationality), (iii) prevent market manipulation (incentive compatibility) and (iv) preserve budget feasibility. Then, we propose two alternative greedy algorithms that efficiently solve the offloading problem, even for large-size network scenarios. The simulation results show the efficiency and robustness of our proposed methods for cellular data offloading.

Index Terms

Multi-item auction, mobile data offloading, heterogeneous networks, robust optimization.

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I. Introduction

The rapid growth of mobile data traffic raises big challenges to cellular network. Global mobile data traffic grew 63 percent and reached 7.2 exabytes per month in 2016, which is 18-fold over the past 5 years [1]. The huge amount of mobile data traffic exceeds the capacity of cellular network and reduces the network quality [2]. To address such challenges, one simple solution is to increase the capacity of cellular network, which is inefficient and expensive due to the corresponding expensive investments in radio access networks and the core infrastructure. One promising solution, namely mobile data offloading, is to offload cellular traffic to other kinds of networks, e.g. WiFi access points and femtocell; this can solve the cellular traffic overload problem [3, 4].

Although mobile data offloading can significantly reduce cellular traffic, the task of developing a comprehensive and reliable mobile data offloading system remains challenging. A key challenge is how to achieve an efficient data offloading coordination among multiple mobile devices. By opportunistic utilization of lower cost access points, mobile subscribers will have better wireless access service with lower cost. In contrast, mobile network operators (MNOs) who have deployed these access points (APs) want to maximize the revenue by selling bandwidth. Thus, how to effectively allocate this bandwidth to mobile devices effectively becomes a key problem to be solved.

Given the limited number of APs deployed in a mobile data offloading (MDO) system, when demands of mobile devices exceed supply in the MDO system, MNO needs an efficient mechanism to decide which devices to serve in order to achieve the highest revenue. Auction theory represents a flexible and efficient approach towards allocation of AP bandwidth [5–10]. Different from simple allocation schemes based on fixed pricing, an auction is economically efficient, automatically discovers the real market value, and assigns bandwidth to users who value it the most.

In this paper, we focus on designing an efficient auction mechanism for allocating AP bandwidth among multiple mobile devices; this is considered as a multi-item auction problem. MNO which owns the network infrastructure acts as the auctioneer and sells bandwidth to mobile devices through an auction. We formulated the auction problem based on robust optimization which models the desirable properties (budget feasibility, incentive compatibility, and individual rationality) of optimal auctions enabling the auctioneer to use historical data or prior knowledge of valuations. The uncertainty of item valuations is modeled as an uncertainty set, which is constructed based on limit theorems of probability theory. The optimal auction mechanism with reservation price has the structure of a Vickrey-Clarke-Groves (VCG) mechanism [11].

The main contributions of this paper can be summarized as follows:

- We propose and analyze multi-item auction to implement an incentive marketplace for maximizing the MNO's revenue and offloading the maximum amount of data traffic from mobile subscribers (MSs).
- Since the optimal multi-item auction problem is NP-hard, we propose two greedy algorithms that solve efficiently (i.e., in polynomial time) the allocation problem.
- We perform numerical analysis and comparative evaluation of the proposed optimal and greedy allocation algorithms, considering realistic network scenarios.
- We illustrate that the proposed offloading mechanisms can improve cellular data offloading performance and has higher robustness compared to Myerson auction.

The rest of the paper is organized as follows. Section 2 presents related work. Section 3 presents the system model. Section 4 formulates the multi-item auction as a robust optimization problem. Section 5 and Section 6 propose the optimal and greedy auction mechanisms, to solve the optimization problem, respectively. Section 7 illustrates and analyzes the numerical results. Section 8 concludes the paper.

II. RELATED WORK

To cope with the growth of cellular traffic, some previous contributions have studied efficient data offloading methods from the perceptive of data offloading decision making. Jung et al. [12] proposed a WiFi based offloading model to maximize per-user throughput by collecting network information, e.g., the number of MSs and their data demands. Cheung et al. [13] proposed a Markov decision process based network selection algorithm for delay-tolerant applications under the setting of a single MS. Barbarossa et al. [14] proposed a centralized scheduling algorithm to jointly optimize the communication and computation resource allocations among multiple users with latency requirements. Kang et al. [15] studied the offloading problem from MNO's perspective and proposed a usage-based charging model to maximize MNO's revenues.

Other contributions have investigated data offloading problems based on auction theory or game theory. Chen et al. [16] studied the scenario where multiple users can access the same wireless base station, and designed a decentralized offloading mechanism that ensures the scalability of the proposed mechanism with the number of mobile users. Zhou et al. [17] proposed a reverse auction based incentive framework for cellular traffic offloading, and provided a prediction model for WiFi data offloading potential. Cheng et al. [18] took into consideration users' mobility information and proposed an auction based offloading mechanism to maximize MSs' social welfare and improve MNO's revenues. Lee et al. [19] proposed a two-stage sequential game to model the interaction between MNO and MSs, and demonstrated, via simulations, that WiFi offloading is economically beneficial for both MNO and MSs. Paris et al. [20] proposed a reverse auction based offloading algorithm leasing WiFi access points, owned by third parties, to allocate bandwidth to multiple mobile users.

All the existing offloading studies have not considered budget constraints of MSs when designing offloading mechanisms. Also, they assumed that MNO has a full knowledge of MSs' valuation information, which is not practical in many situations [21]. To motivate mobile users to participate in the cellular traffic offloading, we propose an multi-item auction based offloading framework. Different from existing contributions, in this paper, we consider the budget feasibility, incentive compatibility and individual rationality simultaneously. Moreover, we consider that MNO does not have a full knowledge of MSs' valuation information; we model the uncertainty of valuation with an uncertainty set.

III. System Model

In this section, we present the economic definitions and network model that are considered in our multi-item auction mechanism; the objective of this mechanism is to implement efficient mobile data offloading. A scenario of data offloading among multiple APs and MSs is shown in Figure 1, where MSs, in the coverage area of APs, engage in an auction to acquire bandwidth (in WiFi network). We first model the uncertainty of MNO's beliefs on MS's valuations using uncertainty set. Then, we introduce the general economical definitions for multi-item auction.

Let $\mathcal{N}$ denote the set of MSs, and $\mathcal{M}$ denote the set of APs owned by MNO, where $|\mathcal{N}| = n$ and $|\mathcal{M}| = m$. MS i has a private valuation for the unit bandwidth usage associated with AP j, denoted by v_{ij} which is unknown to MNO. Let $\boldsymbol{v} = \{v_{ij} | i \in \mathcal{N}, j \in \mathcal{M}\}$ denote the private valuation matrix. Thus, for AP j, $\boldsymbol{v}_j = (v_{1j}, \ldots, v_{nj})$ denotes the column vector of private valuation matrix $\mathcal{P}$. Moreover, MS i is budget constrained and the available budget is denoted by $B_i, i \in \mathcal{N}$, while AP j is bandwidth constrained and the available bandwidth is denoted by

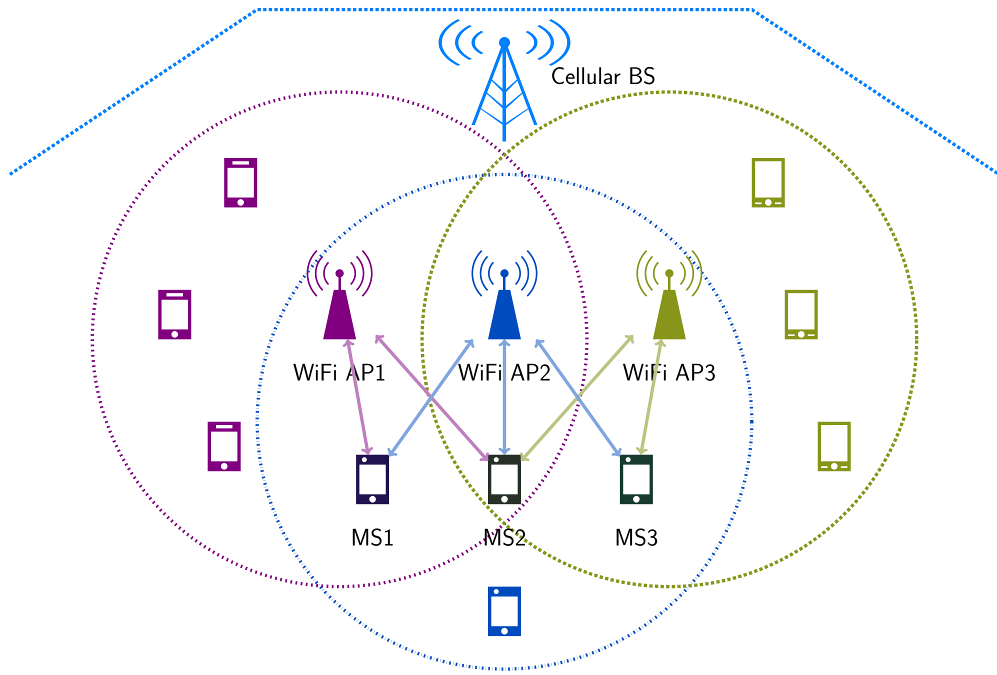


Fig. 1. An illustration of data offloading auction model. WiFi APs are managed by a single MNO that provides network access to its mobile subscribers (e.g., MS1). The network capacity of WiFi access points (e.g., AP1) is allocated to MSs for data traffic offloading. In this scenario, MS1, MS2 and MS3 bid for bandwidth (i.e., AP1, AP2 and AP3) with different valuations. Considering the coverage area of each AP, MS2 can bid for three APs, while MS1 and MS3 can bid for two APs. MNO who is the auctioneer allocates different AP bandwidth to MSs. The winning MSs can use bandwidth determined by MNO.

$C_j, j \in \mathcal{M}$. In this paper, we consider that the valuation information is private (only known to MS) and budget information is public (known to all including MNO).

For AP j, since the private valuations of MSs are hidden from MNO, we model MNO's beliefs on the valuations of n MSs using uncertainty set $\mathcal{U}_j$, where the valuation vector $\boldsymbol{v}_j \in \mathcal{U}_j$. MNO's belief on valuations for all APs is denoted as $\mathcal{U} = \{\mathcal{U}_j\}_{j \in \mathcal{M}}$.

We observe that the valuations of AP j increase with the number of MSs, due to the competition between multiple MSs. Motivated by this observation, we define the valuation of MS i for AP j as follows,

$$v_{ij} = f_j(k) + y_{ij}, \tag{1}$$

where $f_j(k)$ is a non-decreasing function on the number of MSs participating in the auction to ask for bandwidth of AP j (i.e., $k = |\mathcal{N}_j|$, $\mathcal{N}_j$ includes all MSs that want to use AP j) and y_{ij} is a random variable describing the historical valuation of AP j by MS i without competition. We assume that the valuations for AP j are independent and identically distributed, as well as the expectation and deviation of AP j are μ_j and δ respectively. Based on the central limit theory, the distribution of

$$\frac{\sum_{i=1}^{n} y_{ij} - n \cdot \mu_j}{\sqrt{n} \cdot \delta_j}$$

is approximately a standard normal distribution when $n \to \infty$. Thus, the uncertainty set $\mathcal{U}_j$ can be constructed as follows.

$$\mathcal{U}_j = \left\{ (v_{1j}, \ldots, v_{nj}) \;\middle|\; \begin{array}{c} v_{ij} = f_j(k) + y_{ij}, \forall i \in \mathcal{N} \\ \\ \underline{F}_j \leq f_i(k) \leq \overline{F}_j \\ \\ -\Gamma \leq \frac{\sum_{i=1}^{n} y_{ij} - n \cdot \mu_j}{\sqrt{n} \cdot \delta_j} \leq \Gamma \end{array} \right\}, \tag{2}$$

where $\underline{F}_j$ and $\overline{F}_j$ are the lower bound and upper bound of the competition function $f_i(k)$, respectively. Γ is a parameter that controls the conservativeness of the historical valuations. For example, under the central limit theorem, the probability that $(\hat{y}_{1j}, \cdots, \hat{y}_{nj})$ belongs to

$$-\Gamma \leq \frac{\sum_{i=1}^{n} y_{ij} - n \cdot \mu_j}{\sqrt{n} \cdot \delta_j} \leq \Gamma$$

can be calculated by

$$\mathbb{P}\big((\hat{y}_{1j}, \cdots, \hat{y}_{nj})\big) = 2\Phi(\Gamma) - 1, \tag{3}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal. If we set Γ to 1,2 and 3, then $\mathbb{P}((\hat{y}_{1j}, \cdots, \hat{y}_{nj}))$ is 0.683, 0.955 and 0.997, respectively. A smaller Γ makes MNO consider only those valuations with higher probability. A larger Γ makes MNO consider a larger range of valuations, which increases the accuracy of auction at the cost of computational complexity. Thus, MNO needs to choose a proper Γ to balance the accuracy and computational complexity of the auction.

IV. Problem Statement

In this section, we formulate the multi-item auction based data offloading problem as a robust optimization problem. Our objective is to maximize the total revenue of MNO for all valuations in the uncertainty set $\mathcal{U}$. We first introduce the decision variables that represent the allocation rule and the payment rule. Then, we define the properties that the allocation and payment rules should satisfy in order to implement an efficient auction. The notations used in this paper are described in Table I.

TABLE I

NOTATION USED IN THE PAPER

$\mathcal{N}$	Set of Mobile Subscribers
$\mathcal{M}$	Set of Access Points
$\mathcal{U}$	Uncertainty set of $\boldsymbol{v}$
$\boldsymbol{B} = \{B_i\}_{i\in\mathcal{N}}$	MS budget constraints
$\boldsymbol{D} = \{D_i\}_{i\in\mathcal{N}}$	MS bandwidth demand
$\boldsymbol{C} = \{C_i\}_{i\in\mathcal{M}}$	AP bandwidth constraints
$\boldsymbol{v} = \{v_{ij}\}_{i\in\mathcal{N},j\in\mathcal{M}}$	Bid matrix
$\boldsymbol{v}_k = \{v_{kj}\}_{j\in\mathcal{M}}$	Bid vector of MS k
$\boldsymbol{v}_{-k} = \{v_{ij}\}_{i\in\mathcal{N}\setminus\{k\},j\in\mathcal{M}}$	Bid vector except for MS k
$\boldsymbol{z} = \{z_{ij}\}_{i\in\mathcal{N},j\in\mathcal{M}}$	Worst case bid vector
$\boldsymbol{x}^* = \{x^*_{ij}\}_{i\in\mathcal{N},j\in\mathcal{M}}$	Nominal allocation in worst case
$\boldsymbol{r}^* = \{r^*_{ij}\}_{i\in\mathcal{N},j\in\mathcal{M}}$	Reservation prices in worst case
$\boldsymbol{y}^{\boldsymbol{v}} = \{y^{\boldsymbol{v}}_{ij}\}_{i\in\mathcal{N},j\in\mathcal{M}}$	Adapted allocation
$\boldsymbol{y}^{\boldsymbol{v}_{-k}} = \{y^{\boldsymbol{v}_{-k}}_{ij}\}_{i\in\mathcal{N}\setminus\{k\},j\in\mathcal{M}}$	Adapted allocation without MS k
$\boldsymbol{a}^{\boldsymbol{v}} = \{a^{\boldsymbol{v}}_{ij}\}_{i\in\mathcal{N},j\in\mathcal{M}}$	Real allocation
$\boldsymbol{p}^{\boldsymbol{v}} = \{p^{\boldsymbol{v}}_i\}_{i\in\mathcal{N}}$	Real payments

A. Allocation and Payment Rules

The decision variable $\boldsymbol{x}^{\boldsymbol{v}} = \{x^{\boldsymbol{v}}_{ij}\}_{i\in\mathcal{N},j\in\mathcal{M}}$ describes APs' bandwidth allocation among multiple MSs based on the valuation matrix $\boldsymbol{v}$, that is, if the valuation matrix is $\boldsymbol{v}$, MNO will allocate $x^{\boldsymbol{v}}_{ij}$ bandwidth of AP j to MS i. If MS i is not in the coverage area of AP j, then $x^{\boldsymbol{v}}_{ij} = 0$. The decision variable $\boldsymbol{p}^{\boldsymbol{v}} = \{p^{\boldsymbol{v}}_i\}_{i\in\mathcal{N}}$ denotes the payment of MSs according to current valuation matrix $\boldsymbol{v}$, where $p^{\boldsymbol{v}}_i$ is the total payment of MS i for using the bandwidth of APs. Thus, $p^{\boldsymbol{v}}_i \geq 0$.

Given the allocation variable $\boldsymbol{x}^{\boldsymbol{v}}$ and payment variable $\boldsymbol{p}^{\boldsymbol{v}}$, we can derive the utility (i.e., the difference of total valuation and payment) of MS i as follows,

$$U^{\boldsymbol{v}}_i = \sum_{j\in\mathcal{M}} v_{ij} \cdot x^{\boldsymbol{v}}_{ij} - p^{\boldsymbol{v}}_i, \quad i \in \mathcal{N}, \quad \boldsymbol{v} \in \mathcal{U}. \tag{4}$$

The allocation and payment variables should satisfy the following properties in order to implement an efficient multi-item auction.

- Individual Rationality (IR). This property ensures nonnegative utilities (i.e., the payment of

MS should be less than his obtained valuation) for MSs who bid truthfully. Formally,

$$p_i^{\boldsymbol{v}} \leq \sum_{j \in \mathcal{M}} v_{ij} \cdot x_{ij}^{\boldsymbol{v}}, \forall i \in \mathcal{N}, \forall \boldsymbol{v} \in \mathcal{U}. \tag{5}$$

- Budget Feasibility (BF). This property ensures the payment of each MS is within his budget constraint. Formally,

$$p_i^{\boldsymbol{v}} \leq B_i, \forall i \in \mathcal{N}, \forall \boldsymbol{v} \in \mathcal{U}, \tag{6}$$

  where B_i is the limited budget of MS i.
- Incentive Compatibility (IC). This property ensures that MS cannot improve his utility by bidding untruthfully. Thus, the utility of MS under truthful bidding is higher than untruthful biddings; this allows avoiding market manipulation by MSs. Formally,

$$U_i^{(\boldsymbol{v}_i, \boldsymbol{v}_{-i})} \geq U_i^{(\boldsymbol{u}_i, \boldsymbol{v}_{-i})}, \forall i \in \mathcal{N}, \forall (\boldsymbol{v}_i, \boldsymbol{v}_{-i}) \in \mathcal{U}, \forall (\boldsymbol{u}_i, \boldsymbol{v}_{-i}) \in \mathcal{U}, \tag{7}$$

  where $\boldsymbol{v}_i = \{v_{ij}\}_{j \in \mathcal{M}}$ is the truthful valuation of MS i and $\boldsymbol{u}_i = \{u_{ij}\}_{j \in \mathcal{M}}$ is a possible valuation of MS i. $\boldsymbol{v}_{-i} = \{v_{kj}\}_{k \in \mathcal{N} \setminus \{i\}, j \in \mathcal{M}}$ denotes the valuation matrix obtained by omitting the valuations from MS i. By substituting Eq. (4) into Eq. (7), we have

$$\begin{gathered} \sum_{j \in \mathcal{M}} v_{ij} \cdot x_{ij}^{(\boldsymbol{v}_i, \boldsymbol{v}_{-i})} - p_i^{(\boldsymbol{v}_i, \boldsymbol{v}_{-i})} \geq \sum_{j \in \mathcal{M}} v_{ij} \cdot x_{ij}^{(\boldsymbol{u}_i, \boldsymbol{v}_{-i})} - p_i^{(\boldsymbol{u}_i, \boldsymbol{v}_{-i})}, \\ \forall i \in \mathcal{N}, \forall (\boldsymbol{v}_i, \boldsymbol{v}_{-i}) \in \mathcal{U}, \forall (\boldsymbol{u}_i, \boldsymbol{v}_{-i}) \in \mathcal{U}, \end{gathered} \tag{8}$$

  With some mathematical manipulation of Eq. (8), we obtain the following equation.

$$\begin{gathered} \sum_{j \in \mathcal{M}} v_{ij} \cdot \left(x_{ij}^{(\boldsymbol{u}_i, \boldsymbol{v}_{-i})} - x_{ij}^{(\boldsymbol{v}_i, \boldsymbol{v}_{-i})} \right) + p_i^{(\boldsymbol{u}_i, \boldsymbol{v}_{-i})} - p_i^{(\boldsymbol{v}_i, \boldsymbol{v}_{-i})} \geq 0, \\ \forall i \in \mathcal{N}, \forall (\boldsymbol{v}_i, \boldsymbol{v}_{-i}) \in \mathcal{U}, \forall (\boldsymbol{u}_i, \boldsymbol{v}_{-i}) \in \mathcal{U}. \end{gathered} \tag{9}$$

B. Optimal auction problem

The optimal auction design problem, based on the above property constraints, is formulated as a robust optimization problem, with the objective to maximize the revenue of MNO for all the valuations in set $\mathcal{U}$. Since MNO's beliefs on MSs' valuations are modeled as an uncertainty set, we focus on maximizing the worst case revenue. The network constraints, including APs' bandwidth constraints and MSs' demand constraints, are also formulated in the optimization problem.

$$\max_{\boldsymbol{x}^{\boldsymbol{v}},\boldsymbol{p}^{\boldsymbol{v}}} \quad W \tag{10a}$$

$$s.t. \quad W - \sum_{i\in\mathcal{N}} p_i^{v} \leq 0, \forall \boldsymbol{v} \in \mathcal{U} \tag{10b}$$

$$p_i^{\boldsymbol{v}} \leq \sum_{j\in\mathcal{M}} v_{ij} \cdot x_{ij}^{\boldsymbol{v}}, \forall i \in \mathcal{N}, \forall \boldsymbol{v} \in \mathcal{U} \tag{10c}$$

$$p_i^{\boldsymbol{v}} \leq B_i, \forall i \in \mathcal{N}, \forall \boldsymbol{v} \in \mathcal{U} \tag{10d}$$

$$\begin{aligned}&\sum_{j\in\mathcal{M}} v_{ij} \cdot \left(x_{ij}^{(\boldsymbol{u}_i,\boldsymbol{v}_{-i})} - x_{ij}^{(\boldsymbol{v}_i,\boldsymbol{v}_{-i})}\right)\\ &+ p_i^{(\boldsymbol{u}_i,\boldsymbol{v}_{-i})} - p_i^{(\boldsymbol{v}_i,\boldsymbol{v}_{-i})} \geq 0, \forall i \in \mathcal{N},\\ &\forall(\boldsymbol{v}_i,\boldsymbol{v}_{-i}) \in \mathcal{U}, \quad \forall(\boldsymbol{u}_i,\boldsymbol{v}_{-i}) \in \mathcal{U}\end{aligned} \tag{10e}$$

$$\sum_{i\in\mathcal{N}} x_{ij}^{\boldsymbol{v}} \leq C_j, \forall j \in \mathcal{M}, \forall \boldsymbol{v} \in \mathcal{U} \tag{10f}$$

$$\sum_{j\in\mathcal{M}} x_{ij}^{\boldsymbol{v}} \leq D_i, \forall i \in \mathcal{N}, \forall \boldsymbol{v} \in \mathcal{U} \tag{10g}$$

$$x_{ij}^{\boldsymbol{v}} \geq 0, \forall i \in \mathcal{N}, \forall j \in \mathcal{M}, \forall \boldsymbol{v} \in \mathcal{U} \tag{10h}$$

$$p_i^{\boldsymbol{v}} \geq 0, \forall i \in \mathcal{N}, \forall \boldsymbol{v} \in \mathcal{U}. \tag{10i}$$

Constraint (10b) ensures the maximization of worst case revenue considering all the possible valuations in the uncertainty set $\mathcal{U}$. Constraints (10c), (10d) and (10e) correspond to IR, BF and IC properties, respectively. Constraint (10f) ensures that the bandwidth allocation should not exceed the available bandwidth of an AP. Constraint (10g) guarantees that each MS cannot obtain over-demanding bandwidth. Note that the demand D_i varies over time due to the stochastic nature of MS traffic. We consider a quasi-static network scenario [22], and analyze the auction mechanism in a data offloading period (e.g., ten seconds), during which D_i remains unchanged for all $i \in \mathcal{N}$. Finally, Constraint (10i) prevents negative allocation and payment for MSs.

V. Optimal auction mechanism

This section presents our proposed optimal auction mechanism used to solve the optimization problem (10) in order to determine an optimal allocation and payment rules. That is, how APs' bandwidth is shared among multiple MSs, and how much MSs are charged for using allocated bandwidth. Our optimal auction mechanism illustrated in Algorithm 1, takes as input the uncertainty set $\mathcal{U}$, MS budget vector $\boldsymbol{B}$, AP constraint vector $\boldsymbol{C}$, MS demand vector $\boldsymbol{D}$ and bid matrix $\boldsymbol{v}$, and calculates as output the real allocation matrix $\boldsymbol{a^v}$ and the payment vector $\boldsymbol{p^v}$. We will refer to Algorithm 1 as Optimal Algorithm in the rest of paper. We first introduce the details of Optimal Algorithm. Then, we show the auction properties of Optimal Algorithm.

Algorithm 1 Optimal offloading auction mechanism

Input: $\mathcal{U}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}, \boldsymbol{v}, \mathcal{M}, \mathcal{N}$

Output: $\boldsymbol{a^v}$, $\boldsymbol{p^v}$

$(\boldsymbol{z}, \boldsymbol{x}^*) \leftarrow$ solving problem (11)
$(\boldsymbol{\xi}^*, \boldsymbol{\eta}^*, \boldsymbol{\lambda}^*, \boldsymbol{\theta}^*) \leftarrow$ solving problem (14)
for $i \in \mathcal{N}$ **do**
    for $j \in \mathcal{M}$ **do**
        $r_{ij}^* = \xi_j^* + (\eta_i^* + \lambda_i^* + \theta_i^*) \cdot z_{ij}$
    end for
end for
$\boldsymbol{y^v} \leftarrow$ solving problem (15)
for $k \in \mathcal{N}$ **do**
    $\boldsymbol{y}^{\boldsymbol{v}_{-k}} \leftarrow$ solving problem (16)
end for
for $i \in \mathcal{N}$ **do**
    for $j \in \mathcal{M}$ **do**
        $a_{ij}^{\boldsymbol{v}} = x_{ij}^* + y_{ij}^{\boldsymbol{v}}$
    end for
end for
for $k \in \mathcal{N}$ **do**
    Calculate $p_k^{\boldsymbol{v}}$ using Eq. (17)
end for

Optimal Algorithm consists of two phases, the phase of nominal allocation (Steps $1-7$) and the phase of final allocation (Steps $8-19$). The phase of nominal allocation calculates the reservation price $\boldsymbol{r}^*=\{r_{ij}\}_{i\in\mathcal{N},j\in\mathcal{M}}$ and the nominal allocation $\boldsymbol{x}^*=\{x_{ij}\}_{i\in\mathcal{N},j\in\mathcal{M}}$. MS i has to bid at least r_{ij} in order to use the bandwidth provided by AP j. $\boldsymbol{x}^*$ represents the best allocation in worst case scenario, which is part of the final allocation calculated in the phase of final allocation. The phase of final allocation calculates the final allocation $\boldsymbol{a}^{\boldsymbol{v}}$ and final payment $\boldsymbol{p}^{\boldsymbol{v}}$ based on the specific bid matrix $\boldsymbol{v}$. The final allocation $\boldsymbol{a}^{\boldsymbol{v}}=\boldsymbol{x}^*+\boldsymbol{y}^{\boldsymbol{v}}$, where $\boldsymbol{y}^{\boldsymbol{v}}=\{y_{ij}^{\boldsymbol{v}}\}_{i\in\mathcal{N},j\in\mathcal{M}}$, called adapted allocation, denotes the best allocation for a specific bid matrix $\boldsymbol{v}$.

A. *Phase of Nominal Allocation*

In the phase of nominal allocation, Step 1 calculates the worst case bid matrix $\boldsymbol{z}$ and reservation price $\boldsymbol{r}^*$ by solving the bilinear optimization problem (11), where the constraints (11b), (11c) and (11d) are derived from constraints (10d), (10f) and (10g), respectively. Constraint (11e) that captures the IC and IR properties of problem (10) is used to calculate the worst case bid matrix $\boldsymbol{z}$, under which the obtained payoff $\sum_{j\in\mathcal{M}} x_{ij}\cdot z_{ij}$ for MS i is minimum. The nominal allocation $\boldsymbol{x}^*$ is a preallocation that corresponds to the worst case bid matrix $\boldsymbol{z}$.

$$\max_{\boldsymbol{v}}\max_{\boldsymbol{x}} \quad \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{M}} x_{ij}v_{ij} \tag{11a}$$

$$s.t. \quad \sum_{j\in\mathcal{M}} x_{ij}\cdot v_{ij} \leq B_i, \forall i\in\mathcal{N}, \tag{11b}$$

$$\sum_{i\in\mathcal{N}} x_{ij} \leq C_i, \forall j\in\mathcal{M}, \tag{11c}$$

$$\sum_{j\in\mathcal{M}} x_{ij} \leq D_j, \forall i\in\mathcal{N}, \tag{11d}$$

$$\sum_{j\in\mathcal{M}} x_{ij}\cdot v_{ij} \leq \sum_{j\in\mathcal{M}} x_{ij}\cdot u_{ij}, \forall \boldsymbol{u}\in\mathcal{U}, \forall i\in\mathcal{N}, \tag{11e}$$

$$\boldsymbol{x}\geq 0,\ \boldsymbol{v}\in\mathcal{U}. \tag{11f}$$

In order to obtain the reservation price $\boldsymbol{r}^*$, We first simplify the problem (11) as a linear programming problem with decision variable $\boldsymbol{x}$ by: 1) replacing variable $\boldsymbol{v}$ with constant $\boldsymbol{z}$

(obtained in Step 1); 2) replacing Constraint (11e) with Eq. (12).

$$\sum_{j\in\mathcal{M}} x_{ij}\cdot v_{ij} \le \sum_{j\in\mathcal{M}} x^*_{ij}\overline{u}^i_j, \quad \forall i\in\mathcal{N}, \tag{12}$$

where

$$\overline{\boldsymbol{u}}^i = arg\min_{\boldsymbol{u}\in\mathcal{U}} \sum_{j\in\mathcal{M}} x^*_{ij}\cdot u_{ij}, \quad \forall i\in\mathcal{N}. \tag{13}$$

Then, we can obtain the dual problem of simplified problem (11) as follows.

$$\min_{\boldsymbol{\xi},\boldsymbol{\eta},\boldsymbol{\lambda},\boldsymbol{\theta}} \quad \sum_{j\in\mathcal{M}} \xi_j C_i + \sum_{i\in\mathcal{N}} \Big(\eta_i B_i + \lambda_i D_i + \theta_i \sum_{j\in\mathcal{M}} x^*_{ij}\overline{u}^i_j\Big) \tag{14a}$$

$$s.t. \quad \xi_j + z_{ij}(\eta_i + \lambda_i + \theta_i) \ge z_{ij}, \forall i\in\mathcal{N}, j\in\mathcal{M}, \tag{14b}$$

$$\xi_j, \eta_i, \lambda_i, \theta_i \ge 0, \forall i\in\mathcal{N}, j\in\mathcal{M}. \tag{14c}$$

The decision variables $\boldsymbol{\xi}^* = \{\xi^*_j\}_{j\in\mathcal{M}}$, $\boldsymbol{\eta}^* = \{\eta^*_i\}_{i\in\mathcal{N}}$, $\boldsymbol{\lambda}^* = \{\lambda^*_i\}_{i\in\mathcal{N}}$ and $\boldsymbol{\theta}^* = \{\theta^*_i\}_{i\in\mathcal{N}}$ correspond to the constraints (11c), (11b), (11d) and (12), respectively. Step 2 calculates the solution of dual problem (14) used to obtain the reservation price $\boldsymbol{r}^*$ in Steps $3-7$, where r^*_{ij} represents the minimum price that MS i should bid in order to use bandwidth of AP j.

B. Phase of Final Allocation

In the phase of final allocation, We first calculates the adapted allocation $\boldsymbol{y}^{\boldsymbol{v}}$ based on bid matrix $\boldsymbol{v}$ in Step 8. The adapted allocation $\boldsymbol{y}^{\boldsymbol{v}}$ is obtained by solving the linear problem (15). The objective function (Eq. (15a)) of this problem maximizes the social welfare (i.e., the total valuations of all MSs) taking into consideration the reservation price $\boldsymbol{r}^*$. Thus, Constraints (15b), (15c) and (15d) are adjusted by considering the impact of nominal allocation $\boldsymbol{x}^*$ and reservation price $\boldsymbol{r}^*$ obtained from the phase of nominal allocation.

$$\max_{\boldsymbol{y}^{\boldsymbol{v}}} \quad \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{M}} y^{\boldsymbol{v}}_{ij}\cdot(v_{ij} - r^*_{ij}) \tag{15a}$$

$$s.t. \quad \sum_{i\in\mathcal{N}} y^{\boldsymbol{v}}_{ij} \le C_i - \sum_{i\in\mathcal{N}} x^*_{ij}, \quad \forall j\in\mathcal{M}, \tag{15b}$$

$$\sum_{j\in\mathcal{M}} y^{\boldsymbol{v}}_{ij}\cdot v_{ij} \le B_i - \sum_{j\in\mathcal{M}} x^*_{ij}\cdot r^*_{ij}, \quad \forall i\in\mathcal{N}, \tag{15c}$$

$$\sum_{j\in\mathcal{M}} y^{\boldsymbol{v}}_{ij} \le D_j - \sum_{j\in\mathcal{M}} x^*_{ij}, \quad \forall i\in\mathcal{N}. \tag{15d}$$

Then we calculate the adapted allocation $\boldsymbol{y}^{\boldsymbol{v}_{-k}}$ without considering the auction participation of MS k in Steps $9-11$. $\boldsymbol{y}^{\boldsymbol{v}_{-k}}$ is used to calculate the final payment of MS k and is obtained by solving the linear problem (16), which is a reduced version of problem (15) by deleting the bidder k from the set of bidders.

$$\max_{\boldsymbol{y}^{\boldsymbol{v}_{-k}}} \quad \sum_{i\in\mathcal{N}\setminus\{k\}} \sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}} \cdot (v_{ij} - r_{ij}^{*}) \tag{16a}$$

$$s.t. \quad \sum_{i\in\mathcal{N}\setminus\{k\}} y_{ij}^{\boldsymbol{v}_{-k}} \leq C_i - \sum_{i\in\mathcal{N}\setminus\{k\}} x_{ij}^{*}, \quad \forall j \in \mathcal{M}, \tag{16b}$$

$$\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}} \cdot v_{ij} \leq B_i - \sum_{j\in\mathcal{M}} x_{ij}^{*} r_{ij}^{*}, \quad \forall i \in \mathcal{N} \setminus \{k\}, \tag{16c}$$

$$\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}} \leq D_j - \sum_{j\in\mathcal{M}} x_{ij}^{*}, \quad \forall i \in \mathcal{N} \setminus \{k\}. \tag{16d}$$

With $\boldsymbol{x}^*$ and $\boldsymbol{r}^*$ obtained in the phase of nominal allocation, as well as $\boldsymbol{y}^{\boldsymbol{v}}$ and $\boldsymbol{y}^{\boldsymbol{v}_{-k}}$ obtained in this phase, we can calculate the final allocation $\boldsymbol{a}^{\boldsymbol{v}}$ and the final payment $\boldsymbol{p}^{\boldsymbol{v}}$ for all $k \in \mathcal{N}$. Steps $12-16$ calculate the final allocation $\boldsymbol{a}^{\boldsymbol{v}}$ that is the sum of nominal allocation $\boldsymbol{x}^*$ and adapted allocation $\boldsymbol{y}^{\boldsymbol{v}}$. Steps $17-19$ calculate the final payment $\boldsymbol{p}^{\boldsymbol{v}}$ using Eq. (17), where $p_k^{\boldsymbol{v}}$ consists of the payment of using $\boldsymbol{a}_k^{\boldsymbol{v}}$ bandwidth and the difference between the optimal value of the objective function obtained with and without the participation of k. This payment scheme guarantees the IR property of Optimal Algorithm. Furthermore, we show that Optimal Algorithm can implement an efficient auction according to Theorem 1.

$$p_k^{\boldsymbol{v}} = \sum_{j\in\mathcal{M}} y_{kj}^{\boldsymbol{v}} \cdot r_{kj}^{*} + \sum_{j\in\mathcal{M}} x_{kj}^{*} \cdot r_{kj}^{*} + \sum_{i\in\mathcal{N}\setminus\{k\}} \sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}} (v_{ij} - r_{ij}^{*}) - \sum_{i\in\mathcal{N}\setminus\{k\}} \sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}} (v_{ij} - r_{ij}^{*}), \quad \forall k \in \mathcal{N}. \tag{17}$$

Theorem 1. *The proposed auction mechanism illustrated in Optimal Algorithm has the properties of incentive compatibility, budget feasibility, individual rationality and worst case optimality.*

The proof of Theorem 1 is illustrated in Appendix A.

C. Solving Optimal Algorithm

In order to solve the optimal multi-item auction mechanism, we need to solve one bilinear optimization problem (11) and three linear optimization problems (14), (15) and (16). The linear problems can be solved using simplex method [23]. The bilinear problem, which is the

Algorithm 2 Bilinear Problem Solver

Input: $\mathcal{U}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{d}, \epsilon, \mathcal{M}, \mathcal{N}$

Output: $\boldsymbol{z}, \boldsymbol{x}^*$

Initialize $\mathcal{Z} \leftarrow \mathcal{U}$ and $\mathcal{X} \leftarrow \{\boldsymbol{x} \geq \mathbf{0} | \boldsymbol{x} \in \mathbb{R}^n\}$
while $\mathcal{X} \neq \varnothing$ **do**
    Call *FindLocalOptimum* $(\mathcal{Z}, \mathcal{X})$
    $\tilde{\boldsymbol{z}} \leftarrow \boldsymbol{x}^m$, $\tilde{\boldsymbol{x}} \leftarrow \boldsymbol{x}^m$
    for all $d_j \in \mathcal{D}$ **do**
        $\theta_j \leftarrow arg\max\{\theta_j | g(\tilde{\boldsymbol{x}} + \theta_i d_j)\} \geq f(\tilde{\boldsymbol{z}}, \tilde{\boldsymbol{x}}) - \epsilon$
    end for
    $G \leftarrow (d_1, \cdots, d_n)$
    $\Delta_x \leftarrow \{\boldsymbol{x} | (\theta_1^{-1}, \cdots, \theta_n^{-1})^T G^{-1}(\boldsymbol{x} - \boldsymbol{x}^*) \geq 1\}$
    $\mathcal{X} \leftarrow \mathcal{X} \cap \Delta_x$
end while
return $\boldsymbol{z} \leftarrow \tilde{\boldsymbol{z}}, \boldsymbol{x}^* \leftarrow \tilde{\boldsymbol{x}}$
procedure FINDLOCALOPTIMUM($\mathcal{Z}, \mathcal{X}$)
    $\boldsymbol{z}_0 \leftarrow arg\min_{z \in \mathcal{Z}} \sum_{i=1}^{m} \sum_{j=1}^{n} v_{ij}$ and $m \leftarrow 1$
    repeat
        $\boldsymbol{x}^m \leftarrow arg\min_{\boldsymbol{x} \in \mathcal{X}} f(\boldsymbol{v}^{m-1}, \boldsymbol{x})$
        $\boldsymbol{z}^m \leftarrow arg\min_{\boldsymbol{z} \in \mathcal{Z}} f(\boldsymbol{v}, \boldsymbol{x}^m)$
    until $f(\boldsymbol{z}^{m-1}, \boldsymbol{x}^m) = f(\boldsymbol{z}^m, \boldsymbol{x}^m)$
end procedure

computation intensive step in the proposed mechanism, is NP-hard [24]. We solve this bilinear problem by a cutting plane method [25], as shown in Algorithm 2.

For simplify, we define a bilinear function $f(\boldsymbol{v}, \boldsymbol{x})$ to represent the objective function of problem (11) as follows:

$$f(\boldsymbol{v}, \boldsymbol{x}) = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{M}} x_{ij} v_{ij}, \quad \forall \boldsymbol{v} \in \mathcal{U},\ \boldsymbol{x} \geq \mathbf{0}. \tag{18}$$

The bilinear function $f(\boldsymbol{v}, \boldsymbol{x})$ can be reduced to a linear function by fixing the value of $\boldsymbol{v}$ or $\boldsymbol{x}$. For example, given the bilinear function derived from Eq. (11), if $\boldsymbol{v}^* = arg\max_{\boldsymbol{z} \in \mathcal{U}} f(\boldsymbol{v}, \boldsymbol{x})$, we get a linear function $g(\boldsymbol{x})$ that can be defined as follows.

$$g(\boldsymbol{x}) = f(\boldsymbol{v}^*, \boldsymbol{x}) = \max_{\boldsymbol{z} \in \mathcal{U}} f(\boldsymbol{z}, \boldsymbol{x}) \tag{19}$$

The aim of Algorithm 2 is to obtain a global ϵ-optimal solution of problem (11), where $\epsilon \geq 0$. In Step 1, we first initialize the feasible regions of valuation matrix $\boldsymbol{z}$ and allocation matrix $\boldsymbol{x}$ based on constraint (11f). The main loop (Steps 2-11) calculates a global ϵ-optimal solution that consists of $\boldsymbol{z}$ and $\boldsymbol{x}^*$. Step 3 uses the procedure *FindLocalOptimum* to obtain a local optimum result $\boldsymbol{z}^m$ and $\boldsymbol{x}^m$. Then, we construct a cut for set $\mathcal{X}$. In Steps 5-7, for direction $d_j (1 \leq j \leq n)$, we find the maximum value of θ_j such that $g(\tilde{\boldsymbol{x}} + \theta_i d_j)\} \geq f(\tilde{\boldsymbol{z}}, \tilde{\boldsymbol{x}}) - \epsilon$. Step 9 calculates the half-space where a better solution exists corresponding to current local optimum $\tilde{\boldsymbol{x}}$. The main loop will end until no better solution is found.

The procedure *FindLocalOptimum* aims to find a local optimum under feasible regions $\mathcal{Z}$ and $\mathcal{X}$. Step 14 initializes $\boldsymbol{z}_0$ that minimizes the valuation matrix. Steps 15-18 calculate $\boldsymbol{x}^m$ and $\boldsymbol{z}^m$ iteratively, until a local optimum $\tilde{\boldsymbol{z}}$ and $\tilde{\boldsymbol{x}}$ is obtained.

VI. Greedy Auction Mechanism

This section proposes two alternative algorithms to solve the multi-item auction based data offloading problem in polynomial time. These two algorithms are based on the concept of two sided matching, where one matching partners are MSs and another matching partners are APs. Each MS can be matched to one AP, while each AP can be matched to multiple MSs. We propose two greedy matching algorithms: the first algorithm is MatchingAP scheme, i.e., it is AP which selects MSs that it will provide network connection to; the second algorithm is MatchingMS scheme, i.e., it is MS which selects appropriate AP for network connection. Then, we show that these two algorithms satisfy the properties of individual rationality and incentive capability.

A. MatchingAP Scheme

The greedy algorithm for MatchingAP scheme, illustrated in Algorithm 3, is composed of two phases, namely, allocation phase and payment phase. The allocation phase aims to select MSs for each AP that can offload mobile data traffic. The payment phase calculates the price paid by each winner by considering the maximum bid from un-winning MSs. This payment scheme is widely used in second price auction to derive a truthful bidding [26].

Algorithm 3 Greedy MatchingAP Scheme

Input: $\boldsymbol{b}, \boldsymbol{d}, \mathcal{M}, \mathcal{N}, \boldsymbol{C}$

Output: $\boldsymbol{a}, \boldsymbol{p}$

1: $\boldsymbol{M} \leftarrow Sort(j \in \mathcal{M}, \frac{C_j}{|\mathcal{N}_j|}, \text{“}non-decreasing''\text{)}$

2: $\boldsymbol{N} \leftarrow \mathcal{N}$

3: **while** $\boldsymbol{M} \neq \varnothing \wedge \boldsymbol{N} \neq \varnothing$ **do**

4: $\quad j \leftarrow Next(\boldsymbol{M})$, $\boldsymbol{M} \leftarrow \boldsymbol{M} \setminus \{j\}$

5: $\quad \boldsymbol{N}_j \leftarrow Sort(i \in \mathcal{N}_j, b_i, \text{“}non-decreasing''\text{)}$

6: $\quad$ **while** $\sum_{i \in \mathcal{N}_j} a_{ij} \leq C_j \wedge \boldsymbol{N}_j \neq \varnothing$ **do**

7: $\qquad i \in Next(\boldsymbol{N}_j)$

8: $\qquad$ **if** $\sum_{j \in \mathcal{M}} a_{ij} = 0 \wedge d_i + \sum_{i \in \mathcal{N}_j} a_{ij} \leq C_j$ **then**

9: $\qquad\quad a_{ij} \leftarrow d_i$

10: $\qquad\quad \boldsymbol{N} \leftarrow \boldsymbol{N} \setminus \{i\}$

11: $\qquad$ **end if**

12: $\quad$ **end while**

13: **end while**

14: **for all** $j \in \mathcal{M}$ **do**

15: $\quad p_k \leftarrow \max_{\{i \in \mathcal{N}_j | a_{ij} = 0\}} b_i$

16: $\quad$ **for all** $i \in \mathcal{N}_j \wedge a_{ij} = d_i$ **do**

17: $\qquad p_i \leftarrow p_j \cdot d_i$

18: $\quad$ **end for**

19: **end for**

In Algorithm 3, Step 1 defines the allocation order for the set of APs. The sorted list $\boldsymbol{M}$ is obtained by sorting all APs participating in the auction in a non-decreasing order of bandwidth per number of covered MSs (i.e., the potential bidders for each AP). The allocation phase (Steps $3-13$) considers APs starting from the first AP in $\boldsymbol{M}$. In MatchingAP scheme, each AP can select MSs under its radio coverage area as potential bidders. Since one AP may have multiple bidders, we define an allocation rule for each AP, which states that the bidder who bids higher value has a higher probability to be served, as shown in Step 5, where MSs under the coverage of AP j are sorted in a non-decreasing order according to the bids submitted by MSs. The bandwidth allocation phase continues until AP j has allocated all its bandwidth or it has no more MSs to be considered (Step 6). For each MS, if it is not allocated to other APs (i.e., served by other APs) and the network demand does not exceed the bandwidth of AP j, it will be allocated to AP j (Steps $8-9$). The payment phase (Steps $14-19$) defines the price paid by each winning MS as the maximum bid value of the set of un-winning MSs. The final payment of MS i is calculated by the market clearing price p_k (obtained in Step 15) and the network demand D_i (Step 17). The computational complexity of Algorithm 3 is $\mathcal{O}(n\log(n)\max\{m,n\})$, where $\mathcal{O}(n\log(n))$ and $\mathcal{O}(\max\{m,n\})$ are due to the inner sort algorithm in Step 5 and outer loop in Step 3, respectively.

B. *MatchingMS Scheme*

In the following, we present the greedy algorithm illustrated in Algorithm 4 for MatchingMS scheme; it has the same algorithm structure as MatchingAP scheme. It also includes allocation and payment phases. Particularly, MatchingMS scheme has same payment rule as MatchingAP scheme.

In Algorithm 4, Step 1 sorts the set of MSs by the maximum bid in a non-decreasing order. Since we aim to maximize the revenue of MNO, MSs are considered according to the allocation order obtained in $\boldsymbol{N}$. The allocation phase (Steps $3-13$) terminates until all MSs or APs are considered. In the inner loop, MS selects one AP that can provide network connection to it. APs that cover MS i are sorted in the list $\boldsymbol{M}_i$ according to bandwidth (Step 5). The network selection phase continues until MS i has selected one AP or it has no more APs to consider (Step 6). For each AP, if it has enough bandwidth to satisfy the demand of MS, it will be selected by MS (Steps $8-9$). The payment phase (Steps $14-19$) is the same as that in Algorithm 3. The computational complexity of Algorithm 4 is $\mathcal{O}(m\log(m)\max\{m,n\})$, where $\mathcal{O}(m\log(m))$ and $\mathcal{O}(\max\{m,n\})$ are due to the inner sort algorithm in Step 5 and outer loop in Step 3, respectively.

These two algorithms satisfy the properties of individual rationality and incentive capability, since they adopted the similar auction structure used in [20]. The budget feasibility is satisfied by the fact that the payment of each MS will not be greater than its bid, i.e., if MS i selects bid $b_i \le \frac{B_i}{d_i}$, then its final payment satisfies $p_i \le b_i$.

Algorithm 4 Greedy MatchingMS Scheme

Input: $\boldsymbol{b}, \boldsymbol{d}, \mathcal{M}, \mathcal{N}, \boldsymbol{C}$

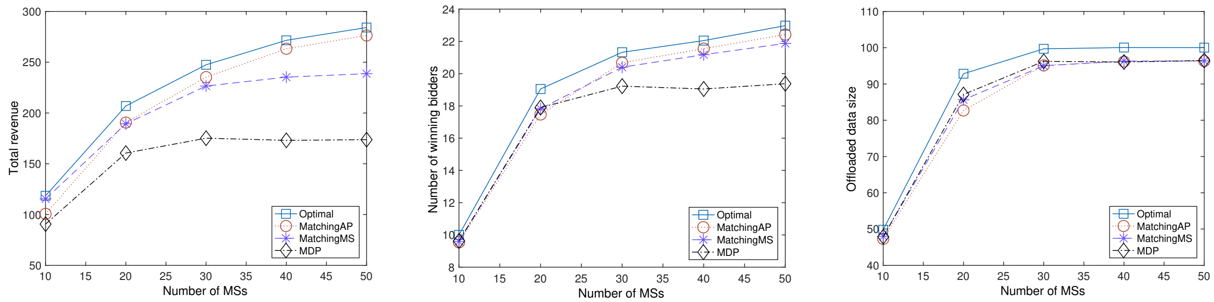
Output: $\boldsymbol{a}, \boldsymbol{p}$

$\boldsymbol{N} \leftarrow Sort(i \in \mathcal{N}, \max_{j \in \mathcal{M}} b_{ij}, \text{“}non-decreasing''\text{)}$
$\boldsymbol{M} \leftarrow \mathcal{M}$
while $\boldsymbol{M} \neq \varnothing \wedge \boldsymbol{N} \neq \varnothing$ **do**
  $i \leftarrow Next(\boldsymbol{N})$, $\boldsymbol{N} \leftarrow \boldsymbol{N} \setminus \{i\}$
  $\boldsymbol{M}_i \leftarrow Sort(j \in \mathcal{M}_i, C_j, \text{“}non-decreasing''\text{)}$
  while $\sum_{j \in \mathcal{M}} a_{ij} < d_i \wedge \boldsymbol{M}_i \neq \varnothing$ **do**
    $j \in Next(\boldsymbol{M}_i)$
    if $d_i + \sum_{i \in \mathcal{N}_j} a_{ij} \leq C_j$ **then**
      $a_{ij} \leftarrow d_i$
      $\boldsymbol{M} \leftarrow \boldsymbol{M} \setminus \{j\}$
    end if
  end while
end while
for all $j \in \mathcal{M}$ **do**
  $p_k \leftarrow \max_{\{i \in \mathcal{N}_j | a_{ij}=0\}} b_i$
  for all $i \in \mathcal{N}_j \wedge a_{ij} = d_i$ **do**
    $p_i \leftarrow p_j \cdot d_i$
  end for
end for

VII. Numerical Results

In this section, we evaluate the performance of the proposed auction mechanism for selling AP bandwidth to MSs in proximity. More specifically, we aim to evaluate the impact of AP density, budget constraint and uncertainty set of valuation on the performance of the proposed mechanisms in order to implement an effective mobile data offloading marketplace. We first introduce the parameter settings, then we illustrate and discuss the numerical results achieved by the proposed offloading algorithms.

In our evaluation, we use the network topology described in [20]. More specifically, we



(a) Revenue (b) Number of Winners (c) Offloaded Traffic

Fig. 2. Performance comparison with low AP density ($m = 5$).

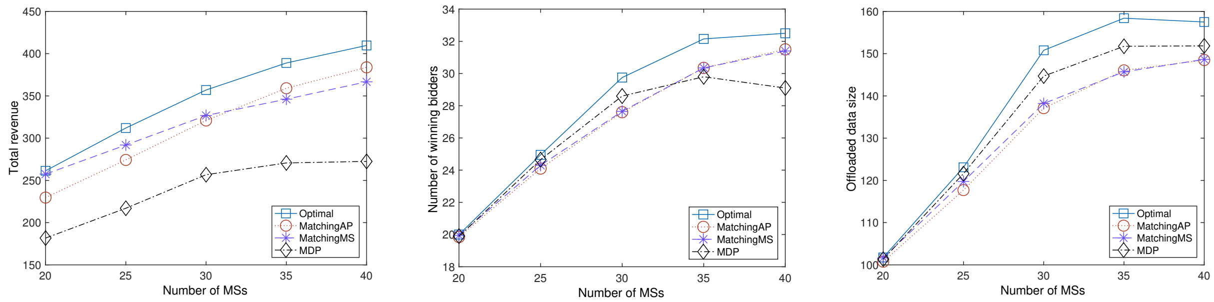
consider a scenario composed of 4 cell sites and the distance between each site is set to 500 meters. All Base Stations (BSs) are installed on some central site and can serve 3 sectors, resulting in 12 sectors in total. The number of APs installed in a sector, denoted by m, belongs to $\{2, \cdots, 20\}$. The number of MSs scattered in a sector, denoted by n, belongs to $\{10, \cdots, 60\}$. In each sector, the locations of APs and MSs are drawn from a uniform distribution. Each MS submits a bid drawn from a normal distribution with mean value equal to 2 monetary units (e.g., US dollars) and derivation equal to 1 unit. The maximum bandwidth of each AP is in the range of $[5Mbps, 40Mbps]$, while the bandwidth demand of each MS is in the range of $[2Mbps, 10Mbps]$. The budget of MS is selected from the range of $[10, 20]$. We use the convex function $f_j(k) = bk^a$ to construct the uncertainty set $\mathcal{U}_j$. As an example, we randomly adopt a and b within the range of $[0.04, 0.08]$ and $[0.8, 1.2]$, respectively.

We compare our proposed schemes, namely optimal scheme, MatchingAP scheme and MatchingMS scheme, with the work in [27, 28], denoted as MDP scheme, since this work aims to maximize MNO's revenue and the amounts of offloaded data based on Markov Decision Process. The following performance metrics are considered in the evaluation.

- Total revenue. The total payoff of MNO.
- Offloaded traffic. The amount of traffic that can be offloaded.
- Winning MSs. The number of MSs that win the auction.

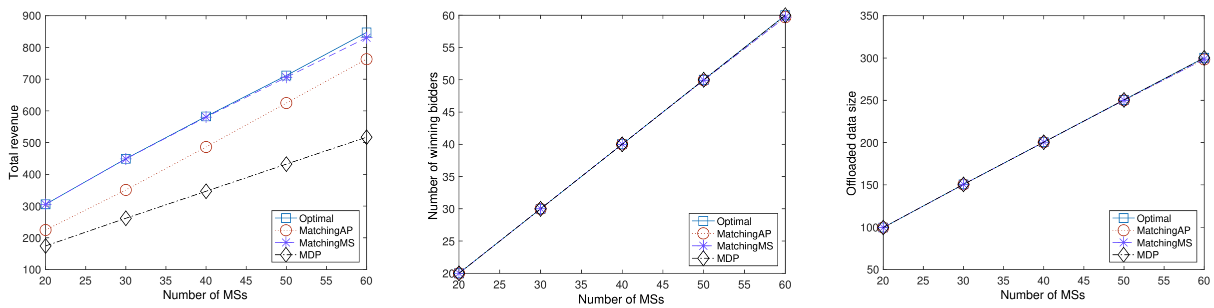
A. *Impact of AP Density in Homogeneous Networks*

We first evaluate the impact of the number of APs on the performance of our multi-item auction mechanism. In order to simulate different AP density levels, we consider three levels of



(a) Revenue (b) Number of Winners (c) Offloaded Traffic

Fig. 3. Performance comparison with medium AP density ($m = 10$).

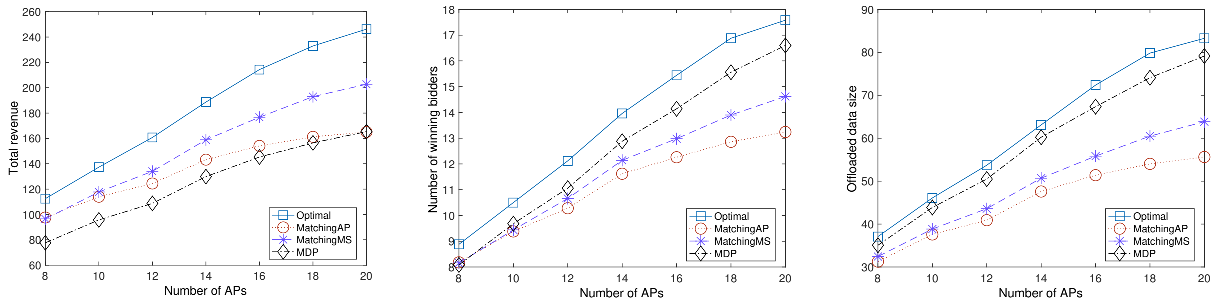


(a) Revenue (b) Number of Winners (c) Offloaded Traffic

Fig. 4. Performance comparison with high AP density ($m = 20$).

device density, with the number of APs equal to 5, 10, and 20, respectively. In all scenarios, the number of MS varies in the range of $[10, 60]$, The demanding bandwidth of MS varies in the range of $[2Mbps, 10Mbps]$. The simulation results are shown in Figs. 2, 3 and 4.

Figs. 2(a), 3(a), and 4(a) show the variation of the MNO's revenue with the number of MSs. We observe that the total revenue increases with the number of MSs; this is expected since the offloading demand increases with the number of MSs resulting in higher payment. Optimal scheme (Optimal Algorithm) outperforms the two greedy schemes, i.e., MatchingAP and MatchingMS, and MDP scheme in all scenarios. On average, a large number of MSs makes MatchingAP achieves higher revenue than MatchingMS; this can be explained by the fact that MatchingAP takes advantage of the competition among MSs to obtain higher revenue, e.g., MatchingAP outperforms MatchingMS when the number of MSs is bigger than 30 and 35 in Figs. 2(a) and 3(a), respectively. The results also indicate that MatchingAP is more suitable than MatchingMS in high MS density scenarios. However, with higher AP density, MatchingMS



(a) Revenue (b) Number of Winners (c) Offloaded Traffic

Fig. 5. Performance comparison with low capacity ($C_j = 5$ Mbps)

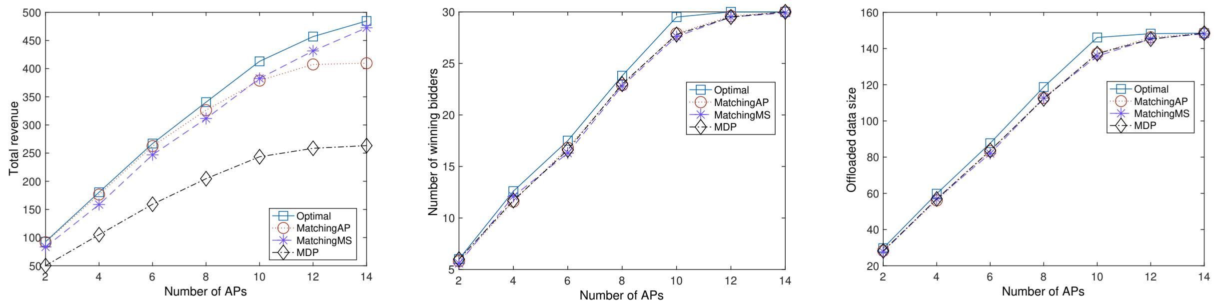
performs better than MatchingAP when the number of MSs is small. Especially, in high AP density scenario, MatchingMS outperforms MatchingAP and achieves nearly optimal revenues. Generally, a larger AP density reduces the competition among MSs, reducing the performance of MatchingAP.

Figs. 2(b), 3(b) and 4(b) show the number of winning MSs versus the number of MSs. We observe that the optimal scheme has the largest number of winning MSs; this indicates that the optimal scheme can implement fairness allocation among multiple MSs. Figure 4(b) illustrates that the number of winning MSs of all schemes increase linearly with the increase of the number of MSs. This is because higher AP density guarantees enough bandwidth for data offloading demand from MSs. However, it is not the same case in Figs. 2(b) and 3(b), where the total APs' bandwidth is not sufficient to support a large number of MSs, resulting in a nearly stable number of winning MSs when the number of MSs is large enough.

In order to evaluate the data offloading performance, we plot the offloaded traffic versus the number of MSs in Figs. 2(c), 3(c) and 4(c). Figs. 2(b) and 3(b) show that the optimal scheme achieves the highest size of offloaded traffic, and the MDP scheme outperforms MatchingAP and MatchingMS schemes. Fig. 4(c) shows that all the schemes achieve the same size of offloaded traffic, since all demands of MSs are satisfied (see Fig. 4(b)).

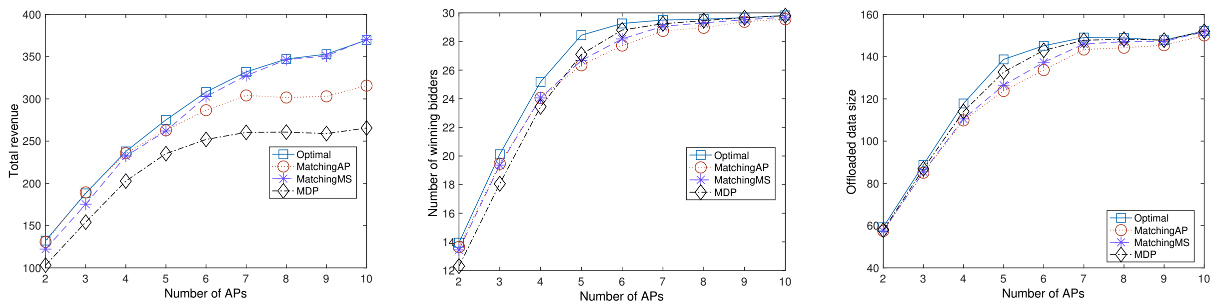
B. Impact of AP Bandwidth in Heterogeneous Networks

We evaluate the effect of AP bandwidth on the performance of our proposed schemes. We consider three levels of bandwidth C, namely low, medium and high, corresponding to 5, 25, and $40 Mbps$, respectively. In all scenarios, the number of APs varies in the range of $[2, 20]$.



(a) Revenue (b) Number of Winners (c) Offloaded Traffic

Fig. 6. Performance comparison with medium capacity ($C_j = 25$ Mbps)



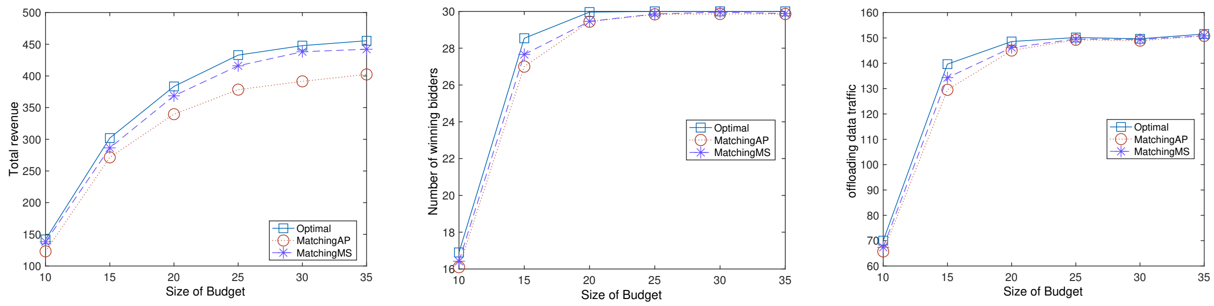
(a) Revenue (b) Number of Winners (c) Offloaded Traffic

Fig. 7. Performance comparison with high capacity ($C_j = 40$ Mbps)

The number of MSs varies in the range of $[30, 40]$ and the demand of MS varies in the range of $[3Mbps, 10Mbps]$. The simulation results are shown in Figs. 5, 6 and 7.

Figs. 5(a), 6(a) and 7(a) show the variation of revenue with the number of APs. We observe that the revenues of all schemes increase with the number of APs. As more APs participate in the auction, MNO has more bandwidth provided to MSs, leading to higher revenue. We find that the optimal scheme outperforms the other schemes in all scenarios. MatchingMS achieves higher revenue than MatchingAP when AP bandwidth is low, as shown in Figure 5(a). Note that with low bandwidth 5 Mbps, each AP can serve one MS at most, since the minimum demand of MS is 3 Mbps. In this scenario, the final payment of winning MS is the same as its bid value, since the only bidder is the winning MS itself. According to the sorting rule of MatchingMS (see Algorithm 4), MS with a higher bid value has higher chance of winning the auction, resulting in a higher revenue.

However, the situation changes when AP bandwidth increases to 25 Mbps, as shown in Figure



(a) Revenue (b) Number of Winners (c) Offloaded Traffic

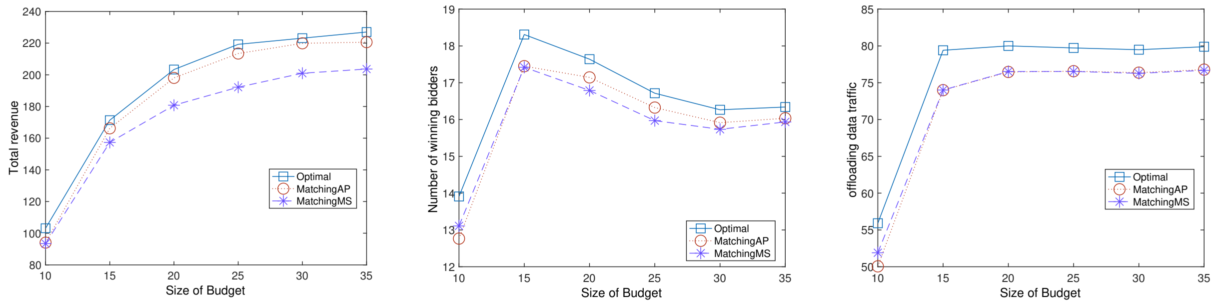
Fig. 8. Performance comparison with budget constraint ($n = 30, m = 10$)

6(a), where one AP can serve multiple MSs. In this scenario, MatchingAP achieves higher revenue than MatchingMS when $m < 10$. This is because MatchingAP selects AP based on its average bandwidth for each MS; larger AP bandwidth can serve more MSs and lead to higher competition among MSs, achieving higher revenue. While MatchingMS simply decides winning MSs based on bid value, without considering the introduction of more competition among MSs. Particularly, in the high bandwidth scenario, when AP bandwidth is 40 Mbps, MatchingAP achieves higher revenue than MatchingAP when $m < 4$. This is because the benefit of competition among bidders is decreased with sufficient bandwidth provided by a large number of APs.

Figs. 5(b), 6(b) and 7(b) show that the number of winning MSs increase with the the number of APs, since large number of APs increases the potential of satisfying the demand of MSs. We observe that the optimal scheme has the highest number of winning MSs. The curves, as shown in Figs. 5(c), 6(c) and 7(c), follow similar trends as Figs. 5(b), 6(b) and 7(b), respectively, due to the fact that the offloaded traffic increases with the number of winning MSs.

We summarize that the optimal scheme outperforms all other schemes in all scenarios. MatchingMS outperforms MatchingAP in the following two scenarios:

- High AP density: In this case, choosing MS with higher value generates higher revenue, since its demand can always be satisfied;
- Low AP bandwidth: This leads to a special case of data offloading, where one AP is connected to at most one MS at a time.



(a) Revenue (b) Number of Winners (c) Offloaded Traffic

Fig. 9. Performance comparison with budget constraint ($n = 30, m = 5$).

C. Impact of Budget Constraint

We evaluate the effect of budget constraint on the performance of our proposed schemes. We consider two scenarios based on whether the aggregate bandwidth of APs can satisfy the bandwidth demands of MSs or not. Fig. 8 shows the result when the aggregate bandwidth of APs is sufficient, i.e., $n = 10$, while Fig. 9 shows the result when the aggregate bandwidth of APs cannot satisfy all the demands from MSs, i.e., $n = 5$. We observe that the optimal method can obtain the highest revenue in all cases, as shown in Figs. 8(a) and 9(a). MatchingMS outperforms MatchingAP when the aggregate bandwidth of APs is sufficient, while MatchingAP outperforms MatchingMS when the aggregate bandwidth of APs is small.

In Fig. 8, we further observe that the total revenue increases with the value of budget. When $B_i \geq 25$, all MSs win the auction (see Fig. 8(b)) and bandwidth demands are satisfied (see Fig. 8(c)). Thus, the number of winning bidders and the offloaded traffic cannot increase with the value of budget when $B_i \geq 25$. However, the total revenue still increases when $B_i \geq 25$, since higher budget indicates higher valuation from MSs.

Fig. 9 shows the scenario where the total bandwidth demands of MSs is larger than the aggregate bandwidth of APs. As shown in Fig. 9(c), when $B_i \geq 15$, the offloaded traffic cannot increase the value of budget. This implies that all bandwidth of APs have been allocated. We observe that, when $B_i \geq 15$, the increase of budget leads to higher revenue (see Fig. 9(a)) and smaller number of winning bidders (see Fig. 9(b)). It is because higher budget increases the winning probability of MSs who have higher valuations and larger bandwidth demands. Thus, the total revenue increases while the number of winning bidders decreases when $B_i \geq 15$.

D. Robustness and Scalability Analysis

Now we illustrate the robustness and scalability of the proposed optimal offloading method. In order to show the robustness of the proposed method, we consider the scenario where the assumed distributions of MSs' valuations differ from the practical distributions, i.e., the MNO's belief on the value of μ_j and δ_j is different from the realized value of μ_j^* and δ_j^*. We compare the proposed method with Myerson auction [29] that is an optimal auction with reservation price. Myerson auction calculates the reservation price by solving the following equation.

$$1 - F_j(v_j) = v_j * f_j(v_j), \tag{20}$$

where $F_j(.)$ and $f_j(.)$ are the cumulative distribution function and probability density function, respectively, of the probability distribution that the valuation v_j is sampled from. Note that our method calculates the reservation price by solving the bilinear programming problem (11). Thus, the reservation prices obtained by Myerson auction are different from that calculated by our proposed method in most cases.

We consider a simple scenario where valuation v_j follows the normal distribution with parameters $\mu_j^* = 3$ and $\delta_j^* = 2$, for all $j \in \mathcal{M}$, where μ_j^* and δ_j^* are the practical expectation and deviation of the normal distribution, respectively. The number of APs is 10 and the number of MSs is 30.

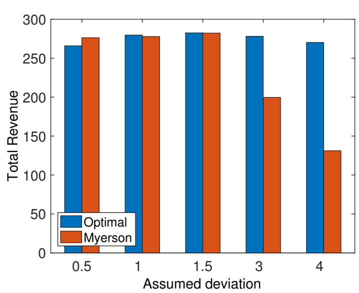


Fig. 10. Robustness comparison with different deviation

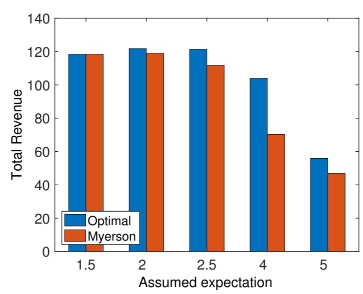


Fig. 11. Robustness comparison with different expectation

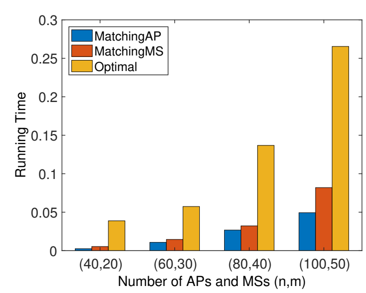


Fig. 12. Scalability comparison among different schemes

We first investigate the revenue achieved by MNO when the assumed deviation δ_j is different from the practical deviation δ_j^*. To evaluate the impact of different deviations, we choose $\delta_j \in \{0.5, 1, 1.5, 3, 4\}$. Fig. 10 shows the total revenue obtained by Myerson auction and our proposed method. The larger value of δ_j, the lower revenue that the Myerson auction can obtain. For example, when $\delta_j = 4$, the proposed optimal auction outperforms the Myerson auction by 56%.

This is because the reservation price used in Myerson auction depends on the assumed distribution. Thus, a misspecified (e.g., non-realistic) distribution reduces the performance of Myerson auction. Furthermore, our proposed auction can achieve better performance due to its insensitivity to the assumed distribution.

We further evaluate how the assumed expectation μ_j affects the total revenue when using the Myerson auction and the proposed auction. Fig. 11 shows the total revenue obtained by Myerson auction and the proposed auction, when the value of μ_j is chosen from $\{1.5, 2, 2.5, 4, 5\}$. We observe that both methods achieve good performance when $\mu_j < \mu_j^*$. However, the situation changes when $\mu_j > \mu_j^*$, e.g., $\mu_j = 5$, where both methods achieve lower revenue due to the misspecification of μ_j.

We conclude that both Myerson auction and the proposed auction are sensitive to the misspecification of μ_j. Furthermore, Myerson auction is sensitive to the misspecification of δ_j, especially when $\delta_j > \delta_j^*$, while the proposed method is insensitive to the misspecification of δ_j. Thus, the proposed method has stronger robustness than Myerson auction when the deviation of normal distribution is misspecified.

Lastly, we evaluate the running time of the proposed schemes on an Intel (R) Core(TM) i7-2620M CPU 2.70GHz processor with RAM of 16.00 GB and 64-bit Linux operating system. We measure the running time (seconds) of different schemes with different numbers of APs and MSs. In Fig. 12, we observe that MatchingAP achieves the lowest running time in all cases. The running time of optimal scheme increases faster than the two other schemes with the number of APs and MSs. Note that when the number of APs is 50 and the number of MSs is 100, the running time of optimal scheme is 0.265 seconds, which is a reasonable value, since the auction is executed every ten seconds.

VIII. Conclusion

This paper proposed a new trading marketplace where mobile operators can sell bandwidth made available by their own Access Points to offload data traffic of their mobile subscribers. The offloading problem was formulated as a multi-item auction based robust optimization approach to guarantee individual rationality, incentive capability and budget feasibility for realistic scenarios in which only part of the valuation information of MSs is known to MNO. In order to solve efficiently (i.e., in polynomial time) the offloading problem for large-scale network scenarios, we also proposed two greedy algorithms. Numerical results show that the proposed schemes capture well the economical and networking essence of the problem, thus representing a promising solution to implement a trading marketplace for next-generation access networks composed of heterogeneous systems.

APPENDIX A
PROOF OF THE PROPERTIES OF THE PROPOSED AUCTION MECHANISM

In this appendix, we present the proof that our proposed auction mechanism has the following properties in sequence, i.e., incentive compatibility (see Lemma 2), budget feasibility (see Lemma 3), individual rationality (see Lemma 4) and worst case optimality (see Lemma 5).

Lemma 1. *If $\boldsymbol{z}$ and $\boldsymbol{x}^*$ are the optimal solution of problem* (11)*, then $\boldsymbol{z}$ and $\boldsymbol{x}^*$ satisfy the following conditions:*

$$\sum_{j\in\mathcal{M}} x_{ij}^* \cdot z_{ij} \le B_i, \quad \forall i \in \mathcal{N}, \tag{21}$$

$$\sum_{i\in\mathcal{N}} x_{ij}^* \le C_j, \quad \forall j \in \mathcal{M}, \tag{22}$$

$$\sum_{j\in\mathcal{M}} x_{ij}^* \ge D_i, \quad \forall i \in \mathcal{N}, \tag{23}$$

$$\sum_{j\in\mathcal{M}} x_{ij}^* \cdot z_{ij} \le \sum_{j\in\mathcal{M}} x_{ij}^* \cdot u_{ij}, \quad \forall \boldsymbol{u} \in \mathcal{U},\ \forall i \in \mathcal{N}. \tag{24}$$

$$\sum_{k\in\mathcal{N}} p_k^{\boldsymbol{z}} = \sum_{k\in\mathcal{N}} \sum_{j\in\mathcal{M}} x_{kj}^* \cdot z_{kj}^* \tag{25}$$

Proof. Lemma 1 can be proved by considering a reduced version of problem (11), where we set $\boldsymbol{v} = \boldsymbol{z}$. Thus, the original bilinear optimization problem (11) is reduced to a new linear optimization problem, since the only variable is $\boldsymbol{x}$. The relations (21), (22), (23) and (24) that $\boldsymbol{z}$ and $\boldsymbol{x}^*$ satisfy are derived directly from the constraints (11b), (11c), (11d) and (11e), respectively. Eq. (25) is derived from the objective function of problem (11). □

Lemma 2. *The proposed auction mechanism with final allocation matrix $\boldsymbol{a}^v$ and payment vector $\boldsymbol{p}^v$, satisfies the property of incentive compatibility. That is $U_k^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})} \ge U_k^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}$, which means that MS k gets higher utility with truthful bidding $\boldsymbol{v}_k$.*

Proof. We assume that the private valuation for MS k is $\boldsymbol{v}_k \in \mathbb{R}^m$, and the private valuation for the rest $(n-1)$ MSs is $\boldsymbol{v}_{-k} \in \mathbb{R}^{(n-1)} \times \mathbb{R}^m$. Now if MS k chooses to bid with valuation

$\boldsymbol{u}_k \in \mathbb{R}^m$ instead of $\boldsymbol{v}_k$; using Eq. (4), where the utility $U_k^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}$ is the difference of payoff and payment, we obtain the utility of MS k as follows:

$$U_k^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})} = \sum_{j\in\mathcal{M}} a_{kj}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})} \cdot v_{kj} - p_k^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}. \tag{26}$$

With the fact that $a_{ij}^{\boldsymbol{v}} = x_{ij}^* + y_{ij}^{\boldsymbol{v}}$ (Step 14 in Optimal Algorithm) and Eq. (17), Eq. (27) can be rewritten as

$$\begin{aligned} U_k^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})} = &\sum_{j\in\mathcal{M}} y_{kj}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})} \cdot v_{kj} + \sum_{j\in\mathcal{M}} x_{kj}^* \cdot v_{kj} - \sum_{j\in\mathcal{M}} y_{kj}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})} \cdot r_{kj}^* - \sum_{j\in\mathcal{M}} x_{kj}^* \cdot r_{kj}^* \\ &- \sum_{i\in\mathcal{N}\setminus\{k\}} \sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}}(v_{ij} - r_{ij}^*) + \sum_{i\in\mathcal{N}\setminus\{k\}} \sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}(v_{ij} - r_{ij}^*). \end{aligned} \tag{27}$$

By substituting the following identical equation

$$\sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}(v_{ij} - r_{ij}^*) \equiv \sum_{j\in\mathcal{M}} y_{kj}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})} \cdot v_{kj} - \sum_{j\in\mathcal{M}} y_{kj}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})} \cdot r_{kj}^* + \sum_{i\in\mathcal{N}\setminus\{k\}} \sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}(v_{ij} - r_{ij}^*), \tag{28}$$

into Eq. (27) and some mathematical manipulations, we have

$$U_k^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})} = \sum_{j\in\mathcal{M}} x_{kj}^* \cdot v_{kj} - \sum_{j\in\mathcal{M}} x_{kj}^* \cdot r_{kj}^* - \sum_{i\in\mathcal{N}\setminus\{k\}} \sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}}(v_{ij} - r_{ij}^*) + \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}(v_{ij} - r_{ij}^*). \tag{29}$$

Similarly, we get the utility $U_k^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})}$ when MS k bid truthfully as follows:

$$U_k^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})} = \sum_{j\in\mathcal{M}} x_{kj}^* \cdot v_{kj} - \sum_{j\in\mathcal{M}} x_{kj}^* \cdot r_{kj}^* - \sum_{i\in\mathcal{N}\setminus\{k\}} \sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}}(v_{ij} - r_{ij}^*) + \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})}(v_{ij} - r_{ij}^*). \tag{30}$$

By subtracting Eq. (29) from Eq. (30), we have

$$U_k^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})} - U_k^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})} = \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})}(v_{ij} - r_{ij}^*) - \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}(v_{ij} - r_{ij}^*). \tag{31}$$

Note that $y_{ij}^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})}$ is the optimal solution of problem (15), while $y_{ij}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}$ is a feasible solution of problem (15). Thus, we obtain

$$\sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})}(v_{ij} - r_{ij}^*) \geq \sum_{j\in\mathcal{M}} y_{ij}^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}(v_{ij} - r_{ij}^*), \tag{32}$$

which demonstrates that $U_k^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})} \geq U_k^{(\boldsymbol{u}_k,\boldsymbol{v}_{-k})}$, due to Eq. (31). $\square$

Lemma 3. *The proposed auction mechanism with final allocation matrix $\boldsymbol{a}^{\boldsymbol{v}}$ and payment vector $\boldsymbol{p}^{\boldsymbol{v}}$, satisfies the property of budget feasibility. That is $p_k^{\boldsymbol{v}} \leq B_k$, which implies that the payment of MS k is smaller than its budget.*

Proof. We first construct an allocation matrix $\tilde{\boldsymbol{y}}^{\boldsymbol{v}} \in \mathbb{R}^{n \times m}$ based on $\boldsymbol{y}^{\boldsymbol{v}_{-k}} \in \mathbb{R}^{(n-1) \times m}$, where

$$\tilde{y}_{ij}^{\boldsymbol{v}} = \begin{cases} y_{ij}^{\boldsymbol{v}_{-k}}, & \forall i \in \mathcal{N} \setminus \{k\},\ \forall j \in \mathcal{M}, \\ 0, & i = k,\ \forall j \in \mathcal{M}. \end{cases} \tag{33}$$

Thus, we can obtain the following identical equation:

$$\sum_{i \in \mathcal{N} \setminus \{k\}} \sum_{j \in \mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}} (v_{ij} - r_{ij}^*) \equiv \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{M}} \tilde{y}_{ij}^{\boldsymbol{v}} (v_{ij} - r_{ij}^*). \tag{34}$$

Note that $\tilde{\boldsymbol{y}}^{\boldsymbol{v}}$ is a feasible solution to problem (16). That is, $\tilde{\boldsymbol{y}}^{\boldsymbol{v}}$ satisfies all the constraints of problem (16). From Eq. (16b), we obtain

$$\sum_{i \in \mathcal{N}} \tilde{y}_{ij}^{\boldsymbol{v}} \leq C_i - \sum_{i \in \mathcal{N}} x_{ij}^*, \quad \forall j \in \mathcal{M}. \tag{35}$$

From Eq. (16c), we obtain that $\forall i \in \mathcal{N} \setminus \{k\}$,

$$\sum_{j \in \mathcal{M}} \tilde{y}_{ij}^{\boldsymbol{v}} \cdot v_{ij} \leq B_i - \sum_{j \in \mathcal{M}} x_{ij}^* \cdot r_{ij}^*. \tag{36}$$

Note that

$$\sum_{j \in \mathcal{M}} \tilde{y}_{kj}^{\boldsymbol{v}} \cdot v_{kj} = 0 \leq B_i - \sum_{j \in \mathcal{M}} x_{ij}^* \cdot r_{ij}^*. \tag{37}$$

By combine Eqs. (36) and (37), we obtain

$$\sum_{j \in \mathcal{M}} \tilde{y}_{ij}^{\boldsymbol{v}} \cdot v_{ij} \leq B_i - \sum_{j \in \mathcal{M}} x_{ij}^* \cdot r_{ij}^*, \quad \forall i \in \mathcal{N}. \tag{38}$$

Similarly, we can obtain

$$\sum_{j \in \mathcal{M}} \tilde{y}_{ij}^{\boldsymbol{v}} \leq D_j - \sum_{j \in \mathcal{M}} x_{ij}^*, \quad \forall i \in \mathcal{N}. \tag{39}$$

From Eqs. (35), (38) and (39), we show that $\tilde{\boldsymbol{y}}^{\boldsymbol{v}}$ is a feasible solution to problem (15), since it satisfies the constraints (15b), (15c) and (15d). Note that $\boldsymbol{y}^{\boldsymbol{v}}$ is the optimal solution to problem (15), Thus, we obtain

$$\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{M}} \tilde{y}_{ij}^{\boldsymbol{v}} (v_{ij} - r_{ij}^*) \leq \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{M}} y_{ij}^{\boldsymbol{v}} (v_{ij} - r_{ij}^*). \tag{40}$$

By substituting Eq. (28) into Eq. (17), we have

$$p_k^{\boldsymbol{v}} = \sum_{j\in\mathcal{M}} y_{kj}^{\boldsymbol{v}} \cdot v_{kj} + \sum_{j\in\mathcal{M}} x_{kj}^* \cdot r_{kj}^* + \sum_{i\in\mathcal{N}\setminus\{k\}} \sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}}(v_{ij} - r_{ij}^*) - \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}}(v_{ij} - r_{ij}^*). \tag{41}$$

By substituting Eq. (34) into Eq. (41), we have

$$p_k^{\boldsymbol{v}} = \sum_{j\in\mathcal{M}} y_{kj}^{\boldsymbol{v}} \cdot v_{kj} + \sum_{j\in\mathcal{M}} x_{kj}^* \cdot r_{kj}^* + \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{M}} \tilde{y}_{ij}^{\boldsymbol{v}}(v_{ij} - r_{ij}^*) - \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}}(v_{ij} - r_{ij}^*). \tag{42}$$

Due to Eq. (40), we have

$$p_k^{\boldsymbol{v}} \le \sum_{j\in\mathcal{M}} y_{kj}^{\boldsymbol{v}} \cdot v_{kj} + \sum_{j\in\mathcal{M}} x_{kj}^* \cdot r_{kj}^*. \tag{43}$$

From Eq. (15c), we obtain that $p_k^{\boldsymbol{v}} \le B_k$. □

Lemma 4. *The proposed auction mechanism with final allocation matrix $\boldsymbol{a}^{\boldsymbol{v}}$ and payment vector $\boldsymbol{p}^{\boldsymbol{v}}$, satisfy the property of individual rationality. That is, if MS k bids truthfully with valuation vector $\boldsymbol{v}_k$, it will get a nonnegative utility $U_k^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})} \ge 0$.*

Proof. By substituting Eq. (34) into Eq. (40), we obtain

$$\sum_{i\in\mathcal{N}\setminus\{k\}} \sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}}(v_{ij} - r_{ij}^*) \le \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}}(v_{ij} - r_{ij}^*). \tag{44}$$

From Eqs. (30) and (44), we obtain

$$U_k^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})} \ge \sum_{j\in\mathcal{M}} x_{kj}^* \cdot v_{kj} - \sum_{j\in\mathcal{M}} x_{kj}^* \cdot r_{kj}^*. \tag{45}$$

From Lemma 1, we get

$$\sum_{j\in\mathcal{M}} x_{kj}^* \cdot v_{kj} \ge \sum_{j\in\mathcal{M}} x_{kj}^* \cdot z_{kj} = \sum_{j\in\mathcal{M}} x_{kj}^* \cdot r_{kj}^*. \tag{46}$$

This implies that $U_k^{(\boldsymbol{v}_k,\boldsymbol{v}_{-k})} \ge 0$. □

Lemma 5. *The proposed auction mechanism with final allocation matrix $\boldsymbol{a}^{\boldsymbol{z}}$ and payment vector $\boldsymbol{p}^{\boldsymbol{z}}$, satisfies the property of worst case optimality. That is $\sum_{k=1}^n \boldsymbol{p}_k^{\boldsymbol{z}} \le \sum_{k=1}^n \boldsymbol{p}_k^{\boldsymbol{v}}$, which means that the revenue of MNO under valuation matrix $\boldsymbol{z}$ (obtained by Optimal Algorithm) is smaller than that under $\boldsymbol{v} \in \mathcal{U}$.*

Proof. We first construct an allocation matrix $\tilde{\boldsymbol{y}}^{\boldsymbol{v}} \in \mathbb{R}^{(n-1)\times m}$ based on $\boldsymbol{y}^{\boldsymbol{v}} \in \mathbb{R}^{n\times m}$, where

$$\tilde{y}_{ij}^{\boldsymbol{v}_{-k}} = y_{ij}^{\boldsymbol{v}}, \quad \forall i \in \mathcal{N} \setminus \{k\},\ \forall j \in \mathcal{M}. \tag{47}$$

Thus, we have the identical equation

$$\sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}}(v_{ij}-r_{ij}^*) \equiv \sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} \tilde{y}_{ij}^{\boldsymbol{v}_{-k}}(v_{ij}-r_{ij}^*). \tag{48}$$

In the following, we show that $\tilde{\boldsymbol{y}}^{\boldsymbol{v}}$ is a feasible solution to problem (15). From Eq. (15b), we obtain that $\forall j \in \mathcal{M}$,

$$\sum_{i\in\mathcal{N}\setminus\{k\}} \tilde{y}_{ij}^{\boldsymbol{v}_{-k}} = \sum_{i\in\mathcal{N}\setminus\{k\}} y_{ij}^{\boldsymbol{v}} \le C_i - \sum_{i\in\mathcal{N}\setminus\{k\}} x_{ij}^*. \tag{49}$$

From Eq. (15c), we obtain that $\forall i \in \mathcal{N}\setminus\{k\}$,

$$\sum_{j\in\mathcal{M}} \tilde{y}_{ij}^{\boldsymbol{v}_{-k}} \cdot v_{ij} = \sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}} \cdot v_{ij} \le B_i - \sum_{j\in\mathcal{M}} x_{ij}^* \cdot r_{ij}^*. \tag{50}$$

From Eq. (15d), we obtain that $\forall i \in \mathcal{N}\setminus\{k\}$,

$$\sum_{j\in\mathcal{M}} \tilde{y}_{ij}^{\boldsymbol{v}_{-k}} = \sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}} \le D_j - \sum_{j\in\mathcal{M}} x_{ij}^*. \tag{51}$$

From Eqs. (49), (50) and (51), we show that $\tilde{\boldsymbol{y}}^{\boldsymbol{v}}$ is a feasible solution to problem (16), since it satisfies constraints (16b), (16c) and (16d). Notice that $\boldsymbol{y}^{\boldsymbol{v}_{-k}}$ is the optimal solution to problem (16). Thus, we have

$$\sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} \tilde{y}_{ij}^{\boldsymbol{v}_{-k}}(v_{ij}-r_{ij}^*) \le \sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}}(v_{ij}-r_{ij}^*). \tag{52}$$

From Eqs. (48) and (52), we derive

$$\sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}}(v_{ij}-r_{ij}^*) \le \sum_{i\in\mathcal{N}\setminus\{k\}}\sum_{j\in\mathcal{M}} y_{ij}^{\boldsymbol{v}_{-k}}(v_{ij}-r_{ij}^*). \tag{53}$$

From Eqs. (17) and (53), we derive

$$p_k^{\boldsymbol{v}} \ge \sum_{j\in\mathcal{M}} y_{kj}^{\boldsymbol{v}} \cdot r_{kj}^* + \sum_{j\in\mathcal{M}} x_{kj}^* \cdot r_{kj}^*. \tag{54}$$

From Lemma 1, we know that

$$\sum_{k\in\mathcal{N}} p_k^{\boldsymbol{z}} = \sum_{k\in\mathcal{N}}\sum_{j\in\mathcal{M}} x_{kj}^* \cdot r_{kj}^*. \tag{55}$$

Finally, we get that $\sum_{k\in\mathcal{N}} p_k^{\boldsymbol{z}} \le \sum_{k\in\mathcal{N}} p_k^{\boldsymbol{v}}$. $\square$

With Lemma $2-5$, we complete the proof of Theorem 1.

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