**Petersen Ch. 3 Exercises**

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**7. Parallel Ricci tensor implies constant scalar curvature.** • If $\nabla \mathrm{Ric} = 0$, then for any $p \in M$ and $X, Y, Z \in T_pM$,

$$(\nabla_Z R(E_i, \cdot, \cdot, E_i))(X, Y) = \sum_i Z(R(E_i, X, Y, E_i)) - R(E_i, \nabla_Z X, Y, E_i) - R(E_i, X, \nabla_Z Y, E_i) = 0,$$

where $(E_i)$ is an orthonormal basis for $T_pM$. Thus by Prop. 3.1.1,

$$Z\left(\sum_{i,j} R(E_i, E_j, E_i, E_j)\right) = \sum_{i,j} 2R(E_i, \nabla_Z E_j, E_j, E_i).$$

But we can assume $(E_i)$ is a parallel frame by Exercise 2.5.19. Therefore the righthand term vanishes, so the scalar curvature is constant.