1. $n = 120$ $\qquad z_{\alpha/2} = z_{0.015} = 2.17$

$\bar{x} = 1575$

$\sigma = 215$ $\qquad \bar{x} \pm z_{\alpha/2}\, \sigma/\sqrt{n}$

1

$1575 \pm 2.17\ 215/\sqrt{120}$

1575 ± 42.590

$\boxed{(1532.410,\ 1617.590)}$

1 1

b) $w = 2(50) = 100$ 0.5

$$n \geq \left[2 z_{\alpha/2} \frac{\sigma}{w} \right]^2$$

$$= \left[\frac{2(2.17)(215)}{100} \right]^2$$

$= 87.067$ 0.5

$\boxed{n = 88}$ 1

1

c) Yes, there is a reason to believe they are different since \$1650 lies outside of our 97% confidence interval.

1

2. $n = 15.$

$$2\lambda \sum x_i \sim \chi^2_{2n}$$

so from $df = 30$ row of χ^2 table
$\chi^2_{0.025}$ and $\chi^2_{0.975}$ are 16.791 and 46.979

so $P\left(16.791 \leq 2\lambda \sum x_i \leq 46.979\right) = 0.95$

$$\frac{16.791}{\sum x_i} \leq 2\lambda \leq \frac{46.976}{\sum x_i}$$

$$\frac{16.791}{2\sum x_i} \leq \lambda \leq \frac{46.976}{2\sum x_i}$$

since $\mu = 1/\lambda$ ...

$$\frac{2\sum x_i}{46.976} \leq \mu \leq \frac{2\sum x_i}{16.791}$$

$$\frac{2(63.2)}{46.976} \leq \mu \leq \frac{2(63.2)}{16.791}$$

$$2.691 \leq \mu \leq 7.528.$$

1 1

0.5 0.5 0.5

∴ I am 95% confident that the true mean lifetime of heatpumps is between 2.691 and 7.528 years.

0.5

b) $99\% \rightarrow \alpha = 0.01$

$\alpha/2 = 0.005$

So use $\chi^2_{0.005,30}$ and $\chi^2_{0.995,30}$

$\Rightarrow$ 53.672 and 13.787.

1 1

c) $V[X] = 1/\lambda^2$ when X is exponential, so the standard deviation is $1/\lambda$, the same as the mean.

$\Rightarrow$ same interval.

0.25 0.25

$\therefore$ We are 95% confident that the true standard deviation of the lifetime of heatpumps is between 2.691 and 7.528 years.

0.25

0.5

3. $$w = \frac{2 z_{\alpha/2} \sqrt{\hat{p}\hat{q}/n + z^2_{\alpha/2}/4n^2}}{1 + z^2_{\alpha/2}/n}$$ 0.5

$$w^2 = \frac{4 z^2_{\alpha/2} \left(\hat{p}\hat{q}/n + z^2_{\alpha/2}/4n^2\right)}{\left(1 + z^2_{\alpha/2}/n\right)^2}$$

Let $z_{\alpha/2} = z$ for simplicity

$$w^2\left(1 + 2z^2/n + z^4/n^2\right) = 4z^2 \frac{\hat{p}\hat{q}}{n} + \frac{4z^4}{4n^2}$$

$$w^2n^2 + 2z^2w^2n + w^2z^4 = 4z^2\hat{p}\hat{q}n + z^4$$

$$\underbrace{w^2}_{a}n^2 + \underbrace{(2z^2w^2 - 4z^2\hat{p}\hat{q})}_{b}n + \underbrace{z^4(w^2-1)}_{c} = 0$$

0.5 0.5 0.5

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(2z^2w^2 - 4z^2\hat{p}\hat{q}) \pm \sqrt{(2z^2w^2 - 4z^2\hat{p}\hat{q})^2 - 4w^2z^4(w^2-1)}}{2w^2}$$

$$= \frac{4z^2\hat{p}\hat{q} - 2z^2w^2 \pm \sqrt{\cancel{4z^4w^4} - 16z^4w^2\hat{p}\hat{q} + 16z^4\hat{p}^2\hat{q}^2 - \cancel{4z^4w^4} + 4z^4w^2}}{2w^2}$$

$$= \frac{2z^2\hat{p}\hat{q} - z^2w^2 \pm \sqrt{4z^4\hat{p}\hat{q}(\hat{p}\hat{q} - w^2) + z^4w^2}}{2w^2} \quad 1$$

4. $n = 34$ $\quad t_{\alpha, n-1} = t_{0.02, 33} = 2.139$ 1

$\bar{x} = 51.676$

$s = 9.204$

$$\bar{x} - t_{\alpha, n-1} \, {}^{s}/_{\sqrt{n}}$$

$$51.676 - 2.139 \, {}^{9.204}/_{\sqrt{34}}$$

$$51.676 - 3.376$$

$$\boxed{48.3}$$ 1

0.5

∴ I am 98% confident that the true mean increase in heart rate after a 10 minute moderate Zumba workout is greater than 48.3 bpm.

0.5 0.5 0.5

b) $t_{\alpha/2, n-1} = t_{0.025, 33} = 2.037$ 1

$$\bar{x} \pm t_{\alpha/2, n-1} \, {}^{s}/_{\sqrt{n}}$$

$$51.676 \pm 2.037 \, {}^{9.204}/_{\sqrt{34}}$$

$$51.676 \pm 3.215$$

$$\boxed{(48.461, 54.891)}$$

0.5 0.5

c) $$\bar{x} \pm t_{\alpha/2, n-1} \, s \sqrt{1 + \frac{1}{n}}$$

1

$$51.676 \pm 2.037 \, (9.204) \sqrt{1 + \frac{1}{34}}$$

$$51.676 \pm 19.022$$

$$\boxed{(32.654, 70.698)}$$

0.5 0.5 0.5

∴ I predict with 95% confidence that the actual increase in heart rate for a randomly selected individual after a moderate 10-min Zumba workout will be between 32.654 and 70.698 bpm.

0.5 0.5 0.5

5. $n = 240$ $\quad z_{\alpha/2} = z_{0.015} = 2.17$

$x = 96$

$n\hat{p} = 96 \geq 10$

$n(1-\hat{p}) = 144 \geq 10$ $\implies$ can use traditional

<u>option 1</u>.

$$\hat{p} \pm z_{\alpha/2}\sqrt{\hat{p}\hat{q}/n}$$

0.5

$$0.4 \pm 2.17\sqrt{0.4(0.6)/240}$$

$$0.4 \pm 0.069$$

$$\boxed{(0.331,\ 0.469)}$$

1 1

0.5

$\therefore$ I am 97% confident that the true prop'n of large companies that provide on-site health facilities is between 0.331 and 0.469.

0.5 0.5

<u>option 2</u>

$$\tilde{p} = \frac{\hat{p} + z_{\alpha/2}^2/2n}{1 + z_{\alpha/2}^2/n} = \frac{0.4 + 2.17^2/2(240)}{1 + 2.17^2/240}$$

$$= 0.402$$

$$\tilde{p} \pm z_{\alpha/2}\frac{\sqrt{\hat{p}\hat{q}/n + z_{\alpha/2}^2/4n^2}}{1 + z_{\alpha/2}^2/n}$$

0.5

$$0.402 \pm 2.17\frac{\sqrt{0.4(0.6)/240 + 2.17^2/4(240)^2}}{1 + 2.17^2/240}$$

$$0.402 \pm 0.068$$

$$\boxed{(0.334,\ 0.470)}$$

1 1

either is <u>correct</u>

0.5

$\therefore$ I am 97% confident that the true prop'n of large companies that provide on-site health facilities is between 0.334 and 0.470

0.5 0.5

6. a) $n=25$

$s^2=5200$

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,\,n-1}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,\,n-1}}$$

$$\frac{24(5200)}{39.364} \le \sigma^2 \le \frac{24(5200)}{12.401}$$

0.5 0.5

$$3170.410 \le \sigma^2 \le 10063.705$$

0.5 0.5

0.5

∴ I am 95% confident that the true variance of the lives of all such lightbulbs is between 3170.410 and 10063.705 square hours.

0.5 0.5 0.5

b) 1 4200 square hours is in the interval therefore @ 95% confidence, it is a plausible value.

1