

$$1. \quad n = 120 \quad Z_{\alpha/2} = Z_{0.015} = 2.17$$

$$\bar{x} = 1575$$

$$\sigma = 215$$

$$\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$1575 \pm 2.17 \frac{215}{\sqrt{120}}$$

$$1575 \pm 42.590$$

$$(1532.410, 1617.590)$$

①

①

$$b) \quad w = 2(50) = 100 \quad (OS)$$

$$n \geq \left[ 2 Z_{\alpha/2} \frac{\sigma}{w} \right]^2$$

$$= \left[ 2(2.17) \frac{215}{100} \right]^2$$

$$= 87.067 \quad (OS)$$

$$n = 88$$

①

c) Yes, there is a reason to believe they are different since 1650 lies outside of our 97% confidence interval.

①

$$2. n = 15.$$

$$2 \sum \bar{x}_i \sim \chi^2_{2n}$$

so from  $df = 30$  row of  $\chi^2$  table

$\chi^2_{0.025}$  and  $\chi^2_{0.975}$  are 16.791 and 46.979

$$\text{so } P(16.791 \leq 2 \sum \bar{x}_i \leq 46.979) = 0.95$$

$$\frac{16.791}{\sum \bar{x}_i} \leq \lambda \leq \frac{46.979}{\sum \bar{x}_i}$$

$$\frac{16.791}{2 \sum \bar{x}_i} \leq \lambda \leq \frac{46.979}{2 \sum \bar{x}_i}$$

$$\text{since } \mu = \lambda \dots$$

$$\frac{2 \sum \bar{x}_i}{46.979} \leq \mu \leq \frac{2 \sum \bar{x}_i}{16.791}$$

$$\frac{2(63.2)}{46.979} \leq \mu \leq \frac{2(63.2)}{16.791}$$

$$2.691 \leq \mu \leq 7.528$$

(1)

(0.5)

(0.5)

(0.5)

∴ I am 95% confident that the true mean lifetime  
of heatpumps is between 2.691 and 7.528 years.

(0.5)

b) 99%  $\rightarrow \alpha = 0.01$

$$\alpha/2 = 0.005$$

so use  $Z_{0.005, 30}$  and  $Z_{0.995, 30}$   
 $\Rightarrow 53.672$  and  $13.787$ .

(1)

(2)

c)  $V[X] = 1/\lambda^2$  when  $X$  is exponential, so the standard deviation is  $1/\lambda$ , the same as the mean.  
 $\Rightarrow$  same interval.

0.25

0.25

$\therefore$  we are 95% confident that the true standard deviation of the lifetime of heatpumps is between 2.691 and 7.528 years.

(3)

3.  $w = Z_{\alpha/2} \sqrt{\hat{p}\hat{q}/n + Z_{\alpha/2}^2/4n^2}$  O.S.

$$w^2 = 4Z_{\alpha/2}^2 \left( \frac{\hat{p}\hat{q}/n + Z_{\alpha/2}^2/4n^2}{1 + Z_{\alpha/2}^2/n} \right)$$

Let  $Z_{\alpha/2} = z$   
for simplicity

$$w^2 (1 + 2z^2/n + z^4/n^2) = 4z^2 \frac{\hat{p}\hat{q}}{n} + \frac{4z^4}{4n^2}$$

$$w^2 n^2 + 2z^2 w^2 n + w^2 z^4 = 4z^2 \hat{p}\hat{q} n + z^4$$

$$a n^2 + b n + c = 0$$

O.S.

O.S.

O.S.

$$\sigma = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= -\frac{(2z^2\omega^2 - 4z^2pq) \pm \sqrt{(2z^2\omega^2 - 4z^2pq)^2 - 4\omega^2 z^4(\omega^2 - 1)}}{2\omega^2}$$

$$= \frac{4z^2pq - 2z^2\omega^2 \pm \sqrt{4z^4\omega^4 - 16z^4\omega^2pq + 16z^4pq^2 - 4z^4\omega^4 + 4z^4\omega^2}}{2\omega^2}$$

$$= \frac{2z^2pq - z^2\omega^2 \pm \sqrt{4z^4pq(pq - \omega^2) + z^4\omega^2}}{2\omega^2} \quad (1)$$

■

$$4. \quad n = 34 \quad t_{\alpha/2, n-1} = t_{0.02, 33} = 2.139 \quad (1)$$

$$\bar{x} = 51.676$$

$$s = 9.204$$

$$\bar{x} - t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} \\ 51.676 - 2.139 \frac{9.204}{\sqrt{34}}$$

$$51.676 - 3.376$$

$$48.3 \quad (1)$$

(0.5)

∴ I am 98% confident that the true mean increase in heart rate after a 10 minute moderate Zumba workout is greater than 48.3 bpm.

(0.5)

(0.5)

$$b) \quad t_{\alpha/2, n-1} = t_{0.025, 33} = 2.037 \quad (1)$$

$$\bar{x} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} \\ 51.676 \pm 2.037 \frac{9.204}{\sqrt{34}}$$

$$51.676 \pm 3.215$$

$$(48.461, 54.891)$$

(0.5)

(0.5)

$$c) \quad \bar{x} \pm t_{\alpha/2, n-1} s \sqrt{1 + \frac{1}{n}} \\ 51.676 \pm 2.037 (9.204) \sqrt{1 + \frac{1}{34}}$$

$$51.676 \pm 19.022$$

$$(32.654, 70.698)$$

(0.5)

(0.5)

(0.5)

∴ I predict with 95% confidence that the actual increase in heart rate for a randomly selected individual after a moderate 10-min Zumba workout will be between 32.654 and 70.698 bpm.

(0.5)

(0.5)

(0.5)

$$5. n = 240 \quad Z_{\alpha/2} = Z_{0.005} = 2.17$$

$$x = 96$$

$$\hat{np} = 96 \geq 10 \quad \Rightarrow \text{can use traditional}$$
$$n(1-\hat{p}) = 144 \geq 10$$

option 1.

$$\hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$
$$0.4 \pm 2.17 \sqrt{\frac{0.4(0.6)}{240}}$$

$$0.4 \pm 0.069$$

$$(0.331, 0.469)$$

①

①

∴ I am 97% confident that the true propn of large companies that provide on-site health facilities is between 0.331 and 0.469.

option 2

$$\tilde{p} = \frac{\hat{p} + Z_{\alpha/2}^2 / 2n}{1 + Z_{\alpha/2}^2 / n} = \frac{0.4 + 2.17^2 / 2(240)}{1 + 2.17^2 / 240}$$
$$= 0.402$$

$$\tilde{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n} + \frac{Z_{\alpha/2}^2}{4n^2}}$$
$$0.402 \pm 2.17 \sqrt{\frac{0.4(0.6)}{240} + \frac{2.17^2}{4(240)^2}}$$
$$1 + \frac{2.17^2}{240}$$

$$0.402 \pm 0.068$$

$$(0.334, 0.470)$$

①

either is correct

∴ I am 97% confident that the true propn of large companies that provide on-site health facilities is between 0.334 and 0.470

6. a)  $n = 25$   
 $s^2 = 5200$

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha/2, n-1}}$$
$$\frac{24(5200)}{39.364} \leq \sigma^2 \leq \frac{24(5200)}{12.401}$$
$$3170.410 \leq \sigma^2 \leq 10063.705$$

(0.5)  $\therefore$  I am 95% confident that the true variance of  
the lives of all such lightbulbs is between  
3170.410 and 10063.705 square hours.

b) (1) 4200 square hours is in the interval therefore @ 95%.  
confidence, it is a plausible value.  
(1)