

ENHANCING USER DETECTION

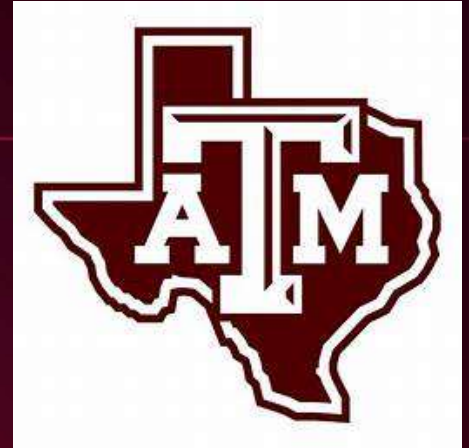
An Undergraduate Research Scholarship

by

HOLLY ROOPER

Submitted to the IAT

in partial fulfillment



# Team 64: Enhancing User Detection Final Presentation

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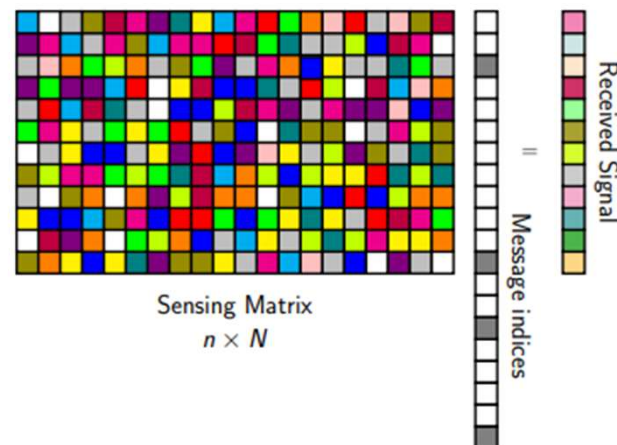
TA: Max Lesser

# Problem Overview

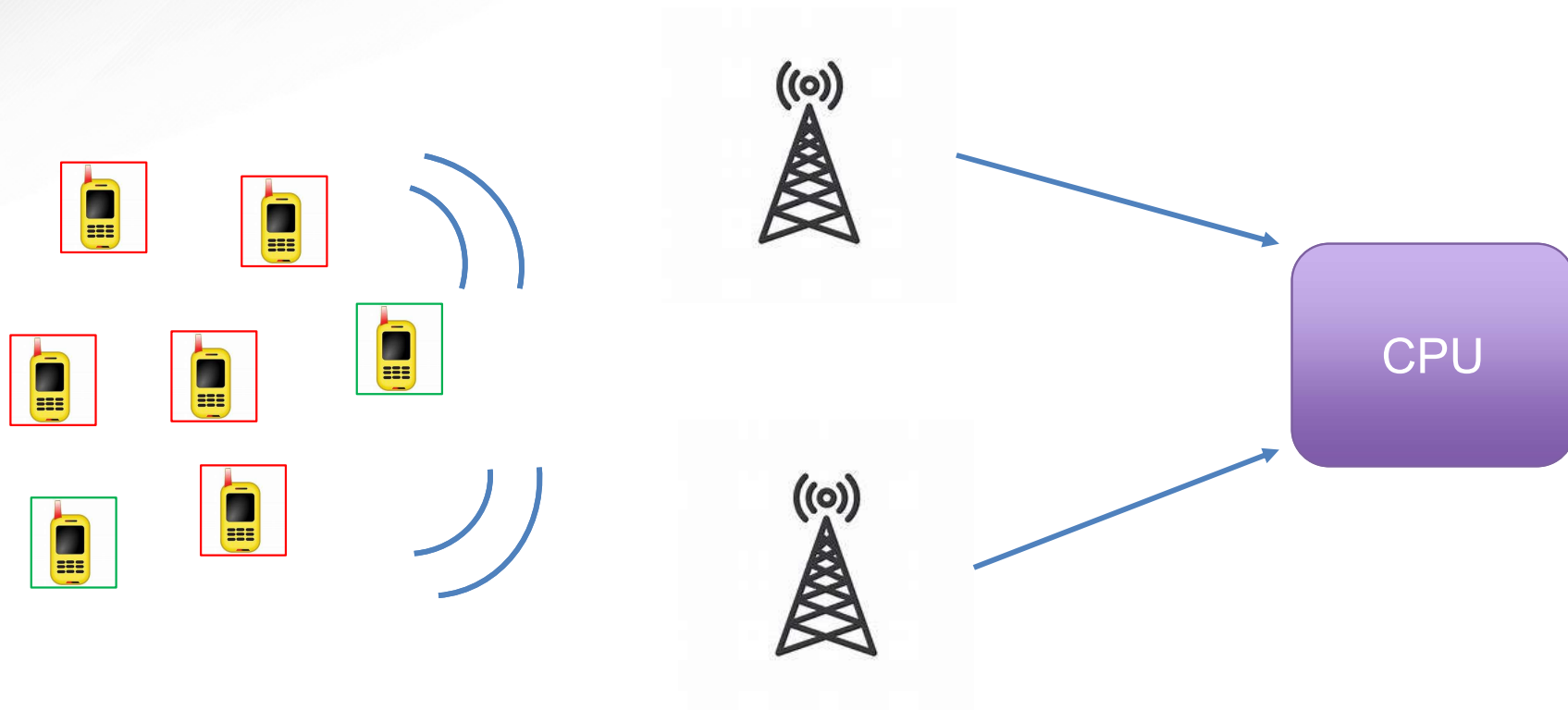
Explored two things:

1. The performance of Learned Iterative Soft Thresholding Algorithm (LISTA) compared to traditional baselines from Approximate Message Passing (AMP) and Iterative Soft Thresholding Algorithm (ISTA)
2. The impact of utilizing two receivers that work together to recover a signal from a user, compared to a single receiver approach.

Tackling user detection as a compressed sensing problem.



# Integrated Project Diagram



$k$  of  $N$  users transmit signals

Base stations receive the signals

They share data with a common CPU to recover the signal



# Agenda

- ISTA
- AMP
- LISTA
- Comparison – Misdetection/False Alarms
- Complexity
- Conclusion



# ISTA

Implemented Complex-ISTA

Calculated MSE vs. Iterations  
and SNR to test performance

Calculated MD/FA to evaluate  
how its effectiveness at user  
detection

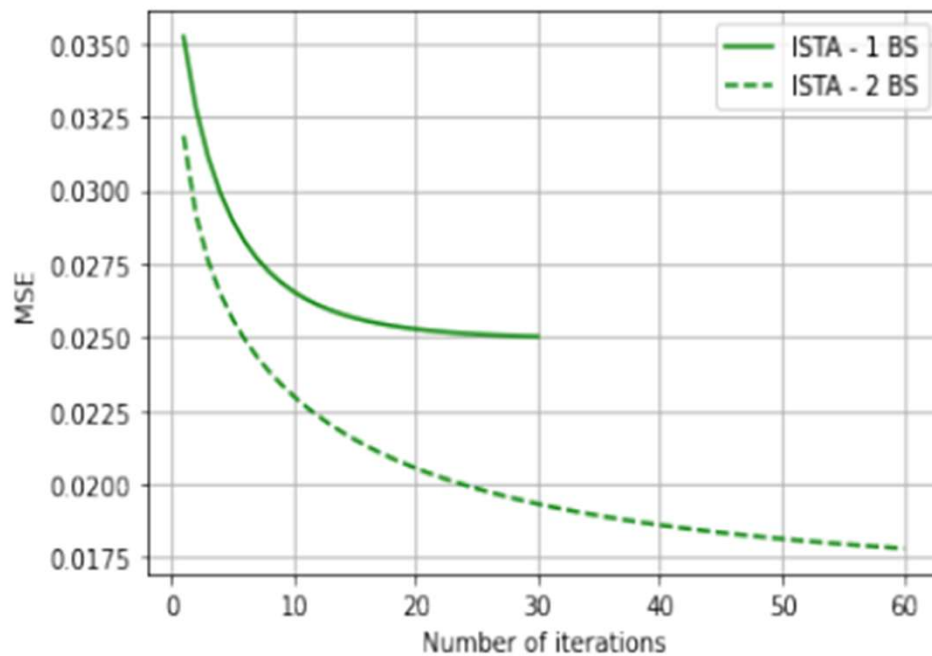
$$\mathbf{x}^{l+1} = \eta\left(\left(\mathbf{I} - \frac{1}{L}\mathbf{A}^T\mathbf{A}\right)\mathbf{x}^l + \frac{1}{L}\mathbf{A}^T\mathbf{y}, \frac{\alpha}{L}\right)$$

$$\eta(u; T) = \begin{cases} \angle u |u - T| & \text{if } u \geq T \\ \angle u |u + T| & \text{if } u \leq T \\ 0 & \text{otherwise} \end{cases}$$

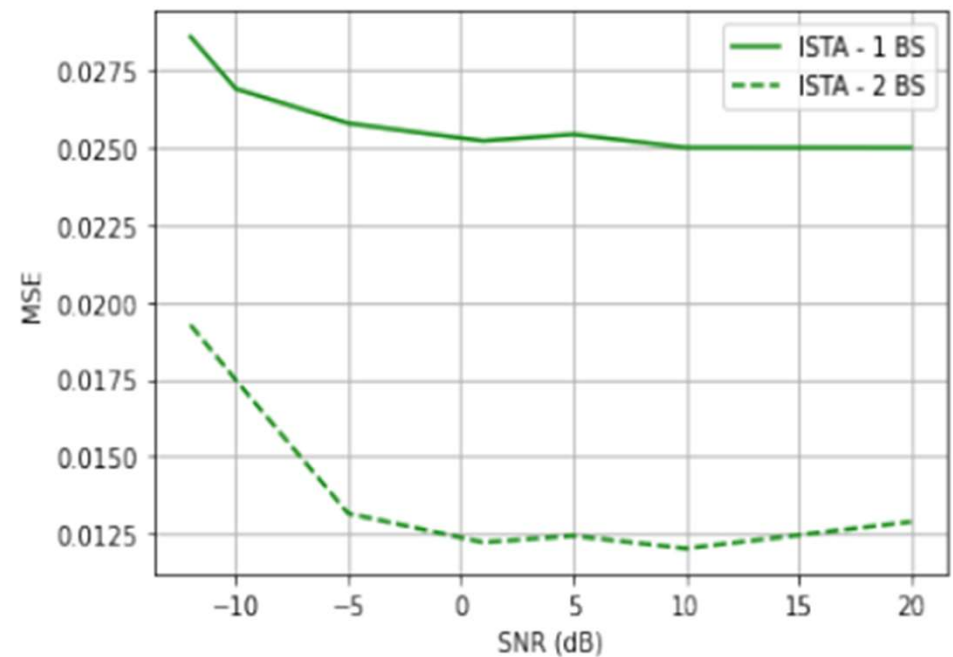


# ISTA

ISTA: MSE vs iterations



ISTA: MSE vs SNR



For 2 BS case, an SNR of 5dB means each BS receives a signal with 5dB.

Implemented Complex-AMP

Calculated MSE vs. Iterations and SNR to test performance

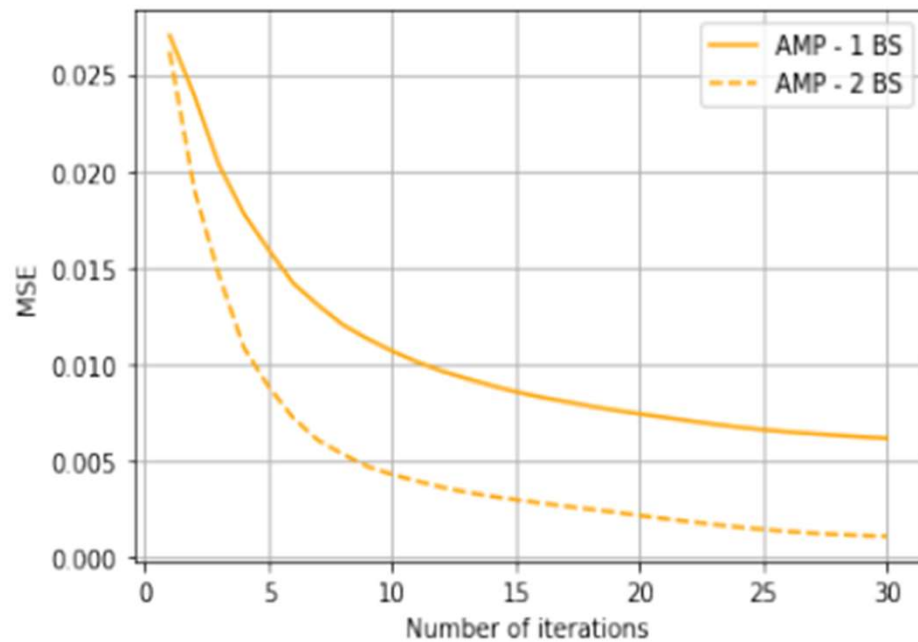
Calculated MD/FA to evaluate how its effectiveness at user detection

$$\mathbf{z}^t = \mathbf{y} - \mathbf{A}\mathbf{x}^t + \frac{1}{2\delta} z^{t-1} \left( \left\langle \frac{\partial \eta^R}{\partial x_R} (x^{t-1} + A^H z^{t-1}; \lambda^{t-1}) \right\rangle + \left\langle \frac{\partial \eta^I}{\partial x_I} (x^{t-1} + A^H z^{t-1}; \lambda^{t-1}) \right\rangle \right)$$

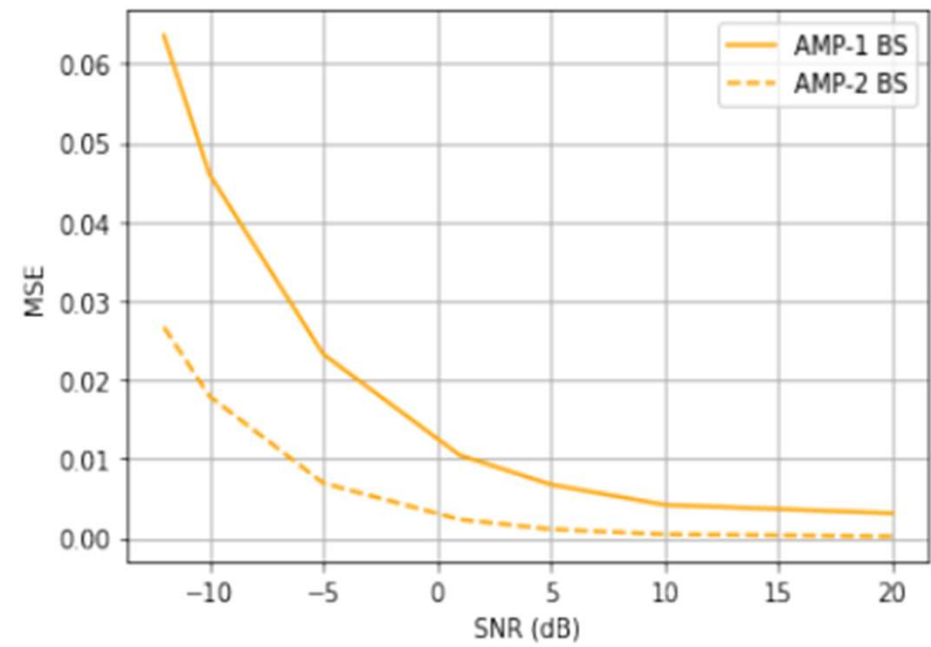
$$\eta(u + iv; \lambda) = (u + iv - \frac{\lambda(u + iv)}{\sqrt{u^2 + v^2}}) I_{\{u^2 + v^2 \geq \lambda\}} \quad \langle \eta'(\alpha; \lambda) \rangle = \frac{\sum_{i=1}^N \eta'(\alpha_i; \lambda)}{N}$$

$$\mathbf{x}^{t+1} = \eta(\mathbf{x}^t + \mathbf{A}^H \mathbf{z}^t; \lambda^t)$$

AMP: MSE vs iterations



AMP: MSE vs SNR





# LISTA

Converted ISTA to a neural network

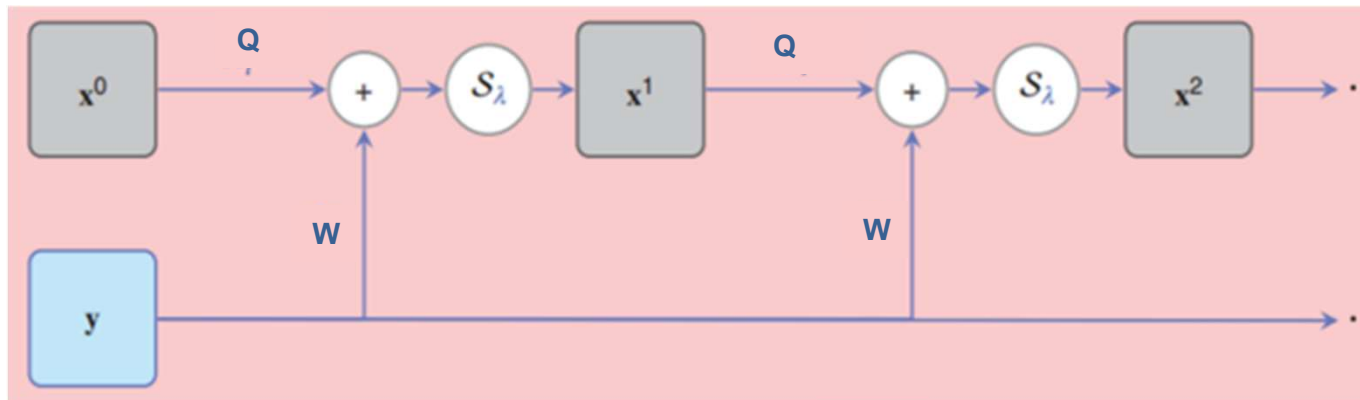
$$\mathbf{x}^{l+1} = \eta \left( \left( \mathbf{I} - \frac{1}{L} \mathbf{A}^T \mathbf{A} \right) \mathbf{x}^l + \frac{1}{L} \mathbf{A}^T \mathbf{y}, \frac{\alpha}{L} \right)$$

Learned  $\alpha$ ,  $Q$ ,  $W$

$$\mathbf{Q} = \mathbf{I} - \frac{\mathbf{A}^T \mathbf{A}}{L} \quad \mathbf{W} = \frac{\mathbf{A}}{L}$$

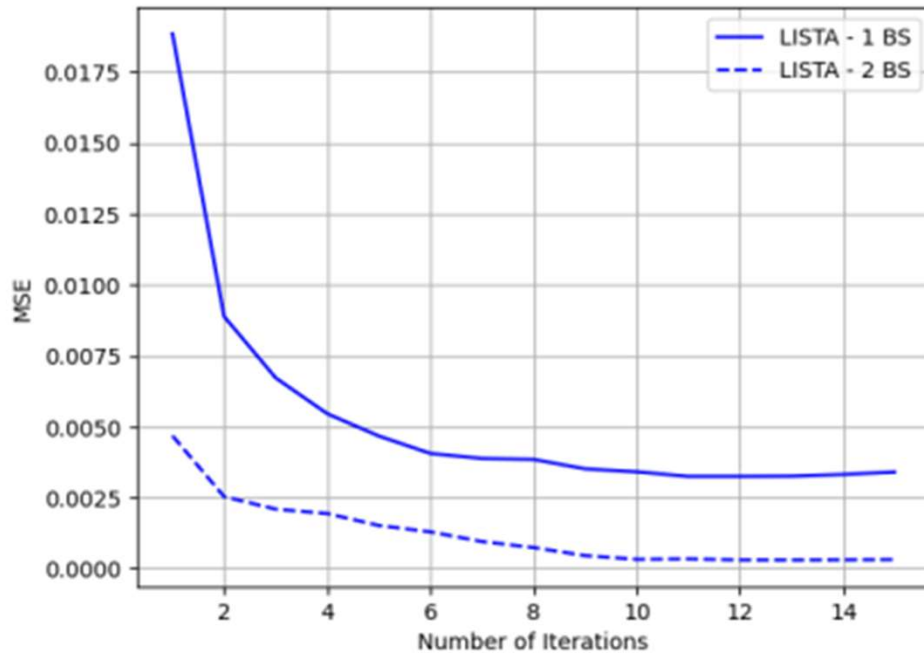
Created data set with various noise and fading realizations

$$Loss = \frac{1}{2} \|\mathbf{y} - \mathbf{A} \hat{\mathbf{x}}\|_2^2 + \alpha \|\hat{\mathbf{x}}\|_1$$

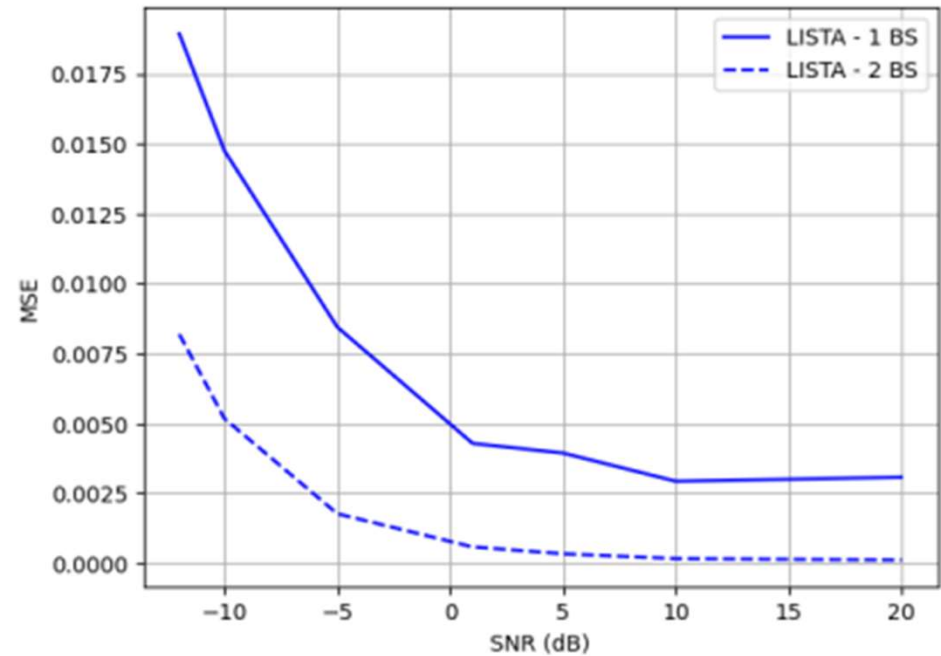


# LISTA

LISTA: MSE vs. Iterations

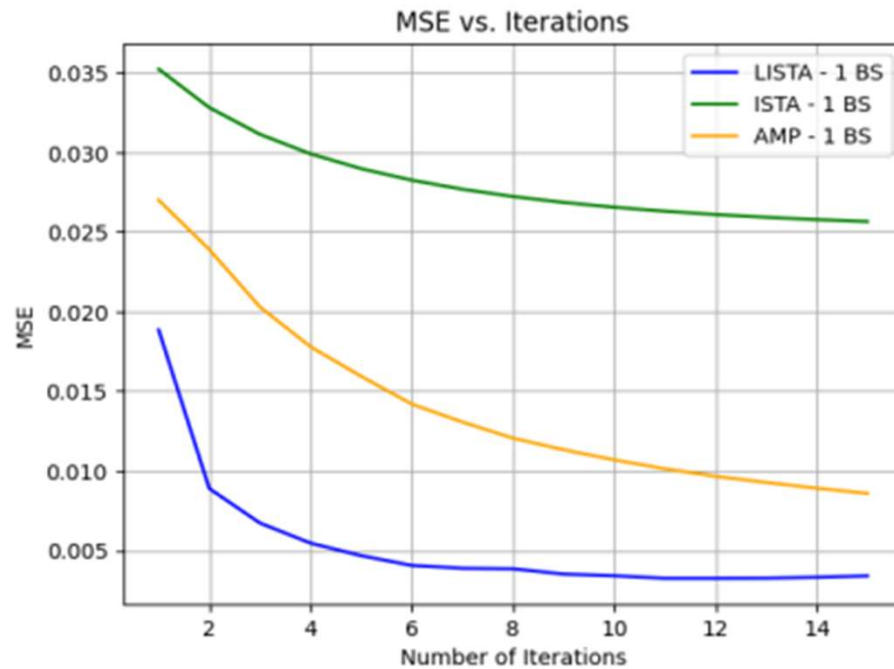


LISTA: MSE vs. SNR

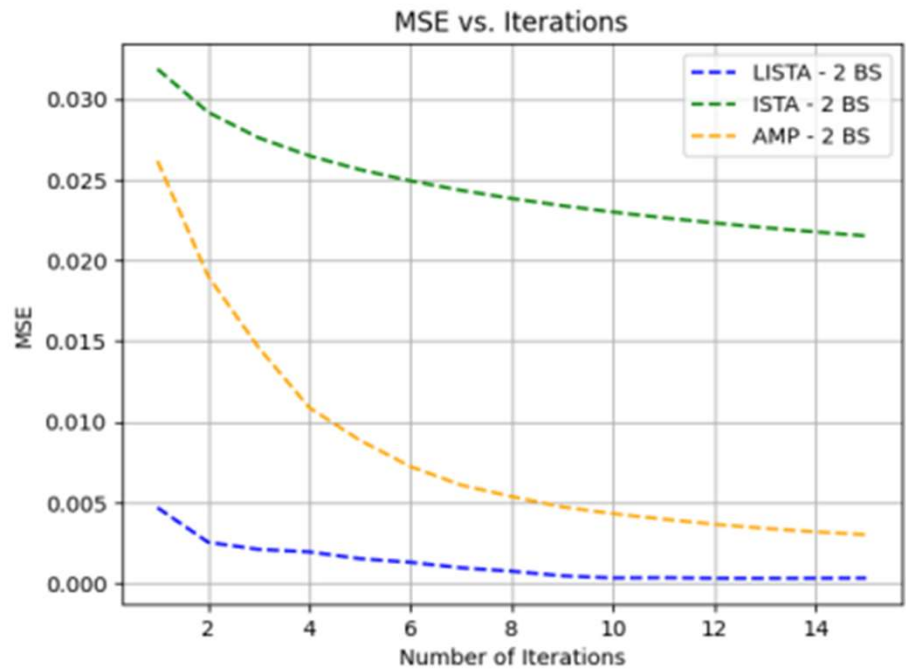


# Comparing All Three

## 1 Base Station



## 2 Base Stations





# Misdetection and False Alarms

A misdetection is failing to notice that a given user is active (i.e. a false negative, or statistical type II error), and a false alarm is claiming a user is active when they are not (i.e. a false positive, or statistical type I error)

We had 40 active users

Took the results after 15 iterations, averaged over 100 trials

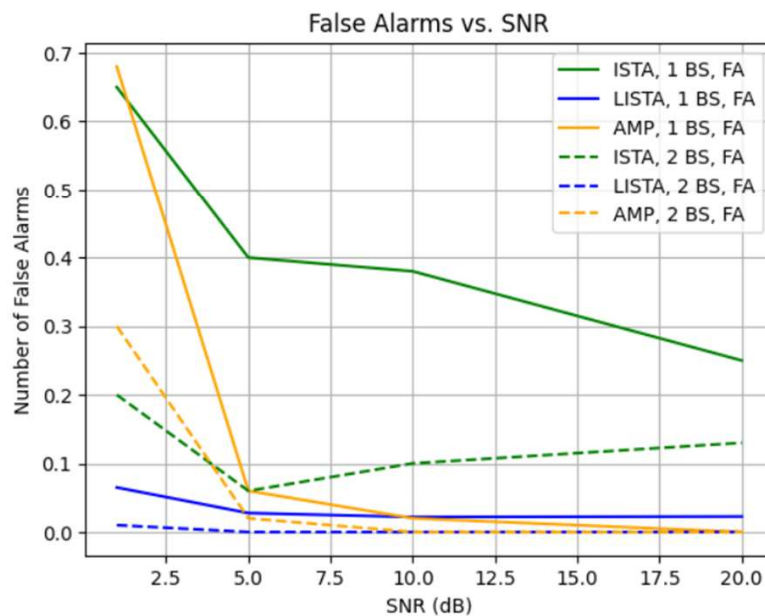
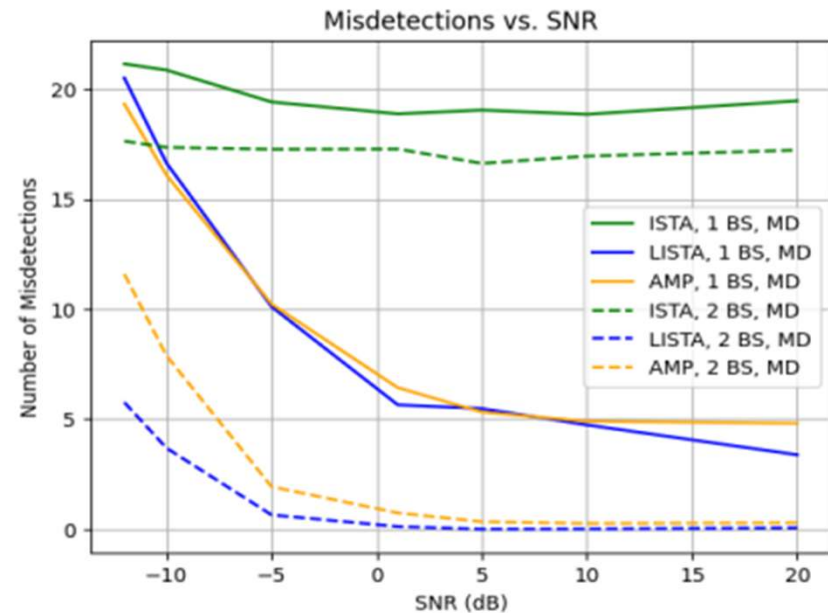
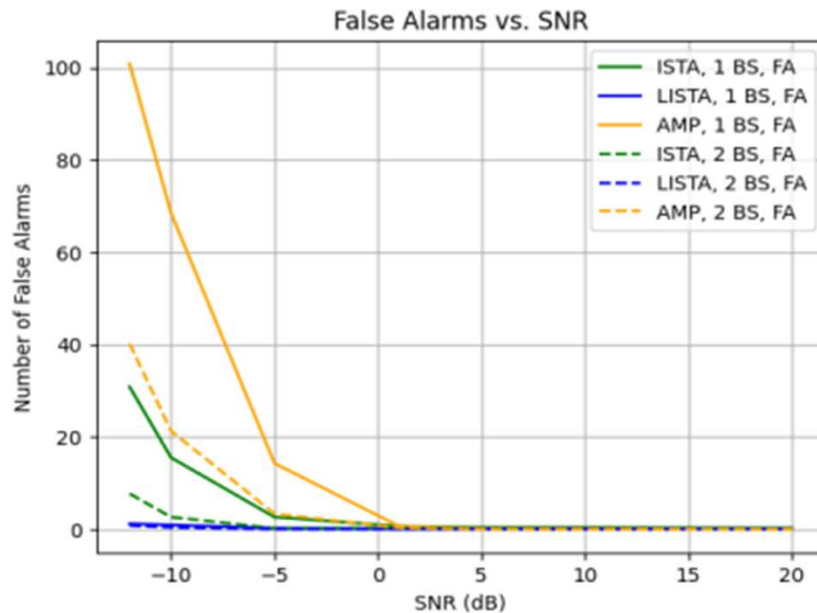
Determined a user was active if the value at their index was greater than a certain threshold

ISTA: 0.2

AMP: 0.4

LISTA: 0.45

# Comparing All Three



Left is a zoomed in view of FA plot to better see the error towards higher SNRs





# Complexity

Matrix multiplication in ISTA has  $\mathcal{O}(nN)$

Thresholding operation has  $\mathcal{O}(N)$

Combining this for  $T$  iterations yields  $\mathcal{O}(TnN)$

The highest order step in AMP is a matrix product with  $\mathcal{O}(n^2)$

Combining this for  $T$  iterations yields  $\mathcal{O}(Tn^2)$

The highest order step in LISTA is a matrix product with  $\mathcal{O}(N^2)$

Combining this for  $T$  iterations yields  $\mathcal{O}(TN^2)$

There is a range of under-sampling to sparsity ratios for which AMP will converge, where the under-sampling ratio is  $\frac{n}{N} = \delta$  and the sparsity ratio is  $\frac{k}{N} = \rho$ . Knowing this, the complexities of AMP and LISTA simplify to  $\mathcal{O}(TnN)$  which is equivalent to ISTA.



# Conclusions

- Two base station scenario outperforms the single base station case
- LISTA outperforms ISTA and AMP, converges quicker and has a lower noise floor
- LISTA can perform at a lower SNR
  - Less transmit power needed -> increases battery life of devices. (Good news for mMTC)
  - Cell free paradigm seeks to increase the minimum service quality received -> users on the edge of the cell would receive higher service quality than they did previously with LISTA
  - Maintain the same service, but widen the area of cells and require less base stations to service a region

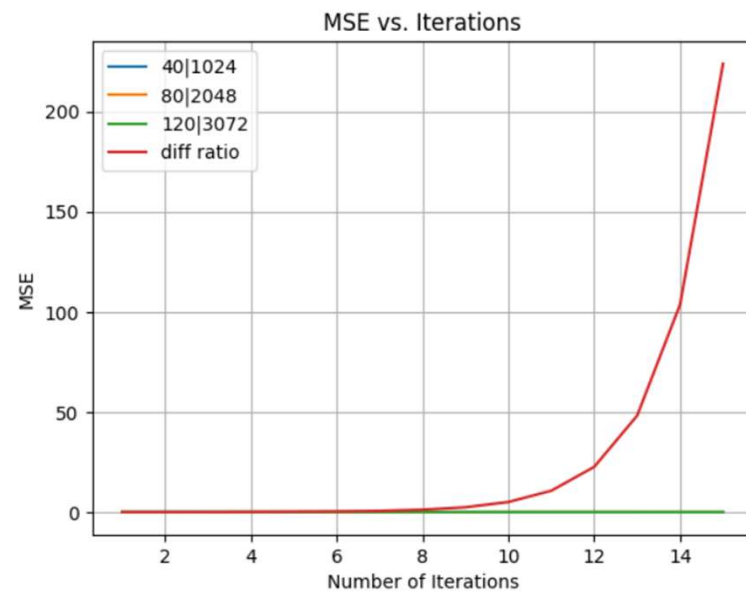
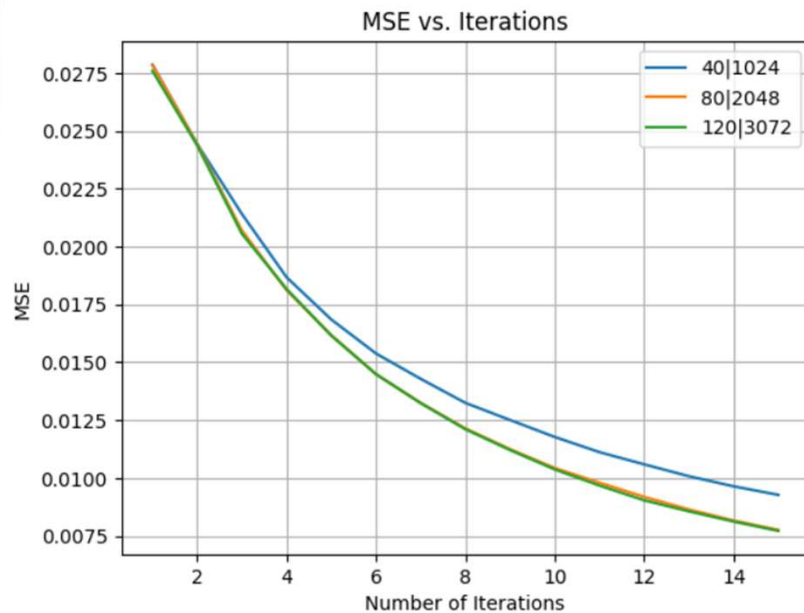


# Thank you



# Complexity ratios

AMP



Proof that if under-sampling and sparsity ratios remain the same, the complexities of the three algorithms match.

When ratio is not maintained, things blow up.

A given performance only holds for a set ratio.