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Test ultrametricity criterion:

① Metric D_1 is NOT ultrametric: let's look at sequences X_1, X_2, X_3

$$d(X_1 \& X_2) = 0.3$$

$$d(X_1 \& X_3) = 0.7$$

$$d(X_2 \& X_3) = 0.6$$

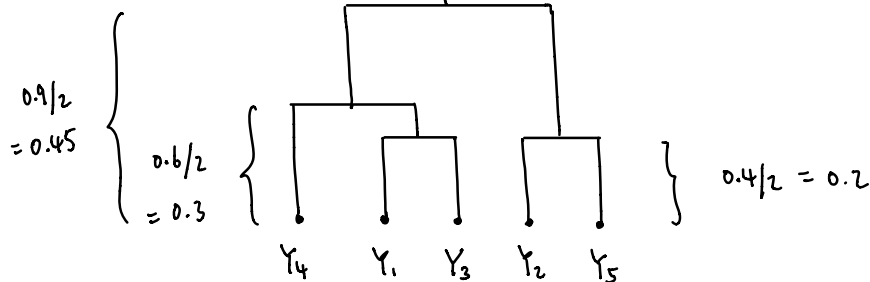
If Metric D_1 is ultrametric, two distances would be the same, and the third would be less than the other two.

However, all three sequence differences are not the same.

② Metric D_2 is ultrametric

Smallest values on the table: 0.4 \rightarrow $\begin{cases} Y_1 \leftrightarrow Y_3 \text{ cluster} \\ Y_2 \leftrightarrow Y_5 \text{ cluster} \end{cases}$

Second smallest value: 0.6 $\begin{cases} Y_1 - Y_4 \\ Y_3 - Y_4 \end{cases} \Rightarrow Y_4 - Y_1 Y_3 \text{ cluster}$
closer in similarity



Test additivity criterion:

Both D_1 and D_2 are additive, but only D_1 is not ultrametric.

① Look at the values in D_1 : X_1, X_2, X_3, X_4 combinations

$$d_{12} + d_{34} = 0.3 + 0.6 = 0.9 \quad \leftarrow \text{smallest value}$$

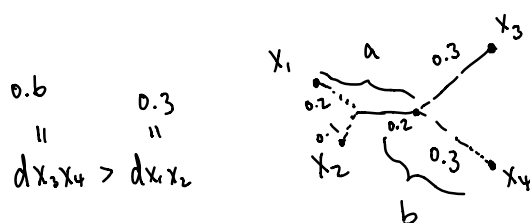
↓

$$d_{13} + d_{24} = 0.7 + 0.6 = 1.3$$

X_1 & X_2 - NJ

$$d_{14} + d_{23} = 0.7 + 0.6 = 1.3$$

X_3 & X_4 - NJ



$$d_{X_2X_3} = 0.6 \quad d_{X_1X_3} = 0.7$$

$$d_{X_1X_4} = 0.7 \quad d_{X_2X_4} = 0.6$$

joint - X_2 is shorter

joint - X_1 is longer

$$a = \frac{d_{X_1X_3} + d_{X_1X_4} - d_{X_2X_4}}{2}$$

$$= \frac{0.7 + 0.7 - 0.6}{2} = 0.4$$

$$b = \frac{d_{X_1X_4} + d_{X_2X_4} - d_{X_1X_3}}{2}$$

$$= \frac{0.7 + 0.6 - 0.3}{2} = 0.5$$

$$X_3, X_4 \text{ joins at } d_{X_1X_3} - a = 0.7 - 0.4 = 0.3$$

distance away from X_3

$$d_{X_1X_4} - a = 0.7 - 0.4 = 0.3$$

joining point to $X_3, X_4 \rightarrow$ same

$$X_1, X_2 \text{ joins at } d_{X_1X_4} - b$$

$$= 0.7 - 0.5 = 0.2$$

distance away from X_1

$$d_{X_2X_4} - b = 0.6 - 0.5$$

$$= 0.1 \text{ distance}$$

away from X_2

2(c).

Scanning through the table:

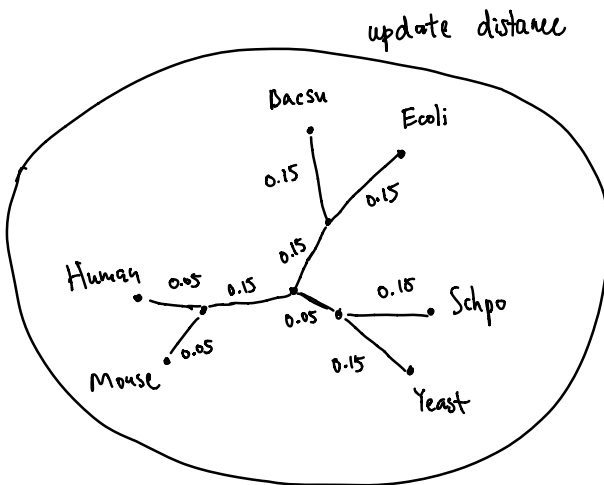
Smallest values: $0.1 + 0.3 = 0.4$

$$\text{Human Mouse} + \text{Yeast Schpo} = 0.1 + 0.3 = 0.4$$

$$\text{Human Yeast} + \text{Mouse Schpo} = 0.4 + 0.4 = 0.8$$

$$\text{Human Schpo} + \text{Mouse Yeast} = 0.4 + 0.4 = 0.8$$

Yeast
Schpo can be replaced
by BACSU and Ecoli
 $HM + BE = 0.4$
 $HP + ME = 0.3 + 0.5 = 1$
 $HE + BM = 1$



$$\frac{HY + MY - HM}{2} = \frac{0.4 + 0.4 - 0.1}{2} = 0.35$$

$$HY - 0.35 = 0.05$$

$$MY - 0.35 = 0.05$$

Merge point of HM called H.M

$$H.M E = 0.5 - 0.05 = 0.45$$

$$H.M B = 0.5 - 0.05 = 0.45$$

$$\frac{0.35 + 0.35 - 0.3}{2} = 0.2 \text{ is}$$

the distance between

Y.S and H.M

$$E.B.E = 0.5 - 0.05 - 0.3$$

$$= 0.15$$

$$B.B.E = 0.15$$

$$\frac{0.45 + 0.45 - 0.3}{2} = 0.3 \text{ is the}$$

distance between B.E and H.M

$$S.S.Y = 0.4 - 0.05 - 0.2$$

$$= 0.15$$

$$Y.S.Y = 0.15$$

distance btwn B.E and Y.S: $0.5 - 0.15 \times 2$

$$= 0.2$$

$$\frac{0.3 + 0.2 - 0.2}{2} = \boxed{0.15}$$