

1. Electron Energy Equation for UEDGE

The dynamical fluid equations for particle continuity, parallel (to the B-field) ion velocity, and separate ion and electron temperatures are given below. The perpendicular ion velocities come from both algebraic equations involving other variables (classical drifts) and diffusion/convection coefficients that are user-specified to mimic anomalous (turbulence-driven) ion/electron fluxes (*i.e.*, the turbulence fluxes are assumed ambipolar, *i.e.*, giving equal ion and electron fluxes). The velocity \mathbf{u} is the portion of the total velocity, \mathbf{V} , that yields a nonzero divergence when multiplied by the species density, n .

For the poloidal ion velocity the various components are

$$u_{ex} = \frac{B_x}{B} v_{i\parallel} + v_{x,E} + v_{ex,\nabla B}, \quad (1)$$

where the second term is the $\mathbf{E} \times \mathbf{B}/B^2$ drift and third term is the sum of the curvature and ∇B drifts scaling in tokamaks as $-eT_e/RB$; here e is the electron charge, and R is the major radius of the tokamak. Note that we only include those drift terms giving a finite divergence of plasma fluxes since they will appear inside divergence terms in the transport equations.

For the radial ion velocity

$$u_{ey} = -\frac{D_a}{n_i} \frac{\partial n_i}{\partial y} + V_a + v_{y,E} + v_{ey,\nabla B}, \quad (2)$$

where D_a and V_a are the anomalous transport coefficients characterizing turbulence-driven transport (assumed ambipolar). The third and fourth terms in Eq. 2 are the radial components of the cross-field drifts. A more detailed discussion of the cross-field drifts terms used is given in Rognlien *et al.*, Phys. Plasmas **6** (1999) 1851 and references therein.

The electron energy equation from Braginskii is

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{3}{2} n_e T_e \right) + \frac{\partial}{\partial x} \left[\frac{5}{2} n_e u_{ex} T_e + q_{ex} \right] + \frac{\partial}{\partial y} \left(\frac{5}{2} n_e u_{ey} T_e + q_{ey} \right) \\ = \mathbf{V}_e \cdot \nabla P_e + Q_e + S_{Ee} \\ = \mathbf{V}_e \cdot \nabla P_e + \mathbf{J} \cdot \mathbf{R}_e / (en_e) - Q_\Delta + S_{Ee}. \end{aligned} \quad (3)$$

where the Braginskii $Q_e = \mathbf{J} \cdot \mathbf{R}_e / (en_e) - Q_\Delta$ has been used. The collisional energy exchange with ions is given by $Q_\Delta = n_e \nu_{eqp} (T_e - T_i)$. The S_{Ee} term represents atomic physics losses from neutral and ion ionization, excitation, and recombination of neutral hydrogen gas and impurities. Equation 3 is the form of the electron energy equation used presently in UEDGE.

The explicit form of R_e is given by Braginskii as

$$\mathbf{R}_e = en_e(\mathbf{J}_{\parallel}/\sigma_{\parallel} + \mathbf{J}_{\perp}/\sigma_{\perp}) - 0.71n_e\nabla_{\parallel}T_e \approx en_e\mathbf{J}_{\parallel}/\sigma_{\parallel} - 0.71n_e\nabla_{\parallel}T_e \quad (4)$$

The σ are conductivities with the parallel one being about twice that of conductivity. Because we assume $J_{\parallel} \gg J_{\perp}$, only the parallel component of \mathbf{R}_e is retained in UEDGE. In more detail for UEDGE, both terms in $V_e \cdot \nabla P_e + J_{\parallel}R_{e\parallel}$ are included in the array `seec(ix,iy)`, computed in subroutine `pandf`. The force term R_e in UEDGE is call `frice(ix,iy)`. Note that the Joule heating between electrons and ions is included in the $\mathbf{J} \cdot \mathbf{R}_e$ so separate term is not needed for this physical process.

Separate details for UEDGE coding in file `bbb/oderhs.m` of UEDGE-V8041: First, note that using a finite-volume method in UEDGE, we first integrate the electron energy equation (Eq. 3) over the volume of the computational cells in order to convert the divergence terms on the LHS into fluxes evaluated at the boundaries of the cell. The terms on the RHS of Eq. 3 then are volume-intergrated values, i.e., the RHS becomes:

$$RHS = (\mathbf{V}_e \cdot \nabla P_e + \mathbf{J} \cdot \mathbf{R}_e)/(en_e) - Q_{\Delta} + S_{Ee} * vol \quad (5)$$

where vol is the UEDGE volume element of a computational cell. These contributions are contained in UEDGE as follows:

- The $(\mathbf{V}_e \cdot \nabla P_e)vol$ is present in the `bbb.seec` array as shown in line 2485 (should add $v_{ey}\nabla_y P_e$ as well as $v_{ex}\nabla_x P_e$).
- The $(J_{\parallel}R_{e\parallel}/en_e)vol$ contribution to `bbb.seec` is present as shown in line 2527-8.
- The $-Q_{\Delta}vol$ term is given by `bbb.w0` in line 4529 and the $S_{Ee}vol$ term is `bbb.seec` in line 4530.
- Finally, the section computing a separate Joule heating term `wjdotr` beginning on line 4743 should be removed as it duplicates a portion of the RHS already included in `bbb.seec` (it was also missing the volume element mentioned above). Also, the related switch `bbb.jhswitch` should be removed.