Leveraging Convex Relaxations for Safe, Efficient and Versatile Autonomy

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I. Introduction

The near future promises an unprecedented level of human-robot interaction, particularly in the field of mobile robots. In emergent collaborative environments (streets, factories, hospitals, etc.), it will be critical to develop autonomy that is not only efficient, but also *inherently* safe. Moreover, given the dynamic nature of these environments, it will also become essential for autonomous agents become more versatile, learning and adapting at an unprecedented scale. These three features – safety, efficiency, and versatility – now represent the 'holy grail' of modern robotics algorithms and pose a unique set of challenges to robotics researchers. My research seeks to leverage powerful tools from optimization theory (namely convex relaxations) to realize a new paradigm of algorithms that are inherently safe, while still enabling versatility through end-to-end learning and remaining computationally efficient.

II. Background

Traditionally, development of robotic algorithms has largely focused on computational efficiency. Robotics problems, though often computationally large-scale, typically exhibit a large amount of structure. Many of the advances in speed over recent decades have depended largely on the exploitation of the inherent structure of these problems. For example, workhorse algorithms for state-estimation – such as localization and simultaneous localization and mapping (SLAM) – are now capable of estimating hundreds of thousands of states on a single processor in real time [31]. SLAM exploits the high degree of sparsity inherent in the graph of measurement data (or *factor graph*) to achieve this feat [2, 11, 8].

Such levels of performance also typically rely on local optimization methods (e.g., Gauss-Newton), which exhibit super-linear convergence properties. Although efficient, these methods can unexpectedly converge to spurious, local solutions rather than the true global minima. This danger is present in all levels of the robotic software stack (state estimation, planning, control, etc.) and can lead to sub-optimal performance or even catastrophic results.

As in other disciplines, the versatility of machine learning approaches has brought them to the center stage of robotics. A new paradigm, leveraging advances in so-called *differentiable optimization*, now enables roboticists to combine learning with fast traditional methods, yielding an *end-to-end learning* framework [28, 33].

The benefits of this approach are manifold, allowing practitioners to capitalize on the respective advantages of model-based and learning approaches while also mitigating their disadvantages [29, 22, 7]. However, since this approach depends on optimization, it is similarly prone to convergence to local optima, leading to incorrect gradients and corrupting the training/optimization process.

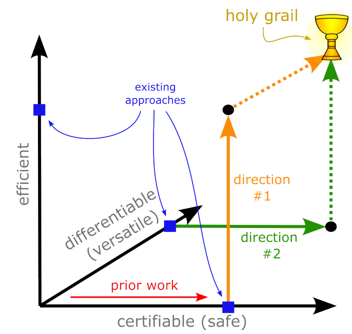

Fig. 1. Future directions for my research: direction #1 uses sparsity to improve efficiency of certifiable approaches with redundant constraints; direction #2 applies certifiable methods to differentiable optimization. Combining both directions leads to a 'holy grail' of autonomy.

As a counterpoint to the issue of local minima, there has been a surge of interest in so-called *certifiable methods*. These methods use convex relaxations to either: (1) globally solve a given robotics problem as a convex semidefinite program (SDP) or; (2) verify that a given solution is correct. Particularly in the subfield of perception and state estimation, there is a large range of problems for which certifiable methods already exist. To name a few, certifiable methods for sensor calibration [34, 13], robust state estimation [36, 37], inverse kinematics [13], image segmentation [16], pose-graph optimization [30], multiple-point-set registration [6, 17], range-only localization [10], and range-aided SLAM [26] have all been explored.

III. Prior Work

To date, my research has focused on the extension of certifiable methods to different problems in robotic state estimation, but the key contributions are *generally* applicable to a broad class of robotics problems.

A. Fast Certification of Structured Problems

Though certifiable methods undoubtedly improve the inherent safety of robotics algorithms, much of the current research interest has centered around recent improvements in efficiency of these methods. Among the most performant of certifiable algorithms is SE-Sync, which solves *pose-graph optimization* (PGO) [30]. My first project extended SE-Sync to include estimation of a set of landmark positions, a problem known as landmark-based SLAM [14]. At approximately the same time, I was also involved with project that certified continuous-time range-only localization, which localizes a mobile robot using measured distances to points of known positions, called anchors [10].

In both cases, we were able to show that by leveraging the underlying *structure* of the problem, our approach maintained the same (linear) complexity as a sparse, local, Gauss-Newton solver, but was also able to certify the solution. Achieving this level of efficiency was a key step in demonstrating that certifiable methods can be adopted in modern robotics pipelines.

B. Certifying More Complex Problems

In general, there is no guarantee that a given problem formulation will have a convex relaxation that can be used to certify its global solution. After these two initial projects, I was interested investigating this limitation and explored the extension of certifiable localization and SLAM problems to the case in which the observation models included stereo vision measurements [15]. Although this extension is readily handled by local methods [3], it led to a fundamental change in the problem formulation that rendered certifiable methods useless.

Fortunately, this underlying brittleness had a remedy: many problems that are not initially certifiable can be made so by introducing so-called *redundant constraints*. This technique has been applied to good effect in the certification of several vision and state-estimation problems [38, 12, 5, 12, 4, 37, 36] and in the controls community [25, 20, 27].

In exploring this technique, our group discovered a means of *automatically* generating redundant constraints for arbitrary problem formulations [9]. Prior to this discovery, redundant constraints had to be tediously constructed by hand based on the intuition of the practitioner. This method also allowed to systematically determine whether it was *possible* to certify a given problem by finding all such redundant constraints. My main contributions to this project were the development of the codebase and a computational method that ensured the discovered redundant constraints were sparse, which was a critical step in making this method efficient enough to be feasible in practice.

In addition to demonstrating this approach on key problems in the literature, we also applied this method to trajectory estimation using a novel Lie algebra formulation [**?**].

IV. Ongoing and Future Work

My ongoing and future work focuses on two independent thrusts towards the development of safe, efficient and versatile autonomy. These directions are visualized in Figure 1 and are explained in the sections below.

A. Speed and Scalability

Our approach in [9], along with other more general approaches [23], provide certification methods that help to guarantee safety of a broad class of robotics algorithms. However, one crucial caveat is that the application of redundant constraints precludes *fast certification methods*, such as those used in SE-Sync. In principle, problems with redundant constraints can still be solved in polynomial time using interior-point solvers [32], but these solvers are typically intractable for large-scale robotics applications. Improving the efficiency of these techniques is therefore a critical direction of current research.

Historically, the key algorithms in robotics experienced a huge efficiency boost once the problem structure was appropriately exploited [21, 19, 18]. Interestingly, there has been a similar, recent development for solving sparse SDPs in the optimization literature [39, 40, 24]. The key idea is that *aggregate sparsity patterns* permit a decomposition that breaks the large SDP into a set of smaller subproblems, which can be solved more efficiently.

Curiously, this approach has been underutilized in the robotics literature, despite the fact that robotics problems often exhibit very sparse structure. Our preliminary experimentation has indicates that this decomposition can be applied to several global optimality problems in robotics (e.g., SLAM, Model Predictive Control, robust perception problems [35]) and that it can even make certifiable methods as performant as fast local methods.

My research plan is to apply this SDP decomposition method to the problems mentioned above as well as those mentioned in Section III. My familiarity with exploiting sparsity in the context of SDPs (see Section III-A) will no doubt aid me in this development. The potential of developing efficient online approaches by solving SDPs *incrementally* can then be explored (similar to iSAM2 [19] or Model Predictive Control).

B. Certified Differentiable Optimization

As mentioned in Section II, when differential optimization is applied to non-convex robotics problems, solvers can potentially converge to local minima and yield incorrect gradients. My second research direction involves combining our global optimality results with differentiable optimization.

Since certifiable methods rely on convex relaxations, we can differentiate the convex relaxation using existing differentiable convex optimization techniques, such as CVXPYLayers [1]. My preliminary investigations suggest that the resulting gradients are also valid for the non-relaxed problem and can be used for backpropagation. The resulting technique will efficiently compute certifiably correct gradients, can be encapsulated as a neural network layer, and will allow certifiable optimization to be easily dropped into existing learning-based robotics pipelines.

C. Finding the Holy Grail

Interestingly, the methods described in the preceding sections are, in fact, compatible. That is, sparse problems in robotics can be made certifiable and *safe* by using the techniques described in Section III-B. The resulting solution can be *efficiently* found using the approach described in Section IV-A. Finally, the solution can become *versatile* by embedding it in a end-to-end learning pipeline using the approach described in Section IV-B. Altogether, convex relaxations will allow us to achieve the holy grail of safe, efficient, and versatile autonomy.

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