# Lab Nr. 9, Numerical Calculus

# Cubic Spline Interpolation;

# Least Squares Approximation

**Matlab functions**

- *spline* and *pchip*
- *polyfit* and *polyval*

**Applications**

**1.** Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = \frac{x+1}{3x^2+2x+1}.$$

Plot on the same set of axes the nodes, $f$ and the Lagrange, Hermite (with double nodes) and deBoor spline interpolants at 7 equally spaced nodes in the interval $[-2, 4]$.

**2.** Consider the function $f(x) = x\sin(\pi x)$ and the nodes $\{-1, -1/2, 0, 1/2, 1, 3/2\}$.

**a)** Obtain the deBoor, complete and piecewise Hermite cubic splines of these data;

**b)** Plot the nodes, $f$ and the three spline functions , on the same set of axes, for $x \in [-1, 3/2]$.

**3.** The following data is given:

| $x$ | 0.5 | 1.5 | 2 | 3 | 3.5 | 4.5 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|---|
| $f$ | 5 | 5.8 | 5.8 | 6.8 | 6.9 | 7.6 | 7.8 | 8.2 | 9.2 | 9.9 |

**a)** Draw the scatterplot and find the least squares polynomial approximation that best fits the data;

**b)** Compute the error of this approximation;

**c)** Use this approximation to estimate the value at $x = 4$;

**d)** Plot on the same graph the data and the approximation polynomial.

**4.** The following table contains values of the pressure $P$ of water vapors (measured in bars) as a function of temperature $T$ (measured in $^\circ C$):

| $T$ | 0 | 10 | 20 | 30 | 40 | 60 | 80 | 100 |
|---|---|---|---|---|---|---|---|---|
| $P$ | 0.0061 | 0.0123 | 0.0234 | 0.0424 | 0.0738 | 0.1992 | 0.4736 | 1.0133 |

**a)** Find the quadratic and cubic least squares approximation polynomials of these data. Which approximation is better?

**b)** Approximate the pressure corresponding to a temperature of $T = 45^\circ$;

**c)** Plot on the same set of axes the data and the two approximating polynomials.

**Optional**

**5.** Implement the computation of the *natural* cubic spline approximation. Then plot it together with the complete spline for the function and nodes in Problem **2.**