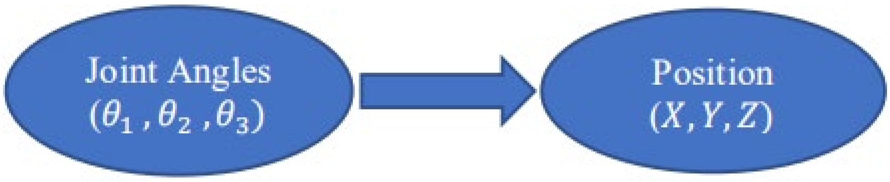
joint	α_i	a_i	d_i	θ_i
1	90°	0	L_1	θ_1
2	0°	0	0	θ_2
3	0°	L_2	0	θ_3
4	0°	L_3	0	0

D-H PARAMETERS TABLE

FORWARD KINEMATIC



To get the forward kinematic solution, Denavit-Hartenberg (D-H) representation is applied. The D-H model of representation is a very simple way of modeling robot links and joints that any robot configuration such as articulated robot configuration that proposed in this project, regardless of its sequence or complexity. To model the robot with the D-H representation, initially assign a local reference frame for each and every joint. Then determine the following parameters :

- θ, represented a rotation about z-axis.
- d, represented the distance on the z-axis between two successive common normal,
- a, represented the length of each common normal, also defined as length of a link,

- α, represented the angle between two successive z axes, also defined as joint twist angle.

Transformation nTn+1 between two successive frames representing the preceding four movement is the product of four basic homogeneous transformation matrices (revolute joints) representing them. The final formula of revolute joints is as shown below:

The final formula of revolution matrix

$$
{}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & s\theta_i a_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Substitute the parameters in D-H Table into the matrix form above to get the matrices for ${}^0_1T, {}^1_2T, {}^2_3T, {}^3_4T$.

Homogeneous transform matrices

$$
{}^0_1T = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^1_2T = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2_3T = \begin{bmatrix} c_3 & -s_3 & 0 & L_2c_3 \\ s_3 & c_3 & 0 & L_2s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{3}_{4}T = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

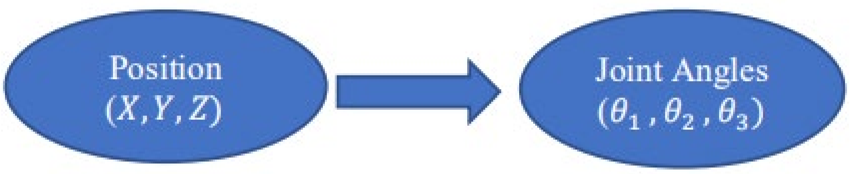
Where c_i is the cosine of the angle θ_i , and s_i represent the sine.

By multiplying the homogeneous transformation matrices in the right order, ${}^{0}_{4}T$ describing the end-effector frame with the reference frame may be obtained.

$$
{}^{0}_{4}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T
$$

$$
{}^{0}_{4}T = \begin{bmatrix} c_1c_{23} - c_1s_{23} & -c_{12}s_3 - c_{13}s_2 & s_1 & c_1c_{23}L_2 + c_1c_{23}L_3 - c_1s_{23}L_2 - c_1s_{23}L_3 \\ s_1c_{23} - s_1s_{23} & -c_2s_{13} - c_3s_{12} & -c_1 & s_1c_{23}L_2 + s_1c_{23}L_3 - s_1s_{23}L_2 - s_1s_{23}L_3 \\ c_{23}s_{23} & c_{23} - s_{23} & 0 & c_2s_3L_3 + c_2s_3L_2 + c_3s_2L_3 + L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1\text{-}1)
$$

INVERSE KINEMATIC



$$
{}^{0}T_{H} = \begin{bmatrix} M_x & N_x & B_x & x \\ M_y & N_y & B_y & y \\ M_y & N_y & B_y & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1-2}
$$

By comparing Eq. (1-1) anf Eq. (1-2)

$$
\begin{aligned}
x &= c_1 c_{23} L_2 + c_1 c_{23} L_3 - c_1 s_{23} L_2 - c_1 s_{23} L_3 \\
y &= s_1 c_{23} L_2 + s_1 c_{23} L_3 - s_1 s_{23} L_2 - s_1 s_{23} L_3 \qquad \text{(Assumed)} \\
z &= c_2 s_3 L_3 + c_2 s_3 L_2 + c_3 s_2 L_3 + L_1
\end{aligned}
$$

$$
\frac{y}{x} = \frac{s_1 c_{23} L_2 + s_1 c_{23} L_3 - s_1 s_{23} L_2 - s_1 s_{23} L_3}{c_1 c_{23} L_2 + c_1 c_{23} L_3 - c_1 s_{23} L_2 - c_1 s_{23} L_3}
$$

$$
= \frac{s_1 \left(c_{23} L_2 + c_{23} L_3 - s_{23} L_2 - s_{23} L_3 \right)}{c_1 \left(c_{23} L_2 + c_{23} L_3 - s_{23} L_2 - s_{23} L_3 \right)}
$$

$$
= \frac{s_1}{c_1} = \tan \theta_1
$$

$$
\theta_1 = a \tan 2 \left(\frac{y}{x} \right) \tag{1-3}
$$

$$
\frac{M_y}{N_y} = \frac{s_{23}}{c_{23}} = \tan \theta_{23}
$$

$$\theta_{23} = a\tan 2\left(\frac{M_y}{N_y}\right) \tag{1-4}$$

$$z = c_2 s_3 L_3 + c_2 s_3 L_2 + c_3 s_2 L_3 + L_1$$

$$s_2 = \frac{z - c_2 s_3 L_3 - c_2 s_3 L_2 - L_1}{c_3 L_3} \tag{1-5}$$

$$x = c_1 c_{23} L_2 + c_1 c_{23} L_3 - c_1 s_{23} L_2 - c_1 s_{23} L_3$$

$$x = c_1\left(c_{23} L_2 + c_{23} L_3 - s_{23} L_2 - s_{23} L_3\right)$$

$$c_2 = \frac{\frac{x}{c_1} + s_{23} L_2 + s_{23} L_3}{c_3 L_2 + c_3 L_3} \tag{1-6}$$

From Eq. (1-5) and Eq. (1-6)

$$\frac{s_2}{c_2} = \tan\theta_2 = \frac{\frac{z - s_{23} L_3 - L_1}{L_2}}{\frac{\frac{x}{c_1} - c_{23} L_3}{L_2}}$$

$$\theta_2 = a\tan 2\left(\frac{\frac{z - c_2 s_3 L_3 - c_2 s_3 L_2 - L_1}{c_3 L_3}}{\frac{\frac{x}{c_1} + s_{23} L_2 + s_{23} L_3}{c_3 L_2 + c_3 L_3}}\right)$$

From Eq. assumed

Summing up squares of x y z

$$c_3 = \frac{x^2 + y^2 + z^2 - \left(L_1{}^2 + L_2{}^2 + L_3{}^2\right) - 2L_1\left(z - L_1\right)}{2L_2L_3} \qquad (1\text{-}7)$$

From pythagorean trigonometric

$$s_3 = \pm\sqrt{1 - c_3} \qquad (1\text{-}8)$$

From Eq. (1-7) and Eq. (1-8)

$$\frac{s_3}{c_3} = \tan\theta_3$$

$$\theta_3 = a\tan 2\left(\frac{s_3}{c_3}\right)$$