Assignment 3
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Problem 1:

a)

$(x \vee 0') \wedge (x' \vee 0) = x'$
$= (x \vee 1) \wedge x'$ (identity)
$= (x \wedge x') \vee (1 \wedge x')$ (Complementation)
$= 0 \vee x'$ (idenity)
$= x'$ (identity)

b) $x, y: (x \vee y) \wedge x = x$
$= (x \vee y) \wedge (x \vee 0)$ *(identity)*
$= x \vee (y \wedge 0)$ *(distributive)*
$= x \vee \left(y \wedge (y \wedge y')\right)$ *(complementation)*
$= x \vee ((y \wedge y) \wedge y')$ *(associative)*
$= x \vee (y \wedge y')$ *(Idempotent)*
$= x \vee 0$ *(complementation)*
$= x$

c) $x, y: y' \vee \left((x \wedge y) \vee x'\right) = 1$
$= y' \vee ((x \vee x') \wedge (y \vee x'))$ (distributive)
$= y' \vee (1 \wedge (y \vee x'))$ *(complementation)*
$= y' \vee (y \vee x')$ *(identity)*
$= (y' \vee y) \vee (x')$ *(accociative)*
$= 1 \vee x'$ *(complementation)*
$= 1$ *(complementation)*

Problem 2.

a)

$\left((p \wedge q) \to r\right) \equiv \left(p \to (q \to r)\right)$

For left part that we have table:

p	q	r	$p \wedge q$	$(p \wedge q) \rightarrow R$
F	F	F	F	F
F	F	T	F	T
F	T	F	T	F
F	T	T	T	F
T	F	F	T	F
T	F	T	T	F
T	T	F	T	F
T	T	T	T	F

For right part that we have table:

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$
F	F	F	F	F
F	F	T	T	T
F	T	F	F	F
F	T	T	F	F
T	F	F	F	F
T	F	T	T	F
T	T	F	F	F
T	T	T	F	F

So that 2 tables' final columns are same. The equation is true

b) ((p → q) → r) ≡ (p → (q → r))

For left part we have table:

p	q	r	$p \rightarrow q$	$(p \rightarrow q) \rightarrow r$
F	F	F	F	F
F	F	T	F	T
F	T	F	T	F
F	T	T	T	F
T	F	F	F	F
T	F	T	F	T
T	T	F	F	F
T	T	T	F	T

For right part we have table:

p	q	r	q → r	$p \rightarrow (q \rightarrow r)$
F	F	F	F	F
F	F	T	T	T
F	T	F	F	F
F	T	T	F	F
T	F	F	F	F
T	F	T	T	F
T	T	F	F	F
T	T	T	F	F

So that 2 tables' final columns are not same. The equation is False

$c)\quad ((p \lor (q \lor r)) \land (r \lor p)) \equiv ((p \land q) \lor (r \lor p))$

Left part we have:

$\big((p \lor (q \lor r)) \land (r \lor p)\big)$

$= (p \lor q \lor r) \land (r \lor p)$ (associative)

$= (p \lor q \lor r) \land r \lor (p \lor q \lor r) \land p$ (distributive)

$= r \land (r \lor p \lor q) \lor (p \lor q \lor r) \land p$ (commutative)

$= r \lor (p \lor q \lor r) \land p$ (absorption)

$= r \lor p \land (p \lor q \lor r)$ (commutative)

$= r \lor p$ (absorption)

Right part we have:

$\big((p \land q) \lor (r \lor p)\big)$

$= (p \lor q) \lor r \lor p$ (associative)

$= p \lor (p \land q) \lor r$ (commutative)

$= p \lor r$ (absorption)

So that left = right. The equation is True

Problem 3.

a)

$dual(\psi \to \varphi)$
$= dual(\neg\varphi \lor \psi)$
$= dual(\neg\varphi) \land dual(\psi)$
$= \neg\, \mathrm{dual}(\varphi) \land \mathrm{dual}(\psi)$
$= \neg(\mathrm{dual}(\varphi) \lor \neg\mathrm{dual}(\psi))$
$= (\neg\, \mathrm{dual}(\psi) \to \mathrm{dual}(\varphi))$

$dual(\psi \leftrightarrow \varphi)$
$= dual\,\big((\varphi \to \psi) \land (\psi \to \varphi)\big)$
$= dual(\varphi \to \psi) \lor dual(\psi \to \varphi)$
$= dual(\neg\varphi \lor \psi) \lor dual(\neg\psi \lor \varphi)$
$= \neg\big(\mathrm{dual}(\varphi) \to \mathrm{dual}(\psi)\big) \lor \neg(\mathrm{dual}(\psi) \to dual(\varphi))$
$=(\neg\big(\mathrm{dual}(\varphi) \leftrightarrow \mathrm{dual}(\psi)\big))$

b) Let a DNF present as following format:

$\psi = (a_1^1 \wedge a_2^1 \ldots \wedge a_m^1) \vee \ldots \vee (a_1^n \wedge a_2^n \wedge \ldots \wedge a_m^n)$

$dual(\psi) = dual(a_1^1 \wedge a_2^1 \ldots \wedge a_m^1) \wedge dual$

$dual(\psi) = (a_1^1 \vee a_2^1 \ldots \vee a_m^1) \wedge \ldots \wedge (a_1^n \vee a_2^n \vee \ldots \vee a_m^n)$

$flip \circ dual(\psi) = flip(a_1^1 \vee a_2^1 \ldots \vee a_m^1) \wedge \ldots \wedge flip\ (a_1^n \vee a_2^n \vee \ldots \vee a_m^n)$

$= (flip(a_1^1) \vee flip(a_2^1)) \vee \ldots \vee (flip(a_1^n) \vee flip(a_m^n))$

$= (a_1^1 \vee a_2^1 \vee \ldots \vee a_n^1) \wedge \ldots \wedge (a_1^n \vee a_1^n \ldots a_m^n)$

c)

Assume $\varphi = \bot$

$= flip \circ dual(\neg\bot)$

$= flip \circ dual(\top)$

$= flip\ (\bot)$

$= \bot$

Assume $\psi = \top$

$= flip \circ dual(\neg\top)$

$= flip \circ dual(\bot)$

$= flip(\bot)$

$= \top$

$\varphi \equiv flip\ \circ dual(\neg\varphi)$

Proof : $\neg\varphi = flip\ \circ dual\ (\neg\neg\varphi)$ $\quad$ $[flip \circ dual(\neg\top)]$

$=flip(\neg dual\ (\neg\varphi)$

$= \neg flip \circ dual\ (\neg\varphi)$

$= \varphi$

$\varphi \vee \psi$: $flip \circ dual\ \big(\neg(\varphi \vee \psi)\big)$

$= \ flip \circ dual(\ \big((\neg\varphi \vee \neg\psi)\big)$

$= flip \circ (\neg dual(\varphi \vee \psi)$

$= flip \circ \neg\big(dual(\varphi) \wedge dual\ (\psi)\big)$

$= (flip \circ dual(\neg\varphi) \vee (flip \circ dual(\neg\psi))$ $\quad$ $[flip \circ dual(\neg\bot)]$

$= \varphi \vee \psi$

$\varphi \to \psi$: $flip \circ dual\big(\neg(\varphi \to \psi)\big)$

$= flip \circ dual\ \big((\neg\varphi \to \neg\psi)\big)$

$= flip \circ dual(\psi \to \varphi)$

$= flip \circ \neg(dual(\psi) \wedge dual(\varphi))$

$= flip \circ dual(\neg\psi) \vee flip \circ dual(\neg\varphi)$

$= \psi \vee \varphi$

$\varphi \leftrightarrow \psi$: $flip \circ dual\big(\neg(\varphi \leftrightarrow \psi)\big)$

$= flip \circ \neg(\neg\big(\mathrm{dual}(\varphi) \to \mathrm{dual}(\psi)\big) \vee\ \neg(\mathrm{dual}(\psi) \to dual(\varphi)))$

$= flip \circ \neg\big(\ \mathrm{dual}(\varphi) \to \mathrm{dual}(\psi)\big) \vee flip \circ \neg\big(\mathrm{dual}(\psi) \to dual(\varphi)\big)$

$= (\psi \wedge \varphi) \vee (\varphi \wedge \psi)$

$= \psi \wedge \varphi$

Problem 4

a) Assume set T$\in$ {0,1,a} $0 \neq 1 \neq a$

Since Complementation law:

$0' = 1, 1' = 0\ , x' \neq 0, x' \neq 1, x' = x$

So that $x \vee x' = x \vee x$

Since complementation law, $x \vee x' = 1$

So that there is no complementation for 3 elements.

Problem 5

a)

(i) Define variables for 10 houses in 2 colors.

Red house 1: R_1, Red House 2: R_2, Red House 3: R_3, Red House 4: R_4, Red House 5: R_5

Blue house 1: B_1, Blue House 2: B_2, Blue House 3: B_3, Red House 4: R_4, Blue House 5: B_5

(ii) Define any propositional formulas.

One house must and can only have one color: $(R_n \wedge B_n) \wedge \neg(\neg R_n \wedge \neg\, B_n): 1 \leq n \leq 5, n \in \mathbb{Z}$

If house are not neighbors, then colors are not same. are same color then tow houses are neighbors. Name all cases as c_1, c_2, c_3, c_4, c_5

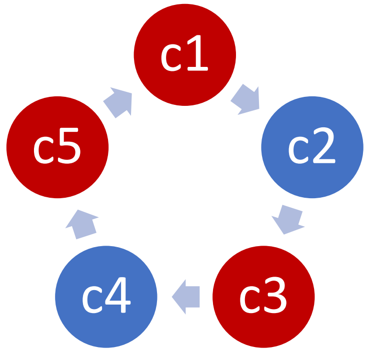
$c_1: (R_1 \to B_4) \wedge (B_1 \to R_4);\ c_2: (R_4 \to B_2) \wedge (B_4 \to R_2);\ c_3(R_2 \to B_5) \wedge (B_2 \to R_5);$

$c_4(R_5 \to B_3) \wedge (B_5 \to R_3);\ c_5(R_3 \to B_1) \wedge (B_3 \to R_1)$

(iii) Assume $c_1 \wedge c_2 \wedge c_3 \wedge c_4 \wedge c_5$.

From previous question we have this graph is a circle. So that it can have $c_1 \wedge c_2 \wedge c_3 \wedge c_4 \wedge c_5$ when the elements in c_n is not painted color inside.

Then in this question. We need paint graph with 2 color. So the circle will be like below:



Obviously, a odd number circle map needs 3 chromatic number. Show above picture, this picture cannot satisfy $c_1 \wedge c_2 \wedge c_3 \wedge c_4 \wedge c_5$.

Problem 6

a) $P_1(n+1) = \frac{1}{2}P_1(n) + \frac{1}{3}P_2(n) + \frac{1}{3}P_5(n)$

$$P_2(n+1) = \frac{1}{4}P_1(n) + \frac{1}{3}P_2(n) + \frac{1}{3}P_3(n)$$

$$P_3(n+1) = \frac{1}{3}P_3(n) + \frac{1}{3}P_3(n) + \frac{1}{3}P_4(n)$$

$$P_4(n+1) = \frac{1}{3}P_3(n) + \frac{1}{3}P_4(n) + \frac{1}{3}P_5(n)$$

$$P_5(n+1) = \frac{1}{4}P_1(n) + \frac{1}{3}P_4(n) + \frac{1}{5}P_5(n)$$

b) From previous question we have the answer of equation answer:

$$P_1 = \frac{1}{4}, P_2 = \frac{3}{16}, P_3 = \frac{3}{16}, P_4 = \frac{3}{16}, P_5 = \frac{3}{16}.$$

c) $E = \frac{1}{4}D_{1-1} + \frac{3}{16}D_{1-2} + \frac{3}{16}D_{1-3} + \frac{3}{16}D_{1-4} + \frac{3}{16}D_{1-5}$

$$= \frac{1}{4} * 0 + \frac{3}{16} * 1 + \frac{3}{16} * 1 + \frac{3}{16} * 2 + \frac{3}{16} * 2$$

$$= \frac{9}{8}$$

Problem 7

a) Let:
left number of node =0 , right number of node = n-1

left number of node =1 , right number of node = n-2

……

left number of node =n-1 , right number of node = 0

So that , $T(n) = T(0) * T(n-1\ ) + T(n) * T(n-2) + \cdots + T(n-1) * T(0)$

$= \sum_{m=0}^{n-1} T(m) * T(n-m-1)$

b)
From Assignment 2, we have $Count(T) = 2 \times leaves(T) + helf - leaves(T) - 1$
Since the tree is Full-binary tree. So that the $half\ - leaves(T) = 0$
So that, $Count(T) = 2\ leves(T) - 1$
So that the number of node is a odd number

c) For B(n). We can get $B(n) = B(1) * B(n-2) + B(3) * B(n-4) + \cdots + B(n-2) * B(1)$
So that $\mathrm{B}(1) = 1, \mathrm{B}(2) = 0, \mathrm{B}(3) = 1, \mathrm{B}(5) = 2, \mathrm{B}(7) = 5$
Since $T(0) = 1\ , T(2) = 1, T(3) = 5$
So that, $\mathrm{B(n)} = \mathrm{T}(\frac{\mathrm{n}-1}{2})$

d) Since the tree is a FBT and n can only be numbers of $leaves\ node$.
Since we have $Count(T) = 2\ leves(T) - 1$ and $Internal\ Node\ =\ leaves - 1$
Internal Node = n-1

$F(n) = 2^{n-1} * B(2n-1) * 2^{n} * n! = T(n-1) * 2^{2n-1} * n!$