Assignment 2

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Problem 1

a. Reflexive

Reflexive for All $x \in S(x, x) \in R$
In this question will be $x \in S(x, x) \in R_1 \cap R_2$
Since $R_1, \; R_2 \subseteq S \times S$. for all $x \in S, xR_1x, xR_2x, \ldots$
So that $(x, y) \in R_1 \cap R_2$

symmetric

Let $(x, y) \in R_1 \cap R_2$, then $(x, y) \in R_1$ and $(x, y) \in R_2$
Since symmetric of set. We will have $(y, x) \in R_1$ and $(y, x) \in R_2$
So that when $(y, x) \in R_1 \cap R_2$. It has symmetric.

transitive

Proof $(x, z) \in R_1 \cap R_2$, when
Let $(x, y) \in R_1 \; and \; (y, z) \in R_1$
Then will have, $(x, z) \in R_1$by transitive of sets.
Let $(x, y) \in R_2 \; and \; (y, z) \in R_2$
Then will have, $(x, z) \in R_2$by transitive of sets.
So that. When $(x, y) \in R_1 \cap R_2$ there is $(z, y) \in R_1 \cap R_2$

b. $y \in [x]_1 \cap [x]_2$
From above question. $y \in [x]$ iff $(x, y) \in R_1 \cap R_2$.
Since $(x, y) \in R_1 \cap R_2$,.We have $(x, y) \in R_1$ and $(x, y) \in R_2$
Since $(x, y) \in R_1$, we have $y \in [x]_1$
Since $(x, y) \in R_2$ we have $y \in [x]_2$
So that $y \in [x]_1 \cap [x]_2$

c. Counterexample:
Let$(x, y), (y, z) \in R_1 \cup R_2$
Then must have $(x, y) \in R_1$, and $(y, z) \in R_2$
Let S= {1,2,3,4}
R_1 = {(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(4,4)}
R_1 = {(1,1),(1,2),(1,4),(2,1),(2,2),(2,4),(3,3),(4,1),(4,2),(4,4)}
So that, there doesn't have (3,4) in $R_1 \cup R_2$

Problem 2

a. Since $R_1, R_2 \subseteq S \times S$ and R_1, R_2 are reflexive
So that there is $(a, b) \in R_1$ and $(b, c) \in R_2$
So that $(a, c) \in R_1; R_2$

b. Counterexample:
If $R_1 = \{(1,3), (3,1)\}$,$R_2 = \{(2,3), (3,2)\}$
Then $R_1; R_2 = \{(1,2)\}$
So that R_1, R_2 is not symmetric

c. Counterexample:
$R_1 = \{(1,4), (2,5)\}$ $R_2 = \{(4,2), (5,3)\}$
So that $R_1; R_2 = \{(1,2), (2,3)\}$
There is not transitive

Problem 3

a. Let $P(j)$ be propositional statement that $R^j = R^i$ for all $j \geq i$
Base step:
$P(i): R^i = R^i$
Inductive step:
Assume $P(k)$ is true for some $k \geq i$, therefore $R^k = R^i$
So that $R^{k+1} := R^k \cup \left(R; R^k\right)$
When $R^k = R^i$
$$R^{k+1} := R^i \cup \left(R; R^k\right)$$
$$R^{k+1} := R^{k+1}$$
When $R^i = R^{i+1}$
$$R^{k+1} := R^i$$
So that $P(k)$ implies $P(k+1)$ as a result, $R^j = R^i$ for all $j \geq i$

b. From above (a) when $j = i$ $R^j = R^i$ so that $R^j \subseteq R^i$
Since $R^{n+1} \coloneqq R^n \cup (R; R^n)$ for $n \geq 0$ we can get $R^0 \subseteq R^1 \subseteq R^2 \subseteq \cdots ... \subseteq R^i$
So that $R^j \subseteq R^i$

c. Since $R \subseteq S \times S$, so $|R| \leq k^2$
$$|R^0| + 1 \leq |R^1|$$
$$|R^1| + 1 \leq |R^2|$$
$$\dots\dots$$
$$|R^i| + 1 \leq |R^k|$$

So that, $k + i + 1 \leq k^2$ which is $i \leq k^2 - k - 1$. Therefore, $i \leq k^2$

d. Let $P(n)$ be the proposition that for all $m \in \mathbb{N}$: $R^n; R^m = R^{n+m}$.

Base Step:

$P(0): R^0; R^m = R^m$

Inductive Step:

Let $P(k)$ holds $R^k; R^m = R^{k+m}$

$R^k; R^m = [R^k \cup (R; R^k)]; R^m$---------------------

$=(R^k; R^m) \cup (R; R^k); R^m$------------- $(R_1 \cup R_2); R_3 = (R_1; R_3) \cup (R_2; R_3)$

$= R^{k+m} \cup (R; R^{k+m})$-------------------- $(R_1; R_2); R_3 = R_1; (R_2; R_3)$

$= R^{k+m+1}$

e. Prove that $if\ (a,b) \in R^{k^2}, (b,c) \in R^{k^2}\ there\ \ is\ (a,c) \in R^{k^2}$

From above (d): we have$(a,c) \in R^{k^2}; R^{k^2}$which is $(a,c) \in R^{2k^2}$

Since above (a) and (c) ,$(a,c) \in R^2\ and\ R^{2k} = R^i$ we can get $(a,c) \in R^{k^2}$

f. From above $(R \cup R^{\leftarrow})^0 \subseteq (R \cup R^{\leftarrow})^k$

Reflexive:

Since $(x,x) \in (R \cup R^{\leftarrow})^{k^2}$

$(x,x) \in (R \cup R^{\leftarrow})^0 \ \subseteq (R \cup R^{\leftarrow})^{k^2}$

Symmetric:

Let $P(n)$ be the proposition statement that $(R \cup R^{\leftarrow})^n$ has symmetric

Base Step:

$(R \cup R^{\leftarrow})^0$

Inductive Step:

prove that $(R \cup R^{\leftarrow})^{k+1}$is symmetric

LET $(R \cup R^{\leftarrow})$ be set A

Since $(x,y) \in A^k$ so that $(y,x) \in A^k$ then $(y,x) \in A^{k+1}$

Since $(x,y) \in A; A^k$ exit $(x,z) \in A$, $(z,y) \in A^k$ then we get,$(x,z) \in A, (y,z) \in A^k$

Since $(x,z) \in A, (y,z) \in A^k$, we get $(y,x) \in A^k; A$

So that $(y,x) \in A^{k+1}$

Transitive:

From above (e) that we R^{k^2} is transitive based on a binary relation of $R \subseteq S \times S$

And binary relation $(x, y) \in (R \cup R^{\leftarrow})^{\text{k^2}}$

So that $(R \cup R^{\leftarrow})^{\text{k+1}}$ is transitive

g.

Problem 4

a. we known $f(n) = f\left(\left\lfloor\frac{n}{3}\right\rfloor\right) + 3f\left(\left\lfloor\frac{n}{5}\right\rfloor\right) + n \; for \; n \geq 1$
$f(n) \leq f\left(\left\lfloor\frac{n}{3}\right\rfloor\right) + 3f\left(\left\lfloor\frac{n}{5}\right\rfloor\right) + n \; for \; n \geq 1$
$f(n) \leq n + \frac{14}{15}n + \frac{14^2}{15^2}\text{n} + \ldots.$
So that O(n)

Problem 5

a. $count\ (\ T) = 0$
$count(T_1, T_2) = 1 + \ count(T_1)\ + count(T_2)$

b. $leaves(T)\ = 0$
$leaves([T_1, T_2])\ =\ 1$ where T_1, T_2 are empty.
$leaves([T_1, T_2])\ =\ leafe\ (T_1)\ +\ leafe(T_2)$ where T_1, T_2 are not empty

c. $Half\ leaves(T) = 1$
$Half\ leaves([T_1, T_2]) = 0$ where $T_1 and\ T_2\ are\ leaves\ node$
$Half\ leaves([T_1, T_2])\ =\ half\ leaves(T_1)\ +\ half\ leaves(T_2)$

d. Base T is empty $count(T)\ =\ 0$
So that $0 = 2 \times 0 + 1 - 1$
Base2 $[T_1, T_2]$ $count(T)\ =\ 0$
So that $1 = 2 \times 1 + 0 - 1$

Inductive case $[T_1, T_2]$ $exit$ T_1, T_2 $that\ count(T_1) = 2leaves(T_1) + half\ leaves(T_1) -$ 1 and $count(T_2) = 2leaves(T_2) + half\ leaves(T_2) - 1$
To prove $count([T_1, T_2]\quad) = 2leaves([T_1, T_2]) + half\ leaves([T_1, T_2]) - 1$

Substate above's $[T_1, T_2]$ will get
$1 + \ count(T_1) + count(T_2) = 2\big(leaves(T_1) + \ leaves(T_2)\big) + half\ leaves(T_1) +$ $half\ leaves(T_2)$ -1
So that $1 + 0 + 0 = 2 \times (0 + 0) + 1 + 1 - 1$
$1 = 0 + 2 - 1$

Problem 6

a. $O(n^2)$
   2 for loop.
b. $O(n^3)$
   2 for loop + 1 (0-n)
c. Since $SW + TY, SX + TZ, UW + VY, UX + VZ\ are\ O(n^2)$
   And each multiplication is $T(\frac{n}{2})$ In matrix AB there are 8 $T(\frac{n}{2})$

   So that we get $T(n) = 8T\left(\frac{n}{2}\right) + O(n^2)$

d. $T(n) \in O(n^3)$