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Index

Symbols

Ab, category of abelian groups, 99
Alg_R^{alt}, category of alternating R -algebras, 126
Alg_K^{fg}, category of finitely generated \mathbb{K} -algebras, 244
Alg_R, category of associative R -algebras, 100
Alg_{R,1}, category of associative R -algebras with identity, 110
Alt_R(M^q, N), alternating mappings, 86
alt, alternation, 87
Ann, annihilator, 268
AS_K, category of \mathbb{K} -algebraic sets, 245
Aut $R(M)$, module automorphisms, 31
 $\mathcal{B}_{X,R}$, presheaf of bounded functions, 140
 $\mathcal{C}^{\mathbb{K},k}_X(U; E)$, sections of E over U , 198
 $\mathcal{C}^{\mathbb{K},k}_X$, structure sheaf of the manifold X , 171
 $\mathcal{C}^{\mathbb{K},k}_X \otimes V$, locally free module sheaf, 169
CAlg_R, category of commutative R -algebras, 100
C_{C-Vect}^{diff}, category, 101
C_{C-Vect}^k, category, 101
 χ_φ , characteristic polynomial, 51
C_{K-Vect}^ω, category, 101
 $\mathcal{C}^{\mathbb{K},\omega}_{X,V}$, sheaf of analytic V -valued functions, 140
 $\mathcal{C}^{\mathbb{K},k}_{X,V}$, sheaf of differentiable V -valued functions, 140
CMet, category of complete metric spaces, 124
C_{Φ,Γ}, category of an algebraic structure, 100
C_{R-Vect}^{diff}, category, 101
C_{R-Vect}^k, category, 101
CRing₁, category of commutative rings with identity, 99
 $\mathcal{C}_{X,R}$, sheaf of continuous functions, 140
 df , differential of f , 196

$D(f)$, standard open set in the maximal spectrum, 258
 $D(f)$, standard open set in the spectrum, 266
 Dx , directional derivative, 304
 $Df(p)$, Jacobian matrix of f , 192
 $\left(\frac{\partial y_I}{\partial x_I}\right)$, Jacobian matrix in multi-index notation, 211
 $\frac{\partial}{\partial x_j}$, basis field, 205
 $\frac{\partial}{\partial x_j}|_p$, derivation in coordinates, 185
 $\mathcal{D}(U)$, differential operators on U , 9
 ∂M , boundary of M , 223
 ∂^α , higher partial derivatives, 8
 ∂_j , partial derivatives, 8
 Δ_X , diagonal in X , 262
 $d\alpha$, exterior derivative of α , 212
 d_g , distance function, 291
 df_p , derivative of f , 190
 dx_I , differential forms in multi-index notation, 210
 $dx_j(p)$, φ -basis for T_p^*M , 196
Der(A), derivations of A , 206
Der(A, B), derivations from $A \subseteq B$, 206
Der_Φ(A, B), Φ -derivations from A to B , 206
Der($\mathcal{C}^{\mathbb{K},k}_M$), derivation sheaf on M , 207
Der($\mathcal{C}^{\mathbb{K},k}_M, \mathcal{C}^{\mathbb{K},k-1}_M$), derivation sheaf on M , 207
dim $\mathbb{K}(X)$, dimension of X , 173
 $D_V(f)$, standard open set, 252
 $dx_1(p)$, φ -basis for the cotangent space, 197
deg(f), degree of a polynomial, 8
 E_p , fiber of a vector bundle, 198
 $\mathcal{E}(X, C)$, category of étalé spaces, 156
 \mathcal{E}_π , sheaf of sections, 199
 $\mathcal{E}(\mathcal{F})$, étalé space of a sheaf, 155

- $\mathbf{\acute{E}Mod}_{\mathcal{O}_X}$, étalé-category of \mathcal{O}_X -modules, 167
 $\text{End } R(M)$, module endomorphisms, 31
 $\text{End}\mathbf{C}(X)$, endomorphisms of a category, 98
 ev , evaluation functor, 129
 ev , evaluation mapping, 92
 ev_a , evaluation mapping, 240
 ev_x , evaluation, 11
 ev_φ , evaluation homomorphism, 48
 $F(E)$, free module over E , 41
 ${}_R F(E)$, free left- R -module over E , 41
 \mathbf{Field} , category of fields, 100
 $\gcd(a_1, \dots, a_k)$, gcd for rings, 20
 $\mathbb{G}_{k,n}$, Grassmann manifold, 178
 $G\mathbf{Mod}_R$, category of G -modules, 125
 \mathbf{Grp} , category of groups, 99
 $\mathbf{Grp}_{\text{top}}$, category of topological groups, 101
 H^\bullet , representing functor, 128
 H_\bullet , representing functor, 129
 H_x , stalk functor in x , 145
 \mathbb{H} , quaternions, 6
 H^X , represented by X functor, 106
 H_X , represented by X functor, 106
 $\text{Hol}_{X,V}$, sheaf of holomorphic V -valued functions, 140
 $\text{Hom}_R(M, N)$, module homomorphisms, 29
 $\text{Hom}\mathbf{C}(X, Y)$, morphisms of a category, 98
 Hom_A , Hom-functor, 107
 $\text{Hom}_{\mathcal{O}_X}(\mathcal{F}, \mathcal{G})$, \mathcal{O}_X -modules of homomorphisms, 168
 $I(X)$, vanishing ideal of a set, 241
 $\iota_{\mathfrak{X}}\alpha$, contraction, 215
 I , ideal for the alternating algebra, 84
 \mathbf{ID} , category of integral domains, 100
 \mathbf{ID}_m , subcategory of integral domains, 122
 $\text{im } (\varphi)$, image of a module homomorphism, 30
 Ind_H^G , induction functor, 125
 J , ideal for the symmetric algebra, 77
 J_q , q -part of J , 82
 $\ker(\varphi)$, kernel of a module homomorphism, 30
 $\ell_g(\gamma)$, curve length, 290
 $L(V_1, \dots, V_s; V_0)$, bundle of s -linear mappings, 203
 $L_{\mathfrak{X}}\alpha$, Lie derivative of a differential form, 215
 \mathcal{L}_X^1 , presheaf of integrable functions, 140
 $\bigwedge T^*M$, form bundle, 211
 $\bigwedge^k T^*M$, k -form bundle, 210
 $\Lambda(M)$, exterior algebra, 84
 $\Lambda(\psi)$, exterior algebra functor, 85
 $\Lambda_q(M)$, antisymmetric tensors, 81
 $\Lambda^q(M)$, q -th exterior power of M , 86
 lcm , least common multiple, 112
 \mathbf{LieAlg}_R , category of Lie algebras over R , 127
 \varprojlim , categorical limit, 113
 \varinjlim , categorical colimit, 113
 $L_R(M_1, \dots, M_n; P)$, multilinear mappings, 56
 $L_R(M_\lambda; P)$, multilinear mappings, 56
 \mathfrak{m}_a , vanishing ideal of a , 246
 $\mathbf{Man}_{\mathbb{K},k}$, category of manifolds, 172
 $\mathbf{Mat}_{\mathbb{K}}$, category of \mathbb{K}^n 's, 136
 \mathbf{Met} , category of metric spaces, 124
 $\mathbf{Mod}_{\mathcal{O}_M}^{\text{lff}}$, category of locally free module sheaves, 200
 \mathbf{Mod}_R , category of right- R -modules, 100
 $\mathbf{Mod}_{\mathcal{O}_X}$, category of \mathcal{O}_X -modules, 166
 ${}_R\mathbf{Mod}$, category of left- R -modules, 100
 $\nabla_{\mathfrak{X}}$, covariant derivative, 303
 $\text{ob}(\mathbf{C})$, objects of a category, 98
 Ω_M^ℓ , sheaf of ℓ -forms, 199
 $\Omega^r(M)$, differential forms of degree r , 210
 Ω_M , sheaf of differential forms, 199, 210, 212
 $\text{Or}(M)$, orientation bundle, 306
 $P(a, \varrho)$, open polycylinder, 235
 $P(a, \varrho)^-$, closed polycylinder, 235
 \mathbb{P}_R^n , projective space over R , 272
 $\mathbf{PS}(X, \mathbf{C})$, category of presheaves, 141
 φ -basis, 184, 187
 φ_{ij} , transition map, 173
 $\mathbb{P}_{\mathbb{K}}^n$, projective space, 178
 $\mathbb{P}_{\mathbb{K}}^n$, projective space, 178
 $\mathbf{PVar}_{\mathbb{K}}$, category of prevarieties, 258
 $Q(R)$, quotient field, 15
 \mathbb{Q}_p , p -adic numbers, 118
 $\text{rad}(I)$, radical of an ideal, 245
 Res_H^G , restriction functor, 125
 $\text{rest}_{]c,d[}$ restriction, 11
 ρ_U , section functor, 149
 $\rho_{V,U}$, restriction mappings, 140
 \mathbf{Ring} , category of rings, 99
 \mathbf{Ring}_1 , category of rings with identity, 99
 \mathbf{RSp}_R , category of R -ringed spaces, 160
 $\mathbf{RSp}_{\text{loc},R}$, category of locally R -ringed spaces, 164
 S , section functor, 200
 $(S\varphi)_U$, section functor, 156
 $S(E, X, \pi)$, section functor, 156
 $S(X, \mathbf{C})$, category of sheaves, 144
 \mathbf{Sch} , category of schemes, 266
 $\mathbf{Sch}^{\text{aff}}$, category of affine schemes, 266
 \mathbf{Set} , category of sets, 98
 Sheaf , sheafification functor, 148
 $\text{sign}(\sigma)$, signature of a permutation, 81
 $S_{k,n}$, Stiefel manifold, 178
 $S(M)$, symmetric algebra, 77
 \mathbb{S}^n , n -sphere, 175
 $\text{Spec}(R)$, spectrum of a ring, 161
 $S(\varphi)$, symmetric algebra functor, 78

$\text{Spm}(A)$, maximal spectrum, 258
 S_q , symmetric group on q letters, 79
 $S^q(M)$, q -th symmetric power of M , 79
 $S_q(M)$, symmetric tensors, 81
 $\text{Sym}_R(M^q; N)$, symmetric mappings, 79
 sym , symmetrization mapping, 81
 TM , tangent bundle, 193
 Tf , derivative of f , 195
 $T(a, \varrho)$, determining surface of a polycylinder, 235
 T^*M , cotangent bundle, 196
 $T_p M$, geometric tangent space, 183
 $T_p^* M$, cotangent space, 196
 $T_p^{\text{alg}} M$, algebraic tangent space, 185
 $T_p^{\text{geo}} M$, geometric tangent space, 183
 $T_p^{\text{phy}} M$, physicist's tangent space, 184
 $T(M)$, tensor algebra, 73
 $T(\varphi)$, tensor algebra functor, 74
 $T^n(M)$, tensors of n -th level, 72
 \mathcal{T}_M^* , cotangential sheaf of M , 199
 \mathcal{T}_M , tangential sheaf of M , 199
Top, category of topological spaces, 100
 $T_\Phi(X)$, term algebra, 92
 $U(L)$, universal enveloping algebra, 127
 $\text{Unit}(R)$, units of R , 4
 $V(I)$, vanishing set in $\text{Spec}(R)$, 161
 $\mathbf{V}(I)$, vanishing set, 258
 $\mathbf{V}(I)$, zero set of an ideal, 240
 $\mathbf{V}(f)$, vanishing set in the spectrum, 266
 $v_\varphi^{(j)}$, φ -basis, 184
 $\mathbf{Var}_{\mathbb{K}}^{\text{aff}}$, category of affine varieties, 259
 $\mathbf{Var}_{\mathbb{K}}$, category of varieties, 262
 $\mathbf{VB}_{M}^{\mathbb{K}, k}$, category of vector bundles over M , 199
 $\mathbf{Vect}_{\mathbb{K}}^{\text{fin}}$, category of finite-dimensional \mathbb{K} -vector spaces, 102
 $\mathcal{X}(M)$, vector fields on M , 205
 \mathbb{Z}_p , p -adic integers, 118
 $\mathbb{Z}[i]$, Gaussian integers, 5
 $\mathbb{Z}/n\mathbb{Z}$, residue class ring, 5
 $\langle E \rangle$, submodule generated by E , 32
 $\{ \cdot, \cdot, \cdot \}$, Jordan triple product, 94
 $R[X_1, \dots, X_k]$, polynomial ring, 8
 $R[X_1, \dots, X_k]_d$, homogeneous polynomials of degree d , 8
 $R[[X_1, \dots, X_k]]$, formal power series, 7
 $[\cdot, \cdot, \cdot]$, Lie triple product, 94
 $[\cdot, \cdot]$, Lie bracket, 93
 $[\gamma]_p$, equivalence class of curves through p , 183
 $[\gamma_j]_p$, φ -basis, 183
 $[a, b]$, Lie bracket, 127
 $[m]$, equivalence class, 95

$[s]$, germs of a section, 155
 $[x]$, coset, 12
 $[\mathbf{A}, \mathbf{B}]$, functor category, 127
 $\mathbb{K}[V]$, coordinate ring of V , 241
 $\mathbb{K}[V]_f$, localization of $\mathbb{K}[V]$ at f , 252
 $\mathbb{K}[\varphi]$, subalgebra of $\text{End}_{\mathbb{K}}(V)$, 48
 $\sigma \cdot t$, permutation of a tensor, 81
 $g \cdot m$, group action, 96
 $G \circ F$, functor composition, 107
 $X \cong_{\mathbf{C}} Y$, isomorphism in a category, 103
 \cong , isomorphism, 13
 $A_X(U)$, locally constant A -valued functions on U , 143
 E_x , fiber in étalé space, 154
 \mathcal{X}_x , value of \mathfrak{X} at x , 205
 φ_U , presheaf morphism, 141
 φ_x , induced morphism of stalks in x , 145
 s_x , germ of s in x , 145
 $\mathcal{F}_\pi(U)$, section sheaf of an étalé space, 154
 \mathcal{F}_x , stalk in x , 145
 1_X , identity (morphism), 98
 φ^\flat , sheaf morphism, 153
 $\widehat{D}(i)$, diagram presheaves, 150
 $f^{-1}(\mathcal{F}')$, inverse image of a sheaf, 151
 $f^{<-1>}(\mathcal{F}')$, inverse presheaf image, 151

$$\begin{array}{ccc} & F & \\ \text{A} & \xrightarrow{\quad} & \text{B} \\ & \perp \Gamma & \\ & G & \end{array}$$

, adjoint functors, 122

$$\begin{array}{ccc} & F & \\ \text{A} & \xrightarrow{\quad} & \text{B} \\ & \Downarrow \Phi & \\ & F' & \end{array}$$

, natural transformation, 127
 $F \dashv G$, adjoint functors, 122
 $I \trianglelefteq R$, ideal in a ring, 10
 $\mathbf{S} \hookrightarrow \mathbf{C}$, subcategory, 100
 $\mathbf{S} \xhookrightarrow{v} \mathbf{C}$, full subcategory, 100
 $\bigoplus_{\lambda \in \Lambda} M_\lambda$, direct sum of modules, 36
 $M \otimes_R N$, tensor product over R , 58
 $M^{\otimes n}$, tensor power of M , 72
 $\bigotimes_r^s TM$, tensor bundle, 202
 $\bigotimes_r T_p M$, tensor space, 202
 $\bigotimes_r^s \mathcal{T}_M^{(r,s)}$, tensor sheaf of level (r, s) , 203
 $\varphi \otimes \psi$ tensor product of homomorphisms, 59
 $\mathcal{F} \otimes_{\mathcal{O}_X} \mathcal{G}$, tensor product of \mathcal{O}_X -modules, 168
 $\bigotimes_{\lambda \in \Lambda} M_\lambda$, tensor product of the M_λ , 65
 $f(m)$, evaluation of a ring element, 258
 $\mathbf{C}(X)$, endomorphisms of a category, 98
 $\mathbf{C}(X, Y)$, morphisms of a category, 98
 $x + I$, coset, 12

- $\coprod_{i \in I} X_i$, categorical sum, 109
- $\prod_{i \in I} X_i$, categorical product, 108
- $\prod_{\lambda \in \Lambda} M_\lambda$, direct product of modules, 36
- ψ^\sharp , sheaf morphism, 153
- $F \simeq F'$, naturally isomorphic functors, 128
- M_\sim , space of equivalence classes, 95
- \sim_x , stalk relation in x , 145
- M/N , quotient module, 31
- $f_*\mathcal{F}$, direct image of a presheaf, 142
- $f'(p)$, derivative of f , 190
- $\mathbf{A} \times \mathbf{B}$, product category, 103
- $(\tilde{U}, \tilde{\varphi})$, associated chart on TM , 195
- $(\tilde{U}, \tilde{\varphi})$, associated chart on T^*M , 197
- $(\tilde{U}, \tilde{\varphi})$, associated chart on $\bigotimes_r^s TM$, 202
- $\tilde{\mathfrak{X}}$, vector field as differential operator, 206
- $E \times_X E'$, fiber product, 158
- $X_1 \times \cdots \times X_n$, categorical product, 108
- F^{op} , opposing functor, 113
- M^0 , the set $\{\emptyset\}$, 91
- M^G , G -invariants, 125
- M^{adj} , adjoint matrix, 247
- $S^{-1}R$, localization, 17
- \mathbf{C}^{op} , opposite category, 102
- \mathcal{F}^+ , sheafification of \mathcal{F} , 145
- M^\vee , dual module, 85, 107
- φ^\vee , dual morphism, 107
- $m \vee m'$, infimum of m and m' , 112
- \mathcal{F}^\vee , dual module sheaf, 169
- $^\vee$, duality functor, 107
- $|\alpha|$, length of the multi-index α , 8
- \wedge , exterior product for differential forms, 212
- \wedge , exterior product, 84
- $m \wedge m'$, supremum of m and m' , 112
- A**
- Abelian
 - category, 281
- Action, 97
 - trivial, 97
- Addition, 3
 - pointwise, 4
- Adjoint action, 292
- Adjoint matrix, 247
- Affine
 - connection, 303
 - scheme, 266
 - subset, 260
 - variety, 258
- Algebra
 - associative, 71
 - commutative, 71
 - homomorphism, 72
 - over a commutative ring, 71
 - over a ring, 246
 - symmetric, over a module, 76
- Algebraic
 - analysis, 314
 - field extension, 249
 - group, 284
 - set, 240
 - structure, 92
 - topology, 279
- Algebraically
 - closed, 51
 - independent, 247
- Almost-complex structure, 293
 - canonical, 294
 - integrable, 294
- Alternating, 126
 - forms, 210
 - mapping, 86
- Analytic mappings, 174
- Antisymmetric
 - mapping, 86
 - tensor, 81
- Arrows of a category, 98
- Associated
 - chart
 - of the cotangent bundle, 197
 - of the tangent bundle, 195
 - elements of a ring, 20
- Atiyah, Michael (1929–2019), 313
- Atiyah-Singer index theorem, 313
- Atlas, 173
 - oriented, 217, 221
 - for a manifold with boundary, 224
- Automorphism in a category, 103
- B**
- Basis
 - field, 205
 - of a module, 33
 - of a topology, 171
- Bimodule, 60
 - homomorphism, 61
- Boundary of a manifold, 223
- Bundle
 - associated, 309
 - horizontal, 310
 - of k -forms, 210
 - trivial, 306
 - vertical, 310

C

Cartan, Henri (1904–1986), 215
 Cartan-identity, 215
 Categorical
 coproduct, 109
 product, 108
 sum, 109
 Category, 98
 abelian, 281
 derived, 283
 exact, 281
 locally small, 128
 opposite, 102
 preadditive, 281
 small, 102
 Cauchy, Augustin Louis (1789–1857), 229
 integral
 formula, 234
 formula, in several variables, 235
 theorem, 231
 product, on formal power series, 7
 Cauchy-Riemann differential equations, 229
 Center of a polycylinder, 235
 Chain complex, 282
 Characteristic
 class, 313
 polynomial, 51
 Chart
 of a manifold, 172
 neighborhood, 172
 Chern, Shiing-shen (1901–2004), 313
 Chern-Weil
 form, 313
 Chevalley, Claude (1909–1984), 53
 Chinese remainder theorem, 12
 Class of a mapping, 172
 Clausen, Dustin, 283
 Closed differential form, 213
 Co-adjoint action, 292
 Cofunctor, 107
 Cokernel of a morphism, 281
 Colimit, 113
 Commutator product, 127
 Complex structure, 293
 Composition of morphisms, 98
 Condensed mathematics, 283
 Congruence relation, 95
 Connected, 179
 component, 180
 topological space, 173
 Connection, 310
 Constant sheaf, 143
 Contraction of a differential form, 215
 Contravariant functor, 107

Coordinate
 neighborhood, 172
 ring of an algebraic set, 241
 Coprime, 20
 Coproduct, 109
 Coset, 5, 96
 Cotangent
 bundle, 196
 sheaf, 199
 space, 196
 Covariant
 derivative, 303
 functor, 107
 Curvature, 305
 form, 312
 Cyclic
 module, 26
 vector, 50

D

Darboux, Gaston (1842–1917), 294
D-cocone, 113
D-colimit, 113
D-cone, 112
 Degree
 of a differential form, 210
 functor, 19
 of a polynomial, 8
 De Rham cohomology, 281
 De Rham, Georges (1903–1990), 209
 Derivation, 185
 of an algebra, 206
 Derived category, 283
 Determinant, 86
 Determining surface of a polycylinder, 235
 Diagram
 commutative, 35
 of the form **I**, 112
 Differentiable structure, 174
 Differential
 form, 85, 199, 210
 of a mapping, 190
 operators
 with constant coefficients, 9
 with smooth coefficients, 9
 Dimension of a manifold, 173
 Direct
 image of presheaves, 142
 image of sheaves, 150
 product of modules, 37
 sum of modules, 37
 Distributions, 315
 Distributivity, 3

Division ring, 6
 Division with remainder in a Euclidean ring, 19
 Divisor, 20
D-limit, 113
 Dolbeault cohomology, 297
 Dolbeault, Pierre (1924–2015), 297
 Dual module, 107

E

Ellipticity, 316
 Epimorphism, 103
 Equationally defined class, 92
 equivalence of categories, 134
 Essentially surjective, 134
 étalé space, 153
 Euclid (ca. 325–265 BC), 19
 Euclidean
 ring, 19
 space, 290
 Evaluation of a ring homomorphism, 49
 Exact
 category, 281
 Exterior
 algebra, 83
 derivative, 212
 power of a module, 86

F

Factor module, 31
 Faithful functor, 131
 Fiber
 bundle, 306
 product, 115
 of a vector bundle, 198
 Field
 of fractions, 15
 Finitely
 generated ideal, 255
 generated *R*-algebra, 246
 Finite *R*-algebra, 246
 Floer, Andreas (1956–1991), 294
 Floer homology, 294
 Foliation, 286
 Formal
 deformation, 301
 star product, 301
 Formality theorem, 301
 Frölicher, Alfred (1927–2010), 312
 Frölicher-Nijenhuis bracket, 312
 Frame bundle
 of a vector bundle, 307

Free group action, 307
 Fubini, Guido (1897–1943), 296
 Fubini-Study metric, 296
 Full
 functor, 131
 subcategory, 99
 Fully faithful functor, 132
 Functor
 category, 127
 contravariant, 107
 covariant, 107
 fully faithful, 132
 opposite, 113
 Functoriality, 59
 Fundamental theorem
 of algebra, 52
 of arithmetic, 21
 of calculus, 225
 on finitely generated abelian groups, 45

G

G-action, 97
 Galois, Evariste (1811–1832), 279
 Galois theory, 279
 Gauß, Carl Friedrich (1777–1855), 5
 Gaussian integers, 5
 Geodesic, 291, 304
 Germ of a presheaf, 145
 Gluing, 116
 data, 144
 of sheaves, 144
G-module, 125
 Gradient, 291
 symplectic, 292
 Grassmann, Hermann (1809–1877), 178
 Grassmann manifolds, 178
 Greatest common divisor, 20
 Group, 93
 algebraic, 284
 object, 284
 structure, 93
 Groupoid, 286
G-set, 96
G-structure, 308

H

Hamilton, William Rowan (1805–1865), 7
 equations of motion, 292
 function, 291
 vector field, 292, 299
 Harmonic, 297
 Hermite, Charles (1822–1901), 295

- Hermitian structure, 296
 - Hilbert, David (1862–1943), 246
 - basis theorem, 256
 - Nullstellensatz, 246
 - Hodge structure, 298
 - Hodge, William (1903–1975), 297
 - Hörmander, Lars (1931–2012), 314
 - Holonomy group, 312
 - Homogeneous polynomial, 8
 - Homological algebra, 280
 - Homology, 279
 - Homomorphism
 - of local rings, 164
 - of modules, 29
 - of rings, 9
 - Homotopy, 279
 - category, 283
 - of chain complexes, 283
 - Hopf, Heinz (1894–1971), 291
 - Horizontal
 - bundle, 310
 - form, 313
 - Hyperfunctions, 314
 - Hyperplane, 177
- I**
- Ideal, 10
 - in an algebra, 72
 - finitely generated, 255
 - generated by a set, 11
 - maximal, 17
 - prime, 17
 - Identity
 - as functor, 104
 - of a group, 5
 - in a ring, 6
 - I -holomorphic curve, 294
 - Image of a module homomorphism, 30
 - Inclusion of a categorical sum, 109
 - Independence in a module, 33
 - Induction, 125
 - Inductive
 - limit, 116
 - system, 116
 - Infimum in a partially ordered set, 112
 - Integrable sub-bundle, 312
 - Integral domain, 14
 - Integral of a differential form, 222
 - Internal direct sum, 40
 - Invariance of basis length, 35
 - Inverse
 - additive, 6
 - image
 - of an étalé space, 159
 - of a sheaf, 151
 - multiplicative, 6
 - Irreducible
 - components of an algebraic set, 272
 - element of an integral domain, 23
 - topological space, 272
 - Isomorphism
 - of algebraic sets, 242
 - of categories, 103, 134
 - of modules, 29
 - of rings, 10
 - Isomorphism theorem for modules, 32
- J**
- Jacobi, Carl-Gustav (1804–1851), 93
 - Jacobi identity, 93
 - Jordan
 - algebra, 94
 - block, 51
 - identity, 94
 - normal form, 52
 - triple system, 94
 - Jordan, Camille (1838–1922), 51
 - Jordan-Chevalley decomposition, 53
 - Jordan, Pascual (1902–1980), 94
- K**
- Kähler, Erich (1906–2000), 296
 - Kähler manifold, 296
 - Kashiwara, Masaki (*1947), 314
 - Kawai, Takahiro (*1945), 314
 - Kernel
 - of a module homomorphism, 30
 - of a morphism, 281
 - of a ring homomorphism, 10
 - Kontsevich, Maxim (*1964), 301
 - \mathbb{K} -rational points, 274
 - \mathbb{K} -valued points, 274
- L**
- Laplace operators, 297
 - Laplace, Pierre-Simon (1749–1827), 297
 - Lattice, 112
 - Laurent, Pierre (1813–1854), 271
 - Laurent polynomials, 271
 - Leading coefficient, 8
 - Left ideal, 26
 - Level
 - of a tensor bundle, 202
 - of a tensor field, 202

Levi-Civita connection, 305
 Levi-Civita, Tullio (1873–1941), 305
 Lie, Sophus (1842–1899), 93
 algebra, 93
 of vector fields, 208
 derivative of differential forms, 215
 groups, 284
 supergroups, 285
 triple system, 94
 Lift of a G -structure, 309
 Limit, 113
 direct, 116
 inductive, 116
 inverse, 116
 projective, 116
 Line
 bundle, 306
 integral, 229
 Linear combination in a module, 32
 Line integral
 complex, 231
 Local
 ring, 163
 ring homomorphism, 164
 trivialization, 306
 Locally
 closed, 263
 finite covering, 220
 ringed space, 163
 Lorentz, Hendrik (1853–1928), 290
 manifold, 290
 metric, 290

M
 Manifold, 171
 with boundary, 223
 differentiable, 171
 oriented, 218
 Microdifferential operators, 316
 Microfunctions, 317
 Minimal polynomial, 49
 Minkowski, Hermann (1864–1909), 290
 Minkowski space, 290
 Module, 26
 cyclic, 26
 dual, 107
 finitely generated, 33
 free, 33
 homomorphism, 29
 left, 26
 right, 28
 sheaf, 166
 Möbius, August Ferdinand (1790–1868), 218

Möbius strip, 218
 Monic polynomial, 8
 Monomorphism, 103
 Morphism, 160, 164
 of a category, 98
 of étalé spaces, 156
 of locally ringed spaces, 164
 of manifolds, 172
 of module sheaves, 166
 of presheaves, 141
 of ringed spaces, 160
 of sheaves, 144
 of vector bundles, 199
 Multi-index, 7, 8
 Multilinear
 form, 56
 mapping, 56
 Multiplication, 3
 pointwise, 4
 Multivector field, 299

N
 n -ary operation, 91
 Natural
 isomorphism, 128
 transformation, 127
 Newlander, August, 294
 Nijenhuis, Albert (1926–2015), 312
 Nijenhuis tensor, 294
 Nirenberg, Louis (1925–2020), 294
 Noether
 normalization, 248
 property, 255
 ring, 255
 spaces, 257
 Noether, Emmy (1882–1935), 255
 Noether, Max (1844–1921), 248
 Non-commutative geometry, 286
 Non-standard analysis, 314
 Normalized polynomial, 8
 Normal subgroup, 96
 Nullstellensatz, 246

O
 Object of a category, 98
 \mathcal{O}_X -module
 dual, 169
 free, 169
 locally free, 169
 of finite type, 169
 1-form, 85, 198

- operation
 - n-ary, 91
- Opposite
 - category, 102
 - functor, 113
- Orbifold, 286
- Orbit, 97
- Orientation, 222
 - bundle, 306
 - induced, 225

- P**
- p*-adic
 - integers, 118
 - numbers, 118
- Paracompact, 220
- Parallel transport, 304
- Parametrization of a manifold, 172
- Partition of unity, 218
 - subordinate to a cover, 218
- Perelman, Grigori (*1966), 305
- Poincaré, Henri (1854–1912), 181
- Pointwise
 - addition, 4
 - multiplication, 4
- Poisson, Siméon (1781–1840), 289
 - bracket, 298
 - manifold, 298
 - tensor, 299
- Polycylinder, 235
- Polyfold, 295
- Polynomial, 8
 - division, 18
 - function, 241
 - homogeneous, 8
 - mapping, 242
 - normalized, 8, 49, 52
- Polyradius, 235
- Power series, 7
- Preadditive, 281
- Presheaf, 140
- Prevariety, 255, 258
- Prime element, 20
- Principal
 - fiber bundle, 307
 - ideal, 11
- Principal ideal
 - domain, 19
- Product
 - categorical, 108
 - category, 103
 - of manifolds, 177
 - in a module, 33
 - topology, 109
- Projection
 - canonical, on submodules, 40
 - of a categorical product, 108
 - stereographic, 175
- Projective
 - limit, 116
 - space, 178
 - over a ring, 272
 - system, 116
- Pseudo-differential operators, 317
- Pseudo-holomorphic curve, 294
- Pseudo-Riemannian
 - manifold, 290
 - metric, 290
- Pullback, 115
 - of differential forms, 216
 - of a fiber bundle, 310
- Pushout, 116

- Q**
- Quaternions, 7
- Quotient
 - group, 26
 - module, 31
 - ring, 12
 - structure, 95

- R**
- Radical, 245
 - ideal, 245
- Radius of a polycylinder, 235
- Rank
 - of a module, 36
 - of a vector bundle, 198
- Rational normal form of a linear mapping, 50
- Reduced, 241
- Reduction of the structure group, 308
- Refinement of a cover, 220
- Regular function, 259
- Regular mapping
 - of affine varieties, 260
 - of prevarieties, 260
- Residue
 - class, 5
 - ring, 5
 - field of a local ring, 163
- Restriction, 140
- Riemann, Bernhard (1826–1866), 229
 - surface, 180

Riemannian

- curvature, 305
- manifold, 290
- metric, 290

Right ideal, 30

Ring, 3

- commutative, 4
- Euclidean, 19
- extension, 29
- factorial, 21
- with identity, 4
- Noetherian, 255

Ringed space, 160

Rinow, Willi (1907–1979), 291

S

Sato, Mikio (1928–2023), 314

Scheme, 266

- affine, 266
- over a ring, 274

Scholze, Peter (*1987), 283

Schouten, Jan (1883–1971), 299

Schouten-Nijenhuis bracket, 299

Section

- of a fiber bundle, 306
- functor, 149
- of a presheaf, 140
- sheaf, 154
- of a vector bundle, 198

Separated prevariety, 262

Sheaf, 143

- of sections, 199

Short exact sequence, 43

Singer, Isadore (1924–2021), 313

Singular set, 317

Skew-field, 6

Skew-symmetric

- mapping, 86

Skyscraper sheaf, 144

Small category, 102

Smooth function, 174

- on singular sets, 223

Solution curve, 27

Spectrum of a ring, 161

Sphere, 175

Spin structure, 309

Stack, 286

Stalk

- functor, 145
- of a presheaf, 145

Standard open set, 252

- of an affine scheme, 266

Stiefel, Eduard (1909–1978), 178

Stiefel manifold, 178

Stokes, George Gabriel (1819–1903), 225

theorem, 225

Structure

- algebraic, 92
- constants of a Lie algebra, 300
- group of a principal fiber bundle, 307
- local, 137
- sheaf of a ringed space, 160

Study, Eduard (1862–1930), 296

Subalgebra, 72

Subcategory, 99

- full, 99

Submanifold, 178

Submodule, 29

- generated by a set, 32

Subring, 29

Sum

- categorical, 109
- topological, 110

Supremum in a partially ordered set, 112

Symbol, 316

Symmetric

- mapping, 79
- power of a module, 79
- tensor, 81

Symplectic

- coordinates, 293
- foliation, 300
- form, 291
- gradient, 292
- manifold, 291
- topology, 294
- vector space, 293

T

Tangent

- bundle, 193
- functor, 196
- sheaf, 199
- space
 - algebraic, 185
 - geometric, 183
 - physicist's, 184
- vector, 184

Tensor, 199

- algebra over a module, 72
- bundle, 202
- field, 75, 202
- product
 - of algebras, 110
 - of modules, 57, 64
 - of \mathcal{O}_X -modules, 168

Terminal object of a category, 284

Theorem

of Darboux, 293

of de Rham, 283

of Hodge, 297

of Hopf-Rinow, 291

of Newlander-Nirenberg, 294

Thurston, Bill (1946–2012), 305

Topological sum, 110

Transition function, 201, 308

Transitive group action, 307

Type of a manifold, 171

U

Unital ring homomorphisms, 122

Unit of a ring, 4

Universal algebra, 32

Universal property

direct sum of modules, 37

exterior algebra, 84

free modules, 34

product of modules, 37

symmetric algebra, 76

tensor algebra, 72

tensor product, 57, 64

Universe, 128

V

Vanishing ideal, 241

Variety, 262

affine, 258

projective, 265

quasiprojective, 265

Vector

bundle, 198

field, 27, 203, 205

subbundle, 209

Vector-valued differential forms, 312

Vertical

bundle, 310

vector, 310

Volterra, Vito (1860–1940), 181

W

Wavefront set, 317

Weak solution, 316

Weil, André (1906–1998), 313

Weyl algebra, 9

Weyl, Hermann (1885–1955), 9

Y

Yoneda, Nobuo (1930–1996), 130

embedding, 132

lemma, 130

Z

Zariski, Oscar (1899–1986), 161

Zariski topology, 161, 240, 242, 258

Zero

of an additive group, 5

divisor, 14

object, 281

Zorn, Max (1906–1993), 33

Zorn's lemma, 33