Framework for liquid crystal based particle models

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***Abstract*—Long-range e.g. Coulomb-like interactions for (quantized) topological charges in liquid crystals are observed experimentally, bringing open question this article is exploring: how far can we take this resemblance with particle physics? Uniaxial nematic liquid crystal of ellipsoid-like molecules can be represented using director field $\vec{n}(x)$ of unitary vectors. It has topological charge quantization: integrating field curvature over a closed surface $\mathcal{S}$, we get 3D winding number of $\mathcal{S} \to S^2$, which has to be integer - getting Gauss law with built-in missing charge quantization if interpreting field curvature as electric field. This article proposes a general mathematical framework, combining Landau-de Gennes and skyrmion models, to extend this similarity with particle physics to biaxial nematic, getting surprising agreement with the Standard Model. Specifically, recognising intrinsic twist of uniaxial nematic e.g. to propagate angular momentum also in this direction, allows hedgehog configurations with one of 3 distinguishable axes: having the same topological charge, but different energy/mass - getting similarity with 3 leptons (also neutrinos, quarks as living in 3D). Vacuum dynamics extends electromagnetism from 3D rotation dynamics, with Klein-Gordon-like equation for twists corresponding to quantum phase. If extending to 4D field with 0th axis, vacuum dynamics to SO(1,3) by boosts, we also get second set of Maxwell equations for GEM (gravitoelectromagnetism) approximation of general relativity.**

Keywords: field theory, topological solitons, liquid crystals, Landau-de Gennes model, skyrmions, long-range interaction, EM, gravitomagnetism, particle physics, Standard Model

I. Introduction

While particles are usually treated in perturbative way, this is only approximation of full nonperturbative QFT: asking for field configurations, to finally consider their Feynman ensembles in 2nd quantization. This fundamental question is now asked nearly only for baryons in lattice QCD [1], while we should try to understand field structures of all particles.

Single electromagnetic field from nonperturbative Feynman ensemble naively has two issues we should repair:

1) missing **charge quantization**: Gauss law should only return integer multiplicities of e (+confined fractional),
2) missing **regularization**: infinite energy of electric field of charge, bounded by 511keV released in annihilation.

In liquid crystals there are experimentally realized quantized topological charges with long-range interactions, e.g. resembling quadrupole-quadrupole [2], dipole-dipole [3], Coulomb [4] or even stronger [5] interactions - suggesting resolution to both problems on classical field level, for example using Faber's approach ([6], [7], [8], [9], [10]) this article extends on - summarized in Figure 1 (gathered materials[1]):

1) Interpret curvature of e.g. field vector $\vec{n}$ as electric field, making **Gauss law counts its topological charge**,
2) **Use Higgs-like potential** e.g. $V = (\|\vec{n}\|^2 - 1)^2$, allowing for e.g. $\vec{n} \to 0$ regularization of singularities.

[1] https://github.com/JarekDuda/liquid-crystals-particle-models/

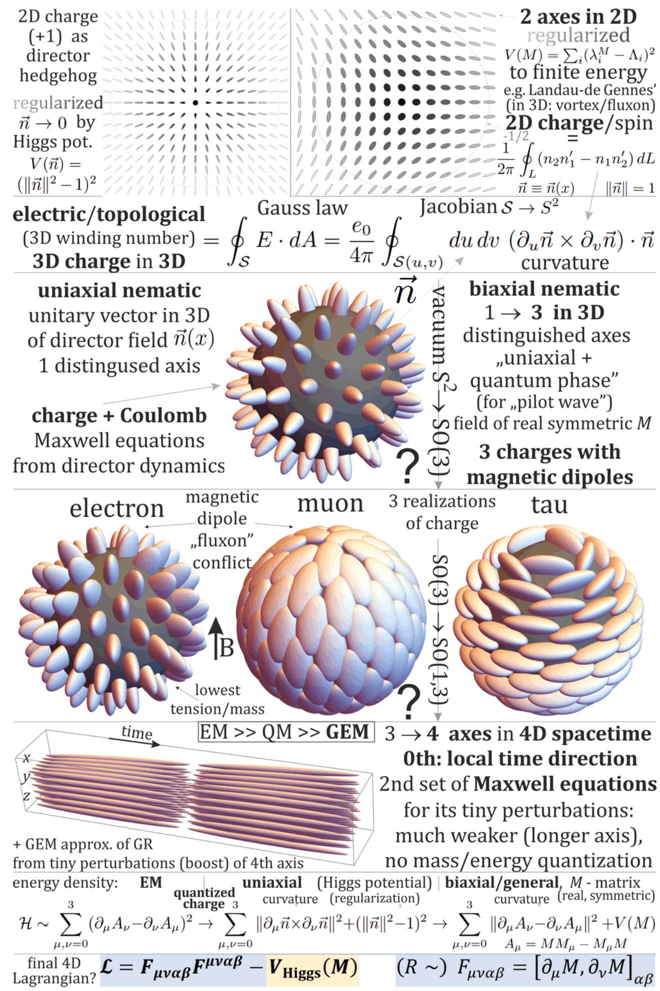

Figure 1. Unitary vector $\vec{n} \equiv \vec{n}(x)$ "director" field has **quantization of topological charge**, for which in liquid crystal experiments there were obtained **long-range interactions** - interpreting curvature as electric field, we can get **Maxwell equations** for their dynamics. Living in 3D suggests to extend it to 3 distinguishable axes representing rotating object - realized e.g. in biaxial nematic, getting **3 types of hedgehog** with the same charge, but different energy (like 3 leptons), also magnetic singularity due to the hairy ball theorem. We model such ellipsoid field like stress-energy tensor: with field of real symmetric matrices $M(x) \equiv M = ODO^T$ (orthogonal $OO^T = I$, $D = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)$), which **prefers some shape as set of eigenvalues due to Higgs-like potential** e.g. $V(M) = \sum_i (\lambda_i - \Lambda_i)^2$ for some fixed model parameters: $\Lambda_1 > \Lambda_2 > \Lambda_3$ - allowing to regularize singularity (discontinuity in the center) to finite energy as in top diagrams. Reminding that we live in 4D spacetime suggests to add 0th time axis as the longest ($D = \mathrm{diag}(\lambda_0, \lambda_1, \lambda_2, \lambda_3)$) - having the strongest tendency to be aligned in parallel, acting as local time direction. Mass/energy should enforce tiny perturbations as curvature of this time axis, with dynamics given by **second set of Maxwell equations for GEM** approximation of the general relativity, for boosts in vacuum extension from SO(3) to SO(1,3) Lorentz group.

Considering Feynman ensembles of such fields allows to resolve infinite energy issue before 2nd quantization: through field regularization corresponding to later renormalization -

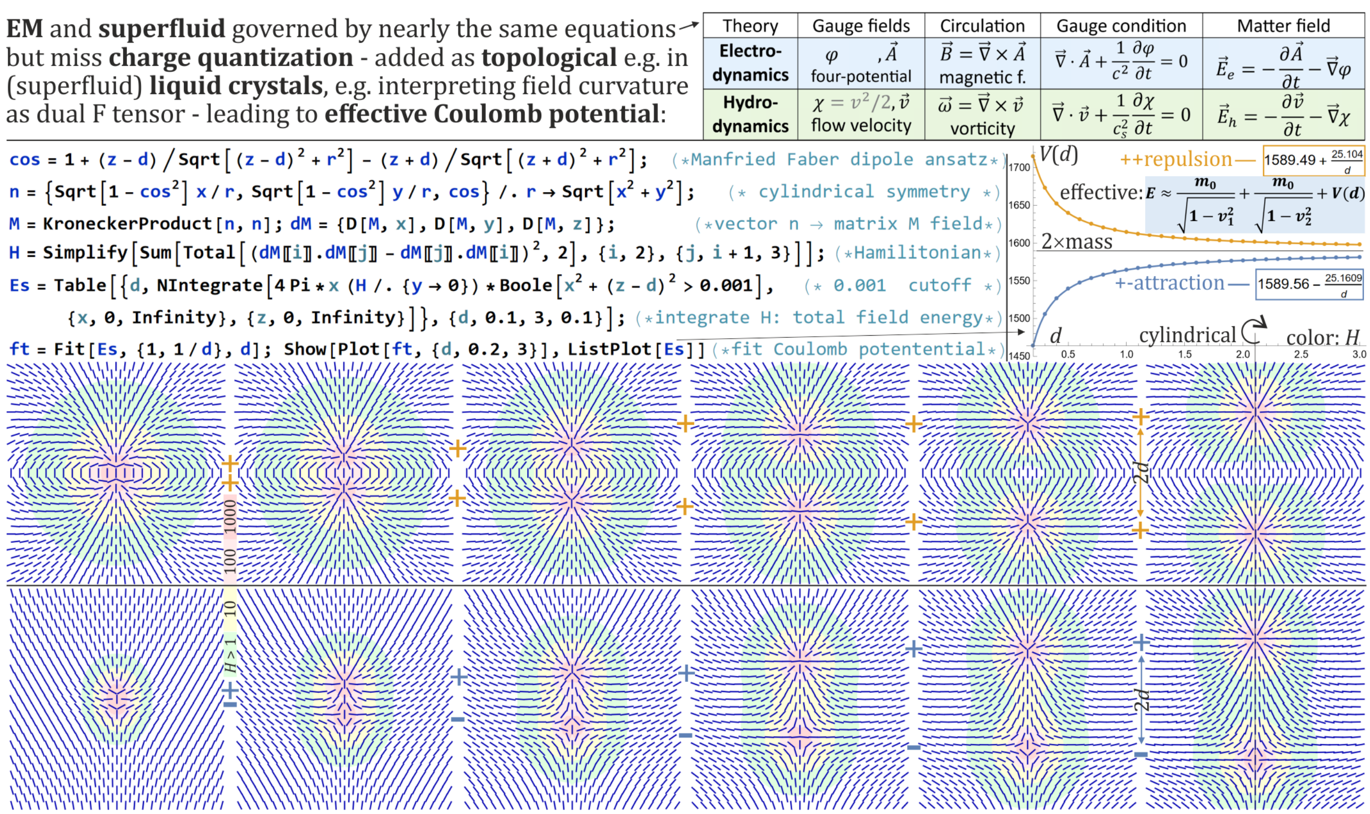

Figure 2. EM-hydrodynamics analogy [11] (top) missing charge quantization in Gauss law, added as topological - below calculation of Coulomb effective potential with shown Mathematica source (extended in GitHub). For "+-" or "++" pair of topological charges, there was postulated ansatz [7]: cylindrically symmetric configuration of unitary vector field n in agreement with electric field of two elementary charges (in GitHub verified satisfaction of variation equation). Such various distance configurations are shown with visualized H density. Seen as uniaxial nematic it corresponds to $M = nn^T$ matrix field, as discussed here with static energy density $H = \sum_{1\leq i<j\leq 3} \|[\partial_i M, \partial_j M]\|^2$. Integrating this energy density with cutoff around two singularities, there was numerically obtained $E(d) \approx 1590 \pm 25/d$ distance-energy dependence as in Coulomb law (shown values and fit). Finally the two singularities are to be regularized with Higgs-like potential e.g. $(1 - \|n\|^2)^2$, due to Lorentz invariance leading to $m_0 \to m_0/\sqrt{1-v^2}$ energy scaling. To avoid infinite energy of singularities in charges there was used cutoff above, which finally should be replaced with regularization by Higgs-like potential - as discussed and calculated in [10], leading to deformations of Coulomb interaction in tiny distances in agreement with the running coupling effect.

allowing to understand the difference between e.g. electron's field configuration and of (infinite energy) perfect point charge.

Liquid crystals use ellipsoid-like molecules, which if cylindrically symmetric (uniaxial nematic) can be represented with director field $\vec{n} \equiv \vec{n}(x)$ of unitary vectors, this way allowing for (quanitzed) topological charges e.g. hedgehog-like configurations. They get long-range e.g. **Coulomb-like interaction** as in Figure 2: total energy of the field (as integrated energy density: Hamiltonian) for two charges in various distances behaves as in Coulomb potential.

Such 3D topological charge as 3D winding number [12] of $\vec{n}$ restricted to $\mathcal{S} \to S^2$, can be calculated by integrating over closed surface $\mathcal{S}$ the Jacobian of this function - which turns out curvature of this field. Therefore, interpreting (e.g. vector) field curvature as electric field, we get **Gauss law with built-in charge quantization as topological**.

The center of such quantized topological charge naively has field discontinuity, which would mean infinite energy - like of electric field around point charge. To prevent it, we could make a cutoff as in Fig. 2, in liquid crystals we can imagine there is no molecule in the center of e.g. hedgehog. However, for a field there should be value everywhere - we need to **regularize** it, deform to finite energy, e.g. at most 511 keVs for electron - released in annihilation. There can be used $V(\vec{n}) = (\|\vec{n}\|^2 - 1)^2$ Higgs potential: preferring unitary vectors, also allowing to

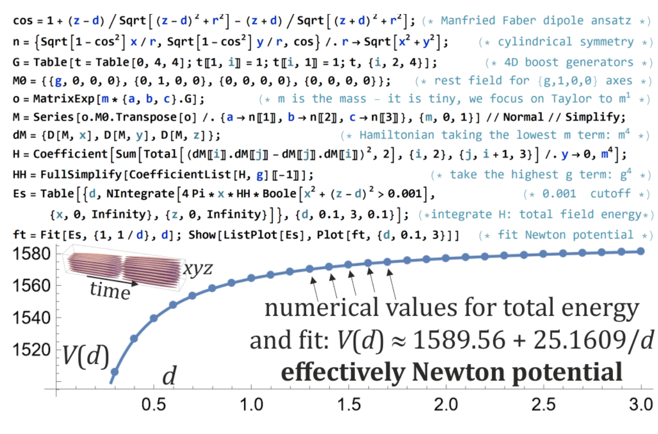

Figure 3. Example of simplified calculation of Newton effective potential (in GitHub) - analogously as in Fig. 2, but this time instead of large spatial rotations, using tiny boosts of 0th time axis for gravity (no mass quantization). Spherically symmetric curvature sources would have increased energy with reduced distance, hence to get attraction there was used dipole ansatz for microscopic scenarios like pair creation, hopefully averaging to spherically symmetric gravity. Final calculations will need further work.

deform e.g. to $\vec{n} = 0$ in the center of singularity to prevent infinity. Massless dynamics of this vacuum (Goldstone bosons) can be chosen to resemble electromagnetism by interpreting curvature as electric field. Experimental consequence of such

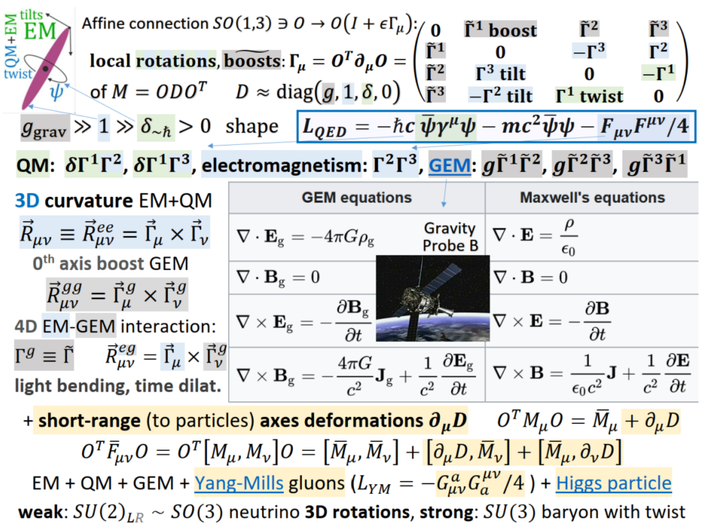

Figure 4. Interaction summary: long range from vacuum dynamics: of SO(1,3) Lorentz group, having 3 different energy scales due to $g \gg 1 \gg \delta > 0$ vacuum eigenvalues of M - δ low energy twists for U(1) phase evolution energy contribution e.g. in QED Lagrangian, 1 for EM from S^2 spatial tilts/rotations, and g for gravity from boosts. As we use Lorentz invariant Lagrangian, Coulomb force implies magnetism, Newton gravitomagnetism (GEM) - confirmed by Gravity Probe B (diagram). EM-GEM interaction e.g. slows down EM propagation in gravitational field - leading to gravitational time dilation, and lensing through Fermat principle. Additionally, there are degrees of freedom deforming these $(g, 1, \delta, 0)$ eigenvalues preferred by Higgs-like potential, activated mainly near particles for regularization, which resemble e.g. Yang-Mills Lagrangian contributions.

regularization to finite energy is deformation of Coulomb interaction in tiny distances, which agrees with known **running coupling** effect [10]. For regularization of a more general field, we need potential with topologically nontrivial minimum, e.g. S^2: $\|\vec n\| = 1$ for uniaxal nematic, SO(3) for biaxial nematic, like in Landau-de Gennes model [13], further extended to SO(1,3) to add gravity as in Fig. 4.

Above director field n does not recognize twist of this vector, hence not need to conserve such angular momentum. To repair it, we can use generic objects in 3D with 3 distinguishable axes, SO(3) rotations - like molecules in experimentally challenging biaxial nematic liquid crystals ([13], [14], [15]). Similarly to Landau-de Gennes model, we will represent such unknown rotation recognizing deeper field, using field of real symmetric matrices $M(x) \equiv M = ODO^T$ (orthogonal $OO^T = I$, $D = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)$): which due to potential prefers some fixed set of eigenvalues $\Lambda_1 > \Lambda_2 > \Lambda_3$, but allows for their (λ_i) deformation/regularization to prevent infinite energy in singularity. For example using $V(M) = \sum_i(\lambda_i - \Lambda_i)^2$ Higgs-like potential, as in top-right diagram in Fig. 1. Instead of integer, this 2D configuration has 2D topological charge +1/2 due to symmetry: that ellipsoid rotated by π is the same ellipsoid, what also agrees with quantum rotation operator: "rotating spin s particle by ϕ angle, rotates phase by $s\phi$" - suggesting to **interpret 2D topological charge as spin, 3D as electric charge** and use symmetric $n \equiv -n$ field to allow spin 1/2. Additionally, fluxons in superconductor are well known 2D topological charges - like spin associated with magnetic field. Also, voricity works analogously to magnetic field e.g. in Aharonov-Bohm ([16], [17]) and Zeeman effect [18].

Distinguishing two types of rotation (of Λ shape): twist

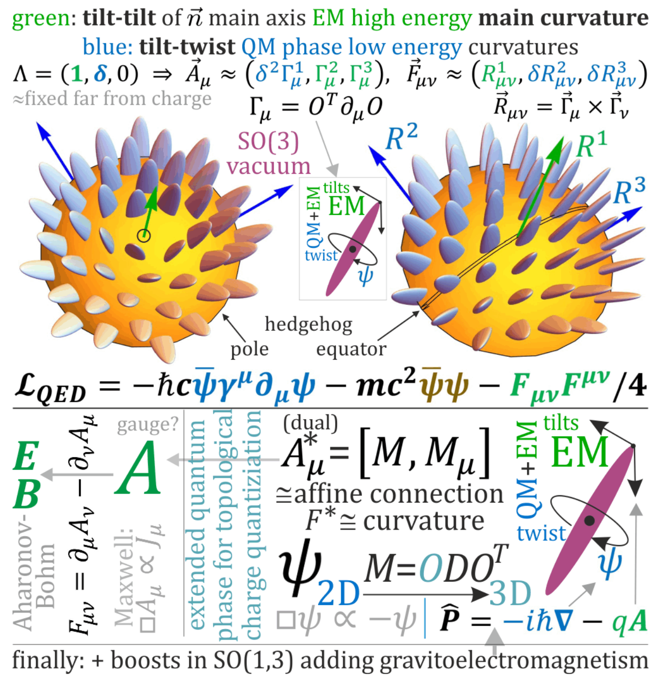

Figure 5. **Top**: parts of hedgehog of the longest $\vec n$ **main axis** of biaxial-nematic-like field. This axis can **tilt** in two directions already in uniaxial nematic, in biaxial there are additionally recognized its **twists** - here tilts correspond to high energy EM dynamics, twists to low energy QM phase dynamics. Such local rotation is affine connection $\Gamma_\mu = O^T(\partial_\mu O)$: antisymmetric matrix we can interpret as local rotation vector $\vec\Gamma_\mu := ((\Gamma_\mu)_{32}, (\Gamma_\mu)_{13}, (\Gamma_\mu)_{21})$. It corresponds to A_μ four-vector weighted with shape $\lambda_i \approx \Lambda_i$ (far from charge fixed by potential e.g. $V = \sum_i(\lambda_i - \Lambda_i)^2$) distinguishing high energy tilts from low energy twists. Curvatures $\vec R_{\mu\nu} = \vec\Gamma_\mu \times \vec\Gamma_\nu$ after weighting with shape become dual $F^*_{\mu\nu}$ tensor: containing high energy tilt-tilt component $R^1_{\mu\nu}$ corresponding to EM, and low energy tilt-twist $R^2_{\mu\nu}, R^3_{\mu\nu}$ corresponding to QM phase like in QED-like Lagrangian. **Bottom**: due to Aharonov-Bohm-like arguments, there is belief that A four-vector is more fundamental than E, B fields, however, it leaves gauge freedom. It also allows for non-integer charges like half-electron - to prevent that, **there is postulated more fundamental field M which (quantized) topological charge is calculated in Gauss law**. This deeper field can be seen as extended quantum phase: from low energetic evolution of U(1) quantum phase (twists), to SO(3) evolution (+tilts) including also electromagnetism with built-in (topological) charge quantization, getting simple EM+QM unification (+GEM with 4D field, SO(1,3) vacuum).

of biaxial nematic, and two tilts as in Fig. 5, will allow to assign them different energy scales. Going to 4D, we can imagine additional much longer 0th time axis undergoing tiny perturbations - naively rotations of SO(4), but Lorentz invariance suggests to use SO(1,3) with boosts instead. Finally the discussed approach allows to **unify 3 types of vacuum dynamics** (far from particles/singularises):

- **electromagnetism** (EM) of relatively high energy - governed by Maxwell (wave-like) equations, corresponding to tilts, already in uniaxial nematics (e.g. Coulomb: [4]),
- **quantum phase** ($\arg(\psi)$) evolution of much lower energies ($\hbar c$ in QED Lagrangian) - corresponding to twists, governed by Klein-Gordon-like (wave-like) equation,
- **gravitoelectromagnetism** (GEM)[2] approximation of general relativity - 2nd set of (wave-like) Maxwell equations for tiny perturbations (boosts) of 0th time axis.

Such field of 3 distinguishable axes allows to construct hedgehog-like configuration of one of 3 axes as in Fig. 1 - they have the same 3D topological charge acting as electric

[2] GEM: https://en.wikipedia.org/wiki/Gravitoelectromagnetism

charge, but require different regularization/deformation - should have different mass/energy, resembling 3 leptons. Additionally, the hairy ball theorem [19] says that we cannot continuously align such axes on the sphere - requiring additional spin-like singularities resembling fluxons, which should correspond to magnetic dipole moment of leptons. In particle physics three families are very common: for leptons, neutrinos, quarks - here as just consequence of living in 3D.

The difference between uniaxial and biaxial nematic can be imagined as recognizing intrinsic rotation (referred as twist) of elongated molecule - here adding to electromagnetism (referred as tilts) single low energy vacuum degree of freedom (crucial for angular momentum conservation), which seems to correspond to quantum phase, pilot wave. Quantum mechanical phase evolution $\exp(iEt/\hbar)$ in relativistic e.g. Dirac equation requires $E = mc^2$, leading to caused by mass itself $\omega = mc^2/\hbar$ frequency oscillations already for resting particles, for example of neutrinos between flavors. For electron it is $\sim 10^{21}$ Hz and was originally postulated by Louis de Broglie, is also called Zitterbewegung, and has multiple experimental confirmations, e.g. direct [20], or in Bose-Einstein condensate analogs [21]. Hence the discussed corresponding configurations should enforce such periodic process, like spin precession [22] or rather rotation (twist) of this additional degree of freedom - leading to "pilot wave" coupled with such electron. For 3D case there is obtained Klein-Gordon-like equation, but missing gravitational mass - added in 4D considerations: due to spacetime signature, there appear subtle negative energy Hamiltonian terms (shown further in Fig. 8), exactly as required to propel such oscillations.

Analogous view on wave-particle duality has also allowed for experimental realizations of hydrodynamical analogs of many quantum phenomena with **hydrodynamical wave-particle duality objects**: walking droplets. For example double slit interference [23] (corpuscle travels one trajectory, its coupled wave travels all - affecting corpuscle trajectory), unpredictable tunneling [24] (depending on complex history of the field), Landau orbit quantization [25] (coupled wave has to become standing wave for resonance condition in analogy to stationary Schrödinger equation for orbital quantization), Zeeman-like splitting [18] of such quantized orbits (using Coriolis force as analogue of Lorentz force, vorticity as magnetic field ([16])), double quantization [26] (in analogy to (n, l) for atomic orbitals), recreating quantum statistics with averaged trajectories [27], Elitzur-Vaidman bomb testing [28], or Bell violation [29]. There are also known hydrodynamical analogs for Casimir [30] and Aharonov-Bohm effects ([16], [17]). For fluxons as 2D topological charges in superconductor there was experimentally realized e.g. interference [31], tunneling [32] and Aharonov-Bohm [33] effect.

Maximal Entropy Random Walk (MERW) also suggests quantum-like statistics for objects undergoing complex dynamics. Standard diffusion models turn out to only approximate the (Jaynes) maximal entropy principle, necessary for statistical physics models - lacking Anderson-like localization, observed also for neutrons [34]. MERW finally does this optimization, getting stationary probability distribution exactly as quantum ground state, with its localization properties ([35], [36], [37], [38]). E.g. for $[0, 1]$ range standard diffusion predicts uniform $\rho = 1$ stationary probability distribution, while QM and MERW predict localized $\rho \propto \sin^2$.

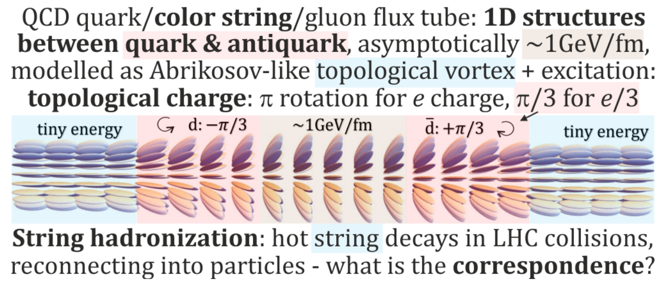

Figure 6. The central object of QCD is quark string, also called color or gluon flux tube. It connects quark-antiquark pairs, and has asymptotically linear $\sim$ 1 GeV/fm energy density, confining the quarks - which in pairs can be created/annihilated on such string. This 1D structure is very stable, and often modeled as topological Abrikosov vorteices [40], fluxons, in liquid crystals called disclinations. To add quarks as fractional charge excitations of such vortex, interpreting electric charge as topological here, we can do it by inward/outward field rotation: by π would be elementary charge, hence for quarks there should be used fractional - as in above diagram, leading to conflict between them, asymptotically with linear energy/length as required. Gauss law for a region cutting such string, has (regularized) singularity in this point. As shown further in Fig. 11, 12, tendency to make such rotation is required by suggested baryon structure. Additionally, one of two hadronization models used e.g. in LHC collider (http://www.scholarpedia.org/article/Parton_shower_Monte_Carlo_event_generators) is string hadronization [41] - assuming creation of such hot string in collision, and analyzing results of its decay through reconnections. Therefore, to understand field configurations of particles, we should search for correspondence between topological vortex decay, and collision observations - constraining toward the discussed approach.

As we live in 4D spacetime, it is tempting to extend from 3 to 4 distinguishable axes by just going from 3×3 to 4×4 real symmetric matrix field M - like the stress-energy tensor, for which M might be microscopic extension. This way the field recognizes not only SO(3) rotations, but also boosts going to SO(1,3) vacuum, what is required for Lorentz invariance. The 0th axis should be the longest - having the strongest tendency to align in nearly parallel way. This way dynamics of its tiny perturbations (boosts) is governed by additional set of Maxwell equations - with goal to obtain e.g. GEM: confirmed by Gravity Probe B approximation of general relativity. Such tiny perturbation/spatial curvature can be caused e.g. by EM-GEM interaction or activating potential to give particles also gravitational mass. Slowing down of EM propagation through EM-GEM interaction could explain gravitational time dilation and lensing [39]. In contrast to charge corresponding to complete spherical angle, this time we have only tiny curvatures - there is no mass quantization.

In liquid crystals, superfluids, superconductors there are also unavoidable 1D topological structures, called Abrikosov's vortex, fluxon, disclination. Searching for its correspondence in particle physics, the only candidate seem quark strings being at heart of QCD, briefly summarized in Fig. 6. They are believed to be decaying during string hadronization process in colliders like LHC, simplifying the task to search for correspondence between such results and decay of topological vortices. As discussed further, it automatically leads to looking perfect at least qualitative agreement.

Like electromagnetism, the discussed approach is viscosity-free, hence complete experimental realizations would require e.g. superfluid like in famous Volovik "Universe in a helium droplet" book [42]. However, simplified experimental settings could allow to get some interesting correspondence, like vortices going out of biaxial nematic topological charge due to the hairy ball theorem (no spin-less charge).

Related skyrmion models ([43], [44]) use similar 4th order kinetic term, also aiming particle correspondence - mainly nuclei, instead of electric charge interpreting topological charge as baryon number. They lack long-range EM interaction due to potential with single minimum, repaired here with Higgs-like. Instead of electric charge conservation, they cannot violate the baryon number - what is questioned e.g. due to lack of Gauss law for baryon number, and violation required e.g. in baryogenesis (creation of more baryons than antibaryons) or Hawking radiation (massless from originally baryons).

While the presented general view was already discussed by the author (the first version of [36], [45]), this article finally introduces looking proper mathematical framework. The current version is update of work in progress, planned to be extended in the future. The goal is not to compete with the Standard Model, but to understand its (nonperturbative) field configurations, and this way also unanswered questions and various issues. For example neutrino masses were originally assumed to be zero, while here they are automatically predicted nonzero. Finally, it should allow to reduce the number of parameters from ≈ 20 to ≈ 3 (QM, gravity energy scales and in Higgs-like potential).

II. General quantitative framework

This main section first introduces to obtaining electromagnetism, with built-in charge quantization and regularization, to director field in analogy to Faber approach ([6], [7], [8], [9], [10]). Further there is discussed generalization to biaxial nematic case using field of real symmetric matrices (like stress-energy tensor) preferring shape as (Λ_i) set of eigenvalues.

A. Gauss law with built-in (topological) charge quantization

Imagine a continuous (director) field of unitary vectors $\vec{n}:\mathbb{R}^d\to S^{d-1}$. Restricting it to a closed surface $\mathcal{S}\subset\mathbb{R}^d$ gives $\vec{n}:\mathcal{S}\to S^{d-1}$ function, which has some integer number of coverings/windings of this sphere - called topological charge.

This (generalized) winding number, multiplied by sphere area, can be obtained by integrating Jacobian (as determinant of Jacobian matrix) over this closed surface - we would like to interpret this Jacobian e.g. as electric field, making that Gauss law counts this winding number - getting missing built-in charge quantization as topological charge.

In 2D case, analogous e.g. to argument principle in complex analysis, integrating derivative of angle over loop gives 2π times topological charge ($\vec{n}'=d\vec{n}/dL$, $\vec{n}=(n_1,n_2)$):

$$\text{2D topological charge} = \frac{1}{2\pi}\oint_L (n_2 n_1' - n_1 n_2')\, dL$$

This way we e.g. get quantization of magnetic field in superconductors as fluxon/Abrikosov vortex [46] being 2D topological charge, which also resembles spin as in quantum rotation operator: saying that rotating spin s particle by φ angle rotates quantum phase by $s\varphi$. Observe that, as in top of Fig. 1, with liquid crystals we can get spin 1/2 this way, as rotating by π radians we get the same ellipsoid due to $\vec{n}\equiv-\vec{n}$ symmetry to S^2/Z_2 (as in projective space).

2D topological charges in 3D (Abrikosov vortex, fluxon) seem related with magnetic field lines (also resembling spin), there is experimentally confirmed analogy between magnetic field and vorticity ([16], [17], [18]). In contrast, 3D topological charge in 3D is nearly point-like and in liquid crystals get long-range interactions due to nontrivial vacuum dynamics of director field ([2], [3], [4], [5]). We would like to propose Lagrangian recreating standard electromagnetism for them.

To calculate winding number of $\vec{n}:\mathcal{S}\to S^2$ in 3D by integration, we need to calculate the Jacobian. Let $u\perp v$ be unitary vectors in some point of surface $\mathcal{S}\subset\mathbb{R}^3$, transformed to $\vec{n}_u=\partial_u\vec{n}$, $\vec{n}_v=\partial_n\vec{n}$ perpendicular to $\vec{n}$, so the Jacobian is:

$$\det[\vec{n},\vec{n}_\mu,\vec{n}_\nu]=\vec{n}\cdot(\vec{n}_\mu\times\vec{n}_\nu)=\pm\|(\vec{n}\times\vec{n}_\mu)\times(\vec{n}\times\vec{n}_\nu)\| \quad (1)$$

allowing to calculate **3D topological charge** like in Gauss law:

$$Q_{el}(\mathcal{S})=\frac{e_0}{4\pi}\oint_{\mathcal{S}(u,v)} du\,dv\ (\partial_u\vec{n}\times\partial_v\vec{n})\cdot\vec{n} \quad (2)$$

Defining $\vec{\Gamma}_\mu=\vec{n}\times\vec{n}_\mu$ affine connection, it can be imagined as axis of local rotation for μ direction transport, with length determining its speed. Then the Jacobian becomes the curvature:

$$\vec{\Gamma}_\mu=\vec{n}\times\partial_\mu\vec{n} \qquad \vec{R}_{\mu\nu}=\vec{\Gamma}_\mu\times\vec{\Gamma}_\nu \quad (3)$$

Hence, to make $Q_{el}(\mathcal{S})=\oint_{\mathcal{S}}E\cdot dA$ Gauss law count topological charge, we need to define electric field E as curvature. To include magnetic field B, Faber ([6], [7], [8], [9]) suggests to define dual (*) EM tensor with these curvatures (choose $c=1$):

$${}^*\vec{F}_{\mu\nu}=\frac{-e_0}{4\pi\epsilon_0 c}\vec{R}_{\mu\nu}\sim\begin{pmatrix}0 & B_1 & B_2 & B_3\\ -B_1 & 0 & E_3 & -E_2\\ -B_2 & -E_3 & 0 & E_1\\ -B_3 & E_2 & -E_1 & 0\end{pmatrix} \quad (4)$$

where dual means exchanging magnetic and electric field in standard F tensor: (norms of) space-space curvature corresponds to electric field, space-time to magnetic (like in vorticity-magnetic field analogy [16], [17]). Using standard EM Hamiltonian $\mathcal{H}_{ED}\propto\sum_{\mu\nu=0}^{3}\|\vec{R}_{\mu\nu}\|^2$ leads to electromagnetism, Maxwell equations for such topological charges. For regularization there is added Higgs-like potential preferring unitary vectors for $\vec{n}$, also allowing e.g. for $\vec{n}=0$ in the center of such topological singularity e.g. hedgehog-like configuration. This field deformation to finite energy leads to Coulomb deformation in agreement with the running coupling effect [10].

B. Curvature for field of rotations: orthogonal matrices

Wanting to generalize the above vector field curvature to rotations of more complex objects, let us start with describing it for orthogonal rotation matrices: $O\equiv O(x)$ field satisfying $OO^T=O^TO=I$. Transporting ϵ size step in μ direction, in linear term we get affine connection describing local rotation:

$$O\to O(I+\epsilon\Gamma_\mu) \quad \text{for} \quad \Gamma_\mu=O^T\partial_\mu O\equiv O^TO_\mu \quad (5)$$

which is now anti-symmetric matrix $\Gamma_\mu=-\Gamma_\mu^T$ (from $0=\partial_\mu I=\partial_\mu(O^TO)=O^T_\mu O+O^TO_\mu$).

For 3×3 matrices in space, or 4×4 with added 0-th coordinate as time, let us use standard notation for anti-symmetric matrix (SO(4) generator later (39) replaced with

SO(1,3) generator by using only positive $\vec{\Gamma}^g_\mu$ for boosts):

$$\Gamma_\mu = O^T O_\mu = \begin{pmatrix} 0 & \vec{\Gamma}^{g1}_\mu & \vec{\Gamma}^{g2}_\mu & \vec{\Gamma}^{g3}_\mu \\ -\vec{\Gamma}^{g1}_\mu & 0 & -\vec{\Gamma}^3_\mu & \vec{\Gamma}^2_\mu \\ -\vec{\Gamma}^{g2}_\mu & \vec{\Gamma}^3_\mu & 0 & -\vec{\Gamma}^1_\mu \\ -\vec{\Gamma}^{g3}_\mu & -\vec{\Gamma}^2_\mu & \vec{\Gamma}^1_\mu & 0 \end{pmatrix} \tag{6}$$

for the two rotation vectors built of $\Gamma_\mu = O^T O_\mu$ coordinates:

$$\vec{\Gamma}_\mu := ((\Gamma_\mu)_{32}, (\Gamma_\mu)_{13}, (\Gamma_\mu)_{21})$$

$$\vec{\Gamma}^g_\mu := ((\Gamma_\mu)_{01}, (\Gamma_\mu)_{02}, (\Gamma_\mu)_{03}) \tag{7}$$

Analogously to Faber, we would like to define dual F EM tensor as proportional to $\vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$, and for GEM analogously using $\vec{R}^{gg}_{\mu\nu} = \vec{\Gamma}^g_\mu \times \vec{\Gamma}^g_\nu$ as curvature of space: submanifold perpendicular to this 0-th axis. There are also curvatures between them corresponding to EM-GEM interaction, finally we have various types of curvatures here:

$$\vec{R}_{\mu\nu} \equiv \vec{R}^{ee}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu \qquad \vec{R}^{gg}_{\mu\nu} = \vec{\Gamma}^g_\mu \times \vec{\Gamma}^g_\nu \tag{8}$$

$$\vec{R}^{eg}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}^g_\nu \qquad \vec{R}^{ge}_{\mu\nu} = \vec{\Gamma}^g_\mu \times \vec{\Gamma}_\nu = -\vec{R}^{eg}_{\nu\mu}$$

Commutator of such connection matrices $[\Gamma_\mu, \Gamma_\nu] = \Gamma_\mu\Gamma_\nu - \Gamma_\nu\Gamma_\mu$ can be expressed with these curvatures:

$$[\Gamma_\mu, \Gamma_\nu] = \begin{pmatrix} 0 & -\vec{R}^{eg}_{\mu\nu} + \vec{R}^{eg}_{\nu\mu} \\ \vec{R}^{eg}_{\mu\nu} - \vec{R}^{eg}_{\nu\mu} & \vec{R}^{ee}_{\mu\nu} + \vec{R}^{gg}_{\mu\nu} \end{pmatrix} := \tag{9}$$

$$\begin{pmatrix} 0 & -\vec{R}^{eg1}_{\mu\nu} + \vec{R}^{eg1}_{\nu\mu} & -\vec{R}^{eg2}_{\mu\nu} + \vec{R}^{eg2}_{\nu\mu} & -\vec{R}^{eg3}_{\mu\nu} + \vec{R}^{eg3}_{\nu\mu} \\ \vec{R}^{eg1}_{\mu\nu} - \vec{R}^{eg1}_{\nu\mu} & 0 & -\vec{R}^{ee3}_{\mu\nu} - \vec{R}^{gg3}_{\mu\nu} & \vec{R}^{ee2}_{\mu\nu} + \vec{R}^{gg2}_{\mu\nu} \\ \vec{R}^{eg2}_{\mu\nu} - \vec{R}^{eg2}_{\nu\mu} & \vec{R}^{ee3}_{\mu\nu} + \vec{R}^{gg3}_{\mu\nu} & 0 & -\vec{R}^{ee1}_{\mu\nu} - \vec{R}^{gg1}_{\mu\nu} \\ \vec{R}^{eg3}_{\mu\nu} - \vec{R}^{eg3}_{\nu\mu} & -\vec{R}^{ee2}_{\mu\nu} - \vec{R}^{gg2}_{\mu\nu} & \vec{R}^{ee1}_{\mu\nu} + \vec{R}^{gg1}_{\mu\nu} & 0 \end{pmatrix}$$

with shortened first matrix notation, expanded in the latter.

However, in flat spacetime this commutator would need to vanish, e.g. enforcing spatial curvature $\vec{R}$ with gravitational $\vec{R}^{gg}$:

$$0 = \partial_\mu\partial_\nu O - \partial_\nu\partial_\mu O = \partial_\mu(O\Gamma_\nu) - \partial_\nu(O\Gamma_\mu) = O[\Gamma_\mu, \Gamma_\nu]$$

$$\partial_\nu\Gamma_\mu - \partial_\mu\Gamma_\nu = [\Gamma_\mu, \Gamma_\nu] \tag{10}$$

It no longer vanishes if replacing field of unitary vectors with field of rotated objects discussed next.

III. Extension to bixial nematic as ellipsoid field

Vector field does not recognize twists of such vectors, hence would allow violation of angular momentum in this direction. In liquid crystals it is repaired by going from (uniaxial) Oseen-Frank model seen as approximation, to full (biaxial) Landau-de Gennes model [13] - field of ellipsoids of potentially three different axes, offering simple general description of field recognizing all SO(3) rotations (also allowing further search "of what" for even more fundamental models), we will further expand with boosts to SO(1,3).

Like e.g. stress-energy tensor, such objects can be represented using real symmetric matrix/tensor field $M \equiv M(x)$ with chosen shape as preferred set of eigenvalues representing lengths of the 3 axes, its eigenvectors point directions of these 3 axes:

$$M = ODO^T \qquad \text{for } OO^T = I,\ D = \text{diag}(\lambda_1, \lambda_2, \lambda_3) \tag{11}$$

for O orthogonal matrix field as in the previous subsection. We will focus on 3D case now, but it naturally generalizes to 4D case using 4×4 matrices, $D = \text{diag}(\lambda_0, \lambda_1, \lambda_2, \lambda_3)$, and O from SO(1,3) containing boost (no longer orthogonal).

The diagonal matrix D should prefer some shape: e.g. fixed $(\Lambda_0 \geq)\ \Lambda_1 \geq \Lambda_2 \geq \Lambda_3$. However, it also requires a possibility of regularization of singularities to finite energy (like top of Fig. 1 in 2D), what again can be obtained using Higgs-like potential, this time with SO(3) minimum, for example:

$$V(M) = \sum_i (\lambda_i - \Lambda_i)^2 \quad \text{or e.g.} \quad \sum_{k=1}^{3} (\text{Tr}(M^k) - c_k)^2 \tag{12}$$

for $c_k = \sum_i (\Lambda_i)^k$ like in Landau-de Gennes potential [13]:

$$V_{LG}(M) = a\,\text{Tr}(M^2) - b\,\text{Tr}(M^3) + c\,\left(\text{Tr}(M^2)\right)^2 \tag{13}$$

This potential is supposed to be activated mainly near particles to prevent infinity (regularization) - corresponds to weak/strong interaction, hence the choice of its details remains a difficult open question requiring simulations.

Let us now focus on the $D \approx \text{diag}(\Lambda_1, \Lambda_2, \Lambda_3)$ vacuum behavior. Thermally it should also contain tiny perturbations, which might correspond to dark energy/matter in analogy to 2.7K cosmic microwave background radiation.

Derivative in μ direction of our tensor field is:

$$M_\mu := \partial_\mu M = O_\mu D O^T + O D O^T_\mu + O D_\mu O^T \tag{14}$$

$$O^T M_\mu O = O^T O_\mu D + D O^T_\mu O + D_\mu = \Gamma_\mu D - D\Gamma_\mu + D_\mu$$

for $\Gamma_\mu = O^T O_\mu$ affine connection being anti-symmetric matrix as in the previous subsection.

From dynamics of rotation part O (later in 4D including boosts), as in Faber model we would like to define EM field as its curvature to make Gauss law count winding number. However, for a few reasons like distinguishing rotations of various energy (EM $\gg$ pilot wave $\gg$ GEM). Therefore, instead of O as previously, this time we would like to directly work on $M(x) \equiv M = ODO^T$ field.

A. Curvature analogue of electromagnetic F tensor

While there might be a better choice, for now let us focus on a simple one: try to just replace discussed previously $[\Gamma_\mu, \Gamma_\nu]$ commutator with $[M_\mu, M_\nu] = M_\mu M_\nu - M_\nu M_\mu$:

$$F_{\mu\nu} := [M_\mu, M_\nu] \quad = -F_{\nu\mu} = -F^T_{\mu\nu} \tag{15}$$

for $M_\mu := \partial_\mu M$. Looking at (14), focusing on vacuum dynamics $D = \text{diag}(\Lambda_1, \Lambda_2, \Lambda_3)$ and conveniently transforming:

$$O^T F_{\mu\nu} O = O^T [M_\mu, M_\nu] O \approx [\Gamma_\mu D - D\Gamma_\mu, \Gamma_\nu D - D\Gamma_\nu] = \tag{16}$$

$$(\Lambda_1 - \Lambda_2)(\Lambda_3 - \Lambda_1)(\Lambda_2 - \Lambda_3) \begin{pmatrix} 0 & \frac{-\vec{R}^3_{\mu\nu}}{\Lambda_1 - \Lambda_2} & \frac{\vec{R}^2_{\mu\nu}}{\Lambda_3 - \Lambda_1} \\ \frac{\vec{R}^3_{\mu\nu}}{\Lambda_1 - \Lambda_2} & 0 & \frac{-\vec{R}^1_{\mu\nu}}{\Lambda_2 - \Lambda_3} \\ \frac{-\vec{R}^2_{\mu\nu}}{\Lambda_3 - \Lambda_1} & \frac{\vec{R}^1_{\mu\nu}}{\Lambda_2 - \Lambda_3} & 0 \end{pmatrix}$$

for $\vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$ and $\vec{\Gamma}_\mu := ((\Gamma_\mu)_{32}, (\Gamma_\mu)_{13}, (\Gamma_\mu)_{21})$ as in the previous subsection. Calculating topological charge through integration using such matrix curvature, we get charge quantization for its each coordinate.

Wanting to interpret $F_{\mu\nu}$ EM tensor with such curvature, instead of single number (or vector in Faber approach), it is now anti-symmetric matrix (or SO(1,3) generator in 4D), requiring to replace $\|F_{\mu\nu}\|$ with a matrix norm. A natural generalization is Frobenius inner product and norm, treating matrix as vector for Euclidean norm:

$$A \bullet B = \mathrm{Tr}(AB^T) = \sum_{ij} A_{ij} B_{ij} \qquad \|A\|_F = \sqrt{A \bullet A} \quad (17)$$

Prolate uniaxial nematic director field case can be imagined as $\Lambda_1 > \Lambda_2 = \Lambda_3$ limit, leaving single curvature $\|R_{\mu\nu}\|_F^2 \propto (\vec{R}_{\mu\nu,1})^2$, making $R^1_{\mu\nu} \equiv \vec{R}_{\mu\nu,1}$ proportional to electric field for spatial $1 \le \mu < \nu < 3$, and to magnetic field for temporal $\mu = 0$ and spatial $\nu \in \{1,2,3\}$.

For promising similar biaxial nematic case $\Lambda_1 > \Lambda_2 > \Lambda_3$, we would like $\Lambda_2 \approx \Lambda_3$ being much closer as in Fig. 1,5 - adding to electromagnetism (R^1 with dominant first axis), low energy degrees of freedom: $R^2_{\mu\nu} \equiv \vec{R}_{\mu\nu,2}, R^3_{\mu\nu} \equiv \vec{R}_{\mu\nu,3}$, hopefully to agree with quantum phase for pilot wave, propelled by particle configuration (de Broglie clock).

B. Lagrangian, four-potential, uniaxial as special case

Let us postulate the **Lagrangian** in analogy to EM:

$$\mathcal{L} = \sum_{\mu=1}^{3} \|F_{\mu 0}\|_F^2 - \sum_{1\le\mu<\nu\le 3} \|F_{\mu\nu}\|_F^2 - V(M) \qquad (18)$$

*1) **Four-potential** A_μ analogue:* The mentioned suggestion to directly use $F_{\mu\nu} = [M_\mu, M_\nu]$ in Lagrangian is inconvenient due to products of derivatives. Hence let us introduce A_μ to work as EM four-potential, this time being matrices (3×3 or 4×4 with gravity). $A_\mu = M\,\partial_\mu M$ would already give $\partial_\mu A_\nu - \partial_\nu A_\mu = [M_\mu, M_\nu]$, but let us anti-symmetrize it for reduced dimension and direct interpretation:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad \text{for} \quad A_\mu := M M_\mu - M_\mu M \approx \quad (19)$$

$$\approx O \begin{pmatrix} 0 & \vec{\Gamma}^3_\mu(\Lambda_1-\Lambda_2)^2 & -\vec{\Gamma}^2_\mu(\Lambda_1-\Lambda_3)^2 \\ -\vec{\Gamma}^3_\mu(\Lambda_1-\Lambda_2)^2 & 0 & \vec{\Gamma}^1_\mu(\Lambda_2-\Lambda_3)^2 \\ \vec{\Gamma}^2_\mu(\Lambda_1-\Lambda_3)^2 & -\vec{\Gamma}^1_\mu(\Lambda_2-\Lambda_3)^2 & 0 \end{pmatrix} O^T$$

where the A_μ approximation is again for $D = \mathrm{diag}(\Lambda_1,\Lambda_2,\Lambda_3)$ vacuum situation. Using commutation of derivatives we get analogue of Maurer-Cartan structural equation:

$$\partial_\mu A_\nu - \partial_\nu A_\mu = 2(\partial_\mu M\,\partial_\nu M - \partial_\nu M\,\partial_\mu M) =: 2F_{\mu\nu} \quad (20)$$

We can calculate variation, which is real anti-symmetric matrix:

$$\frac{\partial \|F_{\mu\nu}\|_F^2}{\partial(\partial_\alpha A_\beta)} = \frac{1}{2}\frac{\partial\|\partial_\mu A_\nu - \partial_\nu A_\mu\|_F^2}{\partial(\partial_\alpha A_\beta)} = (\delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\beta}\delta_{\nu\alpha})F_{\alpha\beta} \quad (21)$$

*2) **Uniaxial nematic** as degenerate case:* Director $\vec{n}$ field can be obtained using e.g. $M = \vec{n}\,\vec{n}^T$ corresponding to $\Lambda_1 = \|\vec{n}\|^2$ (constant), $\Lambda_2 = \Lambda_3 = 0$ case:

$$A_\mu = \|\vec{n}\|^2 \begin{pmatrix} 0 & (\vec{n}\times\vec{n}_\mu)_3 & -(\vec{n}\times\vec{n}_\mu)_2 \\ -(\vec{n}\times\vec{n}_\mu)_3 & 0 & (\vec{n}\times\vec{n}_\mu)_1 \\ (\vec{n}\times\vec{n}_\mu)_2 & -(\vec{n}\times\vec{n}_\mu)_3 & 0 \end{pmatrix}$$

For which $\partial_\mu A_\nu - \partial_\nu A_\mu$ gives curvature as in Faber approach:

$$\partial_\mu(\vec{n}\times\vec{n}_\nu) - \partial_\nu(\vec{n}\times\vec{n}_\mu) = 2\,\vec{n}_\mu \times \vec{n}_\nu$$

Here we have 2 vacuum degrees of freedom rotating $\vec{n}$, in general case we slightly separate Λ_2 and Λ_3 by adding one low energy degree of freedom for twists of $\vec{n}$, supposed to work as quantum phase. To see this generalization as perturbation, we will replace $\Lambda = (1,0,0)$ case with $\Lambda = (1,\delta,0)$ for tiny δ related with Planck constant, and focus on low order δ terms.

IV. General equations of motion for 3D case

Let us now derive equations of motion from Lagrangian optimization - zeroing of its variation. For EM it is usually done with variation of A field - we will start with as simpler. However, e.g. to get built-in charge quantization, there was proposed more fundamental field M (which topological charge is calculated by Gauss law) - we further consider its variation.

A. Simplification: Euler-Lagrange equations for A field

Equations of motion for electromagnetism are usually derived with Euler-Lagrange equations for A field (21) - let us start here as simpler, bringing valuable intuitions:

$$\frac{\partial\mathcal{L}}{\partial A_\alpha} = \frac{d}{dx_0}\frac{\partial\mathcal{L}}{\partial(\partial_0 A_\alpha)} + \sum_{i=1}^{3}\frac{d}{dx_i}\frac{\partial\mathcal{L}}{\partial(\partial_i A_\alpha)}$$

$$\frac{\partial V}{\partial A_\alpha} = \partial_0 F_{0\alpha} - \sum_{i=1}^{3}\partial_i F_{i\alpha} = \Box A_\alpha - \partial_\alpha\left(\partial_0 A_0 - \sum_i \partial_i A_i\right) \quad (22)$$

for $\Box = \partial_{00} - \sum_{i=1}^3 \partial_{ii}$ d'Alembertian.

In vacuum the potential vanishes, getting Maxwell-like equations for $\vec{E} = (F_{23}, F_{31}, F_{12})$, $\vec{B} = (F_{01}, F_{02}, F_{03})$ dual analogs of electric and magnetic fields, but this time with each component being a matrix, in vacuum satisfying $\frac{\partial V}{\partial A_\alpha} = 0 = \Box F_{\mu\nu}$ wave equation with $c = 1$ propagation speed, and with built-in charge quantization as topological.

As in EM Lorentz gauge condition, the $\partial_0 A_0 - \sum_i \partial_i A_i = [M, \Box M]$ term should be zero from integration by parts (assuming fields vanish in infinity), leading to $\frac{\partial V}{\partial A_\alpha} = \Box A_\alpha$.

Regarding the **potential**, its choice remains difficult main open question, which will require simulations e.g. aiming agreement with electron, 3 leptons.

While there was mentioned potential $V(M)$ directly preferring shape as (λ_i) (similarity to Faber), here we get $\frac{\partial\mathcal{L}}{\partial A_\alpha}$ suggesting to use $V(A)$ instead - as in (19) using differences of (λ_i), this time multiplied by derivative in Γ.

Both choices have M derivative dependence in Γ, which if preferring some values with Higgs-like potential, enforce nonzero M derivatives - what might be the source e.g. of Zitterbewegung intrinsic periodic process of electron [20].

Ideally would be not having to fix shape (Λ_i) as parameters of the model, but to make them automatically emerge from a simple e.g. $V(A) = (\sum_\mu \|A_\mu\|_F^2 - 1)^2$ Higgs-like potential, with additional e.g. volume constraint $\det(M) = \prod_i \lambda_i =$ const to prevent using only long axes which allow for low curvature (hence energy).

Hamiltonian (energy density) derivation is analogous to EM:

$$\mathcal{H} = \sum_{\mu=1}^{3}\frac{\partial\mathcal{L}}{\partial(\partial_0 A_\mu)}\partial_0 A_\mu - \mathcal{L} = \sum_{\mu=1}^{3} F_{0\mu}\bullet(2F_{0\mu} + \partial_\mu A_0) - \mathcal{L}$$

$$\mathcal{H} = \sum_{0\leq\mu<\nu\leq 3} \|F_{\mu\nu}\|_F^2 + V \quad + \sum_{\mu=1}^{3} F_{0\mu} \bullet \partial_\mu A_0 \tag{23}$$

The last sum vanishes in EM due to integration by parts (assuming fields vanish in infinity) to shift derivative to F, getting divergence of electric field without sources. Here it becomes $-A_0 \bullet \sum_{\mu=1}^3 \partial_\mu F_{0\mu}$, which from above Euler-Lagrange equation vanishes at least in vacuum.

B. Proper equations of motion: variation of M field

In the discussed approach, as in Fig. 5, we assume there is more fundamental field M - e.g. to make Gauss law count its topological charge, enforcing charge quantization.

To get equations of motion we consider its variation. In 3D we have 3 rotation generators G:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{24}$$

(plus 3 in 4D), and 3 (+1 in 4D) axis elongation generators:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{25}$$

Now for $M = ODO^T$ field, generator as matrix G (one of 3+3 above, 3+3+4 in 4D), infinitesimal $\epsilon \in \mathbb{R}$, $\eta : \mathbb{R}^4 \to \mathbb{R}$ function, let us consider variation in convenient form where generators are directly acting on D:

$$O \to O(I + \epsilon\eta G) \tag{26}$$

$$M \to O(I+\epsilon\eta G)\, D\, (I+\epsilon\eta G^T)O^T \approx M+\epsilon\eta O(GD+DG^T)O^T$$

$$M \to M + \epsilon\eta O G' O^T \qquad \text{for} \qquad G' = GD + DG^T \tag{27}$$

$$M_\mu \to M_\mu + \epsilon O(\eta_\mu G' + \eta[\Gamma_\mu, G'] + \eta G'_\mu)O^T \tag{28}$$

plus $\mathsf{O}(\epsilon^2)$ neglected higher order term.

We will work on $G' = GD + DG^T$ 6 (or 10 in 4D) generators, which for rotations have two $+1$ coefficients, for elongations we can take $G' = G$.

1) Vacuum case derivation - fixed D, only SO(3) rotations: For simplicity let us focus first on vacuum case: fixed $\lambda_i = \Lambda_i$ minimizing potential, only 3 rotation generators (3+3 in 4D). We also use $G'_\mu = 0$, but nonzero is included in the final formula (35).

Using $\Gamma_\mu = O^T O_\mu = -O_\mu^T O$, for rotations only we can get conveniently transformed versions: $\overline{M}_\mu, \overline{A}_\mu, \overline{F}_{\mu\nu}$ with commutators:

$$M_\mu = \partial_\mu(ODO^T) = O\overline{M}_\mu O^T \quad \text{for} \quad \overline{M}_\mu = [\Gamma_\mu, D] \tag{29}$$

$$A_\mu = [M, M_\mu] = O\overline{A}_\mu O^T \quad \text{for} \quad \overline{A}_\mu = [D, \overline{M}_\mu] \tag{30}$$

$$F_{\mu\nu} = [M_\mu, M_\nu] = O\overline{F}_{\mu\nu} O^T \quad \text{for} \quad \overline{F}_{\mu\nu} = [\overline{M}_\mu, \overline{M}_\nu] \tag{31}$$

Applying variation (26) and denoting $G' := [G, D]$:

$$M \to M + \epsilon\eta\, O[G, D]O^T = O(D + \epsilon\eta G')O^T \tag{32}$$

plus $\mathsf{O}(\epsilon^2)$. As $\overline{M}_\mu := [\Gamma_\mu, D]$, its ∂_μ derivative is:

$$\partial_\mu M = O\overline{M}_\mu O^T \to O\left(\overline{M}_\mu + \epsilon\eta[\Gamma_\mu, G'] + \epsilon\eta_\mu G'\right) O^T \tag{33}$$

plus $\mathsf{O}(\epsilon^2)$. Using $F_{\mu\nu} = [M_\mu, M_\nu]$, $\overline{F}_{\mu\nu} = O^T F_{\mu\nu} O$:

$$\overline{F}_{\mu\nu} \to \left[\, \overline{M}_\mu + \epsilon\eta[\Gamma_\mu, G'] + \epsilon\eta_\mu G',\ \overline{M}_\nu + \epsilon\eta[\Gamma_\nu, G'] + \epsilon\eta_\nu G' \right]$$
$$= \overline{F}_{\mu\nu} + \epsilon\eta([\overline{M}_\mu, [\Gamma_\nu, G']] - [\overline{M}_\nu, [\Gamma_\mu, G']])+$$
$$+\epsilon\eta_\nu[\overline{M}_\mu, G'] - \epsilon\eta_\mu[\overline{M}_\nu, G'] + \mathsf{O}(\epsilon^2)$$

Lagrangian (18) needs $-\mathrm{Tr}(F_{\mu\nu}F_{\mu\nu}^T) = \mathrm{Tr}(F_{\mu\nu}F_{\mu\nu}) \to$

$$\mathrm{Tr}(F_{\mu\nu}F_{\mu\nu}) + 2\epsilon\eta \mathrm{Tr}\left(\overline{F}_{\mu\nu}\left([\overline{M}_\mu, [\Gamma_\nu, G']] - [\overline{M}_\nu, [\Gamma_\mu, G']]\right)\right) +$$
$$+2\epsilon \mathrm{Tr}\left(\eta_\nu \overline{F}_{\mu\nu}[\overline{M}_\mu, G'] - \eta_\mu \overline{F}_{\mu\nu}[\overline{M}_\nu, G']\right) + \mathsf{O}(\epsilon^2)$$

using $\mathrm{Tr}(F_{\mu\nu}F_{\mu\nu}) = \mathrm{Tr}(\overline{F}_{\mu\nu}\overline{F}_{\mu\nu})$, $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$. Lagrangian (18) sums 6 $\mathrm{Tr}(F_{\mu\nu}F_{\mu\nu})$ terms. As in the minimum necessary condition, we get the least action if the ϵ term vanishes. Like in derivation of Euler-Lagrange equation, we need first to apply integration by parts to shift η derivatives (assuming η vanishes at some boundary). Using $\partial_\mu M_\nu = \partial_\nu M_\mu$:

$$\partial_\nu \overline{M}_\mu = \partial_\nu\left(O^T M_\mu O\right) = O_\nu^T M_\mu O + O^T(\partial_\nu M_\mu)O + O^T M_\mu O_\nu$$
$$= \Gamma_\nu^T \overline{M}_\mu + O^T(\partial_\nu M_\mu)O + \overline{M}_\mu \Gamma_\nu = O^T(\partial_\nu M_\mu)O + [\overline{M}_\mu, \Gamma_\nu]$$

$$\partial_\nu \overline{M}_\mu - \partial_\mu \overline{M}_\nu = [\overline{M}_\mu, \Gamma_\nu] - [\overline{M}_\nu, \Gamma_\mu] \tag{34}$$

Finally the $\delta\mathcal{L} = 0$ equations of motion are:

$$0 = \sum_{\mu\nu} d_{\mu\nu} \mathrm{Tr}\left(\overline{F}_{\mu\nu}\left([\overline{M}_\mu, [\Gamma_\nu, G']] - [\overline{M}_\nu, [\Gamma_\mu, G']]\right)\right) -$$
$$-\mathrm{Tr}\left(\overline{F}_{\mu\nu}\left([[\overline{M}_\mu, \Gamma_\nu], G'] - [[\overline{M}_\nu, \Gamma_\mu], G']\right)\right))$$
$$-\mathrm{Tr}\left(\overline{F}_{\mu\nu,\mu}[\overline{M}_\nu, G'] - \overline{F}_{\mu\nu,\nu}[\overline{M}_\mu, G']\right)$$

for $\overline{F}_{\mu\nu,\mu} = \partial_\nu \overline{F}_{\mu\nu}$, $d_{\mu\nu} = 1$ for $\mu\nu \in \{10, 20, 30\}$ and $d_{\mu\nu} = -1$ for $\mu\nu \in \{23, 31, 12\}$ and 0 otherwise. Using Jacobi identity $([[A, B], C] + [[B, C], A] + [[C, A], B] = 0)$ we can simplify the first two lines:

$$0 = \sum_{\mu\nu} d_{\mu\nu} \mathrm{Tr}\left(\overline{F}_{\mu\nu}\left([\Gamma_\nu, [\overline{M}_\mu, G']] - [\Gamma_\mu, [\overline{M}_\nu, G']]\right)\right) +$$
$$+\mathrm{Tr}\left(\overline{F}_{\mu\nu,\nu}[\overline{M}_\mu, G'] - \overline{F}_{\mu\nu,\mu}[\overline{M}_\nu, G']\right) \tag{35}$$

In the general case there there is additional $G'_\mu = \partial_\mu G' = GD_\mu + D_\mu G$ term in (28) depending on evolution of diagonal $\partial_\mu D = D_\mu$. It needs additional $\overline{F}_{\mu\nu}([M_\mu, G'_\nu] - [M_\nu, G'_\mu])$ term in (35), and including potential V.

2) Simplification: : These equations for 3 generators G are still quite complex. To simplify as in (16), denote 3x3 antisymmetric matrices using $\vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$ vectors. Then denote EB fields as coordinates of $R = F^*$ tensor:

$$\vec{B}_1 = \vec{R}_{01} \qquad \vec{B}_2 = \vec{R}_{02} \qquad \vec{B}_3 = \vec{R}_{03} \tag{36}$$
$$\vec{E}_1 = \vec{R}_{32} \qquad \vec{E}_2 = \vec{R}_{13} \qquad \vec{E}_3 = \vec{R}_{21}$$

each of them is now 3D vector, which coordinates correspond to different energies as in Fig. 5 - let us denote them with superscript e.g. $\vec{B}_i = (B_i^1, B_i^2, B_i^3)$.

Let us now choose Λ eigenvalues as $D = \mathrm{diag}(1, \delta, 0)$. For $\delta = 0$ we get uniaxial nematic Faber's case. Here we assume δ is tiny positive, with twist corresponding to quantum phase - hence δ should be related with Planck constant. Neglecting higher order δ terms, the 3 EB coordinates correspond to $\approx (1, \delta, \delta)$ energies: first coordinate $(B^1 = (B_1^1, B_2^1, B_3^1), E^1 =$

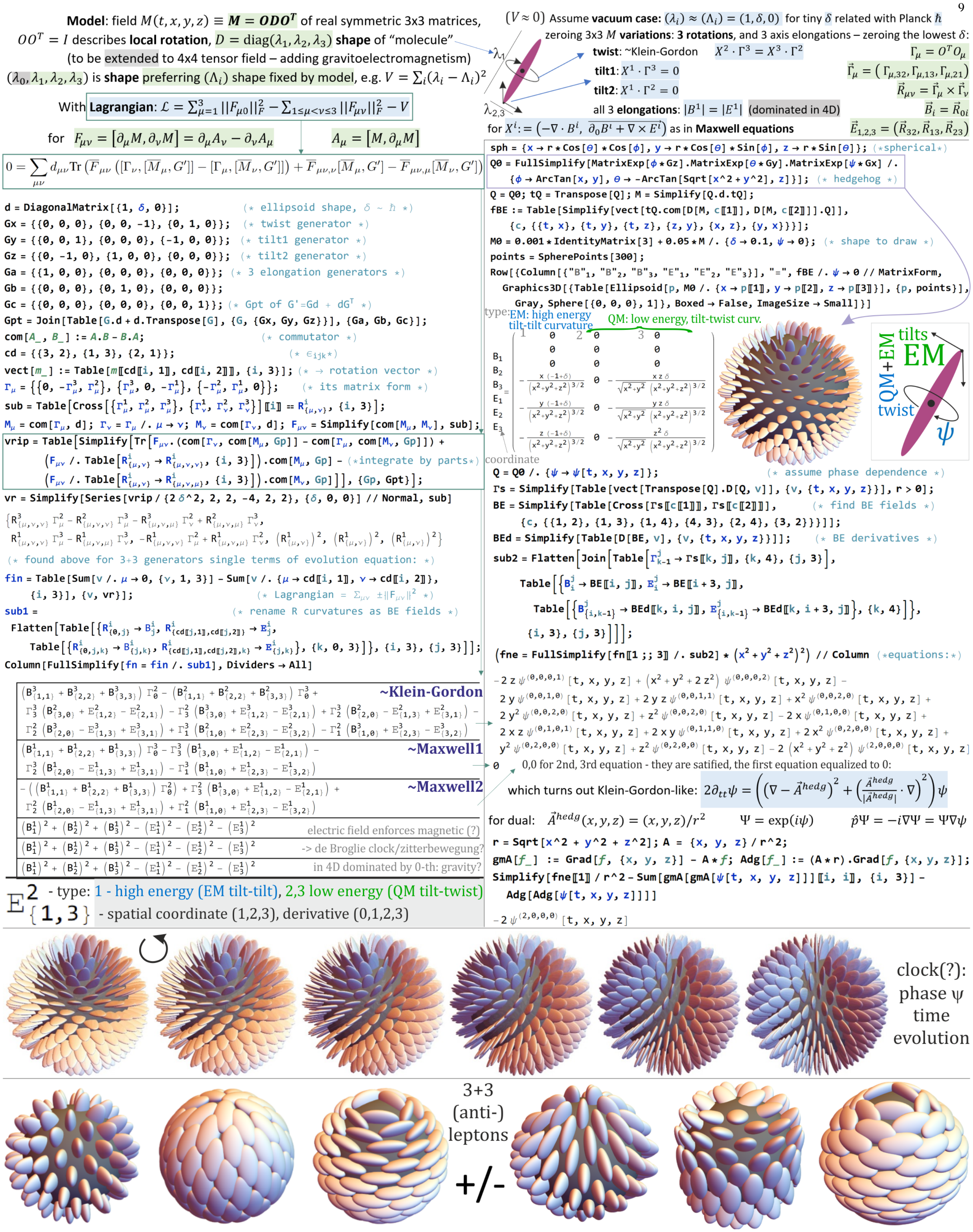

Figure 7. Used Mathematica source for 3D case and $\Lambda = (1, \delta, 0)$ shape (available in https://github.com/JarekDuda/liquid-crystals-particle-models). Application of written evolution equation (35) M_μ in vacuum case ($V = 0, G'_\mu = 0$), 6 generators G, and neglecting higher δ terms. Right: application for basic hedgehog case with ψ function defining local twist (wavefunction $\Psi = \exp(i\psi)$), deriving Klein-Gordon-like equation. Bottom: visualization of expected phase evolution for de Broglie clock/Zitterbewegung, and of 3 leptons, anti-leptons.

(E_1^1, E_2^1, E_3^1)) to standard electromagnetism, the remaining two to low energy quantum phase, hopefully to recreate relativistic QM like Klein-Gordon, QED Lagrangian.

To find the final equations (35), neglecting higher order δ terms, there is provided used Mathematica source (GitHub, Fig. 7), leading to terms for 3 rotation generators:

$$\text{twist:}\quad \Gamma^2_\mu \partial_\nu R^3_{\mu\nu} - \Gamma^3_\mu \partial_\nu R^2_{\mu\nu} - \Gamma^2_\nu \partial_\mu R^3_{\mu\nu} + \Gamma^3_\nu \partial_\mu R^2_{\mu\nu}$$

$$\text{2 tilts:}\quad \Gamma^3_\mu \partial_\nu R^1_{\mu\nu} - \Gamma^3_\nu \partial_\mu R^1_{\mu\nu}, \quad \Gamma^2_\nu \partial_\mu R^1_{\mu\nu} - \Gamma^2_\mu \partial_\nu R^1_{\mu\nu}$$

Such terms need to be summated by μ, ν and equalized to 0, leading to Maxwell-like equation terms for EM. Denoting:

$$X^i = (-\nabla \cdot B^i, \overrightarrow{\partial_0 B^i + \nabla \times E^i}), \tag{37}$$

the (35) equations for 3 rotation generators become:

$$X^2 \cdot \Gamma^3 = X^3 \cdot \Gamma^2 \qquad \sim \text{Klein-Gordon for twist(phase)} \tag{38}$$

$$X^1 \cdot \Gamma^3 = 0,\ X^1 \cdot \Gamma^2 = 0 \qquad \text{Maxwell equation for two tilts}$$

Figure 7 contains this implementation, also applying these equations to hedgehog ansatz (model of lepton), getting Klein-Gordon-like equation for the twist (phase). Figure 9 contains implementation deriving 4D equations, getting 2nd set of Maxwell-like equations for GEM. Figure 2 calculates Coulomb effective potential for such two topological charges, Figure 3 suggests a way to analogously get Newton law for 4D field.

Further work is planned to extend this agreement, also parametrization to moduli space, finally maybe hydrodynamical simulations. The first difficulty is getting angular momentum, clock for charge (electron), hopefully through regularization by including Higgs-like potential, as most of mass/energy of particle is localized in its center - where potential is nonzero.

V. 4D case: Lorentz invariance and gravity

Previously we were focused on 3D case as for biaxial nematic, briefly mentioning 4D case e.g. as SO(4) rotation in (6). In contrast, there is a general belief for Lorentz invariance in 4D, replacing SO(4) with SO(1,3) Lorentz group. The previous antisymmetric rotation generator (6) with mixed below - all positive $\vec{\Gamma}^g_\mu$ being boost generator for chosen rapidity:

$$\Gamma_\mu = O^T O_\mu = \begin{pmatrix} 0 & \vec{\Gamma}^{g1}_\mu & \vec{\Gamma}^{g2}_\mu & \vec{\Gamma}^{g3}_\mu \\ \vec{\Gamma}^{g1}_\mu & 0 & -\vec{\Gamma}^3_\mu & \vec{\Gamma}^2_\mu \\ \vec{\Gamma}^{g2}_\mu & \vec{\Gamma}^3_\mu & 0 & -\vec{\Gamma}^1_\mu \\ \vec{\Gamma}^{g3}_\mu & -\vec{\Gamma}^2_\mu & \vec{\Gamma}^1_\mu & 0 \end{pmatrix} \tag{39}$$

Matrix $O \equiv O_\alpha^\beta$ contains rotation and boost, can be calculated by exponentiation of above generator matrix. It is no longer orthogonal, but still we can use $M = ODO^T$ as rotation and boost of some fundamental anisotropic object.

In Euler-Lagrange equations for vacuum we should use 6 generators: 3 antisymmetric for 3D rotation (24), and 3 symmetric for boosts (+ 4 axis elongation generators):

$$\begin{pmatrix} 0&1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&0 \end{pmatrix}$$

Another change is in matrix products - they are unchanged for $O \equiv O_\alpha^\beta$ up-down indexes. However, our fundamental tensor field is rather symmetric as $M \equiv M_{\alpha\beta}$ (equivalently could be both upper indexes), requiring $M_{\alpha\beta} \to M_\alpha^\beta$ by product with $\xi = \text{diag}(-1,1,1,1)$ spacetime signature matrix, e.g. transforming:

$$[M_\mu, M_\nu] \to M_\mu \xi M_\nu - M_\nu \xi M_\mu \qquad \text{for} \quad \xi = \text{diag}(-1,1,1,1) \tag{40}$$

For Lagrangian we further took Frobenius norm of such commutator, which for Lorentz invariance becomes:

$$\|X\|_F^2 \to \|X\|_\xi^2 := \text{Tr}(X\xi X^T \xi) = \sum_{\mu\nu} X_{\mu\nu} X^{\mu\nu} \tag{41}$$

with also negative contributions, well known e.g. in EM Lagrangian:

$$\|F\|_\xi^2 = \sum_{\mu\nu} F_{\mu\nu} F^{\mu\nu} = 2 \sum_{1\leq\mu<\nu\leq 3} (F_{\mu\nu})^2 - 2\sum_{\mu=1}^3 (F_{0\mu})^2$$

Finally Lorentz invariant Lagrangian can be chosen like 4th order term in skyrmion model as (we can use $[\partial_\mu M, \partial_\nu M] = \partial_\mu M\, \xi\, \partial_\nu M - \partial_\nu M\, \xi\, \partial_\mu M$ operating on matrices):

$$\mathcal{L} = -\sum_{\alpha\beta\mu\nu} F_{\mu\nu\alpha\beta} F^{\mu\nu\alpha\beta} - V(M) \tag{42}$$

$$\text{for} \qquad F_{\mu\nu\alpha\beta} = [\partial_\mu M, \partial_\nu M]_{\alpha\beta}$$

with electromagnetic F tensor extended to include also quantum phase governed by Klein-Gordon-like equation, and second set of Maxwell equations for gravity.

Figure 8 contains practical approximation of $\bar{F}_{\mu\nu}$ tensor as $F_{\mu\nu}$ with reversed rotation and boost. It is for vacuum case with $D = \text{diag}(g, 1, \delta, 0)$ shape for $g \gg 1 \gg \delta > 0$ formula (16) for O containing rotation + boost and $\Gamma_\mu = O^T O_\mu$. Affine connection, curvature without tilde $\Gamma, R \equiv R^e$ is for spatial rotations, with tilde:x $\tilde{\Gamma}, \tilde{R} \equiv R^g$ for the 0th time axis.

Hamiltonian can be calculated as previously (23), replacing Frobenius norm with Lorentz invariant one - having both positive spatial energy contributions, but surprisingly also negative $\alpha 0$ energy contributions (as Legendre transform changes only 2 out of 4 indexes of curvature tensor - corresponding to derivation directions, not SO(1,3) generators inside):

$$\mathcal{H} = \sum_{0\leq\mu<\nu\leq 3} F_{\mu\nu\alpha\beta} F^{\alpha\beta}_{\mu\nu} + V(M) \tag{43}$$

$$\mathcal{H} = 2\sum_{0\leq\mu<\nu\leq 3} \left(\sum_{1\leq\alpha<\beta\leq 3} (F_{\mu\nu\alpha\beta})^2 - \sum_{\alpha=1}^3 (F_{\mu\nu\alpha 0})^2 \right) + V(M)$$

Spatial part of $\bar{F}_{\mu\nu}$ in Figure 8 contains sum of spatial curvature R, as previously (e.g. Fig. 5) multiplied by δ for 2nd and 3rd coordinate (without $R^1_{\mu\nu} = \Gamma^2_\mu \Gamma^3_\nu - \Gamma^3_\mu \Gamma^2_\nu$). It is summed with curvature of time axis $R^g \equiv \tilde{R}$ corresponding to GEM. Energy density (Hamiltonian) contains squares of such sums. Gravity fluctuations are usually many orders of magnitude slower, hence expanding such positive energy term into expected values:

$$E[(R + g^2\tilde{R})^2] = E[R^2] + E[(g^2\tilde{R})^2] + 2E[g^2 R\tilde{R}]$$

we can treat electromagnetism (minimizing first term) and gravity (second) as nearly independent, Fig. 9 derives Maxwell-like equations for GEM. Then EM-GEM interaction: dependence through the $E[R\tilde{R}]$ last term should slow down EM propagation

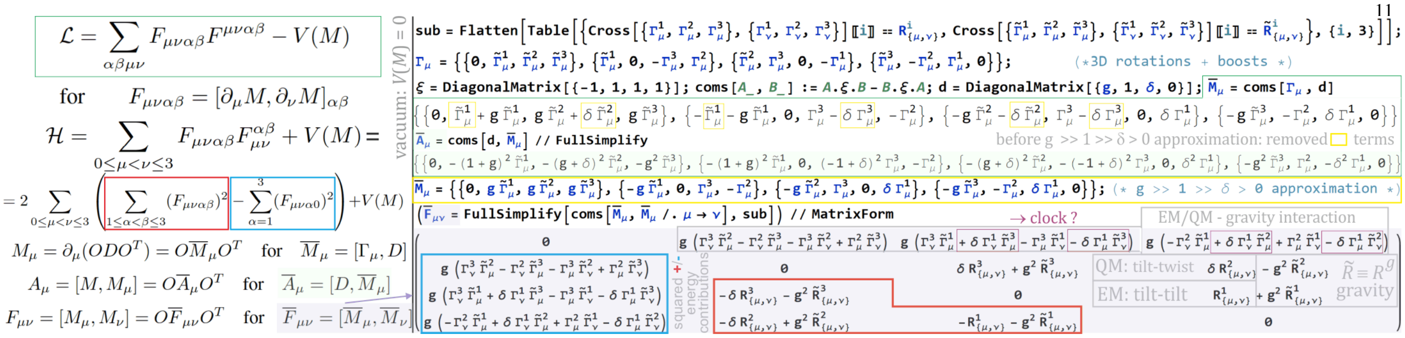

Figure 8. Gathered formulas for suggested 4D model, $M = ODO^T$, $D = \mathrm{diag}(g, 1, \delta, 0)$ with calculated (in GitHub) vacuum $V(M) = 0$, $g \gg 1 \gg \delta > 0$ approximation for $\bar{F}_{\mu\nu}$. With tilde there are noted time axis/gravitational parts, $\vec{R}_{\mu\nu} = \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$. Surprisingly, the Hamiltonian turns out having not only positive (red), but also negative (blue) contributions. Positive energy contributions (red) for separate EM and GEM are as expected ($\sim E^2 + B^2$), would lead to Maxwell equations for independent each of them. However, combined suggest tendency for opposite curvatures - nearly unsatisfied due to very rigid time axis direction (with tilde). The negative energy contributions (blue) give tendency to increase imbalance, such freedom is mostly in $\Gamma_0^1 \tilde{\Gamma}_i$ for $i = 1, 2, 3$ terms (violet) for temporal derivative of twist as in de Broglie clock, with rotation of temporal axis in spatial directions.

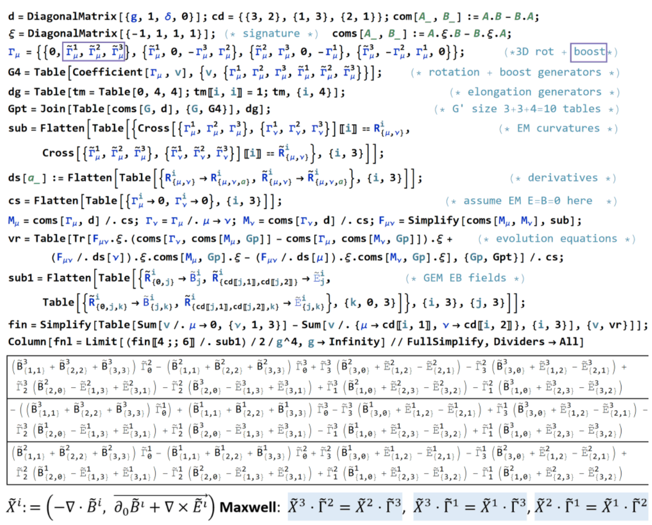

Figure 9. Basic source to derive equations for 4D case (available in GitHub) with $\Lambda = (g, 1, \delta, 0)$ shape and $g \gg 1$. Now we have 10 generators: 3 for 3D rotations corresponding to EM, 3 for boosts as rotations of 0th axis (symmetric vs antisymmetric generators) - leading to GEM, and 4 equations for elongations of 4 axes ($\tilde{\Gamma} \equiv \Gamma^g$, $\tilde{R} \equiv R^{gg}$). Such equations become much more complex, there is derived GEM-only case with zeroed 3D rotations, getting Maxwell-like equations for GEM.

in $\tilde{R}$ gravitational field as in variable speed of light [39] - leading e.g. to gravitational time dilation, or refractive index for light bending through Fermat principle.

The most interesting are negative energy contributions to Hamiltonian due to spacetime signature, which seem unavoidable as also e.g. in Dirac equation. They are mostly $\Gamma\tilde{\Gamma}$ type, e.g. leading to tendency for spatial rotations (toward time Γ_0 or space $\Gamma_1, \Gamma_2, \Gamma_3$) in presence of gravity. Especially for twists Γ_0^1 in $\Gamma_0^1 \tilde{\Gamma}_i$ terms - exactly as required to propel neutrino oscillations and electron's de Broglie clock by mass itself: local $\tilde{\Gamma}$ boosts caused e.g. to reduce energy activating these negative energy terms, or regularization. The latter might require additional $det(M) =$ const (volume preserving) constraint, deforming 0th axis in presence of field regularization in particles.

In contrast, $\Gamma\Gamma$, $\tilde{\Gamma}\tilde{\Gamma}$ same type products have positive energy contributions, leading to energetic tendency for only single nonzero affine connection of given type - prevented e.g. by matter or various types of noise like cosmic microwave background radiation, and noise of other degrees of freedom - which should be thermalized, but difficult to observe, hence acting as dark energy/matter. Energy levels (temperature) of such noise weakens throughout the history of the Universe, what might be crucial in cosmological models. Generally the negative energy contributions should have tendency to create cosmic voids of locally lowered noise levels, and indeed lots of them are observed.

For Newton force naively we would get same sign as for Coulomb this way, what needs to be inversed to make same masses attract. These negative energy contributions seem crucial for this inverse, suggesting e.g. $\Gamma \propto \tilde{\Gamma}$ statistical dependence. Additionally, there are changes of axes lengths (M eigenvalues) - e.g. for noise as dark matter/energy contribution, also necessary in particles for regularization - crucial for their mass, gravity, oscillations. Such derivatives bring additional EM-like energy contributions, which might correspond e.g. to gluons: 8 generators of Yang-Mills theory.

Regarding the Higgs-like potential $V(M)$ minimized for a fixed set of eigenvalues (shape), one approach could be like Landau-de Gennes using traces of powers, now we need to modify it for SO(1,3) to include rotations and boosts. Using $M \equiv M_{\alpha\beta}$, traces of $(M\xi)^p = (-\lambda_0)^p + \lambda_1^p + \lambda_2^p + \lambda_3^p$ are rotation-boost invariants, we could use e.g. $V(M) = \sum_{p=1}^{k} (\mathrm{Tr}((M\xi)^p) - c_p)^2$ potential.

Choosing the details is very difficult, will rather require PDE simulations. The negative energy term should be usually compensated by positive, e.g. due to noises. Obviously the discussed Lagrangian might be incorrect, or simplified e.g. requiring additional terms. Finally such M field represents rotations and boosts of some abstract field - searching for a more concrete one could lead to an even deeper model, e.g. to further reduce the number of parameters.

VI. Correspondence with the Standard Model

Let us try to briefly discuss further correspondence between topological excitations of SO(1,3) vacuum field with the Standard Model, summarized in Fig. 10, 11 and 12.

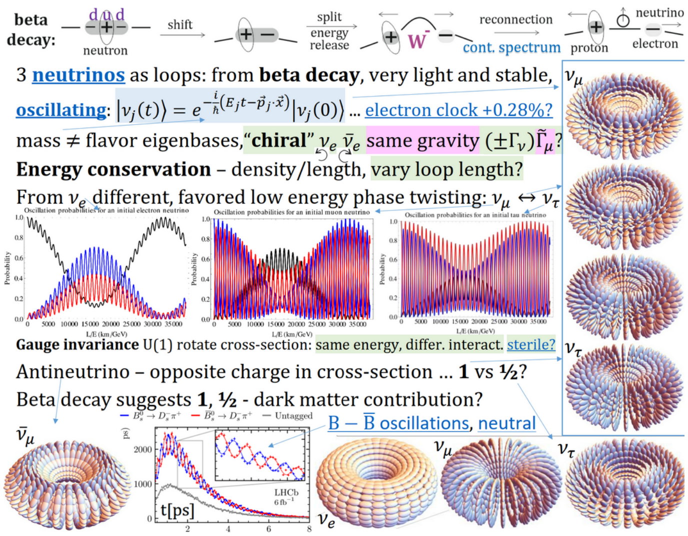

Figure 10. Neutrino model as simple loops of topological vortices - 3 types, very light and stable, unavoidable in (topological) string hadronization, should be created in suggested beta decay mechanism (having continuous spectrum). Also their oscillations (including 3 oscillation plots from neutrino oscillations and one from B-Bbar oscillation Wikipedia articles). The oscillation formulas use the same mechanism as for electron's de Broglie clock [20], but with 3 masses of eigenbasis different from flavor, hence leading to oscillations between flavors - as we can see, mainly between muon and taon neutrino - also in the prosed model. To conserve energy, here mass is proportional to length - such loops could also very length during oscillations. Such vortices along one of 3 axes have always U(1) gauge invariant freedom, however, its rotation should affect interactions - suggested by experiments sterile neutrinos, instead of separate particles, might be a state of standard neutrinos. Antineutrino would have opposite topological charge in cross-section (not Majorana), seems there could be also 1/2 charge, but even more difficult to create and interact - might contribute to dark matter coming e.g. from Big Bang.

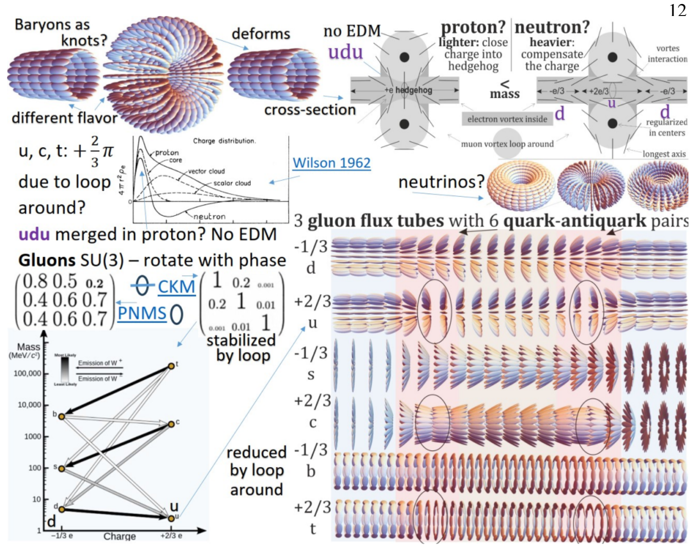

Figure 11. Baryons as the simplest knots, e.g. unavoidable in (topological) string hadronization. As we can see, the outer topological loop enforces charge-like rotation in the inner one - can be fractional for quarks. This way baryons structurally require some charge - proton can enclose it into elementary, while neutron has to compensate it - what is costly, explaining why neutron is heavier. It can have 3 quark structure as required - with positive core, negative shell, as in the shown charge distribution plot from Wilson [47]. In proton such 3 quarks are nearly merged - leading to charge distribution wider than for point charges. There are also shown such 6 quark pairs - as excitations topological vortices along one of 3 axes, of two charges: 2/3 e enforced by loop around - this way reducing energy especially of 'u' up quark, and 1/3 e required to compensate them (energy diagram from Cabibbo-Kobayashi-Maskawa Wikipedia article). There are also shown approximate quark (CKM) and neutrino (PNMS) mixing matrices - the first one is much more diagonal, what can be understood here internal rotations for knots (as baryon) are more difficult than for simple loops (as neutrinos) - due to constraining outer loop.

A. *Topological string hadronization correspondence*

Searching for such correspondence, the best hint seems string hadronization of quark string as Abrikosov vortex like in Fig. 6 - a hot string created in high energy particle collisions e.g. in LHC, decays through reconnections into particle shower.

Thinking about decay possibilities for topological vortices, and searching for correspondence with such particle shower:

- Such vortex can form **simple loops**, which should be very light and stable - the only particle created in colliders agreeing with this description seems neutrino.
- Such vortices could create stable **knots** - these knots might be quite large, and the only known such varying size objects in particle physics are nuclei, with baryon as the simplest knot: vortex loop around another vortex.
- Loops before closing could twist like Möbius strip - such **twisted loops** should be statistically quite frequent, and pions, kaons are dominating in collisions - suggesting to interpret this twist as related to strangeness, also for strange baryons twisting their loop.

The above correspondence seems quite constrained - it is a valuable exercise to search for different possibilities, to realize lack of freedom for its modification. Fortunately, as discussed further, at least qualitatively its consequences seem to agree with experimental view on the Standard Model.

For example here are some observed baryon decay modes - literally releasing some strangeness by pion, or twice larger by kaon - like releasing part of vortex twist to reduce tension as "strangeness decay" in Fig. 12:

$$\text{strangeness 3 to 2 or 1:}\quad \Omega^- \to \Xi + \pi \quad \text{or} \quad \Lambda^0 + K^-$$

$$\text{strangeness 2 to 1:}\quad \Xi^- \to \Lambda^0 + \pi^-, \quad \Xi^0 \to \Lambda^0 + \pi^0$$

$$\text{strangeness 1 to 0:}\quad \Lambda^0 \to p^+ + \pi^- \quad \text{or} \quad n^0 + \pi^0$$

Obviously there are hundreds of observed particles, for which we should search for correspondence e.g. with local energy minima in field configuration space. As they are often very short-lived, these could be shallow minima. Also some are interpretations of perturbative approximation, e.g. Coulomb interaction represented with photon exchange, this way e.g. W, Z bosons could represent dynamical vortex excitations.

B. *Quarks, strings, baryons, nuclei*

Gauss law for a region returns electric charge inside, which should be integer multiplicity of elementary charge e, here enforced by making it topological. However, deep inelastic scattering has shown fractional charges inside baryons - which are believed to be connected by 1D quark strings (what also initiated string theory), often modelled as topological Abrikosov-like vortex [40] - in which there can be added fractional charge excitations through inward/outward field rotation as in Fig. 6. Gauss law for a region cutting such string has (regularized) singularity/conflict in this point, leading to additional energy density per length, which for agreement with QCD should be asymptotically ~ 1 GeV/fm.

Existence of vacuum 1D topological defects as quark string should also have consequences in larger scale, and seems they indeed have. For example in Sun's corona there are seen bright stable lines, called magnetic flux tubes, and suspected to have such topological nature, from [48]: "Vortices in superfluid Helium and superconductors, magnetic flux tubes in solar atmosphere and space, filamentation process in biology and chemistry have probably a common ground, which is to be yet established. One conclusion can be made for sure: formation of filamentary structures in nature is energetically favorable and fundamental process". Also 1D structures are postulated in cosmological scale as cosmic strings, with first claims of experimental confirmation [49].

With suggested baryon as the simplest knot, looking at diagram e.g. in 11, we can see that the loop around deforms structure of internal vortex - exactly into inward/outward field rotation required for charge. If this rotation would be by π, we would get hedgehog corresponding to elementary charge. Here it can be smaller - requiring a conflict on such string, having additional energy density per length - should be ~ 1 GeV/fm for quark string, making it energetically expensive to take quarks apart - confining them. However, its contribution weakens in high energy for asymptotic freedom.

Proton can enclose this structurally enforced fractional charge into hedgehog being elementary charge, while neutron has to compensate it to zero total charge - explaining larger mass. It agrees with required charge distributions from literatures e.g. [47], [50] - neutron as having positive core and negative shell, proton only positive but smeared in comparison to point charge - suggesting 3 quarks very close together.

For deuteron, both baryons require fractional positive charge - can share a single elementary charge to reduce energy for binding, as in Fig. 12. This way it is "+-+" charge distribution - has relatively large electric quadrupole moment, what is known from experiment. In contrast, **deuteron** as just proton + neutron would have no electric quadrupole moment - requiring e.g. shift of quarks. Deformation of quark distribution when nucleons combine into nuclues is generally referred as EMC effect [51], and seems not currently understood - the proposed model could help with. Also spins of both baryons should be aligned as $\mu_d \approx \mu_p + \mu_n$, what agrees with predicted view.

Quark strings e.g. as topological vortices might be crucial to bind larger nuclei against Coulomb repulsion. Especially for halo nuclei [52] - stably binding single (halo) neutrons or protons in a few femtometer distance. Believed to require 3-body forces [53], forming Borromean structures. For example Lithium-11 binding 2 neutrons for miliseconds in a few femotometer distance, larger than of strong force. Binding them through connection by (topological) quark strings could explain both stability in large distance, and 3-body forces. This way we could see larger nuclei as knots of quark strings.

As there is no Gauss law for baryon number, such view on baryon as knot of quark strings could allow to violate baryon number conservation. Such violation is required e.g. for baryogenesis (creation of more baryons after Big Bang), or (massless) Hawking radiation. The latter suggests ultimate energy source by squeezing baryons into black hole, and gathering energy from its evaporation. If possible through black hole, it should be doable also directly by stimulation of e.g. proton decay, for example optimizing parameters for particle collisions, or shooting incoming proton beam with free electron laser. For baryon as knot, e.g. trying to swing to untangle: internal charge of proton with electric field, or twist the magnetic dipole with magnetic - finalizing the proposed model, we could numerically optimize its parameters.

C. Neutrinos

While EM waves are stopped by a centimeter of lead, neutrinos can easily pass the Earth - need some stabilization mechanism, like topological. From the other side, having quark strings it seems unavoidable for them to make loop, which should behave as particle, would be very stable and light - and we know only neutrinos in agreement with these properties.

Also they have 3 flavors - as topological vortices: along one of 3 axes for 3 spatial directions. They can oscillate between flavors, vortices through field rotations, like in Fig. 10. As in this diagram, oscillation for low energy twist should be more likely - and indeed there is dominating between muon and taon neutrino, electron neutrino turns out essentially different.

These oscillations should be propelled by the mass itself, as for electron's clock using $\psi \propto \exp(-iEt/\hbar)$ phase evolution for $E = mc^2$ relativistic mass, also angular momentum (of the field not point particle). These configurations are relatively simple - are planned to be studied in details to understand this propulsion through mass. Field rotations (affine connection) are: around vortex, around loop, temporal for oscillations Γ_0, and also boosts $\tilde{\Gamma}$ from mass - products with boosts have negative energy contribution in Hamiltonian in Fig. 8, however, time dependence has also positive energy contributions from the remaining products. Minimization of their sum should lead to the observed oscillation parameters, allowing to constrain the model. For antineutrino, gravitational mass should be the same, but field rotation around vortex is reversed - what should explain left/right-handedness of (anti)neutrinos.

While electron has only U(1) degree of freedom for field rotations, for vortices additionally we have freedom for SO(3)$\sim$ SU$(2)_L$ (as SU(2) is double covering of SO(3)). Direct electron clock experimental confirmation [20] required 0.28% higher energy than predicted - this difference might come from still 3 types of tendencies (as for neutrino) acting in electron, but kind of being projected (added) into single allowed evolution degree of freedom.

Finally to effectively describe weak interactions, we indeed need U(1)$\times$SU$(2)_L$ symmetry group. Such field rotations for vortices are relatively unconstrained, in contrast to their knots - with loops blocking field rotations, hence making mixing matrices much more diagonal (CKM vs PNMS), enforcing additional phase change during spatial rotations - requiring to use full SU(3) symmetry group for strong interactions. As mentioned in Fig. 4, gluons in Yang-Mills term could be also interpreted as field curvature with one connection up the Higgs potential: changing M eigenvalue.

To oscillate changing flavors, there are needed 3 different masses - suggesting oscillations vary mass, what naively would mean violation of energy conservation. It is repaired here as this is mass per length - allowing to conserve energy by also varying loop length. A basic source of neutrino is beta decay of neutron, which has continuous energy spectrum, suggesting also varying length of created vortex loop, mass of the electron antineutrino. While experimental data finds lighter and lighter neutrinos, maybe we should not treat them as boundaries of the mass - this mass could be varying, as for continuous spectrum we should rather consider its probability distribution.

Cross-section of such vortex loop has topological charge - beta decay suggests it should ± 1. However, spin 1/2 suggests field with $n \equiv -n$ type symmetry, allowing also for vortices with $\pm 1/2$ charge in cross-section, which would be even more difficult to interact with - might contribute to dark matter. Also, U(1) field rotation freedom for vortices of given flavor, while it does not change energy or flavor, seems crucial e.g. for beta decay-like interactions - their probability should depend on this rotation, being less likely could be interpreted as currently required sterile neutrinos - not as separate particles, only less likely interacting intrinsic state of standard neutrinos.

D. Hints for Standard Model issues

The goal of the discussed approach is not replacing the Standard Model, only trying to get its better/deeper understanding - of (nonperturbative) field configurations and their evolutions for all the particles and Feynman diagrams. It for example suggests answer where Standard Model has issues, like the below:

- Standard Model has originally predicted zero neutrino masses, which turned out nonzero - as vortex loops here, they have mass/energy density per length.
- Quark mass is only about 1% of proton mass - here baryon is much larger structure, up quark has reduced mass by vortex loop around.
- Proton spin crisis [54] - while it was expected that quarks carry all proton's spin, experimental data suggest it is barely $4-24\%$. For proton as simple knot, angular momentum is distributed over the entire field configuration,
- Proton radius puzzle - (root mean squared) radius through interactions with electron measured as ≈ 0.877 fm, with muon ≈ 0.842 fm [55]. Here electron and muon should have field rotated by $\pi/2$, what should modify interactions with proton, also e.g. using taon.
- Neutron lifetime puzzle - turns out that cold "bottle" neutrons have ≈ 878 seconds lifetime, while hot "beam" ≈ 887 seconds [56], [57] - neutron as knot has intrinsic excitation modes, which deexcitation time should correspond to this time difference.
- EMC effect - turned out quark distributions in nucleons are modified when binding into nucleus [51] - also here e.g. leading to electric quadrupole moment of deuteron.
- Three-body force, halo nuclei seem not well understood - here rather requiring topological vortices as quark strings connecting the nucleons - binding halo neutrons/protons, and effectively acting as 3-body interactions.
- For dark matter e.g. lighter versions of neutrinos in Fig. 10 are suggested. For dark energy could be noise of degrees of freedom of all interactions like CMBR for EM. Baryons could be created in baryogenesis here. Gravity is automatically unified as GEM.

While these seem valuable suggestions for issues of Standard Model, the details will rather require finalizing such proposed deeper model and performing numerical simulations.

VII. Conclusions and further work

There was briefly presented mathematical framework allowing for EM + pilot wave + GEM unification for topological configurations e.g. in (superfluid) liquid crystals, extending Faber's approach: vectors $\rightarrow$ matrices, to make Γ, R now include shape dependence becoming A, F, this way distinguishing EM, QM, GEM vacuum dynamics, and further leading to promising correspondence with the Standard Model, which seems required for string hadronization using topological vortices.

This article is work in progress, which is planned to be further developed: aiming as good agreement with particle physics as possible - both for better understanding, also maybe to try to recreate some phenomena with liquid crystal experiments, like observation of additional fluxon-like vortex (disclination) for hedgehog configuration in biaxial nematic (no naked charges), transformation between 3 types of vortices as neutrino oscillation analogy, their quark-like excitations, hadronization as decay of such vortex into particles, etc.

One main open question is choosing the potential - e.g. depending only on the shape $V(M)$, or maybe also derivatives $V(A)$, ideally with minimal number of parameters like only preferred eigenvalues (or less). A natural direction is through search for agreement with 3 leptons as hedgehog of one of 3 axes as in Fig. 1: they should form 3 local minima in the space of possible rotations of hedgehog ansatz, probably stabilized by the enforced magnetic vortices.

We should also get neutrino oscillations enforced by mass, and analogous electron's intrinsic periodic process (Zitterbewegung/de Broglie clock [20]) probably due to negative energy terms in Hamiltonian as in Fig. 8. Details are yet to be developed, e.g. gravitational mass might require e.g. fixing $\det(M)$ constraint suggested in [36]. EM-GEM interaction slowing down EM propagation should explain graviational time dilation and lensing.

Calculations like started in Fig. 2, 3, 7 and 9 are crucial development direction, also parametrizations to moduli space, trying to extend correspondence with particle physics, finally performing 2nd quantization aiming agreement with the Standard Model. Preferably also full hydrodynamical simulations to better understand the configurations and dynamics.

To summarize, while the main focus is on QFT pertubative approximations, the real situation is given by nonperturbative QFT - understanding field configurations of particles and Feynman diagrams, before considering their Feynman ensembles. Especially the string hadronization as topological requires quite constrained correspondence, discussed in Section VI and summarized in Fig. 12. While we automatically get looking perfect qualitative agreement, quantitative will require further work: finalizing the model including potential, and performing (numerical) calculations - e.g. to derive ≈ 20 Standard Model parameters from ≈ 3: $\sim \hbar, \sim G$ for QM and gravity energy scales, and in Higgs-like potential. Finally, the discussed M is some abstract field recognizing SO(1,3) dynamics - searching its concrete realization could lead to even deeper model, e.g. to further reduce the number of parameters.

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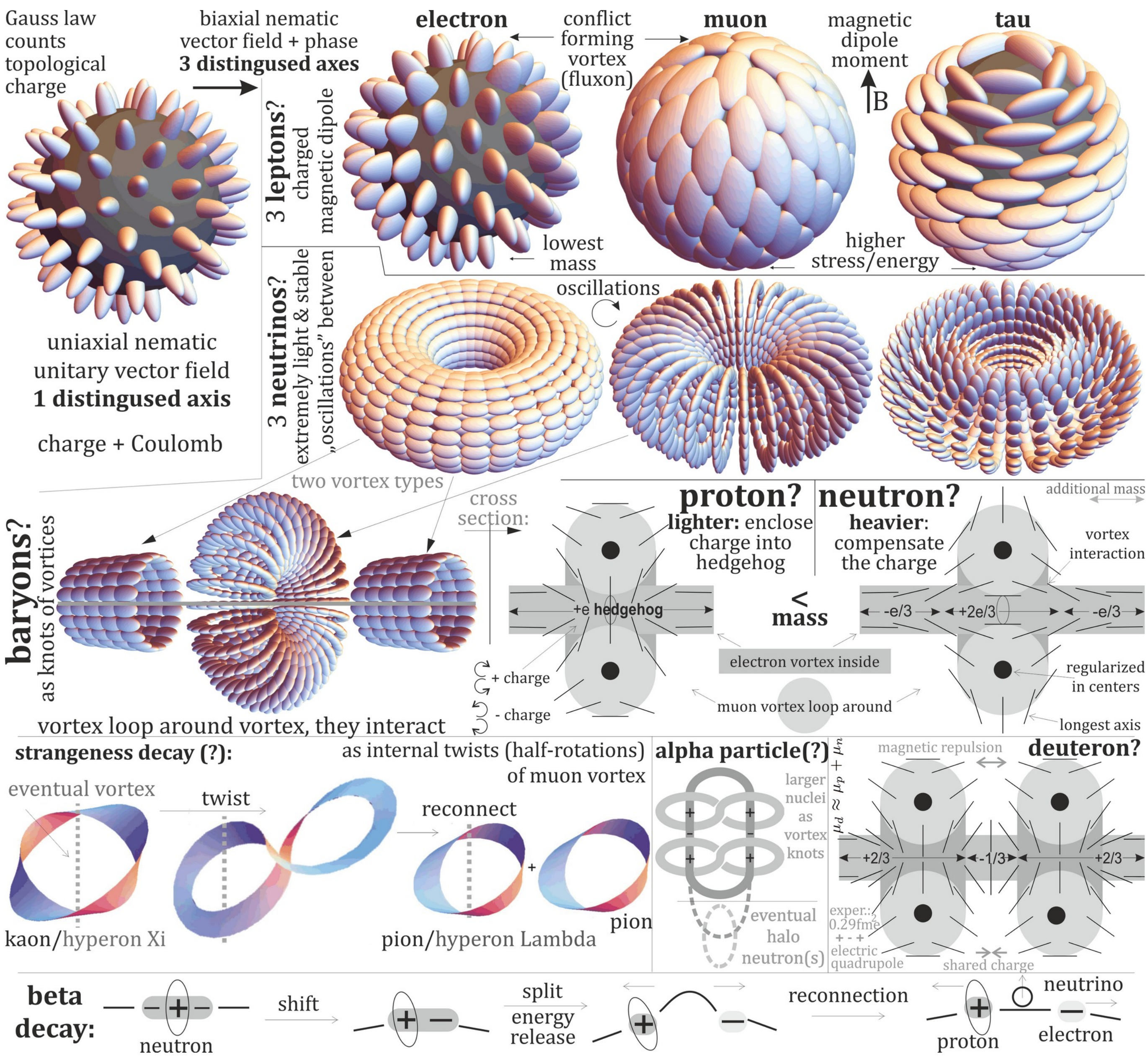

Figure 12. Further particle-like configurations nearly required by string hadronization as topological - discussed in Section VI. Hedgehog configurations of one of 3 axes resemble **3 leptons**: the same electric charge (as topological), but different realization: regularization, mass. Hairy ball theorem says there is a conflict of axes on the sphere - leading to outgoing vortex (2D topological charge) of one of 3 types (along one of 3 axes) - like fluxons in superconductor carrying magnetic field: enforcing magnetic dipole moment. Short loop of such vortex would be very light and difficult to interact with, resembling **3 neutrinos** - with possible oscillations between each other through field rotation (Fig. 10), they should be produced in beta decay. Such vortices, corresponding to "quark strings" in QCD, can further form knots, which resemble baryons, nuclei. As in Fig. 11, interaction between vortices inside such knot with vortex loop around enforces charge-like (hedgehog) configuration inside: makes that **baryon configuration requires some charge** - can be fractional, but all sum to integer charge (confinement). Proton can just enclose this charge (hedgehog), but neutron has to compensate it to zero - suggesting **why neutron is heavier** (naively should be lighter due to charge), through quark-like fractional charge structure. Such concluded: positive core, negative shell charge distribution of neutron is suggested in literature e.g. [47], [50]. It also suggest **binding mechanism for deuteron**: as two baryons sharing required charge - also explaining observed relatively large electric quadrupole moment (experimentally: 0.2859 e fm^2) and aligned spins ($\mu_d \approx \mu_p + \mu_n$). Binding of larger nuclei could additionally use vortices forming stable knots, e.g. **halo nuclei** with neutrons stably bind in large a few femtometer distance. Vortex loop might have additional internal twist, which is quantized ($k\pi/2$) and resembles **strangeness** - relaxed through muon/kaon production in decay of strange baryons.