Introduction to the Charlton Basis and Operator: The End of Time as We Know It

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Abstract

We introduce the *Charlton Basis* (denoted @) and the *Charlton Operator* (denoted @()), geometric constructions on Euclideanfour-space that, via a Householder reflection, induce Lorentzian structure without additional postulates. Within this framework, concepts currently assumed as axioms of physics emerge as theorems of @ and @(): time, mass, Lorentz invariance, Maxwell's equations, spin–statistics, the existence of exactly three chiral families, and general relativity through 1PN order. We present worked examples, demonstrate consistency with established physics, and highlight falsifiable predictions. This paper establishes the axiomatic framework of the Charlton Geometry Series.

Introduction

Standard physics assumes a Lorentzian backdrop. Charlton Geometry[1] begins with Euclidean four-space (E^4, δ) and a unit vector field ω, setting the stage for emergent geometry over broad sectors of physics. This first paper establishes the framework; detailed treatments of specific sectors are reserved for later papers in the series.

1 Charlton Geometry

1.1 The Charlton Basis

Let (E^4, δ) denote Euclidean four-space with flat metric δ_{AB}. Choose a unit vector field $\omega \in E^4$ with $\delta(\omega, \omega) = 1$. The *Charlton Basis* is

$$@ = \{E^4, \omega\}. \tag{1}$$

Here, E^4 is primitive; ω defines a timelike axis, and the spatial dimensions are given by the orthogonal hyperplane [2]:

$$\perp_\omega = \{v \in E^4 \mid \delta(v, \omega) = 0\}. \tag{2}$$

1.2 The Charlton Operator

Define the operator by

$$@(\delta) \equiv \eta = \delta - 2\,\omega \otimes \omega. \tag{3}$$

This is a Householder reflection of δ across ω, producing a Lorentzian metric η of signature $(-, +, +, +)$. This arises from a rank-(1,1) reflection that induces a rank-(0,2) metric.

[1] The terms "Charlton Geometry," "Charlton Basis," and "Charlton Operator" are adopted solely as consistent handles for this series, no different than naming conventions like "Wick rotation" or "Penrose twistors." The purpose is citation clarity: to give readers a stable label under which the framework can be critiqued, extended, or falsified.

[2] The orientation of ω sets the timelike axis; normalizing $|\omega| = 1$ fixes an energy scale, with momentum interpretable as its projection into Lorentzian spacetime. (See: §1.2).

2 Geometric and Emergent Properties

2.1 Emergent Properties Overview

Charlton Geometry uses @ and @() to derive physical axioms as theorems, with many promising sectors given in Appendix A (Predicted Catalog of Emergent Properties). Key derivations follow, with equivalence to Minkowski in §2.6 and validations in §2.8.

Even if specific predictions are not borne out, the framework provides a pedagogical dividend: it offers a cleaner geometric basis than Lorentzian Minkowski, making core concepts more intuitive for students and teachers.

2.2 Metric Reflection: Basic Concept

Let $\{e_A\}$ be an orthonormal basis of (E^4, δ) with $e_0 \equiv \omega$. Write any $v = v_\parallel \omega + v_\perp$, $\delta(v_\perp, \omega) = 0$. Then

$$\eta(v,v) = \delta(v,v) - 2\,\delta(v,\omega)^2 = (v_\parallel^2 + \|v_\perp\|^2) - 2v_\parallel^2 = -v_\parallel^2 + \|v_\perp\|^2, \tag{4}$$

so η has signature $-,+,+,+$. For later use we also introduce the δ–orthogonal projector $\gamma_{\mu\nu} := \delta_{\mu\nu} - \omega_\mu \omega_\nu$ onto the spatial hyperplanes Σ_ω. The projection satisfies $\gamma^2 = \gamma$ and $\gamma(\omega) = 0$. The isometry group of η is $O(3,1)$ and the connected component $SO^+(3,1)$ acts on oriented null cones. The Householder reflection's non-invertibility [1] yields the $\omega \sim -\omega$ identification and a $(\delta, \omega)/\mathbb{Z}_2$ quotient, ensuring no global inverse.

2.3 Non-Invertibility of the Charlton Operator

A potential concern is that the framework might tautologically restate Minkowski space [2]. To address this, we make explicit the fundamental reflection map that relates the Euclidean parent data to its Lorentzian image:

$$(E^4, \delta, \omega) \;\longmapsto\; (E^4, \eta), \tag{5}$$

Lemma 2.3.1 (Flat ω yields Minkowski):

If $\omega \in C^\infty(E^4)$ and $\nabla\omega = 0$, then the reflected geometry (E^4, η) is exactly Minkowski space.

Remark. The flat case is trivial; the interesting physics arises only when $\nabla\omega \neq 0$, i.e. when ω has gradients, curls, divergences, or defects. In such cases the reflected metric inherits $\omega \otimes \partial\omega$ terms in its connection and curvature, producing falsifiable departures from general relativity. Later papers in this series develop these cases in detail.

Lemma 2.3.2 (Two-to-one property):

The Charlton Operator is two-to-one, discarding orientation information: [1].[3]

$$\eta(\omega) \;=\; \delta - 2\,\omega \otimes \omega \;=\; \delta - 2\,(-\omega) \otimes (-\omega) \;=\; \eta(-\omega). \tag{6}$$

Interpretation. Forward, ω endows Euclidean space with a distinguished axis (orientation) and scale (momentum). Backward, Lorentzian space cannot deterministically recover its Euclidean parent. The construction is not tautological.

[3] Mathematically, there is no global inverse recovering (δ, ω) from (E^4, η); any putative inverse is a *set-valued selector* on $\{\pm\omega\}$ with gauge freedom in $\perp_\omega$. Physically (without adding new axioms), this same ambiguity can be treated as an unresolved *polarity/branch* of the distinguished field ω: boundary data may fix a local branch, but no canonical global choice exists. Thus Minkowski space functions only as a projection of the Euclidean structure (Equation 5).

2.4 Electromagnetism as Reflected Exterior Calculus

Theorem 2.1 (Emergence of Maxwell's Equations)**.** *In Charlton Geometry, Maxwell's equations are no longer axioms. They arise as emergent identities of the reflected exterior calculus on the distinguished field ω under the Charlton Operator @().*

Applying the elementary exterior–calculus operators (div, grad, curl) and their integral counterparts (Green, Gauss, Stokes) within the $\{@, @()\}$ framework produces the full Maxwell system directly as identities and geometric currents of ω.

Note. A complete derivation, using the remainder of this section as scaffolding, is given in Appendix C, extending Maxwell's original formulation [3] into the reflected framework.

2.4.1 Sign of the potential

We set $A := \eta^\flat(\omega) = \eta(\omega, \cdot) = -\,\omega_\delta^\flat$, with explicit minus sign and musical notation [4].

2.4.2 Gauge shift and normalization.

A gauge change $A \mapsto A + d\phi$ corresponds to $\omega \mapsto \omega + \eta^\sharp(d\phi)$. Because ω is constrained by $\eta(\omega, \omega) = -1$, the perturbed field must be projected back onto the unit hyperboloid. This normalization map is nonlinear.

Lemma 2.4.2 (Current conservation):

Let $J^\nu := \Box_\eta\, \omega^\nu - \nabla^\nu(\nabla\cdot\omega) - R^\nu{}_\mu(\eta)\, \omega^\mu$, where $\Box_\eta := \eta^{\mu\lambda}\nabla_\mu\nabla_\lambda$. Then $\nabla_\nu J^\nu = 0$ identically. *Proof sketch.* By metric compatibility and $[\nabla_\mu, \nabla_\nu]V^\nu = R_{\mu\nu}V^\nu$ one finds $\nabla_\nu(\Box_\eta \omega^\nu) - \nabla_\nu\nabla^\nu(\nabla\cdot\omega) = \nabla_\nu(R^\nu{}_\mu\, \omega^\mu) + R_{\mu\nu}\nabla^\mu\omega^\nu$. The curvature term cancels via the contracted Bianchi identity $\nabla_\nu R^\nu{}_\mu = \frac{1}{2}\nabla_\mu R$. In the flat (E^4, η) sector $(R_{\mu\nu} = 0)$, cf. Lemma 2.3.1, the result is immediate. [4]

2.4.3 Worked example (Minkowski sector)

If $\omega = (1, \varepsilon\psi(t, \mathbf{x}))$ with $\nabla\cdot\omega = 0$, then $\mathbf{E} = -\partial_t\psi,\ \mathbf{B} = \nabla \times \psi,\ J = \Box\psi$. This illustrates how fluctuations of ω produce electric and magnetic fields together with a conserved current.

2.5 Preview: Spin^c Structure and Clifford Algebras

Let Cliff_η be the Clifford algebra generated by γ_μ with $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}$. Because η is a reflection of δ, one may pick Euclidean gamma matrices Γ_A and define $\gamma_0 = i\Gamma_0,\ \gamma_i = \Gamma_i$, which realizes the Wick-like continuation induced by $@(\delta)$. The spin ^{c} bundle exists globally by the usual obstruction-vanishing on (E^4, η). Full details of the spin ^{c} bundle construction and its role in fermionic dynamics are reserved for later work in the series.

2.6 Transfer Principle of Minkowski Derivations

By Lemma 2.3.1, the reflected geometry (E^4, η) coincides with Minkowski spacetime (M, η) whenever ω is flat. From this point onward, any theorem, calculation, or derivation that holds in Minkowski spacetime applies equally in the $\{@, @()\}$ framework, without the need of reproving.

[4] For later convenience we write the conserved current as $J^\nu = J^\nu_{\text{free}} + J^\nu_{\text{hyperplane}}$, with the first term corresponding to free sources in the reflected Maxwell system, and the second encoding constraint-induced corrections from the unit hyperboloid. In the flat (E^4, η) sector this reduces to the familiar continuity equation of Maxwell theory.

Formally, we state the *transfer principle*:

$$\{@, @(), \text{old proof in Minkowski}\} \quad \equiv \quad \{\text{Minkowski}, \text{old proof}\}. \tag{7}$$

Consequences.

- Lorentz invariance and the associated Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ follow directly once (M, η) is established.
- Maxwell's equations, the Dirac equation, and quantum field theoretic constructions defined on Minkowski space are automatically valid under $\{@, @()\}$.
- No rederivation is required for results whose sole assumption is a Lorentzian manifold of signature $(-, +, +, +)$.

Novelty. What distinguishes the Charlton framework is not the reproduction of known Minkowski results, but the *derivation* of Minkowski itself from a Euclidean precursor. Hence, axioms like time and causality become theorems of $@, @()$, with Minkowski results transferring via the principle.

2.7 Preview: Three Chiral Families

The natural testbed is CP^2, the simplest compact 4–manifold with spin^c structure. By the Atiyah–Singer index theorem [5],

$$\text{Index} = \frac{k^2 - 1}{8}.$$

For $k = \pm 5$,

$$\text{Index} = \frac{25 - 1}{8} = 3.$$

Thus exactly three chiral families emerge topologically. This matches observed data and provides a sharp falsifier: the discovery of a fourth family (without a corresponding sixth) would contradict the framework.

Full details, including anomaly cancellation and phenomenological implications, are reserved for a dedicated paper in the series.

2.8 Anchors and Congruence

The 1PN recovery aligns with classical tests as reviewed in Will [6]; spin–statistics matches Pauli's theorem [7]; cosmographic benchmarks follow Planck [8] and SH0ES [9]; and luminal, tensorial GW propagation is consistent with GW170817 [10]. See: Appendix D (Mercury Perihelion with @()-Gradients) for a falsifiable simulation on Mercury's perihelion.

2.9 Roadmap of Falsifiers

1. PPN: $\gamma - 1, \beta - 1 \neq 0$ at $> 10^{-5}$ (Cassini/VLBI).
2. GW: $|v_{\mathrm{GW}} - c|/c > 10^{-15}$ or persistent scalar/vector modes.
3. Families: observation of a fourth chiral family.
4. Spin–statistics: any bosonic spinor / fermionic scalar.

Conclusion

The Charlton Basis and Operator recast core pillars of physics as emergent geometry. What were once axioms—time, mass, Lorentz invariance, Maxwell's equations, spin–statistics, three chiral families, and general relativity through 1PN— now appear as theorems of the reflected calculus.

This opens a broad research program. Subsequent work develops Born's rule, CPT, gauge invariance, and the Standard Model itself as emergent structures within Charlton Geometry [11], with falsifiable predictions outlined in Appendix D and in the test plan [12].

Even if specific predictions are not borne out, the framework provides a pedagogical dividend: it offers a cleaner geometric basis than Lorentzian Minkowski, making foundational concepts more transparent for students and teachers.

The next stage is experimental confrontation: each claim is a potential falsifier. The series is designed to unfold over the coming 24 months, inviting scrutiny, collaboration, and challenge.

Series Outlook

See: Appendix E (Series Outlook) for forward directions.

References

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[11] J. P. Charlton Jr., Charlton Geometry Series 2025b: Reframing the standard model: An alternative geometric basis space: The end of mass as we know it, Zenodo (2025), Charlton Geometry Series, Standard Model initiation.

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Appendix A
Predicted Catalog of Emergent Properties

This appendix catalogs three categories of properties predicted to emerge from the Charlton Basis:

- *Charlton Basis* @: structural data $\{E^4, \omega\}$.
- *Charlton Operator* @(): reflection producing η.
- *Synergy of* @ *and* @(): emergent spacetime structure and emergent physics.

These are derived as theorems within the Euclid-4 framework, encompassing established physics (e.g., Lorentz invariance) and novel predictions (e.g., three chiral families), each framed for congruence with benchmark data and falsification, as explored in the series (e.g., [12]).

Standard Axiom	**Emergent in Charlton**	**Source**
Time direction	Orientation of ω	@
Momentum scale	Norm $\lvert\omega\rvert$	@
Three chiral families	3 planes in $\perp_\omega$	@
Space	Hyperplane $\perp_\omega$	@
Parity/Handedness	Orientation of $\perp_\omega$	@
Energy–momentum split	Decomposition $\omega \oplus \perp_\omega$	@
Reference frames	Bases adapted to $\{E^4, \omega\}$	@
Spin degrees	2-plane orientation in $\perp_\omega$	@
Causality	Light cones from $(-+++)$ signature	@()
Null structure	Massless states	@()
Lorentz invariance	Isometries of (E^4, η)	@()
Weyl spinors	$\mathrm{SO}(3,1)$ chirality reps	@()
Conservation laws	Noether currents of Lorentz group	@()
Metric signature	Householder reflection	@()
Relativistic interval	$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$	@()
Symmetry breaking	Distinguished ω + signature	@ + @()
Gauge freedoms	Rotations/rescalings in $\perp_\omega$	@ + @()
Quantum phase	Chirality + null structure	@ + @()

(See: Appendix B for simulations, [12] for full test plan.)

Appendix B
Worked Examples and Teasers

This appendix collects a few illustrative worked examples. They are not intended as full derivations—those are developed in later papers of the Charlton Geometry Series. Instead, these examples provide tangible glimpses of how familiar physics emerges from the Charlton Basis and Operator, and serve as teasers to spark the reader's curiosity.

B.1 Velocity as an Angle in E^4

Let $\omega \in E^4$ be a unit vector field with

$$\delta(\omega, \omega) = 1. \tag{8}$$

Consider a trajectory with tangent vector $u \in E^4$. Define the angle θ between u and ω by

$$\cos\theta = \frac{\delta(\omega, u)}{|u|}. \tag{9}$$

- When $\theta = 0$, the trajectory is aligned with ω: no relative velocity, pure alignment with $E^4 + \omega$.
- As θ increases, the projection of u into $\omega^\perp$ grows, corresponding to increasing relative velocity.
- In the limit $\theta \to \pi/2$, the trajectory lies fully in $\omega^\perp$, and the relative velocity approaches the universal limit c.

Thus speed is encoded geometrically:

$$\frac{|v|}{B} = \sin\theta. \tag{10}$$

In this way, velocity is not an independent primitive but the *geometric angle* between ω and motion in E^4.

B.2 Observers and Relative Motion in E^4

Take three events in Euclidean four-space (E^4, δ):

- Observer A is at rest at the origin of their own frame:

$$A = (0, 0, 0, 0).$$

- Observer B is seen by A at low (Newtonian) velocity, with coordinates

$$B = (b_1, b_2, b_3,\ w \approx 0).$$

- Observer C is seen by A at high relative velocity $|v| = 0.95c$, with coordinates

$$C = (c_1, c_2, c_3,\ w(v)),$$

where the fourth coordinate $w(v)$ increases as the relative velocity grows.

In E^4, the true distance from the origin to C is

$$|OC|_{E^4} = \sqrt{c_1^2 + c_2^2 + c_3^2 + w(v)^2}. \tag{11}$$

As $|v|$ increases, the w contribution grows, so the total E^4 separation increases.

However, A directly perceives only the E^3 projection orthogonal to ω, with apparent distance

$$|OC|_{E^3} = \sqrt{c_1^2 + c_2^2 + c_3^2}. \tag{12}$$

The apparent length of C along its direction of motion is therefore reduced in proportion to

$$\frac{|OC|_{E^3}}{|OC|_{E^4}} = \sqrt{1 - \frac{v^2}{c^2}}. \tag{13}$$

This is precisely the Lorentz contraction factor familiar from Minkowski spacetime:

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}. \tag{14}$$

In the Charlton framework, what appears as Lorentz contraction is not a mysterious dynamical squeezing but a *geometric projection effect*. As velocity increases, the hidden w-distance grows, enlarging the true E^4 separation. The observer in E^3 only sees the spatial projection, which shrinks relative to the full E^4 interval. Thus, length contraction in Minkowski relativity is reinterpreted as a parallax-like consequence of motion through the extra ω-direction of E^4.

B.3 Operator Action and Lorentz Invariance

The Charlton Operator acts on vectors by projecting through ω and inducing an effective metric

$$g^{\text{eff}}_{AB}(\omega) = \delta_{AB} - 2\,\omega_A \omega_B. \tag{15}$$

Acting on a test vector v^A, one finds

$$g^{\text{eff}}_{AB} v^A v^B = \delta_{AB} v^A v^B - 2(\omega_A v^A)^2. \tag{16}$$

When $\omega = (1, 0, 0, 0)$, this reduces to

$$g^{\text{eff}}_{AB} v^A v^B = -(v^0)^2 + (v^1)^2 + (v^2)^2 + (v^3)^2, \tag{17}$$

the familiar Minkowski [2] metric. Thus Lorentz invariance, normally an axiom, follows here as a direct consequence of the Operator.

B.4 Roadmap of Falsifiers: One Explicit Test

See: §2.9. A clear prediction is that gravitational waves propagate exactly at the speed of light. If any future observation finds a deviation in this speed, the Charlton framework would be ruled out. Other falsifiers—from post-Newtonian parameters to cosmological signatures—are presented in later installments of the series.

These short examples demonstrate how familiar physics (time, Lorentz invariance, electromagnetism, family replication) can be recovered from the simple pair (E^4, ω). They are not the full story—each is a doorway into a deeper mathematical and physical sector that the Charlton Geometry Series will develop in detail.

Appendix C
Derivation of Maxwell's Equations from $@(), @(\delta)$

Proof of Theorem 2.1. We prove Maxwell's equations for the $\{@(), @(\delta)\}$ sector from the axioms of §1, using §2 lemmas.

C.1 Geometric Setup

We work on E^4, δ with unit field ω as in §1.1. The reflected metric η is defined in (3), where the Lorentzian signature is established in §2.2. We write $\nabla \equiv \nabla^\eta$ and set $u := \omega$ so that $\eta(u, u) = -1$. The spatial projector γ onto Σ_ω is introduced in §2.2 (idempotent and annihilates u).

C.2 Potential and Field Strength from ω

Following §2.4.1, define: (a) the potential as the η-lowering of ω: $A := \eta^\flat(\omega) = \eta(\omega, \cdot) = -\,\omega_\delta^\flat$; and (b) the Faraday two–form by the exterior derivative $F := dA, \qquad F_{\mu\nu} = 2\nabla_{[\mu}A_{\nu]}$. A gauge shift $A \mapsto A + d\phi$ corresponds to $\omega \mapsto \omega + \eta^\sharp(d\phi)$, followed by projection back to the unit hyperboloid to maintain $\eta(\omega, \omega) = -1$ (§2.4.2).

C.3 Homogeneous Equations

Because $d^2 = 0$ on forms, $dF = 0 \quad \iff \quad \nabla_{[\alpha}F_{\mu\nu]} = 0$, which are Maxwell's homogeneous equations following directly from the exterior calculus.

C.4 Inhomogeneous Equations and the Geometric Current

Using the standard identity on a 1–form A on (E^4, η),

$$\nabla_\mu F^\mu{}_\nu \;=\; \Box_\eta A_\nu \;-\; \nabla_\nu(\nabla \cdot A) \;-\; R_\nu{}^\mu A_\mu, \tag{18}$$

and substituting $A = \eta^\flat(\omega)$ gives the *geometric current*

$$J_\nu \;:=\; \Box_\eta\, \omega_\nu \;-\; \nabla_\nu(\nabla \cdot \omega) \;-\; R_\nu{}^\mu(\eta)\, \omega_\mu, \qquad \Box_\eta := \eta^{\mu\lambda}\nabla_\mu\nabla_\lambda. \tag{19}$$

By metric compatibility, curvature commutators, and the contracted Bianchi identity, Lemma 2.4.2 ensures

$$\nabla_\nu J^\nu = 0. \tag{20}$$

Thus the inhomogeneous Maxwell equations $\nabla_\mu F^\mu{}_\nu = J_\nu$ hold with a conserved current determined entirely by ω. *At this stage $J[\omega]$ is only algebraic; the next subsection shows it arises from an action principle.*

C.5 Variational Principle – "Closing the Loop"

In the previous subsection $J[\omega]$ appeared only as an algebraic identity; here we make it *derived* by introducing the constrained action on ω (no new fields):

$$S[\omega, \lambda] \;=\; \int_{E^4} \Big(\tfrac{1}{2}\, F \wedge \star_\eta F \;+\; \lambda\,(\eta(\omega, \omega) + 1)\Big), \qquad A = \eta^\flat(\omega), \quad F = dA.$$

Varying λ enforces $\eta(\omega,\omega) = -1$; varying ω yields the Euler–Lagrange equations equivalent to $\nabla_\mu F^\mu{}_\nu = J_\nu$ with the $J[\omega]$ obtained in C.4. The explicit operator form was already established there; here we show it arises variationally. The continuity equation $\nabla\cdot J = 0$ follows by Noether's theorem for the gauge symmetry $A \mapsto A + d\phi$.

C.6 3+1 Split and Field Interpretation

Relative to $u = \omega$ (the distinguished unit field), define

$$E_\mu := F_{\mu\nu}u^\nu, \qquad B_\mu := \tfrac{1}{2}\,\varepsilon_{\mu\nu\alpha\beta}u^\nu F^{\alpha\beta}, \quad u\cdot E = u\cdot B = 0, \tag{21}$$

and let D_a be the Levi–Civita derivative on the spatial slices (Σ_ω, γ). The 1+3 Maxwell system then reads

$$\operatorname{div} B = 0, \qquad \mathcal{L}_u B + \operatorname{curl} E \;=\; -\Big(\tfrac{2}{3}\theta\, B + \sigma\cdot B - \omega\times E\Big), \tag{22}$$

$$\operatorname{div} E = \rho, \qquad \operatorname{curl} B - \mathcal{L}_u E \;=\; J - \Big(\tfrac{2}{3}\theta\, E + \sigma\cdot E + \omega\times B\Big), \tag{23}$$

with $\rho := -J_\mu u^\mu$ and $J_a := \gamma_a{}^\mu J_\mu$. Using $A = \eta^\flat(\omega)$ and $F = dA$ one obtains

$$E_a = -\,\mathcal{L}_u(\omega_a), \qquad B_a = (\operatorname{curl}\omega)_a, \tag{24}$$

which reproduces the standard interpretation (cf. §2.4 C.5).

C.7 Vacuum vs. Sources

The vacuum sector corresponds to $J_\nu = 0 \iff \Box_\eta\omega_\nu - \nabla_\nu(\nabla\cdot\omega) - R_\nu{}^\mu\omega_\mu = 0$, which still admits wave–type solutions with $F = d(\eta^\flat\omega) \neq 0$. Sourceful sectors arise when the ω–equation is not satisfied; then $J[\omega] \neq 0$ provides the inhomogeneous term in $\nabla\cdot F = J$. This cleanly separates radiative vacuum sectors from sourceful ones, without introducing external currents by fiat.

C.8 Minkowski Sector and Conclusion

When $\nabla\omega = 0$ (Lemma 2.3.1), (E^4, η) is Minkowski and all kinematical terms in the 1+3 system vanish, giving the familiar vector form $\nabla\cdot B = 0$, $\nabla\times E + \partial_t B = 0$, $\nabla\cdot E = \rho$, $\nabla\times B - \partial_t E = J$ in this sector. More generally, within the $\{@(), @(\delta)\}$ scope and starting from §1 data and using only §2 lemmas, we have: $A = \eta^\flat(\omega)$, $F = dA$ (homogeneous equations by identity), $\nabla\cdot F = J[\omega]$ (from the action), U(1) gauge realized, vacuum/source split explicit, units consistent.

$\square$

Future Work Sketches

Later papers in this series will develop the following directions:

- **Gauge Structure and U(1) Realization.** Local shifts $A \mapsto A + d\phi$ define a genuine U(1) principal bundle with curvature F, not merely a formal shift symmetry.
- **Units and Constitutive Relation.** Normalization of the action fixes the vacuum impedance ($c = 1$), reproducing the physical constants ε_0, μ_0; the constitutive law is entirely metric via $\star_\eta$.
- **Regularity, Boundary Terms, and Topology.** Regularity assumptions: C^2 fields, compact support or sufficient falloff for variations, global unit field ω, and global hyperbolicity to apply Stokes' theorem.

Appendix D
Falsifiable Mercury Perihelion Deviations from $\{@, @()\}$: Quadratic Gradient Ansatz and RK4 Benchmarking

This appendix presents the perihelion example from §2.8. Charlton Geometry posits $\eta(x) = \delta - 2\,\omega(x)\otimes\omega(x)$; small spatial variations $\nabla\omega \neq 0$ induce corrections beyond 1PN (cf. [11], Sec. 5.4). We parametrize deviations in Mercury's perihelion advance $\Delta\varpi$ relative to GR at fixed (a, e) by

$$\varepsilon := \ell_\odot\,\|\nabla\omega\| \simeq \ell_\odot\|G\|,$$

with $\ell_\odot$ a solar-system scale and $R_* = 10^5 R_\odot$ fixing G to suppress multipoles. For $\varepsilon = 0$ we recover GR (1PN), $\Delta\varpi_{\mathrm{GR}} \approx 43.03''$/century.

At leading sensitivity via $\{@, @()\}$,

$$\Delta\varpi_{@()}(\varepsilon) = \Delta\varpi_{\mathrm{GR}} \;+\; \kappa\left(\frac{\varepsilon}{\varepsilon_0}\right)^2 + \mathcal{O}(\varepsilon^3), \tag{25}$$

with $\varepsilon_0 = 10^{-5}$ and κ an apparent arcsec/century coefficient encoding the integrated effect of $\nabla\omega$. In full @+@() dynamics, κ reflects orbit-averaged sensitivity to $S(\tau) = u^\mu u^\nu \nabla_\mu \omega_\nu$. Numerically, $\varepsilon_0 = 10^{-5}$ is a part-per-million tilt of ω across solar-system scales. This quadratic ansatz is the first non-trivial CPT-even correction permitted by $\omega \sim -\omega$, consistent with [11], Sec. 5.4.

Benchmark. Set $\kappa = 0.01''$/century (i.e., $+0.01''$/century at $\varepsilon = \varepsilon_0$). Then (25) predicts $\leq 0.09''$ for $\varepsilon \leq 3 \times 10^{-5}$, below classic sensitivity yet testable with modern tracking.

D.1 Numerical Protocol (Geodesic RK4)

- Metric: $\eta_{\mu\nu} = \delta_{\mu\nu} - 2\,\omega_\mu\omega_\nu$, with $\omega_\mu = \omega_\mu^{(0)} + \delta\omega_\mu$ and $\|\nabla\omega\| = \varepsilon/\ell_\odot$.
- Expand Christoffels to $\mathcal{O}(\varepsilon^2)$; integrate $\ddot{x}^\mu + \Gamma^\mu{}_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta = 0$ by RK4 over $\sim 10^3$ orbits.
- Track successive perihelia and fit the slope of $\varpi(t)$.
- Scan $\varepsilon \in [0, 3 \times 10^{-5}]$; compare with (25).

D.2 Results and Falsifiable Target

Table 1 shows $\Delta\varpi$ shifts; a quadratic trend with $\kappa \sim 10^{-2}\,''$/century at $\varepsilon \sim 10^{-5}$ is falsifiable by BepiColombo ($\pm 0.01''$).

ε	$\Delta\varpi_{\mathrm{GR}}$	$\Delta\varpi_{@()}$	Deviation
0.0×10^{-5}	43.03	43.03	+0.00
0.333×10^{-5}	43.03	43.03	+0.001
1.0×10^{-5}	43.03	43.04	+0.01
2.0×10^{-5}	43.03	43.07	+0.04
3.0×10^{-5}	43.03	43.12	+0.09

Table 1: Perihelion advance (arcsec/century) under GR vs. @() with $\kappa = 0.01''$/century. Note: refine κ with detailed $\omega(x)$ in future work.

D.3 Community Challenge

Benchmark with BepiColombo orbital data (2026, r, v precision $\pm 0.01''$); share deviations and RK4 code (Appendix B) via #CharltonGeometry.

Appendix E
Series Outlook

This foundational paper launches the *Charlton Geometry Series* (Charlton2025a...), exploring physics which emerges from the Charlton Basis @ and Operator @() across diverse domains. Eleven sector drafts are in preparation as collaborative scaffolding, with additional papers planned on focused topics. Near-term directions are outlined in Table 2, each aligned with established data.

The invitation is open: domain experts are encouraged to join as co-authors, bringing observational and phenomenological depth to their fields while engaging the geometric framework.

The series is open-ended by design, shaped by encounters with experts as much as by the geometry itself. Later parts of the series, including detailed calibrations of emergent parameters and predictions, are open to refinement based on collaborative input and empirical data.

Priority	Topic/Paper and Subtopics
1	Foundational Core: Light Cones, Non-Invertibility, 3 Families
2	SM Emergence: Yukawas, CP Mixing, Full Lagrangian
3	Quantum Foundations: Born Rule, Pauli Proof
4	Gravity Tests: PPN: Solar System Orbits
5	Gravity Tests: Pulsars: Timing Precision
6	Gravity Tests: GW Propagation: Tensor Modes
7	Gravity Tests: Black Holes: Geodesic Tracing
8	Cosmology: CMB Anomalies, BH Dynamics, E$\hat{4}$ Effects
9	CB/CO Expositions: Recoverability, Coarse-Graining
10	Experimental/Outreach: Double-Slit, Casimir, Phenomena Catalog
11	GUT/TOE: Unification, Full Param Derivation
12	Variational Principles: Action for ω, Conservation Laws, EFT linkages

Table 2: Updated series outlook (11 core papers), prioritized by focus areas.

The table includes Charlton2025r, reserved for a cross-series catalog of falsifiables (e.g., π-flip damping, PPN $\epsilon > 10^{-5}$). Later parts of the series, including detailed calibrations, are open to refinement based on collaborative input and empirical data, ensuring adaptability to new findings.