

# On a link between finite QFT and the standard RG approaches

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**ABSTRACT:** In this note the finite formulation of quantum field theory, which is based on the system of the differential equations which are reminiscent of Callan-Symanzik equations is discussed. This system of equations was previously formulated on the bare language. We rederive these equations on a fully renormalized language. In this language, it was demonstrated for a simple  $\phi^4$  toy model, that with the specific choice of renormalization conditions – on-shell scheme for renormalized mass – the class of such finite renormalization prescriptions is equivalent to the classical renormalization group equation written in Callan-Symanzik-Ovsyannikov form.

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## 1 Introduction

In standard ways to proceed the considerations in quantum field theory (QFT) one always faces the divergent multi-loop Feynman diagrams. However, after the procedure of regularization to extract UV divergencies and subsequent renormalization the final answer for the physical observables becomes finite [1–3].

Nevertheless, it is also possible to think about the whole way of computations in QFT as follows: whatever procedure and calculation method we use, at the end of the day it is all a connection or *map* between finite parameters characterizing the theory and finite Green functions. From this point of view, it looks pretty natural to consider such an approach, where one does not meet any divergent expressions at any stage of the computation.

There are several approaches in QFT, where intermediate divergencies are treated carefully. One of them is the Bogolubov-Parasuk-Hepp-Zimmermann (BPHZ) renormalization procedure [4–6] with the application of R-operation. It is known that this method is perfectly applicable for renormalizable theories. Moreover, recently there have been works [7–9] on how this method can be used for non-renormalizable theories as well.

We focus our attention on another formulation, which is based on *the system of differential equations, which are reminiscent of Callan-Symanzik equations* [10, 11]. This approach was firstly formulated in Refs. [13, 14] as a proof of the validity of the multiplicative renormalization procedure. The idea of this procedure corresponds to *the differentiation* of bare field propagator with respect to *mass*. Application of this operation reduces the degree of divergency of the particular graph. It was explicitly shown, that within this program one can find the quantum corrections to the  $n$ -point Green functions as well as corrections to the effective potential [12, 15–17] in a finite way, i.e. no intermediate divergencies arise at all. Below, we will refer to this method and to system of differential equations as to “CS method” and “CS equations”, respectively.

This work is devoted to the study of some subtlety related to this finite CS method. By the construction, the CS equations were formulated in a “bare” language. The latter particularly may remind the specific choice or redefinition of renormalization factor in each order of perturbation theory as it was considered, for example in Ref. [18], for charge renormalization factor  $Z_3 = 1$  in supersymmetric QED, or the specific finite renormalization procedure in  $d = 4$  space-time [19]. An interesting question arises: whether it is possible to rederive CS equations in terms of fully renormalized language, explicitly introducing the renormalization of the field, the mass, the coupling constant and all correlation function? The answer is “yes”. In this note, such a derivation is done for a simple and clear  $\phi^4$  toy model. Moreover, this rederivation allows to see that *under the certain conditions which are related to the choice of concrete renormalization scheme*, the system of the CS equations is equivalent to the known equation of the renormalization group which is written in the Callan-Symanzik-Ovsyannikov form [10, 11, 20].

This paper is organized as follows. In section 2 we specify the model to work with and introduce all necessary notations. The section 3 is dedicated to the rederivation of the CS equations in terms of renormalized values within chosen  $\phi^4$  toy model and to the discussion of the equivalence of rederived system of differential equations and standard RG

group equation expressed in Callan-Symanzik-Ovsyannikov form, as well as the condition, what is actually related to the choice of specific renormalization scheme, under which this equivalence is clarified. We conclude in section 4.

## 2 Generalities

In this short section chosen setup and notations are discussed. To begin with, let us specify the model we are going to consider. For the simplicity we choose standard  $\phi^4$  toy model, which reads

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi_0\partial^\mu\phi_0 - \frac{m_0^2}{2}\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4, \quad (2.1)$$

where subscript 0 means bare values. The signature of the metric is  $(-, +, +, +)$ .

Later on, we are going to consider equations for *n-point one-particle irreducible (1PI) correlation functions*. To introduce the latter, we firstly remind the definition of Green function  $G_n$ , which reads

$$(2\pi)^4\delta^{(4)}(p_1 + \dots + p_n)G_n(p_1, \dots, p_n) = \int \prod_{i=1}^n d^4x_i e^{-ip_i x_i} \langle \phi_0(x_1) \dots \phi_0(x_n) \rangle, \quad (2.2)$$

for  $n$  external momenta. The notation  $\langle \dots \rangle$  means chronologically ordered product of fields and that only connected graphs are taken into account. Next, to obtain 1PI Green function  $G_n^{1\text{PI}}(p_1, \dots, p_n)$ , one should take in eq. (2.2) only diagrams that cannot be divided into two disconnected parts by cutting any internal line. Removing the external propagators from what remains (i.e. consider amputated diagrams), we obtain the function  $\Gamma^{(n)}$ :

$$\Gamma^{(n)}(p_1, \dots, p_n) \equiv \prod_{i=1}^n \left( \frac{p^2 + m^2}{-i} \right) G_n^{1\text{PI}}(p_1, \dots, p_n). \quad (2.3)$$

For example, two-point Green function at the tree level reads

$$G_2^{\text{tree}}(p) = \frac{-i}{p^2 + m_0^2}. \quad (2.4)$$

that is why  $\Gamma_{\text{tree}}^{(2)}(p)$  is just inverse propagator

$$\Gamma_{\text{tree}}^{(2)}(p) \equiv G_2^{\text{tree}}(p) \times \left( \frac{p^2 + m_0^2}{-i} \right)^2 = i(p^2 + m_0^2). \quad (2.5)$$

One can also obtain four-point 1PI correlation function within theory (2.1) at the tree level:

$$\Gamma_{\text{tree}}^{(4)} = -i\lambda_0. \quad (2.6)$$

Above we have considered the expressions for bare objects only. For our further purposes, we now introduce the standard relations between renormalized and bare correlation

functions as follows

$$\Gamma^{(n)}(\lambda_0, m_0) = Z_n \bar{\Gamma}^{(n)}(\lambda, m), \quad (2.7)$$

where for now we turn to the full (not only tree) correlation function. The relations between bare and renormalized field, coupling and mass are

$$\phi_0 = Z^{1/2} \phi_r, \quad (2.8a)$$

$$\lambda_0 = Z_\lambda \lambda_r, \quad (2.8b)$$

$$m_0 = Z_m m_r. \quad (2.8c)$$

For the simplicity we will write all renormalized values without any subscript “*r*” below. For the theory (2.1) with (2.7) and (2.8), it is known, that

$$Z = Z_2, \quad Z_\lambda = \frac{Z_4}{Z_2^2}, \quad (2.9)$$

and  $Z_2$  and  $Z_4$  are defined in (2.7) with  $n = 2, 4$ , respectively.

The calculation of the mentioned renormalization factors like  $Z_2$ ,  $Z_4$  occurs in a specific renormalization scheme. The physically observable values at the end of the day can not depend on the choice of the particular scheme. This requirement imposes certain constraints on the parameters defined in different renormalization programs and finally leads to the *renormalization group* (RG) approach, see Refs. [1–3] for details. This relates to the RG equation, which for the n-point correlation function has a form [2]

$$\left[ \mu \frac{\partial}{\partial \mu} + \beta(\lambda) \frac{\partial}{\partial \lambda} + \gamma_m(\lambda) m^2 \frac{\partial}{\partial m^2} + n \gamma_n(\lambda) \right] \bar{\Gamma}^{(n)}(\lambda, m, \mu) = 0, \quad (2.10)$$

where

$$\beta \equiv \mu \frac{d\lambda(\mu)}{d\mu} \Big|_{m_0, \lambda_0=\text{fixed}}, \quad (2.11a)$$

$$\gamma_m \equiv \frac{1}{m^2} \frac{dm^2(\mu)}{d\ln \mu} \Big|_{m_0, \lambda_0=\text{fixed}}, \quad (2.11b)$$

$$\gamma_n \equiv \frac{d\ln Z_n}{d\ln \mu} \Big|_{m_0, \lambda_0=\text{fixed}}, \quad (2.11c)$$

being  $\beta$ -function, anomalous dimension of mass and anomalous dimension of the field, respectively. We emphasize once again that these notations (2.11) are introduced with fixed bare  $m_0, \lambda_0$ . The  $\beta$ -function may be expressed in terms of anomalous dimension (what is right up to all perturbation theory’s orders) for (2.1):

$$\beta = \gamma_4 - 2\gamma_2. \quad (2.12)$$

Now, we will specify how we define  $m(\mu)$  and  $\lambda(\mu)$ , since these quantities are not fixed in (2.11), i.e. for now eq. (2.10) is written in arbitrary renormalization scheme. To define

$\lambda$ , one can stick to the *momentum subtraction* scheme at zero external momenta, so that renormalized coupling reads:

$$\bar{\Gamma}^{(4)}(\kappa_i^2 = 0) = -i\lambda, \quad (2.13a)$$

with  $\kappa_i^2 = \{(k_1 + k_2)^2, (k_1 - k_3)^2, (k_1 - k_4)^2\}$  being the Mandelstam variables and they are the sum of incoming and outgoing momenta in three different  $s-$ ,  $t-$ ,  $u-$  channels, respectively.

At the same time, one can define renormalized mass as

$$\bar{\Gamma}^{(2)}(k^2 = \mu^2) = im^2, \quad (2.13b)$$

what is off-shell renormalization condition. If one sticks to the choice of  $\mu^2 = -m^2$ , then renormalized mass is defined in the *on-shell* scheme. Another condition, that fixes the renormalization of the field may be chosen as:

$$\frac{d}{dk^2} \bar{\Gamma}^{(2)}(k^2) \Big|_{k^2=\mu^2} = i. \quad (2.14)$$

Any chosen renormalization conditions are correct up to all orders and they define the renormalization factors  $Z_m$ ,  $Z_2$ , and  $Z_4$  from (2.8) and (2.9).

Having introduced all the necessary notations and expressions, in the next section we turn to the finite CS method of renormalization. Corresponding system of differential equations will be obtained on a fully renormalized language. In this form we will compare CS equations with the RG equation (2.10). This will allow us to understand which renormalization scheme the CS method corresponds to.

### 3 The relation between the system of differential CS equations and RG equation

In this Section we rederive the system of differential CS equations from Refs. [14, 15] on a fully renormalized language, so that  $\beta$ -function and anomalous dimensions explicitly include all renormalization factors (which were introduced in a quite general manner in (2.7) and (2.8)). The logic of the derivation of the CS equations is fully alike with [14, 15].

Firstly, let us briefly recall the main idea of CS method [13–15]. The latter is based on the observation that differentiating a Feynman graph with respect to the mass gives a sum of terms in which each propagator in the graph is doubled. This operation lowers the degree of UV-divergence of the particular graph. In Refs. [13–15] this operation is called “theta-operation” and defined as

$$\Gamma_\theta^{(n)} \equiv -i \times \frac{d}{dm_0^2} \Gamma^{(n)}. \quad (3.1)$$

The definition (3.1) includes bare  $\Gamma^{(n)}$ . For  $n = 2$  (we stick to two-point correlation function in this paper for the simplicity), this bare correlation function relates to renormalized one

through (2.7). The renormalization of the l.h.s. of (3.1) with  $n = 2$  is

$$\Gamma_\theta^{(2)}(\lambda_0, m_0) = Z_2 Z_\theta \bar{\Gamma}_\theta^{(2)}(\lambda, m), \quad (3.2)$$

where  $Z_\theta$  is introduced to renormalize  $\Gamma_\theta^{(2)}(\lambda_0, m_0)$ . However, it is known, that the corresponding tadpole diagram for  $\Gamma^{(2)}$  diverges quadratically.<sup>1</sup> That is why, one needs to apply two theta-operation on this diagram in order to make it finite. To this end, in the framework of CS approach another object should be introduced [13–15]

$$\Gamma_{\theta\theta}^{(2)} \equiv -i \times \frac{d}{dm_0^2} \Gamma_\theta^{(2)}. \quad (3.3)$$

The connection between renormalized  $\bar{\Gamma}_{\theta\theta}$  and bare  $\Gamma_{\theta\theta}$  is

$$\Gamma_{\theta\theta}^{(2)}(\lambda_0, m_0) = Z_2 Z_\theta^2 \bar{\Gamma}_{\theta\theta}^{(2)}(\lambda, m). \quad (3.4)$$

By the same logic, one can introduce expressions for four-point correlation function:

$$\Gamma_\theta^{(4)}(\lambda_0, m_0) = Z_2^2 Z_\theta \bar{\Gamma}_\theta^{(4)}(\lambda, m). \quad (3.5)$$

Next, we rewrite the derivative from (3.1) and (3.3) as follows

$$\frac{d}{dm_0^2} \Big|_{m, \lambda=\text{fixed}} = \frac{\partial}{\partial m_0^2} \left( \frac{m_0^2}{Z_m^2} \right) \frac{\partial}{\partial m^2} + \frac{\partial}{\partial m_0^2} \left( \frac{\lambda_0}{Z_4/Z_2^2} \right) \frac{\partial}{\partial \lambda}, \quad (3.6)$$

where we also use (2.7)–(2.9) and where the derivatives are defined with fixed renormalized mass and coupling. Substituting (2.7)–(2.9), (3.2), (3.4) and (3.6) into the definitions of theta-operation (3.5), (3.3), and (3.1), one arrives to

$$2m^2 i \tilde{\gamma}_m \bar{\Gamma}_\theta^{(4)} = \left[ \left( 2m^2 \frac{\partial}{\partial m^2} + \tilde{\beta} \frac{\partial}{\partial \lambda} \right) + 4\tilde{\gamma}_4 \right] \bar{\Gamma}^{(4)}, \quad (3.7a)$$

$$2im^2 \tilde{\gamma}_m \bar{\Gamma}_{\theta\theta}^{(2)} = \left[ \left( 2m^2 \frac{\partial}{\partial m^2} + \tilde{\beta} \frac{\partial}{\partial \lambda} \right) + 2\tilde{\gamma}_2 + \tilde{\gamma}_\theta \right] \bar{\Gamma}_\theta^{(2)}, \quad (3.7b)$$

$$2m^2 i \tilde{\gamma}_m \bar{\Gamma}_\theta^{(2)} = \left[ \left( 2m^2 \frac{\partial}{\partial m^2} + \tilde{\beta} \frac{\partial}{\partial \lambda} \right) + 2\tilde{\gamma}_2 \right] \bar{\Gamma}^{(2)}, \quad (3.7c)$$

respectively, and where we have introduced the finite  $\beta$ -function and anomalous dimensions

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<sup>1</sup>We also note, that the detailed discussion of higher-loop orders in the framework of the CS method can be found in Ref. [12] and Section 3.1.3 wherein.

with the fixed renormalized  $m, \lambda$  as

$$\tilde{\gamma}_m \equiv \left[ \frac{\partial}{\partial m_0^2} \left( \frac{m_0^2}{Z_m^2} \right) \right]^{-1} Z_\theta \Big|_{m,\lambda=\text{fixed}}, \quad (3.8a)$$

$$\tilde{\beta} \equiv 2m^2 \left[ \frac{\partial}{\partial m_0^2} \left( \frac{m_0^2}{Z_m^2} \right) \right]^{-1} \frac{\partial}{\partial m_0^2} \left( \frac{\lambda_0}{Z_4/Z_2^2} \right) \Big|_{m,\lambda=\text{fixed}}, \quad (3.8b)$$

$$\tilde{\gamma}_2 \equiv m^2 \left[ \frac{\partial}{\partial m_0^2} \left( \frac{m_0^2}{Z_m^2} \right) \right]^{-1} \frac{\partial \ln Z_2}{\partial m_0^2} \Big|_{m,\lambda=\text{fixed}}, \quad (3.8c)$$

$$\tilde{\gamma}_4 \equiv m^2 \left[ \frac{\partial}{\partial m_0^2} \left( \frac{m_0^2}{Z_m^2} \right) \right]^{-1} \frac{\partial \ln Z_4}{\partial m_0^2} \Big|_{m,\lambda=\text{fixed}}, \quad (3.8d)$$

$$\tilde{\gamma}_\theta \equiv 2m^2 \left[ \frac{\partial}{\partial m_0^2} \left( \frac{m_0^2}{Z_m^2} \right) \right]^{-1} \frac{\partial \ln Z_\theta}{\partial m_0^2} \Big|_{m,\lambda=\text{fixed}}. \quad (3.8e)$$

It is important to note, that the coefficients in the expansion (in each order of perturbation theory) of beta-function and anomalous dimensions given by (3.8) *does not depend on the renormalization scheme*. For  $\beta$ -function and anomalous dimensions formulated with the corresponding derivatives taken with respect to renormalized mass and coupling but at fixed bare mass  $m_0$  and  $\lambda_0$  the situation is the opposite and the coefficients depend on the particular scheme, see a textbook discussion in [2] or an example within some specific supersymmetric QED setup with one charge in Ref. [18].

The system of equations (3.7) allows to derive  $\bar{\Gamma}^{(2)}$  in a fully finite way [13–15]:  $\bar{\Gamma}_{\theta\theta}^{(2)}$  and  $\bar{\Gamma}_\theta^{(4)}$  are already finite by the construction,  $\bar{\Gamma}_\theta^{(2)}$  can be found from (3.7b) and then  $\bar{\Gamma}^{(2)}$  – from (3.7c), while  $\bar{\Gamma}^{(4)}$  – from (3.7a). Surely, for the completeness, the system of CS equations (3.7) must be supplemented by some *ad hoc* boundary conditions, which actually correspond to the choice of concrete renormalization scheme. In principle, this method works for any choice for the external momentum scale where the boundary conditions are formulated, see discussion in [12, 15]. Moreover, in [21] authors also have shown that the proof of renormalizability through the use of the Callan-Symanzik equation can be done without imposing any normalization conditions.

### The choice of renormalization conditions

The difference between equations (3.7) with (3.8) and what was used in [14, 15] is as follows. Though the equations (3.7) themselves are fully the same as the CS equations from [14, 15] (up to some notations), now all  $\beta$ -functions and anomalous dimensions (3.8) explicitly include the renormalization factors  $Z_\theta, Z_m, Z_2$ , and  $Z_4$ , which we have introduce in a quite general manner. To find  $Z_\theta, Z_m, Z_2$ , and  $Z_4$  and then  $\beta$ -function and anomalous dimensions in each order of perturbation theory one should stick to some renormalization scheme or conditions to define the renormalized mass and coupling constant, as well as the scale at which they are defined. So, in order to understand which renormalization scheme the equations (3.7) with (3.8) correspond to, let us now consider to the renormalization group equation (2.10).

As it was mentioned in section 2, the RG equation (2.10) is written in arbitrary renormalization scheme until the renormalized mass  $m(\mu)$  and coupling  $\lambda(\mu)$  are defined. Let us

define mass  $m(\mu)$  and coupling  $\lambda(\mu)$  through the conditions (2.13). Moreover, we take  $\mu$  scale as  $\mu^2 = -m^2$  in (2.13b). *This choice can be related as momentum subtraction scheme at zero external momenta for coupling constant and on-shell scheme for mass.*

Now, we have come close to the main point of this paper: if one sticks to the mentioned scheme as well as substitutes the specific scale  $\mu^2 = -m^2$  into the RG equation (2.10) for  $n = 2, 4$ , then one easily finds out, that (2.10) with  $n = 2, 4$  becomes just *equivalent* (term by term) to the system of equations (3.7).<sup>2</sup> Indeed, consider, for example, the following term from (3.7c) and proceed the mentioned substitutions:

$$\begin{aligned} 2m^2 i\tilde{\gamma}_m \bar{\Gamma}_\theta^{(2)} &= 2m^2 i\tilde{\gamma}_m \frac{\Gamma_\theta^{(2)}(\lambda_0, m_0)}{Z_2 Z_\theta} \\ &= 2m^2 \left[ \frac{\partial}{\partial m_0^2} \left( \frac{m_0^2}{Z_m^2} \right) \right]^{-1} Z_\theta \Big|_{m, \lambda=\text{fixed}} \frac{(d\Gamma^{(2)}(\lambda_0, m_0)/dm_0^2)}{Z_2 Z_\theta} \\ &= 2m^2 \frac{\partial \bar{\Gamma}^{(2)}(\lambda, m)}{\partial m^2} \Big|_{m_0, \lambda_0=\text{fixed}}. \end{aligned} \quad (3.9)$$

Here we emphasize, that the factor  $Z_m$  just cancels from the final expression in (3.9). The latter is finally equivalent to the  $\gamma_m(\lambda)m^2 \frac{\partial}{\partial m^2} \bar{\Gamma}^{(2)}$  from (2.10) with (2.11b) as well as with substituted  $\mu^2 = -m^2$ ; in this scheme anomalous dimension of mass (2.11b) from (2.10) becomes just a constant.

By the same logic, one may show such an equivalence for the rest of the terms in the CS system of equations and RG equations. We emphasize that during the explicit substitution of  $\mu^2 = -m^2$  into the RG equation (2.10) and the subsequent comparison with CS equations (3.7), one should bear in mind, that  $\beta$ -function and anomalous dimensions are defined differently for RG equation and for the system of CS equations.

In the next section we discuss what exactly this mentioned equivalence means and what conclusions can be drawn from this.

## 4 Discussion

In this note we have shown that renormalization group equation in the Callan-Symanzik-Ovsyannikov form [10, 11, 20] is equivalent to the system of differential equations which are used in finite QFT formulation [13–15]. However, this equivalence arises explicitly only when one chooses the specific renormalization scheme. Particularly, the choice of on-shell scheme for renormalized mass is needed for the mentioned equivalence. So, we conclude, that within massive  $\phi^4$  toy model the on-shell renormalization scheme for mass is somehow distinguished for CS method.

The applications of on-shell schemes for mass are known for many different examples and sometimes these schemes are indeed distinguished for some reason. Such a scheme was used in the case of QED, for instance, in [22, 23]; in supersymmetric QED in [24]; and

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<sup>2</sup>Since we find  $\bar{\Gamma}_\theta^{(2)}$  from equation (3.7b), then one should compare only one CS equation (3.7c) with RG equation (2.10) with  $n = 2$  within chosen renormalization scheme. Obviously, in the case of four-point correlation function, one should compare (2.10) with  $n = 4$  and (3.7a).

in quantum chromodynamics (QCD), e.g., in [25]. However, the requirement of on-shell renormalization condition for mass is subtle to implement in the theories with asymptotic freedom such as QCD. In such models the on-shell mass can be defined in a gauge-invariant way by using a specific gauge-invariant procedure analogous to subtraction scheme at zero momenta (or at a specific subtraction point), however the calculations become difficult within such gauge-invariant procedure, see [25] for details. From this point of view related to the difficulty of calculations, the realization of finite QFT formulation for QCD requires additional delicate consideration.

Finally, as it was mentioned above, the coefficients in the expansion for beta-function and anomalous dimensions (3.8) (which appear in CS method) does not depend on the renormalization scheme, while for  $\beta$ -function and anomalous dimensions (2.11) (which appear in RG equation) the situation is the opposite and the coefficients depend on the particular scheme [2, 3]. So, it would be interesting to further explore these dependence and independence on the particular scheme choice. Within  $\phi^4$  toy model, this study, however, requires calculating the anomalous dimension in a two-loop approximation and the  $\beta$ -function in a three-loop approximation, so that the evaluations and results from, e.g., Ref. [26] may be useful for the further researches.

## Acknowledgments

The authors are grateful to D. I. Kazakov and N. V. Krasnikov, and in particular to M. E. Shaposhnikov for useful comments and fruitful discussions. The work of YA has been supported by Russian Science Foundation Grant No. 24-72-00121, <https://rscf.ru/project/24-72-00121/>.

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