

Higher Curvature Inflation and the Species Scale

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Abstract

We study the scalar potentials that arise from higher curvature corrections in general $f(R)$ theories of gravity and their connection to a dynamical species scale. Starting from general considerations in arbitrary dimensions, we show that at large field values, the scalar potential generated by an infinite series of curvature terms and the field dependent species scale arising from circle compactification both decay exponentially, in complementary ways. We identify conditions under which these two effects precisely balance out, giving rise to exponentially flat, plateau-like potentials. We additionally find a precise embedding of Starobinsky inflation consistent with the Swampland program, and we discuss possible implications the mechanism proposed could have for M and string theory.

1 Introduction

Cosmological inflation provides a compelling mechanism for addressing several phenomenological issues of standard cosmology, such as the horizon and flatness problems, while simultaneously generating nearly scale-invariant primordial fluctuations in agreement with CMB data [1, 2]. Among the broad landscape of inflationary models, the Starobinsky model [3], remains as one of the most robust and elegant scenarios. In its simplest form, the addition of an R^2 term to the Einstein-Hilbert action gives rise to a scalar degree of freedom whose potential naturally exhibits a plateau at large field, as well as a Minkowski minimum around which the inflaton can enter a reheating phase. The resulting predictions for the spectral tilt n_s are in remarkable agreement with data, and the predicted values for the tensor-to-scalar ratio r are within the next generation of B-mode polarization experiments [4, 5]. The Starobinsky model naturally connects to the broader class of $f(R)$ and more general modified theories of gravity. These are often studied as extensions of general relativity within an effective field theory perspective (for a review, see e.g. [6]).

It has also been long understood that a renormalizable theory of gravity requires extending the Einstein-Hilbert action by higher curvature operators [7, 8]. From the Wilsonian perspective of effective field theory, the R^2 term should not be isolated, but rather part of an infinite tower of higher-derivative terms [9]. An immediate question is then whether the inflationary plateau can remain once such corrections are included.

This is closely tied to our recent understanding of quantum gravity in the context of the Swampland program, which attempts to distinguish effective field theories consistent with quantum gravity from those which are not [10]. In particular, the Swampland Distance Conjecture (SDC) [11] (and its refinement [12]) suggests that large scalar field excursions are necessarily accompanied by towers of light states, which invalidate the naive low-energy effective description. A key role in this is played by the *species scale* Λ_{sp} [13, 14, 15], which sets the effective cutoff of the theory in the presence of a large number of light species, and, as has been recently explored in the literature [16, 17, 18], can be identified as the scale suppressing higher curvature corrections to any EFT coupled to gravity. The species scale can generally be a field dependent quantity [16], for instance in Kaluza-Klein compactifications where it depends exponentially on the volume modulus of the compact space.

The role of the swampland program in Starobinsky inflation was also studied in [19] (see also [20]), where it was argued that, in UV-complete theories of gravity, the R^2 term should necessarily be weighted by the quantum gravitational cutoff. This suggests that

inflation would occur at energy scales where the weakly coupled EFT cannot be trusted. Moreover, as large field displacements occur during inflation, the scalar potential itself is expected to vary significantly, thereby destabilizing the plateau even in the absence of explicit higher curvature corrections.

In this work, we find that at large field values, the species scale coming from compactification on a circle and the scalar potential coming from an infinite curvature series may decay exponentially in complementary ways. We take this as motivation to study how the interplay of higher curvature corrections and the species scale can contribute to generate scalar plateau-like scalar potentials and whether they can be made consistent with swampland constraints.

The structure of this paper is as follows: In Section 2 we revisit the construction of scalar potentials from general $f(R)$ theories in arbitrary dimension. We recall how plateau-like potentials can only emerge for specific choices of asymptotics, and generically one has exponentially decaying potentials. In Section 3 we analyze how the scalar potential can be uplifted by compactification on a circle, which induces an exponential dependence of the cutoff scale on the scalar field that can exactly flatten the potential. In Section 4 we discuss the implications for inflationary cosmology and for string and M-theory, assuming these allow for backgrounds with constant curvature. Finally, in Section 5 we conclude.

2 Scalar Potentials from $f(R)$ Gravity

We start by reviewing how modified gravity theories of the form $f(R)$ in d dimensions are recast as Einstein-Hilbert gravity coupled to an additional scalar degree of freedom¹.

Consider the action for $f(R)$ gravity:

$$S = \frac{M_{\text{Pl}}^{d-2}}{2} \int d^d x \sqrt{-g} f(R^{(J)}), \quad (2.1)$$

where $R^{(J)}$ denotes the scalar curvature in the Jordan-frame, as opposed to Einstein-frame. We can write the action in the form:

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\text{Pl}}^{d-2}}{2} F R - U \right], \quad (2.2)$$

where the functions F and U are defined as

$$F = \frac{\partial f}{\partial R}, \quad U = \frac{M_{\text{Pl}}^{d-2}}{2} (F R - f). \quad (2.3)$$

¹In appendix A we also show that during a quasi de-Sitter phase one can neglect other types of curvature invariants.

To proceed to the Einstein-frame, we perform a Weyl rescaling of the metric:

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} = e^{2\varphi} g_{\mu\nu}, \quad (2.4)$$

under which the determinant and Ricci scalar transform as

$$\sqrt{-g} = \Omega^{-d} \sqrt{-\tilde{g}}, \quad (2.5)$$

$$R = \Omega^2 \left(\tilde{R} + 2(d-1)\tilde{\square}\varphi - (d-1)(d-2)\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \right). \quad (2.6)$$

Choosing the conformal factor as

$$F = \Omega^{d-2}, \quad (2.7)$$

one obtains the Einstein-frame action

$$S = \int d^d x \sqrt{-\tilde{g}} \left[\frac{M_{\text{Pl}}^{d-2}}{2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]. \quad (2.8)$$

Here we have defined

$$\phi = \sqrt{(d-1)(d-2)} M_{\text{Pl}}^{\frac{d-2}{2}} \varphi, \quad (2.9)$$

such that

$$V(\phi) = \frac{U}{F^{d/(d-2)}}, \quad F = e^{\sqrt{\frac{d-2}{d-1}} \frac{\phi}{M_{\text{Pl}}^{\frac{d-2}{2}}}}. \quad (2.10)$$

While F can always be written explicitly in terms of ϕ , obtaining an analytic form of $V(\phi)$ is generally only feasible for specific cases of $f(R)$ [21]. As we will attempt to connect the two, we briefly discuss asymptotically flat and exponentially decaying potentials arising from these sort of theories.

2.1 Asymptotically Flat Potentials from $f(R)$ Gravity

To identify the functional forms of $f(R)$ that yield asymptotically flat scalar potentials, we analyze the large-field behavior of $V(\phi)$ in the limit $\phi \rightarrow \infty$. In this regime, we demand that

$$\lim_{\phi \rightarrow \infty} V(\phi) = c_1, \quad \lim_{\phi \rightarrow \infty} (FR - f) = c_2 F^{\frac{d}{d-2}}, \quad (2.11)$$

$$\lim_{\phi \rightarrow \infty} \frac{dV}{d\phi} = 0, \quad \lim_{\phi \rightarrow \infty} \left[\frac{R}{F^{2/(d-2)}} \left(\frac{2}{d} - \frac{f}{RF} \right) \right] = 0, \quad (2.12)$$

for constants $c_1, c_2 > 0$. Solving this system leads to a unique large- R behavior:

$$f(R) \propto R^{d/2}. \quad (2.13)$$

A minimal realization of this is

$$f(R) = R + \frac{R^{d/2}}{M^{d-2}}, \quad (2.14)$$

with M some energy scale. More generally, any function of the form

$$f(R) = R + \frac{R^{d/2}}{M^{d-2}} g(R), \quad (2.15)$$

where $g(R) \rightarrow \text{const} > 0$ as $R \rightarrow \infty$, leads to an asymptotically flat potential.

As a concrete example, consider a scalar potential with an exponential plateau

$$V(\phi) \simeq V_0 \left(1 - e^{-\gamma \sqrt{\frac{d-2}{d-1}} \phi} \right). \quad (2.16)$$

This corresponds, in the large- ϕ limit, to an $f(R)$ theory of the form:

$$f(R) \simeq R^{d/2} + R^{1-\frac{d-2}{2}(\gamma-1)}, \quad (2.17)$$

where, only for $\gamma = 1$, one has theory with an Einstein-Hilbert term in the Jordan-frame.

2.2 Exponential Potentials from $f(R)$ Gravity

When the leading term in the curvature expansion is of order n

$$f(R) = R + M_0^2 (R/M_0^2)^n, \quad (2.18)$$

we will have an exponential potential of the form

$$V(\phi) = \frac{1}{2} M_0^2 M_{\text{Pl}}^{d-2} (n-1) \left(\frac{1}{n} \right)^{\frac{n}{n-1}} e^{-\sqrt{\frac{d^2}{(d-2)(d-1)}} \phi} \left(e^{\sqrt{\frac{d-2}{d-1}} \phi} - 1 \right)^{\frac{n}{n-1}}. \quad (2.19)$$

At large ϕ we have

$$V(\phi) \simeq \frac{1}{2} M_0^2 M_{\text{Pl}}^{d-2} (n-1) \left(\frac{1}{n} \right)^{\frac{n}{n-1}} e^{-\frac{1}{\sqrt{(d-2)(d-1)}} \frac{2n-d}{n-1} \phi} \left(1 - \frac{n}{n-1} e^{-\sqrt{\frac{d-2}{d-1}} \phi} \right). \quad (2.20)$$

and the potential decays exponentially at a rate

$$2\gamma = \frac{1}{\sqrt{(d-2)(d-1)}} \frac{2n-d}{(n-1)}. \quad (2.21)$$

For integer $n < d/2$ we simply have that the potential is exponentially bounded from both sides. For $n > d/2$, γ satisfies

$$0 \leq \gamma \leq \frac{1}{\sqrt{(d-2)(d-1)}}, \quad (2.22)$$

with the upper bound saturated when $n \rightarrow \infty$. We can argue that this maximum decay rate will be generic for infinite series. To this effect consider a potential characterized by two exponential scales:

$$V(\phi) \simeq \frac{M_0^2 M_{\text{Pl}}^{d-2}}{2} e^{-2\gamma\phi} (1 - a_0 e^{-\lambda\phi}) , \quad (2.23)$$

coming from a curvature series

$$f(R) = \sum_{n=1} c_n M_0^2 \left(\frac{R}{M_0^2} \right)^n . \quad (2.24)$$

The exponential parameter γ is related to the asymptotic form of $f(R)$ via

$$\gamma = -\frac{1}{2} \left(\frac{1}{V} \frac{dV}{d\phi} \right)_{\phi \rightarrow \infty} = \frac{1}{\sqrt{(d-2)(d-1)}} \frac{1 - \frac{d}{2} \frac{f}{RF}}{1 - \frac{f}{RF}} \bigg|_{R \rightarrow M_0^2} , \quad (2.25)$$

where we have assumed that the infinite distance limit of ϕ corresponds to $R \rightarrow M_0^2$. This is equivalent to demanding that F diverges at M_0^2

$$\sum_{n=1} c_n n = \infty . \quad (2.26)$$

Given this, and assuming that the series is such that its partial sums are bounded from below²

$$\sum_{n=1}^N c_n > -\infty , \forall N \quad (2.27)$$

it follows that

$$\lim_{R \rightarrow M_0^2} \frac{f}{RF} \rightarrow 0 , \quad (2.28)$$

and

$$\gamma = \frac{1}{\sqrt{(d-2)(d-1)}} . \quad (2.29)$$

As such, under some assumptions we obtain that an infinite series of curvature corrections leads to an exponentially decaying scalar field with the same decay rate as one would obtain from compactification on a circle [23], independently on the number of dimensions. For rational coefficients, if we assume some asymptotic spectrum $c_n = n^\alpha$, one can explicitly check:

$$\gamma = \frac{1}{\sqrt{(d-2)(d-1)}} , \quad \text{for } \alpha \geq -2, \quad (2.30)$$

²This condition is, for example, weaker than requiring the series to contain only a finite number of negative coefficients, and it is further supported by positivity constraints on irrelevant operators [22].

since for $\alpha < -2$, F is convergent at $R = M_0^2$.

To determine the second exponent λ , we define the rescaled potential

$$\tilde{V} = e^{2\gamma\phi} V(\phi), \quad (2.31)$$

and compute

$$\lambda = \left. \frac{d\tilde{V}/d\phi}{\frac{M_0^2 M_{\text{Pl}}^{d-2}}{2} - \tilde{V}} \right|_{\phi \rightarrow \infty} = \left. \frac{\sqrt{\frac{d-2}{d-1}}}{1 + (M_0^2 - R)^{\frac{F}{f}}} \right|_{R \rightarrow M_0^2}. \quad (2.32)$$

For a monomial spectrum $c_n = n^\alpha$ this is:

$$\lambda = \begin{cases} \sqrt{\frac{d-2}{d-1}} \cdot \frac{1}{2+\alpha} & \text{for } \alpha \geq -1, \\ \sqrt{\frac{d-2}{d-1}} & \text{for } -2 < \alpha \leq -1. \end{cases} \quad (2.33)$$

Then λ is upper-bounded by $\sqrt{\frac{d-2}{d-1}}$, and will saturate this bound whenever the coefficients satisfy $1/n^2 \leq c_n \leq 1/n$. Apart from these, we also briefly discuss other types of curvature series in appendix B.³

In an EFT approach, and also the typical expectation from string theory, one expects that the scale controlling the higher order terms in the series corresponds to the UV cutoff of the series. This cutoff has been identified in various contexts as the species scale $\Lambda_{\text{sp}} = M_{\text{Pl}}/N_{\text{sp}}^{\frac{1}{d-2}}$, where N_{sp} is the number of light degrees of freedom in the theory. This is generally a moduli dependent quantity, but has been conjectured to also hold on a general scalar field space⁴ [32]. It is then somewhat remarkable that potentials with exponential decay rates given by (2.29) are exactly what one would obtain from compactifying a theory one dimension higher with a cosmological constant $\Lambda = M_0^2/2$ on a circle. Also in that same setup, the species scale after compactification will satisfy $\Lambda_{\text{sp}} \simeq \sqrt{2V(\phi)}$. Motivated by this, we explore the relation further, to try and obtain asymptotically flat potentials.

3 $f(R)$ gravity on a circle

In this section we study how simple Kaluza–Klein reductions on a circle affect the $f(R)$ action in the lower-dimensional theory, and how the resulting radion dependence can nat-

³We expect the coefficients to be constrained by the analyticity and unitarity of graviton scattering amplitudes as in [24], $c_n^{\frac{1}{2n-2}} M \lesssim \Lambda_{\text{sp}}$, so the coefficients cannot be arbitrarily large; in the limit $M \simeq \Lambda_{\text{sp}}$, we expect them to be strictly non-increasing.

⁴Although there is no unifying picture for the SDC in a general field space, this has been studied from several different perspectives (see e.g. [25, 26, 27, 28, 29, 30, 31]).

urally induce a field-dependent cutoff (or species) scale. A field-dependent gravitational scale arises naturally from Kaluza–Klein (KK) compactification on a circle, $x^d \sim x^d + 2\pi r$. After reduction the lower-dimensional effective theory takes the form of an $f(R)$ model in which the Jordan-frame action is coupled non-minimally to a scalar field. Consider the $D = d + 1$ -dimensional action

$$S_D = \frac{M_{\text{Pl},D}^{D-2}}{2} \int d^D x \sqrt{-G} \left[R_D + M_0^2 \sum_{n=2} c_n \frac{R_D^n}{M_0^{2n}} \right], \quad (3.1)$$

where M_0 is the mass scale controlling higher curvature corrections, $M_{\text{Pl},D}$ is the higher dimensional Planck mass and c_n are dimensionless coefficients.

For the dimensional reduction we use the standard metric ansatz

$$ds^2 = G_{MN} dx^M dx^N = e^{2\alpha\phi} g_{\mu\nu} dx^\mu dx^\nu + e^{2\beta\phi} (dx^d)^2, \quad (3.2)$$

with the radion, ϕ , independent of the internal coordinate x^d . We will consider (nearly) maximally symmetric solutions, such that the universe is in a quasi-de Sitter phase. The slow roll parameter is defined as

$$\varepsilon \equiv -\frac{\dot{H}}{H^2}, \quad (3.3)$$

where H is the Hubble radius. Then, being in a slow roll phase requires

$$\varepsilon \ll 1, \quad (3.4)$$

so that we in fact have quasi-de Sitter. This allows us to neglect higher-derivative kinetic terms in the scalar field⁵. A canonical scalar kinetic term in d dimensions is obtained by choosing

$$\alpha^2 = \frac{1}{(d-1)(d-2)}, \quad \beta = -(d-2)\alpha, \quad (3.5)$$

which fixes the radion normalization. The resulting d -dimensional action becomes at tree level⁶

$$S_d = \frac{M_{P,d}^{d-2}}{2} \int d^d x \sqrt{-g} \left[R + M_0^2 e^{2\alpha\phi} \sum_{n=2} c_n \frac{R^n}{M_0^{2n} e^{2n\alpha\phi}} + (\partial\phi)^2 \right], \quad (3.6)$$

⁵During slow roll we have $(\partial\phi)^2 \simeq \varepsilon H^2$, such that $(\partial\phi)^{2n} \ll (\partial\phi)^2 \ll R$ for $n > 1$.

⁶Higher loop effects can induce an additional curvature series whose characteristic scale is the Kaluza–Klein mass of the compact manifold [33], and in the case of irrelevant operators, these can in some cases dominate over the tree level reduction. In Type IIA/M-Theory this occurs for example for terms of the form $D^n R^4$ for decompactification limits to $d = 10$ [33].

where R denotes the d -dimensional Ricci scalar, and we have identified the lower dimensional Planck mass as $M_{\text{Pl}}^{d-2} = 2\pi R M_{\text{Pl},D}^{d-1}$. The cutoff controlling the higher curvature corrections acquires a ϕ -dependence of the form

$$M(\phi) = M_0 e^{\pm \frac{\phi}{\sqrt{(d-1)(d-2)}}}, \quad (3.7)$$

where the sign indicates whether the species scale grows or decays with ϕ under the chosen convention. Since $M(\phi)$ is now the scale controlling the curvature series we identify it with the species scale Λ_{sp} [17].

It has been conjectured [12, 23, 34] that in string theoretic setups the exponential coefficient γ appearing in the species scale $\Lambda_{\text{sp}} \sim e^{\pm \gamma \phi}$ is necessarily bounded as

$$\frac{1}{\sqrt{(d-1)(d-2)}} < |\gamma_{\text{sp}}| < \frac{1}{\sqrt{d-2}}. \quad (3.8)$$

The lower bound corresponds to a decompactification from $d+1$ to d dimensions, as we have found, while the upper bound signals the emergence of weakly coupled string. We then find that the exponential scaling of the potential and that of the species scale coincide only at the unique value $\gamma = 1/\sqrt{(d-1)(d-2)}$. In short, the exponential decay rate corresponding to a potential arising from an infinite-order curvature expansion can be matched to that of the species scale in a decompactification limit from d to $d+1$.

3.1 Einstein-frame Action

We now perform the conformal transformation

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = e^{\frac{\phi}{\sqrt{(d-1)(d-2)}}} g_{\mu\nu}, \quad (3.9)$$

such that the scalar degree of freedom arising from $f(R)$, the scalaron, coincides with the radion ϕ , similar to the case of Higgs inflation [35]. For simplicity we take $M_{\text{Pl}} = 1$, the Einstein-frame action is⁷

$$S = \int d^d x \sqrt{-g} \left[\frac{1}{2} R + \frac{1}{2} \left(1 + e^{-\sqrt{\frac{d-2}{d-1}} \phi} \right) (\partial\phi)^2 - V(\phi) \right]. \quad (3.10)$$

In the large-field regime $\phi \gtrsim \mathcal{O}(1)$, the kinetic term is approximately canonical:

$$1 + e^{-\sqrt{\frac{d-2}{d-1}} \phi} \simeq 1. \quad (3.11)$$

⁷Here Einstein-frame means that there are no terms depending on R other than the Einstein-Hilbert term.

The potential in this regime is given by

$$V(\phi) = \frac{M(\phi)^2}{2} e^{-\frac{d}{\sqrt{(d-1)(d-2)}}\phi} \sum_{n=1} c_n (n-1) \frac{R(\phi)^n}{M(\phi)^{2n-2}}, \quad (3.12)$$

where $R(\phi)$ is implicitly defined via

$$1 + \sum_{n=2} c_n n \frac{R^{n-1}}{M(\phi)^{2(n-1)}} = e^{\sqrt{\frac{d-2}{d-1}}\phi}. \quad (3.13)$$

Although it is generally difficult to solve this equation analytically for arbitrary coefficients c_n , qualitative features can still be understood from specific choices. As argued in Section 2, under some considerations the exponential rate of the potential is generic for infinite series and it matches precisely the scaling for the species scale resulting from compactification on circle. Then, taking⁸

$$M = M_0 e^{\frac{\phi}{\sqrt{(d-2)(d-1)}}}, \quad (3.14)$$

the exponential factors will cancel out, leading to a flat potential. Such plateau-like potentials at large ϕ obey the parameterization

$$V(\phi) = \frac{1}{2} M_0^2 (1 - e^{-\lambda\phi}), \quad (3.15)$$

with λ determined by the structure of the $f(R)$ series. In this regime, the inflationary scale $H \sim \sqrt{V}$ remains parametrically below the species scale $M(\phi)$. Since we are looking at cosmological solutions one has an Einstein-frame curvature $R \simeq V(\phi)$, which leads to the relation

$$\Lambda_{\text{sp}} \equiv M \simeq M_0 \left(\frac{M_0^2}{M_0^2 - R} \right)^{\frac{1}{d-2}}, \quad (3.16)$$

where we have assumed $c_n \simeq \mathcal{O}(1)$. As such, there are two distinct scales, Λ_{sp} , and M_0 . The former is the scale of quantum gravity, while the later determines the cosmological constant, and they are related exponentially in terms of the displacement of the inflaton. In what follows, we analyze specific choices of $f(R)$ that yield viable plateau potentials after uplifting and we comment on their consistency with Swampland constraints.

3.1.1 $f(R) = R + M^2 \sum_n \frac{R^n}{M^{2n}}$

For a series of curvature corrections in which every term is at the same order $c_n \simeq 1$ we have

$$f(R) = R + M^2 \sum_{n=2} \frac{R^n}{M^{2n}} = \frac{R}{1 - R/M^2}, \quad (3.17)$$

⁸We assume initial conditions $\phi > 0$, $\dot{\phi} < 0$, such that M decreases with time.

with $R < M^2$. The scalar potential is

$$V(\phi) = \frac{1}{2} M^2 e^{-\frac{2}{\sqrt{(d-2)(d-1)}}\phi} \left(1 - e^{-\frac{1}{2}\sqrt{\frac{d-2}{d-1}}\phi} \right)^2, \quad (3.18)$$

where the exponential rates satisfy eq.(2.29), and eq.(2.33) with $\alpha = 0$. Then, if M is identified with the species scale coming from compactification on a circle we have

$$M \rightarrow M_0 e^{\frac{\phi}{\sqrt{(d-2)(d-1)}}}, \quad (3.19)$$

leading to a flat potential

$$V(\phi) = \frac{M_0^2}{2} \left(1 - e^{-\frac{1}{2}\sqrt{\frac{d-2}{d-1}}\phi} \right)^2. \quad (3.20)$$

As such, one could obtain Starobinsky-like models of inflation as the effective scalaron potential coming from an infinite series of curvature corrections, lifted by the exponentially varying species scale coming from the compactification of an internal circle.

3.1.2 $f(R) = R + M^2 \sum_n n^\alpha \frac{R^n}{M^{2n}}$

As discussed in Section 2, the scalar potentials resulting from series with rational coefficients have very constrained asymptotic behavior. Whenever the curvature series has mostly positive terms and the coefficients are bounded as $c_n > 1/n^2$ we will have a potential with overall exponential suppression that can be canceled out after compactification on a circle

$$V \simeq \frac{M_0^2}{2} (1 - a e^{-\lambda\phi}), \quad (3.21)$$

with λ given by eq. (2.33). Given slow roll, inflationary phenomenology will only depend directly on the precise value of λ , so we briefly focus on series that have phenomenology equivalent to that of the Starobinsky model. This corresponds to series with coefficients $c_n \simeq n^\alpha$, with $-2 \leq \alpha \leq -1$, which are upperbounded by

$$V(\phi) = \frac{M_0^2}{2} \left(1 - e^{-\sqrt{\frac{d-2}{d-1}}\phi} (1 + \sqrt{\frac{d-2}{d-1}}\phi) \right) \quad (\alpha = -1), \quad (3.22)$$

$$V(\phi) \simeq \frac{M_0^2}{2} \left(1 - e^{-\sqrt{\frac{d-2}{d-1}}\phi} \right) \quad (\alpha = -2),$$

where for the case of $\alpha = -2$ we are assuming large ϕ to neglect subleading terms. These two potentials (see Fig.1), and any resulting potential with coefficients $1/n^2 < c_n < 1/n$ will have phenomenology equivalent to that of the Starobinsky model to leading order in

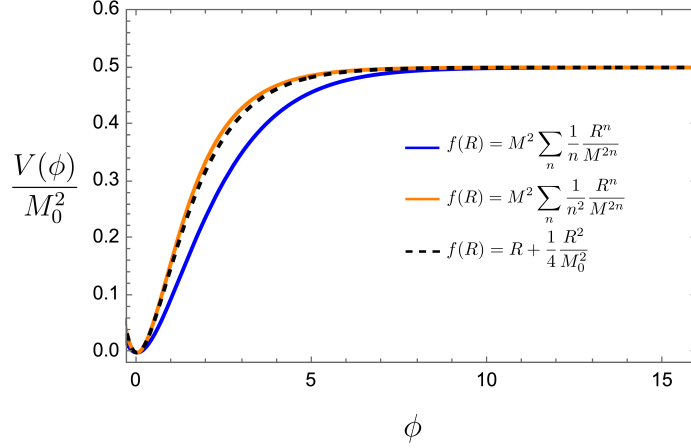


Figure 1: Potentials obtained from series of higher (scalar) curvature corrections to EH gravity after modifying the mass scale as $M(\phi) = e^{\frac{\phi}{\sqrt{(d-2)(d-1)}}}$, for series with decreasing polynomial coefficients $c_n = n^\alpha$. Blue: $\alpha = -1$. Orange: $\alpha = -2$, computed numerically. Black: Starobinsky model presented for comparison.

λ , this is discussed in Section 4. One can also construct a potential that uplifts exactly to Starobinsky after making the exponential dependence on the mass scale explicit

$$M(\phi) = M_0 e^{\sqrt{\frac{1}{(d-1)(d-2)}}\phi} \Big|_{d=4} = M_0 e^{\sqrt{\frac{1}{6}}\phi}, \quad (3.23)$$

this requires that the potential before uplifting is given by

$$V(\phi) = \frac{1}{2} M^2 e^{-2\sqrt{\frac{1}{6}}\phi} \left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2. \quad (3.24)$$

One can express this in terms of a differential equation for $f(R) = \sum c_n (R/M^2)^n$ as

$$f'^2 (R - M^2) - f' (f - 2M^2) = M^2, \quad (3.25)$$

which then gives a recursive relation for the coefficients, after fixing $c_0 = 0, c_1 = 1$,

$$2(n+1)c_{n+1} + \sum_{k=0}^n (n-k+1)c_{n-k+1} [(k-1)c_k - (k+1)c_{k+1}] = 0. \quad (3.26)$$

This leads to the coefficients expressed in closed form

$$c_n = \frac{1}{(2n-1)2^{2n-1}} \binom{2n}{n}, \quad \{c_n\} = (1, \frac{1}{4}, \frac{1}{8}, \dots). \quad (3.27)$$

As expected, the coefficients satisfy $1/n^2 \leq c_n \leq 1/n$, and the potential after uplifting is precisely the same as Starobinsky

$$V(\phi) = \frac{1}{2} M_0^2 \left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2, \quad (3.28)$$

normalized as

$$f(R) = R + \frac{1}{4} \frac{R^2}{M_0^2}. \quad (3.29)$$

It is worth mentioning that the setup proposed here avoids all considerations presented in [19], where it was argued that the scale controlling the R^2 term in Starobinsky inflation must necessarily be $M \simeq \Lambda_{\text{sp}}$. In the effective parameterization of eq. (3.29) we have seen that one can have $M_0 \ll \Lambda_{\text{sp}}$. As such, this could be considered an embedding of the Starobinsky model consistent with the Swampland program.

Finally, we note that our analysis assumes an exponential dependence of the species scale on the canonically normalized field ϕ , as suggested by the Swampland Distance Conjecture [11]. This behavior signals that one is approaching the boundary of moduli (field) space, where an infinite tower of states becomes exponentially light. In our parameterization, the bulk of field space is towards $\phi \gg 1$, where $\Lambda_{\text{sp}} \simeq M_{\text{Pl}}$, and the exponential scaling is expected to break down [16]. On the other hand, at small $\phi \simeq 0$, where $\Lambda_{\text{sp}} \simeq M_0$, the potential settles into its minimum and exits the plateau regime, signaling the end of slow roll. The effective potential is thus expected to be reliable only within an intermediate region of field space, corresponding to a part of the plateau regime, where

$$M_0 \simeq H \ll \Lambda_{\text{sp}} \ll M_{\text{Pl}}. \quad (3.30)$$

4 Phenomenology

We have seen that $f(R)$ theories of gravity in d dimensions lead to potentials that decay exponentially fast as $V \propto e^{-\gamma\phi}$ when $f(R) > R^{d/2}$. By considering the mass scale associated to the model to be an exponential scale, as predicted by the SDC with $M = M_0 e^{\gamma_{\text{sp}}\phi}$ we found that these potentials can develop a plateau when considering $\gamma = \gamma_{\text{sp}}$,

$$V(\phi) = V_0(1 - a \exp(-\lambda\phi))^n. \quad (4.1)$$

A potential of this form leads to the expected leading order values of the scalar spectral tilt and tensor to scalar ratio

$$n_s - 1 \simeq -\frac{2}{N_e}, \quad r \simeq \frac{8}{\lambda^2 N_e^2}, \quad (4.2)$$

such that, at first order, the observables only depend on λ and on the number of e-folds N_e . We would like to constraint the possible potentials by whether they can satisfy experimental bounds [1, 2]

$$n_s - 1 = -0.035 \pm 0.004, \quad r < 0.03, \quad (4.3)$$

For potentials asymptotically reproducing Starobinsky inflation such as eq. (3.22), corresponding to curvature series with coefficients $1/n^2 \leq c_n \leq 1/n$ they have

$$\lambda = \sqrt{\frac{d-2}{d-1}} \quad , \quad \lambda_{(d=4)} = \sqrt{\frac{2}{3}} \quad (4.4)$$

$$n_s - 1 = -0.033 \quad , \quad r = 0.0033, \quad (4.5)$$

independent of n . As such, from the perspective of early universe cosmology, they display the same phenomenology as the original Starobinsky model, and as such easily satisfy experimental bounds.

For a spectrum of coefficients obeying $c_n = n^\alpha$ with $\alpha \geq -1$ we found

$$\lambda = \frac{1}{2+\alpha} \sqrt{\frac{d-2}{d-1}} \quad , \quad \lambda_{(d=4)} = \frac{1}{2+\alpha} \sqrt{\frac{2}{3}} \quad (4.6)$$

$$n_s - 1 = -0.033 \quad , \quad r = (2+\alpha)^2 0.0033. \quad (4.7)$$

So these series can fit the experimental bounds on n_s and r for $\alpha \lesssim 1$.

In summary, the resulting potentials described in Section 3, arising from infinite curvature series with coefficients c_n , together with an appropriately varying UV scale, can fit inflationary bounds as long as the coefficients satisfy $1/n^2 \leq c_n \leq n$.

4.1 Inflation with exponentially decaying potentials

The exponential decay of the species scale is bounded as

$$\frac{1}{\sqrt{(d-2)(d-1)}} \leq |\gamma_{sp}| \leq \frac{1}{\sqrt{d-2}}, \quad (4.8)$$

$$\frac{1}{\sqrt{6}} \leq |\gamma_{sp, d=4}| \leq \frac{1}{\sqrt{2}}, \quad (4.9)$$

while for a general series we have exponentially decaying potential with

$$0 \leq \gamma \leq \frac{1}{\sqrt{(d-2)(d-1)}} \quad (4.10)$$

$$0 \leq \gamma_{d=4} \leq \frac{1}{\sqrt{6}}. \quad (4.11)$$

As discussed already, these two can only be matched at a point, corresponding to a species scale coming from compactification on a circle and an infinite curvature series. In any other case, the resulting potential will decay exponentially with rate $\beta = 2(\gamma - \gamma_{sp})$.

$$V(\phi) = V_0 e^{\beta\phi} (1 - \exp(-\lambda\phi))^n \quad (4.12)$$

Here we assume we have a finite series of curvature corrections and the species scale comes from compactification on a circle $\gamma_{sp} = \frac{1}{\sqrt{6}}$. The spectral tilt and tensor-to-scalar ratio receive corrections of the form [19]

$$n_s - 1 \simeq -\frac{2}{N_e} + \beta\lambda, \quad r \simeq \frac{8}{N_e^2 \lambda^2} + \frac{8\beta}{N_e \lambda}, \quad (4.13)$$

such that at leading order they have no dependence on n . For a number of e-folds $N_e = 60$, experimental bounds on n_s restrict β as

$$-0.007 \leq \beta \leq 0.003, \quad (4.14)$$

independent of λ . This means one can only consider a coupling γ very close to γ_{sp} . For a finite series of curvature corrections one finds that satisfying experimental bounds then requires a leading curvature term R^n with $n \geq \mathcal{O}(100)$.

4.2 Curvature corrections in M-theory and $f(R)$

It is known that higher curvature corrections survive in M-theory and in 10-dimensional supersymmetric string theory. The first such correction in M-theory is given by

$$S = \frac{M_{11}^9}{2} \int d^{11}x \sqrt{-g} \left(R + \frac{(4\pi)^{2/3}}{(2\pi)^4 \cdot 3^3 \cdot 2^{13}} \frac{1}{M_{11}^6} \left(\hat{t}_8 \hat{t}_8 \mathcal{R}^4 - \frac{1}{24} \epsilon_{11} \epsilon_{11} \mathcal{R}^4 \right) \dots \right). \quad (4.15)$$

Higher-order terms of the form $D^n R^p$ for $p \leq 6$, $n \leq 6$ have also been computed [36, 37, 34, 33, 38, 39]. However, even higher-order terms are no longer BPS and are not necessarily protected against radiative corrections. We will assume a generic series of curvature corrections and attempt to infer a relation for their coefficients.

Minkowski space $\mathbb{R}^{1,10}$ is a known solution of M-theory, but $AdS_p \times S^{11-p}$ solutions preserving maximal supersymmetry have also been studied [40]. In particular, these solutions are locally symmetric spacetimes with constant curvature, and the Riemann tensor can be written in block diagonal symmetric form, despite the spacetime not having the maximal number of Killing vectors. Since these spaces typically do not exhibit scale separation one has $R_{adS} = R_{KK} = R$, such that every curvature invariant can be expressed in terms of R . Motivated by this, we assume that M-theory allows for spaces that satisfy:

$$R_{\mu\nu\rho\sigma} = \frac{R}{110} (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) \quad (4.16)$$

Under this ansatz, the \mathcal{R}^4 term simplifies to

$$\frac{(4\pi)^{2/3}}{(2\pi)^4 \cdot 3^3 \cdot 2^{13}} \frac{1}{M_{11}^6} \left(\hat{t}_8 \hat{t}_8 \mathcal{R}^4 - \frac{1}{24} \epsilon_{11} \epsilon_{11} \mathcal{R}^4 \right) \simeq \frac{R^4}{M_{11}^6}. \quad (4.17)$$

Given the existence of an infinite tower of curvature corrections, as argued in [16, 23], the effective action takes the form:

$$S = \frac{M_{11}^9}{2} \int d^{11}x \sqrt{-g} \left(R + \sum_n^N c_n \frac{R^n}{M_{11}^{2n-2}} \right). \quad (4.18)$$

We are working at the low energy limit of M-theory, namely 11-dimensional supergravity, so we should generally expect such an infinite series to be an *asymptotic series*. Here, we assume that the leading term in the series is $N > 11/2$, so that the resulting potential decays exponentially. Depending on the nature of the coefficients, the low energy sector of the theory can then vary considerably. For $c_n > 1/n^2$, we obtain after going to Einstein-frame:

$$S = \frac{M_{11}^9}{2} \int d^{11}x \sqrt{-g} \left(R + (\partial\phi)^2 + V_0 e^{-\gamma\phi} \right), \quad (4.19)$$

with $V_0 \simeq M_{11}^{11}$ and $\gamma > 0$.

Several issues arise in this case. In particular, one can formally take a Minkowski limit by expanding $\phi(x) = \phi_0 + \delta\phi(x)$ and taking ϕ_0 to infinity, where the exponential potential vanishes. However, once the potential drops below any other scale in the theory, $\delta\phi$ effectively behaves as a modulus. This is problematic from the standpoint of M-theory, where the moduli space is expected to be trivial [41, 42]. Indeed, we have started from M-theory and ended up with the same theory plus an additional massless field. Furthermore, this being an infinite-distance limit suggests that we should expect a further decompactification or some other exotic limit, which would perhaps suggest that the curvature series cannot be resummed in the way we have described.

In fact, none of these issues arise when the coefficients obey $c_n < 1/n^2$, since ϕ is not allowed to run to infinity⁹. For illustrative purposes, assuming coefficients of the form $c_n = n^\alpha$, with $\alpha < -2$ there is a single Minkowski minimum at $\phi = 0$, and the field gains a mass:

$$m_\phi = \sqrt{\frac{9}{10} 2^{\alpha+2}} M_{11}. \quad (4.20)$$

In this case, the mass is of the order of the cutoff, which can be integrated out, and one recovers the expected content of M-theory in flat space. We then propose that at least one of the following must hold in M-theory:

- One cannot continuously deform a maximally symmetric universe to flat space.

⁹- ∞ is technically allowed, but an exponential wall prevents ϕ from approaching this limit.

- There is no optimal truncation at order \mathcal{R}^N with $N > 11/2$. In this sense, there is no regime in which one can consider the gravitational series as finite at order $N > 11/2$.¹⁰
- The coefficients of the gravitational expansion must asymptotically satisfy $c_n < 1/n^2$.

If this issue could somehow be overcome, the additional modulus might be interpreted as parameterizing the size of the M-theory circle, making sense after dimensional reduction to $d = 10$. The construction proposed here could then naturally be used to argue for the presence of a 10-dimensional cosmological constant coming from compactification on the M-theory circle. However, this requires assuming that the 11d theory has broken supersymmetry, and is thus not protected against quantum corrections.

When the apparent quintessence field is in a slow-roll phase, the cosmological constant is given by:

$$\Lambda_{10} \simeq M_{11}^2 M_{10}^8. \quad (4.21)$$

After further dimensional reduction to $d = 4$, this becomes:

$$\Lambda_{cc} \simeq M_{11}^2 M_{\text{Pl}}^2 \lesssim \Lambda_{\text{sp}}^2 M_{\text{Pl}}^2. \quad (4.22)$$

It is worth mentioning that this scenario is compatible with the strong version of the (A)dS Distance Conjecture [43], particularly in the case of a tower of weakly coupled string excitation modes.

As such, this would lead to a near species scale cosmological constant unless the scalaron is allowed to have a large field excursion. The validity range of the theory is determined by their separation:

$$\frac{M_{11}}{\Lambda_{\text{sp}}} = e^{-\Delta\phi/\sqrt{6}}. \quad (4.23)$$

For a cosmological constant $\Lambda_{cc} \simeq 10^{-120} M_{\text{Pl}}^4$ and a conservative estimate of the species scale $\Lambda_{\text{sp}} \simeq 10 \text{ TeV}$, we require a field excursion:

$$\Delta\phi \simeq 200 M_{\text{Pl}}, \quad (4.24)$$

which is again in conflict with the Swampland Distance Conjecture.

¹⁰The first term to break all supersymmetry is \mathcal{R}^8 , so it is perhaps expected that the series starts diverging once the terms are no longer BPS protected.

5 Conclusion

In this work we have analyzed how higher curvature corrections in $f(R)$ gravity give rise to scalar potentials and how these connect to the notion of a dynamical species scale. Starting from general considerations of $f(R)$ models in arbitrary dimensions, we found that the generic outcome of infinite curvature series is an exponentially decaying potential at large field values. We then established a connection between these potentials and the exponential behavior of the species scale. When the curvature corrections are controlled by the species scale, cancellations can occur such that the potential remains flat, or is effectively uplifted. In particular, compactification on a circle induces a field-dependent cutoff that scales exponentially with the radion, whose scaling precisely matches the asymptotics of exponential potentials generated by infinite higher curvature terms, leading to a plateau potential and suggesting a relation between the two. In this setup the inflationary scale remains parametrically below the cutoff throughout the relevant field range, ensuring the consistency of the effective description even in the presence of an infinite tower of corrections.

From a phenomenological perspective, the uplifted potentials are nearly indistinguishable from Starobinsky inflation, and we have provided an embedding of the Starobinsky model consistent with the presence of higher curvature corrections. Plateau potentials can thus arise naturally from higher curvature gravity uplifted by compactification, rather than as isolated or finely tuned scenarios. Their predictions for the spectral tilt and tensor-to-scalar ratio are essentially determined by the asymptotic structure of curvature series and the number of e -folds. The precise value of the exponential rate λ is then highly constrained, and may come within experimental reach in the coming decade [4, 5].

We also commented on possible implications in string and M-theory, where series of curvature corrections are expected to arise [37, 36]. While the structure and convergence of such series remain uncertain, we have argued that the resulting potentials can be problematic if the coefficients do not decay sufficiently fast, for instance slower than $c_n \sim 1/n^2$.

To conclude, we have highlighted a simple mechanism linking higher curvature corrections, the species scale, and plateau-like inflationary potentials. It is worth mentioning that along curvature series weighted down by the species scale there is also a series of higher loop corrections weighted down by the KK scale of the compact space, and these have been shown to be the dominant contribution in some cases [33]. This remains subject for future work, as it would be interesting to see if the mechanism we have outlined could be obstructed by higher loop corrections.

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A Higher curvature Terms in Quasi-de Sitter

In this appendix we justify the approximation $f(\mathcal{R}) \simeq f(R)$ during slow-roll inflation and make the scaling with slow-roll parameters explicit.

For an FLRW background with scale factor $a(t)$ and Hubble parameter $H = \dot{a}/a$, the Ricci scalar in d dimensions is

$$R = 2(d-1)\dot{H} + d(d-1)H^2 = d(d-1)H^2 \left(1 - \frac{2}{d}\epsilon\right), \quad (\text{A.1})$$

with the slow roll parameter defined as

$$\epsilon \equiv -\frac{\dot{H}}{H^2}. \quad (\text{A.2})$$

During a slow roll phase, we have $\epsilon \ll 1$, and the background is (nearly) maximally symmetric, with

$$R_{\mu\nu} \simeq \frac{R}{d} + g_{\mu\nu} + \mathcal{O}(\epsilon), \quad R_{\mu\nu\rho\sigma} \simeq \frac{R}{d(d-1)}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}) + \mathcal{O}(\epsilon), \quad (\text{A.3})$$

so that any local invariant made out of k Riemann tensors satisfies

$$\underbrace{\mathcal{R} \cdots \mathcal{R}}_{k \text{ times}} \propto R^k + \mathcal{O}(\epsilon). \quad (\text{A.4})$$

We can also consider operators with higher derivative orders

$$\mathcal{O}_{m,n} \sim (\nabla^2)^m R^n. \quad (\text{A.5})$$

Using $\nabla_\alpha R = \mathcal{O}(\epsilon H^3)$ and $\nabla^2 R = \mathcal{O}(\epsilon H^4)$ on the background, each pair of derivatives brings an extra suppression by ϵ

$$(\nabla^2)^m R^n \sim \epsilon^m H^{2(n+m)} \sim \epsilon^m R^{n+m}. \quad (\text{A.6})$$

Then, for a gravitational EFT of the form

$$\mathcal{L} = \frac{1}{2}R + \sum_{n \geq 2} \frac{c_{0,n}}{\Lambda^{2n-2}} R^n + \sum_{m \geq 1, n \geq 1} \frac{c_{m,n}}{\Lambda^{2(n+m)-2}} (\nabla^2)^m R^n + \dots, \quad (\text{A.7})$$

with Λ the UV cutoff, we will have that on a slow-roll background,

$$\frac{\mathcal{O}_{m,n}}{R^n} \sim \epsilon^m \left(\frac{H}{\Lambda} \right)^{2m}. \quad (\text{A.8})$$

Then, as long as

$$\epsilon \ll 1, \quad \frac{H}{\Lambda} \ll 1, \quad (\text{A.9})$$

the leading derivative corrections will be parametrically suppressed relative to the R^n terms.

B Asymptotic Behavior of Infinite Series

In this appendix we discuss the behavior of infinite series that define $f(R)$ near its finite radius of convergence.

Polynomial coefficients

Consider coefficients of the form $c_n = n^\alpha$. The associated power series has radius of convergence $|R| < M_0^2$ for $\alpha \geq -1$, and $|R| \leq M_0^2$ for $\alpha < -1$, since the series converges also at the boundary point. In these regimes one finds closed forms:

$$f(R) = \frac{M_0^2 P_\alpha \left(\frac{R}{M_0^2} \right)}{\left(1 - \frac{R}{M_0^2} \right)^{\alpha+1}}, \quad \alpha \geq 0, \quad (\text{B.1})$$

$$f(R) = M_0^2 \text{Li}_{-\alpha} \left(\frac{R}{M_0^2} \right), \quad \alpha \leq -1, \quad (\text{B.2})$$

where P_α is a polynomial of degree α and Li_ν denotes the polylogarithm of order ν .

The series in (B.2) in fact converges for $R < M_0^2$ and also at the boundary $R = M_0^2$, however, only for $\alpha > -2$ does the limit $\phi \rightarrow \infty$ correspond to $R \rightarrow M_0^2$. For $\alpha \leq -2$, the potential is supported only up to a finite field range in $\phi > 0$.

Exponential coefficients

If $c_n = a e^{\mu n}$ with fixed $a, \mu \in \mathbb{R}$, then

$$\sum_{n \geq 1} c_n \left(\frac{R}{M_0^2} \right)^n = a \sum_{n \geq 1} \left(e^\mu \frac{R}{M_0^2} \right)^n = a \sum_{n \geq 1} \left(\frac{R}{M_0'^2} \right)^n, \quad M_0'^2 \equiv M_0^2 e^{-\mu}.$$

Thus an exponential weight can be absorbed into a rescaling of the reference scale M_0 ; the large- R behavior is unchanged, only the effective radius of convergence shifts.

Super-exponential coefficients and optimal truncation

If c_n grows faster than any $e^{\mu n}$ (e.g. factorial growth), the radius of convergence is zero and the series is asymptotic, with optimal truncation at order

$$|c_{N+1} \left(\frac{R}{M_0^2} \right)^{N+1}| \approx |c_N \left(\frac{R}{M_0^2} \right)^N|. \quad (\text{B.3})$$

In such cases, one may consider an optimal truncation of the series at some finite order $n \leq N$ [44, 45].

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