Part I
Algebraic Structures

We begin the part on algebraic structures with an introductory chapter on rings. In Chap. 2 we then present a rich algebraic structure, the *module* over a *ring*. We describe several examples, prove a number of basic facts about this structure and also a prototypical theorem about a special case, the finitely generated modules over Euclidean rings. We also show some non-trivial applications, especially to the linear mappings known from elementary linear algebra. Later we will also see applications to the integration of functions in several variables.

In Chap. 3 we explain several constructions in the context of modules. Also in this chapter, the selection of material follows the two basic criteria: relevance in mathematical practice and the model character for general mathematical procedures. My choice fell on multilinear algebra, which is treated rather stepmotherly in the common curricula despite its omnipresence in all parts of mathematics.

To make the model character of the presented structures and constructions clear to the readers, we introduce basic concepts of universal algebra and category theory in Chap. 4. Using many examples, we sketch how the use of the categorical language in particular unifies algebraic structure considerations. In the last two sections, we then show that the constructions presented in Chaps. 2 and 3 are natural in a precise sense, and even inevitable in a certain way.