

\mathcal{R}^2 effectively from Inflation to Dark Energy

Philippe Brax and Pierre Vanhove¹

¹*CEA, DSM, Institut de Physique Théorique, IPhT, CNRS,
MPPU, URA2306, Saclay, F-91191 Gif-sur-Yvette, France*

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We consider the single-parameter $\mathcal{R} + c\mathcal{R}^2$ gravitational action and use constraints from astrophysics and the laboratory to derive a natural relation between the coefficient c and the value of the cosmological constant. We find that the renormalisation of c from the energy of the inflationary phase to the infrared, where the acceleration of the expansion of the Universe takes place, is correlated with the evolution of the vacuum energy. Our results suggest that the coefficient of the \mathcal{R}^2 term may provide an unexpected bridge between high-energy physics and cosmological phenomena such as inflation and dark energy.

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I. INTRODUCTION

Our understanding of the force of gravity impacts the way we view the shape and the dynamics of the observable Universe. The current cosmological paradigm relies on General Relativity, Einstein's theory of gravity [1, 2] crafted as a relativistic theory of curved space-time. Gravity is inferred to be universal, i.e. it couples equally to all types of matter and energy, from simple requirements such as Lorentz invariance [3]. Einstein proposed three classical tests of this theory of gravity [4]: the perihelion precession of Mercury, the deflection of light by the Sun and the gravitational redshift of light. These classical tests illustrate the ubiquity of the gravitational force. In addition, several modern tests are routinely performed in order to test general relativity as well as effects that, in principle, could occur in a theory of gravitation different from Einstein's theory of gravity [5].

Einstein's General Relativity has stood the test of time and remains unscathed after more than a century of intense investigations. In the last 20 years, the emergence of the

acceleration of the expansion of the Universe [6–8] has led to various attempts to understand why gravity does not appear to be attractive on large cosmological scales. This could lead to intricate models of dark energy although the recent observation of the neutron star merger by the LIGO/Virgo consortium [9, 10] has reinforced the strength of the claim that, so far, the best candidate as an explanation for the cosmic acceleration is dark energy in the form of a constant vacuum energy, whose archetype is the original cosmological constant introduced by Einstein in 1917 and leading to the de Sitter space-time of 1919.

In order to explain the expansion of the observable Universe a cosmological constant is added (with the signature $(- +++)$)

$$S_{EH-\Lambda} = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} \mathcal{R} - \Lambda^4 + g^{\mu\nu} T_{\mu\nu}^{\text{matter}} \right], \quad (1)$$

with a value today given by [11]

$$\Lambda_0^4 = \Omega_{\Lambda 0} 3(H_0 M_{\text{Pl}})^2 \simeq 2.4 \times 10^{-47} \text{GeV}^4 \simeq (2.2 \times 10^{-3} \text{eV})^4, \quad (2)$$

where we have introduced $\Omega_{\Lambda 0} \sim 0.7$ as the fraction of dark energy in the Universe and $\rho_c = 3M_{\text{Pl}}^2 H_0^2$ is the critical density of the Universe. Here H_0 is the Hubble rate today and $M_{\text{Pl}} = \sqrt{\hbar c/(8\pi G_N)}$ is the reduced Planck mass.

The smallness of the cosmological constant is still an open problem. This is the “old” cosmological constant problem [12] whereby all the massive particles of the Universe contribute to the vacuum energy with threshold corrections which are quartic in their masses. Phase transitions, and at least the electro-weak and the quantum chromodynamics ones, also contribute to an alarming level. These large contributions should cancel in order to match with observations for a determination of (2) solely from the framework of particle physics.

Concretely, the quantum corrections do read

$$\Lambda_0^4 = \Lambda^4(m_e) - 2 \sum_{f=1}^3 \frac{m_f^4}{64\pi^2} \ln \frac{m_e^2}{m_f^2} + \delta\Lambda_0^4. \quad (3)$$

where $\Lambda^4(m_e)$ encapsulates the contributions from the tower of particle masses above the electron mass, i.e. it captures the high energy physics effects which are relevant to the calculation of the dark energy in the deep IR. We have singled out the contributions from the neutrino, because the neutrino masses are known to be at most in the milli-eV range

from the Planck 2018 constraints [13–18]. We have added a component $\delta\Lambda_0^4$ coming from new physics at low energy below the electron mass.

Cosmological constraints: The neutrino contribution to the vacuum energy

$$\delta\Lambda_{\text{neutrino}}^4 = -2 \sum_{f=1}^3 \frac{m_f^4}{64\pi^2} \ln \frac{m_e^2}{m_f^2}. \quad (4)$$

can be estimated using the Planck data [13] to be a large contribution

$$6 \times 10^{-7} \text{eV}^4 \leq |\delta\Lambda_{\text{neutrino}}^4| \leq 6 \times 10^{-6} \text{eV}^4 \quad (5)$$

which exemplifies the nature of the cosmological constant problem even at low energy¹.

Astrophysical constraints: We constrain the energy densities $\Lambda^4(m_e)$ and $\delta\Lambda_0^4$ from astrophysical considerations. The X-ray emitting gas of a galaxy cluster has a typical temperature of $T_X \sim 1$ keV, in regions of total baryonic and dark matter density of about 500 times the mean density of the Universe. These systems typically appeared at a redshift $z \gtrsim 0.1$ and already have a lifetime of the order of the age of the Universe. In such clusters the neutrinos, coming either from the early Universe with an energy of order 10^{-4} eV or from astrophysical processes such as the burning of stars with an energy around 100 keV, have a very small cross section with matter and decouple from the physics inside the clusters, which can then be described by non-relativistic matter particles (such as electrons and protons) and General Relativity augmented with a vacuum energy. Under the strong assumption that the latter only takes into account all the physics for energy scales greater than T_X , then the vacuum energy $|\Lambda^4(m_e)|$ cannot be too large otherwise it would significantly affect the dynamics within the cluster. In a spherical approximation, the cluster would behave as a separate universe [19], with its own vacuum energy $\Lambda^4(m_e)$. To ensure small dynamical effects, we have the conservative bound

$$|\Lambda^4(m_e)| \lesssim 200 \Lambda_0^4. \quad (6)$$

This is the Weinberg bound which guarantees that the existence of galaxies is not prevented by the cosmological constant. Larger values would prevent the existence of galaxies and therefore clusters. The factor of 200 is obtained using the spherical collapse approximation for a separate Universe and reads more precisely [19]

$$|\Lambda^4(m_e)| \lesssim \frac{500}{729} \delta_R \rho_R. \quad (7)$$

¹ The Planck data constrains the sum of the neutrino masses $\sum_\nu m_\nu \leq 0.12 \text{eV}$ at 95 % confidence level.

where ρ_R is the matter density at recombination and δ_R the typical overdensity at the same epoch. This reasoning makes use of the presence at low redshift of hot high-density structures within the cooler and lower-density cosmological background.

II. THE \mathcal{R}^2 GRAVITY

The unknown contribution $\delta\Lambda_0^4$ is there to compensate for the other quantum corrections to the vacuum energy in particular the large value from the neutrinos in (4). The nature of these extra quantum corrections have led to many possible scenarios (supersymmetry, ...) but so far have failed to give a convincing answer. In this text we propose a scenario from the purely gravitational sector by embedding Einstein gravity into an effective field theory framework.

It is very natural to embed the classical Einstein theory of gravity into a quantum gravity framework valid at high energy, possibly close to the Planck scale, where the Einstein-Hilbert action (1) is only the first term of a low-energy effective action

$$\mathcal{S}_{\text{eff}} = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} \mathcal{R} - \Lambda^4 + g^{\mu\nu} T_{\mu\nu}^{\text{matter}} + \mathcal{L}_{\text{corrections}} \right]. \quad (8)$$

The contributions $\mathcal{L}_{\text{corrections}}$ arise from such extensions of Einstein gravity induced either by high-energy quantum corrections from a high-energy completion like string theory or new interactions from extra massless fields which are still undetected. For instance in a string context and for curvatures below the string scale, only the curvature corrections up to cubic order are relevant. Higher order corrections in the curvature are suppressed by powers involving the ratio with the string scale [20, 21]. Similarly the cubic term in the curvature is suppressed compared to the quadratic terms as long as the curvature is less than the compactification scale. In summary and as long as one considers scales below the string and compactification scales, models described by quadratic curvature corrections are sufficient. Of course this applies to the description of inflation as long as the inflation scale is low enough compared to the string and compactification scales.

This approach is based on the idea that long range interactions in gravity can be described by a low-energy effective field theory, even if the high-energy behaviour of quantum gravity is still unknown [22–24]. Although the status of the high-energy behaviour of quantum gravity is still open, considering the effective field theory of gravity at low energy does not

pose a problem. One can safely extract low-energy physics from the quantization of the gravitational interactions observables that are independent of the high-energy behaviour. We may quote J. D. Bjorken in [25] who argues that the Einstein-Hilbert term with its universal coupling to matter is naturally the first term of an effective field theory of quantum gravity. To the contrary to what many may think “as an open theory, quantum gravity is arguably our best quantum field theory, not the worst. [...] quantum gravity, when treated [...] as an effective field theory, has the largest bandwidth; it is credible over 60 orders of magnitude, from the cosmological to the Planck scale of distances.” [25]. This is precisely the philosophy that we will be following in this text.

Amongst the terms induced from the unknown high-energy corrections, the scalar \mathcal{R}^2 operator plays a distinctive role

$$S_{R^2}(\mu) = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} \mathcal{R} - \Lambda^4(\mu) + c_0(\mu) \mathcal{R}^2 + c_2(\mu) \left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} \mathcal{R}^2 \right) \right] + S_{\text{matter}}, \quad (9)$$

it has been shown in [26–29] that the coefficient $c_2(\mu)$ is always asymptotically free, since $dc_2(\mu)/d \log \mu^2 > 0$, whereas $c_0(\mu)$ is asymptotically safe, $dc_0(\mu)/d \log \mu^2 < 0$ for the non tachyonic case $c_0(\mu) > 0$, which is the case of interest in this paper. Therefore, at low-energy $c_2(\mu)$ tends to zero whereas $c_0(\mu)$ grows, leading to the hierarchy $c_0(\mu) \gg c_2(\mu)$ at very low energy. Hence the quadratic Ricci scalar term is enhanced as compared with other quadratic and higher order contributions.

Therefore we can restrict ourselves to the scalar \mathcal{R}^2

$$S_{R^2}(\mu) = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} \mathcal{R} - \Lambda^4(\mu) + c_0(\mu) \mathcal{R}^2 \right] + S_{\text{matter}}. \quad (10)$$

At late time, a relevant effective action in the same Wilsonian sense is

$$\mathcal{S}_{\text{late}} = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} \mathcal{R} - \Lambda_0^4 + c_0(\mu_{\text{late}}) \mathcal{R}^2 \right]. \quad (11)$$

Stelle [30] showed that the presence of the \mathcal{R}^2 term leads to the propagation of a scalar massive degree of freedom, the scalaron, of mass

$$m_{\text{scalarm}}^2 = \frac{M_{\text{Pl}}^2}{6c_0(\mu_{\text{late}})}, \quad (12)$$

which could contribute to a fifth force in gravitational experiments

$$V(r) = -\frac{G_N M}{r} \left(1 + \frac{1}{3} e^{-m_{\text{scalarm}} r} \right). \quad (13)$$

The absence of evidence for short range forces in the Eöt-Wash experiment [31–33] provides an upper bound on the range of scalar forces $d \leq 52\mu\text{m}$ corresponding to the strong lower bound

$$m_{\text{scalarmon}} \gtrsim 3.8 \times 10^{-3} \text{ eV}. \quad (14)$$

At this point we notice the observational coincidence that this lower bound in the milli-eV range is strikingly close to the typical energy scale set by the cosmological constant (2) and suggest that the scalaron could lead to the $\delta\Lambda_0^4$ contribution to the vacuum energy (3). Or equivalently that the value of the coefficient of the \mathcal{R}^2 term could be connected to the value of the cosmological constant. The scalaron contributes to the vacuum energy as

$$\delta\Lambda_{\text{scalarmon}}^4 = \frac{m_{\text{scalarmon}}^4}{64\pi^2} \log\left(\frac{m_e^2}{m_{\text{scalarmon}}^2}\right). \quad (15)$$

Assuming that only the scalaron and the neutrinos have a mass smaller than the electron mass, then $\delta\Lambda_0^4 = \delta\Lambda_{\text{scalarmon}}^4$. Combining the condition (6) with the range (5) we deduce that the scalaron mass is bounded from above by the fourth-power mean value of the neutrino masses

$$m_{\text{scalarmon}} \lesssim \bar{m}_\nu = (m_1^4 + m_2^4 + m_3^4)^{\frac{1}{4}} \simeq 0.1 \text{ eV}, \quad (16)$$

leading to the narrow interval on the scalaron mass

$$3.8 \times 10^{-3} \text{ eV} \lesssim m_{\text{scalarmon}} \lesssim 0.1 \text{ eV} \quad (17)$$

or for the coefficient the \mathcal{R}^2 term in the gravity effective action in the infrared (IR)

$$10^{57} \lesssim c_0(\mu_{\text{IR}}) \lesssim 7 \times 10^{59}. \quad (18)$$

III. \mathcal{R}^2 RUNNING FROM INFLATION TO DARK ENERGY

The \mathcal{R}^2 term plays an interesting role in two regimes of interest for cosmology. Its role could be significant from the large energies and early times of the inflationary era to the late time and low energy regime where dark energy sets the scene. Inflation is one of the most successful and elegant theoretical descriptions of the post-Planck early time, since it overcomes in a rigid way most of the shortcomings of the standard Big Bang cosmology. During the inflationary era, the currently best model when compared to data is well described by the effective action with the \mathcal{R}^2 correction [34]

$$\mathcal{S}_{\text{infl.}} = \int d^4x \sqrt{-g} \left(\frac{M_{\text{Pl}}^2}{2} \mathcal{R} + c_0(\mu_{\text{infl}}) \mathcal{R}^2 \right). \quad (19)$$

This so-called Starobinski's model produces a very good fit of the observed spectrum of primordial perturbations. As the coefficient of the \mathcal{R}^2 term is dimensionless, it naturally depends on the energy scale in a Wilsonian sense, here set to be μ_{infl} which can be identified with the nearly-constant Hubble rate during inflation.

Let us now review how to relate the inflation models (19) and the late time effective action (11) by the renormalisation group equation.

The effective action in (10) is an $f(\mathcal{R})$ theory with

$$f(\mathcal{R}) = -\frac{2\Lambda^4(\mu)}{M_{\text{Pl}}^2} + \mathcal{R} + \frac{2c_0(\mu)}{M_{\text{Pl}}^2} \mathcal{R}^2. \quad (20)$$

This $f(\mathcal{R})$ model can be written as scalar-tensor theories with an action

$$S_{\text{scalarmon}} = \int d^4x \sqrt{-g} M_{\text{Pl}}^2 \left(\frac{1}{2} \mathcal{R} - \frac{1}{2} (\partial\varphi)^2 - V(\varphi) \right) + S_{\text{matter}}(e^{\frac{2}{\sqrt{6}}\varphi} g_{\mu\nu}) \quad (21)$$

where φ is dimension less and the coupling to matter is via the Jordan metric $g_{\mu\nu}^J = e^{\frac{2}{\sqrt{6}}\varphi} g_{\mu\nu}$. The potential in the action is given by the relation

$$V(\varphi) = \frac{\mathcal{R} \frac{df(\mathcal{R})}{d\mathcal{R}} - f(\mathcal{R})}{\left(\frac{df(\mathcal{R})}{d\mathcal{R}} \right)^2} \quad (22)$$

whilst the field φ is related to the curvature as

$$\frac{df(\mathcal{R})}{d\mathcal{R}} = \exp \left(-\frac{2}{\sqrt{6}} \varphi \right). \quad (23)$$

The potential for the scalaron reads explicitly

$$V(\varphi) = \frac{M_{\text{Pl}}^2}{8c_0(\mu)} \left(e^{\frac{2\varphi}{\sqrt{6}}} - 1 \right)^2 + 2 \frac{\Lambda^4(\mu)}{M_{\text{Pl}}^2} e^{\frac{4\varphi}{\sqrt{6}}} \quad (24)$$

as a function of the scalaron φ . From this potential we can read-off the effective mass of the scalaron

$$m_{\text{scalarmon}}^2(\mu) = \frac{M_{\text{Pl}}^2}{6c_0(\mu)} + \frac{16}{3} \frac{\Lambda^4(\mu)}{M_{\text{Pl}}^2}. \quad (25)$$

Having fixed the Planck mass M_{Pl}^2 , which is then independent of μ and fixes all the energy, distance and time scales, the parameters in the Wilson effective action (21) are μ dependent. The cosmological constant $\Lambda(\mu)$ and the coefficient $c_0(\mu)$ of the \mathcal{R}^2 term follow the

renormalisation group equations where they evolve each time a particle species of mass m is integrated out [35], i.e. when $\mu \leq m$. In the history of the Universe, particles in the thermal bath are integrated out when the temperature falls below the mass m , leading to

$$\frac{dm_{\text{scalarm}}^2(\mu)}{d \ln \mu} = -\frac{1}{48\pi^2 M_{\text{Pl}}^2} \text{Str}(M^4) \theta(\mu - M). \quad (26)$$

and

$$\frac{d\Lambda^4(\mu)}{d \ln \mu} = -\frac{1}{32\pi^2} \text{Str}(M^4) \theta(\mu - M) \quad (27)$$

in terms of the supertrace of the complete mass matrix M of all the particles in the Universe. The scalaron arising from the gravitational sector of the theory couples to all particles in the spectrum. This implies that the coefficient of the \mathcal{R}^2 term has an evolution linked to the one of the cosmological constant

$$\frac{d(M_{\text{Pl}}^4 c_0(\mu)^{-1} + 28\Lambda^4(\mu))}{d \ln \mu} = 0. \quad (28)$$

The initial values of the renormalisation group are taken at the end of inflation corresponding to the reheat temperature T_{reh} , i.e. larger than any physical masses of the particles in the spectrum of the theory. The previous renormalisation group equations allow one to calculate the evolution of c_0 and Λ from inflation to the infrared. The only new ingredient in this integration is the presence of phase transitions taking place at different values of μ , for instance the QCD phase transition, where the cosmological constant jumps due to the change of vacuum energy during the transitions [36].

The value of $c_0(\mu_{\text{end}})$ at the end of inflation can be deduced from the normalisation of the CMB spectrum as

$$\frac{V(\varphi_{\star})^3}{M_{\text{Pl}}^2 (V'(\varphi_{\star}))^2} \simeq \frac{3e^{-\frac{4\varphi_{\star}}{\sqrt{6}}}}{32c_0(\mu_{\text{end}})} \simeq 2 \times 10^{-11} \quad (29)$$

evaluated at the value of φ_{\star} determined by the spectral index

$$n_s - 1 = 2\eta_{\star} = 2 \frac{V''(\varphi)}{V(\varphi)} \Big|_{\varphi=\varphi_{\star}} \simeq -\frac{8}{3} e^{\frac{2\varphi_{\star}}{\sqrt{6}}}. \quad (30)$$

giving the constraint on the coefficient of the \mathcal{R}^2 from the CMB data

$$c_0^{-1}(\mu_{\text{end}}) \simeq 1.52 10^{-9} (n_s - 1)^2 \simeq 2 \times 10^{-12}, \quad (31)$$

where we used that $n_s - 1 \simeq -0.0351$ according to the Planck data [13].

The cosmological constant $\Lambda^4(\mu_{\text{end}})$ at the end of inflation is not directly determined by the experimental data, but from an ultraviolet (UV) point of view, its value is constrained by the renormalisation group equation (28) reinforcing the fact that the physics at high energy seems to be largely constrained by the physics at low energy. This is the case of the coefficient of the \mathcal{R}^2 term during inflation which is constrained by the CMB data. Here the physics of the vacuum in the IR, i.e. the vacuum stability combined with gravitational tests, determines indirectly the value of the cosmological constant in the UV. Of course, our analysis does not provide any explanation for this value from a top-bottom point of view.

IV. OBSERVATIONAL PREDICTIONS

The gravitational effective action (10) displays an interesting universality behaviour as it can be applied to the inflation regime and the late cosmological times. The effective action has a priori two free parameters the cosmological constant $\Lambda^4(\mu)$ and the coefficient $c_0(\mu)$ of the \mathcal{R}^2 term. We have argued that these two coefficients have an evolution that is related by the renormalisation group equation in (28) and determined the value of $c_0(\mu)$ in the UV and IR from observations.

At late time, the coefficient of the \mathcal{R}^2 term is in the narrow interval for the scalaron mass (17). This prediction can be tested by the Eöt-Wash experiments [31–33].

We remark that this interval is compatible with the value $m_{\text{scalarmon}} \simeq 4.4 \times 10^{-3}$ eV for which the \mathcal{R}^2 induced scalaron could be at the origin of the observed dark matter abundance [37–40]. In this scenario, at low energy compared to the inflation scale, the vacuum expectation value of the scalaron is displaced from the origin by an amount depending on the electroweak scale $v \simeq 250$ GeV because of its coupling to the Higgs field. For more generic initial conditions after inflation taking into account the quantum fluctuations of the scalaron during inflation, the whole interval up to $m_{\text{scalarmon}} \simeq 0.1$ eV could accommodate dark matter. As the electroweak transition begins, the scalaron starts oscillating with a decreasing amplitude eventually converging to the origin. This misalignment mechanism is similar to what happens for axions and leads an abundance of dark matter which fits the observed value for $m_{\text{scalarmon}} \simeq 4.4 \times 10^{-3}$ eV [40]. Combining both scenarios, this would lead to a possible signal in gravitational experiments below a distance $d \lesssim 45$ μm , which is well

within the allowed interval (17)². This strongly supports the exciting possibility of testing the existence of the new gravitational interaction \mathcal{R}^2 , which could play a role in both the late acceleration of the universe and dark matter. All in all, we believe that this is a consequence of the universality of the effective long distance physics of gravity compared to the scales of quantum gravity.

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² The link with scalar dark matter is guaranteed as long as the amplitude of oscillation around the minimum of the scalar potential is small and higher order terms act as corrections to a simple quadratic oscillator. When the quadratic terms are dominant the scalar dark matter is of the fuzzy type [41] whilst corrections to the quadratic Lagrangian can also be interesting astrophysically [42]. We leave a detailed study of the phenomenology for future work.

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