Topologically Stabilized Torsion in Weak-Field Gravity: A Ricci–Flow Framework

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Abstract

We investigate stationary torsional configurations supported by chiral Majorana neutrino currents in linearized gravity. A Ricci–flow–inspired geometric relaxation (with no physical time interpretation) is introduced to drive the metric perturbation toward fixed points sustained by chiral sources while keeping curvature invariants negligible. We show that divergence-free chiral currents can support globally non-trivial torsional holonomy stabilized by topological invariants associated with $\pi_1(S^1)$ and $\pi_3(S^3)$. Toroidal skyrmionic domains emerge when one chirality dominates, whereas a chiral-flip interference sector enables Möbius-type non-orientable bridges between opposite-chirality regions. In the static limit, a Green-function formulation provides a finite-range Yukawa-type response governed by the neutrino coherence length. These results identify a purely torsional mechanism — independent of local curvature — through which coherent chiral currents may influence effective gravitational behavior in neutrino-rich environments.

Keywords: Torsion in gravity; Chiral spinor fields; Majorana neutrinos; Ricci flow; Stationary solutions; Topological structures; Linearized gravity.

1 Introduction

Spinor fields in gravitational theories with torsion couple to the antisymmetric part of the affine connection [1, 2]. Majorana neutrinos, with intrinsic chirality, are natural sources for torsional effects and may form coherent domains. This short communication focuses on *stationary* torsional solutions supported by chiral currents, which are topologically non–trivial while the curvature remains negligible. Coherent neutrino populations are expected to arise in several high-density astrophysical environments — from supernova interiors to the early Universe neutrino background — making torsion-induced effects of potential relevance beyond laboratory scales.

A broader development, including extended classification and topological aspects, appears in a companion study [3]. Related Yukawa–torsion effects are discussed in [6].

1.1 Chiral currents and weak–field setting

We consider the weak–field expansion of the metric,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1, \tag{1}$$

and focus on Majorana spinor fields, whose left- and right-handed components are related by charge conjugation. Their intrinsic chirality makes them natural sources for torsional degrees of freedom.

The left- and right-handed fermionic currents are defined as

$$J_L^\mu = \bar{\psi}\gamma^\mu P_L \psi, \qquad J_R^\mu = \bar{\psi}\gamma^\mu P_R \psi, \qquad P_{L,R} = \frac{1 \mp \gamma_5}{2}, \tag{2}$$

and encode the local transport of chiral charge. In spatial components,

$$J_{L,R}^k = \bar{\psi}_{L,R}\,\sigma^k\,\psi_{L,R}, \tag{3}$$

with σ^k the Pauli matrices.

Torsion is sourced by the chiral imbalance between the left- and right-handed sectors, schematically $(J_L - J_R)$, and by an interference (flip) sector built from mixed bilinears of the form $\bar{\psi}_L \psi_R$. The latter enables the formation of globally non-orientable torsional structures despite locally vanishing curvature.

1.2 Ricci flow evolution

To model the geometric response of the torsional sector to chiral currents, we use a flow parameter τ that evolves the metric perturbation toward stationary configurations without representing physical time. In the weak–field regime, the evolution equation takes the form

$$\frac{\partial h_{\mu\nu}}{\partial \tau} = -2\,R_{\mu\nu}(h), \tag{4}$$

inspired by the classical Ricci flow framework introduced by Hamilton and extended to include entropy functionals by Perelman [4, 5]. Here $R_{\mu\nu}(h)$ is the linearized Ricci tensor and encodes the torsional contribution generated by chiral currents [3].

Using the Levi–Civita structure ϵ_{abc}, the most relevant components can be expressed schematically as

$$\frac{\partial h_{0c}}{\partial \tau} \propto \int dx^b\,\epsilon_{bck}\left(\bar{\psi}_L\sigma^k\psi_L - \bar{\psi}_R\sigma^k\psi_R\right), \tag{5}$$

$$\frac{\partial h_{ac}}{\partial \tau} \propto \int dx^0\,\epsilon_{ack}\left(\bar{\psi}_L\sigma^k\psi_L - \bar{\psi}_R\sigma^k\psi_R\right), \tag{6}$$

$$\left.\frac{\partial h_{ac}}{\partial \tau}\right|_{\text{flip}} \propto \int dx^b\,\epsilon_{bca}\left(\bar{\psi}_L\psi_R - \bar{\psi}_R\psi_L\right), \tag{7}$$

demonstrating that left- and right-handed currents induce torsion with opposite sign, while the flip bilinear couples the two chiral components and enables the formation of globally coherent, possibly non-orientable configurations.

The aim of the flow evolution is to identify geometry–current combinations that satisfy $\partial_\tau h_{\mu\nu} = 0$, i.e. stationary torsional states, which are analyzed in the next section.

1.3 Left/right and chiral-flip contributions

The Levi–Civita tensor ϵ_{abc} in Eqs. (5)–(7) enforces an opposite orientation of torsion generated by left- and right-handed currents. Domains dominated by J_L and J_R therefore induce counter-rotating local frames, realizing geometrically distinct chiral phases with vanishing curvature.

The flip sector, driven by mixed bilinears of the form $\bar{\psi}_L\psi_R$, couples the two chiral components and allows for a continuous interpolation between them. This mechanism acts as a torsional "stitching" that preserves stationarity while modifying the global orientability of the configuration.

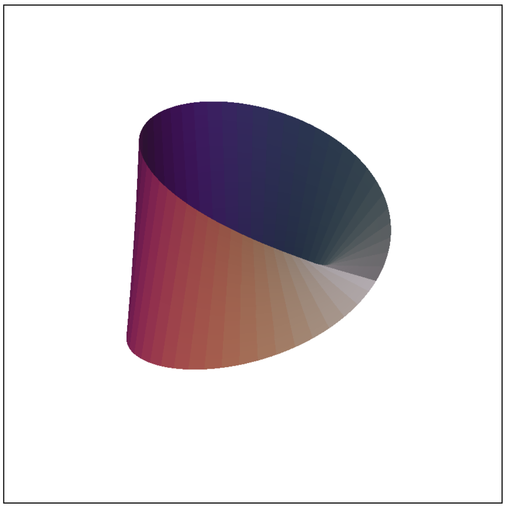

Figure 1: Schematic Möbius-like torsional bridge generated by chiral-flip stitching between domains of opposite chirality. Local curvature remains negligible while the global holonomy is non-trivial.

Coherent domains connected through the flip sector can thus produce globally non-trivial torsional structures — such as toroidal skyrmionic regions or Möbius-like bridges — where all curvature invariants remain arbitrarily small. These configurations will be classified topologically in the next section.

2 Topological characterization

Stationary torsional configurations fall into distinct topological classes depending on whether torsion is confined along a closed line or distributed over a tubular domain. These properties are captured by global invariants associated with the holonomy of the torsional connection and its mapping into an internal chiral space.

2.1 Holonomy and torsional vortices

The torsional sector can be represented by a gravitational vector potential A^g_μ [3], such that parallel transport acquires a phase depending solely on the global structure of the connection. The torsional circulation along a closed loop Γ is quantified by the holonomy:

$$\Phi_T = \oint_\Gamma A^g_\mu \, dx^\mu = 2\pi n, \qquad n \in \mathbb{Z}\,. \tag{8}$$

A non-zero integer n signals the presence of a line-like torsional vortex, classified by the first homotopy group $\pi_1(S^1)$. The spacetime remains locally flat but exhibits a globally non-trivial phase — a torsional analogue of the Aharonov–Bohm effect in gauge theory. Thus, Φ_T detects the existence of a quantized torsional defect.

2.2 Skyrmionic torsion and toroidal domains

Beyond line-like defects, torsion can also form volumetric domains where the orientation of the chiral frame winds non-trivially throughout a 3D region. This is characterized by a unit vector field $\hat{\mathbf{n}}(\mathbf{x})$ associated with the local chiral frame, whose global wrapping defines the topological charge:

$$Q_S = \int d^3x \; \epsilon^{ijk} \, (\partial_i \hat{\mathbf{n}}) \cdot \big(\partial_j \hat{\mathbf{n}} \times \partial_k \hat{\mathbf{n}}\big) \in \mathbb{Z}\,. \tag{9}$$

A non-zero Q_S corresponds to a $\pi_3(S^3)$ classification, ensuring that the torsional domain cannot be continuously unwound without destroying or severing the region that supports it. In this sense, $Q_S \neq 0$ stabilizes toroidal skyrmion-like structures as stationary solutions independent of the holonomy associated with Φ_T.

In summary, Φ_T identifies a torsional vortex and Q_S guarantees that it remains topologically protected: the former detects the defect, the latter prevents it from disappearing.

3 Examples of stationary torsional domains

Stationary solutions supported by chiral currents naturally organize into coherent domains whose topology is fixed by the structure of the currents. Here we present two representative geometries, corresponding to the two distinct topological invariants introduced in the previous section: a toroidal skyrmionic configuration with non-zero $\pi_3(S^3)$ charge, and a Möbius-like torsional bridge associated with a $\pi_1(S^1)$ holonomy induced by the flip sector.

3.1 Toroidal stationary domain (skyrmion-like)

A domain in which left-handed currents circulate around a closed loop yields a tubular region with non-trivial $\pi_3(S^3)$ charge. The torsion remains localized within the torus while curvature stays negligible everywhere. The internal circulation encodes the skyrmionic structure and stabilizes the configuration against smooth deformations.

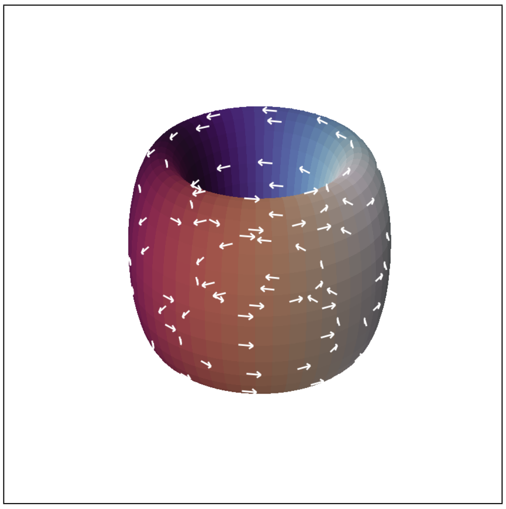

Figure 2: Schematic toroidal torsional domain supported by circulating chiral currents. The torsional geometry is globally stable due to a non-zero skyrmion charge.

3.2 Chiral-flip stitching: Möbius-like bridge

When two regions dominated by opposite chirality are connected through the flip sector, the resulting torsional field may become globally non-orientable while remaining flat locally. The holonomy around the strip leads to a non-trivial $\pi_1(S^1)$ classification, as in Aharonov–Bohm-type defects.

These structures demonstrate that topologically non-trivial torsion can be sustained solely by chiral imbalance, providing explicit realizations of the stationary solutions described above.

4 Static limit and Green-function formulation

The stationary condition in the flow parameter τ does not necessarily imply the absence of residual spatial variations. It is thus useful to complement the Ricci–flow–based description with a static limit, where $\partial_0 h_{\mu\nu} = 0$ and time derivatives vanish in physical coordinates. In this regime, the linearized field equations reduce to an elliptic system

$$\nabla^2 h_{\mu\nu}(\boldsymbol{x}) = -2\, T^{\text{eff}}_{\mu\nu}(\boldsymbol{x}), \tag{10}$$

where $T^{\text{eff}}_{\mu\nu}$ is built directly from localized chiral sources (including the flip interference). This formulation highlights the finite-range response of the torsional sector to the underlying spinor currents.

A screened Green function,

$$G(\boldsymbol{x}, \boldsymbol{x}') = \frac{e^{-\mu|\boldsymbol{x}-\boldsymbol{x}'|}}{4\pi|\boldsymbol{x}-\boldsymbol{x}'|}, \tag{11}$$

accounts for a characteristic coherence length μ^{-1} of the torsion-inducing neutrino currents. The metric perturbation can then be written in compact Yukawa-type form:

$$h_{0c}(\boldsymbol{x}) = \frac{1}{3}\int d^3x'\, G(\boldsymbol{x}, \boldsymbol{x}')\, \epsilon_{bck}\big(\bar{\psi}_L\sigma^k\psi_L - \bar{\psi}_R\sigma^k\psi_R\big)(\boldsymbol{x}'), \tag{12}$$

$$h^{(L/R)}_{ac}(\boldsymbol{x}) = -\frac{1}{3}\int d^3x'\, G(\boldsymbol{x}, \boldsymbol{x}')\, \epsilon_{ack}\big(\bar{\psi}_L\sigma^k\psi_L - \bar{\psi}_R\sigma^k\psi_R\big)(\boldsymbol{x}'), \tag{13}$$

$$h^{(\text{flip})}_{ac}(\boldsymbol{x}) = -\frac{1}{3}\int d^3x'\, G(\boldsymbol{x}, \boldsymbol{x}')\, \epsilon_{bca}\big(\bar{\psi}_L\psi_R - \bar{\psi}_R\psi_L\big)(\boldsymbol{x}')\,. \tag{14}$$

These expressions show that left- and right-handed contributions retain opposite sign, while the flip sector couples chiral domains, thus enabling globally non-orientable stationary configurations such as those depicted in Figs. 2–1. Complementary Yukawa–torsion analysis is developed by the author in [6].

5 Conclusion

We have shown that chiral Majorana neutrino currents can sustain stationary torsional structures in the weak-field regime through a Ricci–flow–inspired evolution of the metric perturbation. These configurations include toroidal domains, stabilized by a skyrmion charge associated with $\pi_3(S^3)$, and Möbius-like bridges where a chiral-flip sector induces a non-orientable global geometry while local curvature remains negligible. The resulting states are characterized by conserved topological invariants (vortex flux and skyrmion number) and a finite-range response encoded by a Yukawa-type Green function. The screened Green–function used here is derived and discussed in [6].

Such torsional stationary domains could, in principle, emerge on scales where coherent neutrino currents are present, potentially influencing the effective gravitational behavior of regions that do not contain ordinary matter. Left-handed configurations may contribute to localized attractive geometric effects, while right-handed sectors provide an opposite response. A network of such stationary torsional structures may therefore play a role in large-scale geometry, possibly contributing to gravitational effects that are not directly associated with visible matter.

A detailed investigation of these broader implications, including their potential relevance in astrophysical and cosmological contexts, is developed in the extended companion study [3].

Acknowledgements

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References

[1] F. W. Hehl, P. von der Heyde, G. D. Kerlick, and J. M. Nester, "General relativity with spin and torsion: Foundations and prospects," *Rev. Mod. Phys.* **48** (1976) 393–416.

[2] I. L. Shapiro, "Physical aspects of the space-time torsion," *Phys. Rept.* **357** (2002) 113–213.

[3] E. Varani, *Geometric Evolution of Torsional Curvature from Majorana Neutrinos: A Ricci Flow Approach*, TJMA, Vol. 13, Issue 1 (2025) 53–104.

[4] R. S. Hamilton, "Three-manifolds with positive Ricci curvature," *J. Differential Geom.* **17** (1982) 255–306.

[5] G. Perelman, "The entropy formula for the Ricci flow and its geometric applications," arXiv:math/0211159.

[6] E. Varani, "Torsional Effects in the Coupling between Gravity and Spinors – Yukawa Gravity," arXiv:2505.17158 [gr-qc] (2025). https://doi.org/10.48550/arXiv.2505.17158.