Rocket Problem Equation Sheet

But how do rockets work?
Rocket Equation!

$$\Delta v = v_e \ln \frac{m_0}{m_f}$$

m_o: initial mass of rocket
m_f: final mass of rocket
v_e: exit velocity

The mass of a rocket is broken into 3 different categories
m_p: mass of the propellant (fuel)
m_s: mass of the structure (casing, engine, electronics)
m_L: mass of payload (satellites, passengers, warhead)

For a single stage rocket
m_o: m_p+ m_s+ m_L
m_f: m_L
v_e: I_{sp}* g_o (g_o is gravity, I_{sp} is specific impulse

For this we are working with a three-stage rocket, so each stage has its own ΔV

$$\Delta v_n = v_{e_n} * \ln\left(\frac{m_{o_n}}{m_{f_n}}\right)$$

The total ΔV is the sum of all Δv_n

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- https://rlg.fas.org/980327-range.htm
  - Ballistic missile parametric equations:
    - $\frac{M_f}{M_0} = e^{(-\frac{V_g}{V_e})}$
    - $R = \frac{V_g^2}{g}$
    - $I_{sp} = \frac{V_e}{g}$
    - $t_b = \frac{V_g - V_e \ln(\frac{M_f}{M_0})}{g}$
    - Equations taken from:
      Range vs. difficulty for ballistic missile development
      Author: R.L. Garwin, IBM Fellow Emeritus
      Published: March 27, 1998
  - Mass ratio of final weight over initial weight and propellant specific impulse determine the range, burn time, and exhaust velocity
    - M_f = final mass (16,000 lbs.)
    - M_0 = initial mass
    - V_g = velocity gain
    - V_e = booster exhaust velocity
    - R = range
    - g = gravity
    - I_{sp} = specific impulse of propellant
    - t_b = burn time
  - Vary I_{sp} is possible
  - Vary Mass ratio of 0.8 to 0.1

- https://en.wikipedia.org/wiki/Tsiolkovsky_rocket_equation

Problem for Teams 1-5

- Given
  - Missile Diameter 1 meter
  - Propulsion stack length 10 meters
  - 3 stages, all solid rocket motors, all same Isp
  - 250 kg payload
  - Assume flat earth
  - Mass/propellant fractions should be stage appropriate
    - Upper stages are more efficient
- What is the "optimized" propulsion stack for Max Range or ΔV
- What is the "optimized" propulsion stack for Max Range or ΔV with First Stage booster that burns for 10 seconds

Problem for Teams 6-10

- Given
  - Missile Diameter .75 meter
  - Propulsion stack length 7.5 meters
  - 3 stages, all solid rocket motors, all same Isp
  - 250 kg payload
  - Assume flat earth
  - Mass/propellant fractions should be stage appropriate
    - Upper stages are more efficient
- What is the "optimized" propulsion stack for Max Range or ΔV
- What is the "optimized" propulsion stack for Max Range or ΔV with First Stage booster that burns for 10 seconds