

Erasmus School of Economics

FEM11074 Advanced Investments



Below you'll find 15 questions, some of them divided into sub-questions. Answer these to the best of your knowledge and ability. Read the entire question before writing down an answer. Explain every part of your answer, a simple 'yes' or 'no' will not be awarded any points. While grading, argumentation and knowledge count equally; unreadable or unintelligible answers will not be graded!

There are 100 points in this exam; each question includes the amount of points that can be earned. Your final grade will be your points scored, divided by 10.
There's a formula list on the last two pages. Good luck!

NB: the answers, as indicated by the highlighted text, are indicative. Alternative explanations are possible, and a less concise writing style is expected.

1. Suppose there are two investors who look at the mean of the return of their portfolio, but in terms of risk, they do not care about the level of risk you normally see in stock markets, they only fear any situation in which the returns drop with more than 10%. Investor A looks at monthly returns, investor B looks at daily returns.

Explain:

- which of the two would see the most risk in stock markets,
 - which of the two has a more realistic model of risk, and
 - give the SDF that is consistent with their preferences.
- (9 points)

On a daily basis, reruns below -10% are extremely rare. In other words, investor B would rarely see any source of risk whatsoever. On a monthly basis, -10% is a lot, but a somewhat regular occurrence in individual stocks. investor A would see more risk.

As a consequence, investor A has the more realistic view of risk, as the threshold and frequency for investor B simply don't match; B would be almost risk-neutral.

SDF: LPM with threshold -10%: $m_{t+1} = a + b \cdot \min(RM - -10\%, 0)$

2. Answer the following questions regarding the Stochastic Discount Factor (SDF):

a) Explain why there are theoretical objections to including a momentum factor in the SDF.
(4 points)

Momentum is based on past returns, while the stochastic discount factor should be based on marginal utility of *future* consumption. Consumption decisions and utility would depend on concurrent events, not past ones. On top of this, future realisations should not depend on past performance if markets are informationally efficient.

b) What is the meaning of m_{t+1} in the SDF formula $p_t = E(m_{t+1}x_{t+1})$? (5 points)

Marginal utility of future consumption: how much do we value returns (to be used for consumption) under the conditions prevailing at the time we get those returns.

3. Suppose the correlation between US and Chinese stocks was positive at first, and then becomes negative. Expected returns remain the same.

a) What would this mean for the portfolio of a based investor based in Germany? Explain your answer in terms of (changes to) portfolio weights, and assume the initial portfolio contained both US and Chinese stocks with positive weights. (5 points)

Diversification benefits increase, as negative correlations allow for a bigger mitigation of risk. It is likely the portfolio weights will change, though we cannot say how, only that with near certainty, we will still invest in both.

b) What would be the implications of this change if there was a ban on short sales in China? (4 points)

None, as you will reduce risk with positive weights if the correlation is negative.

4. Suppose an investor has the following portfolio weights:

- Small stocks, as defined by the returns over the last 12 months: + 100%
- Large stocks, as defined by the returns over the last 12 months: -100%
- Stock with relatively high operating profitability: +50%
- Stock with relatively low operating profitability: -50%
- Risk-free asset: +100%

The risk free rate is 1.5%, the market risk premium (defined as in the CAPM) is 5.0%.

a) According to the CAPM, what is the expected return of this portfolio assuming all stocks have a beta of 1.1? Explain your answer. (4 points)

portion invested in risky stocks: net 0%. Leaves R_f , so 1.5%.

b) Is this investment strategy is in line with the literature regarding anomalies? Explain. (6 points)

Yes. Small stocks tend to outperform, though much less so in later years, and high profitability tstocks tend to outperform the CAPM as well (FF 2014)

5. Explain what you should do to minimize the influence of Roll's critique. (5 points)

Roll's critique is that the market portfolio is unmeasurable, as it should contain every asset. testing the CAPM is therefore a joint test of the appropriateness of the proxy and the CAPM itself. The only way to reduce this problem is to use the broadest possible proxy, so a global index with all kinds of assets.

The next five questions (numbers 6 to 10) require you to explain if the following statements are true or false, **and why you reach that conclusion**:

NB: the explanation is what's graded: you can get partial credit for the wrong option with a partially correct argumentation, but just the correct option will not yield any points.

6. "Investing in a reversal-based strategy would incur more transaction costs than investing based on the size-anomaly." (6 points)

False. Reversal is based on the longer term (losers of the past 3-5 years will become winners), size is based on a characteristic that will also not change dramatically over time. With a moderate investment horizon, reversal will not (clearly) cause more trades therefore it will have similar transaction costs.

NB: if reversal is well understood, but a very long horizon or a high rebalancing frequency is assumed and explained, 'true' can still get full points.

7. "The Sharpe ratio of an investment portfolio that is invested abroad but with the currency exposures hedged, will almost surely be lower than the Sharpe ratio of that same investment portfolio without the currency hedging. (assume you judge the returns in your home currency)" (6 points)

False. The sharpe ratio, return / standard deviation, could decrease or increase, depending on whether the foreign currency is positively or negatively correlated with the returns.

8. "The CAPM violates non-satiation, but this is rarely relevant in practical applications of the CAPM" (6 points)

True, the violation ($SDF < 0$) only occurs for very positive returns, which are quite rare.

9. "An event that erodes the trust investors have in a stock (such as the emissions scandal at Volkswagen) will influence the alpha of an investment portfolio that contains such a stock" (6 points)

yes, but only in so far there isn't a compensation build in the portfolio: if the competition benefits, it might have no net result (diversifiable risk), depending on the weights.

10. "If everyone would invest according to the CAPM, non-diversifiable risk would not exist any longer." (6 points)

False. The risk would still be there. It would not be priced, but individual companies can still go bankrupt, have scandals, great innovations etc. etc.

Below you'll find five questions, each with several possible options / answers. Indicate which option best reflects the answer to the question, and explain why in no more than 90 words.

NB: the explanation is what's graded: you can get partial credit for the wrong option with a partially correct argumentation, but just the correct option will not yield any points.

11. Which asset would have the lowest market beta according to the Fama & French 5 factor model? Explain your answer. (6 points)

- A. A portfolio tracking stocks in the social media sector (e.g. Facebook)
- B. An ETF tracking the S&P500.
- C. A set of call options on the MSCI global index.
- D. A portfolio of government bonds.

D. The beta of gov bonds would be close to zero, as they do not co-move with the market. All others will have positive beta's (A probably just above 1, B of almost exactly 1, C between 0 and 1 depending on moneyness)

12. There is a major rally in the stock markets; they go up by 80%. You estimate the Fama & French 3 factor model on the dataset that consists of the period when this rally took place. Next, the bubble bursts. You want to calculate the expected return for an investment made in the period after this event; the markets lost 40%. You calculate the expected return based on the parameters from the good time.

Which input of the expected return will cause problems if you evaluate the performance of your investment in this way? Explain your answer. (6 points)

- A. The risk free rate
- B. The one of the beta's of the model
- C. The market risk premium
- D. There will not be a problem.

C. Massive overestimation of the MRP in your estimation period; the value will not be applicable in the downturn. One could argue D if one states that the market risk premium in the downturn (so the present observation) should be used and inputs are only seen as the betas.

13. If the risk free rate becomes negative, this will impact which of the following performance measures? Explain your answer. (6 points)

- A. The M2 measure.
- B. The Sharpe ratio.
- C. The Treynor ratio.
- D. All of the above.

D. All, as each involves R_f somehow. [M2: leveraging of the market; Sharpe: use of excess returns, Treynor: calculation of the beta]

14. Suppose the investment universe is described by figure 1 below (the numbers in the table are returns, in percentages).

Figure 1:

	Probability	Asset 1	Asset 2	Asset 3
State 1	10%	10%	-4%	50%
State 2	40%	-14%	3%	-6%
State 3	50%	4%	-2%	-6%

What is the least risky portfolio for an investor with an LPM preference structure (order = 2) and a threshold of -5%? Assume no diversification. Explain your answer, show calculations. (5 points)

- A. Asset 1.
- B. Asset 2.
- C. Asset 3.

LPMs:

asset 1: $0.4 * (-14 - (-5))^2 = 32.4$

asset 2: 0 (no returns below -5%)

asset 3: $0.4 * (-6 - (-5))^2 + 0.5 * (-6 - (-5))^2 = 0.9$

B is therefore the least risky, as it has the lowest LPM.

15. See figure 1 again. Assume there's a risk-free asset available with a return of exactly zero, and short positions are possible too. Assume no diversification. Which of the investment strategies below would an investor, who bases him/herself on the mean-LPM framework (parameters as in the previous question), choose? Explain your answer, show calculations. (5 points)

- A. A net long position in one of assets 1, 2 and 3. (indicate which one)
- B. No investment in all of assets 1, 2 and 3.
- C. A net short position in one assets 1, 2 and 3. (indicate which one)

C. expected returns are all negative. Yet short in B gives +0.2%.

----- END of the exam – the following pages contain the formula list -----

Formula list:

Portfolio optimization: $\min_{\omega} \frac{1}{2} \omega' V \omega + \lambda [\bar{R}_p - \omega' \bar{r}] + \gamma [1 - \omega' e]$

$$\sigma_p^2 = \omega^{*'} V \omega^* \quad \omega^* = \lambda V^{-1} \bar{r} + \gamma V^{-1} e$$

$$\omega^* = \frac{e' V^{-1} e \bar{R}_p - \bar{r}' V^{-1} e}{\bar{r}' V^{-1} \bar{r} e' V^{-1} e - (\bar{r}' V^{-1} e)^2} V^{-1} \bar{r} + \frac{\bar{r}' V^{-1} \bar{r} - \bar{r}' V^{-1} e \bar{R}_p}{\bar{r}' V^{-1} \bar{r} e' V^{-1} e - (\bar{r}' V^{-1} e)^2} V^{-1} e$$

Same, with risk free object:

$$\min_{\omega} \frac{1}{2} \omega' V \omega + \lambda \left\{ \bar{R}_p - [r_f + \omega' (\bar{r} - r_f e)] \right\}$$

$$\omega^* = \frac{\bar{R}_p - r_f}{\bar{r}' V^{-1} \bar{r} - 2 \bar{r}' V^{-1} e (r_f) + e' V^{-1} e (r_f)^2} V^{-1} (\bar{r} - r_f e)$$

$$\sigma_p^2 = \omega^{*'} V \omega^* = \frac{(\bar{R}_p - r_f)^2}{\bar{r}' V^{-1} \bar{r} - 2 \bar{r}' V^{-1} e (r_f) + e' V^{-1} e (r_f)^2}$$

Security Market Line

$$E(r_i) = r_f + (E(r_M) - r_f) \beta_i$$

SDF in a linear factor model:

$$1 = E(m_{t+1} r_i) \quad m = a + b' f$$

$$E(m_{t+1} r_i) = E(m_{t+1}) E(r_i) + \text{cov}(m_{t+1}, r_i)$$

Skewness and Kurtosis:

$$skew = \frac{\sum (x - \bar{x})^3}{(1/n) \left[\sum (x - \bar{x})^2 \right]^{3/2}} \quad kurt = \frac{\sum (x - \bar{x})^4}{(1/n) \left[\sum (x - \bar{x})^2 \right]^2}$$

Arithmetic and Geometric means:

$$\bar{r}_{arithmetic} = \frac{1}{T} \sum_{t=1}^{t=T} r_t \quad \bar{r}_{geometric} = \left[\prod_{t=1}^{t=T} (1 + r_t) \right]^{1/T} - 1$$

Relative and constant risk aversion

$$RRA = - \frac{x U''(x)}{U'(x)} \quad ARA = - \frac{U''(x)}{U'(x)}$$

Lower Partial Moments:

$$LPM_{n,\tau} = \frac{1}{T} \sum_{t=1}^{t=T} \left(\text{Min}[(x - \tau), 0] \right)^n$$

Present value bond:

$$P_{bond} = \sum_{t=1}^{t=T} \frac{coupon_t}{(1 + r_t)^t} + \frac{principal}{(1 + r_T)^T}$$

Duration:

$$D = \sum_{t=1}^{t=T} t \frac{CF_t / (1 + y)^t}{P_{Bond}} \quad \frac{\Delta P_{Bond}}{P_{Bond}} = -D \frac{\Delta y}{1 + y}$$

Convexity:

$$C = \frac{1}{P_{bond}(1 + y)^2} \sum_{t=1}^{t=T} \frac{CF_t(t^2 + t)}{(1 + y)^t} \quad \frac{\Delta P_{Bond}}{P_{Bond}} = -D \frac{\Delta y}{1 + y} + \frac{1}{2} C [\Delta y]^2;$$

Optimal investment, Grossman model:

$$X_i = \frac{E[P_1] + \rho_{i;P_1,y_i}^2 (y_i - E[P_1]) - r_f P_0}{a_i \text{Var}[P_1] (1 - \rho_{i;P_1,y_i}^2)}$$