Below you'll find 15 questions, some of them divided into sub-questions. Answer these to the best of your knowledge and ability. Read the entire question before writing down an answer. Explain every part of your answer, a simple 'yes' or 'no' will not be awarded any points. While grading, argumentation and knowledge count equally; unreadable or unintelligible answers will not be graded!

There are 100 points in this exam; each question includes the amount of points that can be earned. Your final grade will be your points scored, divided by 10.
There's a formula list on the last two pages. Good luck!

1. Assume you have the following Stochastic Discount Factor (SDF):

$m_{t+1} = 0.75 - 2.1*R_m + 0.04*R_m^2$

Explain how this SDF differs from the SDF of the CAPM, and why this is likely to be an improvement to our model in terms of explaning actual return data. ***(6 points)***

There's an additional term, namely Rm squared (this has to be identified). In itself, an extra term will almost surely add to the explanatory power, though if this is significant depends on the data. However, research has shown a skewness factor (which is what Rm square represents) does indeed help explain returns, there is evidence for a skewness preference.

2. Refer to the SDF of question 1. The average return on the market portfolio is 0.056, the historical data for R_m ranges from -0.217 to +0.275. Explain how one should check if the assumptions of nonsatiation and risk aversion are valid for this SDF in combination with this dataset. Then perform that check; show your calculations! ***(7 points)***

To check for nonsatiation, you need to check that the SDF stays positive. In case of a decreasing straight line, you can plug in the upper bound, but in case of a parabola it is more involved, as the minimum might be somewhere between the upper and lower bound for Rm. That actually doesn't happen here, as the SDF is decreasing throughout the allowed range. That's enough.

In more detail: determine the extreme value, which is done by setting the first derivative (*of the SDF, in this case!*) equal to zero. That yields the value for Rm for which the SDF is lowest, in this case 26.25. That is very out outside the range, so the parabola must either be continuously decreasing or increasing within the -0.217 to +0.275 interval. Plugging in both numbers (-0.217 and +0.275) result in a positive value for the SDF. Combined, those two facts (the extreme value lies outside the range, and the begin and end of the range are both in positive territory) are enough for a parabola to be positive everywhere.
NB: alternatively, one could of course solve (with the discriminant formula) the points for which the parabola is zero. But I assume that is a formula most of you would have trouble recalling in a pinch at an exam, hence the other approach.

Risk aversion: SDF must be decreasing. first derivative of the SDF <0 : -2.1 + 0.08* maximum Rm = -2.1 + 0.08*0.275 <0. Also checks out.
NB: the proof/analysis must be robust to the fact that the SDF in this case is a parabola.

3. Suppose that during both a crisis and a bubble in asset prices, correlations between risky assets will increase and become much closer to +1. Explain what such a change means for:

a) The standard deviation of an investment portfolio that is spread over 100 different assets, and keeps investing in the same assets. ***(5 points)***

Diversification possibilities diminish, the returns will move in the same direction to a larger degree, movements will cancel out less. Therefore, the standard deviation will increase.

b) The expected return of an investor that is restricted in terms of the variance of his portfolio (the variance must remain below a certain number). This investor cannot go short. ***(5 points)***

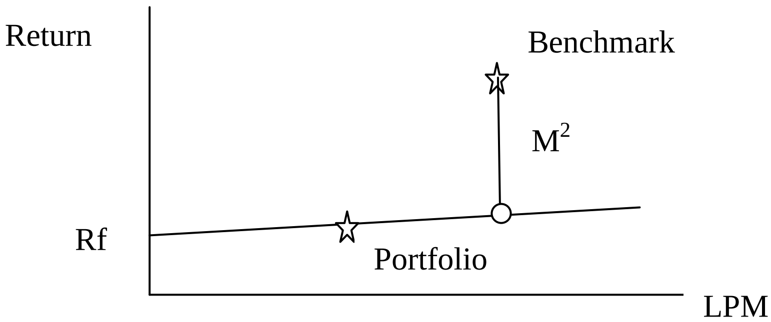
Because of reduced diversification, and the inability to go short, one must flee to safer assets. In general that means the expected return will decrease (can also be illustrated with a bullet graph).

c) The demand for short positions from investors that can go short. ***(5 points)***

Increases, a short position and a long position in 2 assets that have a correlation of nearly +1 will give a lot of return while diversifying away almost all risk. (except model risk, though that observation is not required from the students.)

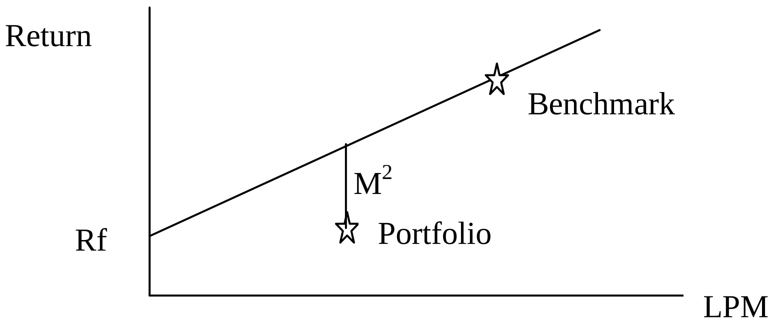
4. Explain how to calculate the (classical) M^2 measure for an investor with preferences that follow an LPM of order 2 with threshold -4%. Illustrate with a graph. ***(6 points)***

There are two ways of doing portfolio evaluation within the general principle (which is: equalize the risk for the two alternatives, and then compare returns): apply leverage on the investment portfolio or on the benchmark. The *classical way* (asked here) requires us to use leverage the portfolio in such a way the combination of risk-free object and portfolio has a risk equal to the benchmark. Then we compare returns, the difference is the M2 measure.



In this case, we see that the levered portfolio (located at the circle) would consist of >100% in the evaluated portfolio and a short position in Rf. Something like +180% and -80% in Rf. This leads to a higher LPM (=the risk measure here). Once the LPM is the same, we can see the difference in return (vertical distance), and it's clear we massively underperformed the benchmark.

Alternatively, we could apply leverage till the LPM of the benchmarkt+Rf combo is equal to the LPM of the actual portfolio. Then compare returns, the difference is the M2 measure.



Different method, same conclusion: if we compare how we invested with a combo of the benchmark and Rf, we still underperformed.

NB: a correct graph is required. Note that the order and threshold aren't strictly speaking needed here.
Also note that in this updated version, the answer is now quite elaborate, more than would be needed at an actual exam.

5. Give an economic reason why a model that incorporates factors from both the stock and the bond markets may achieve higher explanatory power for all asset classes. ***(6 points)***

Stocks and corporate bonds are both claims on the same set of cashflows. Bond holders need to be paid before the stockholders can receive anything. Therefore it does make sense that bond-related factors partially influence stocks, and stock-related factors influence bonds (the stock related factors might indicate risk that the value of the assets might drop below that of the debt level).

The next six questions (numbers 6 to 11) require you to explain if the following statements are true or false, ***and why you reach that conclusion***:
NB: the explanation is what's graded: you can get partial credit for the wrong option with a partially correct argumentation, but just the correct option will not yield any points.

6. "An asset with a higher expected return but the same variance might not necessarily be a better investment, while an asset with a higher alpha will be a better investment." NB: when answering this question, you should assume your asset pricing model is correct. ***(6 points)***

True, the same variance means identical total risk, but much of the total risk might be diversifiable. Only systematic risk is priced. An alpha will relate to the reward for systematic risk. NB: if the first part alone is deemed false based on the argument that the total available capital for an investor is considered, this is also correct.

7. "Any news event should be seen as a contribution to diversifiable risk, or as an expression of systematic risk, but it cannot be a contribution to both." ***(6 points)***

False, it can be both at the same time: diversifiable risk in so far it concerns the company it refers to, but systematic risk if it causes the entire market to shift. The VW case in the German market is an example: the majority might be diversifiable, but the impact through the sector is so big, it becomes partially systematic.

8. "An SDF that depends on several factors will violate risk-aversion because of the presence of multiple factors" ***(6 points)***

False. Risk aversion requires a decreasing SDF. As long as the parameters in the SDF are well chosen given the data, the SDF can meet this criterium regardless of the number of factors.

9. "An increase in the risk-free rate will lower the Sharpe ratio of the market portfolio". Illustrate your answer with a graph! ***(6 points)***

The tangency portfolio will shift. The efficient frontier will be flatter, the market will be further to the right. As the slope of the line is the sharpe ratio, the statement is true.

10. "The Black-Litterman approach should be implemented using returns that are calculated as geometric averages" ***(6 points)***

False. Black-littermann is designed to capture expectations. The expectations should be formed based on arithmetic returns, not geometric ones, as they're for one period only.

11. "A one-period model means that the investment horizon is fixed, and will also be a major determinant of your investment decisions" ***(6 points)***

True, because you optimize over the period inherent in your frequency, you cannot shift the investment horizon, as return distributions change if you adjust the frequency (or horizon).

Below you'll find four questions, each with several possible options / answers. Indicate which option best reflects the answer to the question, and explain why in no more than 90 words.
NB: the explanation is what's graded: you can get partial credit for the wrong option with a partially correct argumentation, but just the correct option will not yield any points.

12. Which model would you expect to have the largest difference in outcomes (in terms of the alpha's) compared to the CAPM, when applied to stocks in the financial sector in the years 2007-2010? <u>Explain your answer</u>. ***(6 points)***

A. Carhart's model
B. The Grossmann model
C. The Lower Partial Moment model (Bawa & Lindenberg)
D. The Fama & French 5 factor model (1993 version)

C is the best answer, A and D are possible for 5 points if argued well, though likely to get 3/4. The negative returns were profound, and had a much bigger impact (assymmetric return distributions), making the LPM model very suitable. Carthart includes momentum, which will help a bit (crisis was prolonged, losers remained losers for a while), D includes bonds factors (could be argued using solvability of banks, but unlikely / less of an impact), B does not yield a model that generates alphas at all.

13. Suppose your financial conditions change, and your preference structure would change as a consequence of that. Which of the items listed below could NOT change as a consequence? Explain your answer. ***(6 points)***

A. The SDF that would be appropriate for your investments.
B. Your coefficient of Relative Risk Aversion.
C. The betas you use to judge your investments.
D. The time-value preference coefficient used to compare consumption now with consumption in a future period.

As your preferences change, that must be visible in the SDF. Relative risk aversion is a way to measure one aspect of your preferences (it uses marginal utility, so the SDF, in the calculation), so should also change. Time-value preference coefficient is another aspect of your preference sctructure (and SDF!). C is the correct answer: the betas are features of the market. If your preferences change, no reason why those would be affected.

14. Suppose the investment universe is described by figure 1 on the next page (the numbers in the table are returns, in percentages. Asset 1 and 2 are stocks, Assets 3 and 4 are long call and put options respectively).

Figure 1:

	Probability	Asset 1	Asset 2	Asset 3	Asset 4
State 1	20%	-3%	-4%	-50%	160%
State 2	50%	+13%	+1%	-12%	0%
State 3	30%	-8%	+15%	+58%	-80%

What is the least risky asset for an investor with an LPM preference structure (order = 2) and a threshold of -5%? Assume no diversification. Explain your answer, show calculations. ***(6 points)***

A. Asset 1.
B. Asset 2.
C. Asset 3.
D. Asset 4.

Asset 1: $0.3 * (-8--5)^2 = 2.7$
Asset 2: 0 (no return below the threshold)
Asset 3: $0.2*(-50--5)^2 + 0.5*(-12--5)^2 = 405 + 24.5 = 429.5$
Asset 4: $0.3*(-80--5)^2 = 1687{,}5$
B, therefore.

15. All problems mentioned below might affect the Sharpe ratio, but which problem from this list will only cause a *lower* Sharpe ratio? Explain your answer. ***(6 points)***

A. Unhedged exposure to foreign currencies.
B. Ignoring the momentum anomaly.
C. A time-varying market risk premium.
D. Home bias in a portfolio.

D. Home bias causes underdiversification, (you're inside the bullet, so the portfolio is inefficient) which automatically means a lower sharpe ratio compared to where there's no home bias.
Exchange rates, momentum and time-varying risk premia could all, under some circumstances, work in an investor's favor and increase the sharpe ratio compared to when those issues are absent.

----------- END of the exam – the following pages contain the formula list ------------

Formula list:

Portfolio optimization: $$\min_{\omega} \frac{1}{2}\omega' V\omega + \lambda[\overline{R_p} - \omega'\bar{r}] + \gamma[1-\omega' e]$$

$$\sigma_p^2 = \omega^{*'} V \omega^{*} \qquad \omega^{*} = \lambda V^{-1}\bar{r} + \gamma V^{-1} e$$

$$\omega^{*} = \frac{e' V^{-1} e \overline{R_p} - \bar{r}' V^{-1} e}{\bar{r}' V^{-1}\bar{r} e' V^{-1} e - \left(\bar{r}' V^{-1} e\right)^2} V^{-1}\bar{r} + \frac{\bar{r}' V^{-1}\bar{r} - \bar{r}' V^{-1} e \overline{R_p}}{\bar{r}' V^{-1}\bar{r} e' V^{-1} e - \left(\bar{r}' V^{-1} e\right)^2} V^{-1} e$$

Same, with risk free object:

$$\min_{\omega} \frac{1}{2}\omega' V\omega + \lambda\left\{\overline{R_p} - [r_f + \omega'(\bar{r} - r_f e)]\right\}$$

$$\omega^{*} = \frac{\overline{R_p} - r_f}{\bar{r}' V^{-1}\bar{r} - 2\bar{r}' V^{-1} e(r_f) + e' V^{-1} e(r_f)^2} V^{-1}(\bar{r} - r_f e)$$

$$\sigma_p^2 = \omega^{*'} V \omega^{*} = \frac{(\overline{R_p} - r_f)^2}{\bar{r}' V^{-1}\bar{r} - 2\bar{r}' V^{-1} e(r_f) + e' V^{-1} e(r_f)^2}$$

Security Market Line

$$E(r_i) = r_f + (E(r_M) - r_f)\beta_i$$

SDF in a linear factor model:

$$1 = E(m_{t+1} r_i) \qquad m = a + b' f$$

$$E(m_{t+1} r_i) = E(m_{t+1})E(r_i) + \operatorname{cov}(m_{t+1}, r_i)$$

Skewness and Kurtosis:

$$skew = \frac{\sum (x-\bar{x})^3}{(1/n)\left[\sum (x-\bar{x})^2\right]^{3/2}} \qquad kurt = \frac{\sum (x-\bar{x})^4}{(1/n)\left[\sum (x-\bar{x})^2\right]^{2}}$$

Arithmetic and Geometric means:

$$\bar{r}_{arithmetic} = \frac{1}{T}\sum_{t=1}^{t=T} r_t \qquad \bar{r}_{geometric} = \left[\prod_{t=1}^{t=T}(1+r_t)\right]^{1/T} - 1$$

Relative and constant risk aversion

$$RRA = -\frac{xU''(x)}{U'(x)} \qquad ARA = -\frac{U''(x)}{U'(x)}$$

Lower Partial Moments:

$$LPM_{n,\tau} = \frac{1}{T}\sum_{t=1}^{t=T}\left(Min\left[(x-\tau),0\right]\right)^{n}$$

Present value bond:

$$P_{bond} = \sum_{t=1}^{t=T}\frac{coupon_t}{(1+r_t)^t} + \frac{principal}{(1+r_T)^T}$$

Duration:

$$D = \sum_{t=1}^{t=T} t\,\frac{CF_t/(1+y)^t}{P_{Bond}} \qquad \frac{\Delta P_{Bond}}{P_{Bond}} = -D\frac{\Delta y}{1+y}$$

Convexity:

$$C = \frac{1}{P_{bond}(1+y)^2}\sum_{t=1}^{t=T}\frac{CF_t(t^2+t)}{(1+y)^t} \qquad \frac{\Delta P_{Bond}}{P_{Bond}} = -D\frac{\Delta y}{1+y} + \frac{1}{2}C\left[\Delta y\right]^2;$$

Optimal investment, Grossman model:

$$X_i = \frac{E\left[P_1\right] + \rho^2_{i;P_1,y_i}\left(y_i - E\left[P_1\right]\right) - r_f P_0}{a_i Var\left[P_1\right]\left(1-\rho^2_{i;P_1,y_i}\right)}$$