Lecture Advanced Investments, October 6th, 2025

Bonds, derivatives & remainder portfolio evaluation

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Program

- Very brief remarks on hedgefunds and market timing
- Asset Pricing models: not for stocks only
  - Bonds: Interest rate risk & default risk
  - Derivatives
- Formula of the week

Hedgefunds (1)

- Estimating excess return is quite difficult when it comes to *hedgefunds* (and more exotic structures in general).

- Hedgefunds often have strategies that are 'market-neutral', meaning they aim for low or zero correlation with R_m, while making returns well above r_f – often with highly leveraged positions.

- These strategies (see BKM) are still risky: they are plays on arbitrage, default risk, convertibility, currency relations….

- So we need to look for risk measures that are appropriate, especially as returns from these strategies are often *extremely non-normal.*

Hedgefunds (2)

- W.r.t. returns one must note that hedgefunds are often secretive regarding their holdings (for legal reasons, but also since it's a highly competitive industry). Also, given the broad discretion managers have, returns may be 'smoothed' – a variant on the January effect.

- Even if we have proper returns, the risk adjustment may (have to) include:
  - Rewards for higher moments (with the inherent problem that we need more coefficients for normative optimization).
  - Alternative risk measures (e.g. downside beta's, see lecture 5 on LPMs).

Market Timing & changing performance ratios

- On top of this, there is the general problem (hedgefunds have this in a larger degree than most, but it also goes for ‘normal’ mutual funds) that *risk exposures aren’t stable over time.*
- Depending on how one measures, a sharpe ratio that has been good for several months may suddenly become quite poor in the next.

- The biggest issue in this respect is that the market risk premium itself is time-changing. This affects all asset prices, but obviously the high-beta stocks are changing most.
- So a fund that buys its assets when the market risk premium is high (prices are therefore low) and sells when the premium has dropped, automatically improves its performance: this is called *market timing*.

- And the question is: was this simply smart timing, or simply luck?

Hierarchical portfolio management (1)

- Finally, we have to distinguish between the performance evaluation of a complete portfolio and of a part of the invested capital.
- Among mutual funds and similar entities, it’s normal to sub-divide investments according to asset class. Country and/or industry. Each manager has its own field of expertise.
- However, if each builds portfolios according to the data in his own remit, the total portfolio may in fact not be optimal! (one potentially ignores correlations between areas, distributes total capital over the sectors with different weights, undiversified ideosyncratic risk)
- In fact, we have a problem of ‘Hierarchical portfolio management’.

Hierarchical portfolio management (2)

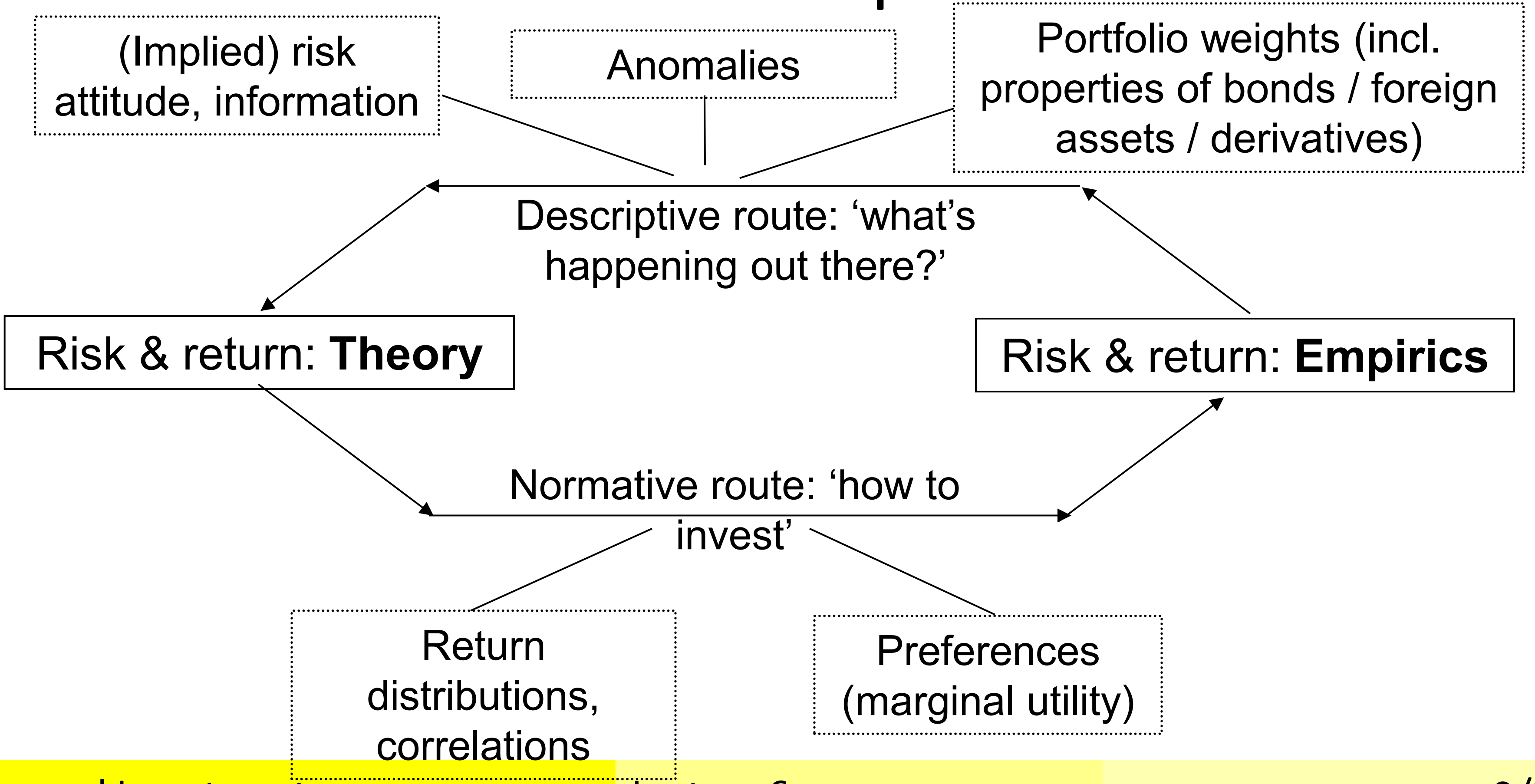
- Two general approaches:
  - Assume each sub-portfolio is well diversified, and treat them as separate assets (optimize again at the next level), or assume that you have so many sub-portfolios that overall the idiosyncratic risk disappears. In that case, the Treynor measure is appropriate.
  - Recognize that sub-portfolios may not be well diversified, and co-ordinate among different sector managers. This means we look at *total* risk again.

- Of course, regardless of how the ultimate portfolio is composed, one needs to track the performance of the area-managers to determine their skill (and bonus)

Intermezzo : top 10 silliest errors

These are 10 of the worst mistakes in history

Roadmap


(Implied) risk attitude, information
Anomalies
Portfolio weights (incl. properties of bonds / foreign assets / derivatives)
Descriptive route: 'what's happening out there?'
Risk & return: **Theory**
Risk & return: **Empirics**
Normative route: 'how to invest'
Return distributions, correlations
Preferences (marginal utility)

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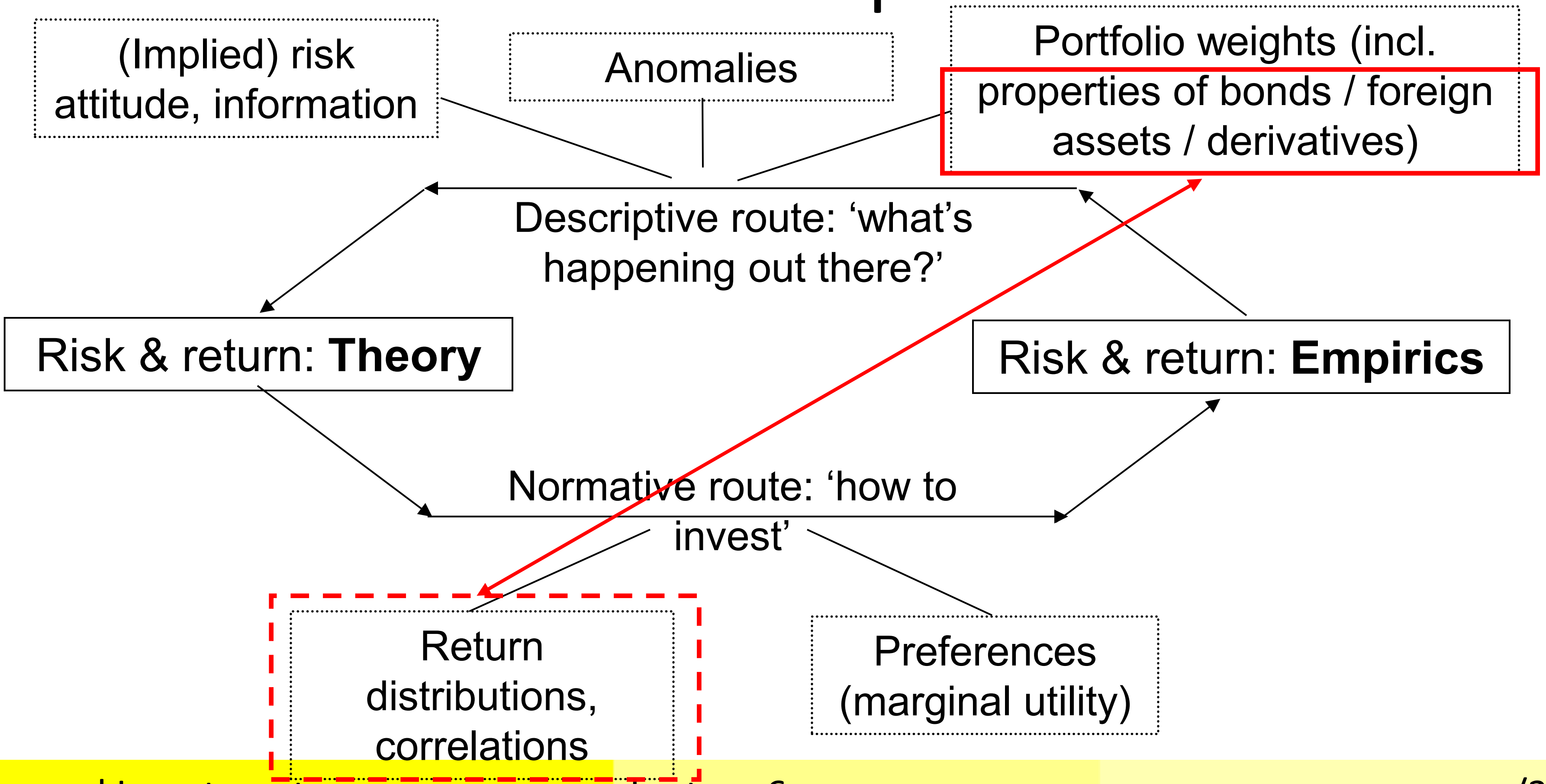
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Asset Pricing models: not just stocks (1)

- So far, we have focused mainly on stocks, in keeping with much of the literature. Yet there are more assets to invest in: bonds, real estate, derivatives, commodities.
- Also, these assets exist abroad too, introducing the effect of exchange rates. Currencies themselves may also qualify as a separate asset.

- In theory, this should not bother us. Models like portfolio theory and the CAPM claim to be valid for all asset classes; the CAPM even *requires* it.
- In reality, these asset classes have their own factors which drive the risk-return trade-off.

- Apart from the theoretical claims, the popularity of stocks as a subject comes mostly from it being the *easiest* asset class to analyze.

Asset Pricing models: not just stocks (2)

- Actually, the picture becomes even more complex if we realize that various asset classes can be bundled.
- Think of a European investor buying a mortgage-backed security in the US. He buys a combination of:
  - A bond (lends money to a bank)
  - Real-estate (the collateral, more precisely it's an option on real estate)
  - Currency (the dollar)
  - Derivatives (mostly short positions, depending on the exact terms. For instance: early repayment(!), repurchase, convertibility)

- *So in a way the theory gets it right: we may need to build a model that prices all risks / assets.*

Bonds (1)

- The first place to start is fixed income (a.k.a. bonds), a market more liquid and often bigger than those for stocks.
- General characteristics:
  - Pays a fixed interest rate; price adjustments cause adjustments in return (a.k.a. yield)
  - Upside potential: limited, realistically to single digits for most bonds; downside potential: -100%.

    This asymmetry is the first complication.
  - Non-linear relation between risk-factors and returns.

    (second complication)
  - Coupon payments need to be re-invested (third complication, but also seen in stocks).

Bonds (2)

- Another difference is that, if we depart from the R_M-explains-all model, we have trouble identifying risk factors for stocks; a large number of factors potentially explains prices. For bonds however, they are obvious:
  - **Interest rate risk**
  - **Default risk**
- No other factors are needed; the price of a bond can change in only 2 ways: either the demand for credit tightens/loosens, affecting all bonds*, or the chance that full repayment is not forthcoming has changed. Management has no discretion in changing the payouts, unless they want to risk receivership.

* Problem: not all bonds way react in the same way, depending on their term structure.

Bonds (3)

- Once again the SDF is useful, it should work for all assets, so also for any combination (only assumptions needed are utility maximization and the existence of preferences).

$$1 = E(m_{t+1} r_i)$$
$$E(m_{t+1} r_i) = E(m_{t+1}) E(r_i) + \text{cov}(m_{t+1}, r_i)$$

- Now if the interest rate doesn't change (or is completely uncorrelated with the SDF) and the same goes for the probability of default:

$$E(r) = \frac{1}{E(m_{t+1})} = r_f$$

- **But of course cov(m_{t+1},r) is not zero**; as we'll see there's a relation between interest rate and default risks and the risk factors we identified for stocks.

Interest rate risk (1)

- The effects of a change in interest rates are not linear:

$$P_{bond} = \sum_{t=1}^{t=T} \frac{coupon_t}{(1+r_t)^t} + \frac{principal}{(1+r_T)^T}$$

- NB: the r_t (a single number) that equates the left-hand and right hand sides of this equation is called the *yield-to-maturity* (YTM).
- Nor are they symmetric: Bond, 5 year maturity, 6% coupon, current interest rate 6%: value = 1000.
  - Interest decreases to 5% for all periods: 1043.3; +43.3
  - Interest increases to 7% for all periods: 959.0; -41.0
- Another source of nonlinearity: time-to-maturity. Long term bonds are more sensitive to interest rate changes.
- High-coupon bonds are less sensitive: a larger part of your cash comes earlier.

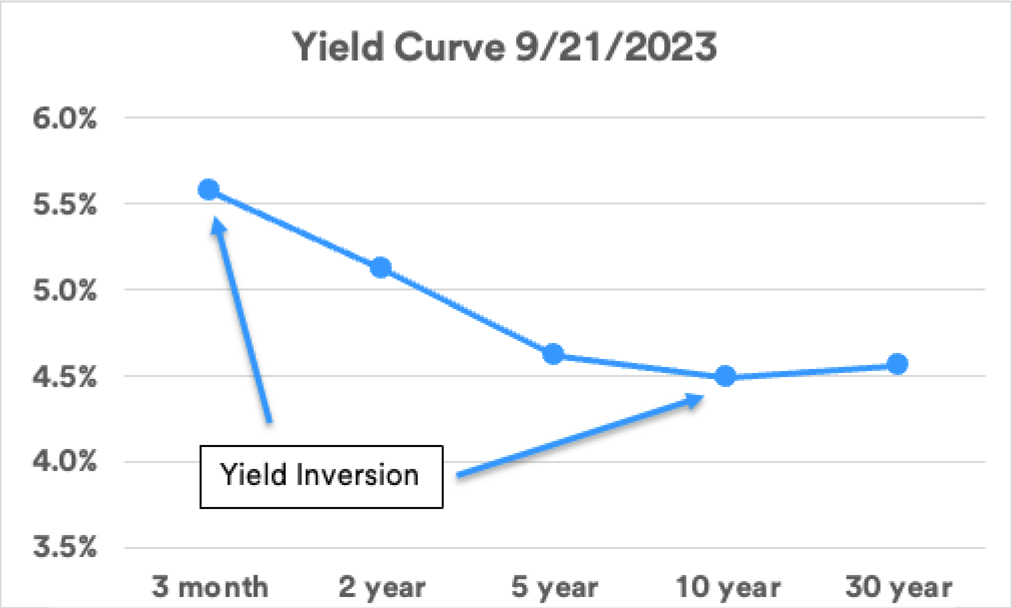
Interest rate risk (2)

- However, the yield to maturity is an artificial construction, a combination of several interest rates over periods in the future. It does matter if the interest rate changes for (e.g.) next year, or the one-year rate we expect over 10 years. (especially if we look at bonds that mature in the period in between)

- Fixed-income analysts look at the *term structure* instead: the collection of all interest rates for bonds with different maturities. (example)
- NB: one must be careful about terminology: ytms and spot rates are only equal for zero-coupon bonds, yet some authors tend to forget that complication.
- Practical advice: *look at the difference between short-term and long-term interest rates! (cf F&F 1993)*

Interest rate risk (update)

- Recently (2022-2023) we've seen interest rates rise considerably again. Historically, t … % range, the peak in the l … after the financial cris …



Source: U.S. Department of the Treasury, via telegraph.co.uk

- Interest rate … imilar to the FF factor) are o … ession risk. An *inverted yiel* … term ones, suggests the … year or two.
- High interest … ashes due to interaction w … sion liabilities.
- Conclusion: R_m … interest rates capture part of it too!

Default risk (1)

- Fixed income is usually rated. Several companies (Moody's, S&P, Fitch) asses the default probability on the bond, rated from AAA to D(efault).
- Government bonds tend to be higher rated due to taxation potential and liquidity.
- The exact default probability is needed to determine the value of a bond. While ratings help, they only give a *range* (and a sometimes obscure one too) and may or may not update the rating in time.
- Empirically, we see that many big investors face the restriction of investing solely in *investment grade* bonds (BBB and above). An adjustment from BBB to BB ('junk bond') can trigger a sell-off.

Default risk (2)

- Bond prices and returns can therefore adjust by larger amounts than implied by merely the default probability.

- Default probabilities are unknown; if the value of a bond is 90% of what it should be were we to discount at the current risk-free rate, the probability of default is much lower than 10% - there's also a risk-premium.

- The risk premium may in turn depend on the credit rating (Kozhemakian 2007), so we'd again have a non-linear effect. The true risk premium is also unknown, even for prices on credit default swaps (which trade default risk in isolation).

- One way of solving this chicken-and-egg story is looking at historical default rates.

Default risk (3)

Standard & Poor's One-, Five-, and Ten-Year Cumulative Issuer-Weighted Default Rates (%) (1981–2005)

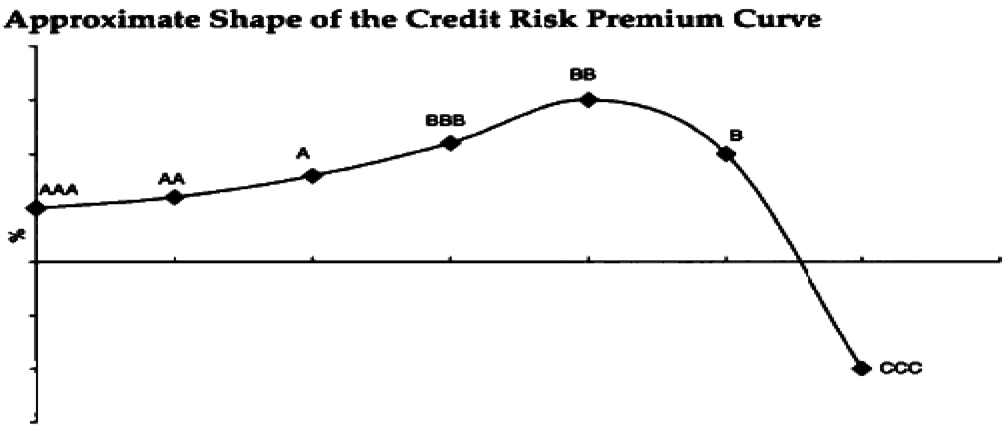
	One-year default rate		Five-year default rate		Ten-year default rate	
	Average	Standard deviation	Cumulative average	Standard deviation	Cumulative average	Standard deviation
AAA	0	0	0.1	0.22	0.44	0.83
AA	0.01	0.04	0.29	0.38	0.81	0.78
A	0.04	0.07	0.59	0.4	1.83	0.93
BBB	0.27	0.29	2.83	1.3	5.82	1.8
BB	1.12	1.12	10.65	4.32	18.29	4.46
B	5.38	3.09	24.16	7.57	32.38	4.32
CCC/C	27.02	12.54	47.56	14.45	53.05	10.37

Source: Standard & Poor's.

- We see that default is not a rare occurrence, especially for the lower segment. Note that bonds tend to move: an AA may default, but it usually takes long enough for the rating agency to make several adjustments.
- NB: strictly speaking, we need to keep an eye on recovery rates as well. Default and a -100% return are 2 different things for bonds.

Default risk (4)

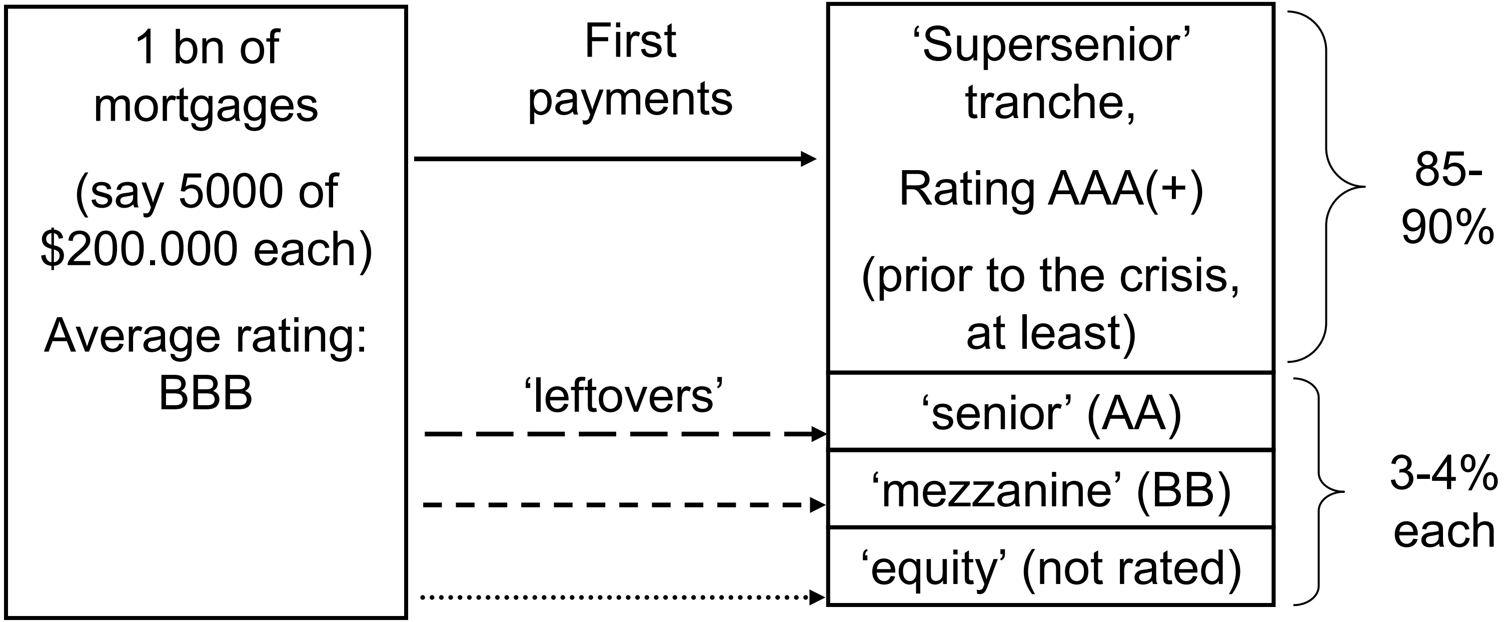
- Even so, investors have a hard time correctly judging risk, these are the risk premia they demanded:



- Explanation: this dataset was affected by 2 recessions, causing a large amount of defaults. Maybe a general economic factor does make sense….

Default risk (5)

- Also, default risk can be *repackaged.* This is / was a common feature of CDOs (Collateralized Debt Obligations), especially when the collateral was a bunch of mortgages....



5 factor model (1)

- To find that general economic factor we need to look beyond $R_{M.}$ Hence research that tries to explain both stocks and bonds (such as Fama & French (1993), required reading for week 3) uses both a terms structure variable and a proxy for defaults.
- NB: this was the 'original' Fama & French 5 factor model.

- In total, that leaves us with a 5 factor model – for both 'SML' and SDF:
  - R_M
  - Size factor (SML)
  - Book-to-market factor (HML)
  - Credit premium (gov bond index minus corporate bond index)
  - Term structure (long term minus short term gov bond returns)

5 factor model (2)

- The bond-related factors also have some explanatory power when *stock* returns are taken to be the dependent variable. (mainly variations, but *not* the level)
- In fact, stocks and bonds are intimately related:

  - Interest rates in general determine the time value of money. With stocks, dividends and proceeds from the sale of the stock are often far of in the future – giving discounting a big impact on the value of the cashflows.

  NB: Empirical proof from this comes not only from regressions, take a look what happens to the stock markets if the FED or ECB announces a major *unexpected* interest rate shift.

  - Corporate bonds have often substantial default probabilities. If a company defaults on its bonds, chances are the stocks are next to worthless (*they're call options on the company's value!*); bondholders receive their stake before stockholders do.

5 factor model (3)

- This also means that hierarchical portfolio construction (separate bond and stock positions) is not a good idea. Portfolio theory also claims to work regardless of the asset type, and in this case it really pays of to check the correlations (or co-LPMs if you go for the LPM framework).
- Here the link with descriptive research is especially strong: bond returns cannot be explained by stock factors only or vice versa, and a comprehensive model works better than 2 smaller ones.
- However, placing 5 factors in the SDF, with at least one of them being only a linear approximation, doesn't suggest a transparent (portfolio formation) nor overly accurate (non-linearity interest risk) model.

Derivatives (1)

- The same SDF approach is valid for derivatives as well. Since derivatives returns are based on their underlying assets, *only the risk factors needed to explain the underlying assets are needed*. So why aren't we using the CAPM/FF/LPM model to value derivatives?

- Big problem: the (portfolio) weights required for replication via the underlying assets change every time the stock price changes!

  Hence the beta of options changes too; drastically and in much shorter time spans than other changes in betas. The risk profile of an option can change very fast indeed.

- *Conclusion*: SDF approaches don't work for derivatives; an SDF needs a longer term investment. Derivatives are more suited for *risk management* and trading strategies based on information / expectations. (expectations about return, or to bet on risk).

Derivatives (2)

- Still, the relation between the SDF approach and derivatives is still useful another reason: *risk neutral probabilities.*

- Now if put-call parity (and even Black-Scholes!) works based on arbitrage alone, you might say we cannot derive any information from option prices. After all, any SDF that is positive will give the same result, right?

- Wrong! If any positive SDF will do, it means a flat SDF ($m_{t+1} = a$) will also be OK. This corresponds to a *risk-neutral investor*.
- Now assume – just as in a binominal tree – that prices for the underlying asset can only go up (denote our return by R_U) or down (we get R_D).

Derivatives (3)

- Add to this that for a risk-neutral investor, the expected return would be r_f. Then the following holds:

$$E(r) = p_{fall} R_D + p_{rise} R_U = r_f; \quad p_{fall} + p_{rise} = 1$$

  As R_f is observable, we can set values for any R_D and R_U (based on option valuation techniques) and then determine the corresponding probabilities p_{fall} and p_{rise} . These are *risk-neutral probabilities*.

- Now the real ('physical') probabilities of a price change corrsponding to R_U or R_D will be different! Yet the risk-neutral probabilities are are *forward looking*. Changes in risk neutral probabilities presumably indicate some shift in the real probabilities as well, *and that can be very useful information*.

Formula of the week

This week, the 'formula of the week' is the 5-factor model (1993) SDF.

$$SDF = a + b_1 R_m + b_2 R_{HML} + b_3 R_{SMB} + b_4 R_{TERM} + b_5 R_{creditpremium}$$

The expansion signals the importance of capturing risk from other sources related to the assets one invests in.

In case of bonds, the term structure (risk of changes in interest rates) and the credit premium (compensation for default risk) are prime candidates.

Derivatives can in theory be priced with the same SDF. In practice the role of the SDF there is limited to ensuring no-arbitrage pricing holds (so SDF>0).