Lecture Advanced Investments, September 15th, 2025

Anomalies

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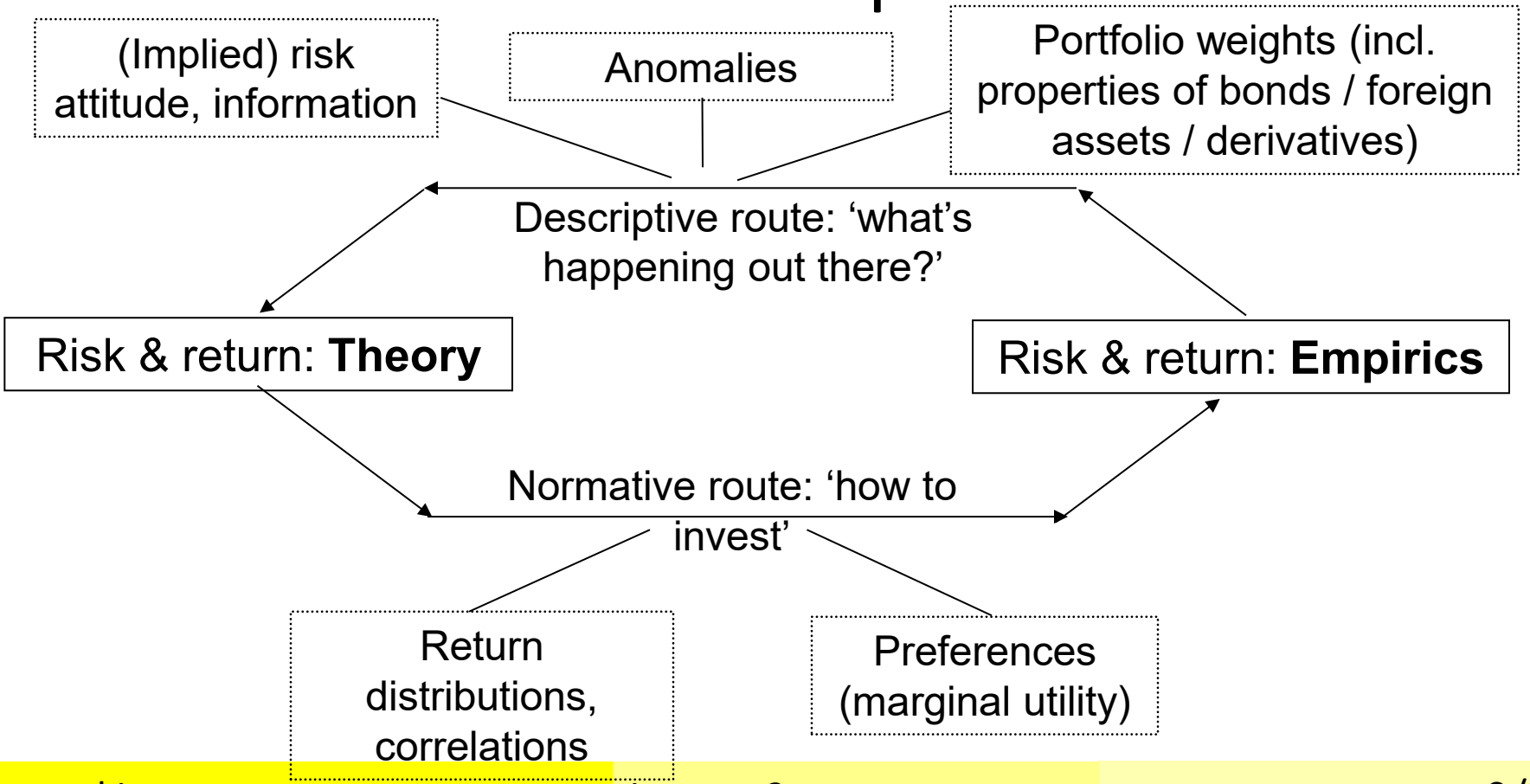
Program

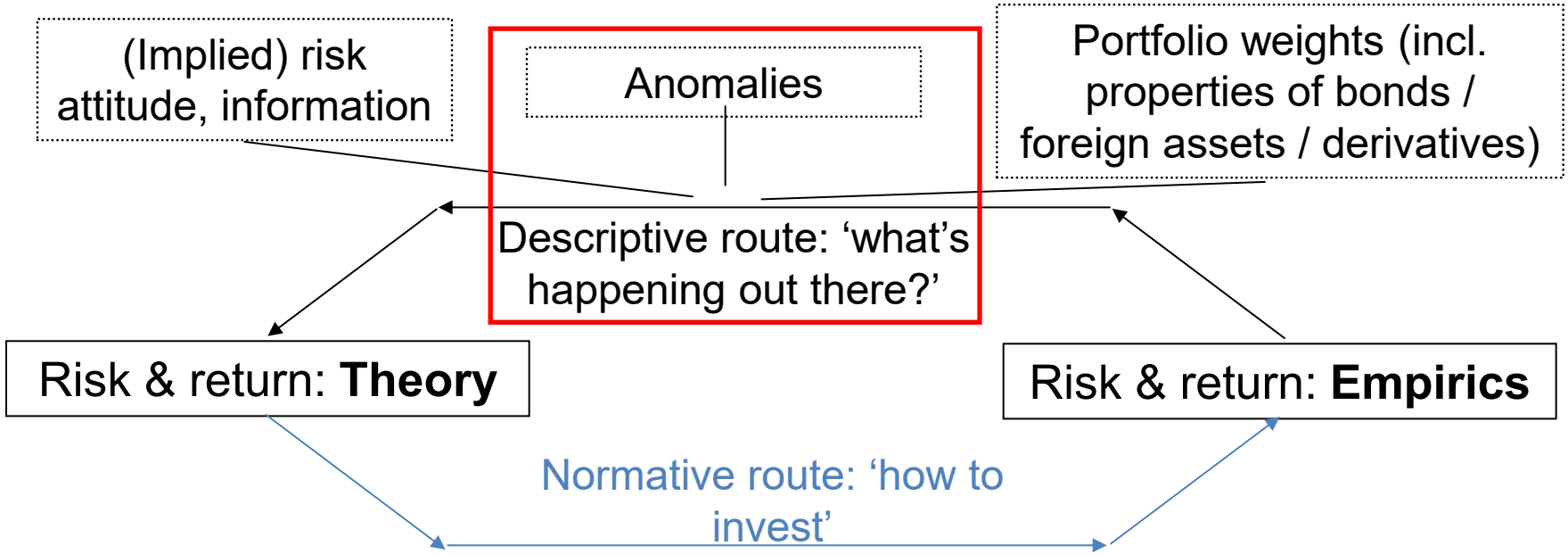
- When is an anomaly anomalous?
  - Why focus on a single anomaly? (also see clip 1 – week 7)
- Empirics (also see clips 2 & 3)
  - Size / value
  - Momentum
  - Investments/profitability (F&F2015)
  - And many more
- Testing for anomalies: (also see clip 4)
  - Fama & MacBeth (from last week, using those slides)
  - sorted portfolios and GRS test
- Trading against an anomaly (see clip 5 / next week)
- Formula of the week

Roadmap


(Implied) risk attitude, information
Anomalies
Portfolio weights (incl. properties of bonds / foreign assets / derivatives)
Descriptive route: 'what's happening out there?'
Risk & return: **Theory**
Risk & return: **Empirics**
Normative route: 'how to invest'
Return distributions, correlations
Preferences (marginal utility)

Roadmap : today

We investigate several well-known anomalies: effects that run counter to the description offered by the CAPM.



What is an anomaly?

- Q: When is an anomaly indeed anomalous?
- A: when there is a *structural, replicable pattern*, that *cannot be explained in the framework of existing (mainstream) theory, but can (potentially) be explained economically.*

- This definition is somewhat loose and quite frankly, literature has not strictly defined the concept.

- Still, it has useful elements:
  - There must be a pattern; one-off events (striking gold) do not qualify, there must be an identifiable characteristic that sets the well-performing stocks apart from their poorer performing peers.

What is an anomaly?

- Elements of an anomaly (cont):
  - It must be an economic anomaly. Any pattern that is claimed but goes unaccompanied by an 'economic rationale' is highly suspect; statistics will occasionally produce results of the correlation-between-storks-and-babies kind
  - Sometimes this can be pretty arcane: companies whose names start with an 'A' may in fact receive closer analyst scrutiny than those staring with a 'Z'.
  - It goes against accepted financial theory. That certain industries have high returns does not constitute an anomaly if their beta's are high as well.
    - 'accepted' is of course open for debate as well. Most researchers agree there is a book-to-market effect, but it is still regarded as an anomaly (vis-à-vis the CAPM).

What is an anomaly?

- Elements of an anomaly (cont):
  - Ideally, we want to consider if anomalies are empirically relevant as well – can money be made from trading on it?
  - If so:
    - The anomaly should either reflect an hitherto unknown aspect of *risk* – no return without risk, no free lunch!
    - Or, a market imperfection that prevents arbitrage from eliminating free lunches.
  - If no money can be made from it, the anomaly isn't that anomalous after all, it has already been reduced so far as market conditions allow.

- Over the past 35 years, several anomalies have been found, mainly in the stock markets.

Single or multiple anomalies?

- Some anomalies are highly dependent on the sample period and/or the markets you choose; a known anomaly without [continued] justification in risk or return characteristics will disappear over time (e.g. size effect in large caps)
  - Consequence: factors that are relevant for some datasets are irrelevant for others. But / Only a factor that's irrelevant for the entire dataset can be taken out.
  - Yet taking all factors into account at once will give you a good idea of the interactions, but a poor overview of the total effect. Preferred solution: first check existence of each, then study if anomalies reinforce each other.
  - Interestingly, most single anomalies – except for momentum – aren't that strong; combinations (e.g. January effect in small caps or less liquid markets) are stronger.

Size & Value anomalies (1)

- Size and value anomalies are (since Fama & French 1992 and onwards) often mentioned in the same breath.
- Size anomaly: *small* firms seem to earn somewhat higher returns than large firms, after correction for their market risk.
- Value anomaly: *value* stocks, that is, stocks with a low market value relative to their book value, seem to earn superior returns over 'growth' stocks, again even after corrections for beta have been made.
- The size anomaly dates back to Banz (1981), the value premium is mostly due to F&F, who also took size into account.
  - One of the reasons for this combination is that if you want to prove the existence of a new anomaly, you better convince the referee it isn't a repackaged old anomaly.

Size & Value anomalies (2)

- Size anomaly: *small* firms seem to earn somewhat higher returns than large firms, after correction for their market risk.
- Magnitude: from negative up to 10% a year, with considerable variation through the decades and between emerging and developed markets.

- Economic motivation: small firms are under less scrutiny from analysts so their prices update less often. This carries a risk, for which compensation would be required.
  - 'alternatively', one could look at it from a liquidity perspective. Small stocks would be less liquid, and therefore more risky (a bigger chance of having to trade against a 'wrong' price)

Size & Value anomalies (3)

- Oddly enough, the size premium has been found, buried, and resurrected?!
- It is quite possible that after the discovery of this anomaly, *investors traded on it and hence reduced the magnitude of the anomaly to insignificant levels.* (in developed markets in particular. Emerging markets almost always had one, but it is plausible that is a segmented market).
  - Yet the last 15 years or so, the anomaly seems to be back?!

Table 1: Size Premium Waves

This table reports average excess returns, volatility, and t-statistic of a size portfolios over different samples. We report these statistics for a portfolio that goes long low-market-equity (the 1^{st} decile) stocks and short high-market-equity stocks (10^{th} decile). The sample period is from July 1926 to December 2017 at a monthly frequency. Returns and volatilities are annualized.

	1926-1940 (1)	1941-1960 (2)	1961-1980 (3)	1981-2000 (4)	2001-2017 (5)
Avg. Return	11.23	9.43	9.12	−4.54	5.31
Volatility	49.55	20.06	17.37	17.93	12.39
t-statistic	0.86	2.10	2.35	−1.13	1.77

Source: Herkovic, Kind & Kung, (2018) working paper

Size & Value anomalies (4)

- However, there's still ample room for disagreement: Blitz & Hanauer (2020) '**Settling the Size Matter**' (ambitious title…) find no size premium outside the US, and the one inside the US is only exploitable if you control for quality and go short in 'junk assets'. No size premium in practice.

- *We can actually learn some lessons from this regarding how science works:*
  - Until it's published, your mileage may vary. The Herkovic, Kind & Kung paper took 5 years to appear in print, *and without the 'wave table'*. Check back on working papers!
  - Science doesn't settle debates easily. A working paper has the lowest status, but not every journal has equal reputation (for quality / strictness). (Blitz & Hanauer have the Journal of Portfolio Management, serious but more practitioner oriented, not very high academic standing), there is seldom a way to say 'now it's settled' as people keep producing papers.
  - They don't mention each other. On the one hand, the field is very wide, on the other hand it's uncommon to publicly disagree. Career scientists are very careful the first 20 years or so. One rarely responds directly if you can avoid it!

Size & Value anomalies (5)

- Value anomaly: value stocks, that is, stocks with a low market value relative to their book value, seem to earn superior returns over 'growth' stocks, again even after corrections for beta have been made.
- Magnitude: 4 to 6% a year

- Economic motivation: returns come from somewhere; ultimately asset prices are determined by (expected) payouts. So since time immemorial (well before portfolio theory, in any case) investors looked at the characteristics of the company to determine if its stock was a good buy.

  Book value is one of these. If market value is close to book value, the firm appears to be in dire shape (no growth opportunities that have any value), and therefore more risky, which in turn raises the required return.

Size & Value anomalies (6)

- The problem is that this 'financial distress hypothesis' is not completely believable, as the factor associated with the value anomaly (more on that later) has low correlations with other measures of distress.
- However, as Lettau & Ludvigson (2001) suggest, the value factor may work primarily in times which are already 'bad' – very low returns in an already bad market are considered even more risky (and require even higher returns) in that view. *Conditionality again!*
- Still, the jury is out, even after over 25 years. Not on whether the F&F factors are relevant, but on what they represent exactly. And as long as that's uncertain, it's quite possible they are just proxies for the true factor(s).
- The reliance on accounting info (see slides on the profitability and investment factors) may be problematic here too. See Lev & Srivastava (2022).

Size & Value anomalies (7)

- As long as the economic rationale behind the F&F factors is unclear, we also have trouble where to look for **improvements** (in a theoretical sense, practioners tend to have less issues with this).
  - Is the assumption that everyone is a price-taker violated? (A10; this corresponds to the illiquidity premium which is relevant if stock prices drop if you're selling)
  - Is there general mispricing (Stambaugh & Yuan, 2016)?
  - Is our understanding of risk (and hence our model of the SDF) flawed?

    "A4. Utility is a function of expected return and variance, and nothing else."

    This is suggested by the 'distress' motivation. But how do we adjust our SDF?

Size & Value anomalies (8)

- How do we adjust our SDF, and how do we proceed from there?
  - Last week, we saw that:

$$1 = E(m_{t+1} r_i) \qquad m = a + b' f$$

    is equivalent to

$$E(r_i) = \gamma + \lambda` \beta_i$$

  - Which means we can simply add the F&F factors to the SDF, and also see them back in the expanded version of the SML. (normally this type of research ignores the SDF, but including it in your version of the SML means you *automatically* incorporate it in the SDF too.)
  - The SDf actually remains $\mathbf{m = a + b' f}$
  - Yet f is now a <u>vector</u> containing R_m, R_{size} and R_{value} (based on the long/short strategy in slide 29)

Size & Value anomalies (9)

- How do we adjust our SDF, and how do we proceed from there?
  - Our 'expanded SML' then becomes:

$$E(r_i) = r_f + \rho_{market}\beta_{M,i} + \rho_{size}\beta_{size,i} + \rho_{value}\beta_{value,i}$$

$$\rho_{market} = ? \qquad \text{but according to theory}: (E(r_M) - r_f)$$

$$\rho_{size} = ?$$

$$\rho_{value} = ?$$

Momentum anomaly (1)

- The momentum anomaly states that assets (stocks) that performed well in the recent past (say 1 year) will outperform the 'losers' for another year.
- Papers: Carhart (1997), Jigadeesh & Titman (1993).
- Magnitude: mostly 4 to 6% a year, but sometimes as high as 2% *a month*.
- Obtained by sorting the stocks on past winners and losers; then (for example) buy the 20% best performing stocks and finance this by shortselling the 20% worst performing stocks.
  - Rinse and repeat

- Careful selection and regular rebalancing is needed, as at some point specific stocks will cease to be 'winners'. Short horizons (3 to 12 months) work best.
- Israel and Moskowitz (2013): present in US from 1927 tot 2012, UK & Europe from 1970s to 2011, across stocks, currencies and commodities.

Momentum anomaly (2)

- The momentum anomaly states that assets (stocks) that have performed well in the recent past (say 1 year) will outperform the 'losers' for another year.
- Technically speaking, you could use this to construct a factor to be included in the SDF
- However, it's so obvious that that this has nothing to do with risk from an economic point of view (especially the 'when do we get returns' part – what's in the past is completely irrelevant) that such a course of action is ill-advised.
- Actually, why doesn't this anomaly disappear rapidly? Trading on it seems relatively easy, and if enough investors do it, returns will adjust.

Momentum anomaly (3)

- Possible explanations:
  - Transaction costs. The short positions are costly to obtain (and even maintain), and without them the profits drop substantially. So after trading costs, there may not be an anomaly after all.
  - It captures illiquidity effects, mostly in small stocks. Illiquidity is a risk of sorts; illiquid stocks may fall a lot further if you want to sell them in a decreasing market.
- Both of these explanations are based in market imperfections – violations of our assumptions.
- Momentum is rather robust to other anomalies, such as size or value premia, or macro economic factors. Jury is still out on illiquidity. (which is rather hard to measure, you're working with risks that often don't materialise). Behavioral explanations may (or may not) help as well.

Momentum anomaly (4)

- Another hypothesis is that of *momentum crashes*. This idea (e.g. Daniel & Moskowitz (2016)) is based on panic states, when the market looses a lot of value amid high volatility; momentum portfolios would be severely affected.
  - It would be akin to a situation where you market beta suddenly goes up at the same time (and for the same reasons?) as the market is going down. In a crash, you don't want a high market beta! Some authors claim this can be hedged though -> unsettled debate within a debate.
- Momentum investments would therefore be subject to infrequent but severe crashes.
  - This clearly relates to the 'when do we like returns', so it may make sense that there is a premium for stocks that may crash at the worst possible moment.
  - It may also create negative skewness in the return distribution (see next week!)
  - And a big loss might have a much more severe impact on utility than a couple of smaller ones (see lecture 5).

Profitability & investment policy (1)

- Fama & French (2015) suggest that a better explanation would be offered by a five factor model that also incorporates (on top of the 3 original factors):

- *4. Profitability*, as measured by a factor that is based on the difference between the most profitable and the least profitable companies.

  Note that this is a *relative* measure. If the economy is in crisis, and no-one really makes a decent profit, the difference will probably still exist between those firms making low profits and those raking up huge losses. The state of the economy is still captured by R_m.
- *5. Investment policy*, again using a relative measure that captures the difference between 'aggressive' (high investment) and 'conservative' (low investment) firms.

Profitability & investment policy (2)

- The results of Fama & French (2015) indicate that these new factors, at least in some datasets, may take the role of the earlier F&F factors, including value.
- In a sense, this is natural, as the 'new' factors have the virtue of being more closely linked with company performance

- What remains to be seen though, is how this type of work relates to 'new economy' type of stocks, such as Amazon.
  https://www.macrotrends.net/stocks/charts/AMZN/amazon/revenue vs
  https://www.macrotrends.net/stocks/charts/AMZN/amazon/net-income

- Also pharmaceuticals / biotech companies would be interesting, as that requires massive investments with possibly years, even decades, of no profit before (hopefully) hitting a jackpot.

And many more (1)

- There are many more anomalies documented over time. New ones – or better said, candidates for new anomalies - are found regularly.
- A brief list of the most important ones:
  - Calender anomalies. Mostly gone now, but influential in some datasets, often in combination with other anomalies. Prime example: January effect.
  - Reversal. Looking at longer timeframes, the momentum effect *reverses* itself. Long term winners become losers and vice versa. Which looks like a natural (but belated) reaction from a market which comes to its senses.
  - Macro-economic factors. Traditionally assumed to be part of R_m, one can wonder if macro-trends (interest rates, QE, but also sustainability) aren't a separate source of risk with (sectoral) biases.

And many more (2)

- Macro-economic factors are (often implicitly) motivated by the fact that our proxy for the market portfolio isn't perfect, and hence the relation between consumption and marginal utility of that consumption, is not as perfect as the CAPM suggests.
- Different ways of operationalizing this:
  - Use of commodity prices (e.g. oil; Driesprong, Jacobsen & Maat, 2008); while there is some truth in this, one wonders if that factor isn't a proxy of something else in itself
  - Using consumption info, or consumption/wealth ratios like Lettau & Ludvigson (2001). Often works just as well as F&F 3-factor. Again, this can be plugged into the SDF!
- However, most macro-economic and consumption data is too low frequency to be really useful.

Testing for anomalies: sorted portfolios (1)

- The methodology of testing deserves attention as well: one needs firm evidence of an anomaly – for trading on it or for a paper.
- Evidence is usually hard to find among the data, which contains returns for thousands of assets, usually over several decades, with possibly an observation for every trade ('tick by tick').
  - We need to make choices regarding which data we're going to use (type of assets/portfolios)
  - We need to make choices regarding the frequency (tick-by-tick, 5 minutes, hourly, daily, weekly, monthly, yearly….)
  - And those choices must leave us with the option of both finding the anomaly, and rejecting it.

Testing for anomalies: sorted portfolios (2)

- A convenient method of *magnifying* the effects of an anomaly is to choose sorted portfolios.
  - You gather information on the characteristics of each asset, especially the characteristic that is related to your anomaly (e.g., the market capitalisation for the size anomaly)
  - You rank all assets according to that characteristic
  - You form portfolios of assets with a similar value, for example you combine the biggest ten percent, then the next 10%, and so on, till you get the 10% smallest companies in your 10^{th} portfolio.
- One can take all of these portfolios to use as a dataset where there at least is considerable spread on the relevant variable.

Intermezzo : top 10 silliest errors

Testing for anomalies: sorted portfolios (3)

- Also, we can construct a *factor* using these portfolios:
  - Go **long** in the portfolio that should have the highest return according to your anomaly (in our example: the 10% smallest ones)
  - Finance this by going **short** in the portfolio that should have the lowest return (here: biggest firms)
  - The result has no net investment, so in theory it should not get you any return at all. If it does, the anomaly exists.
- Such a factor will enter into the time-series and cross-section regressions.

$$r_{i,t} = \alpha + \beta_1 r_{market,t} + \beta_2 r_{factor,t}$$

$$E(r_i) = \rho_0 + \rho_1 \hat{\beta}_1 + \rho_2 \hat{\beta}_2$$

Testing for anomalies: GRS test (1)

- The methodology of testing deservers attention as well: one needs firm evidence of an anomaly – for trading on it or for a paper.
- We saw the Fama & MacBeth two-pass-regression- approach last week. The problem of using estimated betas and hence having errors-in-variables is unavoidable in that way.
- Hence, it pays to look at later alternatives, like the *Gibbons-Ross-Shanken* (1989) test. It's major advantage is that we only need the time-series regressions.
- However, we do need to find a way to combine the results from time-series regressions from different portfolios.

Testing for anomalies: GRS test (2)

- The results are combined by testing for the *joint significance* of the alpha's, also called *pricing errors.*

$$\mathrm{r}_{\mathrm{it}} = \alpha + \beta_1 \mathrm{r}_{\mathrm{factor1,t}} + \beta_2 \mathrm{r}_{\mathrm{factor2,t}} + \varepsilon_{\mathrm{t}}$$

- Each and every alpha should be zero, but ofcourse randomness will cause some deviations. GRS showed that if we take the sum of the squared pricing errors and weigh/standardize them properly, that sum follows an F-distribution.

- This allows for statistical tests: if the weighted squared errors are too large (threshold given by the F-statistic) we know that we have found an anomaly.
- Both positive and negative alphas can be tested in this way, since we have to take squares.

Testing for anomalies: GRS test (3)

- The tricky part is the weighting matrix and standarization. The formula is (what follows on this slide is non-compulsory for the exam, but useful for theses):

$$\frac{T-N-1}{N}\left[1+\left(\frac{E(f)}{\sigma_f}\right)^2\right]^{-1}\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}\sim F_{N,T-N-1}$$

  - T = number of observations in the time-series
  - N = number of cross-sections (assets/portfolios)
  - f = factor (R_M, could be something else too ofcourse)
  - Σ = covariance matrix of the residuals
- NB: See Cochrane 12.1, also for multiple factors

Trading against an anomaly (1)

- With so many different factors it's hard to keep track, and even more to believe in all of them – one author managed to count 99 different ones in the literature! (see e.g. Feng, Giglio and Xiu (2020))
- While many factors in that list might still be highly correlated, even reducing them to a more manageable number (e.g. through principal component analysis or similar methods) will leave us with factors that many investors *will simply not believe in*. Is risk really such a 'multi-dimensional beast'?

- Now, if you do not believe in a factor representing risk, but the other participants do reward it with extra return, that is actually a golden opportunity!

Trading against an anomaly (2)

- Now, if you do not believe in a factor representing risk, but the other participants do reward it with extra return, that is actually a golden opportunity!

- The easiest way, call it a 'plain vanilla' approach, would be to invest in a long-short portfolio:long in stocks with a high exposure to that factor, short in stocks with a low exposure. Add additional leverage according to taste.
- However, such a strategy would have two disadvantages:
  - *Undesirable exposures* to other (priced or unpriced) factors
  - *Moving* exposures or risk premia, *noise* in the input

Trading against an anomaly (3)

- The risk of *undesirable exposures* (a portfolio based on a factor-strategy can still have a non-zero size-beta, value-beta, momentum exposure of even exposure to diversifiable risk) is explained in two recent papers:
  - Ehsani et al, 2020. Quite easy to follow, makes the main points
  - Daniel et al, 2020. More comprehensive, higher quality paper on the same basis, but also more technical
- You should read one of these two papers!

- The actual hedging methods employed will not be further discussed and are not exam material, but the phenomenon that hedging your strategy on how it loads on other risk-factors and even unpriced risk can have a big impact, should be known and understood.

Trading against an anomaly (4)

- A second issue is that exposures may not be constant, the premia for risk factors may not be constant either, or the estimate of the exposure (that is, the relevant beta) may be noisy.

- An example of the latter occurs when you bet against beta (so short high beta stock, go long in low beta stock; the most common example is market beta, but it would also hold for size beta, value beta or others): you base your portfolio weight on an estimate that is uncertain. If you mistakenly thing a stock has a low beta, the strategy will unravel.

- One solution for this is to scale beta's, using $w\beta_{measured} + (1-w)\beta_{assumed}$, with $\beta_{assumed}$ usually set to 1. But better would be to use deciles, so you use the relative value of beta. This reduces noise, especially if you work with lots of stocks.

Trading against an anomaly (5)

- Finally, there may be changing risk premia, for example due to preferences shifting over time (in 2008 there was a ‘flight to quality’), the availability of leverage or liquidity issues.

- Even those can be traded upon, see for example Frazzini & Pedersen (2014) ‘Betting against beta’ (https://www.nber.org/papers/w16601.pdf)

Formula of the week

This week, the 'formula of the week' is the **expanded Security Market Line**, taking anomalies into account. In the example of the F&F 3-factor model, it is:

$$E(r_i) = r_f + \rho_{market}\beta_{M,i} + \rho_{size}\beta_{size,i} + \rho_{value}\beta_{value,i}$$

However, one can add factors for each anomaly (constructed by shorting poorly performing assets according to that anomaly, and going long in the ones that perform well).

Yet if we do so, we also insert that factor into the SDF, meaning it has to have some economic justification as a factor capturing some sort of risk.

NB: with the GRS test we only need the corresponding time-series regressions to test, but the SDF is still adjusted!