Lecture Advanced Investments, September 23rd, 2024

Return and portfolio management

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Program

- Testing: sorted portfolios and GRS (from last week)
- Empirical return distributions / horizon also see clip 1
- Empirical return distributions / normality
  - Role of the normal distribution
  - Testing for normality also see clip 2
- Incorporation of higher moments : Skewness / Kurtosis
- Role of expected return in portfolio formation also see clip 3
  - ‘Round up the usual suspects’
  - Black-Litterman model
  - Excess returns & portfolio evaluation see clip 4
  - Fund management see clip 5

Testing for anomalies: sorted portfolios (1)

- The methodology of testing deserves attention as well: one needs firm evidence of an anomaly – for trading on it or for a paper.
- Evidence is usually hard to find among the data, which contains returns for thousands of assets, usually over several decades, with possibly an observation for every trade (‘tick by tick’).
  - We need to make choices regarding which data we’re going to use (type of assets/portfolios)
  - We need to make choices regarding the frequency (tick-by-tick, 5 minutes, hourly, daily, weekly, monthly, yearly….)
  - And those choices must leave us with the option of both finding the anomaly, and rejecting it.

Testing for anomalies: sorted portfolios (2)

- A convenient method of *magnifying* the effects of an anomaly is to choose sorted portfolios.
  - You gather information on the characteristics of each asset, especially the characteristic that is related to your anomaly (e.g., the market capitalisation for the size anomaly)
  - You rank all assets according to that characteristic
  - You form portfolios of assets with a similar value, for example you combine the biggest ten percent, then the next 10%, and so on, till you get the 10% smallest companies in your 10th portfolio.
- One can take all of these portfolios to use as a dataset where there at least is considerable spread on the relevant variable.

Testing for anomalies: sorted portfolios (3)

- Also, we can construct a *factor* using these portfolios:
  - Go **long** in the portfolio that should have the highest return according to your anomaly (in our example: the 10% smallest ones)
  - Finance this by going **short** in the portfolio that should have the lowest return (here: biggest firms)
  - The result has no net investment, so in theory it should not get you any return at all. If it does, the anomaly exists.
- Such a factor will enter into the time-series and cross-section regressions.

$$\mathrm{r}_{\mathrm{i,t}} = \alpha + \beta_1 \mathrm{r}_{\mathrm{market,t}} + \beta_2 \mathrm{r}_{\mathrm{factor,t}}$$

$$\mathrm{E}(\mathrm{r}_{\mathrm{i}}) = \rho_0 + \rho_1 \hat{\beta}_1 + \rho_2 \hat{\beta}_2$$

Testing for anomalies: GRS test (1)

- The methodology of testing deservers attention as well: one needs firm evidence of an anomaly – for trading on it or for a paper.
- We saw the Fama & MacBeth two-pass-regression- approach last week. The problem of using estimated betas and hence having errors-in-variables is unavoidable in that way.
- Hence, it pays to look at later alternatives, like the *Gibbons-Ross-Shanken* (1989) test. It's major advantage is that <u>we only need the time-series regressions</u>.
- However, we do need to find a way to combine the results from time-series regressions from different portfolios.

Testing for anomalies: GRS test (2)

- The results are combined by testing for the *joint significance* of the alpha's, also called *pricing errors.*

$$r_{it} = \alpha + \beta_1 r_{factor1,t} + \beta_2 r_{factor2,t} + \varepsilon_t$$

- Each and every alpha should be zero, but ofcourse randomness will cause some deviations. GRS showed that if we take the sum of the squared pricing errors and weigh/standardize them properly, that sum follows an F-distribution.

- This allows for statistical tests: if the weighted squared errors are too large (threshold given by the F-statistic) we know that we have found an anomaly.
- Both positive and negative alphas can be tested in this way, since we have to take squares.

Testing for anomalies: GRS test (3)

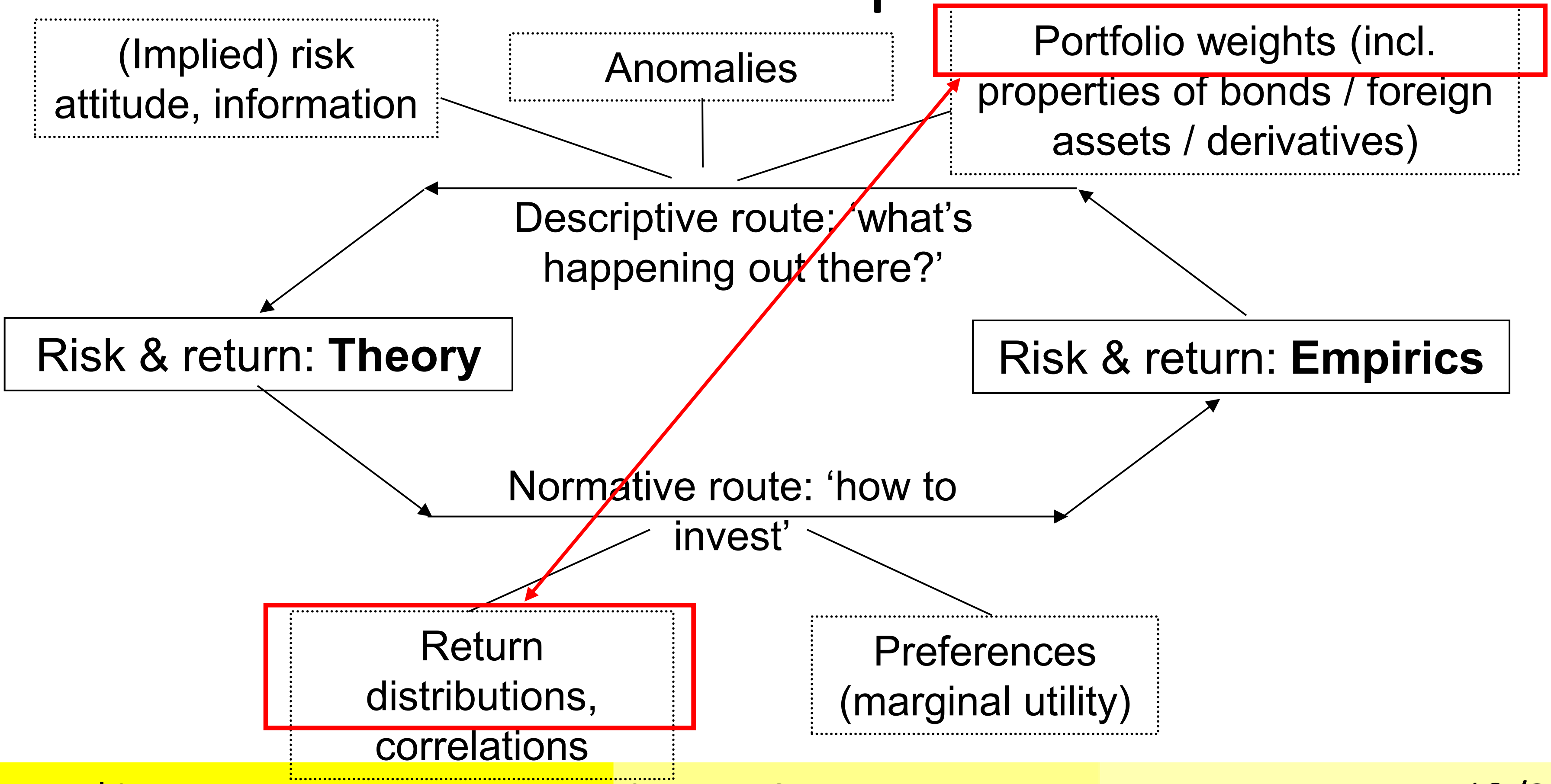
- The tricky part is the weighting matrix and standarization. The formula is (what follows on this slide is non-compulsory for the exam, but useful for theses):

$$\frac{T-N-1}{N}\left[1+\left(\frac{E(f)}{\sigma_f}\right)^2\right]^{-1}\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha} \sim F_{N,T-N-1}$$

  – T = number of observations in the time-series
  – N = number of cross-sections (assets/portfolios)
  – f = factor (R_M, could be something else too ofcourse)
  – Σ = covariance matrix of the residuals
- NB: See Cochrane 12.1, also for multiple factors

Intermezzo : top 10 silliest errors

Roadmap


(Implied) risk attitude, information
Anomalies
Portfolio weights (incl. properties of bonds / foreign assets / derivatives)
Descriptive route: 'what's happening out there?'
Risk & return: **Theory**
Risk & return: **Empirics**
Normative route: 'how to invest'
Return distributions, correlations
Preferences (marginal utility)

Emprical return distributions (1)

- We saw we that we have several data choices to make in our research: "Evidence is usually hard to find among the data, which contains returns for thousands of assets, usually over several decades, with possibily an observation for every trade ('tick by tick')."

- But this problem does not only enter the descriptive research, it also affects the normative part:

  Which investment horizon should I take when constructing portfolios?

- The question 'which frequency should I use?' might seem more logical to the academic, but those questions <u>are one and the same</u>.

Emprical return distributions (2)

- The frequency of the data you use implicitly also sets the investment horizon you're considering, assuming we work with a 1-period model. (assumption A3). So yearly returns means you optimize for an investment for 1 year.

- Of course, investment horizons may be longer or shorter, or uncertain. It would not matter much if return distributions would be the same regardless of the frequency one uses.

- Sadly, they're not. Means and variances change in other ways than 'normal' and so do correlations and higher moments (skewness, kurtosis). Also see the excel file with daily, weekly and monthly returns – the risk doesn't scale with the frequency!

- Actually, this is expected: returns are multiplicative, and the product of 2 normal distributions *is not a normal, but a chi-square distribution.*

Emprical return distributions – extra assignment

- It can be quite instructive to take the excel-file with daily-weekly-monthly returns for those 6 portfolios, and apply the technique of the portfolio optimization-excel to find the optimal portfolio weights. *You'll quickly see that for comparable levels of required return, the portfolio weights are different.* This is due to the changing return distributions once we change the frequency of the data.

- For example:
  - Daily returns, required return 0,09% a day:

    Portfolio weights: -1,3589 / 1,2343 / **0,9986 / 0,2431 / -0,0410** / -0,0761
  - Weekly returns, required return 0,40% a week:

    Portfolio weights: -1,3303 / 1,3266 **/ 0,6267 / 0,4037 / 0,0352** / -0,0619
  - Monthly returns, required return 1,80% a month:

    Portfolio weights: -1,5556 / 1,6916 / **0,4257 / 0,8605 / -0,2445** / -0,1777
- Conclusion: portfolio weights are quite sensitive to this choice!

Role of normal distributions (1)

- Also when working with lognormal distributions (meaning ln(r) is normally distributed, a mathematically more convenient description), we see that reality does not want to help out the troubled researcher/investor.

- A true solution to this problem does not exist. However, it does matter. Simply change one average return in our spreadsheet, and watch the differences…

- This is a general effect: *optimal portfolios are very sensitive to the average / expected return*.

- However, in the end the problem of horizon is dwarfed by that of expectations, as we'll see later today.

Role of normal distributions (2)

- But before we get to the role of expectations, an explanation of why the normal distribution plays an important role in theory is due:

  Normally distributed returns are one way to frame the investment problem as a balance between mean and variance.

- This is an assumption underlying portfolio theory, the CAPM and the vast majority of both scientific and empirical literature.

Role of normal distributions (3)

- The idea behind this is simple: a normal distribution can be described by just two parameters, which happen to be mean and variance. From those two numbers, the entire distribution can be recreated.

- In other words: no other factor can influence the risk-return relation, as there *is* no other factor. Risk has to enter into the return distribution, as risk is nothing else than a return which is different from the average.

- Actually, it's one of 2 main ways to arrive at the MV-framework, the other being an assumption on the utility function (mainly quadratic utility).

Testing for normality (1)

- To judge if a return distribution is normal - which we need to know for 2 reasons:

  1) does our problem exist, and

  2) is it worthwhile to look at the ‘straightforward’ expansions that might solve the issue,

  - one has to look at the *skewness* and *kurtosis.*

- Skewness is a measure of asymmetry, a normal distribution is symmetric around its mean. If the left tail (the lower/negative returns) is more pronounced than the right tail, the distribution has a negative skewness. If vice versa, positive skewness.

Testing for normality (2)

- Formula:

$$skew = \frac{\sum (x - \bar{x})^3}{(1/n)\left[\sum (x - \bar{x})^2\right]^{3/2}}$$

- Kurtosis the degree of peakedness of a distribution, but the actual application of this measure refers to the *tails*. If the kurtosis is higher than 3 (the value for a normal distribution) the tails are fatter. This is typical of financial data, especially at higher frequencies. It causes higher probablities of extreme returns.

- Formula:

$$kurt = \frac{\sum (x - \bar{x})^4}{(1/n)\left[\sum (x - \bar{x})^2\right]^{2}}$$

- Graphic example.

Testing for normality (3)

- The standard test for normality combines the skewness and kurtosis into the Jarque-Bera test:

$$JB = \frac{n}{6}\left(skew^2 + \frac{1}{4}(kurt - 3)^2 \right)$$

- This test statistic has a chi-square distribution with 2 degrees of freedom, so one can use standard testing procedure here.

- Again, one would be hard pressed to find financial data for which the null hypothesis of normality is not rejected.

Incorporating higher moments (1)

- From Kraus-Litzenberger (1976) onwards researchers have tried to include higher moments – such as skewness and kurtosis – in their models.
- Later this was picked up by Harvey & Siddique (2000 - skewness) and Dittmar (2002- kurtosis).
- In fact these are generalisations of the CAPM; if returns are normally distributed (zero skewness and excess kurtosis), the normal CAPM obtains.
- Agents are assumed to dislike negative skewness (and like positive), and dislike kurtosis.

Incorporating higher moments (2)

- Ofcourse, this means adjusting the SDF. To quote Dittmar (2002):

  Specifically, rather than take a stand on the exact form of the pricing kernel, we approximate it using a Taylor series expansion:

$$m_{t+1} = h_0 + h_1 \frac{U''}{U'} R_{W,t+1} + h_2 \frac{U'''}{U'} R_{W,t+1}^2 + \dots, \qquad (2)$$

- Or more precisely,

  Thus, a cubic pricing kernel can be justified under intuitive arguments, which suggests that investors are averse to extreme outcomes in a distribution, as well as utility-based arguments such as standard risk aversion. Consequently, we investigate a version of equation (2) that truncates the expansion at the return on aggregate wealth cubed

$$m_{t+1} = d_0 + d_1 R_{W,t+1} + d_2 R_{W,t+1}^2 + d_3 R_{W,t+1}^3. \qquad (6)$$

Incorporating higher moments (3)

- The only downside to this approach is that it works easily in the context of explaining returns, where we can employ several factors, but that it is more troublesome in the context of optimalization.

- With the SDF we can determine if a portfolio is ex post efficient given our risk attitudes (more on this next week), but an expression for the efficient set is more troublesome – we now need to balance return with variance, skewness and kurtosis.

- Mathematically, we can expand the objective function, but we need information in advance concerning *the relative importance of skewness and kurtosis*. Simply minimizing variance for a given return no longer works.

“Round up the usual suspects” (1)

Last 2 minutes of ‘Casablanca’ (1942).

“Round up the usual suspects” (2)

- Until now, we have assumed that the historical mean and (co)variances offer a good description of the asset market *in the future*, as well as that everyone is equally well informed.

- Historical data is often no more related to the future returns than the ‘usual supsects’ to the shooting in the film. Or, as the Dutch disclaimer says: “in het verleden behaalde rendementen zijn geen garantie voor de toekomst.”

- Similarly, no everyone is equally well informed, for some certain information is in plain view, while others have no idea.

“Round up the usual suspects”- not

- Hence, it makes sense to look at how to incorporate ‘personal views’ into the portfolio decision. We also want to judge if that supposedly superior knowledge leads to a better risk-return trade-off.
  - Efficient Market Hypothesis…..?
- In fact we want to compare the effects of (1) ‘active portfolio management’ (as BKM calls it, one could also use the term *stock picking*), as sole investment strategy or part of a broader one, with (2) an orthodox portfolio formed simply on historical data as the most dependable estimator, as normative theory prescribes.

- Still, even with superior information one wants to spread one’s risks – only rarely can one use ‘arbitrage-like’ strategies to bet on a single stock/event.

Incorporating views: Black-Litterman (1)

- One model that does give weight to expectations *and* historical data is the *Black-Litterman model*.

- The Black-Litterman model roughly works as follows:
  - Get the historical or CAPM predicted returns as a starting point
  - Incorporate the views and their (gu)es(s)timated uncertainty
  - Contrast these with historical data / baseline stimates, <u>and get a weighted average</u>
  - Use these to perform classical portfolio optimization
- This is relatively easy with a few assets. However, the method is of limited use on a stock level, where there are n (e.g. 500) means on which one must have a view, but also n^2 (25.000) covariances.

Incorporating views: Black-Litterman (2)

- As a consequence, this approach works better in two situations:
  - When comparing asset*classes*, for example stocks, bonds, commodities, currencies.

    NB: MV can lead to rather one-sided portfolios in those cases.
  - When focussed on a limited number of stocks, for instance a mutual fund manager focussing on a single industry in a single country.
- Also, division of labor works for analysts as well. It's not illogical to focus on creating expectations for a specific subset of the market.

- However, this leads to the problem of *hierarchical portfolio management* (see later slides).

Mixing differently

- Black-Litterman is based on mixing historical returns with your own expectations. But views can take on different forms as well: maybe your view is that a certain factor is more relevant / deserves more exposure / is excessively rewarded.

- Then you can also mix: Boido & Fasano (2023) attempt to integrate these portfolio construction approaches, by balancing the mean-variance historical optimisation with factor tilting.

- However, with all mixing you need to decide on what the weight of each method in the mix might be. Black-Litterman would do this based on perceived confidence, Boido & Fasano have a utility function-like approach. In all cases, this is where the weak spot is, as it remains fairly arbitrary!

Excess returns

- The next stage is to judge if your strategy did indeed give a superior risk-return trade-off. Ofcourse one can never totally exclude outside influences (e.g. 9/11, credit crisis), but barring such episodes, one has to check if the higher realised return didn't come from taking more risk.

- Return above what is needed to compensate for risk is (also) called excess return, or *alpha*, or *tracking error*.

$$r_{i,t} = \alpha + r_{f,t} + \beta(r_m - r_{f,t}), \textbf{ or } \textbf{tr. error} = r_{i,t} - r_{benchmark}$$

- Actually, one can use these techniques also to judge *in advance* if a particular stock is worth it, as BKM (Ch27, p. 977-978) illustrate, one can translate target prices into alphas – which can be combined with the Black-Litterman approach as well.

Excess returns (2)

- Firstly, we need to distinguish several types or return:
  - Holding period return: the return in the period under consideration: $(P_{end}\text{-}P_{begin)}/P_{begin}$
  - Arithmetic return: the average return – our best estimate of the expected return barring extra information.

$$\bar{r}_{arithmetic} = \frac{1}{T}\sum_{t=1}^{t=T} r_t$$

  - Geometric / time weighted average:

$$\bar{r}_{geometric} = \left[\prod_{t=1}^{t=T}(1+r_t)\right]^{1/T} - 1$$

Excess returns (3)

- If we want to compare performace of portfolios over different time-periods, the geometric returns are normally the better choice, as they track what actually happened. (example: 2 periods, -20% and +20% return). Arithmetic returns are the better estimator for expected returns.

- Next, we need to correct for risk. Performance evaluation based solely on returns ignores the risk altogether.

Excess returns (4) / Assignment 3

- Traditional correction mechanism for risk are all based on the MV framework:
  - Sharpe ratio $(r_p - r_f)/\sigma_p$
  - Treynor ratio $(r_p - r_f)/\beta_p$
  - Information ratio $\alpha_p/\sigma_{\varepsilon,p}$
  - Jensen's alpha [alpha according to the CAPM]

  (See BKM Ch 24, study this yourself)

Also interesting: Ledoit & Wolf, 2008, "Robust Performance Hypothesis Testing with the Sharpe Ratio" – Sharpe ratios even with non-normal returns!

- Assignment for this week: take the spreadsheets from assingment 2a, and calculate the 4 measures above and the M^2 measure for your optimal portfolio with and without shortselling.

  NB: Use an equally weighted average of the 10 basic assets to construct a proxy for the market.

Excess returns (5)

- Somewhat more flexible is the M^2 measure (which has nothing to do with a square):

  $M^2 = r_{p.adj} – r_{market.}$

  The first term stands for the return on a ‘managed portfolio’, a combination of the real evaluated portfolio and the risk-free object, so that the managed portfolio *has the same standard deviation* as your benchmark (usually Rm), making direct comparions of returns possible.

- Ofcourse (and personally I’d prefer that) one can also mix the benchmark return with the T-bill to obtain the same standard deviation as your evaluated portfolio. This also offers a closer analogy to MPT.

Excess returns (6)

- Additional advantage of the 'adjusted' M^2 is that it's a natural comparison of active and passive investment; if the evaluated portfolio has the risk you find desirable, the matched portfolio can be taken to be the optimal portfolio from MPT. (assuming MPT holds)

- In that approach, M^2 gives a direct measure of the return you get from the active investment strategy. (*but remember it is non-standard*!)
- In the traditional way, M^2 is linked to the sharpe-ratio.

- In either version one can take another benchmark (e.g. an industry or counrty index) as well, and in principle it's also applicable for other measures of risk as long as one can find the right managed portfolio (which is relatively easy as long as your risk measure is dependent on a single factor / characteristic.

Formula of the week

This week, the 'formula of the week' is the formula for computing the M2 performance measure.

$$\mathbf{M}^2 = \mathbf{r}_{\text{evaluated portf.}} - \mathbf{r}_{\text{matched portf.}}$$

It shows us several vital points:

- Return needs to be measured properly, and above all in relation to risk
- True skill / superior expectations would show as *return beyond that required by the risk of the portfolio*, and should outperform (on average) a passive strategy.
- The matching portfolio allows much needed flexibily to capture the risk of a portfolio by choosing an appropriate benchmark and/or risk measure.