# CENG 382 - Analysis of Dynamic Systems
# 20221
# Take Home Exam 3 Solutions

YILMAZ, Mert Kaan
e2381093@ceng.metu.edu.tr

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1. (a) $f_1 = -x_1 + x_2^2$
   $f_2 = 3x_1^2 - 2x_2$

$$Df(x) = \begin{bmatrix} -1 & 2x_2 \\ 6x_1 & -2 \end{bmatrix}$$

   $f(x) \approx Df(\tilde{x})(x - \tilde{x}) + f(\tilde{x})$
   $= Df(\tilde{x})x - Df(\tilde{x})\tilde{x}$

   (b) 1) Lyapunov function is continuous and has continuous first partial derivatives.
   2) It has uniqueue minimum at $\tilde{x}$
   3)
   We know that $V(x_1, x_2) = \frac{x_1^2}{2} + \frac{x_2^2}{4}$
   Firs, let's find $\Delta V$.
   $\Delta V = \frac{dV}{dt} = \frac{dV}{dx_1} \cdot \frac{dx_1}{dt} + \frac{dV}{dx_2} \cdot \frac{dx_2}{dt}$
   $= x_1 \cdot (-x_1 + x_2^2) + \frac{x_2}{2} \cdot (-2x_2 + 3x_1^2)$
   $= x_1^2 + x_1x_2^2 - x_2^2 + \frac{3}{2}x_2x_1^2$

2.

3. We know that $V(x_1, x_2) = \frac{x_1^2}{2} + \frac{x_2^2}{2}$
   Firs, let's find $\Delta V$.
   $\Delta V = \frac{dV}{dt} = \frac{dV}{dx_1} \cdot \frac{dx_1}{dt} + \frac{dV}{dx_2} \cdot \frac{dx_2}{dt}$
   $= 2x_1 \cdot (x_1 + x_2 - 4x_1(x_1^2 + x_2^2)) + 2x_2 \cdot (-x_1 + x_2 - 4x_1(x_1^2 + x_2^2))$
   $= 2x_1^2 + 2x_2^2 - 16x_1x_2(x_1^2 + x_2^2)$

4. (a) Let's find fixed points $k \approx \tilde{k}$
   $x(\tilde{k}) = 3 - x(\tilde{k})^2$
   $= x(\tilde{k})^2 + x(\tilde{k}) - 3 = 0$
   We need to find it's roots:
   $\tilde{k}_{1,2} = \frac{-1 \pm \sqrt{13}}{2}$

   (b) $x(k+1) = f(x(k))$
   $x = f^2(x) = f(3 - x^2) = 3 - (3 - x^2)^2$
   $= 3 - (9 - 6x^2 - x^4) = x^4 + 6x^2 - 6$

   (c)