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Answer 1

Picking a start from 10 distinct stars: $\binom{10}{1} = 10$
Picking two habitable planets from 20 habitable planets: $\binom{20}{2}$
Picking eight non-habitable planets from 80 habitable planets: $\binom{80}{8}$
Then from product rule, choosing all the possibilities for selecting: $\binom{10}{1} \cdot \binom{20}{2} \cdot \binom{80}{8}$
Since the order of the planets matter, we should calculate all the permutations of the chosen planets: 10!

Therefore, the answer is: $\binom{10}{1} \cdot \binom{20}{2} \cdot \binom{80}{8} \cdot 10! = 199860952 \cdot 10^{12}$

Answer 2

Reorder the equation: $a_n - 2a_{n-1} - 15a_{n-2} + 36a_{n-3} = 2^n$
Let $a_n = r^3, a_{n-1} = r^2, a_{n-2} = r, a_{n-3} = 1$
The characteristic equation: $r^3 - 2r^2 - 15r + 36 = 0 = (r+4)(r-3)^2$
So $r_1 = -4, r_{2,3} = 3$ (multiplicity of 2)

The solution of the homogeneous recurrence relation:
$a_n^{(h)} = A(-4)^n + (Bn + C)3^n$
Particular solution:
F(n) = 2^n
Since 2 is not a root of the characteristic equation of the associated linear homogeneous recurrence relation, there is a particular solution of the form
$a_n^{(p)} = p_0(2)^n$ and this should satisfy the recurrence relation.
$a_n - 2a_{n-1} - 15a_{n-2} + 36a_{n-3} = 2^n$
$p_0(2)^n - 2p_0(2)^{n-1} - 15p_0(2)^{n-2} + 36p_0(2)^{n-3} = 2^n$ Divide both sides with 2^{n-3}
$8p_0 - 8p_0 - 30p_0 + 36p_0 = 8 \;\rightarrow\; p_0 = 3/4$
Then $a_n^{(p)} = (3/4)(2)^n$

Therefore, general solution for the recurrence relation is:
$a_n = a_n^{(h)} + a_n^{(p)}n = A(-4)^n + (Bn + C)3^n + (3/4)(2)^n$

Answer 3

$a_1 = 5$
There are two mutual exclusive sets of valid activation codes
a. X $\rightarrow$ |a valid code with length n-1| + even digit
b. Y $\rightarrow$ |an invalid code with length n-1| + odd digit
$|X| = a_{n-1} \cdot 5$
$|Y| = (10^{n-1} - a_{n-1}) \cdot 5$
Then,
$a_n = |X| + |Y|$
$a_n = 5 \cdot a_{n-1} + 5 \cdot (10^{n-1} - a_{n-1})$
$a_n = 5 \cdot 10^{n-1}$

Answer 4

The equation can be written as: $a_k - 3 \cdot a_{k-1} + 3 \cdot a_{k-2} - a_{k-3} = 0$
Let $f_{(x)}$ denote the generating function for the sequence a_k; that is, $f_{(x)} = \sum_{k\geq 0}^{n} a_k \cdot x^k$
Taking the first equation, multiply each term by x^k and sum each term over all positive k≥3.
$\sum_{k\geq 3}^{n} a_k \cdot x^k - 3 \cdot \sum_{k\geq 3}^{n} a_{k-1} \cdot x^k + 3 \cdot \sum_{k\geq 3}^{n} a_{k-2} \cdot x^k - \sum_{k\geq 3}^{n} a_{k-3} \cdot x^k = 0$

Manipulate each term so that we can write them as expressions in terms of the generating function $f_{(x)}$ and known series representations.

$\sum_{k\geq 3}^{n} a_k \cdot x^k = \sum_{k\geq 3}^{n} a_k \cdot x^k + a_0 + a_1 x + a_2 x^2 - a_0 - a_1 x - a_2 x^2 = \sum_{k\geq 0}^{n} a_k \cdot x^k - a_0 - a_1 - a_2 x^2$
$= f(x) - 1 - 3x - 6x^2$

$\sum_{k\geq 3}^{n} a_{k-1} \cdot x^k = x \cdot \sum_{k\geq 3}^{n} a_{k-1} \cdot x^{k-1} = x \cdot \sum_{k\geq 2}^{n} a_k \cdot x^k = x \cdot (\sum_{k\geq 0}^{n} a_k \cdot x^k - a_1 x - a_0) = x(f(x) - 3x - 1)$

$\sum_{k\geq 3}^{n} a_{k-2} \cdot x^k = x^2 \cdot \sum_{k\geq 3}^{n} a_{k-2} \cdot x^{k-2} = x^2 \cdot \sum_{k\geq 1}^{n} a_k \cdot x^k = x \cdot (\sum_{k\geq 0}^{n} a_k \cdot x^k - a_0) = x^2(f(x) - 1)$

$\sum_{k\geq 3}^{n} a_{k-3} \cdot x^k = x^3 \cdot \sum_{k\geq 3}^{n} a_{k-3} \cdot x^{k-3} = x^3 \cdot \sum_{k\geq 0}^{n} a_k \cdot x^k = x^3 f(x)$

Then, our equation will be:
$f(x) - 1 - 3x - 6x^2 - 3xf(x) + 9x^2 + 3x + 3x^2 f(x) - 3x^2 - x^3 f(x) = 0$
$f(x) - 3xf(x) + 3x^2 f(x) - x^3 f(x) = 1$
$f(x)(1-x)^3 = 1$
$f(x) = \frac{1}{(1-x)^3}$

From book, page 542: $\frac{1}{(1-x)^n} = \sum_{k=0}^{n} C(n+k-1, k) \cdot x^k and a_k = C(n+k-1, k)$
Therefore, for n = 3
$a_k = C(2+k, k)$

Answer 5

a. For the given relation to be an equivalence class, it should be reflexive, symmetric, and transitive.
Reflexivity
Consider (a,b)R(a,b), (a,b) on $\mathbb{Z}^+$x $\mathbb{Z}^+$
Apply given relation condition: a+b = b+a
Since this is true for all (a,b), R is reflexive

Symmetry
Given (a,b)R(c,d) such that a+d=b+c
Consider (c,d)R(a,b) on $\mathbb{Z}^+$x $\mathbb{Z}^+$
Given relation condition says that: c+b=d+a.
Because this satisfies the given relation condition, R is symmetric.

Transitivity
Let (a,b)R(c,d) and (c,d)R(m,n), and all those pairs are in the set of $\mathbb{Z}^+ x \mathbb{Z}^+$
Apply given relation condition:
c+n=d+m (*)
a+d=b+c (can also be written as a-c=b-d(**))
Add (*) and (**) side by side, a-c+c+n = b-d+d+m
a+n=b+m $\rightarrow$ (a,b)R(m,n) also satisfies the condition.
Therefore R is transitive.

Since we showed that R is reflexive, symmetric, and transitive, R is an equivalence relation.

b. This class consist of all real pairs (a,b) satisfying (a,b)R(1,2).
Apply the given relation condition: 1+b = 2+a $\rightarrow$ b = a+1
Therefore, the equivalence class of (1,2): $[(1,2)] = \{(a,a+1)|a \in \mathbb{Z}^+\}$