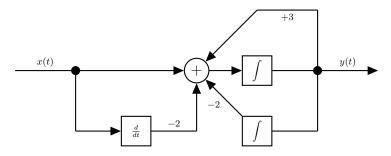
Spring 2022 Homework 2

## Regulations:

- Grouping: You are strongly encouraged to work in pairs.
- Drawing Plots: Clearly label the coordinate axes and make sure that your plots are not open to different interpretations.
- Submission: You need to submit a pdf file named 'hw2.pdf' to the odtuclass page of the course. You need to use the given template 'hw2.tex' to generate your pdf files. Otherwise you will receive zero.
- Deadline: 23:55, 17 April, 2022 (Sunday).
- Late Submission: Not allowed.
- 1. (30 pts) Consider an LTI system given by the following block diagram:



- (a) (15 pts) Find the differential equation which represents this system.
- (b) (15 pts) Find the output y(t), when the input  $x(t) = (e^{-t} + e^{-2t})u(t)$ . Assume that the system is initially at rest.
- 2. (10 pts) Evaluate the following convolutions.
  - (a) (5 pts) Given  $x[n] = \delta[n-1] + 3\delta[n+2]$  and  $h[n] = 2\delta[n+2] \delta[n+1]$ , compute and  $\underline{\operatorname{draw}}\ y[n] = x[n] * h[n]$ .
  - (b) (5 pts) Given x[n] = u[n+1] u[n-2] and h[n] = u[n-4] u[n-6], compute and  $\underline{\text{draw}}\ y[n] = x[n] * h[n]$ .
- 3. (10 pts) Evaluate the following convolutions.
  - (a) (5 pts) Given  $h(t) = e^{-\frac{1}{2}t}u(t)$  and  $x(t) = e^{-t}u(t)$ , find y(t) = x(t) \* h(t).
  - (b) (5 pts) Given  $h(t) = e^{-3t}u(t)$  and x(t) = u(t) u(t-4), find y(t) = x(t) \* h(t).
- $4.\ \ (20\ \mathrm{pts})\ \mathrm{Consider}\ \mathrm{a}\ \mathrm{continuous}\ \mathrm{LTI}\ \mathrm{system},\ \mathrm{initially}\ \mathrm{at}\ \mathrm{rest},\ \mathrm{given}\ \mathrm{by}\ \mathrm{the}\ \mathrm{following}\ \mathrm{input}\ \mathrm{and}\ \mathrm{output}\ \mathrm{equation};$

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 3) d\tau \tag{1}$$

- (a) (10 pts) Find and plot the impulse response h(t).
- (b) (10 pts) Find the response of the system when the input x(t) is shown as:

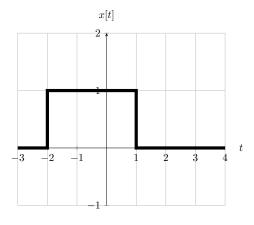


Figure 1: t vs. x(t).

5. (30 pts) Consider a discrete LTI system represented by the following block diagram which is initially at rest:



where

$$h_1^{-1}[n] = (\frac{1}{2})^n u[n] \tag{2}$$

and the overall impulse response of the system is:

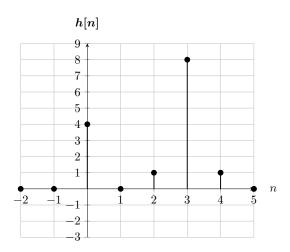


Figure 2: n vs. h[n].

- (a) (10 pts) Find  $h_1[n] * h_1[n]$ .
- (b) (10 pts) Find the impulse response  $h_0[n]$ .
- (c) (10 pts) Consider a discrete LTI system represented by the following block diagram which is initially at rest:

$$\begin{array}{c|c} \hline & x[n] \\ \hline & \\ \hline \end{array} \begin{array}{c|c} \hline & y[n] \\ \hline \end{array}$$

What is the response of this system when  $x[n] = \delta[n] + \delta[n-2]$ ?