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Answer 1

a)

Expected value for blue dice:
2·P(2) + 3·P(3) + 4·P(4)
$= 2\cdot\frac{4}{6} + 3\cdot\frac{1}{6} + 4\cdot\frac{1}{6} = \frac{15}{6} = 2.5$

Expected value for yellow dice:
1·P(1) + 2·P(2) + 3·P(3)
$= 1\cdot\frac{2}{6} + 2\cdot\frac{2}{6} + 3\cdot\frac{2}{6} = \frac{12}{6} = 2$

Expected value for red dice:
1·P(1) + 2·P(2) + 3·P(3) + 5·P(5)
$= 1\cdot\frac{2}{8} + 2\cdot\frac{2}{8} + 3\cdot\frac{3}{8} + 5\cdot\frac{1}{8} = \frac{20}{8} = 2.5$

b)

By using expected values that are calculated in part a, Let's say expected values of blue, yellow, and red dice are E_B, E_Y, E_R respectively. Then,

Expected values of 2 red and 1 yellow dice is: $2\cdot E_R + E_Y = 2\cdot 2.5 + 2 = 7$
Expected values of 2 yellow and 1 blue dice is: $2\cdot E_Y + E_B = 2\cdot 2 + 2.5 = 6.5$

Since expected value of first option is higher than the second one, I would pick "2 red and 1 yellow" (first option) to maximize total value.

c)

Basically, we say $E_B = 4$. Then, expected value of second option will be E_Y+4 . Since expected value of the second option will be 8, which is higher than the first one, choosing second option would give me the maximum total number.

d)

R, B, and Y are events of choosing red, blue and yellow dice respectively.
We know that, we got outcome of 3 from a dice. Also, our sample space is "getting 3 from blue OR red OR yellow dice". Then, getting a 3 from a red dice probability can be calculated as:
$\frac{P(R) \cdot P(3|R)}{P(B) \cdot P(3|B) + P(Y) \cdot P(3|Y) + P(R) \cdot P(3|R)} = \frac{\frac{1}{3} \cdot \frac{3}{8}}{\frac{1}{3} \cdot \frac{1}{6} + \frac{1}{3} \cdot \frac{2}{6} + \frac{1}{3} \cdot \frac{3}{8}} = \frac{3}{7}$

e)

Possibilities are:
(Red: 3 , Yellow: 3) and (Red: 5 , Yellow: 1)
$P_A(B)$ is the possibility of B outcome for dice A.
Possibility of the total value 6: $P_R(3) \cdot P_Y(3) + P_R(5) \cdot P_Y(1) = \frac{3}{8} \cdot \frac{2}{6} + \frac{1}{8} \cdot \frac{2}{6} = \frac{1}{6}$

Answer 2

a)

There are no electric outages in Ankara and two electric outages in Istanbul: a = 0 and i = 2. Therefore,
P(A=0, I=2) = 0.17

b)

There are two electric outages in Ankara and no electric outages in Istanbul: a = 2, i = 0.
Since 'a' value can take either 0 or 1, there is no possibility for 'a' to take 2. Therefore P(A=2, I=0) = 0

c)

There are two electric outages in total: a + i = 2
Possible values of a and i = [(a=0, i=2), (a=1, i=1)]
Therefore the result is summation of that two outcomes:
P(A=0, I=2) + P(A=1, I=1) = 0.17 + 0.11 = 0.28

d)

Single electric outage in Ankara: Since a is 0 or 1, a can only take value 1 and i can be anything(0, 1, 2, 3). Therefore,
the probability is: P(A=1, I=0) + P(A=1, I=1) + P(A=1, I=2) +P(A=1, I=3) = 0.12 + 0.11 + + 0.22 + 0.15 = 0.60

e)

All possible values of total number of outages is between 0 and 4(both are included). Let's say Y = A + I, then,
$P_Y(0)$ = P{Y = 0} = P(A=0, I=0) = 0.08
$P_Y(1)$ = P{Y = 1} = P(A=0, I=1) + P(A=1, I=0) = 0.13 + 0.12 = 0.25
$P_Y(2)$ = P{Y = 2} = P(A=0, I=2) + P(A=1, I=1) = 0.17 + 0.11 = 0.28
$P_Y(3)$ = P{Y = 3} = P(A=0, I=3) + P(A=1, I=2) = 0.02 + 0.22 = 0.24
$P_Y(4)$ = P{Y = 4} = P(A=1, I=3) = 0.15

f)

They are not independent and to show that, we need to find an example which holds P(A∩I)≠P(A)·P(I). For example, let number of outages as A = 1 and I = 0. Then P(A=1 ∩ I=1) = 0.11 where P(A=1) = $\frac{1}{2}$ and P(I=1) = $\frac{1}{4}$ since there are 2 options for 'a' and 4 options for 'i'.
Therefore, P(A=1)·P(I=1) = $\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$ = 12.5.

Because P(A=1∩I=1)≠P(A=1)·P(I=1), we can say that electric outages in Ankara and Istanbul are not independent.