CENG 384 - Signals and Systems for Computer Engineers
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Homework 1

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1. (a) To use $\bar{z}$ in equation, first we need to find it by taking the conjugate of z.
$\bar{z} = x - yj$
Now we can put corresponding values into given equation.
$2(x + yj) - 9 = 4j - (x - yj)$
$3x + yj = 9 + 4j$
Therefore $x = 3$ and $y = 4$
Put x and y values in z, we get: $z = 3 + 4j$
$|z| = \sqrt{(3)^2 + (4)^2} = 5$
$|z|^2 = 25$
z on the complex plane:

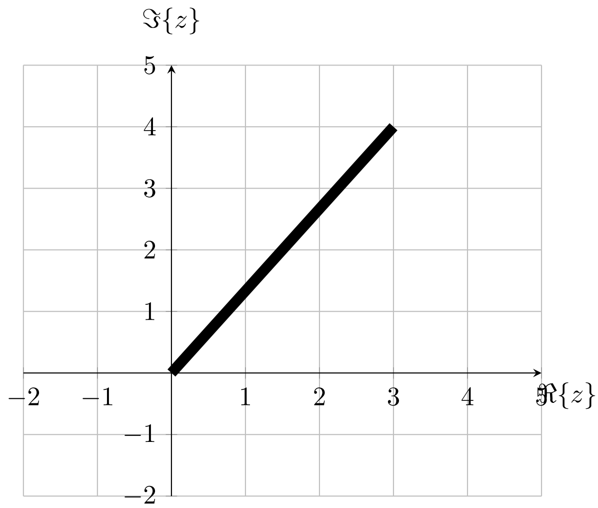

Figure 1: $z = 3 + 4j$

(b) We know $z = re^{j\theta}$
and $z^3 = -27j$ is given.
Therefore, $r^3 e^{3j\theta} = -27j$
From this equation, we can find that
$r = -3$,
$e^{3j\theta} = j$
From Euler's Formula:
$e^{j(3\theta)} = \cos\left(\frac{\pi}{2}\right) + j\sin\left(\frac{\pi}{2}\right) = j$
So,
$3\theta = \frac{\pi}{2}$ then $\theta = \frac{\pi}{6}$
Now let's write z in polar form
$z = -3 \cdot (\cos\left(\frac{\pi}{6}\right) + j\sin\left(\frac{\pi}{6}\right))$
$z = -3e^{(j\frac{\pi}{6} + \frac{2\pi}{3}m)}$ for m=1,2,3,...

(c) Multiply both denominator and numerator with conjugate of $\sqrt{3}+j$
$z = \frac{(1+j)(\sqrt{3}-j)^2}{(\sqrt{3}+j)(\sqrt{3}-j)} = \frac{(1+j)(\sqrt{3}-j)^2}{2} = 2+\sqrt{3}-j\sqrt{3}+2j$
Magnitude of z:
$\sqrt{(2+\sqrt{3})^2+(2-\sqrt{3})^2} = \sqrt{14}$
Angle of z:
$\theta = \arctan(\frac{2-\sqrt{3}}{2+\sqrt{3}}) = 4.1074 degrees$

(d) $z = -(1+j)^8 e^{j\frac{\pi}{2}}$ is given.
$z = -(1+j)^8 e^{j\frac{\pi}{2}} = z = -(2j)^4 e^{j\frac{\pi}{2}} = z = -16e^{j\frac{\pi}{2}}$

2. (a) Energy: E $= \sum_{n=-\infty}^{\infty} |nu[n]|$
E $= \sum_{n=-\infty}^{-1} 0 + \sum_{n=0}^{\infty} n = 0+1+2+3+...$
E $= \infty$ which does not hold $0 < E < \infty$. Therefore it's not an energy signal.
Power: P $= \lim_{N\to\infty} \frac{1}{2N+1}[\sum_{n=-N}^{N} |nu[n]|]$
P $= \lim_{N\to\infty} \frac{1}{2N+1}[\sum_{n=0}^{N} |n|]$
P $= \infty = \lim_{N\to\infty} \frac{N(N+1)}{2(2N+1)}$
Therefore P is not a power signal.
To conclude, we can say that the signal x[n] is neither energy nor power signal.

(b) Energy: E $= \lim_{T\to\infty} \int_{-T}^{T} (e^{-2t})^2 u(t)dt = \lim_{T\to\infty} \int_{0}^{T} (e^{-4t})dt$
E $= \lim_{T\to\infty} \frac{e^{-4t}}{-4}|_0^T = \lim_{T\to\infty}(\frac{e^{-4T}}{-4} + \frac{1}{4})$
E $= \frac{1}{4}$. Since equation $0 < E < \infty$ holds, we can say that it's an energy signal.
Power: P $= \lim_{T\to\infty} \frac{(\frac{e^{-4t}}{-4}+\frac{1}{4})}{2T} = 0$ and since P=0, it's not a power signal.
Conclusion: signal x(t) is an energy signal but not a power signal.

3. The signal of $y(t) = \frac{1}{2}x(-\frac{1}{3}t+2)$

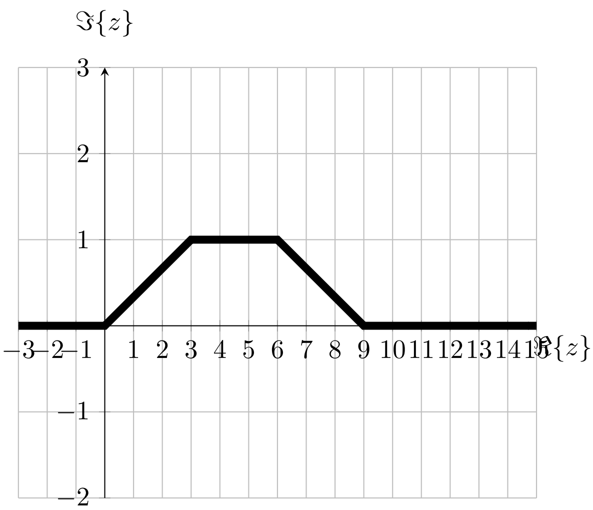

Figure 2: $y(t) = \frac{1}{2}x(-\frac{1}{3}t+2)$

4. (a) To get $x[-2n]$, we simply take symmetry wrt. y axis and shrink it by 2.
For $x[n-2]$, we move the graph by 2 to right.
After those steps, we sum them up and draw $x[-2n]+x[n-2]$

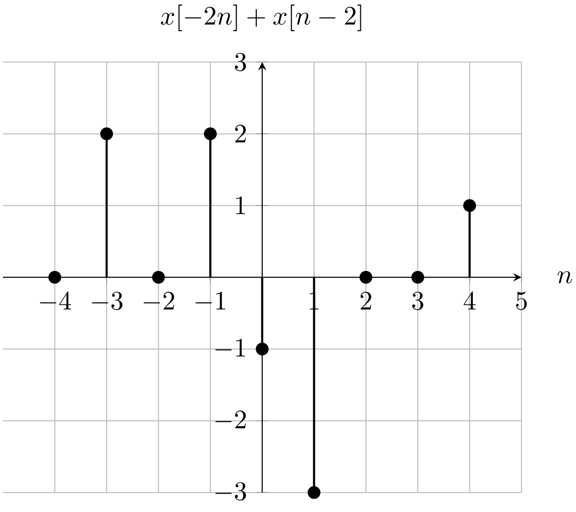

Figure 3: n vs. $x[-2n]+x[n-2]$.

(b) $x[-2n]+x[n-2] = 2\delta(n+3)+2\delta(n+1)-\delta(n)-3\delta(n-1)+\delta(n-4)$

5. (a) It's periodic.
$\frac{e^{j3t}}{-j} = \frac{e^{j3(t+T)}}{-j}$
$\frac{e^{j3t}}{-j} = \frac{e^{j3(t)}}{-j} \cdot e^{j3(T)}$
$e^{j3(T)} = 1$
Since $e^{2\pi k} = 1$ and $e^{j3(T)} = e^{2\pi k}$ we can say that $2\pi k = 3T$
Therefore $T = \frac{2\pi k}{3}$ for all k.
The fundamental period is the smallest positive value for k, so we can put 1 at k to get the fundamental period as $T_0 = \frac{2\pi}{3}$.

(b) It's periodic.
Let's say $\frac{1}{2}sin[\frac{7\pi}{8}n] + 4cos[\frac{3\pi}{4}n - \frac{\pi}{2}] = x_1[n] + x_2[n]$,
$x_1[n] = \frac{1}{2}sin[\frac{7\pi}{8}n]$ and $x_2[n] = 4cos[\frac{3\pi}{4}n - \frac{\pi}{2}]$
So,
For x_1:
$\frac{1}{2}sin[\frac{7\pi}{8}n] = \frac{1}{2}sin[\frac{7\pi}{8}(n+N)]$
$\frac{7\pi}{8}n + 2\pi k = \frac{7\pi}{8}n + \frac{7\pi}{8}N$
$N = \frac{16k}{7}$ for k=7,14,21...
Then $N_1 = 16$.
For x_2:
$4cos[\frac{3\pi}{4}n - \frac{\pi}{2}] = 4cos[\frac{3\pi}{4}(n+N) - \frac{\pi}{2}]$
$\frac{3\pi}{4}n - \frac{\pi}{2} + 2\pi k = \frac{3\pi}{4}n + \frac{3\pi}{4}N - \frac{\pi}{2}$
$N = \frac{8k}{3}$ for k=3,6,9...
Then $N_2 = 8$.
Fundamental period = N_0 = LCM(16, 8)= 16

6. (a) Because the signal is not symmetric about the origin, we can say that it's not odd. Additionally, since the signal is not symmetric about the y-axis, it's not an even signal either. Therefore, we can conclude that the signal is neither even nor odd.

(b) Even decomposition: $Even\{x(t)\} = \frac{(x(t)+x(-t))}{2}$
Odd decomposition: $Odd\{x(t)\} = \frac{(x(t)-x(-t))}{2}$

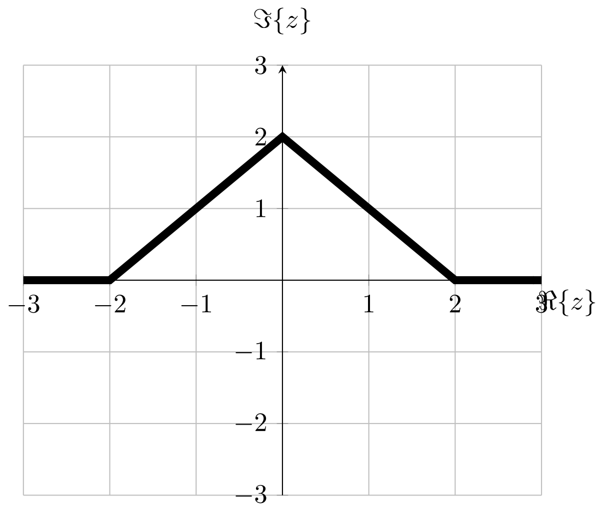

Figure 4: $Even\{x(t)\} = \frac{(x(t)+x(-t))}{2}$

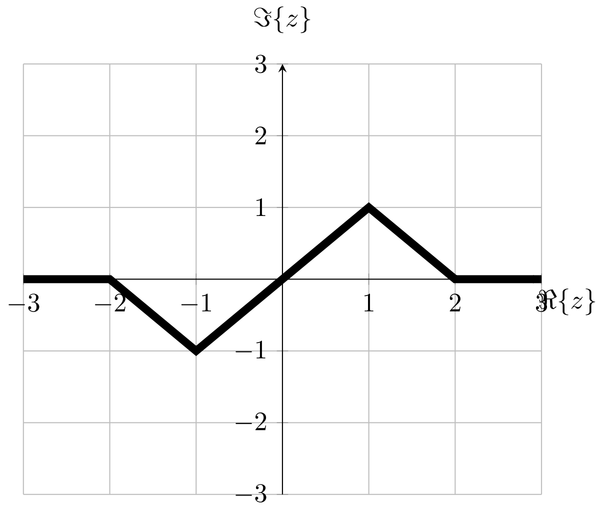

Figure 5: $Oddx(t) = \frac{(x(t)-x(-t))}{2}$

7. (a) $x(t) = 3u(t+3) - 3u(t+1) + 2u(t-2) - 4u(t-4) + 3u(t-6)$

(b) We know that $\frac{du(t)}{dt} = \delta(t)$
$\frac{du(t)}{dt} = 3\delta(t+3) - 3\delta(t+1) + 2\delta(t-2) - 4\delta(t-4) + \delta(t-6)$

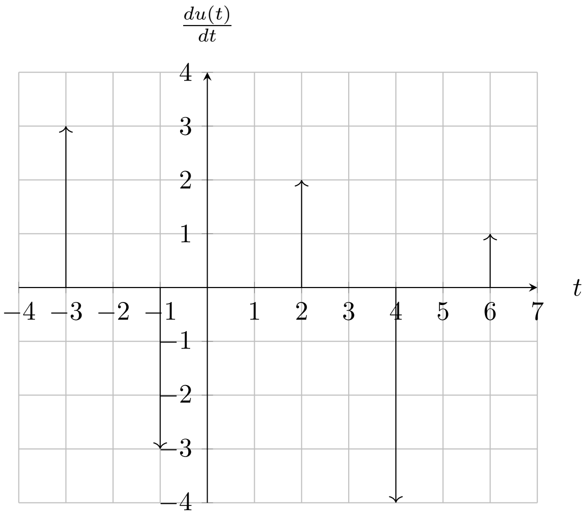

Figure 6: t vs. $\frac{du(t)}{dt}$.

8. (a)
   i. Memory: It has memory, for example y[1] = x[0].
   ii. Stability: It is stable, because all bounded inputs result in bounded outputs.
   iii. Causality: It is not causal, because output depends on future output. y[5] = x[8]
   iv. Linearity: We can say it's linear, because superposition holds.
   (Superposition means that $y_1(t) + y_2(t) = h(x_1(t) + x_2(t))$.)
   v. Invertibility: It's not invertible, because $x[n] = y[\frac{n+2}{2}]$ is not defined for all values of n.
   vi. Time-invariance: We can say that it's time varying. $x[2n - 2n_0 - 2] \neq [2n - n_0 - 2]$

(b)
   i. Memory: It has memory, for example $y(2) = 2x(0)$
   ii. Stability: It's not stable, because bounded inputs does not always create bounded outputs (y(t) depends on t that is unbounded).
   iii. Causality: It's causal, because output does not depend on future inputs.
   iv. Linearity: It's linear, because superposition property holds.
   (Superposition means that $y_1(t) + y_2(t) = h(x_1(t) + x_2(t))$.)
   v. Invertibility: It's not invertible, because $x(t) = \frac{y(2t+2)}{2t+2}$ is not defined for $t = -1$.
   vi. Time-invariance: It's time varying. $tx(\frac{t-t_0}{2} - 1) \neq (t - t_0)x(\frac{t-t_0}{2} - 1)$