Middle East Technical University Department of Computer Engineering

CENG 223

Discrete Computational Structures

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Homework 3

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Question 1

For the Fermat's Little Theorem, $a^{p-1} \equiv 1(mod\, p)$, if p is prime and a cannot be divided by p. Also for every integer a, $a^p \equiv a(mod\, p)$.
Then,
$(2^{22} + 4^{44} + 6^{66} + 8^{80} + 10^{110})(mod\, 11)$
$\equiv\ (2^2 \cdot (2^2)^{10} + 4^4 \cdot (4^4)^{10} + 6^6 \cdot (6^6)^{10} + (8^8)^{10} + (10^{11})^{10})(mod\, 11)$
$\equiv\ (4 \cdot 1 + 16 \cdot 16 \cdot 1 + 36 \cdot 36 \cdot 36 \cdot 1 + 1 + 1)(mod\, 11)$
$\equiv\ (4 + 5 \cdot 5 + 3 \cdot 3 \cdot 3 + 1 + 1)(mod\, 11)$
$\equiv\ 58(mod\, 11)$
$\equiv 3$

Question 2

To find gcd(5n + 3, 7n + 4) we will use The Euclidean Algorithm.

$7n + 4 = 1 \cdot (5n + 3) + (2n + 1)$
$5n + 3 = 2 \cdot (2n + 1) + (n + 1)$
$2n + 1 = 1 \cdot (n + 1) + n$
$n + 1 = 1 \cdot (n) + 1$

Then gcd(5n + 3, 7n + 4) = 1

Question 3

We know that x is a prime number and, $m^2 = n^2 + k \cdot x$ equation holds for m,n,k integer values.
Then, we can write this equation as,
$m^2 - n^2 = k \cdot x$
$(m + n) \cdot (m - n) = k \cdot x$
For this equality to be hold, x should be the prime factor of either (m-n) or (m+n).
So this means $x|(m + n)\ or\ x|(m - n)$

Question 4

Let P(n) be the proposition that the sum of all the integers that

P(1) is true, because $1 = \frac{1 \cdot (3 \cdot 1 - 1)}{2}$. For the inductive part, we assume that P(k) holds for an arbitrary positive integer k.
Namely, we assume that,
$1 + 4 + 7 + ... + (3 \cdot k - 2) = \frac{k \cdot (3 \cdot k - 1)}{2}$
Under this assumption, we should show that P(k+1) is true.
Namely,
$1 + 4 + 7 + ... + (3 \cdot k - 2) + (3 \cdot k + 1) = \frac{(k+1) \cdot (3 \cdot (k+1) - 1)}{2} = \frac{(k+1) \cdot (3 \cdot k + 2)}{2}$
is also true.
We add both sides $(3 \cdot k + 1)$.
$1 + 4 + 7 + ... + (3 \cdot k - 2) + (3 \cdot k + 1) = \frac{k \cdot (3 \cdot k - 1)}{2} + (3 \cdot k + 1)$
$1 + 4 + 7 + ... + (3 \cdot k - 2) + (3 \cdot k + 1) = \frac{k \cdot (3 \cdot k - 1) + 2 \cdot (3 \cdot k + 1)}{2}$
$1 + 4 + 7 + ... + (3 \cdot k - 2) + (3 \cdot k + 1) = \frac{(3 \cdot k + 2) \cdot (k + 1)}{2}$
This last equation shows that P(k+1) is true under the assumtion that P(k) is true. This completes the inductive step.

In other words, we have proven $1 + 4 + 7 + ... + (3 \cdot n - 2) = \frac{n \cdot (3 \cdot n - 1)}{2}$