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Lab 4

Data Analysis

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In this lab the goal was to take four initial value problems and use three methods for approximating their values at later points on an interval. The problems given were:

a) x' = x/1+t with x(0)= 1. Find x(2). (Analytical solution: x= t+1)

b) x' = t + x with x(0)= 1. Find x(2). (Analytical solution: x= −t+2et−1)

c) x' = t - x with x(1)= 2. Find x(3). (Analytical solution: x= t+2e^(1−t)−1)

d) x’ = t-x/t+x with x(2)= 1. Find x(4). (Analytical solution: x= -t +sqrt(2t^2 +1))

First we were to use Maple to determine analytical solutions for each problem. We then created a program to numerically solve the problem with three separate methods, Euler’s Method, Heun’s Method, and the Runge-Kutta Method. Each was done over 10 iterations and 100 iterations. We can see that as the methods progress and the number of iterations increases, the accuracy of our approximations increases and the relative error decreases. In order from a to d, with increments of 10 to 100 we have the following tables…

n =10	Euler's Method	n =100	a)
t	x	t	x
0.2000000000	1.2000000000	0.0200000000	1.0200000000
0.4000000000	1.4000000000	0.0400000000	1.0400000000
0.6000000000	1.6000000000	0.0600000000	1.0600000000
0.8000000000	1.8000000000	0.0800000000	1.0800000000
1.0000000000	2.0000000000	0.1000000000	1.1000000000
1.2000000000	2.2000000000	0.1200000000	1.1200000000
1.4000000000	2.4000000000	0.1400000000	1.1400000000
1.6000000000	2.6000000000	0.1600000000	1.1600000000
1.8000000000	2.8000000000	0.1800000000	1.1800000000
2.0000000000	3.0000000000	2.0000000000	3.0000000000
Relative Error	0.0000000000	Relative Error	0.0000000000

n =10	Heun's Method	n =100	a)
t	x	t	x
0.2000000000	1.2000000000	0.0200000000	1.0200000000
0.4000000000	1.4000000000	0.0400000000	1.0400000000

0.6000000000	1.6000000000	0.0600000000	1.0600000000
0.8000000000	1.8000000000	0.0800000000	1.0800000000
1.0000000000	2.0000000000	0.1000000000	1.1000000000
1.2000000000	2.2000000000	0.1200000000	1.1200000000
1.4000000000	2.4000000000	0.1400000000	1.1400000000
1.6000000000	2.6000000000	0.1600000000	1.1600000000
1.8000000000	2.8000000000	0.1800000000	1.1800000000
2.0000000000	3.0000000000	2.0000000000	3.0000000000
Relative Error	0.0000000000	Relative Error	0.0000000000

n =10	Runge-Kutta Method	n =100	a)
t	x	t	x
0.2000000000	1.2000000000	0.0200000000	1.0200000000
0.4000000000	1.4000000000	0.0400000000	1.0400000000
0.6000000000	1.6000000000	0.0600000000	1.0600000000
0.8000000000	1.8000000000	0.0800000000	1.0800000000
1.0000000000	2.0000000000	0.1000000000	1.1000000000
1.2000000000	2.2000000000	0.1200000000	1.1200000000
1.4000000000	2.4000000000	0.1400000000	1.1400000000
1.6000000000	2.6000000000	0.1600000000	1.1600000000
2.0000000000	3.0000000000	2.0000000000	3.0000000000
Relative Error	0.0000000000	Relative Error	0.0000000000

n =10	Euler's Method	n =100	b)
t	x	t	x
0.2000000000	1.2000000000	0.0200000000	1.0200000000
0.4000000000	1.4800000000	0.0400000000	1.0408000000
0.6000000000	1.8560000000	0.0600000000	1.0624160000
0.8000000000	2.3472000000	0.0800000000	1.0848643200
1.0000000000	2.9766400000	0.1000000000	1.1081616060
1.2000000000	3.7719680000	0.1200000000	1.1323248390
1.4000000000	4.7663616000	0.1400000000	1.1573713350
1.6000000000	5.9996339200	0.1600000000	1.1833187620
1.8000000000	7.5195607040	0.1800000000	1.2101851370
2.0000000000	9.3834728450	2.0000000000	11.4892922400
Relative Error	0.2033126630	Relative Error	0.0245217540

n =10	Heun's Method	n =100	b)
t	x	t	x
0.2000000000	1.2400000000	0.0200000000	1.0204000000
0.4000000000	1.5768000000	0.0400000000	1.0416160800

0.6000000000	2.0316960000	0.0600000000	1.0636647250
0.8000000000	2.6306691200	0.0800000000	1.0865627520
1.0000000000	3.4054163260	0.1000000000	1.1103273200
1.2000000000	4.3946079180	0.1200000000	1.1349759320
1.4000000000	5.6454216600	0.1400000000	1.1605264460
1.6000000000	7.2154144260	0.1600000000	1.1869970800
1.8000000000	9.1748055990	0.1800000000	1.2144064210
2.0000000000	11.6092628300	2.0000000000	11.7761712300
Relative Error	0.0143358600	Relative Error	0.0001647950

n =10	Runge-Kutta Method	n =100	b)
t	x	t	x
0.2000000000	1.2428000000	0.0200000000	1.0204026800
0.4000000000	1.5836359200	0.0400000000	1.0416215480
0.6000000000	2.0442129130	0.0600000000	1.0636730930
0.8000000000	2.6510416520	0.0800000000	1.0865741350
1.0000000000	3.4365022730	0.1000000000	1.1103418360
1.2000000000	4.4401438770	0.1200000000	1.1349937030
1.4000000000	5.7102717310	0.1400000000	1.1605475970
1.6000000000	7.3058858920	0.1600000000	1.1870217420
2.0000000000	11.7777784800	2.0000000000	11.7781121600
Relative Error	0.0000283336	Relative Error	0.0000000035

n =10	Euler's Method	n =100	c)
t	x	t	x
1.2000000000	1.8000000000	1.0200000000	1.9800000000
1.4000000000	1.6800000000	1.0400000000	1.9608000000
1.6000000000	1.6240000000	1.0600000000	1.9423840000
1.8000000000	1.6192000000	1.0800000000	1.9247363200
2.0000000000	1.6553600000	1.1000000000	1.9078415940
2.2000000000	1.7242880000	1.1200000000	1.8916847620
2.4000000000	1.8194304000	1.1400000000	1.8762510670
2.6000000000	1.9355443200	1.1600000000	1.8615260450
2.8000000000	2.0684354560	1.1800000000	1.8474955240
3.0000000000	2.2147483650	3.0000000000	2.2652391120
Relative Error	0.0246280560	Relative Error	0.0023920050

n =10	Heun's Method	n =100	c)
t	x	t	x
1.2000000000	1.8400000000	1.0200000000	1.9804000000
1.4000000000	1.7448000000	1.0400000000	1.9615840800

1.6000000000	1.7027360000	1.0600000000	1.9435367150
1.8000000000	1.7042435200	1.0800000000	1.9262426880
2.0000000000	1.7414796860	1.1000000000	1.9096870830
2.2000000000	1.8080133430	1.1200000000	1.8938552790
2.4000000000	1.8985709410	1.1400000000	1.8787329440
2.6000000000	2.0088281720	1.1600000000	1.8643060320
2.8000000000	2.1352391010	1.1800000000	1.8505607730
3.0000000000	2.2748960630	3.0000000000	2.2707072040
Relative Error	0.0018609030	Relative Error	0.0000161353

n =10	Runge-Kutta Method	n =100	c)
t	x	t	x
1.2000000000	1.8374666670	1.0200000000	1.9803973470
1.4000000000	1.7406485420	1.0400000000	1.9615788780
1.6000000000	1.6976336500	1.0600000000	1.9435290670
1.8000000000	1.6986692570	1.0800000000	1.9262326930
2.0000000000	1.7357704760	1.1000000000	1.9096748360
2.2000000000	1.8023998150	1.1200000000	1.8938408740
2.4000000000	1.8932048080	1.1400000000	1.8787164710
2.6000000000	2.0038032170	1.1600000000	1.8642875780
3.0000000000	2.2706790970	3.0000000000	2.2706705670
Relative Error	0.0000037570	Relative Error	0.0000000005

n =10	Euler's Method	n =100	d)
t	x	t	x
2.2000000000	1.0666666670	2.0200000000	1.0066666670
2.4000000000	1.1360544220	2.0400000000	1.0133627020
2.6000000000	1.2075434560	2.0600000000	1.0200873360
2.8000000000	1.2806854470	2.0800000000	1.0268398240
3.0000000000	1.3551491410	2.1000000000	1.0336194470
3.2000000000	1.4306850550	2.1200000000	1.0404255100
3.4000000000	1.5071020390	2.1400000000	1.0472573400
3.6000000000	1.5842513610	2.1600000000	1.0541142870
3.8000000000	1.6620156700	2.1800000000	1.0609957200
4.0000000000	1.7403012140	4.0000000000	1.7441548290
Relative Error	0.0024426940	Relative Error	0.0002337650

n =10	Heun's Method	n =100	d)
t	x	t	x
2.2000000000	1.0680272110	2.0200000000	1.0066813510
2.4000000000	1.1383616230	2.0400000000	1.0133915570

2.6000000000	1.2105122590	2.0600000000	1.0201298680
2.8000000000	1.2841160770	2.0800000000	1.0268955570
3.0000000000	1.3588994520	2.1000000000	1.0336879230
3.2000000000	1.4346525910	2.1200000000	1.0405062890
3.4000000000	1.5112121500	2.1400000000	1.0473499970
3.6000000000	1.5884491790	2.1600000000	1.0542184120
3.8000000000	1.6662606010	2.1800000000	1.0611109210
4.0000000000	1.7445630830	4.0000000000	1.7445626470
Relative Error	0.0000002499	Relative Error	0.0000000000

n =10	Runge-Kutta Method	n =100	d)
t	x	t	x
2.2000000000	1.0680269320	2.0200000000	1.0066813510
2.4000000000	1.1383612090	2.0400000000	1.0133915570
2.6000000000	1.2105117840	2.0600000000	1.0201298670
2.8000000000	1.2841155780	2.0800000000	1.0268955570
3.0000000000	1.3588989510	2.1000000000	1.0336879230
3.2000000000	1.4346520970	2.1200000000	1.0405062890
3.4000000000	1.5112116700	2.1400000000	1.0473499970
3.6000000000	1.5884487150	2.1600000000	1.0542184120
4.0000000000	1.7445626520	4.0000000000	1.7445626470
Relative Error	0.0000000030	Relative Error	0.0000000003

List of items learned

- Euler’s method
- Heun’s method
- Runge-Kutta Method
- C++ practice
- Calculating relative error practice