ANALYSIS

To determine the speed of light we must establish a relationship between this speed and the displacement point of our light source. Doing so we must know the rate of rotation of our rotating mirror (ω), the distance between our rotating mirror (M_R) and our spherical mirror (M_F), and the magnification of L₁, which depends on the positions of L₁, L₂, and M_F.

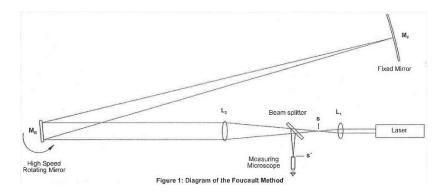
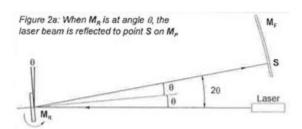


FIGURE 2: Setup for Foucault's Method of Measuring the Speed of Light

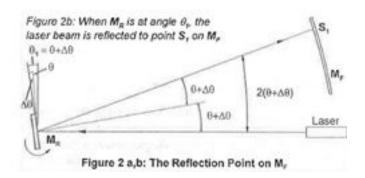
The initial pulse of light will strike M_R while it is at some angle θ . Since the angle of incidence is equal to the angle of reflection, the angle between the incident and reflected rays is 2θ . A pulse of light reflecting off M_R at this angle will strike M_F at a certain point S.



A pulse of light leaving slightly later will reflect off M_R at a different angle θ_1 , which depends on how far the mirror has rotated, $\Delta\theta$.

$$\theta_1 = \theta + \Delta\theta \tag{1}$$

This means that the total angle between the incident and reflected ray is $2(\theta_1)$, and the beam will strike M_F at a different position, S_1 .

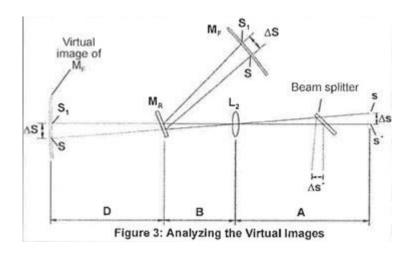


Treating S and S_1 and the endpoints of 2 arcs, we can find the distance between them by finding the difference between their respective arc lengths. In this situation, the radius is the length between M_R and M_F , which we will call D.

$$\Delta S = S_1 - S = D(2\theta_1 - 2\theta) = D[2(\theta + \Delta\theta) - 2\theta] = 2D\Delta\theta \tag{2}$$

Using the thin lens theory, we can state that

$$\Delta s' = \Delta s = \left(\frac{i}{o}\right) \Delta S = \frac{A}{D+B} \Delta S \tag{3}$$



In this case, the negative sign in the thin lens theory is ignored since we aren't concerned that the image in inverted.

Substituting in (2), we have

$$\Delta s' = \frac{2DA\Delta\theta}{D+B} \tag{4}$$

The angle M_R turns, $\Delta\theta$, will be how far the mirror can turn in the time it takes light to bounce between the two mirrors, a distance of 2D.

$$\Delta\theta = \frac{2D\omega}{c} \tag{5}$$

Substituting (5) into (4) gives

$$\Delta s' = \frac{4AD^2\omega}{c(D+B)} \tag{6}$$

We can now solve for the speed of light

$$c = \frac{4AD^2\omega}{(D+B)\Delta s'} \tag{7}$$

We can then adjust this equation to account for taking data in both the clockwise and counter-clockwise directions.

$$\omega = 2\pi \left(\frac{Rev}{s}_{CW} + \frac{Rev}{s}_{CCW}\right) \tag{8}$$

$$\Delta s' = s'_{CW} - s'_{CCW} \tag{9}$$

Substituting (8) and (9) gives a final equation for the speed of light (c)

$$c = \frac{8\pi AD^2 \left(\frac{Rev}{s} _{CW} + \frac{Rev}{s} _{CCW}\right)}{(D+B)\left(s'_{CW} - s'_{CCW}\right)}$$
(10)