Approximating Magnetically Levitated Graphite Resonator As A Driven Damped Harmonic Oscillator*

Lingqiang He[1, †]

[1] *Physics Department, Purdue University*

(Dated: December 31, 2022)

The magnetic levitated graphite structure is composed of a checkerboard of 2 N-pole and 2 S-pole square-shaped magnets and a graphite square with thickness w floated by the magnetic field $\vec{B}(\vec{x},\vec{y},\vec{z})$ due to the checkerboard where $\vec{x}$ and $\vec{y}$ are the horizontal coordinates of the plane and $\vec{z}$ is the vertical displacement from the plane, and the origin is at the center of the checkerboard. The Hamiltonian of such a system without any external force is [1]

$$H = \frac{p^2}{2m} + mgz + w\int_S dxdy \frac{-1}{2\mu_0}\chi B_z(x,y,z)^2 \tag{1}$$

where w is the width of the checkerboard plates, μ_0 is the magnetic permeability of vacuum, S is the area of the graphite chip, χ is the magnetic susceptibility is explicitly written as

$$\chi = \frac{-1}{1 + k_B T/\Delta} \tag{2}$$

if we ignore the stacking patterns of Carbon atoms.

$$\Delta = \frac{g_v g_s}{6}(\frac{v}{c})^2 \frac{q_e^2}{4\pi\epsilon_0 d} \tag{3}$$

is the characteristic energy scale, g_v and g_s are gyromagnetic constants (?), c is the speed of light, q_e is the charge of electron. And $\Delta \approx 0.03$ MeV.

The magnetic field is produced by the arrangement of four NdFeB blocks with side length b (can be b_x, b_y or b_z), the individual contribution to the total field is

$$\begin{aligned} B_z^{(1)}(x,y,z) = -\frac{B_0}{2\pi}[&F_1(-x,y,z) + F_1(-x,y,-z) \\ &+F_1(-x,-y,z) + F_1(-x,-y,-z) \\ &+F_1(x,y,z) + F_1(x,y,-z) \\ &+F_1(x,-y,z) + F_1(x,-y,-z)] \end{aligned} \tag{4}$$

with

$$F_1(x,y,z) = tan^{-1}\left[\frac{(x+\frac{b_x}{2})(y+\frac{b_y}{2})}{(z+\frac{b_z}{2})\sqrt{(x+\frac{b_x}{2})^2 + (y+\frac{b_y}{2})^2 + (z+\frac{b_z}{2})^2}}\right] \tag{5}$$

where B_0 is the magnetic field strength at the surface, let's set $B_0 = 0.5$ T for convenience. The poles of the magnet is at $z = \pm\frac{b_z}{2}$.

To obtain the equilibrium position, take $\frac{dU}{dz} = 0$; if the graphite plate is thin enough, one should find that $\left\langle \frac{dB_z^2}{dz} \right\rangle_S = 2\mu_0 \frac{\rho g}{\chi}$. For instance, the floating height $z_{lev} \approx 0.65$ mm for $b = 7.5$ mm. (See reference [1] for detail.)

* Special thanks to Gerard Higgins at Univ. of Vienna.

† he614@purdue.edu

From above information, we can start to simplify the problem in order to make an order of magnitude check. Assume that our graphite plate has lengths of half of the length of checkerboard and Maclaurin expanding arctan( ) inside $B_z(x, y, z)$ with respect to z, the leading term will be a linear function of z and it dominates because a large portion of the levitated plate is supported by this magnetic field. If we look at Fig.4 subplot a) in reference [1] and imagine that the corners of the graphite plate lie above the maximum and the side length is comparable to the sides of the checkerboard, then $B_z(x, y, z)$ is approximately linear w.r.t. to z and the graphite plate is firmly levitated at near the equilibrium point without severe rotations. I hope the reader is now convinced that under this measure the system behaves like a harmonic oscillator along and realize that the "spring constant" can be found by rearranging the constants in eq. (1) and (4).

Since the equation of motion of a driven damped harmonic oscillator (damping is due to interaction with air in most cases) is

$$\ddot{z} + \gamma\dot{z} + \omega_0^2 z = \frac{F_{ext}}{m} \tag{6}$$

where m is the mass of the moving object as a point mass, b_{air} is the damping coefficient, k is the spring constant, $\gamma = \frac{b_{air}}{m}$, $\omega_0^2 = \frac{k}{m}$ and F_{ext} is the external force. [2]

The analogous "spring constant" for our levitating graphite system is resembled by

$$k = \frac{w\,|\chi|\,B_0^2}{8\pi^2\mu_0} \tag{7}$$

in an order of 10^{-4} for $w = 10^{-3}\ m$, $\chi = 10^{-4}$ $B_0 = 0.5\ T$, $\mu = 4\pi \times 10^{-7}\ \frac{H}{m}$. Here we shall define the quality factor $Q = \frac{\omega_0}{\gamma} = \frac{m\omega_0}{b_{air}}$. If the graphite plate weight $10^{-5}\ kg$, $\omega_0 = 10^1 Hz$.

The Q-factor has an expression noted by Chen et al. in high vacuum condition is [3]:

$$Q = \frac{80\pi\omega_{res}\rho_r((\rho_g - \rho_e) + \frac{\rho_e}{V_f})}{(C_r d)^2\nabla^2 B} \tag{8}$$

where ω_{res} is the resonance frequency, ρ_r is the resistivity, ρ_g is the density of graphite, ρ_e is the density of epoxy, and C_r is the effective particle size factor. Since the Q-factor the reference [3] provides is in high ($10^3 \sim 10^5$), the resonance frequency is close to the natural frequency due to $\omega_{res} = \omega_0\sqrt{1 - \frac{1}{4Q^2}}$.

[1] M. Fujimoto and M. Koshino, "Diamagnetic levitation and thermal gradient driven motion of graphite," (2019).

[2] J. R. Taylor, *Classical mechanics* (University Science Books, 2005).

[3] X. Chen, S. K. Ammu, K. Masania, P. G. Steeneken, and F. Alijani, Advanced Science **9**, 2203619 (2022).