Exercise 3

First function:

g(n) := $1 + \frac{1}{n}$

$\sim \boldsymbol{g(n)} := \quad \boldsymbol{f(n) = 1}$

Proof:

$$\lim_{n\to\infty}\left(\frac{g(n)}{f(n)}\right) = \lim_{n\to\infty}\left(\frac{1+\frac{1}{n}}{1}\right) = \lim_{n\to\infty}\left(1+\frac{1}{n}\right) = 1$$

Second function:

g(n) := $(1 + \frac{1}{n})(1 + \frac{2}{n})$

$\sim \boldsymbol{g(n)} := \quad \boldsymbol{f(n) = 1}$

Proof:

$$\lim_{n\to\infty}\left(\frac{g(n)}{f(n)}\right) = \lim_{n\to\infty}\left(\frac{\left(1+\frac{1}{n}\right) * (1+\frac{2}{n})}{1}\right) = \lim_{n\to\infty}\left(1+\frac{1}{n}\right) = 1$$

Third function:

g(n) := $2n^3 - 15n^2 + n$

$\sim \boldsymbol{g(n)} := \quad \boldsymbol{f(n) = 2n^3}$

Proof:

$$\lim_{n\to\infty}\left(\frac{g(n)}{f(n)}\right) = \lim_{n\to\infty}\left(\frac{2n^3 - 15n^2 + n}{2n^3}\right) = \lim_{n\to\infty}\left(1 - \frac{15n^2}{2n^3} + \frac{n}{2n^3}\right) = \lim_{n\to\infty}\left(1 - \frac{15}{2n} + \frac{1}{2n^2}\right) = 1$$

Fourth function: [1]

g(n) := $\frac{log(2n)}{log(n)}$

$$\sim \boldsymbol{g(n)} := \quad \boldsymbol{f(n) = 1}$$

Proof:

$$\lim_{n\to\infty}\left(\frac{g(n)}{f(n)}\right) = \lim_{n\to\infty}\left(\frac{\log(2n)/\log(n)}{1}\right) = \lim_{n\to\infty}\left(\frac{\log(2)}{\log(n)} + \frac{\log(n)}{\log(n)}\right) = \lim_{n\to\infty}\left(\frac{\log(2)}{\log(n)} + 1\right) = 1$$

Fifth function:

g(n) := $\frac{log(n^2+1)}{log(n)}$

$$\sim \boldsymbol{g(n)} := \quad \boldsymbol{f(n) = 2}$$

Proof:

Because:

$$\lim_{n\to\infty}\left(\log(n^2+1)/\log(n)\right) = 2$$

whereas we want = 1, so we try to divide g(n) by 2:

$$\lim_{n\to\infty}\left(\frac{g(n)}{f(n)}\right) = \lim_{n\to\infty}\left(\frac{\log(n^2+1)/\log(n)}{2}\right) = \lim_{n\to\infty}\left(\frac{\log(n^2+1)}{2 * \log(n)}\right) = 1$$

Sixth function:

g(n) := $\frac{n^{100}}{2^n+1}$

$$\sim \boldsymbol{g(n)} := \quad \boldsymbol{f(n) = \frac{n^{100}}{2^n}}$$

Proof:

$$\lim_{n\to\infty}\left(\frac{g(n)}{f(n)}\right) = \lim_{n\to\infty}\left(\frac{n^{100}/(2^n+1)}{n^{100}/2^n}\right) = \lim_{n\to\infty}\left(\frac{n^{100} * 2^n}{n^{100} * (2^n+1)}\right) = \lim_{n\to\infty}\left(\frac{2^n}{2^n+1}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2^n}{2^n * (1+1/2^n)}\right) = \lim_{n\to\infty}\left(\frac{1}{1+2^{-n}}\right) = \lim_{n\to\infty}\left(\frac{1}{1+0}\right) = 1$$

[1] log = natürlicher Logarithmus = eulersche Zahl als Basis = 2,718....