$$= a^2 \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos\theta \, . \cos\theta \, d\theta$$

$$= a^2 \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta \, d\theta$$

$$= a^2 \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1+\cos2\theta}{2}\right) d\theta$$

$$= \frac{a^2}{2} \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (1+\cos2\theta)\, d\theta$$

$$= \frac{a^2}{2} \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} d\theta + \frac{a^2}{2} \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos2\theta \, d\theta$$

$$= \frac{a^2}{2} [\theta]_{\frac{-\pi}{2}}^{\frac{\pi}{2}} + \frac{a^2}{2}\left[\frac{\sin2\theta}{2}\right]_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{a^2}{2}\left[\frac{\pi}{2}+\frac{\pi}{2}\right] + \frac{a^2}{4}\left[\sin2\left(\frac{\pi}{2}\right) - \sin2\left(\frac{-\pi}{2}\right)\right]$$

$$= \frac{a^2}{2}\left(\frac{\pi+\pi}{2}\right) + \frac{a^2}{4}(0+0)$$

$$= \frac{a^2}{2}\left(\frac{2\pi}{2}\right)$$

$$= \frac{a^2\pi}{2} \text{ Sq. units} \qquad \text{Ans.}$$

EXERCISE 3.8

Q.1 Check that each of the following equations written against the differential equation in its solution.

(i) $x\dfrac{dy}{dx} = 1 + y$ $\qquad$ $y = cx - 1$

(ii)	$x^2(2y+1)\dfrac{dy}{dx} - 1 = 0$	$y^2 + y = c - \dfrac{1}{x}$
(iii)	$y\dfrac{dy}{dx} - e^{2x} = 1$	$y^2 = e^{2x} + 2x + c$
(iv)	$\dfrac{1}{x}\dfrac{dy}{dx} - 2y = 0$	$y = ce^{x^2}$
(v)	$\dfrac{dy}{dx} = \dfrac{y^2+1}{e^{-x}}$	$y = \tan(e^x + c)$

Solution:

(i) $x\dfrac{dy}{dx} = 1 + y$ ——— **(1)** **(Lhr. Board 2007)**

$$y = cx - 1$$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = c$$

Put in equation (1)

$$xc = 1 + e^x - 1$$

$$xc = cx$$

Which is true

$\therefore$ $y = cx - 1$ is the solution of $x\dfrac{dy}{dx} = 1 + y$

(ii) $x^2(2y+1)\dfrac{dy}{dx} - 1 = 0$ ——— **(1)**

$$y^2 + y = c - \frac{1}{x}$$

Diff. w.r.t. 'x'

$$2y\frac{dy}{dx} + \frac{dy}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx}(2y+1) = \frac{1}{x^2}$$

$$\frac{dy}{dx} = \frac{1}{x^2(2y+1)}$$

Put in equation (1)

$$x^2(2y+1)\frac{1}{x^2(2y+1)} - 1 = 0$$

$$1 - 1 = 0$$

$$0 = 0$$

Which is true

$\therefore$ $y^2 + y = c - \dfrac{1}{x}$ is the solution of $x^2(2y+1)\dfrac{dy}{dx} - 1 = 0$

(iii) $\mathbf{y\dfrac{dy}{dx} - e^{2x} = 1}$ ——— **(1)**

$y^2 = e^{2x} + 2x + c$

Diff. W.r.t. 'x'

$2y\dfrac{dy}{dx} = 2e^{2x} + 2$

$\dfrac{dy}{dx} = \dfrac{2e^{2x}+2}{2y} = \dfrac{2(e^{2x}+1)}{2y} = \dfrac{e^{2x}+1}{y}$

Put in equation (1)

$y\left(\dfrac{e^{2x}+1}{y}\right) - e^{2x} = 1$

$e^{2x} + 1 - e^{2x} = 1$

$1 = 1$

Which is true

$\therefore$ $y^2 = e^{2x} + 2x + c$ is the solution of $y\dfrac{dy}{dx} - e^{2x} = 1$

(iv) $\mathbf{\dfrac{1}{x}\dfrac{dy}{dx} - 2y = 0}$ ——— **(1)**

$y = ce^{x^2}$

diff. w.r.t. 'x'

$\dfrac{dy}{dx} = cxe^{x^2}.\ 2x$

$= 2cxe^{x^2}$

Put in equation (1)

$\dfrac{1}{x}\ .\ 2\,cxe^{x^2} - 2\,(ce^{x^2}) = 0$

$2\,ce^{x^2} - 2ce^{x^2} = 0$

$0 = 0$

Which is true

$\therefore$ $y = ce^{x^2}$ is the solution of $\dfrac{1}{x}\dfrac{dy}{dx} - 2y = 0$

(v) $\mathbf{\dfrac{dy}{dx} = \dfrac{y^2+1}{e^{-x}}}$ ——— **(1)**

$y = \tan(e^x + c)$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \sec^2(e^x + c) \cdot e^x$$

$$= \frac{1 + \tan^2(e^x + c)}{e^{-x}}$$

$$= \frac{1 + y^2}{e^{-x}}$$

$\therefore$ From equation (1)

$$\frac{y^2 + 1}{e^{-x}} = \frac{y^2 + 1}{e^{-x}}$$

Which is true

$\therefore$ $y = \tan(e^x + c)$ is the solution of $\dfrac{dy}{dx} = \dfrac{y^2 + 1}{e^{-x}}$

Q.2 $\mathbf{\dfrac{dy}{dx} = -y}$

Solution:

$$\frac{dy}{dx} = -y$$

Separate variables

$$\frac{dy}{y} = -\,dx$$

Integrate

$$\int \frac{dy}{y} = -\int dx$$

$$\ln y = -x + \ln c_1$$

$$e^{\ln y} = e^{-x + c_1}$$

$$y = e^{-x} \cdot e^{c_1} \qquad \boxed{\because \; y = e^{\ln y}}$$

$$\boxed{y = ce^{-x}} \qquad \because e^{c_1} = c \qquad \text{Ans.}$$

Q.3 $\mathbf{y\,dx + x\,dy = 0}$ **(Guj. Board 2006)**

Solution:

$$ydx + xdy = 0$$

Separate variables

$$ydx = -xdy$$

$$\frac{dy}{y} = \frac{-dx}{x}$$

Integrate

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

$$\ln y = -\ln x + \ln c$$

$$\ln y = \ln \frac{c}{x}$$

$$y = \frac{c}{x}$$

$$\boxed{xy = c} \quad \text{Ans}$$

Q.4 $\dfrac{dy}{dx} = \dfrac{1-x}{y}$ **(Lhr. Board 2008)**

Solution:

$$\frac{dy}{dx} = \frac{1-x}{y}$$

Separate variables

$$ydy = (1-x)\,dx$$

Integrate

$$\int ydy = \int dx - \int xdx$$

$$\frac{y^2}{2} = x - \frac{x^2}{2} + c$$

$$y^2 = 2(x - \frac{x^2}{2}) + 2c \qquad \boxed{\because \ 2c_1 = }$$

$$y^2 = 2x - x^2 + c$$

$$y^2 = x(2-x) + c \qquad \text{Ans.}$$

Q.5 $\dfrac{dy}{dx} = \dfrac{y}{x^2}$, **(y > 0)** **(Guj. Board 2008)**

Solution:

$$\frac{dy}{dx} = \frac{y}{x^2}, \ (y > 0)$$

Separate variables

$$\frac{dy}{y} = \frac{dx}{x^2}$$

Integrate

$$\int \frac{dy}{y} = \int \frac{dx}{x^2}$$

$$\ell ny = \frac{x^{-1}}{-1} + \ell nc$$

$$\ell ny = \frac{-1}{x} + \ell nc$$

$$e^{\ell ny} = e^{\frac{-1}{x} + \ell nc}$$

$$y = e^{\frac{-1}{x}} e^{\ell nc} \qquad \boxed{\because \; y = e^{\ell ny}}$$

$$y = ce^{\frac{-1}{x}} \qquad \text{Ans.}$$

Q.6 **siny cosec x $\frac{dy}{dx}$ = 1** **(Guj. Board 2005, 2008)**

Solution:

$$\text{siny cosecx} \frac{dy}{dx} = 1$$

Separate variables

$$\text{siny dy} = \frac{dx}{\text{cosecx}}$$

$$\text{siny dy} = \text{sinx dx}$$

Integrate

$$\int \text{sinydy} = \int \text{sinxdx}$$

$$-\cos y = -\cos x + c$$

$$\cos y = \cos x + c \qquad \text{Ans.}$$

Q.7 **xdy + y (x – 1) dx = 0** **(Lhr. Board 2007)**

Solution:

$$xdy + y(x-1)dx = 0$$

Separate variables

$$xdy = -y(x-1)dx$$

$$\frac{dy}{y} = \frac{-x+1}{x}dx$$

$$\frac{dy}{y} = \left(\frac{-x}{x} + \frac{1}{x}\right)dx$$

Integrate

$$\int \frac{dy}{y} = \int dx + \int \frac{dx}{x}$$

$$\ell ny = -x + \ell nx + \ell nc \qquad \boxed{\because \; y = e^{\ell ny}}$$

$$e^{\ell ny} = e^{-x + \ell nx + \ell nc}$$

$$y = e^{-x} \,.\, e^{lnx} \,.\, e^{lnc}$$

$$\boxed{y = cxe^{-x}} \qquad \text{Ans.}$$

Q.8 $\mathbf{\dfrac{x^2+1}{y+1} = \dfrac{x}{y} \,.\, \dfrac{dy}{dx}, \ (x, y > 0)}$ **(Guj. Board 2006)**

Solution:

$$\frac{x^2+1}{y+1} = \frac{x}{y} \,.\, \frac{dy}{dx}$$

Separate variables

$$\frac{x^2+1}{x}\,dx = \frac{y+1}{y}\,dy$$

$$\left(\frac{y}{y} + \frac{1}{y}\right)dy = \left(\frac{x^2}{x} + \frac{1}{x}\right)dx$$

$$dy + \frac{dy}{y} = xdx + \frac{dx}{x}$$

Integrate

$$\int dy + \int \frac{dy}{y} = \int xdx + \int \frac{dx}{x}$$

$$y + lny = \frac{x^2}{2} + lnx + lnc$$

$$e^{y+lny} = e^{\frac{-x^2}{2} + lnx + lnc}$$

$$e^{y} \,.\, e^{lny} = e^{\frac{x^2}{2}} \,.\, e^{lnx} \,.\, e^{lnc}$$

$$ye^{y} = cxe^{\frac{x^2}{2}} \qquad \text{Ans.}$$

$$\boxed{\because \ y = e^{\ell ny}}$$

Q.9 $\mathbf{\dfrac{1}{x}\dfrac{dy}{dx} = \dfrac{1}{2}\,(1+y^2)}$ **(Lhr. Board 2008)**

Solution:

$$\frac{1}{x}\frac{dy}{dx} = \frac{1}{2}\,(1+y^2)$$

Separate variables

$$\frac{dy}{1+y^2} = \frac{x}{2}\,dx$$

Integrate

$$\int \frac{dy}{1+y^2} = \frac{1}{2}\int xdx$$

$$\tan^{-1}y = \frac{1}{2} \,.\, \frac{x^2}{2} + c$$

$$\tan^{-1}y = \frac{x^2}{4} + c \qquad \text{Ans.}$$

Q.10 $\mathbf{2x^2y\frac{dy}{dx} = x^2 - 1}$

Solution:

$$2x^2y\frac{dy}{dx} = x^2 - 1$$

Separate variables

$$2ydy = \frac{x^2 - 1}{x^2}\ dx$$

$$2ydy = \left(\frac{x^2}{x^2} - \frac{1}{x^2}\right)dx$$

$$2ydy = dx - x^{-2}\ dx$$

Integrate

$$2\int ydy = \int dx - \int x^{-2}dx$$

$$\frac{2y^2}{2} = x - \frac{x^{-1}}{-1} + c$$

$$y^2 = x + \frac{1}{x} + c \qquad \text{Ans.}$$

Q.11 $\mathbf{\frac{dy}{dx} + \frac{2xy}{2y + 1} = x}$ **(Lhr. Board 2009 (S))**

Solution:

$$\frac{dy}{dx} + \frac{2xy}{2y + 1} = x$$

Separate variables

$$\frac{dy}{dx} = x - \frac{2xy}{2y + 1}$$

$$\frac{dy}{dx} = x\left(1 - \frac{2y}{2y + 1}\right)$$

$$\frac{dy}{dx} = x\left(\frac{2y + 1 - 2y}{2y + 1}\right)$$

$$\frac{dy}{dx} = \frac{x}{2y + 1}$$

$$(2\ y + 1)\ dy = xdx$$

Integrate

$$2\int ydy + \int dy = \int xdx$$

$$\frac{2y^2}{2} + y = \frac{x^2}{2} + c$$

$$y^2 + y = \frac{x^2}{2} + c$$

$$y(y + 1) = \frac{x^2}{2} + c \qquad \text{Ans.}$$

Q.12 $\mathbf{(x^2 - yx^2)\frac{dy}{dx} + y^2 + xy^2 = 0}$ **(Lhr. Board 2006, 2011)**

Solution:

$$(x^2 - yx^2)\frac{dy}{dx} + y^2 + xy^2 = 0$$

Separate variables

$$x^2(1 - y)\frac{dy}{dx} = -y^2 - xy^2$$

$$x^2(1 - y)\frac{dy}{dx} = -y^2(1 + x)$$

$$\left(\frac{1 - y}{-y^2}\right)dy = \left(\frac{1 + x}{x^2}\right)dx$$

$$\left(\frac{-1}{y^2} + \frac{y}{y^2}\right)dy = \left(\frac{1}{x^2} + \frac{x}{x^2}\right)dx$$

$$-y^{-2}\,dy + \frac{dy}{y} = x^{-2}dx + \frac{dx}{x}$$

Integrate

$$-\int y^{--2}\,dy + \int \frac{dy}{y} = \int x^{-2}\,dx + \int \frac{dx}{x}$$

$$-\frac{y^{-1}}{-1} + lny = \frac{x-1}{-1} + lnx + c$$

$$\frac{1}{y} + lny = \frac{-1}{x} + lnx + c$$

$$lny + \frac{1}{y} = lnx - \frac{1}{x} + c \qquad \text{Ans}$$

Q.13 $\mathbf{sec^2x\, tany\, dx + sec^2y\, tanx\, dy = 0}$ **(Lhr. Board 2005)**

Solution:

$$sec^2x\ tany\ dx + sec^2y\ tanx\ dy = 0$$

$$sec^2x\ tany\ dx + sec^2y\ tanx\ dy = 0$$

Separate variables

$$sec^2y\ tanx\ dy = -sec^2x\ tany\ dx$$

$$\frac{\sec^2 y}{\tan y}\, dy = \frac{-\sec^2 x}{\tan x}\, dx$$

Integrate

$$\int \frac{\sec^2 y}{\tan y}\, dy = -\int \frac{\sec^2 x}{\tan x}\, dx$$

$$ln\,(\tan y) = -ln\,(\tan x) + ln c$$

$$ln\,(\tan y) = ln \frac{c}{\tan x}$$

$$\tan y = \frac{c}{\tan x}$$

$$\tan x \tan y = c \qquad \text{Ans.}$$

Q.14 $\left(\mathbf{y - x\frac{dy}{dx}}\right) = \mathbf{2\left(y^2 + \frac{dy}{dx}\right)}$

Solution:

$$\left(y - x\frac{dy}{dx}\right) = 2\left(y^2 + \frac{dy}{dx}\right)$$

Separate variables

$$y - x\frac{dy}{dx} = 2y^2 + 2\frac{dy}{dx}$$

$$y - 2y^2 = 2\frac{dy}{dx} + x\frac{dy}{dx}$$

$$\frac{dy}{dx}(2 + x) = y\,(1 - 2y)$$

$$\frac{dy}{y(1-2y)} = \frac{dx}{2+x}$$

Integrate

$$\int \frac{dy}{y\,(1-2y)} = \int \frac{dx}{2+x}$$

$$I = ln\,(2 + x) + c_1 \qquad (1)$$

$$I = \int \frac{dy}{y(1-2y)}$$

Let

$$\frac{1}{y\,(1-2y)} = \frac{A}{y} + \frac{B}{1-2y} \qquad (2)$$

Multiplying y (1 – 2y) on both sides in eq. (2)

$$1 = A\,(1 - 2y) + By \qquad (3)$$

To find A

Put $y = 0$ in eq. (3)

$$\boxed{1 = A}$$

To find B

Put

$$1 - 2y = 0$$
$$2y = 1$$
$$y = \frac{1}{2} \text{ in eq. (3)}$$
$$1 = B\left(\frac{1}{2}\right)$$
$$\boxed{B = 2}$$

$\therefore$ From equation (2)

$$\frac{1}{y(1-2y)} = \frac{1}{y} + \frac{2}{1-2y}$$

Integrate

$$\int \frac{dy}{y(1-2y)} = \int \frac{dy}{y} + \int \frac{2}{1-2y}\, dy$$
$$I = \ln y - \int \frac{2}{2y-1}\, dy$$
$$= \ln y - \ln(2y-1) + c_2$$

Put in eq. (1)

$$\ln y - \ln(2y-1) + c_2 = \ln(2+x) + c_1$$
$$\ln \frac{y}{2y-1} = \ln(2+x) + c_1 - c_2$$
$$\ln \frac{y}{2y-1} = \ln(2+x) + \ln c, \text{ where } c_1 - c_2 = \ln c$$
$$\ln \frac{y}{2y-1} = \ln c(2+x)$$
$$\frac{y}{2y-1} = c(x+2) \qquad \text{Ans.}$$

Q.15 $\mathbf{1 + \cos x \tan y \dfrac{dy}{dx} = 0}$

Solution:

$$1 + \cos x \tan y \frac{dy}{dx} = 0$$

Separate variables

$$\cos x \tan y \frac{dy}{dx} = -1$$

$$\tan y \, dy = \frac{-1}{\cos x} dx$$

$$\frac{-\sin y}{\cos y} dy = \sec x \, dx$$

Integrate

$$\int \frac{-\sin y}{\cos y} dy = \int \sec x \, dx$$

$$ln(\cos y) = ln(\sec x + \tan x) + ln c$$

$$ln(\cos y) = ln\, c(\sec x + \tan x)$$

$$\cos y = c(\sec x + \tan x) \qquad \text{Ans.}$$

Q.16 $\mathbf{y - x \frac{dy}{dx} = 3\left(1 + x \frac{dy}{dx}\right)}$

Solution:

$$y - x \frac{dy}{dx} = 3\left(1 + x \frac{dy}{dx}\right)$$

Separate variables

$$y - x \frac{dy}{dx} = 3 + 3x \frac{dy}{dx}$$

$$y - 3 = 3x \frac{dy}{dx} + x \frac{dy}{dx}$$

$$y - 3 = (3x + x) \frac{dy}{dx}$$

$$\frac{dx}{4x} = \frac{dy}{y - 3}$$

Integrate

$$\int \frac{dy}{y - 3} = \frac{1}{4} \int \frac{dx}{x}$$

$$ln(y - 3) = \frac{1}{4} ln x + ln c$$

$$ln(y - 3) = \frac{1}{4} ln x^{\frac{1}{4}} + ln c$$

$$ln(y - 3) = ln\, c x^{\frac{1}{4}}$$

$$y - 3 = c x^{\frac{1}{4}} \qquad y = 3 + c x^{\frac{1}{4}} \qquad \text{Ans.}$$

Q.17 $\mathbf{secx + tany\frac{dy}{dx} = 0}$

Solution:

$$\sec x + \tan y\frac{dy}{dx} = 0$$

Separate variables

$$\tan y\frac{dy}{dx} = -\sec x$$

$$-\tan y\, dy = \sec x\, dx$$

$$\frac{-\sin y}{\cos y}dy = \sec x dx$$

Integrate

$$\int \frac{-\sin y}{\cos y}\, dy = \int \sec x dx$$

$$ln(\cos y) = ln(\sec x + \tan x) + ln c$$

$$ln(\cos y) = ln c(\sec x + \tan x)$$

$$\cos y = c(\sec x + \tan x) \qquad \text{Ans.}$$

Q.18 $\mathbf{(e^x + e^{-x})\frac{dy}{dx} = e^x - e^{-x}}$ **(Lhr. Board 2011)**

Solution:

$$(e^x + e^{-x})\frac{dy}{dx} = e^x - e^{-x}$$

Separate variables

$$dy = \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$$

Integrate

$$\int dy = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$$

$$y = ln(e^x + e^{-x}) + c \qquad \text{Ans.}$$

Q.19 Find the general solution of the equation $\mathbf{\frac{dy}{dx} - x = xy^2}$. Also find the particular solution if y = 1 when x = 0.

Solution:

$$\frac{dy}{dx} - x = xy^2$$

To find General Solution

$$\frac{dy}{dx} - x = xy^2$$

Separate variables

$$\frac{dy}{dx} = xy^2 + x$$

$$\frac{dy}{dx} = x\,(y^2 + 1)$$

$$\frac{dy}{y^2 + 1} = xdx$$

Integrate

$$\int \frac{dy}{y^2 + 1} = \int xdx$$

$$\tan y^{-1}\, y = \frac{x^2}{2} + c \qquad \text{———} \qquad (1)$$

To find particular solution

Put $y = 1, \quad x = 0$ in equation (1)

$$\tan^{-1}(1) = \frac{(0)^2}{2} + c$$

$$c = \frac{\pi}{4}$$

Put $c = \frac{\pi}{4}$ in equation (1)

$$\tan^{-1}y = \frac{x^2}{2} + \frac{\pi}{4} \qquad \text{Ans.}$$

Q.20 Solve the differential equation $\frac{dx}{dt}$ = 2x given x = 4 when t = 0.

Solution:

$$\frac{dx}{dt} = 2x$$

Separate variable

$$\frac{dx}{x} = 2dt$$

Integrate

$$\int \frac{dx}{x} = 2\int dt$$

$$\ln x = 2t + \ln c$$

$$e^{\ln x} = e^{2t + \ln c}$$

$$e^{\ln x} = e^{2t} \cdot e^{\ln c}$$

$$x = ce^{2t} \quad \text{———} \quad (1)$$

Put $x = 4, \quad t = 0$ in equation (1)

$$4 = ce^{2(0)}$$

$$c = 4$$

Put $c = 4$ in equation (1)

$$x = 4e^{2t} \quad \text{Ans.}$$

Q.21 Solve the differential equation $\dfrac{ds}{dt} + 2st = 0$. Also find the particular solution if s = 4e, when t = 0.

Solution:

$$\frac{ds}{dt} + 2st = 0$$

Separate variables

$$\frac{ds}{dt} = -2st$$

$$\frac{ds}{s} = -2tdt$$

Integrate

$$\int \frac{ds}{s} = -2\int tdt$$

$$l\text{ns} = -\frac{2t^2}{2} + l\text{nc}$$

$$e^{l\text{ns}} = e^{-t^2} + l\text{nc}$$

$$s = e^{-t^2} \cdot e^{l\text{nc}}$$

$$\boxed{s = ce^{-t^2}} \quad \text{———} \quad (1)$$

To find particular solution

Put $s = 4e, \quad t = 0$ in equation (1)

$$4e = ce^{o}$$

$$c = 4e$$

Put $c = 4e$ in equation (1)

$$s = 4e \cdot e^{-t^2}$$

$$\boxed{s = 4e^{1-t^2}} \quad \text{Ans.}$$

Q.22 In a culture bacteria increases at the rate proportional to the number of bacteria present. If bacteria are 200 initially and are doubled in 2 hours, find the number of bacteria present four hours later.

Solution:

Let P be the number of bacteria present at time t.

$$\frac{dp}{dt} = kP \qquad (k > 0)$$

Separate variable

$$\frac{dp}{p} = kdt$$

Integrate

$$\int \frac{dp}{p} = k\int dt$$

$$\ln p = kt + c_1 \quad \text{———} \quad (1)$$

Put $p = 200$, $t = 0$ in equation (1)

$$\ln 200 = 0 + c_1$$

$$c_1 = \ln 200$$

Put $c_1 = \ln 200$ in equation (1)

$$\ln p = kt + \ln 200 \quad \text{———} \quad (2)$$

Put $P = 400$, $t = 2$ hour in equation (2)

$$\ln 400 = 2k + \ln 200$$

$$\ln 400 - \ln 200 = 2k$$

$$k = \frac{1}{2}\ln\left(\frac{400}{200}\right)$$

$$k = \frac{1}{2}\ln 2$$

Put $k = \frac{1}{2}\ln 2$ in equation (2)

$$\ln P = \frac{1}{2}\ln 2t + \ln 200 \quad \text{———} \quad (3)$$

To find the number of bacteria presents four hour later

Put $t = 4$ hour in equation (3)

$$\ln P = \frac{1}{2}\ln 2 \times 4 + \ln 200$$

$$\ln P = 2\ln 2 + \ln 200$$

$$\ln P = \ln 2^2 + \ln 200$$

$$\ln P = \ln(4 \times 200)$$

$\boxed{P = 800}$ Ans.

Q.23 A ball is thrown vertically upward with a velocity of 2450 cm/sec. Neglecting air resistance, find
(i) velocity of ball at any time t. (ii) distance travelled in any time t.
(iii) maximum height attained by the ball.

Solution:

(i) Let 'v' be the velocity at any time 't' then by Newton's law of motion

$$\frac{dv}{dt} = -g$$

Separate variables

$$dv = -g\,dt$$

Integrate $\int dv = -g\int dt$

$$v = -gt + c_1 \quad \text{———} \quad (1)$$

Put $v = 2450$ cm/sec and $t = 0$ in eq. (1)

$$2450 = 0 + c_1$$

$$c_1 = 2450$$

Put $c_1 = 2450$ in equation (1)

$$v = -980t + 2450 \quad \text{———} \quad (2) \quad (\text{taking } g = 980)$$

(ii) Let 'h' be the height attained at any time 't'.

$$\frac{dh}{dt} = 2450 - 980\,t$$

Separate variables

$$dh = 2450\,dt - 980\,t\,dt$$

Integrate $\int dh = 2450\int dt - 980\int t dt$

$$h = 2450t - 980\frac{t^2}{2} + c_2$$

$$h = 2450\,t - 490\,t^2 + c_2 \quad \text{———} \quad (3)$$

Put $h = 0$, $t = 0$ in equation (3)

$$0 = 0 - 0 + c_2$$

$$c_2 = 0$$

Put $c_2 = 0$ in equation (3)

$$h = 2450t - 490t^2 \quad \text{———} \quad (4)$$

(iii) The maximum height attained by the ball when $v = 0$

$$-980\,t + 2450 = 0$$

$$980t = 2450$$

$$t = \frac{2450}{980} = 2.5 \text{ Sec}$$

Put $t = 2.5$ sec in equation (4)

$$h = 2450\,(2.5) - 490\,(2.5)^2$$

$$h = 6125 - 490\,(6.25) = 6125 - 3062.5$$

$$h = 3062.5 \text{ cm} \qquad \text{Ans.}$$