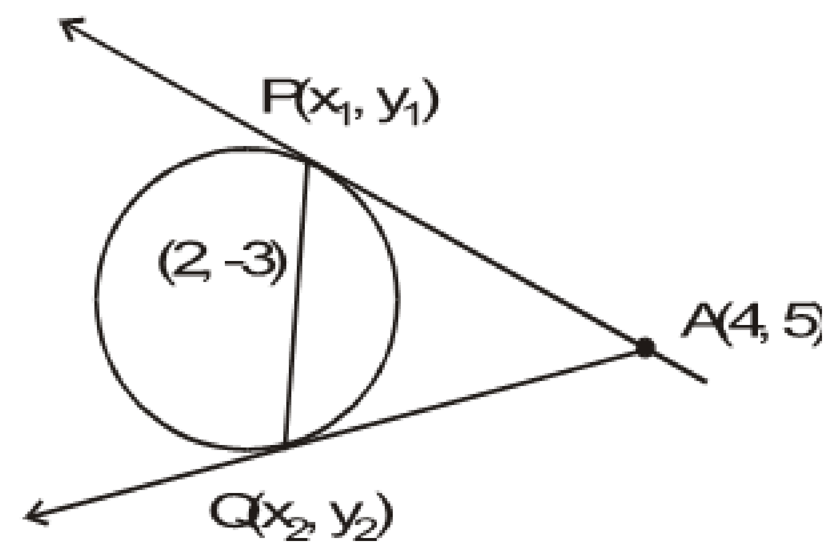
$$x^2 + y^2 - 4x + 6y + \frac{21}{2} = 0$$

Now Let points of contact of the two tangents be p (x, y_1) Q, x_2, y_2) An equation of the tangent at p(x_1, y_1) is



$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0 \quad \text{............ (1)}$$

Since $(-g, -f) = (2, -3)$

$g = -2 \qquad f = 3 \qquad$ Put in I

$$xx_1 + yy_1 - 2(x + x_1) + 3(y + y_1) + \frac{21}{2} = 0 \quad \text{............ (2)}$$

Since it passes through (4, 5)

$$4x_1 + 5y_1 - 2(4 + x_1) + 3(5 + y_1) + \frac{21}{2} = 0$$

$$4x_1 + 5y_1 - 8 - 2x_1 + 15 + 3y_1 + \frac{21}{2} = 0$$

$$2x_1 + 8y_1 + 7 + \frac{21}{2} = 0$$

$$4x_1 + 16y_1 + 14 + 21 = 0$$

$$4x_1 + 16y_1 + 35 = 0 \quad \text{................ (i)}$$

Similarly for point Q (x_2, y_2), we have

$$4x_2 + 16y_2 + 35 = 0 \quad \text{................ (ii)}$$

(i) & (ii) Show that both the points P(x_1, y_1) & Q (x_2, y_2) lie on $4x + 16y + 35 = 0$ and so it is the required equation of the chord of contact.

EXERCISE 6.3

Q.1: Prove that normal lines of a circle pass through the center of the circle. (Lahore Board 2009)

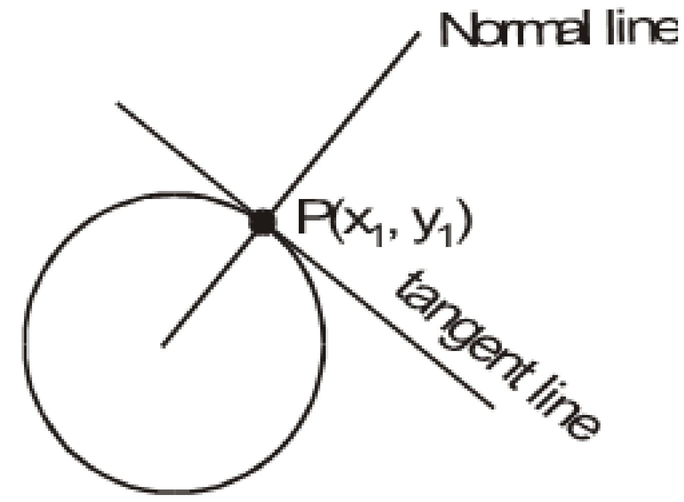
Solution:

Let us consider a circle with center (0, 0) and radius r.

Therefore equation of circle is

$x^2 + y^2 = r^2$

Diff. w.r.t. 'x'



$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$$

$$m = \frac{dy}{dx}\Big|(x_1, y_1) = \text{Slope of tangent} = -\frac{x_1}{y_1}$$

$$m_1 = \text{Slope of normal} = \frac{1}{\frac{-x_1}{y_1}} = \frac{y_1}{x_1} \quad (\text{– ve reciprocal})$$

Thus equation of the normal line passing through $P(x_1, y_1)$ is given by

$$y - y_1 = m_1 (x - x_1)$$

$$y - y_1 = \frac{y_1}{x_1}(x - x_1) \quad \text{.......... (2)}$$

$$x_1 y - x_1 y_1 = xy_1 - x_1 y_1$$

$$x_1 y = y_1 x$$

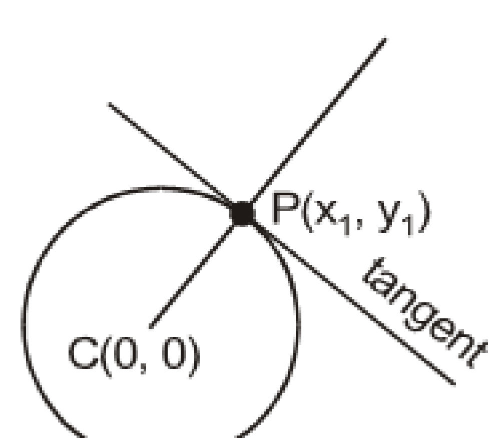
Clearly center of circle (0, 0) satisfy the above equation. Hence normal lines of circles passing through the center of the circle.

Q.2: Prove that the straight line drawn from the center of a circle perpendicular to a tangent passes through the point of tangency.

Solution:

Equation of circle with center (0, 0) & radius r is given by

$x^2 + y^2 = r^2$



Diff. w.r.t. 'x'

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$m = \text{Slope of tangent} = \frac{dy}{dx} = -\frac{x_1}{y_1}$$

Since the required line is perpendicular therefore $m_1 = \dfrac{1}{\frac{-x_1}{y_1}} = \dfrac{y_1}{x_1}$

The equation of the straight line perpendicular to the tangent through (0, 0).

$$y - y_1 = m'(x - x_1)$$

$$y - 0 = \frac{y_1}{x_1}(x - 0)$$

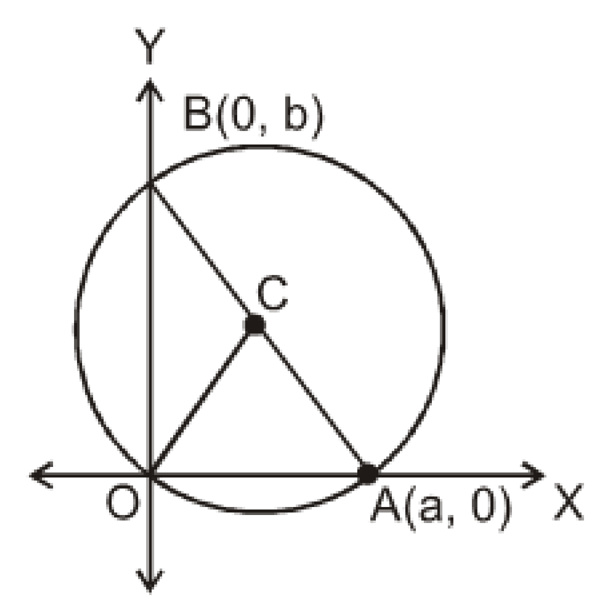
$$x_1 y = y_1 x$$

Thus the straight line drawn from the center and perpendicular to the tangent passes through the point of tangency.

Q.3: Prove that the mid point of the hypogenous of a right-angled triangle is the circum center of the triangle.

Solution:

Let OAB be a right angle triangle with $|OA| = a$ and $|OB| = b$.



Since 'c' be the mid point of $\overline{AB}$.

$\therefore$ By ratio formula coordinates of C are $\left(\frac{a}{2}, \frac{b}{2}\right)$

$$|CA| = \sqrt{\left(\frac{a}{2} - a\right)^2 + \left(\frac{b}{2} - 0\right)^2} = \sqrt{\left(\frac{-a}{2}\right)^2 + \left(\frac{b}{2}\right)^2} = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} \quad .... (1)$$

$$|CB| = \sqrt{\left(\frac{a}{2} - 0\right)^2 + \left(\frac{b}{2} - b\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{-b}{2}\right)^2} = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} \quad .... (2)$$

$$|CO| = \sqrt{\left(\frac{a}{2} - 0\right)^2 + \left(\frac{b}{2}\right)^2} = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} \quad .... (3)$$

From equations (1), (2) and (3)

$|CA| = |CB| = |CO|$

Shows the mid point of hypotenuse of a right triangle is the circum center of the triangle.

Q.4: Prove that the perpendicular dropped from a point of circle on a diameter is a mean proportional between the segments into which its divides the diameter.

Solution:

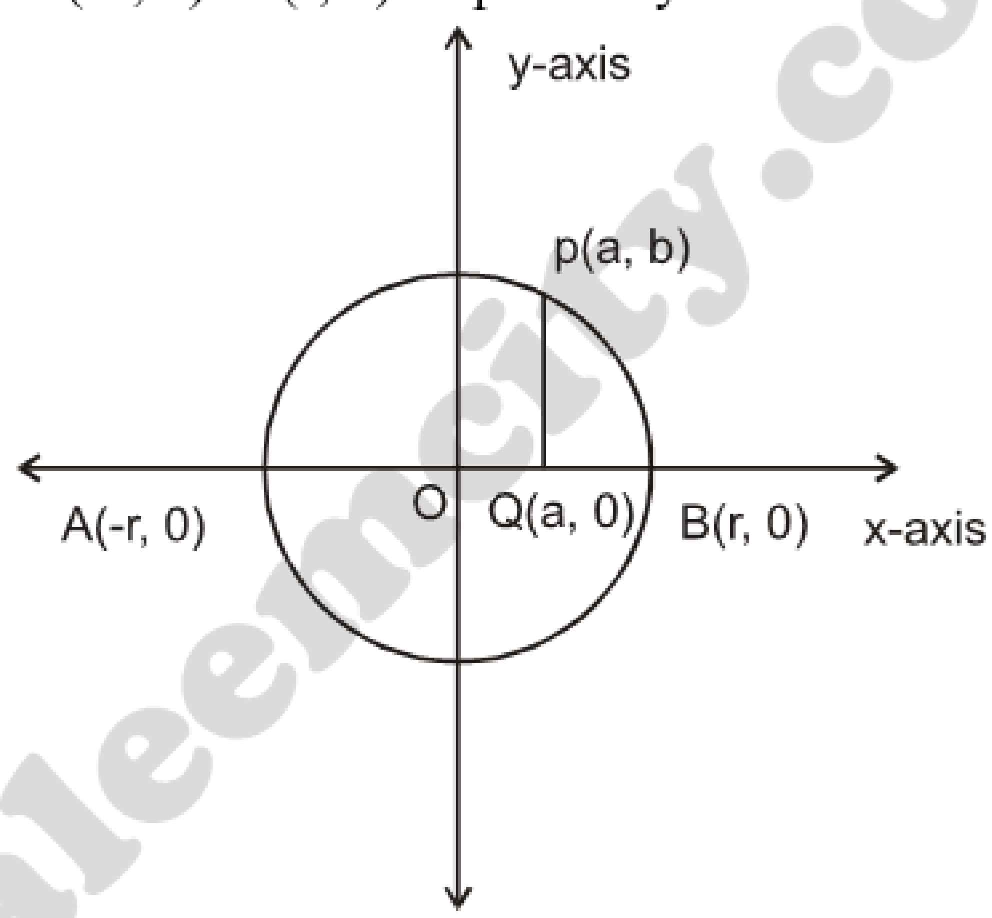
Let us consider a circle with center (0, 0) and radius r.

$\because$ Equation of circle is $x^2 + y^2 = r^2$ .... (1)

Let p (a, b) be any point on the circle then (1) becomes

$a^2 + b^2 = r^2$ .... (2)

From point P drop a perpendicular on the diameter AB at point Q. Therefore coordinates of A & B are (–r, 0) & (r, 0) respectively.



we known that definition of mean proportional.

x : M : : M : y

$xy = M^2 \rightarrow (3)$

$|PQ| = \sqrt{(a-a)^2 + (0-b)^2} = \sqrt{0 + b^2} = \sqrt{b^2} = b$

$|AQ| = \sqrt{[a-(-r)]^2 + (0-0)^2} = \sqrt{(a+r)^2} = \sqrt{a+r} = r + a$

$|QB| = \sqrt{(r-a)^2 + (0-0)^2} = \sqrt{r-a^2} = r - a$

(3) becomes

$|AQ|\,|QB| = |PQ|^2$

$(r + a)(r - a) = b^2$

$r^2 - a^2 = b^2$

$$b^2 = b^2$$

Hence proved.

Parabola: (Lahore Board 2009)

Let L be a fixed line in a plane and F be a fixed point not on the line L. Suppose |PM| denotes the distance of a point P(x, y) from the line L. The set of all points P in the plane such that

$$\frac{|PF|}{|PM|} = 1 \quad \text{is called a parabola}$$

Where fixed line L is directrix.

fixed point F is called Focus

STANDARD FORMS OF PARABOLA

(i) $y^2 = 4ax$

(ii) $y^2 = -4ax$

(iii) $x^2 = 4ay$

(iv) $x^2 = -4ay$

Axis of Parabola

The line through the focus and perpendicular to the directrix is called axis of Parobola.

Vertex

The point where the axis meets the parabola is called vertex.

Chord

A line joining two distinct points on a parabola is called chord of the parabola.

Focal Chord

A chord, which passes through focus is called focal chord.

Latusrectum

The focal chord perpendicular to the axis of the parabola is called latusrectum of the parabola.

Note:

In standard form vertex is at origin (0, 0).

If vertex is not at origin then equations of parabola become

(i) $(y-k)^2 = 4a(x-h)$

(ii) $(y-k)^2 = -4a(x-h)$

(iii) $(x-h)^2 = 4a(y-k)$

(iv) $(x-h)^2 = -4a(y-k)$