$$\Delta = \frac{1}{2} bc \sin\alpha = \frac{1}{2} ab \sin\gamma = \frac{1}{2} ac \sin\beta$$

Case II: When one side & two angles are given

$$\text{Area of triangle} = \frac{a^2 \sin\beta \sin\gamma}{2 \sin\alpha} = \frac{b^2 \sin\alpha \sin\gamma}{2 \sin\beta} = \frac{c^2 \sin\alpha \sin\beta}{2 \sin\gamma}$$

Case III: When three sides are given

Area of triangle by Hero's formula is

$$\Delta = \sqrt{S\ (S-a)\ (S-b)\ (S-c)}$$

where a, b, c sides of triangle ABC

and $S = \frac{a+b+c}{2}$

Exercise 12.7

Q.1 Find the area of triangle ABC, given two sides and their included angle

(i) $a = 200,\ b = 120,\ \gamma = 150^o$ **(Lahore Board 2006, Gujranwala Board 2007)**

(ii) $b = 37,\ C = 45°,\ \alpha = 30^o\ 50'$ **(Gujranwala Board 2005, Lahore Board 2008,2011)**

(iii) $a = 4.33,\ b = 9.25,\ \gamma = 56^o\ 44'$

Solution:

(i) $a = 200,\ b = 120,\ \gamma = 150^o$

$$\Delta = \frac{1}{2} ab \sin\gamma$$

$$= \frac{1}{2} (200) \times 120 \sin 150^o$$

$$= 12000 \times 0.50$$

$$= 6000 \text{ sq. units.}$$

(ii) $b = 37,\ C = 45°,\ \alpha = 30^o\ 50'$

$$\Delta = \frac{1}{2} bc \sin\alpha$$

$$= \frac{1}{2} (37) \times 45 \sin 30^o\ 50'$$

$$= 426.692 \text{ sq. units.}$$

(iii) $\mathbf{a = 4.33,\ b = 9.25,\ \gamma = 56^o\ 44'}$

$$\Delta = \frac{1}{2} ab \sin\gamma$$

$$= \frac{1}{2}(4.33) \times 9.25 \sin 56^o\ 44'$$

$$= 16.745 \text{ sq. units.}$$

Q.2 Find area of triangle ABC, given one side and two angles.

(i) $\mathbf{b = 25.4,\ \gamma = 36^o\ 41',\ \alpha = 45^o\ 17'}$

(ii) $\mathbf{c = 32,\ \alpha = 47^o\ 24',\ \beta = 70^o\ 16'}$

(iii) $\mathbf{a = 4.8,\ \gamma = 83^o\ 42',\ \gamma = 37^o\ 12'}$ **(Lahore Board 2010)**

Solution:

(i) $\mathbf{b = 25.4,\ \gamma = 36^o\ 41',\ \alpha = 45^o\ 17'}$

$$\because\ \alpha + \beta + \gamma = 180°$$

$$\beta = 180^o - 36^o\ 41' - 45^o\ 17'$$

$$\beta = 98^o\ 2'$$

We know that area of triangle

$$\Delta = \frac{1}{2} b^2 \frac{\sin\alpha \times \sin\gamma}{\sin\beta}$$

$$= \frac{1}{2}(25.4)^2 \frac{\sin 45^o\ 17' \sin 36^o\ 41'}{\sin 98^o\ 2'}$$

$$\Delta = 138.29 \text{ sq. units.}$$

(ii) $\mathbf{c = 32,\ \alpha = 47^o\ 24',\ \beta = 70^o\ 16'}$

$$\because\ \alpha + \beta + \gamma = 180°$$

$$\gamma = 180^o - \alpha - \beta$$

$$\gamma = 180^o - 47^o\ 24' - 70^o\ 16'$$

$$\gamma = 62^o\ 20'$$

$$\text{Area of triangle} = \frac{1}{2} c^2 \frac{\sin\alpha \times \sin\beta}{\sin\gamma}$$

$$\Delta = \frac{1}{2}(32)^2 \frac{\sin 47^o\ 24' \sin 70^o\ 16'}{\sin 62^o\ 20'}$$

$\Delta = 400.57$ sq. units.

(iii) **$a = 4.8,\ \gamma = 83^o\ 42',\ \gamma = 37^o\ 12'$**

$\because\ \alpha + \beta + \gamma = 180°$

$\beta = 180^o - \alpha - \gamma$

$= 180^o - 83^o\ 42' - 37^o\ 12'$

$\beta = 59^o\ 6'$

Area of triangle $= \frac{1}{2} a^2 \frac{\sin \beta \times \sin \gamma}{\sin \alpha}$

$\Delta = \frac{1}{2} (4.8)^2 \frac{\sin 59^o\ 6' \sin 37^o\ 12'}{\sin 83^o\ 42'}$

$\Delta = 6.02$ sq. units.

Q.3 Find the area of the triangle ABC, given three sides

(i) $a = 18,\ b = 24,\ c = 30$

(ii) $a = 524,\ b = 276,\ c = 315$

(iii) $a = 32.65,\ b = 42.81,\ c = 64.92$

Solution:

(i) **$a = 18,\ b = 24,\ c = 30$**

$S = \frac{a+b+c}{2} = \frac{18+24+30}{2} = 36$

$S - a = 36 - 18 = 18$

$S - b = 36 - 24 = 12$

$S - c = 36 - 30 = 6$

Required Area $= \sqrt{S\,(S-a)\,(S-b)\,(S-c)}$

$= \sqrt{36 \times 18 \times 12 \times 6} = 216$ sq. units.

(ii) **$a = 524,\ b = 276,\ c = 315$**

$S = \frac{a+b+c}{2} = \frac{524+276+315}{2} = 557.5$

$S - a = 557.5 - 524 = 33.5$

$S - b = 557.5 - 276 = 281.5$

$S - c = 557.5 - 315 = 242.5$

Required Area $= \sqrt{S\,(S-a)\,(S-b)\,(S-c)}$

$= \sqrt{557.5 \times 33.5 \times 281.5 \times 242.5} = 35705.89$ sq. units.

(iii) **a = 32.65, b = 42.81, c = 64.92**

$S = \frac{a+b+c}{2} = \frac{32.65+42.81+64.92}{2} = 70.19$

$S - a = 70.19 - 32.65 = 37.54$

$S - b = 70.19 - 42.81 = 27.38$

$S - c = 70.19 - 64.92 = 5.27$

Required Area $= \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{70.19 \times 37.54 \times 27.38 \times 5.27} = 616.60$ sq. units.

Q.4 **The area of triangle is 2437. If a = 79, c = 97. Find angle β.**

(Lahore Board 2005).

Solution:

$\Delta = 2437$, $a = 79$, $c = 97$, $\beta = ?$

area of triangle $= \frac{1}{2} ac \sin\beta$

$2437 = \frac{1}{2} \times 79 \times 97 \sin\beta$

$\beta = \sin^{-1} \frac{2437}{3831.5}$

$\beta = 39^o\ 30'$

Q.5 **The area of triangle is 121.34. If $\alpha = 32^o\ 15'$, $\beta = 65°\ 37'$ find c and γ.**

(Lahore Board 2004)

Solution:

$\because\ \alpha + \beta + \gamma = 180°$

$\gamma = 180^o - \alpha - \beta$

$= 180^o - 32^o\ 15' - 65^o\ 37'$

$\gamma = 82^o\ 8'$

Area of triangle $= \frac{1}{2} c^2 \frac{\sin\alpha \times \sin\beta}{\sin\gamma}$

$121.34 = \frac{1}{2} c^2 \frac{\sin 32^o\ 15' \sin 65^o\ 37'}{\sin 82^o\ 8'}$

$c^2 = \frac{240.3962}{0.4860}$

$c^2 = 494.64$

c = 22.24 sq. units.

Q.6 One side of a triangular garden is 30m. If its two corner angles are $22^{o}\frac{1}{2}$ and $112^{o}\frac{1}{2}$. Find the cost of planting the grass at the rate of Rs. 5 per square meter.

Solution:

a = 30

$\beta = 22^{o}\frac{1}{2} = 22.5° = 22^{o}\,30'$

$\gamma = 112^{o}\frac{1}{2} = 112^{o}30'$

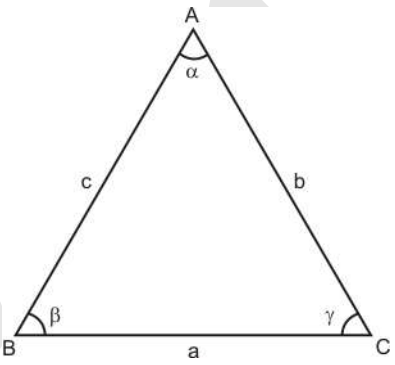
α = ?

$\because\ \alpha + \beta + \gamma = 180°$

$\alpha = 180^{o} - \beta - \gamma$

$= 180^{o} - 22^{o}\,30' - 112^{o}\,30'$

$\alpha = 45^{o}$



Area of triangle $= \frac{1}{2}a^2\frac{\sin\beta\sin\gamma}{\sin\alpha}$

$= \frac{1}{2}(30)^2\frac{\sin 22^{o}\,30'\sin 112^{o}\,30'}{\sin 45^{o}}$

Δ = 225 sq. m

Grass planting @ Rs. 5/sq. m = 225 x 5

= Rs. 1125 Ans.

EXERCISE 12.8

Q.1 Show that

(i) $r = 4R\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}$

(ii) $S = 4R\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$

Solution:

(i) $r = 4R\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}$

R.H.S. $= 4R\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}$

$= 4\frac{abc}{4\Delta}\sqrt{\frac{(S-b)(S-c)}{bc}}\sqrt{\frac{(S-a)(S-c)}{ac}}\sqrt{\frac{(S-a)(S-b)}{ab}}$