EXERCISE 13.2

Q.1 Prove that $\sin^{-1}\frac{15}{13} + \sin^{-1}\frac{7}{25} = \cos^{-1}\frac{253}{325}$

Solution:

$$\sin^{-1}\frac{15}{13} + \sin^{-1}\frac{7}{25} = \cos^{-1}\frac{253}{325}$$

$$\cos\left(\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25}\right) = \frac{253}{325} \qquad \ldots\ldots\ldots (1)$$

Let $\sin^{-1}\frac{5}{13} = \alpha, \quad \sin^{-1}\frac{17}{25} = \beta$

Equation (1) becomes

$$\cos(\alpha+\beta) = \frac{253}{325}$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \frac{253}{325} \qquad \ldots\ldots\ldots (2)$$

$$\sin\alpha = \frac{5}{13}$$

$$\cos\alpha = \sqrt{1-\sin^2\alpha}$$

$$\cos\alpha = \sqrt{1-\frac{25}{169}}$$

$$\cos\alpha = \sqrt{\frac{169-25}{169}}$$

$$\cos\alpha = \sqrt{\frac{144}{169}}$$

$$\cos\alpha = \frac{12}{13}$$

$$\sin\beta = \frac{7}{25}$$

$$\cos\beta = \sqrt{1-\sin^2\beta}$$

$$\cos\beta = \sqrt{1-\frac{49}{625}}$$

$$\cos\beta = \sqrt{\frac{625-49}{625}}$$

$$\cos\beta = \sqrt{\frac{576}{625}}$$

$$\cos\beta = \frac{24}{25}$$

Substitute values in equation (2)

$$\cos(\alpha+\beta) = \frac{12}{13}\times\frac{24}{25} - \frac{5}{13}\times\frac{7}{25}$$

$$= \frac{288}{325} - \frac{35}{325} = \frac{288-35}{325}$$

$$\cos(\alpha+\beta) = \frac{253}{325}$$

Hence proved.

Q.2 $\tan^{-1}\dfrac{1}{4}+\tan^{-1}\dfrac{1}{5} = \tan^{-1}\dfrac{9}{19}$ **(Gujranwala Board 2007)**

$$\boxed{\textbf{Formula}\quad \tan^{-1}A+\tan^{-1}B = \tan^{-1}\frac{A+B}{1-AB}}$$

Solution:

$$\text{L.H.S.} = \tan^{-1}\frac{1}{4}+\tan^{-1}\frac{1}{5}$$

$$= \tan^{-1}\left(\frac{\frac{1}{4}+\frac{1}{5}}{1-\frac{1}{4}\,\frac{1}{5}}\right) = \tan^{-1}\left(\frac{\frac{5+4}{26}}{\frac{20-1}{20}}\right)$$

$$= \tan^{-1}\frac{9}{19} = \text{R.H.S.}\quad \text{Hence proved}$$

Q.3 $2\tan^{-1}\dfrac{2}{3} = \sin^{-1}\dfrac{12}{13}$

Solution:

$$2\tan^{-1}\frac{2}{3} = \sin^{-1}\frac{12}{13}$$

$$\sin\left(2\tan^{-1}\frac{2}{3}\right) = \frac{12}{13} \qquad \ldots\ldots\ldots (1)$$

Let $\tan^{-1}\dfrac{2}{3} = \theta$

Equation (1) becomes

$$\sin 2\theta = \frac{12}{13} \qquad \ldots\ldots\ldots (2)$$

$$\tan\theta = \frac{2}{3}$$

$$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\frac{4}{9}} = \sqrt{\frac{9+4}{9}} = \frac{\sqrt{13}}{3}$$

$$\cos\theta = \frac{3}{\sqrt{13}}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{9}{13}} = \sqrt{\frac{13-9}{13}} = \sqrt{\frac{4}{13}}$$

$$\sin\theta = \frac{2}{\sqrt{13}}$$

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\times\frac{2}{\sqrt{13}}\times\frac{3}{\sqrt{13}} = \frac{12}{13}\quad \text{Hence proved}$$

Q.4 Prove that $\tan^{-1}\frac{120}{119} = 2\cos^{-1}\frac{12}{13}$

Solution:

$$2\cos^{-1}\frac{12}{13} = \tan^{-1}\frac{120}{119}$$

$$\tan\left(2\cos^{-1}\frac{12}{13}\right) = \frac{120}{119} \qquad \text{.........} \ (1)$$

Let $\cos^{-1}\frac{12}{13} = \theta \quad \Rightarrow \quad \cos\theta = \frac{12}{13}$

Equation (1) becomes

$$\tan 2\theta = \frac{120}{119}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{120}{119} \qquad \text{.........} \ (2)$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{114}{169}} = \sqrt{\frac{169-144}{169}} = \sqrt{\frac{25}{169}}$$

$$\sin\theta = \frac{5}{13}$$

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\times\frac{5}{13}\times\frac{12}{13} = \frac{120}{169}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{144}{169} - \frac{25}{169} = \frac{144-25}{169} = \frac{119}{169}$$

$$\tan 2\theta = \frac{\frac{120}{169}}{\frac{119}{169}}$$

$$\tan 2\theta = \frac{120}{119} \quad \text{Hence proved.}$$

Q.5 $\sin^{-1}\dfrac{1}{\sqrt{5}} + \cot^{-1} 3 = \dfrac{\pi}{4}$

Solution:

Let $\sin^{-1}\dfrac{1}{\sqrt{5}} = \alpha$, $\cot^{-1} 3 = \beta$

$\sin\alpha = \dfrac{1}{\sqrt{5}}$ $\cot\beta = 3$

$\cos\alpha = \sqrt{1-\sin^2\alpha}$ $\Rightarrow \tan\beta = \dfrac{1}{3}$

$\cos\alpha = \sqrt{1-\dfrac{1}{5}} = \sqrt{\dfrac{5-1}{5}} = \sqrt{\dfrac{4}{5}}$ $\sec\beta = \sqrt{1+\tan^2\beta}$

$\cos\alpha = \dfrac{2}{\sqrt{5}}$ $\sec\beta = \sqrt{1+\dfrac{1}{9}} = \sqrt{\dfrac{10}{9}} = \dfrac{\sqrt{10}}{3}$

$\cos\beta = \dfrac{3}{\sqrt{10}} \Rightarrow \sin\beta = \sqrt{1-\cos^2\beta} = \sqrt{1-\dfrac{9}{10}} = \sqrt{\dfrac{10-9}{10}} = \dfrac{1}{\sqrt{10}}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha+\beta) = \dfrac{1}{\sqrt{5}}\times\dfrac{3}{\sqrt{10}} + \dfrac{2}{\sqrt{5}}\times\dfrac{1}{\sqrt{10}} = \dfrac{3}{\sqrt{50}} + \dfrac{2}{\sqrt{50}} = \dfrac{5}{\sqrt{50}}$

$\sin(\alpha+\beta) = \dfrac{5}{5\sqrt{2}}$

$\alpha+\beta = \sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$

$\sin^{-1}\dfrac{1}{\sqrt{5}} + \cot^{-1} 3 = \dfrac{\pi}{4}$ Hence proved

Q.6 $\sin^{-1}\dfrac{3}{5} + \sin^{-1}\dfrac{8}{17} = \sin^{-1}\dfrac{77}{85}$ **(Lahore Board 2006, Gujranwala Board 2007)**

Solution: $\boxed{\textbf{Formula } \sin^{-1}A + \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} + B\sqrt{1-A^2}\right)}$

$$\text{L.H.S.} = \sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17}$$

$$= \sin^{-1}\left(\frac{3}{5}\sqrt{1-\frac{64}{289}} + \frac{8}{17}\sqrt{1-\frac{9}{25}}\right)$$

$$= \sin^{-1}\left(\frac{3}{5}\sqrt{\frac{289-64}{289}}+\frac{8}{17}\sqrt{\frac{25-9}{25}}\right)$$

$$= \sin^{-1}\left(\frac{3}{5}\sqrt{\frac{225}{289}}+\frac{8}{17}\sqrt{\frac{16}{25}}\right)$$

$$= \sin^{-1}\left(\frac{3}{5}\times\frac{15}{17}+\frac{8}{17}\times\frac{4}{5}\right)$$

$$= \sin^{-1}\left(\frac{45}{85}+\frac{32}{85}\right) = \sin^{-1}\frac{77}{85}$$

= R.H.S. Hence proved.

Q.7 $\sin^{-1}\frac{77}{85}-\sin^{-1}\frac{3}{5} = \cos^{-1}\frac{15}{17}$ **(Lahore Board 2009, 2010)**

Solution:

$$\sin^{-1}\frac{77}{85}-\sin^{-1}\frac{3}{5} = \cos^{-1}\frac{15}{17}$$

$$\cos\left(\sin^{-1}\frac{77}{85}-\sin^{-1}\frac{3}{5}\right) = \frac{15}{17} \quad \text{.........} (1)$$

Let $\sin^{-1}\frac{77}{85} = \alpha$

$$\sin\alpha = \frac{77}{85}$$

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{5929}{7225}} = \sqrt{\frac{7225-5929}{7225}} = \sqrt{\frac{1296}{7225}} = \frac{36}{85}$$

$\sin^{-1}\frac{3}{5} = \beta$

$$\sin\beta = \frac{3}{5}$$

$$\cos\beta = \sqrt{1-\sin^2\beta} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{25-9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$= \frac{36}{85}\times\frac{4}{5}+\frac{77}{85}\times\frac{3}{5}$$

$$= \frac{144}{425} + \frac{231}{425} = \frac{144+231}{425} = \frac{375}{425} = \frac{15}{17}$$

$$\sin^{-1}\frac{77}{85} - \sin^{-1}\frac{3}{5} = \cos^{-1}\left(\frac{15}{17}\right) \text{ Hence proved.}$$

Q.8 $\mathbf{\cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5} = \sin^{-1}\frac{3}{5}}$

Solution:

$$\cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5} = \sin^{-1}\frac{3}{5}$$

$$\therefore \quad 2\tan^{-1}\frac{1}{5} = \tan^{-1}\frac{2\left(\frac{1}{5}\right)}{1-\frac{1}{25}} = \tan^{-1}\frac{\frac{2}{5}}{\frac{24}{25}}$$

$$2\tan^{-1}\frac{1}{5} = \tan^{-1}\frac{5}{12}$$

Given equation becomes

$$\cos^{-1}\frac{63}{65} + \tan^{-1}\frac{5}{12} = \sin^{-1}\frac{3}{5}$$

$$\Rightarrow \quad \sin\left(\cos^{-1}\frac{63}{65} + \tan^{-1}\frac{5}{12}\right) = \frac{3}{5} \quad \text{......... (1)}$$

Let $\cos^{-1}\frac{63}{65} = \alpha$ $\qquad \tan^{-1}\frac{5}{12} = \beta$

Equation (1):

$$\cos\alpha = \frac{63}{65}\ , \ \tan\beta = \frac{5}{12}$$

$$\sin(\alpha+\beta) = \frac{3}{5} \quad \text{......... (2)}$$

$$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{3969}{4225}}$$

$$= \sqrt{\frac{4225-3969}{4225}} = \sqrt{\frac{256}{4225}}$$

$$\sin\alpha = \frac{16}{65}$$

$$\sec\beta = \sqrt{1+\tan^2\beta} = \sqrt{1+\frac{25}{144}}$$

$$\sec\beta = \sqrt{\frac{144+25}{144}} = \sqrt{\frac{169}{144}} = \frac{13}{12}$$

$$\cos\beta = \frac{12}{13}$$

$$\sin\beta = \sqrt{1-\cos^2\beta}$$

$$= \sqrt{1-\frac{144}{169}} = \sqrt{\frac{169-144}{169}}$$

$$\sin\beta = \sqrt{\frac{25}{169}}$$

$$\sin\beta = \frac{5}{13}$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$= \frac{16}{65}\times\frac{12}{13}+\frac{63}{65}\times\frac{5}{13}$$
$$= \frac{192}{845}+\frac{315}{845} = \frac{507}{845} = \frac{3}{5}$$
$$\alpha+\beta = \sin^{-1}\left(\frac{3}{5}\right)$$
$$\cos^{-1}\frac{63}{65}+\tan^{-1}\frac{5}{12} = \sin^{-1}\frac{3}{5} \quad \text{Hence proved.}$$

Q.9 $\mathbf{\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{5}-\tan^{-1}\frac{8}{19} = \frac{\pi}{4}}$

Solution:

$$\text{L.H.S.} = \tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{5}-\tan^{-1}\frac{8}{19}$$
$$= \tan^{-1}\left[\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{3}{4}\times\frac{3}{5}}\right]-\tan^{-1}\frac{8}{19}$$
$$= \tan^{-1}\frac{\frac{15+12}{20}}{\frac{20-9}{20}}-\tan^{-1}\frac{8}{19}$$
$$= \tan^{-1}\frac{27}{11}-\tan^{-1}\frac{8}{19}$$
$$= \tan^{-1}\left[\frac{\frac{27}{11}-\frac{8}{19}}{1+\frac{27}{11}\times\frac{8}{19}}\right]$$
$$= \tan^{-1}\left[\frac{\frac{513-88}{11\times19}}{\frac{209+216}{11\times19}}\right]$$
$$= \tan^{-1}\frac{425}{425}$$
$$= \tan^{-1}(1) = \frac{\pi}{4}$$
$$= \text{R.H.S. Hence proved.}$$

Q.10 $\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{65}=\frac{\pi}{2}$

Solution:

$$\begin{aligned}
\text{L.H.S.} &= \sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{65}\\
&= \sin^{-1}\left[\frac{5}{13}\sqrt{1-\frac{16}{25}}+\frac{4}{5}\sqrt{1-\frac{25}{169}}\right]+\sin^{-1}\frac{16}{65}\\
&= \sin^{-1}\left[\frac{5}{13}\sqrt{\frac{25-16}{25}}+\frac{4}{5}\sqrt{\frac{169-25}{169}}\right]+\sin^{-1}\frac{16}{65}\\
&= \sin^{-1}\left[\frac{5}{13}\times\frac{3}{5}+\frac{4}{5}\times\frac{12}{13}\right]+\sin^{-1}\frac{16}{65}\\
&= \sin^{-1}\left[\frac{3}{13}+\frac{48}{65}\right]+\sin^{-1}\frac{16}{65}\\
&= \sin^{-1}\frac{63}{65}+\sin^{-1}\frac{16}{65}\\
&= \sin^{-1}\left[\frac{16}{65}\sqrt{1-\frac{3969}{4225}}+\frac{63}{65}\sqrt{1-\frac{256}{4225}}\right]\\
&= \sin^{-1}\left[\frac{16}{65}\times\frac{16}{65}+\frac{63}{65}\times\frac{63}{65}\right]\\
&= \sin^{-1}\left[\frac{256}{4225}+\frac{3969}{4225}\right]\\
&= \sin^{-1}\left[\frac{256+3969}{4225}\right]\\
&= \sin^{-1}\left(\frac{4225}{4225}\right)\\
&= \sin^{-1}(1)\\
&= \frac{\pi}{2} = \text{R.H.S. Hence proved.}
\end{aligned}$$

Q.11 $\tan^{-1}\frac{1}{11}+\tan^{-1}\frac{5}{6}=\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}$ **(Lahore Board 2011)**

Solution:

$$\tan^{-1}\frac{1}{11}+\tan^{-1}\frac{5}{6} = \tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}$$

$$\tan^{-1}\left(\frac{\frac{1}{11}+\frac{5}{6}}{1-\frac{1}{11}\times\frac{5}{6}}\right) = \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\times\frac{1}{2}}\right)$$

$$\tan^{-1}\left[\frac{\frac{6+55}{66}}{\frac{66-5}{66}}\right] = \tan^{-1}\left[\frac{\frac{2+3}{6}}{\frac{6-1}{6}}\right]$$

$$\tan^{-1}\left(\frac{61}{61}\right) = \tan^{-1}\left(\frac{5}{5}\right)$$

$$\tan^{-1}(1) = \tan^{-1}(1)$$

$$\frac{\pi}{4} = \frac{\pi}{4} \quad \text{Hence proved.}$$

Q.12 $\mathbf{2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7} = \frac{\pi}{4}}$

(Gujranwala Board 2005, 2006) (Lahore Board 2006, 2007, 2008)

Solution:

$$\text{L.H.S.} = 2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{\frac{2}{3}}{1-\frac{1}{9}}+\tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right)$$

$$= \tan^{-1}\left[\frac{\frac{21+4}{28}}{\frac{28-3}{28}}\right]$$

$$= \tan^{-1}\frac{25}{25} = \tan^{-1}(1) = \frac{\pi}{4}$$

$$= \text{R.H.S.} \quad \text{Hence proved.}$$

Q.13 Show that $\cos(\sin^{-1} x) = \sqrt{1-x^2}$ (Gujranwala Board 2007)

Solution:

$$\text{L.H.S.} = \cos(\sin^{-1} x)$$

Let $\sin^{-1} x = \alpha \quad \Rightarrow \quad \sin\alpha = x \qquad \alpha \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-x^2}$$

$$\cos(\sin^{-1} x) = \sqrt{1-x^2} = \text{R.H.S.}$$

Hence proved.

Q.14 Show that $\sin(2\cos^{-1} x) = 2x\sqrt{1-x^2}$

Solution:

$$\text{L.H.S.} = \sin(2\cos^{-1} x) \qquad \text{.........} (1)$$

Let $\cos^{-1} x = \alpha \quad \Rightarrow \quad x = \cos\alpha \qquad \alpha \in [0, \pi]$

Equation (1) becomes

$$\sin 2\alpha = 2\sin\alpha\cos\alpha \qquad \text{.........} (2)$$

$$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-x^2}$$

Put values in equation (2)

$$\sin 2\alpha = 2x\sqrt{1-x^2}$$

$\sin 2(\cos^{-1} x) = 2x\sqrt{1-x^2}$ R.H.S. Hence proved.

Q.15 Show that $\cos(2\sin^{-1} x) = 1-2x^2$

Solution:

$$\text{L.H.S.} = \cos(2\sin^{-1} x) \qquad \text{.........} (1)$$

Let $\sin^{-1} x = \alpha \quad \Rightarrow \quad \sin\alpha = x, \qquad \alpha \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Equation (1) becomes

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha \qquad \text{.........} (2)$$

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-x^2}$$

Put in equation (2)

$$\cos 2\alpha = \left(\sqrt{1-x^2}\right)^2 - x^2$$

$$= 1 - x^2 - x^2$$

$$\cos 2(\sin^{-1} x) = 1 - 2x^2$$

$= \text{R.H.S.}$ Hence proved.

Q.16 Show that $\tan^{-1}(-x) = -\tan^{-1} x$

Solution:

$$\tan^{-1}(-x) + \tan^{-1} x = 0$$

$$\text{L.H.S.} = \tan^{-1}\left[\frac{-x+x}{1-(-x)(x)}\right]$$

$$= \tan^{-1}\left[\frac{0}{1+x^2}\right]$$

$$= \tan^{-1} 0$$

$$= 0 = \text{R.H.S. Hence proved}$$

Q.17 Show that $\sin^{-1}(-x) = -\sin^{-1} x$

Solution:

$$\sin^{-1}(-x) + \sin^{-1} x = 0$$

$$\text{L.H.S.} = \sin^{-1}(-x) + \sin^{-1} x$$

$$= \sin^{-1}\left[(-x)\sqrt{1-x^2} + x\sqrt{1-x^2}\right]$$

$$= \sin^{-1}(0)$$

$$= 0 = \text{R.H.S. Hence proved.}$$

Q.18 Show that $\cos^{-1}(-x) = \pi - \cos^{-1} x$

Solution:

$$\cos^{-1}(-x) + \cos^{-1} x = \pi$$

$$\text{L.H.S.} = \cos^{-1}(-x) + \cos^{-1} x$$

$$\boxed{\text{Formula } \cos^{-1} A + \cos^{-1} B = \cos^{-1}\left[AB - \sqrt{(1-A^2)(1-B^2)}\right]}$$

$$\cos^{-1}(-x) + \cos^{-1} x = \cos^{-1}\left[-x \times x - \sqrt{(1-x^2)(1-x^2)}\right]$$

$$= \cos^{-1}\left[-x^2 - \sqrt{(1-x^2)^2}\right]$$

$$= \cos^{-1}(-1)$$

$$= \pi = \text{R.H.S. Hence proved.}$$

Q.19 Show that $\tan(\sin^{-1} x) = \dfrac{x}{\sqrt{1-x^2}}$ (Lahore Board 2008)

Solution:

$$\tan(\sin^{-1} x) \qquad \ldots\ldots\ldots (1)$$

Let $\sin^{-1} x = \alpha$

Equation (1) becomes

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \qquad \ldots\ldots\ldots (2)$$

$$x = \sin\alpha$$

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-x^2}$$

Put in equation (2)

$$\tan\alpha = \frac{x}{\sqrt{1-x^2}}$$

$$\tan(\sin^{-1}x) = \frac{x}{\sqrt{1-x^2}} \quad \text{Hence proved.}$$

Q.20 Given that $x = \sin^{-1}\frac{1}{2}$, find the values of following trigonometric functions sin x, cos x, tan x, cosec x, sec x, cot x.

Solution:

$$x = \sin^{-1}\frac{1}{2}$$

$$\Rightarrow \quad \sin x = \frac{1}{2}$$

$$\Rightarrow \quad \cos x = \sqrt{1-\sin^2 x} = \sqrt{1-\frac{1}{4}} = \sqrt{\frac{4-1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$\cot x = \sqrt{3}$$

$$\sec x = \frac{1}{\cos x} = \frac{2}{\sqrt{3}}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = 2 \quad \text{Ans.}$$