SOLUTION OF EQUATIONS REDUCIBLE TO THE QUADRATIC EQUATION

There are certain types of equations, which do not look to be of degree 2, but they can be reduced to the quadratic form.

Type I

The equations of the form $ax^{2n} + bx^n + c = 0$; $a \neq 0$. Put $x^n = y$ and get the given equation reduced to quadratic equation in y.

Type II

The equations of the form $(x + a)(x + b)(x + c)(x + d) = K$ where $a + b = c + d$.

Type III: EXPONENTIAL EQUATIONS

Equations in which variable occurs in exponent are called exponential equations. For example $2^{2x} - 3.2^{x+2} + 32 = 0$.

Type IV: RECIPROCAL EQUATIONS

An equation, which remains unchanged when x is replaced by $\frac{1}{x}$, is called a reciprocal equation. For example $x^4 - 3x^3 + 4x^2 - 3x + 1 = 0$.

EXERCISE 4.2

Solve the following equations:

Q.1 $\mathbf{x^4 - 6x^2 + 8 = 0.}$

Solution:

$x^4 - 6x^2 + 8 = 0$

Put $x^2 = y$ .... (1)

$y^2 - 6y + 8 = 0$

$\Rightarrow y^2 - 4y - 2y + 8 = 0$

$\Rightarrow y(y - 4) - 2(y - 4) = 0$

$\Rightarrow (y - 4)(y - 2) = 0$

$\Rightarrow$ Either $y - 4 = 0$ or $y - 2 = 0$

$\Rightarrow y = 4$ or $y = 2$

When $y = 2$ equation (1) becomes

$x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

when $y = 4$ equation (1) becomes

$x^2 = 4 \Rightarrow x = \pm 2$

Hence the solution set $= \{\pm\sqrt{2}, \pm 2\}$

Q.2 $\mathbf{x^{-2} - 10 = 3x^{-1}}$. **(Gujranwala Board 2007, Lahore Board 2010)**

Solution:

$$x^{-2} - 10 = 3x^{-1}$$
$$\Rightarrow \quad x^{-2} - 3x^{-1} - 10 = 0$$
$$\Rightarrow \quad (x^{-1})^2 - 3x^{-1} - 10 = 0$$
$$\text{Put } x^{-1} = y \quad \ldots\ldots\ldots (1)$$
$$\Rightarrow \quad y^2 - 3y - 10 = 0$$
$$\Rightarrow \quad y^2 - 5y + 2y - 10 = 0$$
$$\Rightarrow \quad y(y-5) + 2(y-5) = 0$$
$$\Rightarrow \quad (y-5)(y+2) = 0$$
$$\Rightarrow \quad \text{Either } y - 5 = 0 \quad \text{or} \quad y + 2 = 0$$
$$\Rightarrow \quad y = 5, \quad \text{or} \quad y = -2$$

when $y = 5$ equation (1) becomes

$$x^{-1} = 5$$
$$\Rightarrow \quad \frac{1}{x} = 5$$
$$\Rightarrow \quad x = \frac{1}{5}$$

when $y = -2$ equation (1) becomes

$$x^{-1} = -2 \quad \Rightarrow \quad \frac{1}{x} = -2 \Rightarrow x = -\frac{1}{2}$$

$$\text{Hence the solution set } = \left\{-\frac{1}{2}, \frac{1}{5}\right\}$$

Q.3 $\mathbf{x^6 - 9x^3 + 8 = 0}$ **(Gujranwala Board 2007)**

Solution:

$$x^6 - 9x^3 + 8 = 0$$
$$\Rightarrow \quad (x^3)^2 - 9x^3 + 8 = 0$$
$$\text{Put } x^3 = y \quad \ldots\ldots\ldots (1)$$
$$\Rightarrow \quad y^2 - 9y + 8 = 0$$
$$\Rightarrow \quad y^2 - 8y - y + 8 = 0$$
$$\Rightarrow \quad y(y-8) - 1(y-8) = 0$$

$\Rightarrow \quad (y-8)(y-1) = 0$

$\Rightarrow \quad \text{Either} \quad y-8 = 0 \quad \text{or} \quad y-1 = 0$

$\Rightarrow \quad y = 8 \quad \text{or} \quad y = 1$

when $y = 8$, equation (1) becomes

$x^3 = 8$

$\Rightarrow \quad x^3 - 8 = 0 \quad \Rightarrow \quad (x)^3 - (2)^3 = 0$

$\Rightarrow \quad (x-2)(x^2 + 2x + 4) = 0$

$\Rightarrow \quad \text{Either} \quad x - 2 = 0 \quad \text{or} \quad x^2 + 2x + 4 = 0$

$$\Rightarrow \quad x = 2 \quad \text{or} \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4-16}}{2}$$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= -1 \pm \sqrt{-3}$$

when $y = 1$ equation (1) becomes

$x^3 = 1 \quad \Rightarrow \quad (x)^3 - (1)^3 = 0$

$\Rightarrow \quad (x-1)(x^2 + x + 1) = 0$

$\Rightarrow \quad \text{Either} \quad x - 1 = 0 \quad \text{or} \quad x^2 + x + 1 = 0$

$$\Rightarrow \quad x = 1 \quad \text{or} \quad x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

Hence the solution set $= \left\{ 1, 2, \dfrac{-1 \pm \sqrt{-3}}{2}, -1 \pm \sqrt{-3} \right\}$

Q.4 $\quad \mathbf{8x^6 - 19x^3 - 27 = 0}$

Solution:

$8x^6 - 19x^3 - 27 = 0$

$\Rightarrow \quad 8(x^3)^2 - 19x^3 - 27 = 0$

Put $x^3 = y \quad \ldots\ldots\ldots (1)$

$\Rightarrow \quad 8y^2 - 19y - 27 = 0$

$\Rightarrow \quad 8y^2 - 27y + 8y - 27 = 0$

$\Rightarrow \quad y(8y - 27) + 1(8y - 27) = 0$

$\Rightarrow \quad (8y - 27)(y + 1) = 0$

$\Rightarrow \quad$ Either $8y - 27 = 0$ or $y + 1 = 0$

$\Rightarrow \quad y = \dfrac{27}{8}$ or $y = -1$

when $y = \dfrac{27}{8}$ equation (1) becomes

$$x^3 = \frac{27}{8} \Rightarrow 8x^3 = 27 \Rightarrow 8x^3 - 27 = 0$$

$\Rightarrow \quad (2x)^3 - (3)^3 = 0$

$\Rightarrow \quad (2x - 3)(4x^2 + 6x + 9) = 0$

$\Rightarrow \quad$ Either $2x - 3 = 0$ or $4x^2 + 6x + 9 = 0$

$\Rightarrow \quad 2x = 3$ or $x = \dfrac{-6 \pm \sqrt{(6)^2 - 4(4)(9)}}{2(4)}$

$\Rightarrow \quad x = \dfrac{3}{2}$ $\quad = \dfrac{-6 \pm \sqrt{36 - 144}}{8}$

$$= \frac{-6 \pm \sqrt{-108}}{8}$$

$$= \frac{-6 \pm 6\sqrt{-3}}{8}$$

$$= \frac{-3 \pm 3\sqrt{-3}}{4}$$

when $y = -1$ equation (1) becomes

$x^3 = -1$

$\Rightarrow \quad x^3 + 1 = 0$

$\Rightarrow \quad (x + 1)(x^2 - x + 1) = 0$

$\Rightarrow \quad$ Either $x + 1 = 0$ or $x^2 - x + 1 = 0$

$\Rightarrow \quad x = -1$ or $x = \dfrac{1 \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$

$\Rightarrow \quad = \dfrac{1 \pm \sqrt{1 - 4}}{2} = \dfrac{1 \pm \sqrt{-3}}{2}$

Hence the solution set $= \left\{ -1, \dfrac{3}{2}, \dfrac{1 \pm \sqrt{-3}}{2}, \dfrac{-3 \pm 3\sqrt{-3}}{4} \right\}$

Q.5 $\mathbf{x^{2/5} + 8 = 6x^{1/5}}$

Solution:

$x^{2/5} + 8 = 6x^{1/5}$

$x^{2/5} - 6x^{1/5} + 8 = 0$

$(x^{1/5})^2 - 6x^{1/5} + 8 = 0$

Put $x^{1/5} = y$ .... (1)

$\Rightarrow$ $y^2 - 6y + 8 = 0$

$\Rightarrow$ $y^2 - 4y - 2y + 8 = 0$

$\Rightarrow$ $y(y-4) - 2(y-4) = 0$

$\Rightarrow$ $(y-4)(y-2) = 0$

$\Rightarrow$ Either $y - 4 = 0$ or $y - 2 = 0$

$\Rightarrow$ $y = 4$ or $y = 2$

when $y = 4$ equation (1) becomes

$x^{1/5} = 4 \Rightarrow x = (4)^5 \Rightarrow x = 1024$

when $y = 2$ equation (1) becomes

$x^{1/5} = 2 \Rightarrow x = 2^5 \Rightarrow x = 32$

Hence the solution set $= \{32, 1024\}$

Q.6 $\mathbf{(x+1)(x+2)(x+3)(x+4) = 24}$

Solution:

$(x+1)(x+2)(x+3)(x+4) = 24$

As $1 + 4 = 2 + 3$

$\Rightarrow$ we combine

$[(x+1)(x+4)][(x+2)(x+3)] = 24$

$\Rightarrow$ $[x^2 + 4x + x + 4)][(x^2 + 3x + 2x + 6)] = 24$

$\Rightarrow$ $[x^2 + 5x + 4)][(x^2 + 5x + 6)] = 24$

Put $x^2 + 5x = y$ .... (1)

$\Rightarrow$ $[y + 4]\,[y + 6] = 24$

$\Rightarrow$ $y^2 + 6y + 4y = 24 = 24$

$\Rightarrow$ $y^2 + 10y = 0$

$\Rightarrow$ $y\,(y + 10) = 0$

$\Rightarrow$ Either $y = 0$ or $y + 10 = 0$

$\Rightarrow$ or $y = -10$

Put $y = 0$ in equation (1)

$\Rightarrow$ $x^2 + 5x = 0$

$\Rightarrow$ $x\,(x + 5) = 0$

$\Rightarrow$ Either $x = 0$ or $x + 5 = 0$

$\Rightarrow$ or $x = -5$

Put $y = -10$ in equation (1)

$\Rightarrow$ $x^2 + 5x = -10$

$\Rightarrow$ $x^2 + 5x + 10 = 0$

$$\Rightarrow \quad x = \frac{-5 \pm \sqrt{(5)^2 - 4\,(1)\,(10)}}{2\,(1)} = \frac{-5 \pm \sqrt{25 - 40}}{2} = \frac{-5 \pm \sqrt{-15}}{2}$$

Hence the solution set $= \left\{-5, 0, \dfrac{-5 \pm \sqrt{-15}}{2}\right\}$

Q.7 $(x - 1)\,(x + 5)\,(x + 8)\,(x + 2) - 880 = 0$

Solution:

$(x - 1)\,(x + 5)\,(x + 8)\,(x + 2) - 880 = 0$

As $-1 + 8 = 5 + 2$

So we combine

$[(x - 1)\,(x + 8)]\,[(x + 5)\,(x + 2)] - 880 = 0$

$\Rightarrow$ $[x^2 + 8x - x - 8]\,[x^2 + 5x + 2x + 10] - 880 = 0$

$\Rightarrow$ $[x^2 + 7x - 8]\,[x^2 + 7x + 10] - 880 = 0$

Put $x^2 + 7x = y$ ……… (1)

$\Rightarrow$ $[y - 8]\,[y + 10] - 880 = 0$

$\Rightarrow$ $y^2 + 10y - 8y - 80 - 880 = 0$

$\Rightarrow$ $y^2 + 2y - 960 = 0$

$\Rightarrow \quad y^2 + 32y - 30y - 960 = 0$

$\Rightarrow \quad y(y + 32) - 30(y + 32) = 0$

$\Rightarrow \quad (y + 32)(y - 30) = 0$

$\Rightarrow \quad$ Either $y + 32 = 0$ or $y - 30 = 0$

$\Rightarrow \quad y = -32$ or $y = 30$

Put $y = -32$ and $y = 30$ in equation (1)

$\Rightarrow \quad x^2 + 7x = -32$

$\Rightarrow \quad x^2 + 7x + 32 = 0$

$$\Rightarrow \quad x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(32)}}{2(1)}$$

$$\Rightarrow \quad = \frac{-7 \pm \sqrt{49 - 128}}{2}$$

$$\Rightarrow \quad = \frac{-7 \pm \sqrt{-79}}{2}$$

$\Rightarrow \quad x^2 + 7x = 30$

$\Rightarrow \quad x^2 + 7x - 30 = 0$

$\Rightarrow \quad x^2 + 10x - 3x - 30 = 0$

$\Rightarrow \quad x(x + 10) - 3(x + 10) = 0$

$\Rightarrow \quad (x + 10)(x - 3) = 0$

$\Rightarrow \quad$ Either $x + 10 = 0$ or $x - 3 = 0$

$\Rightarrow \quad x = -10$ or $x = 3$

Hence the solution set $= \left\{-10, 3, \dfrac{-7 \pm \sqrt{-79}}{2}\right\}$

Q.8 $\quad (x - 5)(x - 7)(x + 6)(x + 4) - 504 = 0$

Solution:

$(x - 5)(x - 7)(x + 6)(x + 4) - 504 = 0$

As $\quad -5 + 4 = -7 + 6$

So we combine

$\Rightarrow \quad [(x - 5)(x + 4)][(x - 7)(x + 6)] - 504 = 0$

$\Rightarrow \quad [x^2 + 4x - 5x - 20][x^2 + 6x - 7x - 42] - 504 = 0$

$\Rightarrow \quad [x^2 - x - 20][x^2 - x - 42] - 504 = 0$

Put $x^2 - x = y$ .... (1)

$\Rightarrow \quad [y - 20][y - 42] - 504 = 0$

$\Rightarrow \quad y^2 - 42y - 20y + 840 - 504 = 0$

$\Rightarrow \quad y^2 - 62y + 336 = 0$

$\Rightarrow \quad y^2 - 56y - 6y + 336 = 0$

$\Rightarrow \quad y(y - 56) - 6(y - 56) = 0$

$\Rightarrow \quad (y - 56)(y - 6) = 0$

$\Rightarrow \quad$ Either $y - 56 = 0$ or $y - 6 = 0$

$\Rightarrow \quad y = 56$ or $y = 6$

Put $y = 56$ and $y = 6$ in equation (1)

$\Rightarrow \quad x^2 - x = 56$	$\Rightarrow \quad x^2 - x = 6$
$\Rightarrow \quad x^2 - x - 56 = 0$	$\Rightarrow \quad x^2 - x - 6 = 0$
$\Rightarrow \quad x^2 - 8x + 7x - 56 = 0$	$\Rightarrow \quad x^2 - 3x + 2x - 6 = 0$
$\Rightarrow \quad x(x - 8) + 7(x - 8) = 0$	$\Rightarrow \quad x(x - 3) + 2(x - 3) = 0$
$\Rightarrow \quad (x - 8)(x + 7) = 0$	$\Rightarrow \quad (x - 3)(x + 2) = 0$
$\Rightarrow \quad$ Either $x - 8 = 0$ or $x + 7 = 0$	$\Rightarrow \quad$ Either $x - 3 = 0$ or $x + 2 = 0$
$\Rightarrow \quad x = 8$ or $x = -7$	$\Rightarrow \quad x = 3$ or $x = -2$

Hence the solution set $= \{-7, -2, 3, 8\}$

Q.9 $(x - 1)(x - 2)(x - 8)(x + 5) + 360 = 0$

Solution:

$(x - 1)(x - 2)(x - 8)(x + 5) + 360 = 0$

So we combine

$[(x - 1)(x - 2)][(x - 8)(x + 5)] + 360 = 0$

$\Rightarrow \quad [x^2 - 2x - x + 2][x^2 + 5x - 8x - 40] + 360 = 0$

$\Rightarrow \quad [x^2 - 3x + 2][x^2 - 3x - 40] + 360 = 0$

Put $x^2 - 3x = y$ .... (1)

$\Rightarrow \quad [y + 2][y - 40] + 360 = 0$

$\Rightarrow \quad y^2 - 40y + 2y - 80 + 360 = 0$

$\Rightarrow \quad y^2 - 38y + 280 = 0$

$\Rightarrow \quad y^2 - 28y - 10y + 280 = 0$

$\Rightarrow \quad y(y - 28) - 10(y - 28) = 0$

$\Rightarrow \quad$ Either $y - 28 = 0$ or $y - 10 = 0$

$\Rightarrow \quad y = 28$ or $y = 10$

Put $y = 28$ and $y = 10$ in equation (1)

$\Rightarrow \quad x^2 - 3x = 28$

$\Rightarrow \quad x^2 - 3x - 28 = 0$

$\Rightarrow \quad x^2 - 7x + 4x - 28 = 0$

$\Rightarrow \quad x(x-7) + 4(x-7) = 0$

$\Rightarrow \quad (x-7)(x+4) = 0$

$\Rightarrow$ Either $x - 7 = 0$ or $x + 4 = 0$

$\Rightarrow \quad x = 7$ or $x = -4$

$\Rightarrow \quad x^2 - 3x = 10$

$\Rightarrow \quad x^2 - 3x - 10 = 0$

$\Rightarrow \quad x^2 - 5x + 2x - 10 = 0$

$\Rightarrow \quad x(x-5) + 2(x-5) = 0$

$\Rightarrow \quad (x-5)(x+2) = 0$

$\Rightarrow$ Either $x - 5 = 0$ or $x + 2 = 0$

$\Rightarrow \quad x = 5$ or $x = -2$

Hence the solution set $= \{-4, -2, 5, 7\}$

Q.10 $(x+1)(2x+3)(2x+5)(x+3) = 945$ (Lahore Board 2007)

Solution:

$(x+1)(2x+3)(2x+5)(x+3) = 945$

Here we combine

$[(x+1)(x+3)][(2x+3)(2x+5)] = 945$

$\Rightarrow \quad [x^2 + 3x + x + 3][4x^2 + 10x + 6x + 15] = 945$

$\Rightarrow \quad [x^2 + 4x + 3][4x^2 + 16x + 15] = 945$

$\Rightarrow \quad [x^2 + 4x + 3][4(x^2 + 4x) + 15] = 945$

Put $x^2 + 4x = y$ .... (1)

$\Rightarrow \quad [y + 3](4y + 15) = 945$

$\Rightarrow \quad 4y^2 + 15y + 12y + 45 = 945$

$\Rightarrow \quad 4y^2 + 27y + 45 - 945 = 0$

$\Rightarrow \quad 4y^2 + 27y - 900 = 0$

$\Rightarrow \quad 4y^2 + 75y - 48y - 900 = 0$

$\Rightarrow \quad y(4y + 75) - (4y + 75) = 0$

$\Rightarrow \quad (4y + 75)(y - 12) = 0$

$\Rightarrow$ Either $4y + 75 = 0$ or $y - 12 = 0$

$\Rightarrow \quad y = \frac{-75}{4}$ or $y = 12$

Put $y = \frac{-75}{4}$ and $y = 12$ in equation (1)

$\Rightarrow \quad x^2 + 4x = \frac{-75}{4}$

$\Rightarrow \quad x^2 + 4x = 12$

$\Rightarrow \quad 4x^2 + 16x = -75$

$\Rightarrow \quad 4x^2 + 16x + 75 = 0$

$\Rightarrow \quad x = \dfrac{-16 \pm \sqrt{(16)^2 - 4(4)(75)}}{2(4)}$

$\Rightarrow \quad = \dfrac{-16 \pm \sqrt{-944}}{8}$

$\Rightarrow \quad = \dfrac{-16 \pm 4\sqrt{-59}}{8}$

$\Rightarrow \quad = \dfrac{-4 \pm \sqrt{-59}}{2}$

$\Rightarrow \quad x^2 + 4x - 12 = 0$

$\Rightarrow \quad x^2 + 6x - 2x - 12 = 0$

$\Rightarrow \quad x(x + 6) - 2(x + 6) = 0$

$\Rightarrow \quad$ Either $x + 6 = 0$ or $x - 2 = 0$

$\Rightarrow \quad x = -6$ or $x = 2$

Hence the solution set $= \left\{-6, 2, \dfrac{-4 \pm \sqrt{-59}}{2}\right\}$

Q.11 $(2x - 7)(x^2 - 9)(2x + 5) - 91 = 0$

Solution:

$(2x - 7)(x^2 - 9)(2x + 5) - 91 = 0$

$\Rightarrow \quad (2x - 7)(x + 3)(x - 3)(2x + 5) - 91 = 0$

Here we combine

$[(2x - 7)(x + 3)][(x - 3)(2x + 5)] - 91 = 0$

$\Rightarrow \quad [2x^2 + 6x - 7x - 21][2x^2 + 5x - 6x - 15] - 91 = 0$

$\Rightarrow \quad [2x^2 - x - 21][2x^2 - x - 15] - 91 = 0$

Put $2x^2 - x = y$ ……… (1)

$\Rightarrow \quad [y - 21][y - 15] - 91 = 0$

$\Rightarrow \quad y^2 - 15y - 21y + 315 - 91 = 0$

$\Rightarrow \quad y^2 - 36y + 224 = 0$

$\Rightarrow \quad y^2 - 28y - 8y + 224 = 0$

$\Rightarrow \quad y(y - 28) - 8(y - 28) = 0$

$\Rightarrow \quad (y - 28)(y - 8) = 0$

$\Rightarrow \quad$ Either $y - 28 = 0$ or $y - 8 = 0$

$\Rightarrow \quad y = 28$ or $y = 8$

Put $y = 28$ and $y = 8$ in equation (1)

$\Rightarrow \quad 2x^2 - x = 28$

$\Rightarrow \quad 2x^2 - x - 28 = 0$

$\Rightarrow \quad 2x^2 - 8x + 7x - 28 = 0$

$\Rightarrow \quad 2x(x - 4) + 7(x - 4) = 0$

$\Rightarrow \quad (x - 4) + (2x + 7) = 0$

$\Rightarrow \quad$ Either $\quad x - 4 = 0 \quad$ or $\quad 2x + 7 = 0$

$\Rightarrow \quad x = 4 \quad$ or $\quad x = \dfrac{-7}{2}$

$\Rightarrow \quad 2x^2 - x = 8$

$\Rightarrow \quad 2x^2 - x - 8 = 0$

$$\Rightarrow \quad x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-8)}}{2(2)}$$

$$\Rightarrow \quad = \frac{1 \pm \sqrt{1 + 64}}{4}$$

$$\Rightarrow \quad = \frac{1 \pm \sqrt{65}}{4}$$

Hence the solution set $= \left\{-\dfrac{7}{2}, 4, \dfrac{1 \pm \sqrt{65}}{4}\right\}$

Q.12 $(x^2 + 6x + 8)(x^2 + 14x + 48) = 105$

Solution:

$(x^2 + 6x + 8)(x^2 + 14x + 48) = 105$

$\Rightarrow \quad (x^2 + 4x + 2x + 8)(x^2 + 8x + 6x + 48) = 105$

$\Rightarrow \quad [x(x + 4) + 2(x + 4)][(x(x + 8) + 6(x + 8)] = 105$

$\Rightarrow \quad [(x + 4)(x + 2)][(x + 8)(x + 6)] = 105$

$\Rightarrow \quad (x + 4)(x + 2)(x + 8)(x + 6) = 105$

As $\quad 2 + 8 = 4 + 6$

So we combine

$[(x + 2)(x + 8)][(x + 4)(x + 6)] = 105$

$\Rightarrow \quad [x^2 + 8x + 2x + 16][x^2 + 6x + 4x + 24] = 105$

$\Rightarrow \quad [x^2 + 10x + 16][x^2 + 10x + 24] = 105$

Put $x^2 + 10x = y \qquad$ ……… (1)

$\Rightarrow \quad [y + 16][y + 24] = 105$

$\Rightarrow \quad y^2 + 24y + 16y + 384 = 105$

$\Rightarrow \quad y^2 + 40y + 384 - 105 = 0$

$\Rightarrow \quad y^2 + 40y + 279 = 0$

$\Rightarrow \quad y^2 + 31y + 9y + 279 = 0$

$\Rightarrow \quad y(y+31)+9(y+31) = 0$

$\Rightarrow \quad (y+9)(y+31) = 0$

$\Rightarrow \quad$ Either $y+9 = 0$ or $y+31 = 0$

$\Rightarrow \quad y = -9$ or $y = -31$

Put $y = -9$ and $y = -31$ in equation (1)

$\Rightarrow \quad x^2+10x = -9$

$\Rightarrow \quad x^2+10x+9 = 0$

$\Rightarrow \quad x^2+9x+x+9 = 0$

$\Rightarrow \quad x(x+9)+1(x+9) = 0$

$\Rightarrow \quad (x+1)(x+9) = 0$

$\Rightarrow \quad$ Either $x+1 = 0$ or $x+9 = 0$

$\Rightarrow \quad x = -1$ or $x = -9$

$\Rightarrow \quad x^2+10x = -31$

$\Rightarrow \quad x^2+10x+31 = 0$

$\Rightarrow \quad x = \dfrac{-10 \pm \sqrt{(10)^2-4(1)(31)}}{2(1)}$

$\Rightarrow \quad = \dfrac{-10 \pm \sqrt{100-124}}{2}$

$\Rightarrow \quad = \dfrac{-10 \pm \sqrt{-24}}{2}$

$\Rightarrow \quad = \dfrac{-10 \pm 2\sqrt{-6}}{2}$

$\Rightarrow \quad = -5 \pm \sqrt{-6}$

Hence the solution set $= \{-1, -9, -5 \pm \sqrt{-6}\}$

Q.13 $(x^2+6x-27)(x^2-2x-35) = 385$

Solution:

$(x^2+6x-27)(x^2-2x-35) = 385$

$\Rightarrow \quad (x^2+9x-3x-27)(x^2-7x+5x-35) = 385$

$\Rightarrow \quad [x(x+9)-3(x+9)][x(x-7)+5(x-7)] = 385$

$\Rightarrow \quad [(x+9)(x-3)][(x-7)(x+5)] = 385$

$\Rightarrow \quad (x+9)(x-3)(x-7)(x+5) = 385$

As $9-7 = 5-3$

So we combine

$[(x+9)(x-7)][(x+5)(x-3)] = 385$

$\Rightarrow \quad [x^2-7x+9x-63][x^2-3x+5x-15] = 385$

$\Rightarrow \quad [x^2+2x-63][x^2+2x-15] = 385$

Put $x^2 + 2x = y$ .... (1)

$\Rightarrow [y - 63][y - 15] = 385$

$\Rightarrow y^2 - 15y - 63y + 945 = 385$

$\Rightarrow y^2 - 78y + 945 - 385 = 0$

$\Rightarrow y^2 - 78y + 560 = 0$

$\Rightarrow y^2 - 70y - 8y + 560 = 0$

$\Rightarrow y(y - 70) - 8(y - 70) = 0$

$\Rightarrow (y - 70)(y - 8) = 0$

$\Rightarrow$ Either $y - 70 = 0$ or $y - 8 = 0$

$\Rightarrow y = 70$ or $y = 8$

Put $y = 70$ and $y = 8$ in equation (1)

$\Rightarrow x^2 + 2x = 70$

$\Rightarrow x^2 + 2x - 70 = 0$

$\Rightarrow x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(-70)}}{2(1)}$

$\Rightarrow = \dfrac{-2 \pm \sqrt{4 + 280}}{2}$

$\Rightarrow = \dfrac{-2 \pm \sqrt{284}}{2}$

$\Rightarrow = \dfrac{-2 \pm 2\sqrt{71}}{2}$

$\Rightarrow = -1 \pm \sqrt{71}$

$\Rightarrow x^2 + 2x = 8$

$\Rightarrow x^2 + 2x - 8 = 0$

$\Rightarrow x^2 + 4x - 2x - 8 = 0$

$\Rightarrow x(x + 4) - 2(x + 4) = 0$

$\Rightarrow (x + 4)(x - 2) = 0$

$\Rightarrow$ Either $x + 4 = 0$ or $x - 2 = 0$

$\Rightarrow x = -4$ or $x = 2$

Hence the solution set $= \{-4, 2, -1 \pm \sqrt{71}\}$

Q.14 $\mathbf{4.2^{2x+1} - 9.2^x + 1 = 0}$ **(Gujranwala Board 2005)**

Solution:

$4.2^{2x+1} - 9.2^x + 1 = 0$

$4.2^{2x} . 2 - 9.2^x + 1 = 0$

$8(2^x)^2 - 9.2^x + 1 = 0$

Put $2^x = y$ .... (1)

$\Rightarrow 8y^2 - 9y + 1 = 0$

$\Rightarrow \quad 8y^2 - 8y - y + 1 = 0$

$\Rightarrow \quad 8y(y-1) - 1(y-1) = 0$

$\Rightarrow \quad (y-1)(8y-1) = 0$

$\Rightarrow \quad$ Either $y - 1 = 0 \quad$ or $\quad 8y - 1 = 0$

$\Rightarrow \quad y = 1 \quad$ or $\quad y = \frac{1}{8}$

Put $y = 1$ and $y = \frac{1}{8}$ in equation (1)

$\Rightarrow \quad 2^x = 1$	$\Rightarrow \quad 2^x = \frac{1}{8}$
$\Rightarrow \quad 2^x = 2^0$	$\Rightarrow \quad 2^x = \frac{1}{2^3}$
$\Rightarrow \quad x = 0$	$\Rightarrow \quad 2^x = 2^{-3}$
	$\Rightarrow \quad x = -3$

Hence the solution set $= \{-3, 0\}$

Q.15 $\mathbf{2^x + 2^{-x+6} - 20 = 0}$

Solution:

$2^x + 2^{-x+6} - 20 = 0$

$2^x + 2^{-x} . 2^6 - 20 = 0$

$2^x + 64\, x^{-x} - 20 = 0$

Put $2^x = y \quad$ ……… (1)

$\Rightarrow \quad 2^{-x} = y^{-1}$

$\Rightarrow \quad y + 64\, y^{-1} - 20 = 0$

$\Rightarrow \quad y + \frac{64}{y} - 20 = 0$

Multiplying by 'y'

$y^2 + 64 - 20y = 20$

$\Rightarrow \quad y^2 - 20y + 64 = 0$

$\Rightarrow \quad y^2 - 16y - 4y + 64 = 0$

$\Rightarrow \quad y(y-16) - 4(y-16) = 0$

$\Rightarrow \quad (y-16)(y-4) = 0$

$\Rightarrow$ Either $y - 16 = 0$ or $y - 4 = 0$

$y = 16$ or $y = 4$

Put $y = 16$ and $y = 4$ in equation (1)

$\Rightarrow 2^x = 16$ | $\Rightarrow 2^x = 4$

$\Rightarrow 2^x = 2^4$ | $\Rightarrow 2^x = 2^2$

$\Rightarrow x = 4$ | $\Rightarrow x = 2$

Hence the solution set = $\{2, 4\}$

Q.16 $\mathbf{4^x - 3.2^{x+3} + 128 = 0}$

Solution:

$4^x - 3.2^{x+3} + 128 = 0$

$\Rightarrow (2^2)^x - 3.2^x . 2^3 + 128 = 0$

$\Rightarrow 2 - 24.2^x + 128 = 0$

Put $2^x = y$ ……… (1)

$\Rightarrow y^2 - 24y + 128 = 0$

$\Rightarrow y^2 - 16y - 8y + 128 = 0$

$\Rightarrow y(y - 16) - 8(y - 16) = 0$

$\Rightarrow (y - 16)(y - 8) = 0$

$\Rightarrow$ Either $y - 6 = 0$ or $y - 8 = 0$

$\Rightarrow y = 16$ or $y = 8$

Put $y = 16$ and $y = 8$ in equation (1)

$\Rightarrow 2^x = 6$ | $\Rightarrow 2^x = 8$

$\Rightarrow 2^x = 2^4$ | $\Rightarrow 2^x = 2^3$

$\Rightarrow x = 4$ | $\Rightarrow x = 3$

Hence the solution set = $\{3, 4\}$

Q.17 $\mathbf{3^{2x-1} - 12.3^x + 81 = 0}$

Solution:

$3^{2x-1} - 12.3^x + 81 = 0$

$\Rightarrow 3^{2x} . 3^{-1} - 12 . 3^x + 81 = 0$

$\Rightarrow \dfrac{3^{2x}}{3} - 12.3^x + 81 = 0$

Multiplying by '3'

$\Rightarrow \quad 3^{2x} - 36 \,.\, 3^x + 243 = 0$

$\Rightarrow \quad (3^x)^2 - 36 \,.\, 3^x + 243 = 0$

Put $3^x = y$ ……… (1)

$\Rightarrow \quad y^2 - 36y + 243 = 0$

$\Rightarrow \quad y^2 - 27y - 9y + 243 = 0$

$\Rightarrow \quad y(y - 27) - 9(y - 27) = 0$

$\Rightarrow \quad (y - 27) - (y - 9) = 0$

$\Rightarrow$ Either $y - 27 = 0$ or $y - 9 = 0$

$\Rightarrow \quad y = 27$ or $y = 9$

Put $y = 27$ and $y = 9$ in equation (1)

$\Rightarrow \quad 3^x = 27$	$\Rightarrow \quad 3^x = 9$
$\Rightarrow \quad 3^x = 3^3$	$\Rightarrow \quad 3^x = 3^2$
$\Rightarrow \quad x = 3$	$\Rightarrow \quad x = 2$

Hence the solution set $= \{2, 3\}$

Q.18 $\left(x + \frac{1}{x}\right)^2 - 3\left(x + \frac{1}{x}\right) - 4 = 0$

Solution:

$$\left(x + \frac{1}{x}\right)^2 - 3\left(x + \frac{1}{x}\right) - 4 = 0$$

Put $x + \frac{1}{x} = y$ ……… (1)

$y^2 - 3y - 4 = 0$

$\Rightarrow \quad y^2 - 4y + y - 4 = 0$

$\Rightarrow \quad y(y - 4) + 1(y - 4) = 0$

$\Rightarrow \quad (y - 4) + 1(y - 4) = 0 \Rightarrow (y - 4)(y + 1) = 0$

$\Rightarrow$ Either $y - 4 = 0$ or $y + 1 = 0$

$\Rightarrow \quad y = 4$ or $y = -1$

Put $y = 4$ and $y = -1$ in equation (1)

$\Rightarrow \quad x + \frac{1}{x} = +4 \qquad \Rightarrow \quad x + \frac{1}{x} = -1$

$\Rightarrow \quad x + \frac{1}{x} - 4 = 0$

$\Rightarrow \quad x^2 + 1 - 4x = 0$

$\Rightarrow \quad x^2 - 4x + 1 = 0$

$\Rightarrow \quad x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$

$\Rightarrow \quad x = \frac{4 \pm \sqrt{16 - 4}}{2}$

$\Rightarrow \quad x = \frac{4 \pm \sqrt{12}}{2}$

$\Rightarrow \quad x = \frac{4 \pm 2\sqrt{3}}{2}$

$\Rightarrow \quad x = 2 \pm \sqrt{3}$

$\Rightarrow \quad x + \frac{1}{x} + 1 = 0$

$\Rightarrow \quad x^2 + 1 + x = 0$

$\Rightarrow \quad x^2 + x + 1 = 0$

$\Rightarrow \quad x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

$\Rightarrow \quad x = \frac{-1 \pm \sqrt{1 - 4}}{2}$

$\Rightarrow \quad x = \frac{-1 \pm \sqrt{-3}}{2}$

Hence the solution set $= \left\{ 2 \pm \sqrt{3}, \frac{-1 \pm \sqrt{-3}}{2} \right\}$

Q.19 $\mathbf{x^2 + x - 4 + \frac{1}{x} + \frac{1}{x^2} = 0}$

Solution:

$x^2 + x - 4 + \frac{1}{x} + \frac{1}{x^2} = 0$

$\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 4 = 0$

Put $x + \frac{1}{x} = y$ .... (1)

Then $\left(x + \frac{1}{x}\right)^2 = y^2$

$\Rightarrow \quad x^2 + \frac{1}{x^2} + 2 = y^2$

$\Rightarrow \quad x^2 + \frac{1}{x^2} = y^2 - 2$

$\Rightarrow$ given equation becomes

$\Rightarrow \quad y^2 - 2 + y - 4 = 0$

$\Rightarrow \quad y^2 + y - 6 = 0$

$\Rightarrow \quad y^2 + 3y - 2y - 6 = 0$

$\Rightarrow \quad y(y+3) - 2(y+3) = 0$

$\Rightarrow \quad (y+3)(y-2) = 0$

$\Rightarrow$ Either $y + 3 = 0$ or $y - 2 = 0$

$\Rightarrow \quad y = -3$ or $y = 2$

Put $y = -3$ and $y = 2$ in equation (1)

$\Rightarrow \quad x + \frac{1}{x} = -3$

Multiplying by 'x'

$\Rightarrow \quad x^2 + 1 = -3x$

$\Rightarrow \quad x^2 + 3x + 1 = 0$

$$\Rightarrow \quad x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(1)}}{2(1)}$$

$$\Rightarrow \quad x = \frac{-3 \pm \sqrt{9-4}}{2}$$

$$\Rightarrow \quad x = \frac{-3 \pm \sqrt{5}}{2}$$

$\Rightarrow \quad x + \frac{1}{x} = 2$

Multiplying by 'x'

$\Rightarrow \quad x^2 + 1 = 2x$

$\Rightarrow \quad x^2 - 2x + 1 = 0$

$\Rightarrow \quad (x-1)^2 = 0$

$\Rightarrow \quad x - 1 = 0$

$\Rightarrow \quad x = 1$

Hence the solution set $= \left\{1, \frac{-3 \pm \sqrt{5}}{2}\right\}$

Q.20 $\left(x - \frac{1}{x}\right)^2 + 3\left(x + \frac{1}{x}\right) = 0$

Solution:

$$\left(x - \frac{1}{x}\right)^2 + 3\left(x + \frac{1}{x}\right) = 0$$

or $\quad x^2 + \frac{1}{x^2} - 2 + 3\left(x + \frac{1}{x}\right) = 0$

or $\quad x^2 + \frac{1}{x^2} + 3\left(x + \frac{1}{x}\right) - 2 = 0 \quad \text{......... (A)}$

Put $x + \frac{1}{x} = y \quad \text{......... (1)}$

then $\left(x+\frac{1}{x}\right)^2 = y^2$

$\Rightarrow$ $x^2+\frac{1}{x^2}+2 = y^2$

$\Rightarrow$ $x^2+\frac{1}{x^2} = y^2-2$

$\Rightarrow$ Above equation (A) becomes

$y^2-2+3y-2 = 0$

$\Rightarrow$ $y^2+3y-4 = 0$

$\Rightarrow$ $y^2+4y-y-4 = 0$

$\Rightarrow$ $y(y+4)-1(y+4) = 0$

$\Rightarrow$ $(y+4)(y-1) = 0$

$\Rightarrow$ Either $y+4 = 0$ or $y-1 = 0$

$\Rightarrow$ $y = -4$ or $y = 1$

Put $y = -4$ and $y = 1$ in equation (1)

$\Rightarrow$ $x+\frac{1}{x} = -4$

$\Rightarrow$ $x^2+1 = -4x$

$\Rightarrow$ $x^2+4x+1 = 0$

$\Rightarrow$ $x = \frac{-4\pm\sqrt{(4)^2-4(1)(1)}}{2(1)}$

$\Rightarrow$ $x = \frac{-4\pm\sqrt{16-4}}{2}$

$\Rightarrow$ $x = \frac{-4\pm\sqrt{12}}{2}$

$\Rightarrow$ $x = \frac{-4\pm 2\sqrt{3}}{2}$

$\Rightarrow$ $x = -2\pm\sqrt{3}$

$\Rightarrow$ $x+\frac{1}{x} = 1$

$\Rightarrow$ $x^2+1 = x$

$\Rightarrow$ $x^2-x+1 = 0$

$\Rightarrow$ $x = \frac{1\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}$

$\Rightarrow$ $x = \frac{1\pm\sqrt{1-4}}{2}$

$x = \frac{1\pm\sqrt{-3}}{2}$

Hence the solution set $= \left\{-2\pm\sqrt{3}, \frac{1\pm\sqrt{-3}}{2}\right\}$

Q.21 $\mathbf{2x^4 - x^3 + x^2 - 3x + 2 = 0}$

Solution:

$$2x^4 - x^3 + x^2 - 3x + 2 = 0$$

Dividing each by x^2

$$\Rightarrow \quad \frac{2x^4}{x^2} - \frac{3x^3}{x^2} - \frac{x^2}{x^2} - \frac{3x}{x^2} + \frac{2}{x^2} = \frac{0}{x^2}$$

$$\Rightarrow \quad 2x^2 - 3x - 1 - \frac{3}{x} + \frac{2}{x^2} = 0$$

$$\Rightarrow \quad 2x^2 + \frac{2}{x^2} - 3x - \frac{3}{x} - 1 = 0$$

$$\Rightarrow \quad 2\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) - 1 = 0 \qquad \text{.........} \ (A)$$

$$\text{Put } x + \frac{1}{x} = y \qquad \text{.........} \ (1)$$

$$\text{Then} \quad \left(x + \frac{1}{x}\right)^2 = y^2$$

$$\Rightarrow \quad x^2 + \frac{1}{x^2} + 2 = y^2 \qquad \Rightarrow \quad x^2 + \frac{1}{x^2} = y^2 - 2$$

Above equation (A) becomes

$$\Rightarrow \quad 2(y^2 - 2) - 3y - 1 = 0$$

$$\Rightarrow \quad 2y^2 - 3y - 5 = 0$$

$$\Rightarrow \quad 2y^2 - 5y + 2y - 5 = 0$$

$$\Rightarrow \quad y(2y - 5) + 1(2y - 5) = 0$$

$$\Rightarrow \quad (2y - 5)(y + 1) = 0$$

$$\Rightarrow \quad \text{Either } 2y - 5 = 0 \quad \text{or} \quad y + 1 = 0$$

$$\Rightarrow \quad y = \frac{5}{2} \quad \text{or} \quad y = -1$$

Put $y = \frac{5}{2}$ and $y = -1$ in equation (1)

$$\Rightarrow \quad x + \frac{1}{x} = \frac{5}{2}$$

$$\Rightarrow \quad x^2 + 1 = \frac{5}{2}x$$

$$\Rightarrow \quad x + \frac{1}{x} = -1$$

$$\Rightarrow \quad x^2 + 1 = -x$$

$\Rightarrow \quad 2x^2 + 2 = 5x$

$\Rightarrow \quad 2x^2 - 5x + 2 = 0$

$\Rightarrow \quad 2x^2 - 4x - x + 2 = 0$

$\Rightarrow \quad 2x(x-2) - 1(x-2) = 0$

$\Rightarrow \quad (2x-1)(x-2) = 0$

$\Rightarrow \quad x^2 + x + 1 = 0$

$\Rightarrow \quad x = \dfrac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

$\Rightarrow \quad x = \dfrac{-1 \pm \sqrt{1-4}}{2}$

$\Rightarrow \quad x = \dfrac{-1 \pm \sqrt{-3}}{2}$

$\Rightarrow$ Either $2x - 1 = 0$ or $x - 2 = 0$

$\Rightarrow \quad x = \dfrac{1}{2}$ or $x = 2$

Hence the solution set $= \left\{\dfrac{1}{2},\ 2,\ \dfrac{-1 \pm \sqrt{-3}}{2}\right\}$

Q.22 $\mathbf{2x^4 + 3x^3 - 4x^2 - 3x + 2 = 0}$

Solution:

$2x^4 + 3x^3 - 4x^2 - 3x + 2 = 0$

Dividing by x^2

$$\Rightarrow \quad \frac{2x^4}{x^2} + \frac{3x^3}{x^2} - \frac{4x^2}{x^2} - \frac{3x}{x^2} + \frac{2}{x^2} = \frac{0}{x^2}$$

$$\Rightarrow \quad 2x^2 + 3x - 4 - \frac{3}{x} + \frac{2}{x^2} = 0$$

$$\Rightarrow \quad 2x^2 + \frac{2}{x^2} + 3x - \frac{3}{x} - 4 = 0$$

$$\Rightarrow \quad 2\left(x^2 + \frac{1}{x^2}\right) + 3\left(x - \frac{1}{x}\right) - 4 = 0 \quad \ldots\ldots\ldots \text{(A)}$$

Put $x - \dfrac{1}{x} = y \quad \ldots\ldots\ldots$ (1)

Then $\left(x - \dfrac{1}{x}\right)^2 \quad \Rightarrow \quad x^2 + \dfrac{1}{x^2} - 2 = y^2 \quad \Rightarrow \quad x^2 + \dfrac{1}{x^2} = y^2 + 2$

$\Rightarrow$ Above equation (A) becomes

$2(y^2 + 2) + 3y - 4 = 0$

$\Rightarrow \quad 2y^2 + 4 + 3y - 4 = 0$

$\Rightarrow \quad 2y^2 + 3y = 0$

$\Rightarrow \quad y(2y + 3) = 0$

$\Rightarrow \quad$ Either $y = 0 \quad$ or $\quad 2y + 3 = 0$

$\Rightarrow \quad$ or $\quad y = \frac{-3}{2}$

Put $y = 0$ and $y = \frac{-3}{2}$ in equation (1)

$\Rightarrow \quad x - \frac{1}{x} = 0$

$\Rightarrow \quad x^2 - 1 = 0$

$\Rightarrow \quad (x + 1)(x - 1) = 0$

$\Rightarrow \quad$ Either $x + 1 = 0$ or $x - 1 = 0$

$\Rightarrow \quad x = -1$ or $x = 1$

$\Rightarrow \quad x - \frac{1}{x} = -\frac{3}{2}$

$\Rightarrow \quad x^2 - 1 = \frac{-3}{2}x$

$\Rightarrow \quad 2x^2 - 2 = -3x$

$\Rightarrow \quad 2x^2 + 3x - 2 = 0$

$\Rightarrow \quad 2x^2 + 4x - x - 2 = 0$

$\Rightarrow \quad 2x(x + 2) - 1(x + 2) = 0$

$\Rightarrow \quad (x + 2)(2x - 1) = 0$

$\Rightarrow \quad$ Either $x + 2 = 0$ or $2x - 1 = 0$

$\Rightarrow \quad x = -2$ or $x = \frac{1}{2}$

Hence the solution set $= \left\{-2, -1, \frac{1}{2}, 1\right\}$

Q.23 $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

Solution:

$6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

Dividing by x^2

$\Rightarrow \quad \frac{6x^4}{x^2} - \frac{35x^3}{x^2} + \frac{62x^2}{x^2} - \frac{35x}{x^2} + \frac{6}{x^2} = \frac{0}{x^2}$

$\Rightarrow \quad 6x^2 - 35x + 62 - \frac{35}{x} + \frac{6}{x^2} = 0$

$\Rightarrow \quad 6x^2 + \frac{6}{x^2} - 35x - \frac{35}{x} + 62 = 0$

$\Rightarrow \quad 6\left(x^2+\frac{1}{x^2}\right)-35\left(x+\frac{1}{x}\right)+62 = 0 \quad \ldots\ldots\ldots (A)$

Put $x+\frac{1}{x} = y \quad \ldots\ldots\ldots (1)$

Then $\left(x+\frac{1}{x}\right)^2 = y^2 \Rightarrow x^2+\frac{1}{x^2}+2 = y^2 \Rightarrow x^2+\frac{1}{x^2} = y^2-2$

$\Rightarrow$ Above equation (A) becomes

$6(y^2-2)-35y+62 = 0$

$\Rightarrow \quad 6y^2-12-35y+62 = 0$

$\Rightarrow \quad 6y^2-35y+50 = 0$

$\Rightarrow \quad 6y^2-20y-15y+50 = 0$

$\Rightarrow \quad 2y(3y-10)-5(3y-10) = 0$

$\Rightarrow \quad (3y-10)(2y-5) = 0$

$\Rightarrow$ Either $3y-10 = 0$ or $2y-5 = 0$

$\Rightarrow \quad y = \frac{10}{3}$ or $y = \frac{5}{2}$

Put $y = \frac{10}{3}$ and $y = \frac{5}{2}$ in equation (1)

$\Rightarrow \quad x+\frac{1}{x} = \frac{10}{3}$	$\Rightarrow \quad x+\frac{1}{x} = \frac{5}{2}$
$\Rightarrow \quad x^2+1 = \frac{10}{3}x$	$\Rightarrow \quad x^2+1 = \frac{5}{2}x$
$\Rightarrow \quad 3x^2+3 = 10x$	$\Rightarrow \quad 2x^2+2 = 5x$
$\Rightarrow \quad 3x^2-10x+3 = 0$	$\Rightarrow \quad 2x^2-5x+2 = 0$
$\Rightarrow \quad 3x^2-9x-x+3 = 0$	$\Rightarrow \quad 2x^2-4x-x+2 = 0$
$\Rightarrow \quad 3x(x-3)-1(x-3) = 0$	$\Rightarrow \quad 2x(x-2)-1(x-2) = 0$
$\Rightarrow \quad (x-3)(3x-1) = 0$	$\Rightarrow \quad (x-2)(2x-1) = 0$
$\Rightarrow$ Either $x-3 = 0$ or $3x-1 = 0$	$\Rightarrow$ Either $x-2 = 0$ or $2x-1 = 0$
$\Rightarrow \quad x = 3$ or $x = \frac{1}{3}$	$\Rightarrow \quad x = 2$ or $x = \frac{1}{2}$

Hence the solution set $= \left\{2, 3, \frac{1}{2}, \frac{1}{3}\right\}$

Q.24 $\mathbf{x^4 - 6x^2 + 10 - \frac{6}{x^2} + \frac{1}{x^4} = 0}$

Solution:

$$x^4 - 6x^2 + 10 - \frac{6}{x^2} + \frac{1}{x^4} = 0$$

$$\Rightarrow \quad x^4 + \frac{1}{x^4} - 6x^2 - \frac{6}{x^2} + 10 = 0$$

$$\Rightarrow \quad \left(x^4 + \frac{1}{x^4}\right) - 6\left(x^2 + \frac{1}{x^2}\right) + 10 = 0 \qquad \ldots\ldots\ldots \text{(A)}$$

Put $x^2 + \frac{1}{x^2} = y \qquad (1)$

Then $\left(x^2 + \frac{1}{x^2}\right)^2 = y^2 \quad \Rightarrow \quad x^4 + \frac{1}{x^4} + 2 = y^2 \quad \Rightarrow \quad x^4 + \frac{1}{x^4} = y^2 - 2$

$\Rightarrow$ above equation (A) becomes

$$y^2 - 2 - 6y + 10 = 0$$

$$\Rightarrow \quad y^2 - 6y + 8 = 0$$

$$\Rightarrow \quad y^2 - 4y - 2y + 8 = 0$$

$$\Rightarrow \quad y(y-4) - 2(y-4) = 0$$

$$\Rightarrow \quad (y-4)(y-2) = 0$$

$\Rightarrow$ Either $y - 4 = 0$ or $y - 2 = 0$

$\Rightarrow$ $y = 4$ or $y = 2$

Put $y = 4$ and $y = 2$ in equation (1)

$\Rightarrow \quad x^2 + \frac{1}{x^2} = 4$	$\Rightarrow \quad x^2 + \frac{1}{x^2} = 2$
$\Rightarrow \quad x^4 + 1 = 4x^2$	$\Rightarrow \quad x^4 + 1 = 2x^2$
$\Rightarrow \quad x^4 - 4x^2 + 1 = 0$	$\Rightarrow \quad x^4 - 2x^2 + 1 = 0$
$\Rightarrow \quad x^2 = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$	$\Rightarrow \quad (x^2 - 1)^2 = 0$
$\Rightarrow \quad x^2 = \frac{4 \pm \sqrt{16-4}}{2}$	$\Rightarrow \quad x^2 - 1 = 0$
$\Rightarrow \quad x^2 = \frac{4 \pm \sqrt{12}}{2}$	$\Rightarrow \quad (x+1)(x-1) = 0$

$\Rightarrow \quad x^2 = \dfrac{4 \pm 2\sqrt{3}}{2}$

$\Rightarrow \quad x^2 = 2 \pm \sqrt{3}$

$\Rightarrow \quad x = \pm\sqrt{2 \pm \sqrt{3}}$

$\Rightarrow \quad$ Either $x + 1 = 0$ or $x - 1 = 0$

$\Rightarrow \quad x = -1$ or $x = 1$

Hence the solution set $= \left\{ -1, 1, \pm\sqrt{2 \pm \sqrt{3}} \right\}$

TYPE V: RADICAL EQUATIONS

Equations involving radical expressions of the variable are called radical equations. To solve a radical equation, we first obtain an equation free from radicals. Every solution of radical equation is also a solution of the radical free equation but the new equation have solutions that are not solutions of the original radical equation. Such extra solutions are called **extraneous roots**.

There are four types of radical equations.

(i) The equations of the form: $l\,(ax^2 + bx) + m\sqrt{ax^2 + bx + c} = 0$

(ii) The equations of the form: $\sqrt{x + a} + \sqrt{x + b} = \sqrt{x + c}$

(iii) The equations of the form: $\sqrt{ax^2 + bx + c} + \sqrt{px^2 + qx + r} = \sqrt{1x^2 + mx + n}$

(iv) The equations of the form: $\sqrt{ax^2 + bx + c} + \sqrt{px^2 + qx + r} = mx + n$

EXERCISE 4.3

Solve the following equation:

Q.1 $\mathbf{3x^2 + 2x - \sqrt{3x^2 + 2x - 1} = 3}$

Solution:

$3x^2 + 2x - \sqrt{3x^2 + 2x - 1} = 3 \qquad \ldots\ldots\ldots (1)$

Put $\sqrt{3x^2 + 2x - 1} = y \qquad \ldots\ldots\ldots (2)$

$\Rightarrow \quad 3x^2 + 2x - 1 = y^2$

$\Rightarrow \quad 3x^2 + 2x = y^2 + 1$

$\Rightarrow$ equation (1) becomes

$y^2 + 1 - y = 3$

$\Rightarrow \quad y^2 - y + 1 - 3 = 0$

$\Rightarrow \quad y^2 - y - 2 = 0$

$\Rightarrow \quad y^2 - 2y + y - 2 = 0$

$\Rightarrow \quad y\,(y - 2) + 1\,(y - 2) = 0$