$$= \sqrt{(-6)^2 + (6)^2}$$
$$= \sqrt{36 + 36}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2}$$

In centre of a Δ ABC is
$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

$$\Delta\text{ ABC} = \left(\frac{5\sqrt{2}\,(4) + 5\sqrt{2}\,(-2) + 6\sqrt{2}\,(5)}{5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}}, \frac{5\sqrt{2}\,(-2) + 5\sqrt{2}\,(4)}{5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}}\right)$$
$$= \left(\frac{\sqrt{2}\,(20 - 10 + 30)}{16\sqrt{2}}, \frac{\sqrt{2}\,(-10 + 20 + 30)}{16\sqrt{2}}\right)$$
$$= \left(\frac{40}{16}, \frac{40}{16}\right)$$
$$= \left(\frac{5}{2}, \frac{5}{2}\right) \quad \text{Ans.}$$

Q.18: Find the points that divide the line segment joining A (x_1, y_1) and B (x_2, y_2) into four equal parts.

Solution:

A (x_1, y_1), B (x_2, y_2)

Let C, D and E be the required points.

$$\text{Coordinates of C} = \left(\frac{1(x_2) + 3(x_1)}{1+3}, \frac{1(y_2) + 3(y_1)}{1+3}\right)$$
$$= \left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right)$$
$$\text{Coordinates of D} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$\text{Coordinates of E} = \left(\frac{1(x_1) + 3(x_2)}{1+3}, \frac{1(y_1) + 3(y_2)}{1+3}\right)$$
$$= \left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right) \quad \text{Ans.}$$

EXERCISE 4.2

Q.1: The two points P and O/ are given in xy – coordinate system. Find the XY-Coordinates of P referred to the translated axes O/X and O/Y.

(i) P(3, 2) ; O/(1, 3) **(ii) P(–2, 6); O/(–3, 2) (Lhr. Board 2011)**

(iii) P(–6, –8); O/(– 4, – 6) **(iv) P$\left(\frac{3}{2}, \frac{5}{2}\right)$; O/$\left(-\frac{1}{2}, \frac{7}{2}\right)$**

Solution:

(i) $\mathbf{P(3, 2); \quad O'(1, 3)}$

$x = 3, \quad y = 2, \quad h = 1, \quad k = 3$

By using

$X = x - h, \quad Y = y - k$

$X = 3 - 1, \quad Y = 2 - 3$

$X = 2, \quad Y = -1$

$\therefore \quad P(X, Y) = P(2, -1)$ Ans

(ii) $\mathbf{P(-2, 6); O'(-3, 2)}$

$x = -2, \quad y = 6, \quad h = -3, \quad k = 2$

By using

$X = x - h, \quad Y = y - k$

$X = -2 + 3, \quad Y = 6 - 2$

$X = 1, \quad Y = 4$

$\therefore \quad P(X, Y) = P(1, 4)$ Ans

(iii) $\mathbf{P(-6, -8); \quad O'(-4, -6)}$

$x = -6, \quad y = -8, \quad h = -4, \quad k = -6$

By using

$X = x - h, \quad Y = y - k$

$X = -6 + 4, \quad Y = -8 + 6$

$X = -2, \quad Y = -2$

$\therefore \quad P(X, Y) = P(-2, -2)$ Ans

(iv) $\mathbf{P\left(\frac{3}{2}, \frac{5}{2}\right); \; O'\left(\frac{-1}{2}, \frac{7}{2}\right)}$

$x = \frac{3}{2}, \quad y = \frac{5}{2}, \quad h = \frac{-1}{2}, \quad k = \frac{7}{2}$

By using

$X = x - h, \quad Y = y - k$

$X = \frac{3}{2} + \frac{1}{2}, \quad Y = \frac{5}{2} - \frac{7}{2}$

$X = \frac{3+1}{2}, \quad Y = \frac{5-7}{2}$

$X = \frac{4}{2}, \quad Y = \frac{-2}{2}$

$X = 2, \quad Y = -1$

$\therefore \quad P(X, Y) = P(2, -1)$ Ans

Q.2: The xy-coordinate axes are translated through the point O' whose coordinates are given in xy-coordinate system. The coordinates of P are given in the XY-coordinate system. Find the coordinates of P in xy-coordinate system.

(i) $P(8, 10)$; $O'(3, 4)$ **(ii)** $P(-5, -3)$; $O'(-2, -6)$

(iii) $P\left(\frac{-3}{4}, \frac{-7}{6}\right)$; $O'\left(\frac{1}{4}, \frac{-1}{6}\right)$ **(iv)** $P(4, -3)$; $O'(-2, 3)$

Solution:

(i) $P(8, 10)$; $O'(3, 4)$

$X = 8$, $Y = 10$, $h = 3$, $k = 4$

By using

$$X = x - h, \quad Y = y - k$$
$$8 = x - 3, \quad 10 = y - 4$$
$$x = 8 + 3, \quad y = 10 + 4$$
$$x = 11, \quad y = 14$$

$\therefore$ $P(x, y) = P(11, 14)$ Ans

(ii) $P(-5, -3)$; $O'(-2, -6)$

$X = -5$, $Y = -3$, $h = -2$, $k = -6$

By using

$$X = x - h, \quad Y = y - k$$
$$-5 = x + 2, \quad -3 = y + 6$$
$$x = -5 - 2, \quad y = -3 - 6$$
$$x = -7, \quad y = -9$$

$\therefore$ $P(x, y) = P(-7, -9)$ Ans

(iii) $P\left(\frac{-3}{4}, \frac{-7}{6}\right)$; $O'\left(\frac{1}{4}, \frac{-1}{6}\right)$

$X = \frac{-3}{4}$, $Y = \frac{-7}{6}$, $h = \frac{1}{4}$, $k = \frac{-1}{6}$

By using

$$X = x - h, \quad Y = y - k$$
$$\frac{-3}{4} = x - \frac{1}{4}, \quad \frac{-7}{6} = y + \frac{1}{6}$$
$$x = \frac{-3}{4} + \frac{1}{4}, \quad y = \frac{-7}{6} - \frac{1}{6}$$
$$x = \frac{-3+1}{4}, \quad y = \frac{-7-1}{6}$$

$x = \frac{-2}{4}$, $y = \frac{-8}{6}$

$x = \frac{-1}{2}$, $y = \frac{-4}{3}$

$\therefore$ $P(x, y) = P\left(\frac{-1}{2}, \frac{-4}{3}\right)$ Ans.

(iv) P(4, – 3) ; O′(– 2, 3)

$X = 4$, $Y = -3$, $h = -2$, $k = 3$

By using

$X = x - h$, $Y = y - K$

$4 = x + 2$, $-3 = y - 3$

$x = 4 - 2$, $y = -3 + 3$

$x = 2$, $y = 0$

$\therefore$ $P(x, y) = P(2, 0)$ Ans

Q.3: The xy-coordinate axes are rotated about the origin through the indicated angle. The new axes are OX and OY. Find the XY-coordinates of the point P with the given xy-coordinates.

Solution:

(i) P(5, 3) ; θ = 45°

$x = 5$, $y = 3$

By using

$X = x\cos\theta + y\sin\theta$, $Y = y\cos\theta - x\sin\theta$

$X = 5\cos 45^\circ + 3\sin 45^\circ$ $Y = 3\cos 45^\circ - 5\sin 45^\circ$

$X = \frac{5}{\sqrt{2}} + \frac{3}{\sqrt{2}}$, $Y = \frac{3}{\sqrt{2}} - \frac{5}{\sqrt{2}}$

$X = \frac{5+3}{\sqrt{2}}$, $Y = \frac{3-5}{\sqrt{2}}$

$X = \frac{8}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$, $Y = \frac{-2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$X = \frac{8\sqrt{2}}{2}$, $Y = \frac{-2\sqrt{2}}{2}$

$X = 4\sqrt{2}$, $Y = -\sqrt{2}$

$\therefore$ $P(X, Y) = P(4\sqrt{2}, -\sqrt{2})$ Ans

(ii) P(3, – 7) ; θ = 30°

$x = 3$, $y = -7$

By using

$X = x\cos\theta + y\sin\theta$, $Y = y\cos\theta - x\sin\theta$

$$X = 3\cos 30^\circ - 7\sin 30^\circ \qquad Y = -7\cos 30^\circ - 3\sin 30^\circ$$

$$X = \frac{3\sqrt{3}}{2} - \frac{7}{2} \quad , \quad Y = \frac{-7\sqrt{3}}{2} - \frac{3}{2}$$

$$X = \frac{3\sqrt{3}-7}{2} \quad , \quad Y = \frac{-7\sqrt{3}-3}{2}$$

$$\therefore \quad P(X, Y) = P\left(\frac{3\sqrt{3}-7}{2}, \frac{-7\sqrt{3}-3}{2}\right) \text{Ans}$$

(iii) P (11, – 15); $\theta = 60^\circ$

$x = 11, \quad y = -15$

By using

$$X = x\cos\theta + y\sin\theta \quad , \quad Y = y\cos\theta - x\sin\theta$$

$$X = 11\cos 60^\circ - 15\sin 60^\circ \quad , \quad Y = -15\cos 60^\circ - 11\sin 60^\circ$$

$$X = \frac{11}{2} - \frac{15\sqrt{3}}{2} \qquad Y = \frac{-15}{2} - \frac{11\sqrt{3}}{2}$$

$$X = \frac{11-15\sqrt{3}}{2} \qquad Y = \frac{-15-11\sqrt{3}}{2}$$

$$\therefore \quad P(X, Y) = P\left(\frac{11-15\sqrt{3}}{2} \quad \frac{-15-11\sqrt{3}}{2}\right) \quad \text{Ans}$$

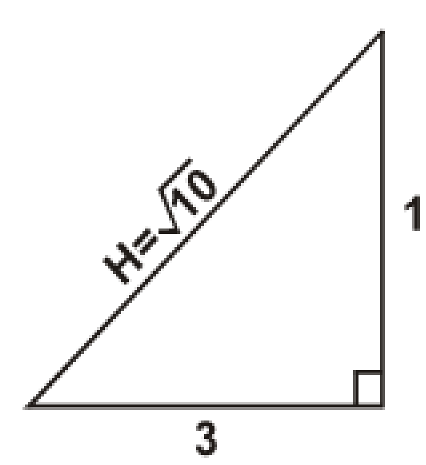
(iv) P(15, 10) ; $\theta = \arctan\frac{1}{3}$

$$x = 15, \quad y = 10 \quad , \quad \theta = \tan^{-1}\frac{1}{3}$$

$$\tan\theta = \frac{1}{3}$$

$$\sin\theta = \frac{1}{\sqrt{10}}, \quad \cos\theta = \frac{3}{\sqrt{10}}$$

$$H^2 = (3)^2 + (1)^2$$
$$H^2 = 9 + 1$$
$$H^2 = 10$$
$$H = \sqrt{10}$$



By using

$$X = x\cos\theta + y\sin\theta \quad , \quad Y = y\cos\theta - x\sin\theta$$

$$X = 15\left(\frac{3}{\sqrt{10}}\right) + 10\left(\frac{1}{\sqrt{10}}\right)$$

$$X = \frac{45}{\sqrt{10}} + \frac{10}{\sqrt{10}}$$

$$X = \frac{45+10}{\sqrt{10}} = \frac{55}{\sqrt{10}}$$

$$Y = 10\left(\frac{3}{\sqrt{10}}\right) - 15\left(\frac{1}{\sqrt{10}}\right)$$

$$Y = \frac{30}{\sqrt{10}} - \frac{15}{\sqrt{10}}$$

$$Y = \frac{30-15}{\sqrt{10}}$$

$$Y = \frac{15}{\sqrt{10}}$$

$\therefore \quad P(X, Y) = P\left(\frac{55}{\sqrt{10}}, \frac{15}{\sqrt{10}}\right)$ Ans.

Q.4: The xy-coordinate axes are rotated about the origin through the indicated angle and the new axes are OX and OY. Find the xy-coordinates of P with the given XY-Coordinates.

(i) $P(-5, 3)$; $\theta = 30^o$ **(ii) $P(-7\sqrt{2}, 5\sqrt{2})$; $\theta = 45^o$**

Solution:

(i) $P(-5, 3)$; $\theta = 30^o$

$X = -5 \quad , \quad Y = 3$

By using $X = x\cos\theta + y\sin\theta \quad , \quad Y = y\cos\theta - x\sin\theta$

$-5 = x\cos 30^o + y\sin 30^o \quad , \quad 3 = y\cos 30^o - x\sin 30^o$

$-5 = \frac{x\sqrt{3}}{2} + \frac{y}{2}, \qquad 3 = \frac{y\sqrt{3}}{2} - \frac{x}{2}$

$3 = \frac{\sqrt{3}y - x}{2}$

$-5 = \frac{\sqrt{3}x + y}{2} \quad , \sqrt{3}\,y - x = 6 \quad \ldots\ldots (2)$

$\sqrt{3}\,x + y = -10 \qquad \ldots. (1)$

Equation (1) + Equation (2) $\times \sqrt{3}$, we get

$$\begin{aligned} \sqrt{3}\,x + y &= -10 \\ 3y - \sqrt{3}\,x &= 6\sqrt{3} \\ \hline 4y &= -10 + 6\sqrt{3} \end{aligned}$$

$$y = \frac{2(-5+3\sqrt{3})}{4}$$

$$y = \frac{-5+3\sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}-5}{2}$$

Put $y = \frac{3\sqrt{3}-5}{2}$ in equation (1)

$$\sqrt{3}\; x + y = -10$$

$$\sqrt{3}\; x = -10 - y$$

$$\sqrt{3}\; x = -10 - \frac{3\sqrt{3}-5}{2}$$

$$\sqrt{3}\; x = \frac{-20-3\sqrt{3}+5}{2}$$

$$x = \frac{-15-3\sqrt{3}}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{3(-5-\sqrt{3})\sqrt{3}}{2(3)}$$

$$= \frac{-5\sqrt{3}-3}{2}$$

$$\therefore \quad P(x, y) = P\left(\frac{-5\sqrt{3}-3}{2}, \frac{3\sqrt{3}-5}{2}\right) \quad \text{Ans}$$

(ii) $\mathbf{P(-7\sqrt{2}, 5\sqrt{2})}$; $\boldsymbol{\theta = 45^{o}}$

$X = -7\sqrt{2}$, $Y = 5\sqrt{2}$

By using

$$X = x\cos\theta + y\sin\theta ,$$

$$-7\sqrt{2} = x\cos 45^{o} + y\sin 45^{o}$$

$$Y = y\cos\theta - x\sin\theta$$

$$Y = y\cos 45^{o} - x\sin 45^{o}$$

$$-7\sqrt{2} = \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} \quad , \quad 5\sqrt{2} = \frac{y}{\sqrt{2}} - \frac{x}{\sqrt{2}}$$

$$-7\sqrt{2} = \frac{x+y}{\sqrt{2}} \quad , \quad 5\sqrt{2} = \frac{y-x}{\sqrt{2}}$$

$$x + y = -14 \quad \ldots..(1) \qquad y - x = 10 \quad \ldots..(2)$$

Adding equation (1) and equation (2), we get

$$\begin{aligned} x + y &= -14 \\ \underline{y - x} &= \underline{10} \\ 2y &= -4 \\ y &= \frac{-4}{2} = -2 \end{aligned}$$

Put $y = -2$ in equation (1)

$$\begin{aligned} x - 2 &= -14 \\ x &= -14 + 2 \\ &= -12 \end{aligned}$$

$\therefore$ $P(x, y) = P(-12, -2)$ Ans

EXERCISE 4.3

Q.1 Find the slope and inclination of the line joining the points:

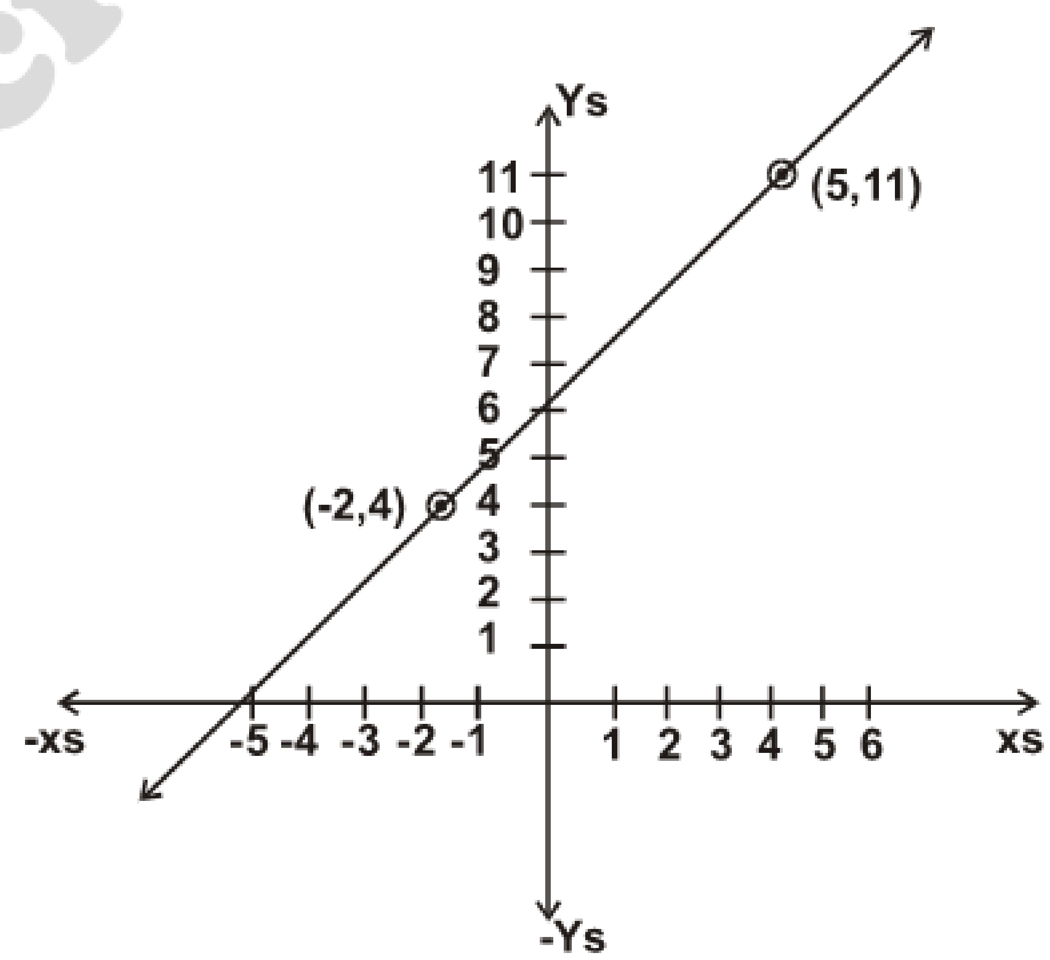
(i) (– 2, 4) ; (5, 11) (ii) (3, – 2) ; (2, 7)

(iii) (4, 6) ; (4, 8)

Solution:

(i) $(-2, 4)$; $(5, 11)$

Let $A(-2, 4)$; $B(5, 11)$



Slope of line AB $= m = \dfrac{11 - 4}{5 + 2}$

$$= \frac{7}{7} = 1$$

$$\tan\alpha = m$$

$$\tan\alpha = 1$$

$$\alpha = \tan^{-1}(1) = 45^\circ$$

(ii) $(3, -2)$; $(2, 7)$

Let $A(3, -2)$, $B(2, 7)$

Slope of line AB $= m = \dfrac{7 + 2}{2 - 3} = \dfrac{9}{-1} = -9$

$$\tan\alpha = m = -9$$

$$\begin{aligned} \alpha &= \tan^{-1}(-9) = 180^\circ - \tan^{-1} 9 \\ &= 180 - 83.66^\circ \\ &= 96.34^\circ \quad \text{Ans.} \end{aligned}$$