Put equation (1) in equation (2), we get

$5\,[9\,(1-r)]^2 = 81\,(1-r^2)$

$5 \times 81\,(1-r)^2 = 81\,(1-r)\,(1+r)$

$5\,(1-r) = 1+r$

$5-5r = 1+r$

$5r+r-5+1 = 0$

$6r-4 = 0 \Rightarrow r = \dfrac{4}{6} \Rightarrow \boxed{r = \dfrac{2}{3}}$

Put $r = \dfrac{2}{3}$ in equation (1), we get

$a = 9\left(1-\dfrac{2}{3}\right)$

$a = 9\left(\dfrac{1}{3}\right) = 3$

so

the required series is

$a + a\,r + a\,r^2 + \ldots\ldots$

i.e. $3 + 3\left(\dfrac{2}{3}\right) + 3\left(\dfrac{2}{3}\right)^2 + 3\left(\dfrac{2}{3}\right)^3 + \ldots..$

or $3 + 2 + \dfrac{4}{3} + \dfrac{8}{9}\ldots\ldots.$

EXERCISE 6.9

Q.1 A man deposits in a bank Rs. 8 in the first year, Rs. 24 in the second year, Rs. 72 in the third year and so on. Find the amount he will have deposited in the bank by the fifth year.

Solution:

Deposited amount is given by

$8 + 24 + 72 + \ldots\ldots$ which is a G.P

Here $a_1 = 8, \quad r = 3, \quad n = 5, \quad S_5 = ?$

As $S_n = \dfrac{a_1\,(r^n-1)}{r-1}$

$S_5 = \dfrac{8\,(3^5-1)}{(3-1)}$

$= \dfrac{8\,(243-1)}{2} = \text{Rs. } 968$

Q.2 A man borrows Rs. 32769 without interest and agrees to repay the loan installments, each installment being twice the proceeding one. Find amount of the last installment, if the amount of the first installment is Rs. 8.

Solution:

Let a_1 be the first installment

then $a_1 = 8$

$\Rightarrow \quad a_2 = 16$ and so on

$\Rightarrow \quad r = 2 \quad , \quad S_n = 32760$

using $\quad S_n = \dfrac{a(r^n - 1)}{r - 1}$

$\Rightarrow \quad 32760 = \dfrac{8(2^n - 1)}{2 - 1} = 8(2^n - 1)$

$\Rightarrow \quad \dfrac{32760}{8} = 2^n - 1$

$\Rightarrow \quad 4095 + 1 = 2^n$

$\Rightarrow \quad 4096 = 2^n$

$\Rightarrow \quad 2^n = 2^{12}$

$\Rightarrow \quad \boxed{n = 12}$

Now we will find a_{12}

As $\quad a_n = ar^{n-1}$

$a_{12} = (8)(2)^{12-1}$

$= 8(2)^{11}\ 8(2048) = 16384$

$a_{12} =$ Rs. 16384

is the amount of last installment

Q.3 The population of a certain village is 62500. What will be its population after 3 years if it increases at the rate of 4% annually?

Solution:

Given population = 62500

After first year = 62500 + 4% of 62500

= 62500 + (0.04) (62500) = 65000

After second year = 65000 + (0.04) (65000) = 67600

After third year = 67600 + (0.04) (67600) = 70304

So the population after 3rd year = 70304

Q.4 The enrollment of a famous school doubled after every eight year from 1970 to 1994. If the enrollment was 6000 in 1994, what was its enrollment in 1970?

Solution:

Let enrollment in 1970 = a

Then enrollment in 1978 = 2a

enrollment in 1986 = 4a

enrollment in 1994 = 8a

But it is given that

Enrolment in 1994 = 6000

$\Rightarrow$ $8a = 6000$

$a = \dfrac{6000}{8}$

$a = 750$

$\Rightarrow$ enrollment in 1970 = a = 750

Q.5 A singular cholera bacteria produces two bacteria in $\frac{1}{2}$ hour. If we start with a colony of bacteria, how many bacteria will we have in n hours?

Solution:

If at the start

Colony of bacteria = A

After every $\frac{1}{2}$ hour interval

number of bacteria are

A, 2A, 4A, 8A, 16A, ...

In one hour interval, number of bacterias are

A, 4A, 16A ……… which is a G.P

$a_1 = A, \quad r = \dfrac{4A}{A} = 4, \quad n = n + 1$

we will find a_{n+1} ¤ As A is present already

As $a_n = a_1 r^{n-1}$

$a_{n+1} = a_1 r^n$

$= A(4)^n = A4^n$

$= A\,2^{2n}$ is Answer

$\Rightarrow$ number of bacteria after n hours = $4^n A$

Q.6 Joining mid points of the sides of an equilateral triangle, an equilateral triangle having half the perimeter of the original triangle is obtained. We form a sequence of rested equilateral triangles in the manner described with the original triangle having perimeter $\frac{3}{2}$. What will be total perimeter of all triangles formed in this way?

Solution:

Perimeter of original triangle $= \frac{3}{2}$

Perimeter of the second triangle $= \dfrac{\frac{3}{2}}{2} = \frac{3}{4}$

Perimeter of the third triangle $= \dfrac{\frac{3}{4}}{2} = \frac{3}{8}$

and so on.

We can write them as

$\frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \ldots\ldots..$ which is a G.P.

we have to find the sum of this infinite G.P.

Here $a_1 = \frac{3}{2}, \quad r = \dfrac{\frac{3}{4}}{\frac{3}{2}} = \frac{1}{2},$

so using

$$S_\infty = \frac{a_1}{1-r}$$

$$= \frac{\frac{3}{2}}{1-\frac{1}{2}} = \frac{\frac{3}{2}}{\frac{1}{2}} = 3$$

$\Rightarrow$ Total perimeter of all the triangles $= 3$.

Harmonic Progression: (H. P.)

A sequence of numbers is called a harmonic sequence or harmonic progression if the reciprocals of its terms are in arithmetic progression.

e.g. $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}$ are in H.P

since

$1, 3, 5, 7$ are in A.P.

nth term or general term of H.P is given by

$$\frac{1}{a_1 + (n-1)d}$$

Harmonic Mean

A number H is said to be the harmonic mean (H.M) between two numbers a and b if a, H, b are in H.P

Also $H = \frac{2ab}{a+b}$

EXERCISE 6.10

Q.1 Find the 9th term of the harmonic sequence

(i) $\mathbf{\frac{1}{3}, \frac{1}{5}, \frac{1}{7} \ldots\ldots\ldots}$ **(ii)** $\mathbf{\frac{-1}{5}, \frac{-1}{3}, -1 \ldots\ldots}$

Solution:

(i) $\mathbf{\frac{1}{3}, \frac{1}{5}, \frac{1}{7} \ldots\ldots\ldots}$

Given sequence

$\frac{1}{3}, \frac{1}{5}, \frac{1}{7}$ .... which is H.P

$3, 5, 7, \ldots\ldots\ldots$ is A.P

$a_1 = 3, \quad d = 5 - 3 = 2,$

As

$$a_n = a + (n-1)d$$

$$a_9 = a_1 + 8d$$

$$= 3 + 8(2) = 3 + 16 = 19$$

$\Rightarrow$ 9th term of A.P $= 19$

$\Rightarrow$ 9th term of H.P $= \frac{1}{19}$

(ii) $\mathbf{\frac{-1}{5}, \frac{-1}{3}, -1 \ldots\ldots}$

Given sequence

$\frac{-1}{5}, \frac{-1}{3}, -1$ ...... which is H.P