Chapter 6

SEQUENCES AND SERIES

SEQUENCE

A sequence is a function whose domain is a subset of the set of natural numbers. First term of the sequence is usually denoted by a_1, second by a_2, third by a_3 and similarly nth term of the sequence is denoted by a_n.

EXERCISE 6.1

Q.1 Write the first four terms of the following sequences, if

(i) $a_n = 2n - 3$ **(ii)** $a_n = (-1)^n n^2$

(iii) $a_n = (-1)^n (2n - 3)$ **(iv)** $a_n = 3n - 5$

(v) $a_n = \dfrac{n}{2n + 1}$ **(vi)** $a_n = \dfrac{1}{2^n}$

(vii) $a_n - a_{n-1} = n + 2,\ a_1 = 2$

(viii) $a_n = n\, a_{n-1},\ a_1 = 1$

(ix) $a_n = (n + 1)\, a_{n-1},\ a_1 = 1$

(x) $a_n = \dfrac{1}{a + (n - 1)\, d}$

Solution:

(i) $a_n = 2n - 3$

Given

$a_n = 2n - 3$

$n = 1, 2, 3, 4$

$n = 1 \Rightarrow a_1 = 2(1) - 3 = 2 - 3 = -1$

$n = 2 \Rightarrow a_2 = 2(2) - 3 = 4 - 3 = 1$

$n = 3 \Rightarrow a_3 = 2(3) - 3 = 6 - 3 = 3$

$n = 4 \Rightarrow a_4 = 2(4) - 3 = 8 - 3 = 5$

So the required terms are $-1, 1, 3, 5$.

(ii) $\mathbf{a_n = (-1)^n n^2}$

Given sequence

$a_n = (-1)^n n^2$

Put $n = 1, 2, 3, 4$

$n = 1 \Rightarrow a_1 = (-1)(1)^2 = -1$

$n = 2 \Rightarrow a^2 = (-1)^2 (2)^2 = 4$

$n = 3 \Rightarrow a_3 = (-1)^3 (3)^2 = (-1)(9) = -9$

$n = 4 \Rightarrow a_4 = (-1)^4 (4)^2 = (1)(16) = 16$

So required terms are $-1, 4, -9, 16$

(iii) $\mathbf{a_n = (-1)^n (2n - 3)}$

Given sequence

$a_n = (-1)^n (2n - 3)$

Put $n = 1, 2, 3, 4$

$n = 1 \Rightarrow a_1 = (-1)^1 (2(1) - 3) = -(2 - 3) = -(-1) = 1$

$n = 2 \Rightarrow a_2 = (-1)^2 (2(2) - 3) = 4 - 3 = 1$

$n = 3 \Rightarrow a_3 = (-1)^3 (2(3) - 3) = (-1)(6 - 3) = -3$

$n = 4 \Rightarrow a_4 = (-1)^4 (2(4) - 3) = 8 - 3 = 5$

So required terms are $1, 1, -3, 5$

(iv) $\mathbf{a_n = 3n - 5}$

Given sequence

$a_n = 3n - 5$

Put $n = 1, 2, 3, 4, 5$

$n = 1 \Rightarrow a_1 = 3(1) - 5 = 3 - 5 = -2$

$n = 2 \Rightarrow a_2 = 3(2) - 5 = 6 - 5 = 1$

$n = 3 \Rightarrow a_3 = 3(3) - 5 = 9 - 5 = 4$

$n = 4 \Rightarrow a_4 = 3(4) - 5 = 12 - 5 = 7$

so the required terms are $-2, 1, 4, 7$

(v) $\mathbf{a_n = \dfrac{n}{2n + 1}}$

Given sequence

$a_n = \dfrac{n}{2n + 1}$

Put n = 1, 2, 3, 4

$n = 1 \Rightarrow a_1 = \frac{1}{2(1)+1} = \frac{1}{2+1} = \frac{1}{3}$

$n = 2 \Rightarrow a_2 = \frac{2}{2(2)+1} = \frac{2}{4+1} = \frac{2}{5}$

$n = 3 \Rightarrow a_3 = \frac{3}{2(3)+1}\frac{3}{6+1} = \frac{3}{7}$

$n = 4 \Rightarrow a_4 = \frac{4}{2(4)+1} = \frac{4}{8+1} = \frac{4}{9}$

so the required terms are $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}$

(vi) $\mathbf{a_n = \frac{1}{2^n}}$

Given sequence

$a_n = \frac{1}{2^n}$

Put n = 1, 2, 3, 4

$n = 1 \Rightarrow a_1 = \frac{1}{2^1} = \frac{1}{2}$

$n = 2 \Rightarrow a_2 = \frac{1}{2^2} = \frac{1}{4}$

$n = 3 \Rightarrow a_3 = \frac{1}{2^3} = \frac{1}{8}$

$n = 4 \Rightarrow a_4 = \frac{1}{2^4} = \frac{1}{16}$

so the required terms are $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$

(vii) $\mathbf{a_n - a_{n-1} = n + 2,\ a_1 = 2}$

Given

$a_n - a_{n-1} = n + 2, \qquad (1) \quad a_1 = 2$

Put n = 2 in (1)

$a_2 - a_1 + 2 + 2$

$a_2 - 2 = 4$

$a_2 = 4 + 2 = 6$

Put $n = 3$ in (1)

$a_3 - a_2 = 3 + 2$

$a_3 - 6 = 5$

$a_3 = 6 + 5 = 11$

Put $n = 4$ in (1)

$a_4 - a_3 = 4 + 2$

$a_4 - 11 = 6$

$a_4 = 6 + 11 = 17$

$\Rightarrow$ required terms are 2, 6, 11, 17

(viii) $\mathbf{a_n = n\, a_{n-1},\ a_1 = 1}$

Given

$a_n = n\, a_{n-1}$, (1) $a_1 = 1$

Put $n = 2$ in (1)

$a_2 = 2a_1 = 2(1) = 2$ ¤ $a_1 = 1$

Put $n = 3$ in (1)

$a_3 = 3a_2 = 3(2) = 6$

Put $n = 4$ in (1)

$a_4 = 4a_3$

$a_4 = 4(6) = 24$

$\Rightarrow$ required terms are 1, 2, 6, 24

(ix) $\mathbf{a_n = (n+1)\, a_{n-1},\ a_1 = 1}$

Given

$a_n = (n+1)\, a_{n-1}$ $a_1 = 1$ (1)

Put $n = 2$ in (1)

$a_2 = (2+1)\, a_1 = 3a_1 = 3(1) = 3$

Put $n = 3$ in (1)

$a_3 = (3+1)\, a_2 = 4a_2 = 4(3) = 12$

Put $n = 4$ in (1)

$a_4 = (4+1)\, a_3 = 5a_3 = 5(12) = 60$

$\Rightarrow$ required terms are 1, 3, 12, 60

(x) $\mathbf{a_n = \dfrac{1}{a + (n-1)\,d}}$

Given

$a_n = \dfrac{1}{a + (n-1)\,d}$

Put $n = 1, 2, 3, 4$

$n = 1 \Rightarrow a_1 = \dfrac{1}{a + (1-1)\,d} = \dfrac{1}{a+0} = \dfrac{1}{a}$

Put $n = 2 \Rightarrow a_2 = \dfrac{1}{a + (2-1)\,d} = \dfrac{1}{a+d}$

Put $n = 3 \Rightarrow a_3 = \dfrac{1}{a + (3-1)\,d} = \dfrac{1}{a+2d}$

Put $n = 4 \Rightarrow a_4 = \dfrac{1}{a + (4-1)\,d} = \dfrac{1}{a+3d}$

Q.2 Find the indicated terms of the following sequences:

(i) **2, 6, 11, 17, ... a_7** **(ii)** **1, 3, 12, 60, ... a_6**

(iii) **$1, \frac{3}{2}, \frac{5}{4}, \frac{7}{8}$, ... a_7** **(iv)** **1, 1, – 3, – 7, ... a_8**

(v) **1, – 3, 5, – 7, 9, – 11, ... a_8**

Solution:

Given

2, 6, 11, 17, ... a_7

from the given sequence, we have

$a_1 = 2$ $a_2 = 2 + 4 = 6$

$a_3 = 6 + 5 = 11$ $a_4 = 11 + 6 = 17$

$a_5 = 17 + 7 = 24$ $a_6 = 24 + 8 = 32$

$a_7 = 32 + 9 = 41$

$\Rightarrow$ required term is $a_7 = 41$

(ii) **1, 3, 12, 60, ... a_6**

Given

1, 3, 12, 60, ….. a_6

from the given sequence, we have

$a_1 = 1$ $a_2 = 1 \times 3 = 3$

$a_3 = 3 \times 4 = 12$ $a_4 = 12 \times 5 = 60$

$a_5 = 60 \times 6 = 360$ $a_6 = 360 \times 7 = 2520$

so the required term $= a_6 = 2520$.

(iii) **$1, \frac{3}{2}, \frac{5}{4}, \frac{7}{8}, \ldots\ldots a_7$**

Given

$1, \frac{3}{2}, \frac{5}{4}, \frac{7}{8}, \ldots\ldots a_7$

from the given sequence

$a_1 = 1$ $\qquad a_2 = \frac{1+2}{1(2)} = \frac{3}{2}$

$a_3 = \frac{3+2}{2(2)} = \frac{5}{4}$ $\qquad a_4 = \frac{5+2}{4(2)} = \frac{7}{8}$

$a_5 = \frac{7+2}{8(2)} = \frac{9}{16}$ $\qquad a_6 = \frac{9+2}{16(2)} = \frac{11}{32}$

$a_7 = \frac{11+2}{32(2)} = \frac{13}{64}$

$\Rightarrow$ required term $= a_7 = \frac{13}{64}$

(iv) **$1, 1, -3, 5, -7, \ldots\ldots a_8$**

Given

$1, 1, -3, 5, -7, \ldots\ldots a_8$

from the given sequence

$a_1 = 1$ $\qquad a_2 = 1$

$a_3 = 1 - 4 = -3$ $\qquad a_4 = 1 + 4 = 5$

$a_5 = -3 - 4 = -7$ $\qquad a_6 = 5 + 4 = 9$

$a_7 = -7 - 4 = -11$ $\qquad a_8 = 9 + 4 = 13$

(v) **$1, -3, 5, -7, 9, -11, \ldots\ldots a_8$**

Given

$1, -3, 5, -7, 9, -11, \ldots\ldots a_8$

From the given sequence

$a_1 = 1$ $\qquad a_2 = -3$

$a_3 = 5$ $\qquad a_4 = -7$

$a_5 = 9$ $\qquad a_6 = -11$

$a_7 = 13$ $\qquad a_8 = -15$

Hence required term $= a_8 = -15$

Q.3 Find the next two terms of the following sequences.

(i) 7, 9, 12, 16, ... **(ii) 1, 3, 7, 15, 31, ...**

(iii) – 1, 2, 12, 40, ... **(iv) 1, – 3, 5, – 7, 9, – 11, ...**

Solution:

(i) 7, 9, 12, 16, ...

Given sequence

7, 9, 12, 16, ...

$\Rightarrow$ $a_1 = 7$ $a_2 = 7 + 2 = 9$

$a_3 = 9 + 3 = 12$ $a_4 = 12 + 4 = 16$

$a_5 = 16 + 5 = 21$ $a_6 = 21 + 6 = 27$

so next two terms are 21, 27

(ii) 1, 3, 7, 15, 31, ...

Given sequence

1, 3, 7, 15, 31, ...

$\Rightarrow$ $a_1 = 1$ $a_2 = 1 + 2 = 3$

$a_3 = 3 + 4 = 7$ $a_4 = 7 + 8 = 15$

$a_5 = 15 + 16 = 31$ $a_6 = 31 + 32 = 63$

$a_7 = 63 + 64 = 127$

so next two terms are 63, 127.

(iii) – 1, 2, 12, 40, ...

Given sequence

– 1, 2, 12, 40, ...

$a_1 = -1 \times -1 = -1$

$a_2 = 1 \times 2 = 2$

$a_3 = 3 \times 4 = 12$

$a_4 = 5 \times 8 = 40$

$a_5 = 7 \times 16 = 112$

$a_6 = 9 \times 32 = 288$

$\Rightarrow$ the next two terms of the sequence are 112, 288

(iv) 1, – 3, 5, – 7, 9, – 11, ...

Given sequence

1, – 3, 5, – 7, 9, – 11, ...

$\Rightarrow$ $a_1 = 1$

$a_2 = -3$

$a_3 = 5$

$a_4 = -7$

$a_5 = 9$

$a_6 = -11$

$a_7 = 13$

$a_8 = -15$

$\Rightarrow$ next two terms are $12, -15$.

Arithmetic Progression (A. P)

A sequence $\{a_n\}$ is an Arithmetic sequence or Arithmetic progression (A.P) if $a_n - a_{n-1}$ is the same number for all $n \in N$ and $n > 1$. The difference of two consecutive terms of an A.P is called common difference and is usually denoted by d. $a_n = a_1 + (n-1)d$ is called the nth term or general term of the A.P.

General form of A.P $a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \ldots..$

EXERCISE 6.2

Q.1 Write the first four terms of the following arithmetic sequence, if

(i) $a_1 = 5$ and other three consecutive terms are 23, 26, 29

(ii) $a_5 = 17$ and $a_9 = 37$

(iii) $a_7 = 7a_4$ and $a_{10} = 33$

Solution:

(i) $a_1 = 5$ and other three consecutive terms are 23, 26, 29

As the given sequence is arithmetic sequence so $d = 26 - 23 = 3$

and $a_1 = 5$ (given)

$\Rightarrow$ $a_2 = a_1 + d = 5 + 3 = 8$

$a_3 = a_2 + d = 8 + 3 = 11$

$a_4 = a_3 + d = 11 + 3 = 14$

$\Rightarrow$ first four terms of the sequence are $5, 8, 11, 14$.

(ii) $a_5 = 17$ and $a_9 = 37$

As $a_5 = 17 \Rightarrow a_1 + 4d = 17$ ..... (1)

$a_9 = 37 \Rightarrow a_1 + 8d = 37$ ..... (2)