EXERCISE 7.2

Q.1 Let A = (2, 5), B (– 1, 1), C (2, – 6) Find (i) $\overrightarrow{AB}$

Solution:

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$= (-1-2)\,\underline{i} + (1-5)\,\underline{j} = -3\underline{i} - 4\underline{j}$$

(ii) $2\overrightarrow{AB} - \overrightarrow{CB}$

Solution:

$$2\overrightarrow{AB} - \overrightarrow{CB}$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$= -3\underline{i} - 4\underline{j}$$

$$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC}$$
$$= (-1-2)\underline{i} + (1+6)\underline{j}$$
$$= -3\underline{i} + 7\underline{j}$$

$$2\overrightarrow{AB} - \overrightarrow{CB} = 2(-3\underline{i} - 4\underline{j}) - (-3\underline{i} + 7\underline{j})$$
$$= -6\underline{i} - 8\underline{j} + 3\underline{i} - 7\underline{j} = -3\underline{i} - 15\underline{j}$$

(iii) $2\overrightarrow{CB} - 2\overrightarrow{CA}$

Solution:

$$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC}$$
$$= (-1-2)\,\underline{i} + (1+6)\,\underline{j} = -3\underline{i} + 7\underline{j}$$

$$\overrightarrow{CA} = \overrightarrow{OA} - \overrightarrow{OC}$$
$$= (2-2)\,\underline{i} + (5+6)\,\underline{j} = 0\underline{i} + 11\underline{j}$$

$$2\overrightarrow{CB} - 2\overrightarrow{CA} = 2(\overrightarrow{CB} - \overrightarrow{CA})$$
$$= 2(-3\underline{i} + 7\underline{j} - 0\underline{i} - 11\underline{j}) = 2(-3\underline{i} - 4\underline{j}) = -6\underline{i} - 8\underline{j}$$

Q.2 **Let $\underline{u} = \underline{i} + 2\underline{j} - \underline{k}$, $\underline{v} = 3\underline{i} - 2\underline{j} + 2\underline{k}$**
$\underline{w} = 5\underline{i} - \underline{j} + 3\underline{k}$. Find the indicated vector or number

(i) $\underline{u} + 2\underline{v} + \underline{w}$

Solution:

$\underline{u} + 2\underline{v} + \underline{w}$

$= (\underline{i} + 2\underline{j} - \underline{k}) + 2(3\underline{i} - 2\underline{j} + 2\underline{k}) + 5\underline{i} - \underline{j} + 3\underline{k}$

$= \underline{i} + 2\underline{j} - \underline{k} + 6\underline{i} - 4\underline{j} + 4\underline{k} + 5\underline{i} - \underline{j} + 3\underline{k}$

$= 12\underline{i} - 3\underline{j} + 6\underline{k}$

(ii) $\underline{v} - 3\underline{w}$

Solution:

$\underline{v} - 3\underline{w}$

$= 3\underline{i} - 2\underline{j} + 2\underline{k} - 3(5\underline{i} - \underline{j} + 3\underline{k})$

$= 3\underline{i} - 2\underline{j} + 2\underline{k} - 15\underline{i} + 3\underline{j} - 9\underline{k} \quad = -12\underline{i} + \underline{j} - 7\underline{k}$

(iii) $|3\underline{v} + \underline{w}|$

Solution:

$|3\underline{v} + \underline{w}|$

$3\underline{v} + \underline{w} = 3(3\underline{i} - 2\underline{j} + 2\underline{k}) + 5\underline{i} - \underline{j} + 3\underline{k}$

$= 9\underline{i} - 6\underline{j} + 6\underline{k} + 5\underline{i} - \underline{j} + 3\underline{k}$

$= 14\underline{i} - 7\underline{j} + 9\underline{k}$

$|3\underline{v} + \underline{w}| = \sqrt{(14)^2 + (-7)^2 + (9)^2}$

$= \sqrt{196 + 49 + 81}$

$|3\underline{v} + \underline{w}| = \sqrt{326}$ Ans.

Q.3 **Find the magnitude of the vector $\underline{v}$ and write the direction cosines of $\underline{v}$.**

(i) $\underline{v} = 2\underline{i} + 3\underline{j} + 4\underline{k}$

Solution:

$\underline{v} = 2\underline{i} + 3\underline{j} + 4\underline{k}$

$|\underline{v}| = \sqrt{(2)^2 + (3)^2 + (4)^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$

direction cosines are

$\left[\dfrac{2}{\sqrt{29}}, \dfrac{3}{\sqrt{29}}, \dfrac{4}{\sqrt{29}}\right]$ Ans.

(ii) $\underline{\mathbf{v}} = \underline{\boldsymbol{i}} - \underline{\mathbf{j}} - \underline{\mathbf{k}}$

Solution:

$\underline{\mathbf{v}} = \underline{\boldsymbol{i}} - \underline{\mathbf{j}} - \underline{\mathbf{k}}$

$|\underline{v}| = \sqrt{(1)^2 + (-1)^2 + (-1)^2} = \sqrt{1+1+1} = \sqrt{3}$

Direction cosines are

$\left[\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right]$ Ans.

(iii) $\underline{\mathbf{v}} = 4\underline{\boldsymbol{i}} - 5\underline{\mathbf{j}}$

Solution:

$\underline{\mathbf{v}} = 4\underline{\boldsymbol{i}} - 5\underline{\mathbf{j}}$

$|\underline{v}| = \sqrt{(4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$

Direction cosines are

$\left[\frac{4}{\sqrt{41}}, \frac{-5}{\sqrt{41}}, 0\right]$ Ans.

Q.4 Find α, so that $|\alpha\underline{i} + (\alpha+1)\underline{j} + 2\underline{k}| = 3$ (Gujranwala Board 2007)

Solution:

$|\alpha\underline{i} + (\alpha+1)\underline{j} + 2\underline{k}| = 3$

$\sqrt{\alpha^2 + (\alpha+1)^2 + (2)^2} = 3$

Taking square on both sides

$\alpha^2 + \alpha^2 + 1 + 2\alpha + 4 = 9$

$2\alpha^2 + 2\alpha + 5 - 9 = 0$

$2\alpha^2 + 2\alpha - 4 = 0$

$\alpha^2 + \alpha - 2 = 0$ (Dividing throughout by 2)

$\alpha^2 + 2\alpha - \alpha - 2 = 0$

$\alpha(\alpha+2) - 1(\alpha+2) = 0$

$(\alpha+2)(\alpha-1) = 0$

$\alpha + 2 = 0 \qquad \alpha - 1 = 0$

$\Rightarrow \quad \alpha = -2 \quad , \quad \alpha = 1$ Ans

Q.5 Find a unit vector in the direction of $\underline{\mathbf{v}} = \underline{\boldsymbol{i}} + 2\underline{\mathbf{j}} - \underline{\mathbf{k}}$

Solution:

$\underline{v} = \underline{i} + 2\underline{j} - \underline{k}$

$|\underline{v}| = \sqrt{(1)^2 + (2)^2 + (-1)^2} = \sqrt{1+4+1} = \sqrt{6}$

Required unit vector is

$\hat{v} = \frac{\underline{v}}{|\underline{v}|} = \frac{\underline{i} + 2\underline{j} - \underline{k}}{\sqrt{6}}$

$$= \frac{1}{\sqrt{6}}\underline{i} + \frac{2}{\sqrt{6}}\underline{j} - \frac{1}{\sqrt{6}}\underline{k} \quad \text{Ans.}$$

Q.6 If $\underline{a} = 3\underline{i} - \underline{j} - 4\underline{k}$, $\underline{b} = -2\underline{i} - 4\underline{j} - 3\underline{k}$ & $\underline{c} = \underline{i} + 2\underline{j} - \underline{k}$. Find a unit vector parallel to $3\underline{a} - 2\underline{b} + 4\underline{c}$ (Gujranwala Board 2004)

Solution:

$3\underline{a} = 3(3\underline{i} - \underline{j} - 4\underline{k}) = 9\underline{i} - 3\underline{j} - 12\underline{k}$

$2\underline{b} = 2(-2\underline{i} - 4\underline{j} - 3\underline{k}) = -4\underline{i} - 8\underline{j} - 6\underline{k}$

$4\underline{c} = 4(\underline{i} + 2\underline{j} - \underline{k}) = 4\underline{i} + 8\underline{j} - 4\underline{k}$

Let $\underline{v} = 3\underline{a} - 2\underline{b} + 4\underline{c} = 9\underline{i} - 3\underline{j} - 12\underline{k} - (-4\underline{i} - 8\underline{j} - 6\underline{k}) + 4\underline{i} + 8\underline{j} - 4\underline{k}$

$= 9\underline{i} - 3\underline{j} - 12\underline{k} + 4\underline{i} + 8\underline{j} + 6\underline{k} + 4\underline{i} + 8\underline{j} - 4\underline{k}$

$\underline{v} = 17\underline{i} + 13\underline{j} - 10\underline{k}$

Now $|\underline{v}| = \sqrt{(17)^2 + (13)^2 + (-10)^2} = \sqrt{289 + 169 + 100} = \sqrt{558}$

$$\hat{\underline{v}} = \frac{\underline{v}}{|\underline{v}|} = \frac{17\underline{i} + 13\underline{j} - 10\underline{k}}{\sqrt{558}} = \frac{17}{\sqrt{558}}\underline{i} + \frac{13}{\sqrt{558}}\underline{j} - \frac{10}{\sqrt{558}}\underline{k} \quad \text{Ans.}$$

Q.7 Find a vector whose

(i) magnitude is 4 and is parallel to $2\underline{i} - 3\underline{j} + 6\underline{k}$

Solution:

Let $\underline{v} = 2\underline{i} - 3\underline{j} + 6\underline{k}$

$|\underline{v}| = \sqrt{(2)^2 + (-3)^2 + (6)^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$

Let $\underline{u}$ be a vector parallel to $\underline{v}$, then

$\underline{u} = \frac{\underline{v}}{|\underline{v}|} = \frac{2\underline{i} - 3\underline{j} + 6\underline{k}}{7}$ (It is a vector whose magnitude is 1 and parallel to $\underline{v}$)

Required vector

$$4\underline{u} = 4\left(\frac{2\underline{i} - 3\underline{j} + 6\underline{k}}{7}\right) = \frac{8}{7}\underline{i} - \frac{12}{7}\underline{j} + \frac{24}{7}\underline{k} \quad \text{Ans.}$$

(ii) magnitude is 2 and is parallel to $-\underline{i} + \underline{j} + \underline{k}$ (Lahore Board 2006)

Solution:

Let $\underline{v} = -\underline{i} + \underline{j} + \underline{k}$

$|\underline{v}| = \sqrt{(-1)^2 + (1)^2 + (1)^2} = \sqrt{3}$

Let $\underline{u}$ is vector parallel to $\underline{v}$

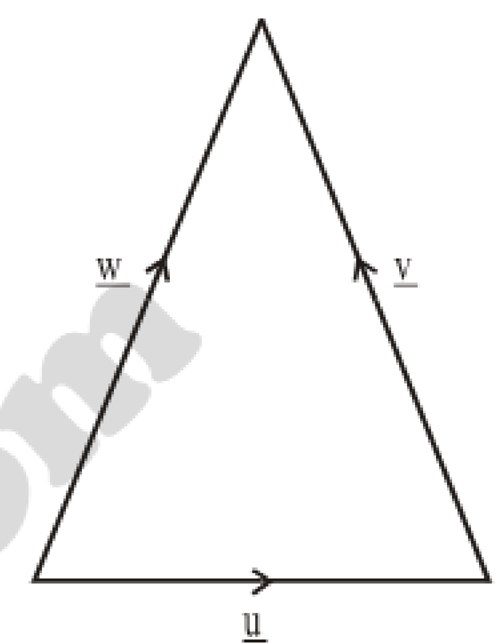
$$\underline{u} = \frac{\underline{v}}{|\underline{v}|} = \frac{-\underline{i} + \underline{j} + \underline{k}}{\sqrt{3}}$$

Required vector

$$2\underline{u} = \frac{2(-\underline{i}+\underline{j}+\underline{k})}{\sqrt{3}} = \frac{-2}{\sqrt{3}}\underline{i} + \frac{2}{\sqrt{3}}\underline{j} + \frac{2}{\sqrt{3}}\underline{k} \qquad \text{Ans.}$$

Q.8 If $\underline{u} = 2\underline{i} + 3\underline{j} + 4\underline{k}$, $\underline{v} = -\underline{i} + 3\underline{j} - \underline{k}$, $\underline{w} = \underline{i} + 6\underline{j} + Z\underline{k}$ represents the sides of a triangle. Find the value of Z.

Solution:



It $\underline{u}$, $\underline{v}$ & $\underline{w}$ represents the sides of a triangle, then by vector addition $\underline{u} + \underline{v} = \underline{w}$

$$2\underline{i} + 3\underline{j} + 4\underline{k} + (-\underline{i} + 3\underline{j} - \underline{k}) = \underline{i} + 6\underline{j} + Z\underline{k}$$

$$2\underline{i} + 3\underline{j} + 4\underline{k} - \underline{i} + 3\underline{j} - \underline{k} = \underline{i} + 6\underline{j} + Z\underline{k}$$

$$\underline{i} + 6\underline{j} + 3\underline{k} = \underline{i} + 6\underline{j} + Z\underline{k}$$

By comparing

$Z = 3$ Ans.

Q.9 The position vectors of the points A, B, C and D are $2\underline{i} - \underline{j} + \underline{k}$, $3\underline{i} + \underline{j}$, $2\underline{i} + 4\underline{j} - 2\underline{k}$ and $-\underline{i} - 2\underline{j} + \underline{k}$ respectively. Show that $\overrightarrow{AB}$ is parallel to $\overrightarrow{CD}$.

Solution:

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$= (3-2)\underline{i} + (1+1)\underline{j} + (0-1)\underline{k}$$

$$\overrightarrow{AB} = \underline{i} + 2\underline{j} - \underline{k}$$

$$\overrightarrow{CD} = \text{Position vector of D} - \text{Position vector of C}$$

$$= (-1-2)\underline{i} + (-2-4)\underline{j} + (1+2)\underline{k}$$

$$= -3\underline{i} - 6\underline{j} + 3\underline{k}$$

$$\overrightarrow{CD} = -3(\underline{i} + 2\underline{j} - \underline{k})$$

$$\overrightarrow{CD} = -3\overrightarrow{AB}$$

Hence $\overrightarrow{AB}$ is parallel to $\overrightarrow{CD}$.

Q.10 Two vectors $\underline{u}$ & $\underline{w}$ in space are parallel, if there is a scalar c such that $\underline{v} = c\underline{w}$. The vectors point in the same direction if c > 0 and the vector point in the opposite direction if c < 0

(a) Find two vectors of length 2 parallel to vector $\underline{v} = 2\underline{i} - 4\underline{j} + 4\underline{k}$

Solution:

$$\underline{v} = 2\underline{i} - 4\underline{j} + 4\underline{k}$$

$$|\underline{v}| = \sqrt{(2)^2 + (-4)^2 + (4)^2} = \sqrt{4 + 16 + 16} = \sqrt{36} = 6$$

$$\Rightarrow \quad \hat{v} = \frac{\underline{v}}{|\underline{v}|} = \frac{2\underline{i} - 4\underline{j} + 4\underline{k}}{6} = \frac{2(\underline{i} - 2\underline{j} + 2\underline{k})}{6} = \frac{\underline{i} - 2\underline{j} + 2\underline{k}}{3}$$

$\therefore$ The two vectors whose length is 2 and parallel to $\hat{v}$ are $2\hat{v}$ & $-2\hat{v}$

i.e; $$2\hat{v} = \frac{2}{3}(\underline{i} - 2\underline{j} + 2\underline{k}) = \frac{2}{3}\underline{i} - \frac{4}{3}\underline{j} + \frac{4}{3}\underline{k} \quad \text{Ans.}$$

$$-2\hat{v} = \frac{-2}{3}(\underline{i} - 2\underline{j} + 2\underline{k}) = \frac{-2}{3}\underline{i} + \frac{4}{3}\underline{j} - \frac{4}{3}\underline{k} \quad \text{Ans.}$$

(b) Find the constant a so that the vectors $\underline{v} = \underline{i} - 3\underline{j} + 4\underline{k}$ and $\underline{w} = a\underline{i} + 9\underline{j} - 12\underline{k}$ are parallel. (Gujranwala Board 2004)

Solution:

Since $\underline{v}$ & $\underline{w}$ are parallel so

$$\underline{w} = c\underline{v}$$

$$a\underline{i} + 9\underline{j} - 12\underline{k} = c(\underline{i} - 3\underline{j} + 4\underline{k})$$

$$a\underline{i} + 9\underline{j} - 12\underline{k} = c\underline{i} - 3c\underline{j} + 4c\underline{k}$$

By comparing

$$a = c, \qquad 9 = -3c, \qquad -12 = 4c$$

$$\Rightarrow \quad \frac{9}{-3} = c \quad \Rightarrow \quad c = -3$$

$$\boxed{a = -3} \quad \text{Ans.}$$

(c) Find a vector of length 5 in the direction opposite that of $\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$.
(Lahore Board 2004)

Solution:

$$\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$$

$$|\underline{v}| = \sqrt{(1)^2 + (-2)^2 + (3)^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$$

$$\hat{v} = \frac{\underline{v}}{|\underline{v}|} = \frac{\underline{i} - 2\underline{j} + 3\underline{k}}{\sqrt{14}}$$

$\therefore$ The vector of length 5 in opposite direction of $\underline{v}$ is

$$-5\hat{\underline{v}} = \frac{-5}{\sqrt{14}}(\underline{i} - 2\underline{j} + 3\underline{k})$$

$$\frac{-5}{\sqrt{14}}\underline{i} + \frac{10}{\sqrt{14}}\underline{j} - \frac{15}{\sqrt{14}}\underline{k} \qquad \text{Ans.}$$

(d) Find a and b so that the vectors $3\underline{i} - \underline{j} + 4\underline{k}$ and $a\underline{i} + b\underline{j} - 2\underline{k}$ are parallel.

Solution:

Since $\underline{v}$ & $\underline{w}$ are parallel so

$$\underline{w} = c\underline{v}$$

$$a\underline{i} + b\underline{j} - 2\underline{k} = c(3\underline{i} - \underline{j} + 4\underline{k})$$

$$a\underline{i} + b\underline{j} - 2\underline{k} = 3c\underline{i} - c\underline{j} + 4c\underline{k}$$

By comparing

$$a = 3c, \qquad b = -c, \qquad -2 = 4c$$

$$\frac{-2}{4} = c$$

$$b = -c \qquad \boxed{\frac{-1}{2} = c}$$

$$\Rightarrow \boxed{b = \frac{1}{2}} \qquad a = 3c \quad \Rightarrow a = 3\left(\frac{-1}{2}\right) \Rightarrow \boxed{a = \frac{-3}{2}}$$

Q.11 Find the direction cosines for the given vectors.

(i) $\underline{v} = 3\underline{i} - \underline{j} + 2\underline{k}$ (Lahore Board 2007)

Solution:

$$\underline{v} = 3\underline{i} - \underline{j} + 2\underline{k}$$

$$|\underline{v}| = \sqrt{(3)^2 + (-1)^2 + (2)^2} = \sqrt{9+1+4} = \sqrt{14}$$

Direction cosines are

$$= \left[\frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{2}{\sqrt{14}}\right]$$

(ii) $\underline{v} = 6\underline{i} - 2\underline{j} + \underline{k}$ (Lahore Board 2006)

Solution:

$$\underline{v} = 6\underline{i} - 2\underline{j} + \underline{k}$$

$$|\underline{v}| = \sqrt{(6)^2 + (-2)^2 + (1)^2} = \sqrt{36+4+1} = \sqrt{41}$$

Direction cosines are $= \left[\frac{6}{\sqrt{41}}, \frac{-2}{\sqrt{41}}, \frac{1}{\sqrt{41}}\right]$ Ans.

(iii) $\overrightarrow{PQ}$, where P (2, 1, 5) & Q = (1, 3, 1)

Solution:

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$
$$= (1-2)\underline{i} + (3-1)\underline{j} + (1-5)\underline{k} = -\underline{i} + 2\underline{j} - 4\underline{k}$$
$$|\overrightarrow{PQ}| = \sqrt{(-1)^2 + (2)^2 + (-4)^2} = \sqrt{1+4+16} = \sqrt{21}$$

Direction cosines are $= \left[\dfrac{-1}{\sqrt{21}}, \dfrac{2}{\sqrt{21}}, \dfrac{-4}{\sqrt{21}}\right]$ Ans.

Q.12 Which of the following triples can be the direction angles of a single vector.

(i) 45°, 45°, 60°

Solution:

If α, β, γ are direction angles of a vector, then it must satisfy $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

L.H.S.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = (\cos 45^\circ)^2 + (\cos 45^\circ)^2 + (\cos 60^\circ)^2$$
$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2} + \frac{1}{2} + \frac{1}{4}$$
$$= \frac{2+2+1}{4} = \frac{5}{4} \neq 1$$

So given triples are not direction angles.

(ii) 30°, 45°, 60°

Solution:

$\boldsymbol{\alpha = 30^\circ, \ \beta = 45^\circ, \ \gamma = 60^\circ}$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma$$
$$= (\cos 30^\circ)^2 + (\cos 45^\circ)^2 + (\cos 60^\circ)^2$$
$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{2} + \frac{1}{4} = \frac{3+2+1}{4} = \frac{6}{4} \neq 1$$

Hence given triples can not be direction angles.

(iii) 45°, 60°, 60°

Solution:

$\boldsymbol{\alpha = 45^\circ, \ \beta = 60^\circ, \ \gamma = 60^\circ}$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = (\cos 45^\circ)^2 + (\cos 60^\circ)^2 + (\cos 60^\circ)^2$$
$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2} + \frac{1}{4} + \frac{1}{4}$$
$$= \frac{2+1+1}{4} = \frac{4}{4} = 1$$

As $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Therefore, given triples can be direction angles of a vector.

The Scalar Product of Two vectors

Definition:

Let two non zero vectors $\underline{u}$ & $\underline{v}$ in the plane or in space, have same initial point. The dot product of $\underline{u}$ and $\underline{v}$, written as $\underline{u} \cdot \underline{v}$, is defined by

$$\underline{u} \cdot \underline{v} = |\underline{u}|\ |\underline{v}| \cos\theta$$ where θ is angle between $\underline{u}$ & $\underline{v}$ and $0 \le \theta \le \pi$.

Orthogonal / Perpendicular vectors:

The two vectors $\underline{u}$ & $\underline{v}$ are orthogonal / perpendicular if and only if $\underline{u} \cdot \underline{v} = o$

Remember:

(i) Dot product, inner product, scalar product are same.

(ii) $\underline{i} \cdot \underline{i} = \underline{j} \cdot \underline{j} = \underline{k} \cdot \underline{k} = 1$

(iii) $\underline{i} \cdot \underline{j} = \underline{j} \cdot \underline{k} = \underline{k} \cdot \underline{i} = 0$

(iv) Scalar product is commutative i.e., $\underline{u} \cdot \underline{v} = \underline{v} \cdot \underline{u}$

EXERCISE 7.3

Q.1 Find the Cosine of the angle θ between $\underline{u}$ and $\underline{v}$.

(i) $\underline{u} = 3\underline{i} + \underline{j} - \underline{k}$ $\quad \underline{v} = 2\underline{i} - \underline{j} + \underline{k}$

Formula

$$\cos\theta = \frac{\underline{u} \cdot \underline{v}}{|\underline{u}|\ |\underline{v}|}$$

Solution:

$$\underline{u} = 3\underline{i} + \underline{j} - \underline{k}, \quad \underline{v} = 2\underline{i} - \underline{j} + \underline{k}$$

$$\underline{u} \cdot \underline{v} = (3\underline{i} + \underline{j} - \underline{k}) \cdot (2\underline{i} - \underline{j} + \underline{k})$$

$$\underline{u} \cdot \underline{v} = 6 - 1 - 1 = 4$$

$$|\underline{u}| = \sqrt{(3)^2 + (1)^2 + (-1)^2} = \sqrt{9+1+1} = \sqrt{11}$$

$$|\underline{v}| = \sqrt{(2)^2 + (-1)^2 + (1)^2} = \sqrt{4+1+1} = \sqrt{6}$$

$$\cos\theta = \frac{\underline{u} \cdot \underline{v}}{|\underline{u}|\ |\underline{v}|} = \frac{4}{\sqrt{11}\sqrt{6}} = \frac{4}{\sqrt{66}}$$

$$\cos\theta = \frac{4}{\sqrt{66}} \qquad \text{Ans.}$$