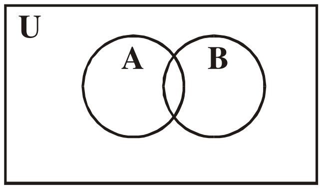
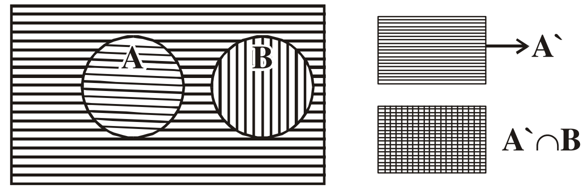
(iv) $\mathbf{A \cup B = A \cup (A' \cap B)}$



A∪B



A∪(A`∩B)

In above two figures the shaded portion is same.

$\Rightarrow \quad A \cup B = A \cup (A' \cap B)$

INDUCTIVE AND DEDUCTIVE LOGIC

Induction

The way of drawing conclusions on the basis of a few basic experiments or observations is called induction.

Deduction

The way of drawing conclusions by accepting some well known facts is called deduction.

Proposition

A declarative statement which may be true or false but not both is called proposition.

Aristotelian and non–Aristotelian Logics:

Deductive logic in which every statement is regarded as true or false and there is no other possibility, is called Aristotelian logic.

Logic in which there is scope for a third or fourth possibility is called non–Aristotelian logic.

Symbolic Logic

Symbol	How to be read	Symbolic expression	How to be read
~ (Negation)	not	$\sim p$	Not p
$\wedge$ (Conjunction)	and	$p \wedge q$	p and q
$\vee$ (Disjunction)	or	$p \vee q$	p or q
$\rightarrow$ (Conditional)	if …. then implies	$p \rightarrow q$	p implies q
$\leftrightarrow$ (Biconditional)	if and only if	$p \leftrightarrow q$	p if and only if q or p is equivalent to q

How to use these Logics

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \vee q$	$p \to q$	$q \to p$	$p \leftrightarrow q$
T	T	F	F	T	T	T	T	T
T	F	F	T	F	T	F	T	F
F	T	T	F	F	T	T	F	F
F	F	T	T	F	F	T	T	T

Converse of $p \to q$ is $q \to p$

Inverse of $p \to q$ is $\sim p \to \sim q$

Contrapositive of $p \to q$ is $\sim q \to \sim p$

Tautology

A statement which is true for all the possible values of the variable involved in it is called a tautology.

Absurdity

A statement which is false for all the possible values of the variable involved in it is called an Absurdity.

Contingency

A statement which can be true or false depending upon the truth values of the variable involved in it is called contingency.

EXERCISE 2.4

Q.1 Write the converse, inverse and contrapositive of the following conditionals.

(i) $\sim p \to q$ **(Gujranwala Board 2007)**

(ii) $q \to p$

(iii) $\sim p \to \sim q$ **(Gujranwala Board 2003, 2007)**

(iv) $\sim q \to \sim p$

Solution:

Given Conditional		Converse	Inverse	Contrapositive
(i)	$\sim p \to q$	$q \to \sim p$	$p \to \sim q$	$\sim q \to p$
(ii)	$q \to p$	$p \to q$	$\sim q \to \sim p$	$\sim p \to \sim q$
(iii)	$\sim p \to \sim q$	$\sim q \to \sim p$	$p \to q$	$q \to p$
(iv)	$\sim q \to \sim p$	$\sim p \to \sim q$	$q \to p$	$P \to q$

(i)

				Given Conditional	Converse	Inverse	Contrapositive
p	q	$\sim p$	$\sim q$	$\sim p \to q$	$q \to \sim p$	$p \to \sim q$	$\sim q \to p$
T	T	F	F	T	F	F	T
T	F	F	T	T	T	T	T
F	T	T	F	T	T	F	T
F	F	T	T	F	T	T	F

(ii)

				Given Conditional	Converse	Inverse	Contrapositive
p	q	$\sim p$	$\sim q$	$q \to p$	$p \to q$	$\sim q \to \sim p$	$\sim p \to \sim q$
T	T	F	F	T	T	T	T
T	F	F	T	T	F	F	T
F	T	T	F	F	T	T	F
F	F	T	T	T	T	T	T

(iii)

				Given Conditional	Converse	Inverse	Contrapositive
p	q	$\sim p$	$\sim q$	$\sim p \to \sim q$	$\sim q \to \sim p$	$p \to q$	$q \to p$
T	T	F	F	T	T	T	T
T	F	F	T	T	F	F	T
F	T	T	F	F	T	T	F
F	F	T	T	T	T	T	T

(iv)

				Given Conditional	Converse	Inverse	Contrapositive
p	q	$\sim p$	$\sim q$	$\sim q \to \sim p$	$\sim p \to \sim q$	$q \to p$	$p \to q$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Q.2 Construct truth tables for the following statements

(i) $(p \rightarrow \sim p) \vee (p \rightarrow q)$ (ii) $(p \wedge \sim P) \rightarrow q$ (Gujranwala Board 2003)

(iii) $\sim (p \rightarrow q) \leftrightarrow (p \wedge \sim q)$

Solution:

(i) $(p \rightarrow \sim p) \vee (p \rightarrow q)$

Required truth table is given below

P	q	$\sim p$	$p \rightarrow \sim p$	$p \rightarrow q$	$(p \rightarrow \sim p) \vee (p \rightarrow q)$
T	T	F	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	T	T

(ii) $(p \wedge \square\, p) \rightarrow q$

Required truth table is given below

p	q	$\sim p$	$p \wedge \sim p$	$(p \wedge \sim p) \rightarrow q$
T	T	F	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(iii) $\sim (P \rightarrow q) \leftrightarrow (P \wedge \sim q)$

Required truth table is given below

P	q	$p \rightarrow q$	$\sim (p \rightarrow q)$	$\sim q$	$p \wedge \sim q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

As entries in the columns of $\sim (p \rightarrow q)$ and $p \wedge \sim q$ are same.

$\Rightarrow \quad \sim (p \rightarrow q) \leftrightarrow p \wedge \sim q$

Q.3 Show that each of the following statement is a tautology:

(i) $(p \wedge q) \rightarrow p$ (ii) $p \rightarrow (p \vee q)$ (Lahore Board 2004, 2009)

(iii) $\sim (p \rightarrow q) \rightarrow p$ (iv) $\sim q \wedge (p \rightarrow q) \rightarrow \sim p$

Solution:

(i) $(p \wedge q) \rightarrow p$

Truth table of $(p \wedge q) \rightarrow p$ is given below

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

As $(p \wedge q) \rightarrow p$ is true for all values of the variable so it is a tautology.

(ii) $p \rightarrow (p \vee q)$

Truth table of $p \rightarrow (p \vee q)$ is given below

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Since $P \rightarrow (P \text{ v } q)$ is true for all values of the variable involved, so it is a tautology.

(iii) $(p \rightarrow q) \rightarrow P$

Truth table of $\sim (p \rightarrow q) \rightarrow P$ is given below.

p	q	$p \rightarrow q$	$\sim (p \rightarrow q)$	$\sim (p \rightarrow q) \rightarrow p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

As $(p \rightarrow q) \rightarrow p$ is true for all values of the variable so it is a tautology.

(iv) $\sim q \wedge (p \to q) \to \sim p$

Truth table of $\sim q \wedge (p \to q) \to \sim p$ is

p	q	~ p	~ q	$(p \to q)$	$\sim q \wedge (p \to q)$	$\sim q \wedge (p \to q) \to \sim P$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

As $\sim q \wedge (p \to q) \to \sim p$ is true for all values of the variable so it is a tautology.

Q.4 Determine whether each of the following is a tautology, a contingency, or an absurdity.

(i) $p \wedge \sim p$ (ii) $p \to (q \to p)$ (iii) q v $(\sim q \vee p)$

(Gujranwala Board 2005)

Solution:

(i) $p \wedge \sim p$

p	~ p	$p \wedge \sim p$
T	F	F
F	T	F

As $p \wedge \sim p$ is false for all values . So it is an absurdity.

(ii) $p \to (q \to p)$

Truth table of $p \to (q \to p)$ is

p	q	$q \to p$	$p \to (q \to P)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

As $p \to (q \to p)$ is true for all values of the variable so it is a tautology.

(iii) $q \vee (\sim q \vee p)$

Truth table of $q \vee (\sim q \vee q)$ is

p	q	~ q	$(\sim q \vee P)$	$q \vee (\sim p \vee p)$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	T

As $q \vee (\sim q \vee p)$ is true for all the values of the variable so it is a tautology.

Q.5 Prove that $p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$ (Lahore Board 2005)

Solution:

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \vee (\sim p \wedge \sim q)$	$P \wedge q$	$p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$
T	T	F	F	F	T	T	F
T	F	F	F	F	T	F	T
F	T	T	T	F	F	F	T
F	F	T	T	T	T	F	T

As entries in the columns of $p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$ and $p \vee (\sim p \wedge \sim q)$ are same.

$\Rightarrow$ $p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$

Hence proved.

EXERCISE 2.5

Convert the following theorems to logical and prove them by constructing truth tables.

Q.1 $(A \cap B)' = A' \cup B'$ (Lahore Board 2004)

Solution:

Its logical form is $\sim (p \wedge q) = \sim p \vee \sim q$ its truth table is given below

p	q	$p \wedge q$	$\sim (p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

As entries in the columns of $\sim (p \wedge q)$ and $\sim p \vee \sim q$ are same.

$\Rightarrow$ $\sim (p \wedge q) = \sim p \vee \sim q$

$\Rightarrow$ $(A \cap B)' = A' \cup B'$