EXERCISE 1.5

Q.1 Draw the graphs of the following equations.

(i) $x^2 + y^2 = 9$ **(ii)** $\dfrac{x^2}{16} + \dfrac{y^2}{4} = 1$

(iii) $y = e^{2x}$ **(iv)** $y = 3^x$

Solution:

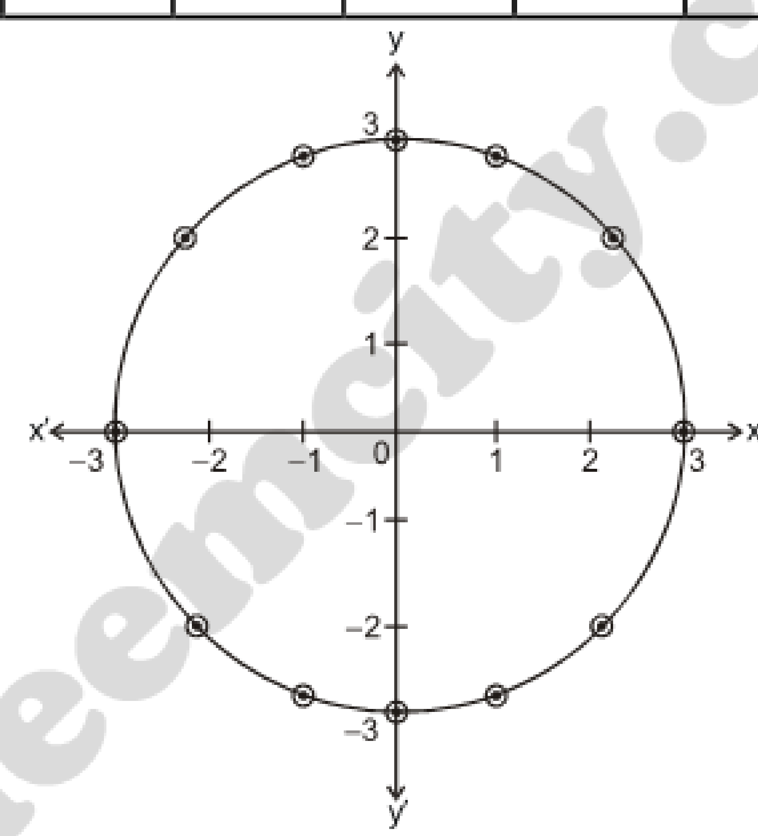
(i) $x^2 + y^2 = 9$

$$y^2 = 9 - x^2$$

$$y = \pm\sqrt{9 - x^2}$$

Its domain is $-3 \le x \le 3+$

x	-3	-2	-1	0	1	2	3
$y = \pm\sqrt{9 - x^2}$	0	± 2.2	± 2.8	± 3	± 2.8	± 2.2	0



(ii) $\dfrac{x^2}{16} + \dfrac{y^2}{4} = 1$

$$\frac{y^2}{4} = 1 - \frac{x^2}{16}$$

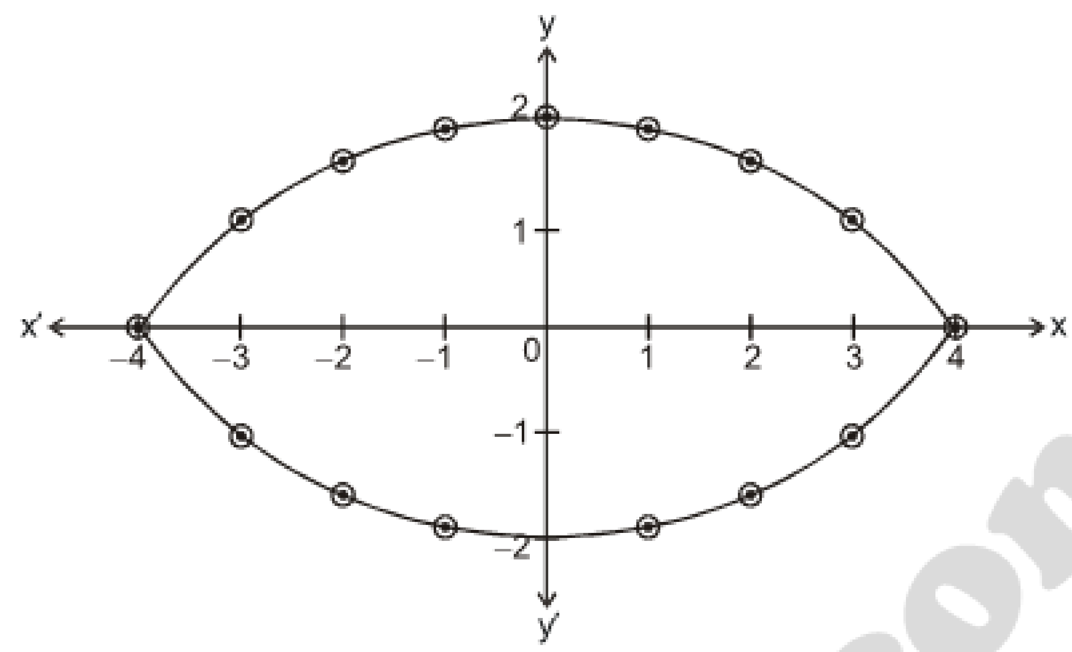
$$y^2 = 4\left(\frac{16 - x^2}{16}\right)$$

$$y^2 = \frac{16 - x^2}{4}$$

$$y = \pm\frac{\sqrt{16 - x^2}}{2}$$

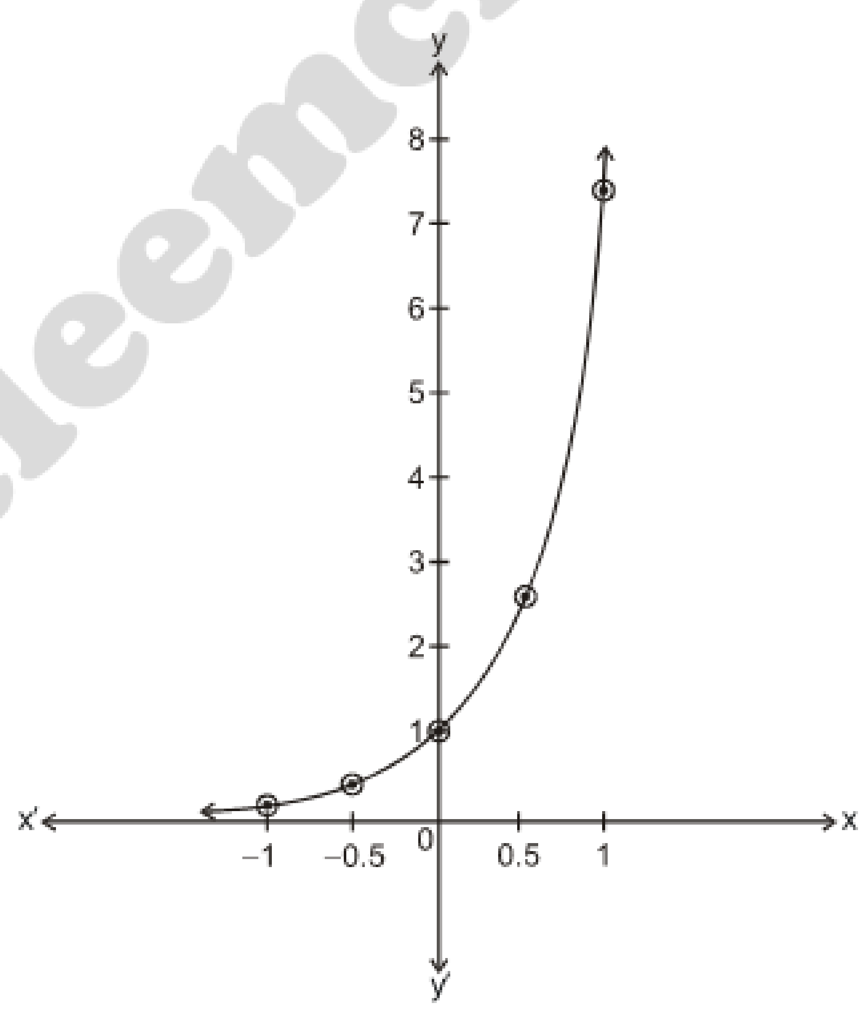
Its domain is $-4 \le x \le 4$.

x	−4	−3	−2	−1	0	1	2	3	4
$y = \pm\frac{\sqrt{9-x^2}}{2}$	0	± 1.3	± 1.7	± 1.9	± 2	± 1.9	± 1.7	± 1.3	0



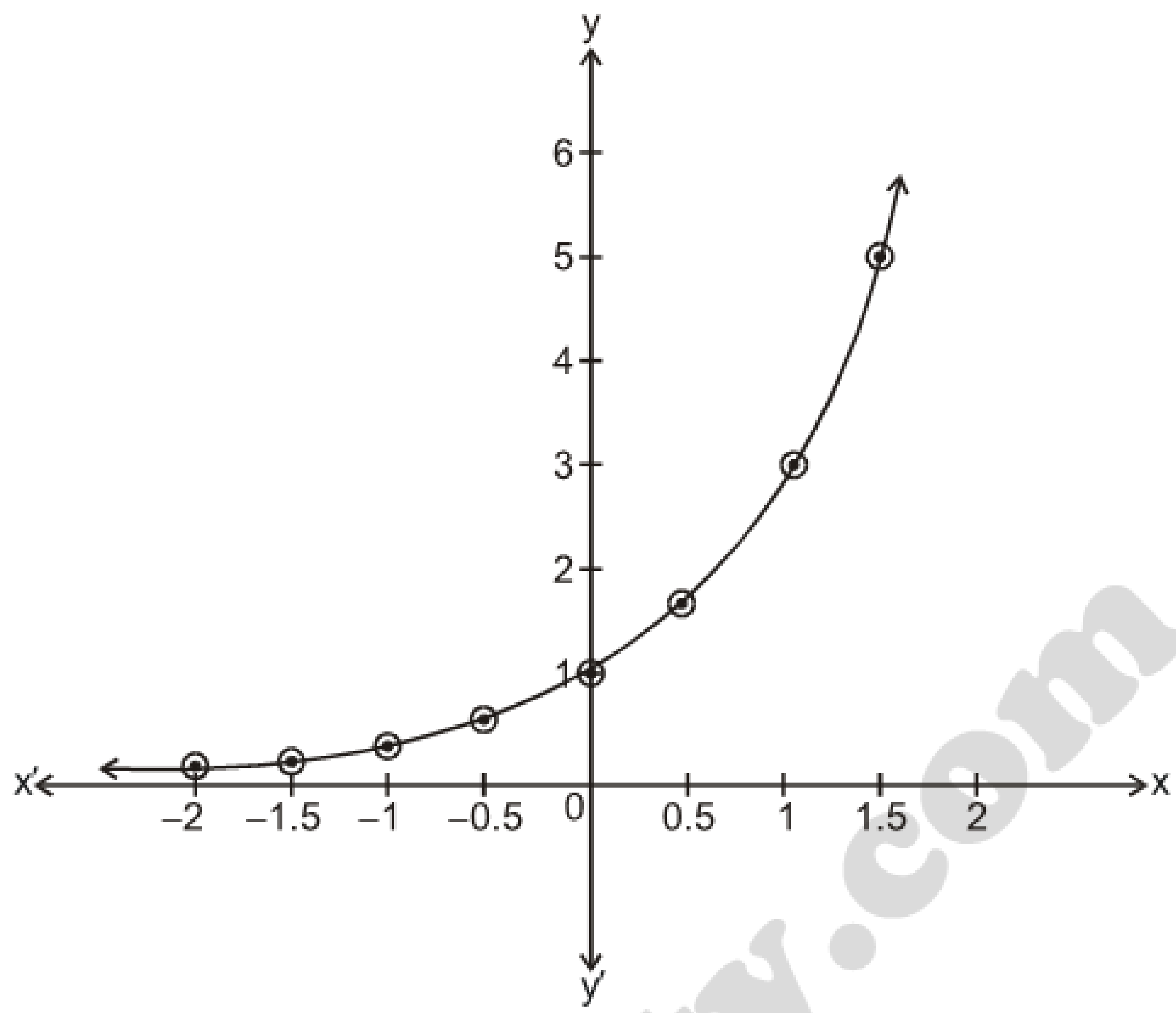
(iii) $\quad y = e^{2x}$

x	−1	−0.5	0	0.5	1
$y = e^{2x}$	0.1	0.4	1	2.7	7.4



(iv) $\quad y = 3^x$

x	−2	−1.5	−1	−0.5	0	0.5	1	1.5
$y = 3^x$	0.1	0.2	0.3	0.6	1	1.7	3	5.2



Q.2 Graph the curves that has the parametric equations given below.

(i) $x = t$, $y = t^2$, $-3 \le t \le 3$ **where 't' is a parameter**

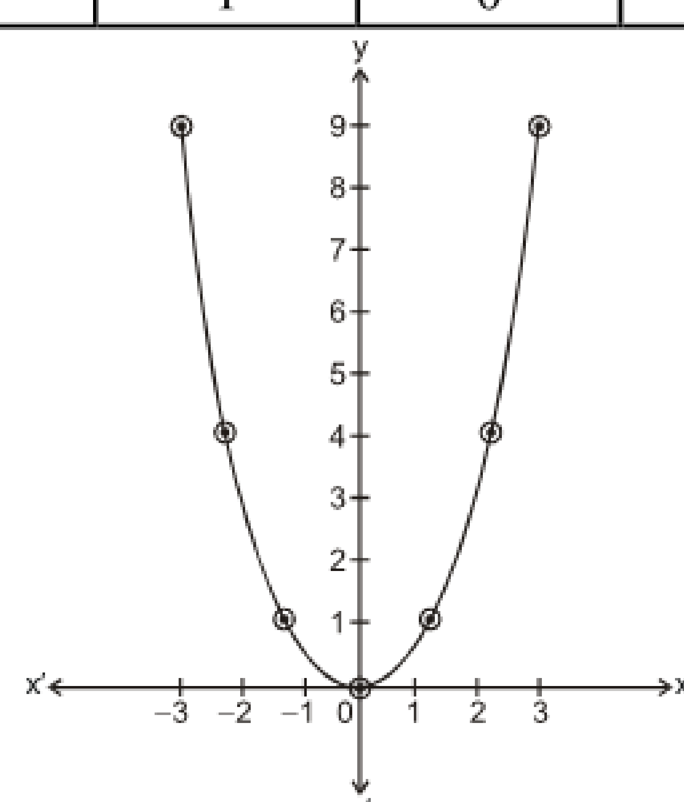
(ii) $x = t - 1$, $y = 2t - 1$, $-1 < t < 5$ **where 't' is a parameter**

(iii) $x = \sec\theta$, $y = \tan\theta$ **where 'θ' is a parameter**

Solution:

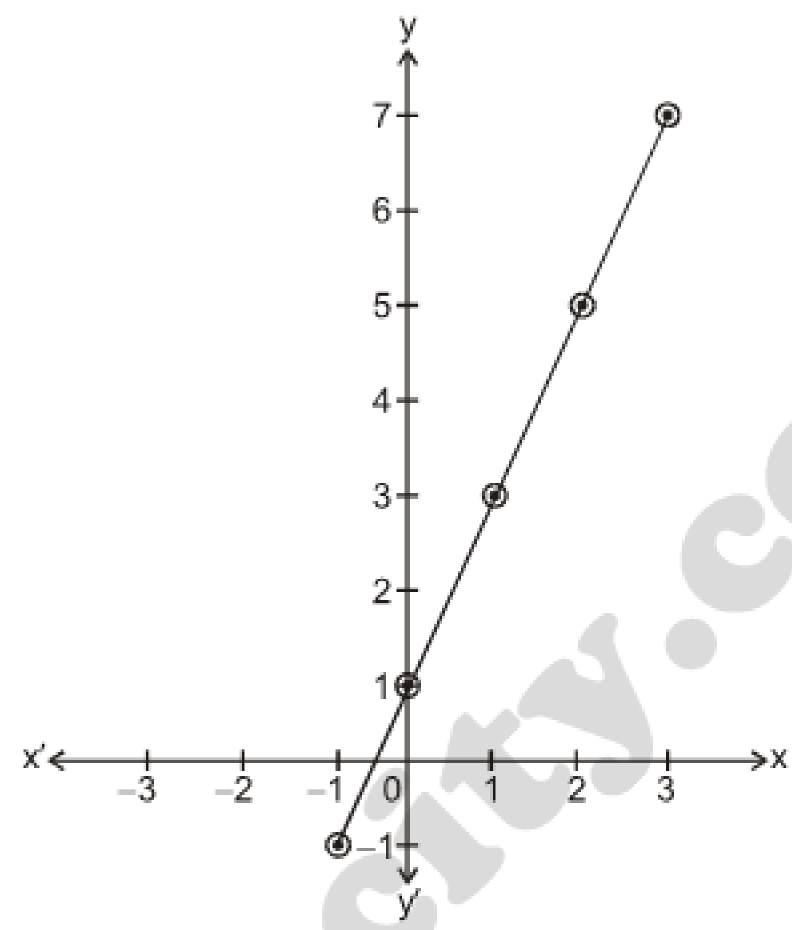
(i) $x = t$, $y = t^2$, $-3 \le t \le 3$ where 't' is a parameter

t	−3	−2	−1	0	1	2	3
x = t	−3	−2	−1	0	1	2	3
$y = t^2$	9	4	1	0	1	4	9



(ii) $x = t - 1$, $y = 2t - 1$, $-1 < t < 5$ where 't' is a parameter

t	0	1	2	3	4
$x = t - 1$	-1	0	1	2	3
$y = 2t - 1$	-1	1	3	5	7



(iii) $x = \sec\theta$, $y = \tan\theta$ where 'θ' is a parameter

$$x^2 = \sec^2\theta , \quad y^2 = \tan^2\theta$$

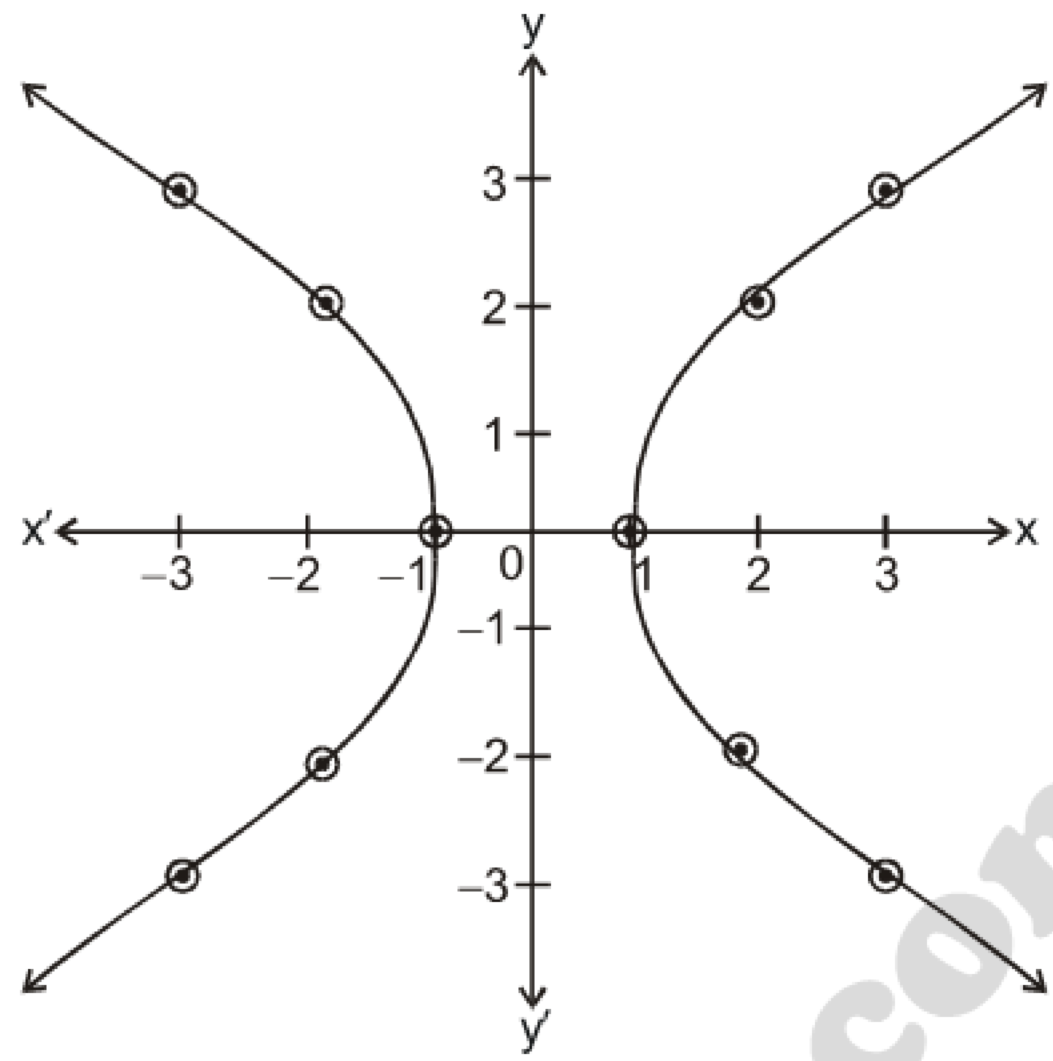
$$x^2 - y^2 = \sec^2\theta - \tan^2\theta$$

$$x^2 - y^2 = 1 \qquad (\because\ 1 + \tan^2\theta = \sec^2\theta \Rightarrow 1 = \sec^2\theta - \tan^2\theta)$$

$$y^2 = x^2 - 1$$

$$y = \pm\sqrt{x^2 - 1}$$

x	-3	-2	-1	1	2	3
$y = \sqrt{x^2 - 1}$	± 2.8	± 1.7	0	0	± 1.7	± 2.8



Q.3 **Draw the graphs of the functions defined below and find whether they are continuous.**

(i) $y = \begin{cases} x-1 & \text{if } x < 3 \\ 2x+1 & \text{if } x \geq 3 \end{cases}$ **(ii)** $y = \dfrac{x^2-4}{x-2}$, $x \neq 2$

(iii) $y = \begin{cases} x+3 & , \; x \neq 3 \\ 2 & , \; x = 3 \end{cases}$ **(iv)** $y = \dfrac{x^2-16}{x-4}$, $x \neq 4$

Solution:

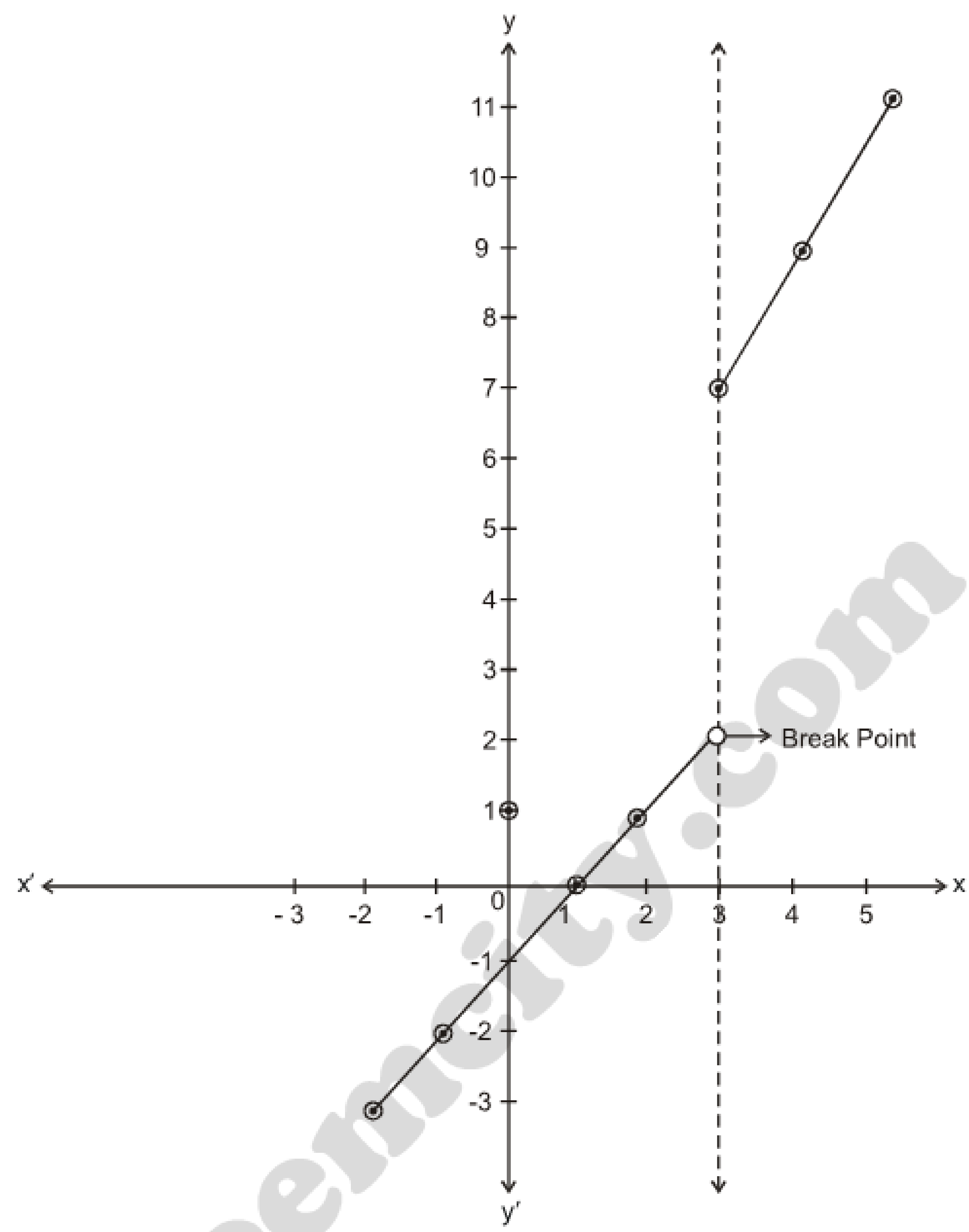
(i) $y = \begin{cases} x-1 & \text{if } x < 3 \\ 2x+1 & \text{if } x \geq 3 \end{cases}$

$y = x - 1 \quad , \quad x < 3$

x	–2	–1	0	1	2
y = x – 1	–3	–2	–1	0	1

$y = 2x + 1 \quad , \quad x \geq 3$

x	3	4	5
y = 2x + 1	7	9	11



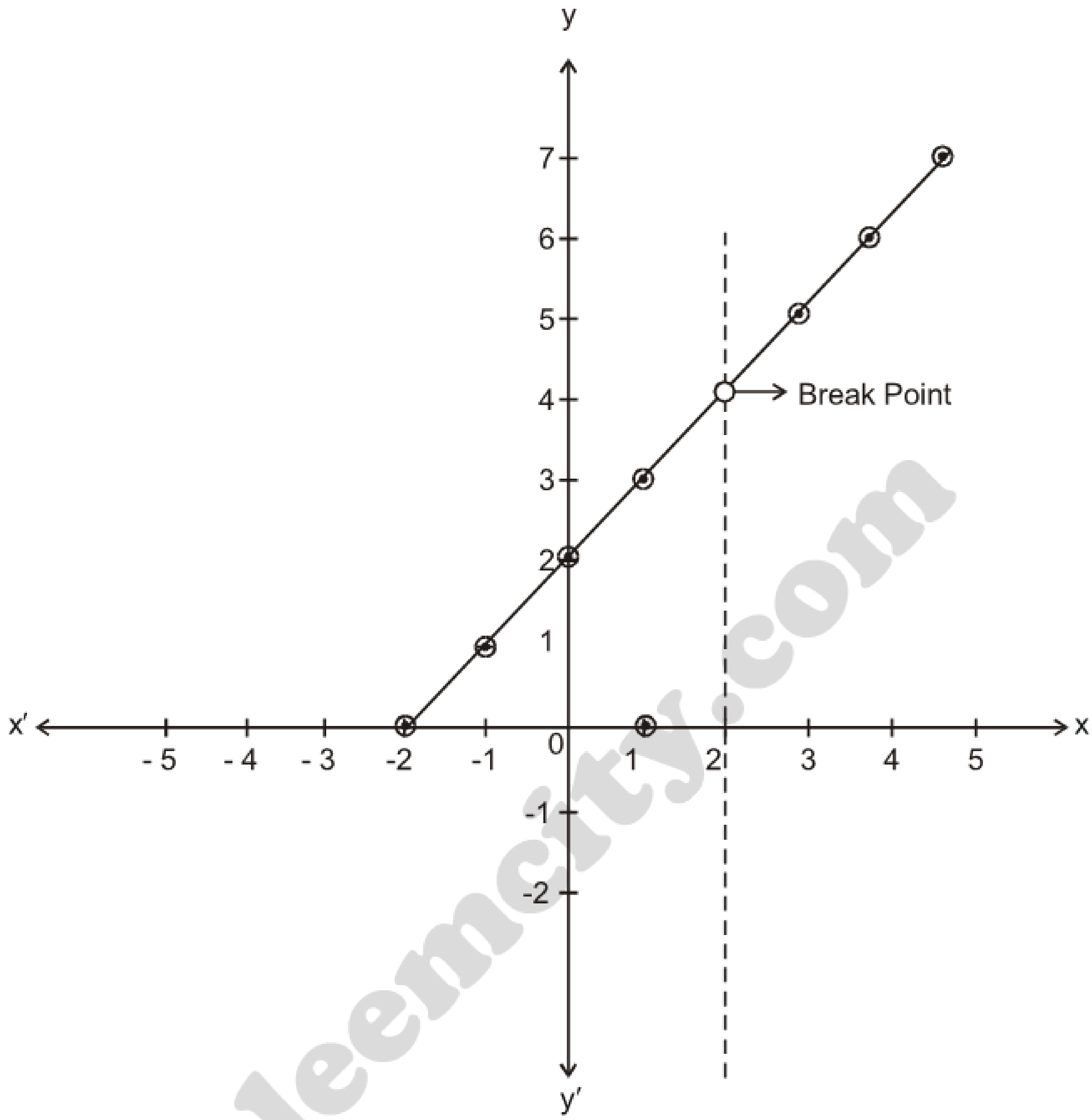
Since there is a break in a graph. So this function is not continuous.

(ii) $\mathbf{y = \dfrac{x^2 - 4}{x - 2} \quad , \quad x \neq 2}$

$$= \frac{(x+2)(x-2)}{x-2} \quad , \quad x \neq 2$$

$$y = x + 2 \ , \quad x \neq 2$$

x	– 3	– 2	– 1	0	1	3	4	5
y	– 1	0	1	2	3	5	6	7



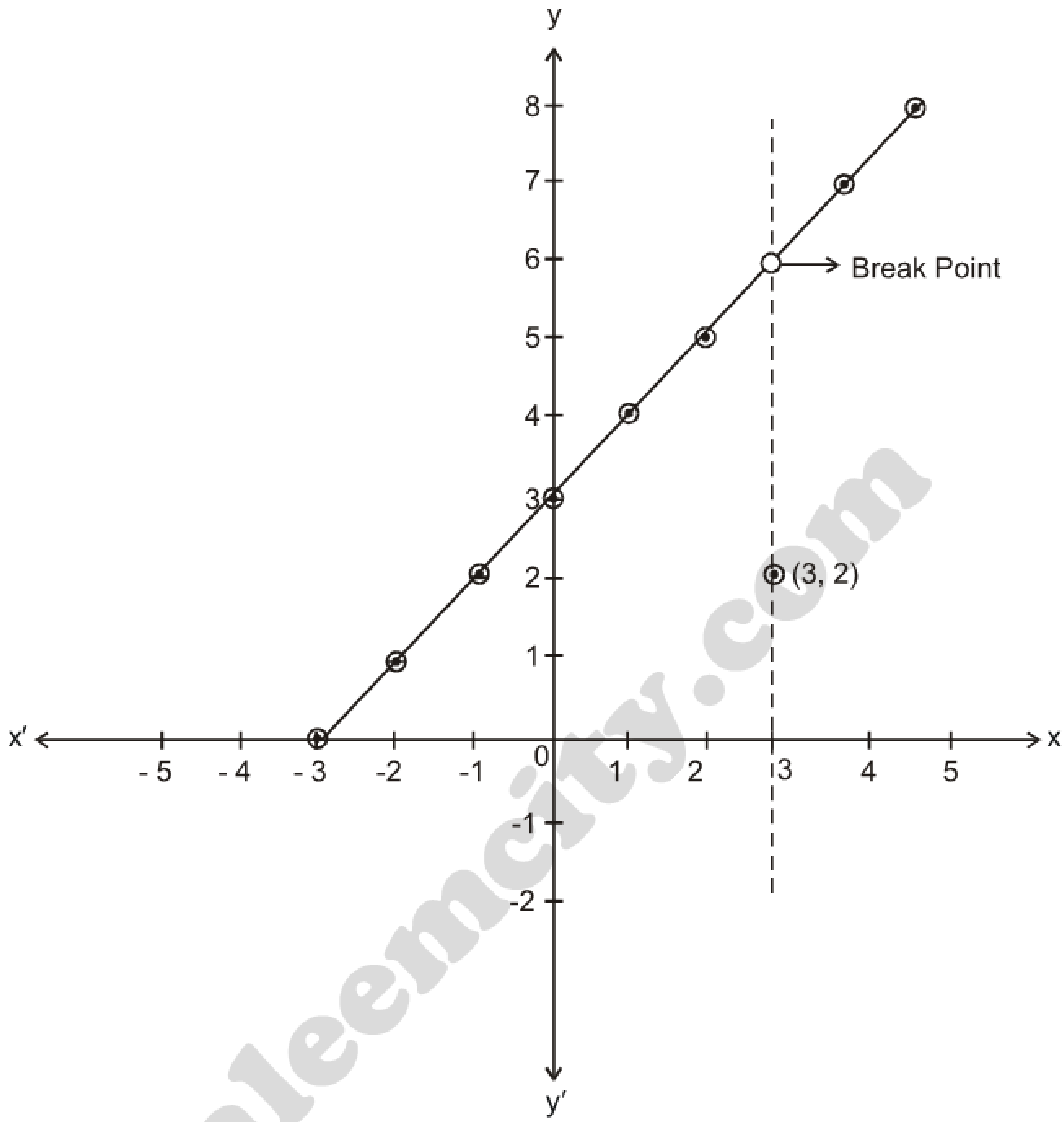
Since there is a break in a graph so this function is not continuous.

(iii) $\mathbf{y = \begin{cases} x + 3 & \text{if} \quad x \neq 3 \\ 2 & \text{if} \quad x = 3 \end{cases}}$

$y = x + 3 \qquad \text{if} \quad x \neq 3$

x	-3	-2	-1	0	1	3	4	5
y	0	1	2	3	4	5	7	8

$y = 2 \qquad \text{if} \quad x = 3$

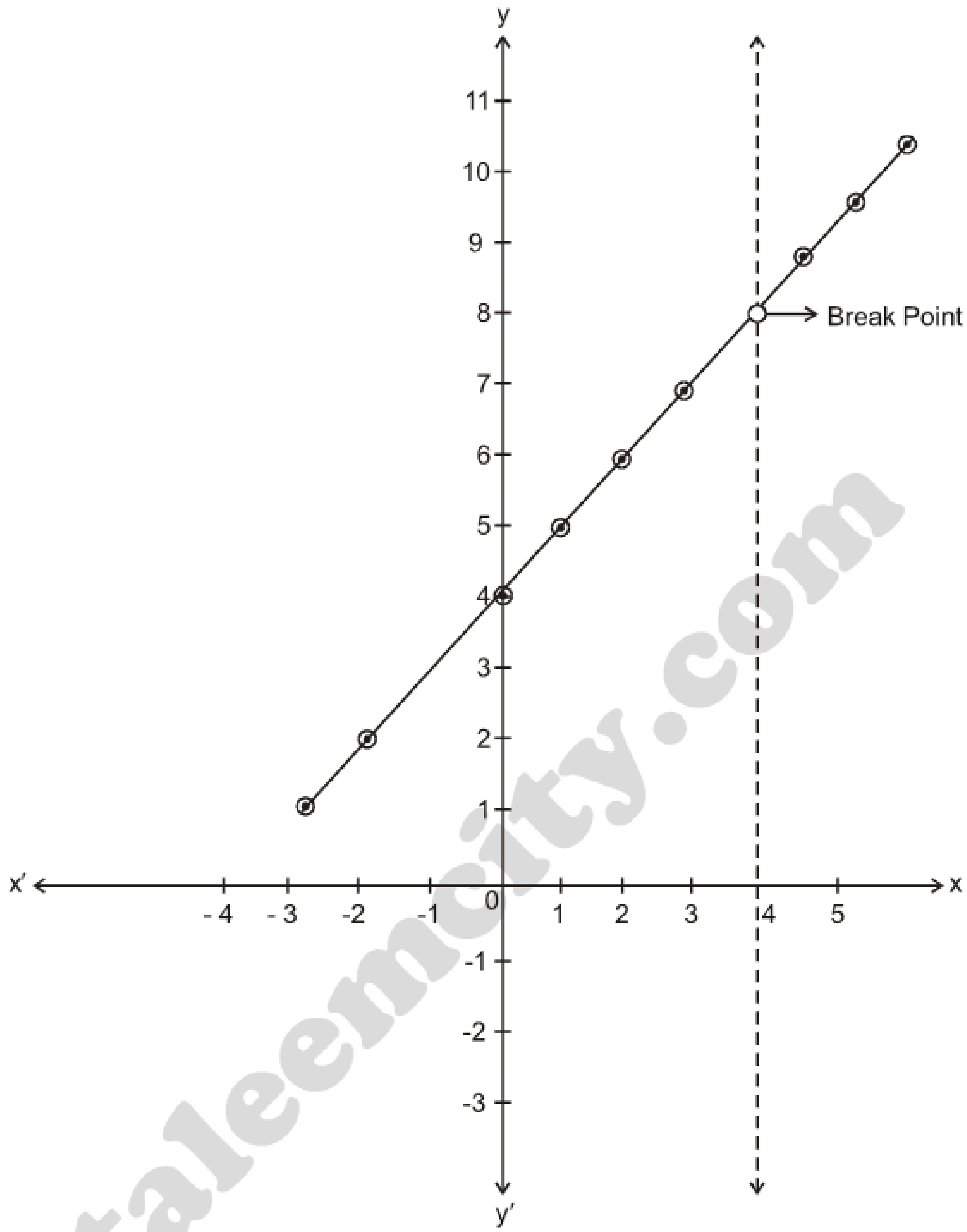


Since there is a break in a graph. So this function is not continuous at x = 3.

(iv) $\mathbf{y} = \dfrac{\mathbf{x^2 - 16}}{\mathbf{x - 4}} \quad , \quad \mathbf{x \neq 4}$

$$= \frac{(x + 4)(x - 4)}{x - 4}, \quad x \neq 4$$

x	– 3	– 2	– 1	0	1	2	3	5	6
y	1	2	3	4	5	6	7	9	10



Since there is a break in a graph. So this function is not continuous at x = 4.

Q.4 Find the graphical solution of the following equations.

(i) $x = \sin 2x$ **(ii)** $\frac{x}{2} = \cos x$ **(iii)** $2x = \tan x$

Solution:

(i) Let $y = x = \sin 2x$

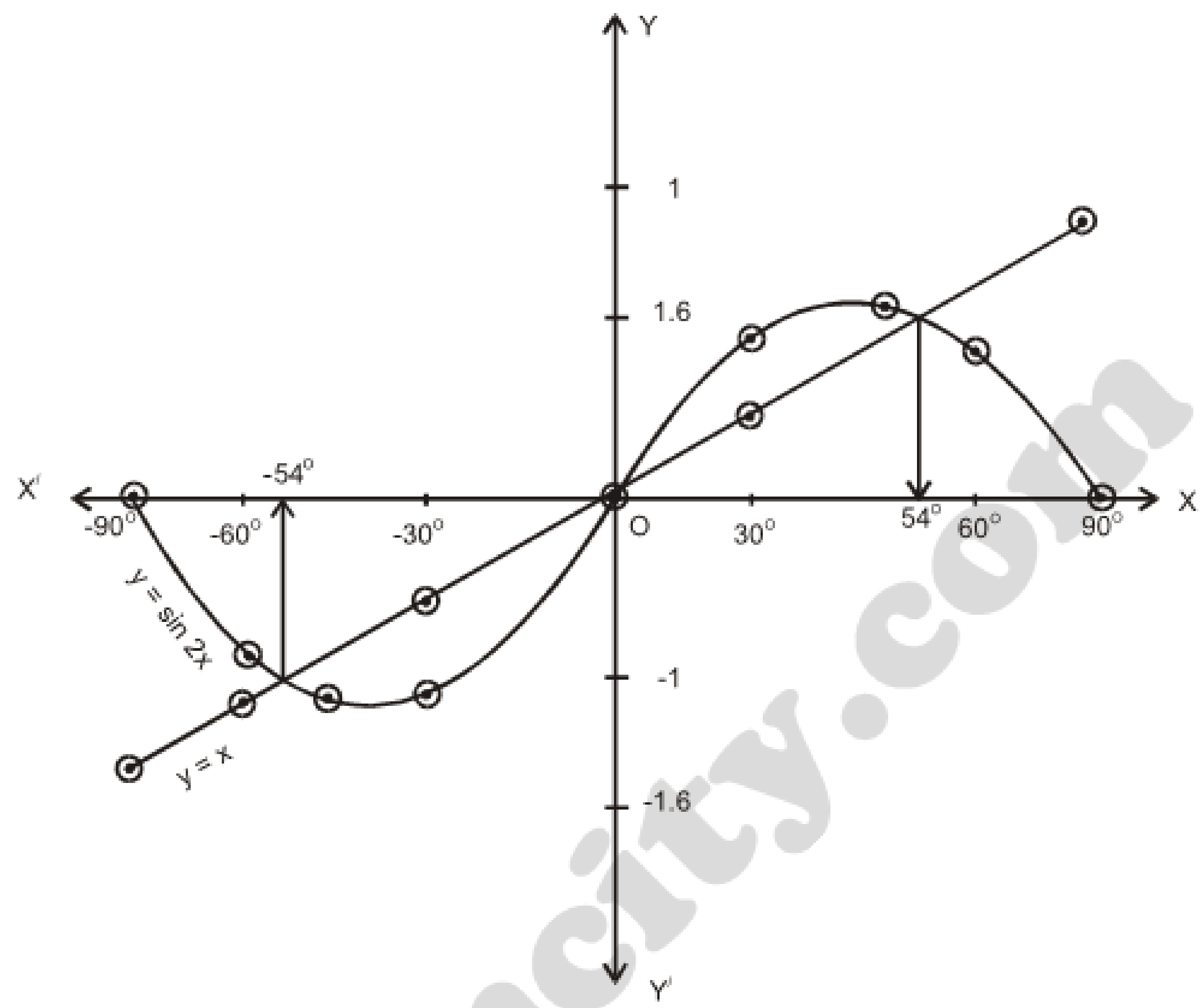
Therefore $y = x$ and $y = \sin 2x$

x	-90^o	-60^o	-30^o	0^o	30^o	60^o	90^o
y = x	$-\pi/2 = -1.6$	$-\pi/3 = -1.05$	$-\pi/6 = -0.52$	0	$\pi/6 = 0.52$	$\pi/3 = 1.05$	$\pi/2 = 1.6$

y = sin 2x

x	– 90º	– 60º	– 30º	0º	30º	60º	90º
y = sin 2x	0	– 0.87	– 0.87	0	0.87	0.87	0

The graphical solution is the points of intersection of two graphs, i.e. x = 0º , 54º



(ii) Let $y = \frac{x}{2} = \cos x$

Therefore $y = \frac{x}{2}$ and $y = \cos x$

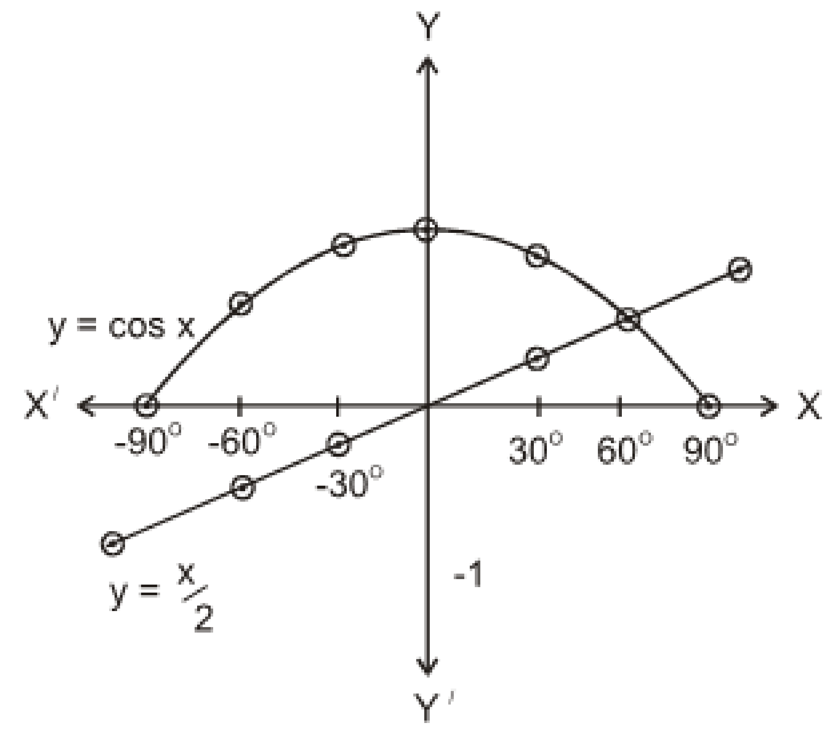
$y = \frac{x}{2}$

x	– 90º	– 60º	– 30º	0º	30º	60º	90º
$y = \frac{x}{2}$	– π/4 = – .79	– π/6 = – 0.52	– π/12 = – 0.26	0	π/6 = 0.26	π/6 = 0.52	π/4 = 0.79

y = cos x

x	– 90º	– 60º	– 30º	0º	30º	60º	90º
y = cos x	0	0.5	0.87	1	0.87	0.5	0

The graphical solution is the point on x-axis, which is just below the point of intersection of two graphs. Hence x = 60º.



(iii) **Let** $\mathbf{y = 2x = \tan x}$

Therefore $y = 2x$ and $y = \tan x$

$y = 2x$

x	– 90º	– 60º	– 30º	0º	30º	60º	90º
y = 2x	$-\pi = -3.14$	$-2\pi/3 = -2.09$	$-\pi/3 = -1.05$	0	$\pi/3 = 1.05$	$2\pi/3 = 2.09$	$\pi = 3.14$

$y = \tan x$

x	– 90º	– 60º	– 30º	0º	30º	60º	90º
y = tan x	∞	– 1.73	– 0.58	0	0.58	1.73	∞

The graphical solution is the point of intersection of two graphs, i.e. x = 0º.