Chapter 1

NUMBER SYSTEMS

The set of real numbers can be written as $R = Q \cup Q'$ where Q is the set of rational numbers and Q' is the set of irrational numbers.

RATIONAL NUMBER

Rational number is a number which can be put in the form $\frac{p}{q}$ where $p, q \in z \wedge q \neq 0$.

Thus $\sqrt{16}$, $\frac{3}{4}$, 2.7 are rational numbers.

Decimal Representation of Rational Numbers

(1) Terminating Decimals

A decimal which has only a finite number of digits in its decimal part, is called a terminating decimal. Terminating decimal represents a rational number. Thus 3.7, 0.0005, 207.9 are rational numbers.

(2) Recurring Decimals

Recurring decimal is a decimal in which one or more digits repeat indefinitely. Every recurring decimal represents a rational number. Thus $0.333\ldots..$, $1.57575757\ldots..$ are rational numbers.

Irrational Number

Irrational number is a number which can not be put into the form $\frac{p}{q}$ where $p, q, \in z \wedge q \neq 0$. Thus $\sqrt{2}$, $\sqrt{3}$, $\sqrt{\frac{5}{6}}$ are irrational numbers.

Note: A non terminating, non recurring decimal represents an irrational number.

Binary Operation

A binary operation in a set A is a rule usually denoted by $*$ that assigns to any pair of elements of A, taken in a definite order, another element of A.

Properties of Real Numbers

1. Addition Laws:

(i) Closure Law of Addition

$\forall\ a, b \in R \quad a + b \in R$

(ii) Associative Law of Addition

$\forall\ a, b, c \in R,\ a + (b + c) = (a + b) + c$

(iii) Additive Identity

$\forall\ a \in R\ ,\ \exists\, 0 \in R$ such that $a + 0 = 0 + a = a$

(v) Commutative Law for Addition

$\forall\ a, b \in R\ ,\ a + b = b + a$

2. Multiplication Laws:

(vi) Closure Law of Multiplication

$\forall\ a, b \in R\ ,\ a\,.\,b \in R$

(vii) Associative Law for Multiplication

$\forall\ a, b, c \in R\ ,\ a\,(bc) = (ab)\,c$

(viii) Multiplicative Identity

$\forall\ a \in R\ ,\ \exists\ 1 \in R$ such that $a\,.\,1 = 1\,.\,a = a$

(ix) Multiplicative Inverse

$\forall\ a\,(\neq 0) \in R\ ,\ \exists\ a^{-1} \in R$ such that $a\,.\,a^{-1} = a^{-1}\,.\,a = 1$

(x) Commutative Law of Multiplication

$\forall\ a, b \in R\ ,\ ab = ba$

3. Multiplication – Addition Law

(xi) Distributive Property

$\forall\ a, b, c \in R$

$a\,(b + c) = ab + ac$ (left distributive property)

$(a + b)\,c = ac + bc$ (Right distributive property)

Any set possessing all the above 11 properties is called a field.

4. Properties of Equality

(i) Reflexive property: $\forall\ a \in R\ ,\ a = a$

(ii) Symmetric property: $\forall\ a, b \in R\ ,\ a = b \Rightarrow b = a$

(iii) Transitive property: $\forall\ a, b, c \in R\ ,\ a = b \wedge b = c \Rightarrow a = c$

(iv) Additive property: $\forall\ a, b, c, \in R\ ,\ a = b \Rightarrow a + c = b + c \wedge c+a = b+c$

(v) Multiplicative property: $\forall\ a, b, c, \in R\ ,\ a = b \Rightarrow ac = bc \wedge ca = cb$

(vi) Cancellation property w.r.t. addition: $\forall\ a, b, c \in R\ ,\ a + c = b + c \Rightarrow a = b$

(vii) Cancellation property w.r.t. Multiplication: $\forall\ a, b, c \in R\ ,\ ac = bc \Rightarrow a = b\ ,\ c \neq 0$

5. Properties of Inequalities

Trichotomy Property

$\forall\ a, b \in R$ either $a = b$ or $a > b$ or $a < b$

Transitive Property

$\forall\ a, b, c \in R$

(i) $a > b \wedge b > c \Rightarrow a > c$

(ii) $a < b \wedge b < c \Rightarrow a < c$

Additive Property

$\forall\ a, b, c \in R$

(a) (i) $a > b \Rightarrow a + c > b + c$ (ii) $a < b \Rightarrow a + c < b + c$

(b) (i) $a > b \wedge c > d \Rightarrow a + c > b + d$ (ii) $a < b \wedge c < d \Rightarrow a + c < b + d$

Multiplicative Properties

(a) $\forall\ a, b, c \in R$ and $c > 0$

(i) $a > b \Rightarrow ac > bc$ (ii) $a < b \Rightarrow ac < bc$

(b) $\forall\ a, b, c \in R$ and $c < 0$

(i) $a > b \Rightarrow ac < bc$ (ii) $a < b \Rightarrow ac > bc$

(c) $\forall\ a, b, c, d \in R$ and a, b, c, d are all positive,

(i) $a > b \wedge c > d \Rightarrow ac > bd$ (ii) $a < b \wedge c < d \Rightarrow ac < bd$

EXERCISE 1.1

Q.1 Which of the following sets have closure property w.r.t. addition and multiplication

(i) {0}

The set is closed w.r.t. addition because $0 + 0 = 0 \in \{0\}$

The set is closed w.r.t. multiplication because $0.0 = 0 \in \{0\}$

(ii) {1}

The set is not closed w.r.t. addition because $1 + 1 = 2 \notin \{1\}$

The set is closed w.r.t. multiplication because $1.1 = 1 \in \{1\}$

(iii) {0, – 1}

+	0	– 1
0	0	– 1
– 1	– 1	– 2

The set is not closed w.r.t. addition because $-2 \notin \{0, -1\}$

•	0	– 1
0	0	0
– 1	0	1

The set is not closed w.r.t. multiplication because $1 \notin \{0, -1\}$

(iv) {1, – 1}

+	1	– 1
1	2	0
– 1	0	– 2

The set is not closed w.r.t. addition because $-2, 0, 2 \notin \{-1, 1\}$

•	1	– 1
1	1	– 1
– 1	– 1	1

The set is closed w.r.t. multiplication.

Q.2 Name the properties used in the following equations (letters, where used, represents real numbers)

Solution:

(i) $4 + 9 = 9 + 4$ Commutative property w.r.t. '+'

(ii) $(a+1)+\frac{3}{4} = a+\left(1+\frac{3}{4}\right)$ Associative property w.r.t. '+'

(iii) $(\sqrt{3}+\sqrt{5})+\sqrt{7} = \sqrt{3}+(\sqrt{5}+\sqrt{7})$ Associative property w.r.t. '+'

(iv) $100+0 = 100$ Additive Identity

(v) $100 \times 1 = 100$ Multiplicative Identity

(vi) $4.1+(-4.1) = 0$ Additive Inverse

(vii) $a-a = 0$ Additive Inverse.

(viii) $\sqrt{2} \times \sqrt{5} = \sqrt{5} \times \sqrt{2}$ Commutative property w.r.t. '.'

(ix) $a(b-c) = ab-ac$ Left distributive property.

(x) $(x-y)z = xz-yz$ Right distributive property.

(xi) $4 \times (5 \times 8) = (4 \times 5) \times 8$ Associative property w.r.t. '.'

(xii) $a(b+c-d) = ab+ac-ad$ Left distributive property

Q.3 Name the properties used in the following inequalities.

Solution:

(i) $-3 < -2 \Rightarrow 0 < 1$ Additive property.

(ii) $-5 < -4 \Rightarrow 20 > 16$ Multiplication property.

(iii) $1 > -1 \Rightarrow -3 > -5$ Additive property.

(iv) $a < 0 \Rightarrow -a > 0$ Multiplicative property.

(v) $a > b \Rightarrow \frac{1}{a} < \frac{1}{b}$ Multiplicative property.

(vi) $a > b \Rightarrow -a < -b$ Multiplicative property.

Q.4 Prove the following Rules of Addition

(i) $\mathbf{\frac{a}{c}+\frac{b}{c} = \frac{a+b}{c}}$

Solution:

L.H.S

$= \frac{a}{c}+\frac{b}{c}$

$= a \cdot \frac{1}{c} + b \cdot \frac{1}{c}$ ¤ $\frac{a}{b} = a \cdot \frac{1}{b}$

$= (a+b) \cdot \frac{1}{c}$ Distributive property

$= \frac{a+b}{c}$ ¤ $a \cdot \frac{1}{c} = \frac{a}{c}$

(ii) $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$

Solution:

L.H.S.	$= \frac{a}{b}+\frac{c}{d}$	
	$= \frac{a}{b}\cdot 1+\frac{c}{d}\cdot 1$	Multiplicative Identity
	$= \frac{a}{b}\cdot\left(d\cdot\frac{1}{d}\right)+\frac{c}{d}\left(b\cdot\frac{1}{b}\right)$	Multiplicative Inverse
	$= \frac{a}{b}\cdot\frac{d}{d}+\frac{c}{d}\cdot\frac{b}{b}$	$\because\ d\cdot\frac{1}{d}=\frac{d}{d},\ b\cdot\frac{1}{b}=\frac{b}{b}$
	$= \frac{ad}{bd}+\frac{cb}{db}$	
	$= \frac{ad}{bd}+\frac{bc}{bd}$	Commutative Property w.r.t. '.'
	$= ad\cdot\frac{1}{bd}+bc\cdot\frac{1}{bd}$	$\because\ \frac{a}{b}=a\cdot\frac{1}{b}$
	$= (ad+bc)\cdot\frac{1}{bd}$	Distributive Property
	$= \frac{ad+bc}{bd}$	$\because\ a\cdot\frac{1}{b}=\frac{a}{b}$
	$=$ R.H.S.	

Q.5 Prove that $-\frac{7}{12}-\frac{5}{18}=\frac{-21-10}{36}$

Solution:

L.H.S.	$= -\frac{7}{12}-\frac{5}{18}$	
	$= -\frac{7}{12}\cdot 1-\frac{5}{18}\cdot 1$	Multiplicative Identity
	$= -\frac{7}{12}\left(3\cdot\frac{1}{3}\right)-\frac{5}{18}\cdot\left(2\cdot\frac{1}{2}\right)$	Multiplicative Inverse
	$= -\frac{7}{12}\cdot\frac{3}{3}-\frac{5}{18}\cdot\frac{2}{2}$	$\because\ a\cdot\frac{1}{b}=\frac{a}{b}$
	$= -\frac{21}{36}-\frac{10}{36}$	

$= -21 \cdot \frac{1}{36} - 10 \cdot \frac{1}{36}$ $\qquad \because \frac{a}{b} = a \cdot \frac{1}{b}$

$= (-21 - 10) \cdot \frac{1}{36}$ $\qquad$ Distributive Property

$= \frac{-21 - 10}{36}$

$=$ R.H.S.

Q.6 Simplify by justifying each step:

(i) $\mathbf{\frac{4 + 16x}{4}}$

Solution:

$\frac{4 + 16x}{4}$

$= \frac{1}{4} \cdot (4 + 16x)$ $\qquad \because \frac{a}{b} = \frac{1}{b} \cdot a$

$= \frac{1}{4} \cdot 4 + \frac{1}{4} \cdot 16\,x$ $\qquad$ Distributive Property

$= \frac{1}{4} \cdot 4 \cdot 4x$ $\qquad$ Multiplicative Inverse

$= 1 + 1 \cdot 4x$ $\qquad$ Multiplicative Inverse

$= 1 + 4x$ $\qquad$ Multiplicative Identity

(ii) $\mathbf{\dfrac{\frac{1}{4} + \frac{1}{5}}{\frac{1}{4} - \frac{1}{5}}}$

Solution:

$\dfrac{\frac{1}{4} + \frac{1}{5}}{\frac{1}{4} - \frac{1}{5}}$

$= \left(\dfrac{\frac{1}{4} + \frac{1}{5}}{\frac{1}{4} - \frac{1}{5}}\right) \cdot 1$ $\qquad$ Multiplicative Identity

$$= \left(\frac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}-\frac{1}{5}}\right).\,20.\,\frac{1}{20}$$ Multiplicative Inverse

$$= \frac{\left(\frac{1}{4}+\frac{1}{5}\right).\,20}{\left(\frac{1}{4}-\frac{1}{5}\right).\,20}$$ $\because\ 20.\,\frac{1}{20} = \frac{20}{20}$

$$= \frac{\frac{1}{4}.\,20+\frac{1}{5}.\,20}{\frac{1}{4}.\,20+\frac{1}{5}.\,20}$$ Distributive Property

$$= \frac{\left(\frac{1}{4}.\,4\right)5+\left(\frac{1}{5}.\,5\right)4}{\frac{1}{4}.\,4.5+\frac{1}{5}.\,5.4} = \frac{1.5+1.4}{1.5-1.4}$$ Multiplicative Inverse

$$= \frac{5+4}{5-4}$$ Multiplicative Identity

$$= \frac{9}{1} = 9$$

(iii) $\mathbf{\dfrac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}}}$

$$= \left(\frac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}}\right).\,1$$ Multiplicative Identity

$$= \left(\frac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}}\right).\,bd.\,\frac{1}{bd}$$ Multiplicative Inverse

$$= \frac{\left(\frac{a}{b}+\frac{c}{d}\right).\,bd}{\left(\frac{a}{b}-\frac{c}{d}\right).\,bd}$$ $\because\ bd.\,\frac{1}{bd} = \frac{bd}{bd}$

$$= \frac{\frac{a}{b}.\,bd+\frac{c}{d}.\,bd}{\frac{a}{b}.\,bd-\frac{c}{d}.\,bd}$$ Distributive Property

$$= \frac{a\,.\,\frac{1}{b}\,.\,b\,.\,d + c\,.\,\frac{1}{d}\,.\,b\,.\,d}{a\frac{1}{b}\,.\,b\,.\,d - c\frac{1}{d}\,.\,b\,.\,d} \qquad \because \quad \frac{1}{b} = a.\frac{1}{b} \;,\; \frac{c}{d} = c\,.\,\frac{1}{d}$$

$$= \frac{a\left(\frac{1}{b}\,.\,b\right)d + c\,.\left(\frac{1}{d}\,.\,d\right)b}{a\left(\frac{1}{b}\,.\,b\right)d - c\left(\frac{1}{d}\,.\,d\right)b} \qquad \text{Commutative Property}$$

$$= \frac{a\,.\,1\,.\,d + c.\,1\,.\,b}{a\,.\,1\,.\,d - c\,.\,1\,.\,b} \qquad \text{Multiplicative Inverse}$$

$$= \frac{ad + cb}{ad - cb} \qquad \text{Multiplicative Identity}$$

(iv) $$\frac{\frac{1}{a} - \frac{1}{b}}{1 - \frac{1}{a}\cdot\frac{1}{b}}$$

$$= \left(\frac{\frac{1}{a} - \frac{1}{b}}{1 - \frac{1}{a}\cdot\frac{1}{b}}\right).\,1 \qquad \text{Multiplicative Identity}$$

$$= \left(\frac{\frac{1}{a} - \frac{1}{b}}{1 - \frac{1}{a}\cdot\frac{1}{b}}\right).\,ab\,.\,\frac{1}{ab} \qquad \text{Multiplicative Inverse}$$

$$= \frac{\left(\frac{1}{a} - \frac{1}{b}\right).\,ab}{\left(1 - \frac{1}{a}\,.\,\frac{1}{b}\right).\,ab} \qquad \because \quad ab\,.\,\frac{1}{ab} = \frac{ab}{ab}\,.$$

$$= \frac{\frac{1}{a}\,.\,ab - \frac{1}{b}\,.\,ab}{ab - \frac{1}{a}\,.\,\frac{1}{b}\,.\,ab} \qquad \text{Distributive Property}$$

$$= \frac{\left(\frac{1}{a}\,.\,a\right)b - \left(\frac{1}{b}\;b\right)a}{ab - \left(\frac{1}{a}\,.\,a\right)\left(\frac{1}{b}.b\right)} \qquad \text{Commutative Property}$$

$$= \frac{1\,.\,b - 1\,.\,a}{ab - 1\,.\,1} \qquad \text{Multiplicative Inverse}$$

$$= \frac{b - a}{ab - 1} \quad \text{Multiplicative Identity}$$