Solution:

(i) {a, b, c}, {1, 2, 3}

Given sets are equivalent.

(ii) The set of the first 10 whole members, {0, 1, 2, 3, …… 9}

The given sets are equivalent and also equal.

(iii) Set of angles of a quadrilateral ABCD set of the sides of the same quadrilateral

The given sets are equivalent.

(iv) Set of the sides of a hexagon ABCDEF, Set of the angles of the same hexagon

The given sets are equivalent.

(v) {1, 2, 3, 4, ……}, {2, 4, 6, 8, ……}

The given sets are equivalent.

(vi) {1, 2, 3, 4, ……}, $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right\}$

The given sets are equivalent.

(vii) {5, 10, 15, 20, …… 55555}, {5, 10, 15, 20, ……}

The given sets are not equivalent.

VENN DIAGRAMS

Venn diagrams are used to describe a relation among the sets. In these diagrams, a rectangular region represents the universal set and circular closed curves represent the subsets.

EXERCISE 2.2

Q.1 Exhibit $A \cup B$ and $A \cap B$ by Venn diagrams in the following cases.

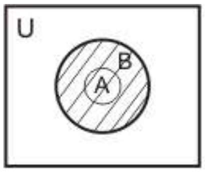
(i) $A \subseteq B$ (ii) $B \subseteq A$ (iii) $A \cup A'$

(iv) A and B are disjoint sets.

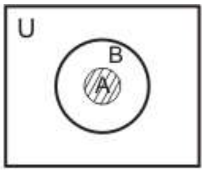
(v) A and B are overlapping sets.

Solution:

(i) $A \subseteq B$

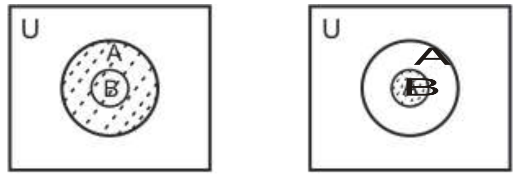


A∪B



A∩B

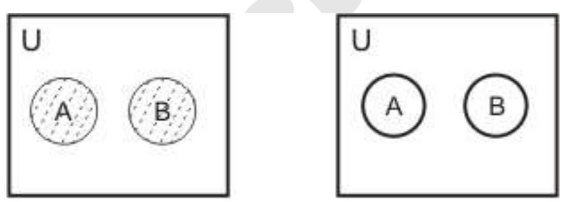
(ii) $B \subseteq A$



$A \cup B$ $A \cap B$

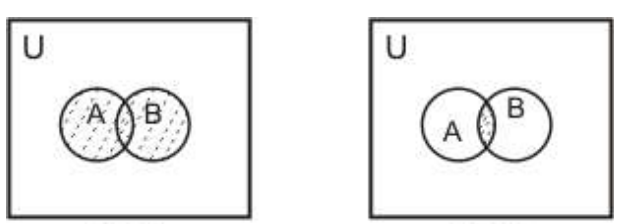
(iii) $A \cup A'$

(iv) A and B are disjoints



$A \cup B$ $A \cap B$

(v) A and B are overlapping sets.

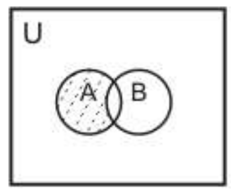


$A \cup B$ $A \cap B$

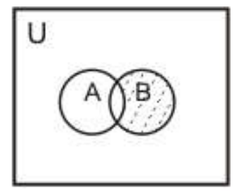
Q.2 Show $A - B$ and $B - A$ by Venn Diagram when
(i) A and B are overlapping sets
(ii) $A \subseteq B$ (iii) $B \subseteq A$

Solution:

(i) A and B are overlapping sets

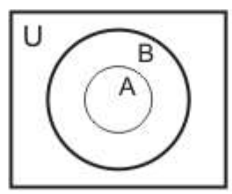


$A - B$

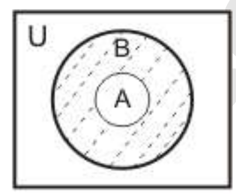


$B - A$

(ii) $A \subseteq B$

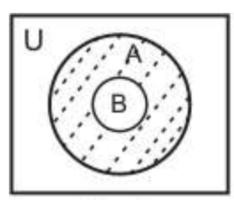


$A - B$

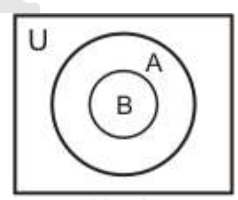


$B - A$

(iii) $B \subseteq A$



$A - B$



$B - A$

Q.3 Under what conditions on A and B are the following statements true?

(i) $A \cup B = A$ **(ii)** $A \cup B = B$ **(iii)** $A - \phi = \phi$
(iv) $A \cap B = B$ **(v)** $n(A \cup B) = n(A) + n(B)$
(vi) $n(A \cup B) = n(A)$ **(vii)** $A - B = A$
(viii) $n(A \cap B) = 0$ **(ix)** $A \cup B = U$
(x) $A \cup B = B \cup A$ **(xi)** $n(A \cap B) = n(B)$
(xii) $U - A = \phi$

Solution:

(i) $A \cup B = A$
if $B \subseteq A$

(ii) $A \cup B = B$
if $A \subseteq B$

(iii) $A - B = A$

if $A \cap B = \phi$

(iv) $A \cap B = B$

if $B \subseteq A$

(v) $n(A \cup B) = n(A) + n(B)$

if $A \cap B = \phi$

(vi) $n(A \cap B) = n(A)$

if $A \subseteq B$

(vii) $A - B = A$

if $A \cap B = \phi$ or $B = \phi$

(viii) $n(A \cap B) = 0$

if $A \cap B = \phi$

(ix) $A \cup B = U$

if $A = B^C$

(x) $A \cup B = B \cup A$

it holds always.

(xi) $n(A \cap B) = n(B)$

if $B \subseteq A$

(xii) $U - A = \phi$

if $A = U$

Q.4 Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,
$A = \{2, 4, 6, 8, 10\}$, $B = \{1, 2, 3, 4, 5\}$ and $C = \{1, 3, 5, 7, 9\}$
List the members of the following sets
(i) A^C (ii) B^C (iii) $A \cup B$ (iv) $A - B$
(v) $A \cap C$ (vi) $A^C \cup C^C$ (vii) $A^C \cup C$ (viii) U^C

Solution:

(i) A^C

$A^C = U - A$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 4, 6, 8, 10\}$

$= (1, 3, 5, 7, 9\}$

(ii) B^C

$B^C = U - B$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 2, 3, 4, 5\}$

$= \{6, 7, 8, 9, 10\}$

(iii) $A \cup B$

$A \cup B = \{2, 4, 6, 8, 10\} \cup \{1, 2, 3, 4, 5\}$

$= \{1, 2, 3, 4, 5, 6, 8, 10\}$

(iv) $A - B$

$A - B = \{2, 4, 6, 8, 10\} - \{1, 2, 3, 4, 5\}$

$= \{6, 8, 10\}$

(v) $A \cap C$

$A \cap C = \{2, 4, 6, 8, 10\} \cap \{1, 3, 5, 7, 9\}$

$= \{\ \}$

(vi) $A^C \cup C^C$

$A^C \cup C^C = (U - A) \cup (U - C)$ .... (1)

$U - A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 4, 6, 8, 10\} = \{1, 3, 5, 7, 9\}$

$U - C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 3, 5, 7, 9\} = \{2, 4, 6, 8, 10\}$

Put in equation (1)

$A^C \cup C^C = \{1, 3, 5, 7, 9\} \cup \{2, 4, 6, 8, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(vii) $A^C \cup C$

$A^C \cup C = (U - A) \cup C$ .... (1)

$U - A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 4, 6, 8, 10\} = \{1, 3, 5, 7, 9\}$

Put in equation (1)

$A^C \cup C = \{1, 3, 5, 7, 9\} \cup \{1, 3, 5, 7, 9\} = \{1, 3, 5, 7, 9\}$

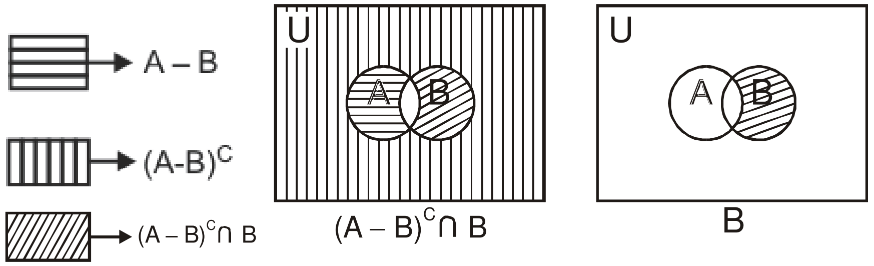
(viii) U^C

$U^C = U - U$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$= \{\ \}$

(ii) $(A - B)^C \cap B = B$



As the shaded portion in above two figures is same $\Rightarrow$ $(A - B)^C \cap B = B$

PROPERTIES OF UNION AND INTERSECTION

(i)	$A \cup B = B \cup A$	Commutative property of union
(ii)	$A \cap B = B \cap A$	Commutative property of intersection
(iii)	$A \cup (B \cup C) = (A \cup B) \cup C$	Associative property of union
(iv)	$A \cap (B \cap C) = (A \cap B) \cap C$	Associative property of intersection
(v)	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributivity of union over intersection
(vi)	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributivity of intersection over union
(vii)	$(A \cup B)' = A' \cap B'$	De Morgan's Laws
(viii)	$(A \cap B)' = A' \cup B'$	

PROOFS OF DE MORGAN'S LAWS AND DISTRIBUTIVE LAWS

(i) $(A \cup B)' = A' \cap B'$

Let $x \in (A \cup B)'$

$\Rightarrow$ $x \notin A \cup B$

$\Rightarrow$ $x \notin A$ and $x \notin B$

$\Rightarrow$ $x \in A'$ and $x \in B'$

$\Rightarrow$ $x \in A' \cap B'$

$\Rightarrow$ $(A \cup B)' \subseteq A' \cap B'$ .... (1)

Now suppose that

$x \in A' \cap B'$

$\Rightarrow$ $x \in A'$ and $x \in B'$

$\Rightarrow$ $x \notin A$ and $x \notin B$

$\Rightarrow$ $x \notin A \cup B$

$\Rightarrow$ $x \in (A \cup B)'$

$\Rightarrow$ $A' \cap B' \subseteq (A \cup B)'$ .... (2)

From (1) and (2), we conclude that $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Let $x \in (A \cap B)'$

$\Rightarrow$ $x \notin A \cap B$

$\Rightarrow$ $x \notin A$ and $x \notin B$

$\Rightarrow$ $x \in A'$ and $x \in B'$

$\Rightarrow$ $x \in A' \cup B'$

$\Rightarrow$ $(A \cap B)' \subseteq A' \cap B'$ ……… (1)

Now suppose that

$x \in A' \cup B'$

$\Rightarrow$ $x \in A'$ or $x \in B'$

$\Rightarrow$ $x \notin A$ or $x \notin B$

$\Rightarrow$ $x \notin A \cap B$

$\Rightarrow$ $x \in (A \cap B)'$

$\Rightarrow$ $A' \cup B' \subseteq (A \cap B)'$ ……… (2)

From (1) and (2), it is verified that $(A \cap B)' = A' \cup B'$

(iii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Let $x \in A \cup (B \cap C)$

$\Rightarrow$ $x \in A$ or $x \in B \cap C$

$\Rightarrow$ if $x \in A$

then $x \in A \cup B$ and $x \in A \cup C$

and if $x \in B \cap C$

$\Rightarrow$ $x \in B$ and $x \in C$

$\Rightarrow$ $x \in A \cup B$ and $x \in A \cup C$

$\Rightarrow$ $x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow$ $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ ……… (1)

Now suppose that

$x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow$ $x \in A \cup B$ and $x \in A \cup C$

Now there are two cases either $x \in A$ or $x \notin A$

if $x \in A$ then $x \in A \cup (B \cap C)$

if $x \notin A$ then $x \in B$ and $x \in C$

$\Rightarrow$ $x \in B \cap C$

$\Rightarrow$ $x \in A \cup (B \cap C)$

$\Rightarrow$ In both cases $x \in A \cup (B \cap C)$ ……… (2)

From (1) and (2)

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Hence proved.

(iv) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Let $x \in A \cap (B \cup C)$

$\Rightarrow$ $x \in A$ and $x \in B \cup C$

$\Rightarrow$ if $x \in A$ and $x \in B$

$\Rightarrow$ $x \in A \cap B \Rightarrow x \in (A \cap B) \cup (A \cap C)$

If $x \in A$ and $x \in C$

$\Rightarrow$ $x \in A \cap C \Rightarrow x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow$ $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ ……… (1)

Now suppose that

$x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow$ $x \in A \cap B$ or $x \in A \cap C$

There are two cases.

Case I if $x \in A \cap B$

$\Rightarrow$ $x \in A$ and $x \in B$

$\Rightarrow$ $x \in A$ and $x \in B \cup C$

$\Rightarrow$ $x \in A \cap (B \cup C)$

Case II if $x \in A \cap C$

$\Rightarrow$ $x \in A$ and $x \in C$

$\Rightarrow$ $x \in A$ and $x \in B \cup C$

$\Rightarrow$ $x \in A \cap (B \cup C)$

$\Rightarrow$ In both cases

$x \in A \cap (B \cup C)$

$\Rightarrow$ $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ ……… (2)

From (1) and (2) we have

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Hence proved.