Chapter 2

SETS, FUNCTIONS AND GROUPS

SET

A well defined collection of distinct objects is named as a set.

The objects in a set are called its members or elements.

There are three different ways of describing a set.

(i) The descriptive method.

(ii) The tabular method.

(iii) Set builder method.

EXERCISE 2.1

Q.1 Write the following sets in set builder notation.

(i) $\{1, 2, 3, ... 1000\}$ **(ii)** $\{0, 1, 2, ... 100\}$

(iii) $\{0, \pm 1, \pm 2, ... \pm 1000\}$ **(iv)** $\{0, -1, -2, ... -500\}$

(v) $\{100, 101, 102, ... 400\}$

(vi) $\{-100, -101, -102, ... -500\}$

(vii) **{Peshawar, Lahore, Karachi, Quetta}**

(viii) **{January, June, July}**

(ix) **The set of all odd natural numbers**

(x) **The set of all rational number**

(xi) **The set of all real number between 1 and 2**

(xii) **The set of all integers between – 100 and 1000**

Solution:

(i) $\{1, 2, 3, ... , 1000\}$

Set builder notation of given set is $\{x \mid x \in N \wedge x \leq 1000\}$

(ii) $\{0, 1, 2, ... 100\}$

Set builder notation of the given set is $\{x \mid x \in W \wedge x \leq 100\}$

(iii) $\{0, \pm 1, \pm 2, ... \pm 1000\}$

Set builder notation of the given set is $\{x \mid x \in z \wedge -1000 \leq x \leq 1000\}$

(iv) $\{0, -1, -2, \ldots\ldots -500\}$

Set builder notation of the given set is $\{x \mid x \in z \wedge -500 \leq x \leq 0\}$

(v) $\{100, 101, 102, \ldots\ldots 400\}$

Set builder notation of the given set is $\{x \mid x \in N \wedge 100 \leq x \leq 400\}$

(vi) $\{-100, -101, -102, \ldots\ldots -500\}$

Set builder notation of the given set is $\{x \mid x \in z \wedge -500 \leq x \leq -100\}$

(vii) {Peshawar, Lahore, Karachi, Quetta}

Set builder notation of the given set is {x | x is the capital of province of Pakistan}

(viii) {January, June, July}

Set builder notation of the given set is {x | x is the name of month starts with "J"}

(ix) The set of all odd natural numbers

Set builder notation of the given set is {x | x is an odd natural number}

(x) The set of all rational number

Set builder notation of the given set is $\{x \mid x \in Q\}$

(xi) The set of all real number between 1 and 2

Set builder notation of the given set is $\{x \mid x \in R \wedge 1 < x < 2\}$

(xii) The set of all integers between – 100 and 1000

Set builder notation of the given set is $\{x \mid x \in z \wedge -100 < x < 1000\}$

Q.2 Write each of the following sets in descriptive and tabular form:

(i) $\{x \mid x \in N \wedge x \leq 10\}$

(ii) $\{x \mid x \in N \wedge 4 < x < 12\}$

(iii) $\{x \mid x \in Z \wedge 5 < x < 5\}$

(iv) $\{x \mid x \in E \wedge 2 < x \leq 4\}$

(v) $\{x \mid x \in Z \wedge -5 < x < 5\}$

(vi) $\{x \mid x \in O \wedge 3 < x < 12\}$

(vii) $\{x \mid x \in E \wedge 4 \leq x \leq 10\}$

(viii) $\{x \mid x \in E \wedge 4 < x < 6\}$

(ix) $\{x \mid x \in O \wedge 5 \leq x \leq 7\}$

(x) $\{x \mid x \in O \wedge 5 \leq x < 7\}$

(xi) $\{x \mid x \in N \wedge x + 4 = 0\}$

(xii) $\{x \mid x \in Q \wedge x^2 = 2\}$

(xiii) $\{x \mid x \in R \wedge x = x\}$

(xiv) $\{x \mid x \in Q \wedge x = -x\}$

(xv) $\{x \mid x \in R \wedge x \neq 2\}$

(xvi) $\{x \mid x \in R \wedge x \notin Q\}$

Solution:

(i) $\{x \mid x \in N \wedge x \leq 10\}$

Descriptive form is: The set of the first ten natural numbers.

The tabular form is: $\{1, 2, 3, \ldots\ldots 10\}$

(ii) $\{x \mid x \in N \wedge 4 < x < 12\}$

Descriptive form is: The set of the natural numbers between 4 and 12.

Tabular form is: $\{5, 6, 7, \ldots\ldots 11\}$

(iii) $\{x \mid x \in Z \wedge -5 < x < 5\}$

Descriptive form is: The set of integers between – 5 and 5.

Tabular form is: $\{-4, -3, -2, \ldots\ldots 4\}$

(iv) $\{x \mid x \in E \wedge 2 < x \leq 4\}$

Descriptive form is: The set of even integers between 2 and 5.

Tabular form is: $\{4\}$

(v) $\{x \mid x \in P \wedge x < 12\}$

Descriptive form is: The set of prime numbers less than 5.

Tabular form is: $\{2, 3, 5, 7, 11\}$

(vi) $\{x \mid x \in O \wedge 3 < x < 12\}$

Descriptive form is: The set of odd integers between 3 and 12.

Tabular form is: $\{5, 7, 9, 11\}$

(vii) $\{x \mid x \in E \wedge 4 \leq x \leq 10\}$

Descriptive form is: The set of even integers from 4 upto 10.

Tabular form is: $\{4, 6, 8, 10\}$

(viii) $\{x \mid x \in E \wedge 4 < x < 6\}$

Descriptive form is: The set of even integers between 4 and 6.

Tabular form is: $\{\ \}$

(ix) $\{x \mid x \in O \wedge 5 \leq x \leq 7\}$

Descriptive form is: The set of odd integers from 5 upto 7.

Tabular form is: $\{5, 7\}$

(x) $\{x \mid x \in O \wedge 5 < x < 7\}$

Descriptive form is: The set of odd integers between 5 and 7.

Tabular form is: $\{\ \}$

(xi) $\{x \mid x \in N \wedge x + 4 = 0\}$

Descriptive form is: The set of the natural numbers x satisfying $x + 4 = 0$.

Tabular form is: $\{\ \}$

(xii) $\{x \mid x \in Q \wedge x^2 = 2\}$

Descriptive form is: The set of rational numbers x satisfying $x^2 = 2$.

Tabular form is: $\{\ \}$

(xiii) $\{x \mid x \in R \wedge x = x\}$

Descriptive form is: The set of real numbers x satisfying $x = x$.

Tabular form is: The set of real numbers.

(xiv) $\{x \mid x \in Q \wedge x = -x\}$

Descriptive form is: The set of rational numbers x satisfying $x = -x$.

Tabular form is: $\{0\}$

(xv) $\{x \mid x \in R \wedge x \neq 2\}$

Descriptive form is: The set of real numbers x satisfying $x \neq 2$.

Tabular form is: $R - \{2\}$

(xvi) $\{x \mid x \in R \wedge x \notin Q\}$

Descriptive form is: The set of real numbers x which are not rational.

Tabular form is: Q'

FINITE AND INFINITE SETS

If the number of elements in a set are finite then it is called finite set and if the number of elements in a set are infinite then it is called an infinite set.

Q.3 Which of the following sets are finite and which of these are infinite.

(i) The set of students of your class

(ii) The set of all schools in Pakistan

(iii) The set of natural numbers between 3 and 10

(iv) The set of rational numbers between 3 and 10

(v) The set of real numbers between 0

(vi) The set of rationales between 0 and 1

(vii) The set of whole numbers between 0 and 1

(viii) The set of all leaves of trees in Pakistan

(ix) P (N)

(x) P {a, b, c}

(xi) {1, 2, 3, …….}

(xii) {1, 2, 3, ……., 100000000}

(xiii) $\{x \mid x \in R \wedge x \neq x\}$

(xiv) $\{x \mid x \in R \wedge x^2 = -16\}$

(xv) $\{x \mid x \in Q \wedge x^2 = 5\}$

(xvi) $\{x \mid x \in Q \wedge 0 \le x \le 1\}$

Solution:

(i) Finite set.

(ii) Finite set.

(iii) Finite set.

(iv) Infinite set.

(v) Infinite set.

(vi) Infinite set.

(vii) Finite set.

(viii) Infinite set.

(ix) Infinite set.

(x) Finite set.

(xi) Infinite set.

(xii) Finite set.

(xiii) Finite set.

(xiv) Finite set.

(xv) Finite set.

(xvi) Infinite set.

SUBSET

If every element of a set A is an element of set B, then A is said to be a subset of B i.e. $A \subseteq B$.

PROPER SUBSET

If A is a subset of B and B contains at least one element which is not an element of A, then A is said to be a proper subset of B.

IMPROPER SUBSET

If A is subset of B and $A = B$, then we say that A is an improper subset of B.

Q.4 Write two proper subsets of each of the following sets

(i) $\{a, b, c\}$ **(Gujranwala Board 2007)**

(ii) $\{0, 1\}$ **(Lahore Board 2006)**

(iii) N **(iv)** z

(v) Q **(vi)** $\Re$

(vii) W **(viii)** $\{x \mid x \in Q \wedge 0 < x \leq 2\}$

Solution:

(i) Two proper subsets are $\{a\}, \{b\}$

(ii) Two proper subsets are $\{0\}, \{1\}$

(iii) Two proper subsets are $\{10\}, \{4\}$

(iv) Two proper subsets are $\{1\}, \{3\}$

(v) Two proper subsets are $\{1\}, \{2\}$

(vi) Two proper subsets are $\{0\}, \{1\}$

(vii) Two proper subsets are $\{1\}, \{5\}$

(viii) Two proper subsets are $\{1\}, \{1, 2\}$

Q.5 Is there any set which has no proper subset? If so name that set?

(Lahore Board 2009)

Solution:

Yes, there is a set which has no proper subset and it is ϕ or $\{\ \}$ i.e. empty set.

Q.6 What is the difference between {a, b} and {{a, b}}.

Solution:

$\{a, b\}$ contains two elements but $\{\{a, b\}\}$ contains only one element $\{a, b\}$.

Q.7 Which of the following sentences are true and which of them are false.

(i) $\{1, 2\} = \{2, 1\}$ **(ii)** $\phi \subseteq \{\{a\}\}$

(iii) $\{a\} \supseteq \{\{a\}\}$ **(iv)** $\{a\} \in \{\{a\}\}$

(v) $a \in \{\{a\}\}$ **(vi)** $\phi \in \{\{a\}\}$

Solution:

(i)	$\{1, 2\} = \{2, 1\}$	True
(ii)	$\phi \subseteq \{\{a\}\}$	True
(iii)	$\{a\} \subseteq \{\{a\}\}$	False
(iv)	$\{a\} \in \{\{a\}\}$	True
(v)	$a \in \{\{a\}\}$	False
(vi)	$\phi \in \{\{a\}\}$	False

POWER SET

The power set of a set S denoted by P (S) is the set containing all the possible subsets of S.

If number of elements in S = m then number of elements in P (s) = 2^m.

or

if n (S) = m

then n P (S) = 2^m.

Q.8 What is the number of elements of the power set of each of the following sets?

(i)	**{ }**	**(ii)**	**{0, 1}**
(iii)	**{1, 2, 3, 4, 5, 6, 7}**	**(iv)**	**{0, 1, 2, 3, 4, 5, 6, 7}**
(v)	**{a, {b, c}}**	**(vi)**	**{{a, b}, {b, c}, {d, e}}**

Solution:

(i) { }

Let S = { } $\Rightarrow$ n (s) = 0

$\Rightarrow$ n p (s) = 2^0 = 1

(ii) {0, 1}

Let s = {0, 1} $\Rightarrow$ n (s) = 2

$\Rightarrow$ n p (s) = 2^2 = 4

(iii) {1, 2, 3, 4, 5, 6, 7}

Let s = {1, 2, 3, 4, 5, 6, 7} $\Rightarrow$ n (s) = 7

$\Rightarrow$ n p (s) = 2^7 = 128

(iv) {0, 1, 2, 3, 4, 5, 6, 7}

Let s = {0, 1, 2, 3, 4, 5, 6, 7} $\Rightarrow$ n (s) = 8

$\Rightarrow$ n p (s) = 2^8 = 256

(v) {a, {b, c}}

Let s = {a, {b, c}} $\Rightarrow$ n (s) = 2

$\Rightarrow$ n p (s) = 2^2 = 4

(vi) {{a, b}, {b, c}, {d, e}}

Let s = {{a, b}, {b, c}, {d, e}} $\Rightarrow$ n (s) = 3

$\Rightarrow$ n p (s) = 2^3 = 8

Q.9 Write down the power set of each of the following sets:

(i) {9, 11} **(ii) {+, –, x, ÷}**

(iii) {ϕ} **(iv) {a, {b, c}}**

Solution:

(i) {9, 11}

Let S = {9, 11}

P (S) = {ϕ, {9}, {11}, {9, 11}}

(ii) {+, –, x, ÷}

Let S = {+, –, x, ÷}

P (S) = {ϕ, {+}, {–}, {x}, {÷}, {+, –}, {+, x}, {+, ÷}, {–, x}
{–, ÷}, {x, ÷}, {+, –, x}, {+, –, ÷}, {–, x, ÷},
{+, x, ÷}, {+, –, x, ÷}}

(iii) {ϕ}

Let S = {ϕ}

$\Rightarrow$ P (S) = {ϕ, {ϕ}}

(iv) {a, {b, c}}

Let S = {a, {b, c}}

$\Rightarrow$ P (S) = {ϕ, {a}, {{b, c}}, {a, {b, c}}}

EQUAL SETS

Two sets A and B are equal iff they have the same elements.

EQUIVALENT SETS

Two sets A and B are equivalent if a (1 – 1) correspondence can be established between A and B.

Q.10 Which pair of sets are equivalent? Which of them are also equal?

(i) {a, b, c}, {1, 2, 3}

(ii) The set of the first 10 whole numbers, {0,1,2, 3, …9}

(iii) Set of angles of a quadrilateral ABCD, set of the sides of the same quadrilateral

(iv) Set of the sides of a hexagon ABCDEF, Set of the angles of the same hexagon

(v) {1, 2, 3, 4, ……}, {2, 4, 6, 8, ……}

(vi) {1, 2, 3, 4, ……}, $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right\}$

(vii) {5, 10, 15, 20, …… 55555}, {5, 10, 15, 20, ……}

Solution:

(i) {a, b, c}, {1, 2, 3}

Given sets are equivalent.

(ii) The set of the first 10 whole members, {0, 1, 2, 3, …… 9}

The given sets are equivalent and also equal.

(iii) Set of angles of a quadrilateral ABCD set of the sides of the same quadrilateral

The given sets are equivalent.

(iv) Set of the sides of a hexagon ABCDEF, Set of the angles of the same hexagon

The given sets are equivalent.

(v) {1, 2, 3, 4, ……}, {2, 4, 6, 8, ……}

The given sets are equivalent.

(vi) {1, 2, 3, 4, ……}, $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots..\right\}$

The given sets are equivalent.

(vii) {5, 10, 15, 20, …… 55555}, {5, 10, 15, 20, ……}

The given sets are not equivalent.

VENN DIAGRAMS

Venn diagrams are used to describe a relation among the sets. In these diagrams, a rectangular region represents the universal set and circular closed curves represent the subsets.

EXERCISE 2.2

Q.1 Exhibit $A\cup B$ and $A\cap B$ by Venn diagrams in the following cases.

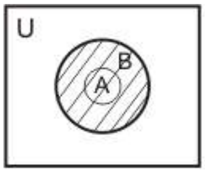
(i) $A \subseteq B$ (ii) $B \subseteq A$ (iii) $A\cup A'$

(iv) A and B are disjoint sets.

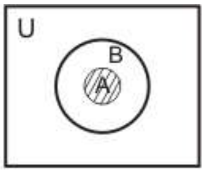
(v) A and B are overlapping sets.

Solution:

(i) $A \subseteq B$



$A\cup B$



$A\cap B$