$a + 6(3) = 20$

$a + 18 = 20 \Rightarrow \boxed{a = 2}$

$\Rightarrow \quad a_1 = 2$

$a_2 = a_1 + d = 2 + 3 = 5$

$a_3 = a_1 + 2d = 2 + 2(3) = 8$

$\Rightarrow$ required A.P. is 2, 5, 8, …….

Q.18 If a^2, b^2 and c^2 are in A.P. show that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

Solution:

If a^2, b^2, c^2 are in A.P.

$\Rightarrow \quad b^2 - a^2 = c^2 - b^2$

To show that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P. we will prove that

$$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

or $$\frac{b+c-(c+a)}{(c+a)(b+c)} = \frac{c+a-(a+b)}{(a+b)(c+a)}$$

or $$\frac{b+c-c-a}{b+c} = \frac{c+a-a-b}{a+b}$$

or $$\frac{b-a}{b+c} = \frac{c-b}{a+b}$$

or $(b-a)(b+a) = (c-b)(c+b)$

or $b^2 - c^2 = c^2 - b^2$ (given)

Hence $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

EXERCISE 6.5

Q.1 A man deposits in a bank Rs. 10 in the first month Rs. 15 in the second month, Rs. 20 in the third month and so on. Find how much he will have deposited in the bank by 9th months.

Solution:

Deposited amount is

10, 15, 20, ... which is A.P

Here $a_1 = 10, \quad d = 15 - 10 = 5, \quad n = 9$

We find S_9

As $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$= \frac{9}{2}[2(10) + (9-1)5]$

$= \frac{9}{2}[20 + 40]$

$= \frac{9}{2}\{60\} = 9(30) = \text{Rs. } 270$

Q.2 378 trees are planted in rows in the shape of an isosceles triangle. The number in the successive rows, decreasing by one from the base to the top. How many trees are there in the row which forms the base of the triangle.

Solution:

Let $a_1 = 1$ is the topmost row and a_n is the base row then sequence of trees is

1, 2, 3, ……

$\Rightarrow$ $1 + 2 + 3 + \ldots..a_n = 378 \Rightarrow a_1 = 1, \quad d = 1$

$\Rightarrow$ $S_n = 378$

$\Rightarrow$ $\frac{n}{2}[2a_1 + (n-1)d] = 378$

$\Rightarrow$ $\frac{n}{2}[2(1) + (n-1)(1)] = 378$

$\Rightarrow$ $n[2 + n - 1] = 756$

$\Rightarrow$ $n(n+1) = 756$

$\Rightarrow$ $n^2 + n - 756 = 0$

$\Rightarrow$ $n^2 + 28n - 27n - 756 = 0$

$\Rightarrow$ $n(n+28) - 27(n+28) = 0$

$\Rightarrow$ $(n-27)(n+28) = 0$

$\Rightarrow$ $n = 27 \qquad n = -28$ (not possible)

$\Rightarrow$ we have to find a_{27}.

As $a_1 = 1, \quad a_2 = 2,$ and so on

$\Rightarrow$ $a_{27} = 27$

Q.3 A man borrows Rs. 1100 and agrees to repay with a total interest of Rs. 230 in 14 installments, each installment is less than the preceding by Rs. 10. What should be his first installment?

Solution:

Total amount to repay = Rs. 1100 + 230 = Rs. 1330

$d = -10, \quad n = 14, \quad S_n = 1330, \quad a_1 = ?$

As

$$S_n = \frac{n}{2}\{2a_1 + (n-1)d\}$$

$$1330 = \frac{14}{2}[2a_1 + (14-1)(-10)]$$

$$1330 = 7(2a_1 - 130)$$

$$\frac{1330}{7} = 2a_1 - 130$$

$$190 = 2a_1 - 130$$

$$190 + 130 = 2a_1$$

$$320 = 2a_1$$

$$\boxed{a_1 = 160}$$

$\Rightarrow$

First installment will be Rs. 160.

Q.4 A clock strikes once when its hour hand is at one, twice, when it is at two and so on. How many times a clock strikes in twelve hours. (Lahore Board 2007)

Solution:

The sequence is

1, ,2, 3, ……, 12 (which is an A.P)

$a_1 = 1, \quad d = 1, \quad n = 12, \quad S_{12} = ?$

As $$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$= \frac{12}{2}[2(1) + (12-1)(1)]$$

$$= 6(2 + 11) = 6(13) = 78$$

Q.5 A student saves Rs. 12 at the end of the first week and goes on increasing his saving Rs. 4 weekly. After how many weeks will he be able to save Rs. 2100.

Solution:

The sequence is

12, 16, 20, …… (which is an A.P)

$a_1 = 12 \quad d = 16 \;\; 12 = 4 \quad n = ? \quad S_n = 2100$

As

$\Rightarrow \quad S_n = \frac{n}{2}\,[2a_1 + (n-1)\,d]$

$\Rightarrow \quad 2100 = \frac{n}{2}\,[2\,(12) + (n-1)\,4]$

$\Rightarrow \quad 2100 = \frac{n}{2}\,[24 + 4n - 4]$

$\Rightarrow \quad 2100 = \frac{n}{2}\,[4n + 20]$

$\Rightarrow \quad 2100 = n\,(2n + 10)$

$\Rightarrow \quad 2100 = 2n\,(n + 5)$

$\Rightarrow \quad 1050 = n\,(n + 5)$

$\Rightarrow \quad n^2 + 5n - 1050 = 0$

$\Rightarrow \quad n^2 + 35n - 30n - 1050 = 0$

$\Rightarrow \quad n\,(n + 35) - 30\,(n + 35) = 0$

$\Rightarrow \quad (n - 30)\,(n + 35) = 0$

$\Rightarrow \quad n = 30 \quad \text{or} \quad n = -35 \quad \text{(not possible)}$

$\Rightarrow$ no. of weeks = 30 weeks.

Q.6 An object falling from rest, fall 9 meter during the first second, 27 meters during next second, 45 meters during the third second and so on.

(i) How far will it fall during the fifth second?

(ii) How far will it fall upto the fifth second?

Solution:

Given that

(i) The sequence is

9, 27, 45, ……. (which is an A.P.)

$a_1 = 9,\ d = 18,\ n = 5,\ a_5 = ?$

As $a_n = a_1 + (n-1)d$

$a_5 = 9 + (5-1)(18) = 9 + 4(18) = 9 + 72 = 81\text{m}$

(ii) To find S_5

As $S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$

$$S_5 = \frac{5}{2}\left[2(9) + (5-1)(18)\right]$$

$$= \frac{5}{2}\left[18 + 4(18)\right] = \frac{5}{2}[90] = 225\text{m}$$

Q.7 An investor earned Rs. 6000 for year 1980 and Rs. 12000 for the year 1990 on the same investment. If his earning have increased by the same amount each year, how much income he has received from the investment over the past eleven years?

Solution:

Earning in 1980 $= a_1 =$ Rs. 6000

Earning in 1990 $= a_{11} =$ Rs. 12000

As $a_{11} = a_1 + 10d$

$\Rightarrow$ $12000 = 6000 + 10d$

$12000 - 6000 = 10d$

$6000 = 10d$

$d = 600$

Now we find S_{11}

As $S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$

$$S_{11} = \frac{11}{2}\left[2(6000) + (11-1)(600)\right]$$

$$= \frac{11}{2}\left[12000 + 10(600)\right]$$

$$= \frac{11}{2}[18000] = 99000$$

$\Rightarrow$ Total earning in 11 years = Rs. 99000

Q.8 The sum of interior angles of polygons having sides 3, 4, 5, ... etc. form an A.P. Find the sum of the interior angles for a 16 sided polygon.

Solution:

Sum of interior angles of 3–sided polygon (triangle) $= \pi$

Sum of interior angles of 4–sided polygon $= 2\pi$

Sum of interior angles of 16–sided polygon $= ?$

The sequence is $\pi, 2\pi, 3\pi, \ldots\ldots$ (which is an A.P.)

$a_1 = \pi, \quad d = 2\pi - \pi = \pi, \quad n = 14, \quad a_{14} = ?$

As $a_n = a_1 + (n-1)d$

$a_{14} = \pi + (14-1)(\pi) = \pi + 13\pi$

$a_{14} = 14\pi$

$\Rightarrow$ sum of interior angles of 14–sided polygon $= 14\pi$

Q.9 The prize money of Rs. 60,000 will be distributed among the eight teams according to their positions determined in the match–series. The award increases by the same amount for each higher position. If the last place team is given Rs. 4000, how much will be awarded to the first place team?

Solution:

The last place team is given $= a_8 =$ Rs. 4000

$S_8 = 60{,}000, \; n = 8, \; a_1 = ?$

As $S_n = \frac{n}{2}[a_1 + a_n]$

$S_8 = \frac{8}{2}[a_1 + a_8]$

$60{,}000 = 4[a_1 + 4000]$

$15000 = a_1 + 4000$

$15000 - 4000 = a_1$

$a_1 = 1100$

First place team will be awarded = Rs. 1100

Q.10 An equilateral triangular base is filled by placing eight balls in the first row, 7 balls in the second row and so on with one ball in the last row and so on with one ball at the row. After this base layer, second layer is formed by placing 7 balls in its first row, 6 balls in its second row and so on with one ball in its last row. Continuing this process, a pyramid of balls is formed with one ball on top. How many balls are there in the pyramid?

Solution:

Balls in the 1^{st} layer $= 8+7+6+5+4+3+2+1 = 36$

Balls in the 2^{nd} layer $= 7+6+5+4+3+2+1 = 28$

Balls in the 3^{rd} layer $= 6+5+4+3+2+1 = 21$

Balls in the 4^{th} layer $= 5+4+3+2+1 = 15$

Balls in the 5^{th} layer $= 4+3+2+1 = 10$

Balls in the 6^{th} layer $= 3+2+1 = 6$

Balls in the 7^{th} layer $= 2+1 = 3$

Balls in the 8^{th} layer $= 1 = 1$

Total Balls $= 36+28+21+15+10+6+3+1 = 120$

GEOMETRIC PROGRESSION OR SEQUENCE (G.P):

A sequence $\{a_n\}$ is a geometric sequence or geometric progression if $\frac{a_n}{a_{n-1}}$ is the same non–zero number for all $n \in N$ and $n > 1$. The quotient $\frac{a_n}{a_{n-1}}$ is usually denoted by r and is called common ratio of the G.P.

General Term of G.P.

General term or nth term of G.P. is given by

$a_n = a_1 r^{n-1}$

EXERCISE 6.6

Q.1 Find the 5^{th} term of G.P. 3, 6, 12, ...

(Lahore Board 2006, Gujranwala Board 2007)

Solution:

Given sequence

3, 6, 12, ...

$a_1 = 3, \quad r = \frac{6}{3} = 2, \quad n = 5$

As $a_n = a_1 r^{n-1}$

$a_5 = (3)(2)^{5-1} = 3(2)^4 = 3(16) = 48$

Q.2 Find the 11^{th} term of the sequence $1+i, 2, \frac{4}{1+i}$ (Lahore Board 2011)

Solution:

Given sequence

$1+i, \ 2, \ \frac{4}{1+i}, \ldots\ldots.$

$a_1 = 1+i, \quad r = \frac{1}{1+i}, \quad n = 11$