Chapter 3

MATRICES AND DETERMINANTS

Matrix

A rectangular array of numbers enclosed by a pair of brackets such as:

$$\begin{bmatrix} 2 & -2 & 3 \\ -5 & 4 & 7 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 5 \\ 2 & 7 & 9 \\ 4 & 1 & 2 \end{bmatrix}$$

(i) (ii)

is called a matrix. The horizontal lines of numbers are called rows and the vertical lines of numbers are called columns.

Order of a Matrix

If a matrix has m rows and n columns then its order is $m \times n$.

In above, matrix (i) has order 2×3 and matrix (ii) has order 4×3.

Addition of Matrices

Two matrices A and B can be added if A and B have same order.

The sum of A and B, $A + B$ can be obtained by adding their corresponding elements.

For example if

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 9 \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 & 3 \\ 1 & 2 \end{bmatrix}$$

then

$$A + B = \begin{bmatrix} 2 & 1 \\ 3 & 9 \end{bmatrix} + \begin{bmatrix} 7 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+7 & 1+3 \\ 3+1 & 9+2 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 4 & 11 \end{bmatrix}$$

Subtraction of Matrices

A matrix B can be subtracted from a matrix A if A and B have same order. Subtraction of B from A, $A - B$ can be obtained by subtracting each element of matrix B from the corresponding element of matrix A.

If

$$A = \begin{bmatrix} 3 & 1 \\ 9 & 7 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 4 \\ 9 & 8 \end{bmatrix}$$

then

$$A + B = \begin{bmatrix} 3 & 1 \\ 9 & 7 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 9 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2 & 1-4 \\ 9-9 & 7-8 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 0 & -1 \end{bmatrix}$$

Scalar Multiplication

If A is a matrix of order $m \times n$ and k is a scalar, then the product k and A, denoted by kA, is the matrix formed by multiplying each entry of A by k, and this process is called scalar multiplication.

If $A = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$ and $2 \in R$

then

$$2A = 2\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 \times 4 & 2 \times 2 \\ 2 \times 3 & 2 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 \\ 6 & 2 \end{bmatrix}$$

Multiplication of Two Matrices

Two matrices A and B are said to be conformable for the product AB if

The number of columns of A = The number of rows of B.

If A is a matrix of order $m \times n$ and B is a matrix of order $n \times p$ then we can find $AB = C$ (say) and order of matrix C will be $m \times p$.

For example

If

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

then

$$AB = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} (2)(2)+(1)(1) & (2)(1)+(1)(2) & (2)(3)+(1)(4) \\ (3)(2)+(4)(1) & (3)(1)+(4)(2) & (3)(3)+(4)(4) \end{bmatrix}$$

$$= \begin{bmatrix} 4+1 & 2+2 & 6+4 \\ 6+4 & 3+8 & 9+16 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 10 \\ 10 & 11 & 25 \end{bmatrix}$$

Transpose of A Matrix

If A is a matrix of order $m \times n$ then an $n \times m$ matrix obtained by interchanging the rows and columns of A, is called the transpose of A and it is denoted by A^t.

For example if $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then $A^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

Determinant of 2×2 Matrix

We can associate a unique number with every square matrix A over R or C, this number is known as the determinant of A.

For example if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then

$$\det A = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Singular and Non–singular Matrices

A matrix A is singular if $|A| = 0$

A matrix A is non–singular if $|A| \neq 0$

Adjoint of 2×2 Matrix

The adjoint of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is denoted by adj A and is defined as

$$\text{adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Inverse of 2×2 Matrix

Let A be a non–singular square matrix of order 2. If there exists a matrix B such that $AB = BA\ I_2$ where $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then B is called the multiplicative inverse of A and is usually denoted by A^{-1}. i.e. $B = A^{-1}$

$$\Rightarrow \quad A A^{-1} = A^{-1} A = I_2$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

EXERCISE 3.1

Q.1 If $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix}$ then show that

(i) $4A - 3A = A$ (ii) $3B - 3A = 3(B - A)$

Solution:

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix}$$

(i) To show

$4A - 3A = A$

L.H.S.

$$\begin{aligned}
4A - 3A &= 4\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} - 3\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \\
&= \begin{bmatrix} (4)(2) & (4)(3) \\ (4)(1) & (4)(5) \end{bmatrix} - \begin{bmatrix} (3)(2) & (3)(3) \\ (3)(1) & (3)(5) \end{bmatrix} \\
&= \begin{bmatrix} 8 & 12 \\ 4 & 20 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix} \\
&= \begin{bmatrix} 8-6 & 12-9 \\ 4-3 & 20-15 \end{bmatrix} \\
&= \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} = A = \text{R.H.S.}
\end{aligned}$$

(ii) To show $3B - 3A = 3(B - A)$

L.H.S.

$$\begin{aligned}
3B - 3A &= 3\begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix} - 3\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \\
&= \begin{bmatrix} (3)(1) & (3)(7) \\ (3)(6) & (3)(4) \end{bmatrix} - \begin{bmatrix} (3)(2) & (3)(3) \\ (3)(1) & (3)(5) \end{bmatrix} \\
&= \begin{bmatrix} 3 & 21 \\ 18 & 12 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix} \\
&= \begin{bmatrix} 3-6 & 21-9 \\ 18-3 & 12-15 \end{bmatrix} \\
&= \begin{bmatrix} -3 & 12 \\ 15 & -3 \end{bmatrix}
\end{aligned}$$

R.H.S.

$$\begin{aligned}
3(B - A) &= 3\left(\begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}\right) \\
&= 3\begin{bmatrix} 1-2 & 7-3 \\ 6-1 & 4-5 \end{bmatrix} \\
&= 3\begin{bmatrix} -1 & 4 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} (3)(-1) & (3)(4) \\ (3)(5) & (3)(-1) \end{bmatrix} \\
&= \begin{bmatrix} -3 & 12 \\ 15 & -3 \end{bmatrix} = \text{R.H.S.}
\end{aligned}$$

$\Rightarrow \quad 3B - 3A = 3(B - A)$

Q.2 **If $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$, show that $A^4 = I_2$.**

Solution:

To show that $A^4 = I^2$ where $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Take

$$A^2 = A.A. = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$= \begin{bmatrix} (i)\,(i) + (0)\,(1) & (i)\,(0) + (0)\,(-i) \\ (1)\,(i) + (-i)\,(1) & (1)\,(0) + (-i)\,(-i) \end{bmatrix}$$

$$= \begin{bmatrix} i^2 + 0 & 0 - 0 \\ i - i & 0 + i^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \because\ i^2 = -1$$

Now

$$A^4 = A^2 . A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (-1)\,(-1) + (0)\,(0) & (-1)\,(0) + (0)\,(-1) \\ (0)\,(-1) + (-1)\,(0) & (0)\,(0) + (-1)\,(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

Hence proved.

Equal Matrices

Two matrices of the same order said to be equal if their corresponding elements are equal.

Q.3 **Find x and y if**

(i) $\begin{bmatrix} x + 3 & 1 \\ -3 & 3y - 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} x + 3 & 1 \\ -3 & 3y - 4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$

Solution:

(i) $\begin{bmatrix} x + 3 & 1 \\ -3 & 3y - 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

As the two matrices are equal so their corresponding elements are equal so

$x + 3 = 2$ and $3y - 4 = 2$

$\Rightarrow$ $x = 2 - 3$ and $3y = 4 + 2$

$\Rightarrow$ $x = -1$ and $3y = 6$

$y = 2$

(ii) $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$

As the two matrices are equal so their corresponding elements are equal

so

$\Rightarrow$ $x + 3 = y$ (i) and $3y - 4 = 2x$ (ii)

Put $y = x + 3$ from equation (i) in equation (ii)

$3(x + 3)\ 4 = 2x$

$\Rightarrow$ $3x + 9 - 4 = 2x$

$\Rightarrow$ $3x + 5 = 2x$

$\Rightarrow$ $3x + 5 - 2x = 0$

$\Rightarrow$ $x + 5 = 0$

$\Rightarrow$ $\boxed{x = -5}$

Put this value in equation (i)

$y = -5 + 3 = -2$

$\Rightarrow$ $\boxed{y = -2}$

Q.4 If $A = \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix}$

find the following matrices.

(i) 4A – 3B (ii) A + 3 (B – A)

Solution:

(i) to find $4A - 3B$

$$4A - 3B = 4\begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} - 3\begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 8 & 12 \\ 4 & 0 & 8 \end{bmatrix} - \begin{bmatrix} 0 & 9 & 6 \\ 3 & -3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -4-0 & 8-9 & 12-6 \\ 4-3 & 0-(-3) & 8-6 \end{bmatrix} = \begin{bmatrix} -4 & -1 & 6 \\ 1 & 3 & 2 \end{bmatrix}$$

(ii) A + 3 (B – A)

$$A + 3(B - A) = \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} + 3\left(\begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} + 3\begin{bmatrix} 0-(-1) & 3-2 & 2-3 \\ 1-1 & -1-0 & 2-2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -3 \\ 0 & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1+3 & 2+3 & 3+(-3) \\ 1+0 & 0+(-3) & 2+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 & 0 \\ 1 & 3 & 2 \end{bmatrix}$$

Q.5 Find x and y if

$$\begin{bmatrix} \mathbf{2} & \mathbf{0} & \mathbf{x} \\ \mathbf{1} & \mathbf{y} & \mathbf{3} \end{bmatrix} + \mathbf{2}\begin{bmatrix} \mathbf{1} & \mathbf{x} & \mathbf{y} \\ \mathbf{0} & \mathbf{2} & \mathbf{-1} \end{bmatrix} = \begin{bmatrix} \mathbf{4} & \mathbf{-2} & \mathbf{3} \\ \mathbf{1} & \mathbf{6} & \mathbf{1} \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & x & y \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + \begin{bmatrix} 2 & 2x & 2y \\ 0 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2+2 & 0+2x & x+2y \\ 1+0 & y+4 & 3-2 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2x & x+2y \\ 1 & y+4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

As the two matrices are equal so their corresponding elements are equal.

$\Rightarrow \quad 2x = -2 \quad , \quad y + 4 = 6$

$\Rightarrow \quad x = -1 \quad , \quad y = 6 - 4 = 2$

$\Rightarrow \quad \boxed{x = -1}$ and $\boxed{y = 2}$

Q.6 If $A = [a_{ij}]_{3x3}$ show that

(i) $\lambda(\mu A) = (\lambda\mu)A$ (ii) $(\lambda + \mu)A = \lambda A + \mu A$

(iii) $\lambda A - A = (\lambda - 1)A$

Solution:

(i) To show $\lambda(\mu A) = (\lambda\mu)A$

Where, $A = [a_{ij}]_{3x3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Take L.H.S.

$$\lambda(\mu A) = \lambda\left(\mu\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\right)$$

$$= \lambda\begin{bmatrix} \mu a_{11} & \mu a_{12} & \mu a_{13} \\ \mu a_{21} & \mu a_{22} & \mu a_{23} \\ \mu a_{31} & \mu a_{32} & \mu a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda\mu a_{11} & \lambda\mu a_{12} & \lambda\mu a_{13} \\ \lambda\mu a_{21} & \lambda\mu a_{22} & \lambda\mu a_{23} \\ \lambda\mu a_{31} & \lambda\mu a_{32} & \lambda\mu a_{33} \end{bmatrix}$$

Now take R.H.S.

$$(\lambda\mu)A = (\lambda\mu)\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda\mu a_{11} & \lambda\mu a_{12} & \lambda\mu a_{13} \\ \lambda\mu a_{21} & \lambda\mu a_{22} & \lambda\mu a_{23} \\ \lambda\mu a_{31} & \lambda\mu a_{32} & \lambda\mu a_{33} \end{bmatrix} = \text{L.H.S.}$$

$\Rightarrow$ $\lambda(\mu A) = (\lambda\mu)A$

(ii) $(\lambda + \mu)A = \lambda A + \mu A$

To show $(\lambda + \mu)A = \lambda A + \mu A$

L.H.S.

$$(\lambda + \mu)A = (\lambda + \mu)\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} (\lambda + \mu)a_{11} & (\lambda + \mu)a_{12} & (\lambda + \mu)a_{13} \\ (\lambda + \mu)a_{21} & (\lambda + \mu)a_{22} & (\lambda + \mu)a_{23} \\ (\lambda + \mu)a_{31} & (\lambda + \mu)a_{32} & (\lambda + \mu)a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda a_{11} + \mu a_{11} & \lambda a_{12} + \mu a_{12} & \lambda a_{13} + \mu a_{13} \\ \lambda a_{21} + \mu a_{21} & \lambda a_{22} + \mu a_{22} & \lambda a_{23} + \mu a_{23} \\ \lambda a_{31} + \mu a_{31} & \lambda a_{32} + \mu a_{32} & \lambda a_{33} + \mu a_{33} \end{bmatrix}$$

Now take R.H.S.

$$\lambda A + \mu A = \lambda \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} + \mu \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ \lambda a_{21} & \lambda a_{22} & \lambda a_{23} \\ \lambda a_{31} & \lambda a_{32} & \lambda a_{33} \end{bmatrix} + \begin{bmatrix} \mu a_{11} & \mu a_{12} & \mu a_{13} \\ \mu a_{21} & \mu a_{22} & \mu a_{23} \\ \mu a_{31} & \mu a_{32} & \mu a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda a_{11} + \mu a_{11} & \lambda a_{12} + \mu a_{12} & \lambda a_{13} + \mu a_{13} \\ \lambda a_{21} + \mu a_{21} & \lambda a_{22} + \mu a_{22} & \lambda a_{23} + \mu a_{23} \\ \lambda a_{31} + \mu a_{31} & \lambda a_{32} + \mu a_{32} & \lambda a_{33} + \mu a_{33} \end{bmatrix}$$

$$= \text{L.H.S.}$$

$\Rightarrow \quad (\lambda + \mu) A = \lambda A + \mu A$

(iii) $\lambda A - A = (\lambda - 1) A$

To show $\lambda A - A = (\lambda - 1) A$

$$\text{L.H.S.} = \lambda \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} - \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ \lambda a_{21} & \lambda a_{22} & \lambda a_{23} \\ \lambda a_{31} & \lambda a_{32} & \lambda a_{33} \end{bmatrix} - \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda a_{11} - a_{11} & \lambda a_{12} - a_{12} & \lambda a_{13} - a_{13} \\ \lambda a_{21} - a_{21} & \lambda a_{22} - a_{22} & \lambda a_{23} - a_{23} \\ \lambda a_{31} - a_{31} & \lambda a_{32} - a_{32} & \lambda a_{33} - a_{33} \end{bmatrix}$$

Now take R.H.S.

$$(\lambda - 1) A = (\lambda - 1) \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} (\lambda - 1) a_{11} & (\lambda - 1) a_{12} & (\lambda - 1) a_{13} \\ (\lambda - 1) a_{21} & (\lambda - 1) a_{22} & (\lambda - 1) a_{23} \\ (\lambda - 1) a_{31} & (\lambda - 1) a_{32} & (\lambda - 1) a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda a_{11} - a_{11} & \lambda a_{12} - a_{12} & \lambda a_{13} - a_{13} \\ \lambda a_{21} - a_{21} & \lambda a_{22} - a_{22} & \lambda a_{23} - a_{23} \\ \lambda a_{31} - a_{31} & \lambda a_{32} - a_{32} & \lambda a_{33} - a_{33} \end{bmatrix}$$

$$= \text{L.H.S.}$$

$\Rightarrow \quad \lambda A - A = (\lambda - 1) A.$

Q.7 If $A = [a_{ij}]_{2x3}$ and $B = [b_{ij}]_{2x3}$ show that $\lambda (A + B) = \lambda A + \lambda B$.

Solution:

Given

$$A = [a_{ij}]_{2x3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$B = [b_{ij}]_{2x3} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

To show that

$\lambda (A + B) = \lambda A + \lambda B$

Take L.H.S.

$$\lambda (A + B) = \lambda \left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix} \right)$$

$$= \lambda \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} & a_{13} + b_{13} \\ a_{21} + b_{21} & a_{22} + b_{22} & a_{23} + b_{23} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda (a_{11} + b_{11}) & \lambda (a_{12} + b_{12}) & \lambda (a_{13} + b_{13}) \\ \lambda (a_{21} + b_{21}) & \lambda (a_{22} + b_{22}) & \lambda (a_{23} + b_{23}) \end{bmatrix}$$

$$= \begin{bmatrix} \lambda a_{11} + \lambda b_{11} & \lambda a_{12} + \lambda b_{12} & \lambda a_{13} + \lambda b_{13} \\ \lambda a_{21} + \lambda b_{21} & \lambda a_{22} + \lambda b_{22} & \lambda a_{23} + \lambda b_{23} \end{bmatrix}$$

Now R.H.S.

$$\lambda A + \lambda B = \lambda \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} + \lambda \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ \lambda a_{21} & \lambda a_{22} & \lambda a_{23} \end{bmatrix} + \begin{bmatrix} \lambda b_{11} & \lambda b_{12} & \lambda b_{13} \\ \lambda b_{21} & \lambda b_{22} & \lambda b_{23} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda a_{11} + \lambda b_{11} & \lambda a_{12} + \lambda b_{12} & \lambda a_{13} + \lambda b_{13} \\ \lambda a_{21} + \lambda b_{21} & \lambda a_{22} + \lambda b_{22} & \lambda a_{23} + \lambda b_{23} \end{bmatrix}$$

$=$ L.H.S.

$\Rightarrow \quad \lambda (A + B) = \lambda A + \lambda B$

Q.8 If $A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$ and $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ find the values of b.

Solution:

Given

$$A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$$

$$= \begin{bmatrix} (1)(1) + (2)(a) & (1)(2) + (2)(b) \\ (a)(1) + (b)(a) & (2)(2) + (b)(b) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 + 2a & 2 + 2b \\ a + ab & 4 + b^2 \end{bmatrix} \quad \text{but} \quad A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 1 + 2a & 2 + 2b \\ a + ab & 2a + b^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

As the two matrices are equal so their corresponding elements are equal.

$\Rightarrow \quad 1 + 2a = 0 \qquad 2 + 2b = 0$

$\Rightarrow \quad a = -\dfrac{1}{2} \qquad b = -1$

$\Rightarrow$ required values are $\boxed{a = -\dfrac{1}{2}}$, $\boxed{b = -1}$

Q.9 If $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$ and $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find values of a and b.

Solution:

Given $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}\begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$$

$$= \begin{bmatrix} (1)(1) + (-1)(a) & (1)(-1) + (-1)(b) \\ (a)(1) + (b)(a) & (a)(-1) + (b)(b) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 - a & -1 - b \\ a + ab & -a + b^2 \end{bmatrix}$$

But it is given that

$$\Rightarrow \quad \begin{bmatrix} 1 - a & -1 - b \\ a + ab & -a + b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

As the two matrices are equal so their corresponding elements are equal.

$\Rightarrow \quad 1 - a = 1 \quad$ and $\quad -1 - b = 0$

$\Rightarrow \quad a = 0 \qquad b = -1$

$\Rightarrow$ required values are $\boxed{a = 0}$ and $\boxed{b = -1}$

Q.10 If $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ then show that $(A+B)^t = A^t + B^t$.

Solution:

To show $(A+B)^t = A^t + B^t$

Take

$$(A+B) = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & -1+3 & 2+0 \\ 0+1 & 3+2 & 1-1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 2 \\ 1 & 5 & 0 \end{bmatrix}$$

$$(A+B)^t = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 5 & 0 \end{bmatrix}^t = \begin{bmatrix} 3 & 1 \\ 2 & 5 \\ 2 & 0 \end{bmatrix} \qquad \ldots\ldots\ldots (1)$$

Now

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix}$$

and $B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

$$B^t = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 0 & -1 \end{bmatrix}$$

Now

$$A^t + B^t = \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 0+1 \\ -1+3 & 3+2 \\ 2+0 & 1-1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 5 \\ 2 & 0 \end{bmatrix} \qquad \ldots\ldots\ldots (2)$$

From (1) and (2), it is clear that

$$(A+B)^t = A^t + B^t$$

Q.11 Find A^3 if $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(1)+(1)(5)+(3)(-2) & (1)(1)+(1)(2)+(3)(-1) & (1)(3)+(1)(6)+(3)(-3) \\ (5)(1)+(2)(5)(6)(-2) & (5)(1)+(2)(2)+(6)(-1) & (5)(3)+(2)(6)+(6)(-3) \\ (-2)(1)+(-1)(5)+(-3)(-2) & (-2)(1)+(-1)(2)+(-3)(-1) & (-2)(3)+(-1)(6)+(-3).(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 1+5-6 & 1+2-3 & 3+6-9 \\ 5+10-12 & 5+4-6 & 15+12-18 \\ -2-5+6 & -2-2+3 & -6-6+9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix}$$

Now

$$A^3 = A^2 \cdot A = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} (0)(1)+(0)(5)+(0)(-2) & (0)(1)+(0)(2)+(0)(-1) & (0)(3)+(0)(6)+(0)(-3) \\ (3)(1)+(3)(5)+(9)(-2) & (3)(1)+(3)(2)+(9)(-1) & (3)(3)+(3)(6)+(9)(-3) \\ -(1)(1)+(-1)(5)+(-3)(-2) & (-1)(1)+(-1)(2)+(-3)(-1) & (-1)(3)+(-1)(6)+(-3).(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 0+0-0 & 0+0-0 & 0+0-0 \\ 3+15-18 & 3+6-9 & 9+18-27 \\ -1-5+6 & -1-2+3 & -3-6+9 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Q.12 Find matrix x if

(i) $X\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$

(ii) $\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}X = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$

Solution:

(i) $X\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$

$X A = B$ (say)

$X A A^{-1} = B A^{-1}$

$XI = B A^{-1}$

$X = B A^{-1}$ ……… (1)

To find A^{-1}

Here

$A = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 5 & 2 \\ -2 & 1 \end{vmatrix} = (5)(1) - (-2)(2) = 5 + 4 = 9 \neq 0$

$\text{adJ} = A\begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$

As

$A^{-1} = \dfrac{\text{adJ } A}{|A|}$

$= \dfrac{1}{9}\begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$

Put this value in equation (1)

$X = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$

$= \dfrac{1}{9}\begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$

$= \dfrac{1}{9}\begin{bmatrix} (-1)(1) + (5)(2) & (-1)(-2) + (5)(5) \\ (12)(1) + (3)(2) & (12)(-2) + (3)(5) \end{bmatrix}$

$= \dfrac{1}{9}\begin{bmatrix} -1 + 10 & 2 + 25 \\ 12 + 6 & -24 + 15 \end{bmatrix}$

$$= \frac{1}{9}\begin{bmatrix} 9 & 27 \\ 18 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{9} & \frac{27}{9} \\ \frac{18}{9} & -\frac{9}{9} \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

which is required matrix.

(ii) $\begin{bmatrix} \mathbf{5} & \mathbf{2} \\ \mathbf{-2} & \mathbf{1} \end{bmatrix}\mathbf{X} = \begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{5} & \mathbf{10} \end{bmatrix}$

$$\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} X = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$$

$A X = B$ (say)

$A^{-1}A X = A^{-1}B$

$I X = A^{-1}B$

$X = A^{-1}B$ .... (1)

To find A^{-1}

$$A = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & 2 \\ -2 & 1 \end{vmatrix} = (5)(1) - (-2)(2) = 5 + 4 = 9 \neq 0$$

$$\text{adj}A = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

As

$$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{9}\begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

Put this value in equation (1)

$$X = \frac{1}{9}\begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} (1)(2) + (-2)(5) & (1)(1) + (-2)(10) \\ (2)(2) + (5)(5) & (2)(1) + (5)(10) \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 2 - 10 & 1 - 20 \\ 4 + 25 & 2 + 50 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} -8 & -19 \\ 29 & 52 \end{bmatrix}$$

$$X = \begin{bmatrix} -\frac{8}{9} & -\frac{19}{9} \\ \frac{29}{9} & \frac{52}{9} \end{bmatrix} \text{ is required matrix.}$$

Q.13 Find the matrix A if

(i) $\begin{bmatrix} 5 & -1 \\ 0 & 0 \\ 3 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} A = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$

Solution:

(i) $\begin{bmatrix} 5 & -1 \\ 0 & 0 \\ 3 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$

Let

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is the required matrix.

$$\Rightarrow \begin{bmatrix} 5 & -1 \\ 0 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} (5)(a) + (-1)(c) & (5)(b) + (-1)(d) \\ (0)(a) + (0)(c) & (0)(b) + (0)(d) \\ (3)(a) + (1)(c) & (3)(b) + (1)(d) \end{bmatrix} = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5a - c & 5b - d \\ 0 & 0 \\ 3a + c & 3b + d \end{bmatrix} = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$$

As the matrices are equal so their corresponding elements are equal.

i.e.

$5a - c = 3$ ……… (1)

$5b - d = -7$ ……… (2)

$3a + c = 7$ ……… (3)

$3b + d = 2$ ……… (4)

Adding equation (1) and (3)

$5a - c = 3$

$3a + c = 7$

$8a = 10 \Rightarrow a = \frac{10}{8} \Rightarrow \boxed{\frac{5}{4}}$

Put $a = \frac{5}{4}$ in equation (1)

$$5\left(\frac{5}{4}\right) - c = 3$$

$$\frac{25}{4} - c = 3 \Rightarrow c = \frac{25}{4} - 3$$

$$c = \frac{25-12}{4} = \frac{13}{4}$$

$$\boxed{c = \frac{13}{4}}$$

Now add equation (2) and (4)

$$5b - d = -7$$

$$3b + d = 2$$

$$8b = -5 \Rightarrow \boxed{b = -\frac{5}{8}}$$

Put $b = -\frac{5}{8}$ in equation (2)

$$5\left(-\frac{5}{8}\right) - d = -7$$

$$-\frac{25}{8} - d = -7$$

$$-\frac{25}{8} + 7 = d$$

$$\frac{-25+56}{8} = d$$

$$\frac{31}{8} = d \Rightarrow \boxed{d = \frac{31}{8}}$$

$$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \frac{5}{4} & -\frac{5}{8} \\ \frac{13}{4} & \frac{31}{8} \end{bmatrix}$$ is the required matrix.

(ii) $\begin{bmatrix} \mathbf{2} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{2} \end{bmatrix} \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{-3} & \mathbf{8} \\ \mathbf{3} & \mathbf{3} & \mathbf{-7} \end{bmatrix}$

$B\,A = C$ (say)

$B^{-1}\,B\,A = B^{-1}\,C$

$I\,A = B^{-1}\,C$

$A = B^{-1}\,C$ ……… (1)

To find B^{-1}

Take $B = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$|B| = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = (2)(2) - (-1)(-1) = 4 - 1 = 3 \neq 0$

$\text{adj } B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

As $B^{-1} = \dfrac{\text{adj } B}{|B|}$

$\Rightarrow$ $B^{-1} = \dfrac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

Put this value in (1)

$$A = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} (2)(0)+(1)(3) & (2)(-3)+(1)(3) & (2)(8)+(1)(-7) \\ (1)(0)+(2)(3) & (1)(-3)+(2)(3) & (1)(8)+(2)(-7) \end{bmatrix}$$

$$= \begin{bmatrix} 0+3 & -6+3 & 16-7 \\ 0+6 & -3+6 & 8-14 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -3 & 9 \\ 6 & 3 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{3} & -\frac{3}{3} & \frac{9}{3} \\ \frac{6}{3} & \frac{3}{3} & -\frac{6}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & -2 \end{bmatrix}$$

Q.14 Show that

$$\begin{bmatrix} r\cos\phi & 0 & -\sin\phi \\ 0 & r & 0 \\ r\sin\phi & 0 & \cos\phi \end{bmatrix}\begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -r\sin\phi & 0 & r\cos\phi \end{bmatrix} = rI_3$$

Solution:

$$\begin{bmatrix} r\cos\phi & 0 & -\sin\phi \\ 0 & r & 0 \\ r\sin\phi & 0 & \cos\phi \end{bmatrix}\begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -r\sin\phi & 0 & r\cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} r\cos\phi\,.\cos\phi + 0.0 + (-\sin\phi)(-r\sin\phi) & r\cos\phi\,.\,0 + 0.1 + (-\sin\phi) & r\cos\phi\,.\sin\phi + 0.0 + (-\sin\phi)(r\cos\phi) \\ 0\,.\cos\phi + r\,.\,0 + 0\,.\,(-r\sin\phi) & 0.0 + r.1 + 0.0 & 0\,.\sin\phi + r.0 + 0.r\cos\phi \\ r\sin\phi\,.\cos\phi + 0.0 + \cos\phi\,(-r\sin\phi) & r\sin\phi\,.\,0 + 0.1 + \cos\phi\,.\,0 & r\sin\phi\,.\sin\phi + 0.0 + \cos\phi\,.\,r\cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} r\cos^2\phi + 0 + r\sin^2\phi & 0+0+0 & r\cos\phi\sin\phi + 0 - r\cos\phi\sin\phi \\ 0+0-0 & 0+r+0 & 0+0+0 \\ r\sin\phi\cos\phi + 0 - r\sin\phi\cos\phi & 0+0+0 & r\sin^2\phi + 0 + r\cos^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} r(\cos^2\phi + \sin^2\phi) & 0 & -0 \\ 0 & r & 0 \\ 0 & 0 & r(\sin^2\phi + \cos^2\phi) \end{bmatrix}$$

$$= \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix}$$

$$= r\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = rI_3 = \text{R.H.S.}$$

Hence proved.

EXERCISE 3.2

Q.1 If $A = [a_{ij}]_{3\times4}$ then show that
(i) $I_3A = A$ (ii) $AI_4 = A$

Solution:

Given

$$A = [a_{ij}]_{3\times4} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

(i) To show $I_3A = A$ where $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Take L.H.S.

$$I_3A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$