EXERCISE 7.4

Q.1 Evaluate the following:

(i) $^{12}C_3$ (Lahore Board 2008) (ii) $^{20}C_{17}$ (iii) $^{n}C_4$

Solution:

Using formula $^{n}C_r = \dfrac{n!}{r!\,(n-r)!}$

(i) $^{12}C_3 = \dfrac{12!}{3!\,(12-3)!} = \dfrac{12!}{3!\,9!} = 220$

(ii) $^{20}C_{17} = \dfrac{20!}{17!\,(20-17)!} = \dfrac{20!}{17!\,3!} = 1140$

(iii) $^{n}C_4 = \dfrac{n!}{4!\,(n-4)!} = \dfrac{n\,(n-1)\,(n-2)\,(n-3)\,(n-4)!}{4!\,(n-4)!} = \dfrac{n\,(n-1)\,(n-2)\,(n-3)}{4!}$

Q.2 Find the value of n, when

(i) $^{n}C_5 = {}^{n}C_4$ (Gujranwala Board 2007) (ii) $^{n}C_{10} = \dfrac{12 \times 11}{2!}$

(iii) $^{n}C_{12} = {}^{n}C_6$ (Lahore Board 2007, 2011)

Solution:

(i) $^{n}C_5 = {}^{n}C_4$

$\Rightarrow \dfrac{n!}{5!\,(n-5)!} = \dfrac{n!}{4!\,(n-4)!}$

$\Rightarrow \dfrac{1}{5.4!\,(n-5)!} = \dfrac{1}{4!\,(n-4)\,(n-5)!}$

$\Rightarrow \dfrac{1}{5} = \dfrac{1}{n-4}$

$\Rightarrow n-4 = 5$

$\Rightarrow \boxed{n = 9}$

(ii) $^{n}C_{10} = \dfrac{12 \text{ x } 11}{2!}$

$\dfrac{n!}{10!\,(n-10)!} = \dfrac{12 \text{ x } 11 \times 10!}{10!\,2!}$

$\dfrac{n!}{10!\,(n-10)!} = \dfrac{12!}{10!\,(12-10)!}$

$\Rightarrow {}^{n}C_{10} = {}^{12}C_{10}$

$\Rightarrow \boxed{n = 12}$

(iii) $^nC_{12} = {^nC_6}$ ……… (1)

Using $^nC_r = {^nC_{n-r}}$

$\Rightarrow$ $^nC_{12} = {^nC_{n-12}}$

$\Rightarrow$ $^nC_6 = {^nC_{n-12}}$ from equation (1)

$\Rightarrow$ $6 = n - 12$

$\Rightarrow$ $\boxed{n = 18}$

Q.3 Find the values of n and r, when

(i) $\mathbf{^nC_r = 35}$ **and** $\mathbf{^nP_r = 210}$ **(Lahore Board 2010)**

(ii) $\mathbf{^{n-1}C_{r-1} : {^nC_r} : {^{n+1}C_{r+1}} = 3 : 6 : 11}$

(Gujranwala Board 2003, Lahore Board 2007)

Solution:

(i) Given that

$^nC_r = 35$

$\Rightarrow$ $\dfrac{n!}{r!\,(n-r)!} = 35$

$\Rightarrow$ $n! = 35r!\,(n-r)!$ ……… (1)

Also $^nP_r = 210$

$\Rightarrow$ $\dfrac{n!}{(n-r)!} = 210$

$\Rightarrow$ $n! = 210\,(n-r)!$ ……… (2)

Dividing (2) by (1), we get

$\dfrac{n!}{n!} = \dfrac{210\,(n-r)!}{35r!\,(n-r)!}$

$1 = \dfrac{6}{r!}$

$\Rightarrow$ $r! = 6 = 3!$

$\Rightarrow$ $r! = 3!$

$\Rightarrow$ $\boxed{r = 3}$

Put $r = 3$ in (2), we get

$n! = 210\,(n-3)!$

$\Rightarrow$ $n\,(n-1)\,(n-2)\,(n-3)! = 210\,(n-3)!$

$\Rightarrow$ $n\,(n-1)\,(n-2) = 210 = 7.6.5$

$\Rightarrow$ $\boxed{n = 7}$

(ii) $\quad {}^{n-1}C_{r-1} : {}^{n}C_{r} : {}^{n+1}C_{r+1} = 3 : 6 : 11$

$${}^{n-1}C_{r-1} : {}^{n}C_{r} = 3 : 6$$

$$\Rightarrow \quad \frac{{}^{n-1}C_{r-1}}{{}^{n}C_{r}} = \frac{3}{6}$$

$$\Rightarrow \quad \frac{\dfrac{(n-1)!}{(r-1)!\,(n-1-r+1)!}}{\dfrac{n!}{r!\,(n-r)!}} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{(n-1)!}{(r-1)!\,(n-r)!}\ \frac{r\,(r-1)!\,(n-r)!}{n\,(n-1)!} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{r}{n} = \frac{1}{2}$$

$$\Rightarrow \quad n = 2r \qquad \text{.........} \ (1)$$

Now $\quad {}^{n}C_{r} : {}^{n+1}C_{r+1} = 6 : 11$

$$\frac{{}^{n}C_{r}}{{}^{n+1}C_{r+1}} = \frac{6}{11}$$

$$\Rightarrow \quad \frac{\dfrac{n!}{r!\,(n-r)!}}{\dfrac{(n+1)!}{(r+1)!\,(n+1-r-1)!}} = \frac{6}{11}$$

$$\Rightarrow \quad \frac{n!}{r!\,(n-r)!} \times \frac{(r+1)\,r!\,(n-r)!}{(n+1)\,n!} = \frac{6}{11}$$

$$\Rightarrow \quad \frac{r+1}{n+1} = \frac{6}{11}$$

$$\Rightarrow \quad 6\,(n+1) = 11\,(r+1)$$

$$\Rightarrow \quad 6n + 6 - 11r - 11 = 0$$

$$\Rightarrow \quad 6n - 11r - 5 = 0$$

$$\Rightarrow \quad 6n - 11r = 5 \qquad \text{.........} \ (2)$$

Put $n = 2r$ from (1) in (2), we get

$$\Rightarrow \quad 6\,(2r) - 11r = 5$$

$$\Rightarrow \quad 12r - 11r = 5$$

$$\Rightarrow \quad \boxed{r = 5}$$

Put $r = 5$ in (1), we get

$$\Rightarrow \quad n = 2\,(5)$$

$$\boxed{n = 10}$$

Q.4 How many (a) diagonals and (b) triangles can be formed by joining the vertices of polygon having

(i) 5 sides (ii) 8 sides (iii) 12 sides

Solution:

(a) As we know that the diagonal is a line segment joining two points. So

(i) We will find combinations of 5 taken 2 at a time

$\Rightarrow$ Total number of line segments $= {}^5C_2 = 10$

But Number of sides $= 5$

$\Rightarrow$ Number of diagonals $= 10 - 5 = 5$

(ii) Here we will find combinations of 8 taken two at a time

$\Rightarrow$ Total number of line segments $= {}^8C_2 = 28$

But Number of sides $= 8$

$\Rightarrow$ Number of diagonals $= 28 - 8 = 20$

(iii) Here we will find combinations of 12 taken two at a time

$\Rightarrow$ Total number of line segments $= {}^{12}C_2 = 66$

But Number of sides $= 12$

$\Rightarrow$ Number of diagonals $= 66 - 12 = 54$

(b) As we know that triangle has three vertices

(i) 5–sided

Number of triangles $= {}^5C_3 = 10$

(ii) 8–sided

Number of triangles $= {}^8C_3 = 56$

(iii) 12–sided

Number of triangles $= {}^{12}C_3 = 220$

Q.5 The members of a club are 12 boys and 8 girls. In how many ways can a committee of 3 boys and 2 girls be formed?

Solution:

Number of committees having 3 boys out of 12 and 2 girls out of 8 $= {}^{12}C_3 \,.\, {}^8C_2 = 6160$

Q.6 How many committees of 5 members can be chosen from a group of 8 persons when each committee must include 2 particular persons?

Solution:

If 2 particular persons are in each committee. Then

Number of ways to select 3 persons out of 6 $= {}^6C_3 = 20$

Q.7 In how many ways can a hockey team of 11 players be selected out of 15 players? How many of them will include a particular player?

Solution:

Number of ways to select 11 players out of 15 $= {}^{15}C_{11} = 1365$

If one particular player is to be included in every team, then

Number of ways to select 10 players out of 14 $= {}^{14}C_{10} = 1001$

Q.8 Show that $^{16}C_{11} + {}^{16}C_{10} = {}^{17}C_{11}$.

Solution:

$${}^{16}C_{11} + {}^{16}C_{10} = {}^{17}C_{11}$$

$$\text{L.H.S.} = {}^{16}C_{11} + {}^{16}C_{10}$$

$$= \frac{16!}{11!\,(16-11)!} + \frac{16!}{10!\,(16-10)!}$$

$$= \frac{16!}{11!\,5!} + \frac{16!}{10!\,6!}$$

$$= \frac{16!}{11.10!\,5!} + \frac{16!}{10!\,6.5!}$$

$$= \frac{16!}{10!\,5!}\left[\frac{1}{11} + \frac{1}{6}\right]$$

$$= \frac{16!}{10!\,5!}\left[\frac{6+11}{(11)\,(6)}\right]$$

$$= \frac{16!}{10!\,5!}\;\frac{17}{(11)\,(6)}$$

$$= \frac{17.16!}{11.10!\,6.5!} = \frac{17!}{11!\,6!}$$

$$= \frac{17!}{11!\,(17-11)!} = {}^{17}C_{11} = \text{R.H.S.}$$

Hence proved.

Q.9 There are 8 men and 10 women members of a club. How many committees of 7 can be formed, having;

(i) 4 women (ii) at the most 4 women (iii) at least 4 women?

Solution:

(i) 4 women

Number of committees having 4 women out of 10 and 3 men out of

$8 = {}^{10}C_4 \,.\, {}^{8}C_3 = 11760$

(ii) at the most 4 women

At the most 4 women means

(4W, 3M), (3W, 4M), (2W, 5M), (1W, 6M) (0W, 7M)

So Number of committees $= {}^{10}C_4\ {}^8C_3 + {}^{10}C_3\ {}^8C_4 + {}^{10}C_2\ {}^8C_5 + {}^{10}C_1 \cdot {}^8C_6 + {}^{10}C_0\ {}^8C_7$

$= 11760 + 8400 + 2520 + 280 + 8$

$= 22968$

(iii) at least 4 women

At least 4 women means

(4W, 3M), (5W, 2M), (6W, 1M), (7W, 0M)

So Number of committees $= {}^{10}C_4\ {}^8C_3 + {}^{10}C_5\ {}^8C_2 + {}^{10}C_6\ {}^8C_1 + {}^{10}C_7\ {}^8C_0$

$= 11760 + 7056 + 1680 + 120$

$= 20616$

Q.10 Prove that $\mathbf{{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r}$.

Solution:

$$\text{L.H.S.} = {}^nC_r + {}^nC_{r-1}$$

$$= \frac{n!}{r!\,(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)!}$$

$$= \frac{n!}{r\,(r-1)!\,(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)\,(n-r)!}$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\left[\frac{n-r+1+r}{r\,(n-r+1)}\right]$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\left[\frac{n+1}{r\,(n-r+1)}\right]$$

$$= \frac{(n+1)\,n!}{r\,(r-1)!\,(n-r+1)\,(n-r)!}$$

$$= \frac{(n+1)!}{r!\,(n-r+1)!} = {}^{n+1}C_r = \text{R.H.S.}$$

Hence proved.

PROBABILITY

Probability is the numerical evaluation of a chance that a particular event would occur.