(ii) One of the equations is homogeneous in x and y

If every term in an equation is of the same degree then it is called homogeneous equation.

For example $x^2 - 3xy + 2y^2 = 0$ is homogeneous in x and y.

We shall factorize its L.H.S. and get two linear equations as

$x^2 - 3xy + 2y^2 = 0$ gives

$(x - y)(x - 2y) = 0$

$\Rightarrow$ $x - y = 0$ and $x - 2y = 0$ are the two linear equations.

(iii) Both equations are non–homogeneous

To solve these equations we eliminate the constants and then get a homogeneous equation.

For example: $y^2 - 2xy = 7$ .... (1) $2x^2 - xy = -3$ .... (2)

Multiply (1) by 3 and (2) by 7 and adding to eliminate constants, gives

$\Rightarrow$ $14x^2 - 13xy + 3y^2 = 0$ $\Rightarrow$ $(2x - y)(7x - 3y) = 0$ linear factors.

EXERCISE 4.9

Q.1 $2x^2 = 6 + 3y^2$; $3x^2 - 5y^2 = 7$.

Solution:

Given equations

$2x^2 = 6 + 3y^2$ .... (1)

$3x^2 - 5y^2 = 7$ .... (2)

$\Rightarrow$ $2x^2 - 3y^2 = 6$

Multiplying equation (1) by (3) and equation (2) by (2) and subtracting

$$\begin{array}{l} 6x^2 - 9y^2 = 18 \\ 6x^2 - 10y^2 = 14 \\ - \quad + \qquad - \\ \hline \qquad y^2 = -4 \end{array}$$

$y^2 = 4$

$\Rightarrow$ $y = \pm 2$

Now put $y = \pm 2$ in equation (1)

when $y = 2$

equation (1) $\Rightarrow$

$2x^2 - 3(2)^2 = 6$

$\Rightarrow \quad 2x^2 - 12 = 6$

$\Rightarrow \quad 2x^2 = 18$

$\Rightarrow \quad x^2 = 9$

$\Rightarrow \quad x = \pm 3$

$\Rightarrow \quad (\pm 3, 2)$

or $\quad (3, 2), (-3, 2)$

when $y = -2$

equation (1) $\Rightarrow$

$2x^2 - 3(-2)^2 = 6$

$\Rightarrow \quad 2x^2 - 12 = 6$

$\Rightarrow \quad 2x^2 = 18$

$\Rightarrow \quad x^2 = 9$

$\Rightarrow \quad x = \pm 3$

$\Rightarrow \quad (\pm 3, -2)$

or $\quad (3, -2), (-3, -2)$

Hence solution set $= \{(3, 2), (-3, 2), (3, -2), (-3, -2)\}$

Q.2 $8x^2 = y^2$; $x^2 + 2y^2 = 19$ **(Gujranwala Board 2007)**

Solution:

Given equations

$8x^2 = y^2$ .... (1)

and $8x^2 - y^2 = 0$

$x^2 + 2y^2 = 19$ .... (2)

multiplying equation (1) by 2 and adding in equation (2)

$$
\begin{aligned}
16x^2 - 2y^2 &= 0 \\
x^2 + 2y^2 &= 19 \\
\hline
17x^2 \quad &= 19
\end{aligned}
$$

$x^2 = \dfrac{19}{17}$

$x = \pm\sqrt{\dfrac{19}{17}}$

Put these values in equation (1)

when $x = \sqrt{\dfrac{19}{17}}$

equation (1) $\Rightarrow$

$8 \cdot \dfrac{19}{17} - y^2 = 0$

when $x = -\sqrt{\dfrac{19}{17}}$

equation (1) $\Rightarrow$

$8 \cdot \dfrac{19}{17} - y^2 = 0$

$\Rightarrow \quad y^2 = 8 . \frac{19}{17}$ $\qquad \Rightarrow \quad y^2 = 8 . \frac{19}{17}$

$\Rightarrow \quad y = \pm\sqrt{\frac{8 \text{ x } 19}{19}}$ $\qquad \Rightarrow \quad y = \pm\sqrt{\frac{8 \text{ x } 19}{19}}$

$y = \pm 2\sqrt{\frac{38}{17}}$ $\qquad \Rightarrow \quad y = \pm 2\sqrt{\frac{38}{17}}$

$\Rightarrow \quad \left(\sqrt{\frac{19}{17}}, \pm 2\sqrt{\frac{38}{17}}\right)$ $\qquad \Rightarrow \quad \left(-\sqrt{\frac{19}{17}}, \pm 2\sqrt{\frac{38}{17}}\right)$

Hence solution set $= \left\{\left(\sqrt{\frac{19}{17}}, \pm 2\sqrt{\frac{38}{17}}\right), \left(-\sqrt{\frac{19}{17}}, \pm 2\sqrt{\frac{38}{17}}\right)\right\}$

Q.3 $\mathbf{2x^2 - 8 = 5y^2 , \quad x^2 - 13 = -2 y2.}$

Solution:

Given equations

$2x^2 - 8 = 5y^2$

$2x^2 - 5y^2 = 8 \qquad \text{.........} (1)$

and $\quad x^2 - 13 = -2y^2$

$x^2 + 2y^2 = 13 \qquad \text{.........} (2)$

multiplying equation (2) by 2 and subtracting from equation (1)

$$\begin{array}{r} 2x^2 - 5y^2 = 8 \\ 2x^2 + 4y^2 = 26 \\ - \quad - \qquad - \\ \hline -9y^2 = -18 \end{array}$$

$y^2 = 2$ $\qquad \Rightarrow \quad y = \pm\sqrt{2}$

Put these values in equation (1)

when $\quad y = \sqrt{2}$ $\qquad y = -\sqrt{2}$

equation (1) $\Rightarrow$ $\qquad$ equation (1) $\Rightarrow$

$2x^2 - 5\left(\sqrt{2}\right)^2 = 8$ $\qquad 2x^2 - 5\left(-\sqrt{2}\right)^2 = 8$

$\Rightarrow \quad 2x^2 - 5 (2) = 8$ $\qquad \Rightarrow \quad 2x^2 - 5 (2) = 8$

$\Rightarrow \quad 2x^2 = 18$ $\qquad \Rightarrow \quad 2x^2 - 10 = 8$

$\Rightarrow \quad x^2 = 9$ $\qquad \Rightarrow \quad x^2 = 9$

$\Rightarrow \quad x = \pm 3$ $\qquad \Rightarrow \quad x = \pm 3$

$\left(\pm 3 , \sqrt{2}\right)$ $\qquad \Rightarrow \quad \left(\pm 3 , -\sqrt{2}\right)$

Hence solution set $= \left\{\left(\pm 3 , \sqrt{2}\right), \left(\pm 3 , -\sqrt{2}\right)\right\}$

Q.4 $\mathbf{x^2 - 5xy + 6y^2 = 0; \quad x^2 + y^2 = 45}$

Solution:

Given equations

$x^2 - 5xy + 6y^2 = 0 \quad , \quad x^2 + y^2 = 45$

$x^2 - 3xy - 2xy + 6y^2 = 0$

$x(x - 3y) - 2y(x - 3y) = 0$

$(x - 3y)(x - 2y) = 0$

$\Rightarrow$ Either $x - 3y = 0$ or $x - 2y = 0$

(i) First we solve

$x - 3y = 0 \quad \ldots\ldots\ldots (1)$

and $x^2 + y^2 = 45 \quad \ldots\ldots\ldots (2)$

from equation (1)

$x = 3y \quad \ldots\ldots\ldots (3)$

Put this value in equation (2)

$\Rightarrow \quad (3y)^2 + y^2 = 45$

$\Rightarrow \quad 9y^2 + y^2 = 45$

$\Rightarrow \quad 10y^2 = 45 \quad \Rightarrow \quad y^2 = \frac{45}{10} = \frac{9}{2}$

$\Rightarrow \quad y^2 = \frac{9}{2} \quad \Rightarrow \quad y = \frac{\pm 3}{\sqrt{2}}$

Put these values in equation (3)

when $y = \frac{3}{\sqrt{2}}$

equation (3) $\Rightarrow$

$x = 3 \cdot \frac{3}{\sqrt{2}} = \frac{9}{\sqrt{2}}$

$\Rightarrow \quad \left(\frac{9}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$

when $y = -\frac{3}{\sqrt{2}}$

equation (3) $\Rightarrow$

$x = -3 \cdot \frac{3}{\sqrt{2}}$

$x = -\frac{9}{\sqrt{2}}$

$\Rightarrow \quad \left(\frac{-3}{\sqrt{2}}, \frac{-9}{\sqrt{2}}\right)$

(ii) Now we solve

$x - 2y = 0 \quad \ldots\ldots\ldots (4)$

$x^2 + y^2 = 45 \quad \ldots\ldots\ldots (5)$

Put this value in equation (5)

$\Rightarrow \quad (2y)^2 + y^2 = 45$

$\Rightarrow \quad 4y^2 + y^2 = 45 \quad \Rightarrow 5y^2 = 45$

$y^2 = \frac{45}{5} \quad y^2 = 9 \quad \Rightarrow \quad y = \pm 3$

Put these values in equation (5)

When $y = 3$

equation (5) $\Rightarrow$

$x = 2(3)$

$x = 6$

$\Rightarrow$ $(6, 3)$

when $y = -3$

equation (5) $\Rightarrow$

$x = 2(-3)$

$x = -6$

$\Rightarrow$ $(-6, -3)$

Hence solution set $\left\{\left(\frac{9}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)\left(\frac{-3}{\sqrt{2}}, \frac{-9}{\sqrt{2}}\right), (6, 3)\ (-3, -6)\right\}$

Q.5 $\mathbf{12x^2 - 25xy + 12y^2 = 0;\ \ 4x^2 + 7y^2 = 148.}$

Solution:

Given equations

$12x^2 - 25xy + 12y^2 = 0$ and $4x^2 + 7y^2 = 148$

$12x^2 - 16xy - 9xy + 12y^2 = 0$

$4x(3x - 4y) - 3y(3x - 4y) = 0$

$(3x - 4y)(4x - 3y) = 0$

Either $3x - y = 0$ or $4x - 3y = 0$

First we solve.

$3x - 4y = 0$ ……… (1)

and $4x^2 + 7y^2 = 148$ ……… (2)

from equation (1)

$3x = 4y$

$\Rightarrow$ $x = \frac{4}{3}y$ ……… (3)

Put this value in equation (2)

$\Rightarrow$ $4\left(\frac{4}{3}y^2\right) + 7y^2 = 148$

$\Rightarrow$ $4\left(\frac{16}{9}y^2\right) + 7y^2 = 148$

$\Rightarrow$ $\frac{64}{9}y^2 + 7y^2 = 148$

$\Rightarrow$ $\frac{64y^2 + 63y^2}{9} = 148$

$\Rightarrow$ $127y^2 = 148 \times 9$

$\Rightarrow$ $127y^2 = 1332$ $\Rightarrow$ $y^2 = \frac{1332}{127}$ $\Rightarrow$ $y = \pm\sqrt{\frac{1332}{127}}$

$\Rightarrow$ $y = \pm 6\sqrt{\frac{37}{127}}$

Put these values in equation (3)

when $y = 6\sqrt{\frac{37}{127}}$

equation (3) $\Rightarrow$

$$x = \frac{4}{3}\left(6\sqrt{\frac{37}{127}}\right)$$

$$x = 8\sqrt{\frac{37}{127}}$$

$$\Rightarrow \left(8\sqrt{\frac{37}{127}}, 6\sqrt{\frac{37}{127}}\right)$$

when $y = -6\sqrt{\frac{37}{127}}$

equation (3) $\Rightarrow$

$$\frac{4}{3}\left(-6\sqrt{\frac{37}{127}}\right)$$

$$x = -8\sqrt{\frac{37}{127}}$$

$$\left(-8\sqrt{\frac{37}{127}}, -6\sqrt{\frac{37}{127}}\right)$$

(ii) Now we solve

$$4x - 3y = 0 \quad (4)$$

and $$4x^2 + 7y^2 = 148 \quad (5)$$

from equation (4)

$$4x = 3y$$

$$x = \frac{3y}{4} \quad (6)$$

Put this value in equation (5)

$$\Rightarrow 4\left(\frac{3y}{4}\right)^2 + 7y^2 = 148$$

$$\Rightarrow 4\left(\frac{9y}{16}\right)^2 + 7y^2 = 148$$

$$\Rightarrow \frac{9y^2}{4} + 7y^2 = 148$$

$$\Rightarrow \frac{9y^2 + 28y^2}{4} = 148$$

$$\Rightarrow 37y^2 = 148 \times 4 \Rightarrow 37y^2 = 592$$

$$\Rightarrow y^2 = \frac{592}{37} = 16 \Rightarrow y = \pm 4$$

put these values in equation (6)

when $y = 4$

equation (6) $\Rightarrow$

$$x = \frac{3(4)}{4}$$

$$x = 3$$

$$\Rightarrow (3, 4)$$

when $y = -4$

equation (6) $\Rightarrow$

$$x = \frac{3(-4)}{4}$$

$$x = -3$$

$$\Rightarrow (-3, -4)$$

Hence solution set $= \left\{(3, 4)\ (-3, -4)\left(8\sqrt{\frac{37}{3}}, 6\sqrt{\frac{37}{127}}\right), \left(-8\sqrt{\frac{37}{3}}, -6\sqrt{\frac{37}{127}}\right)\right\}$

Q.6 $12x^2 - 11xy + 2y^2 = 0, \quad 2x^2 + 7xy = 60.$

Solution:

Given equations

$12x^2 - 11xy + 2y^2 = 0$ and $2x^2 + 7xy = 60$

$\Rightarrow \quad 12x^2 - 8xy - 3xy + 2y^2 = 0$

$\Rightarrow \quad 4x(3x - 2y) - y(3x - 2y) = 0$

$\Rightarrow \quad (3x - 2y)(4x - y) = 0$

Either $3x - 2y = 0$ or $4x - y = 0$

(i) First we solve

$3x - 2y = 0$ .... (1)

and $2x^2 + 7xy = 60$ .... (2)

from equation (1)

$3x = 2y$

$\Rightarrow \quad x = \dfrac{2y}{3}$ .... (3)

Put this value in equation (2)

$$\Rightarrow \quad 2\left(\frac{2y}{3}\right)^2 + 7\left(\frac{2y}{3}\right)y = 60$$

$$\Rightarrow \quad 2\left(\frac{4y^2}{9}\right) + \frac{14}{3}y^2 = 60$$

$$\Rightarrow \quad \frac{8y^2}{9} + \frac{14}{3}y^2 = 60$$

$$\Rightarrow \quad \frac{8y^2 + 42y^2}{9} = 60$$

$$\Rightarrow \quad 50y^2 = 9 \times 60$$

$$\Rightarrow \quad 50y^2 = 540$$

$$\Rightarrow \quad y^2 = \frac{540}{50}$$

$$\Rightarrow \quad y^2 = \frac{54}{5} \quad \Rightarrow \quad y = \pm\sqrt{\frac{54}{5}}$$

$$\Rightarrow \quad y = \pm 3\sqrt{\frac{6}{5}}$$

put these values in equation (3)

when $y = 3\sqrt{\dfrac{6}{5}}$ when $y = -3\sqrt{\dfrac{6}{5}}$

equation (3) $\Rightarrow$

$$x = \frac{2}{3}3\sqrt{\frac{6}{5}}$$

$$= 2\sqrt{\frac{6}{5}}$$

$\Rightarrow$ $\left(2\sqrt{\frac{5}{6}}\ ,\ 3\sqrt{\frac{6}{5}}\right)$

equation (3) $\Rightarrow$

$$x = \frac{2}{3}\left(-3\sqrt{\frac{6}{5}}\right)$$

$$= -2\sqrt{\frac{6}{5}}$$

$\left(-2\sqrt{\frac{6}{5}}\ ,\ -3\sqrt{\frac{6}{5}}\right)$

(ii) Now we solve

$4x - y = 0$ ……… (4)

and $2x^2 + 7xy = 60$ ……… (5)

from equation (4)

$y = 4x$ ……… (6)

put this value in equation (5)

$\Rightarrow$ $2x^2 + 7x(4x) = 60$ $\Rightarrow$ $2x^2 + 28x^2 = 60$

$30x^2 = 60 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

put these values in equation (6)

when $x = \sqrt{2}$

equation (6) $\Rightarrow$

$y = 4\sqrt{2}$

$\Rightarrow$ $(\sqrt{2}, 4\sqrt{2})$

when $x = -\sqrt{2}$

equation (6) $\Rightarrow$

$y = -4\sqrt{2}$

$\Rightarrow$ $(-\sqrt{2}, -4\sqrt{2})$

Hence solutions set is

$$\left\{(\sqrt{2}, 4\sqrt{2}),\ (-\sqrt{2}, -4\sqrt{2}),\left(2\sqrt{\frac{6}{5}}\ ,\ 3\sqrt{\frac{6}{5}}\right),\ \left(-2\sqrt{\frac{6}{5}}\ ,\ -3\sqrt{\frac{6}{5}}\right)\right\}$$

Q.7 $x^2 - y^2 = 16$, $xy = 15$.

Solution:

Given equations

$x^2 - y^2 = 16$ ……… (1)

$xy = 15$ ……… (2)

from equation (2)

$y = \frac{15}{x}$ ……… (3)

Put this value in equation (1)

$$x^2 - \left(\frac{15}{x}\right)^2 = 16$$

$$\Rightarrow \quad x^2 - \frac{225}{x^2} = 16$$

$$\Rightarrow \quad \frac{x^4 - 225}{x^2} = 16$$

$$\Rightarrow \quad x^4 - 225 = 16x^2$$

$$\Rightarrow \quad x^4 - 16x^2 - 225 = 0$$

By quadratic formula

$$x^2 = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(-225)}}{2(1)}$$

$$= \frac{16 \pm \sqrt{256 + 900}}{2} = \frac{16 \pm \sqrt{1156}}{2} = \frac{16 \pm 34}{2}$$

$$\Rightarrow \quad x^2 = \frac{16 + 34}{2} \quad \text{or} \quad x^2 = \frac{16 - 34}{2}$$

$$\Rightarrow \quad x^2 = \frac{50}{2} = 25 \qquad x^2 = \frac{-18}{2} = -9$$

$$\Rightarrow \quad x = \pm 5 \qquad x \ \ x = \pm 3i$$

put these values in equation (3)

when $x = 5 \Rightarrow y = \frac{15}{5} = 3 \Rightarrow (5, 3)$

when $x = -5 \Rightarrow y = \frac{15}{-5} = -3 \Rightarrow (-5, -3)$

when $x = 3i \Rightarrow y = \frac{15}{3i} = \frac{5}{i} \cdot \frac{i}{i} = -5i \Rightarrow (3i, -5i)$

when $x = -3i \Rightarrow y = \frac{15}{-3i} = -\frac{5}{i} \cdot \frac{i}{i} = 5i \Rightarrow (-3i, 5i)$

Hence solution set $= \{(5, -3), (-5, 3), (3i, -5i), (-3i, 5i)\}$

Q.8 $\mathbf{x^2 + xy = 9, \quad x^2 - y^2 = 2.}$

Solution:

Given equations

$x^2 + xy = 9$ ……… (1)

$x^2 - y^2 = 2$ ……… (2)

Multiplying equ. (1) by 2 and equ. (2) by 9 and subtracting.

$$\begin{array}{rcl} 2x^2 + 2xy & = & 18 \\ 9x^2 \qquad - 9y^2 & = & 18 \\ - \qquad + \qquad & & - \\ \hline -7x^2 + 2xy + 9y^2 & = & 0 \end{array}$$

$\Rightarrow \quad 7x^2 - 2xy - 9y^2 = 0$

$\Rightarrow \quad 7x^2 - 9xy + 7xy - 9y^2 = 0$

$\Rightarrow \quad x(7x - 9y) + y(7x - 9y) = 0$

$\Rightarrow \quad (7x - 9y)(x + y) = 0$

Either $7x - 9y = 0$ or $x + y = 0$

(i) First we solve

$7x - 9y = 0 \quad \ldots\ldots\ldots (3)$

and $x^2 - y^2 = 2 \quad \ldots\ldots\ldots (4)$

from equation (3)

$7x = 9y \quad \Rightarrow \quad x = \frac{9}{7}y \quad \ldots\ldots\ldots (5)$

Put this value in equation (4)

$\left(\frac{9}{7}y\right)^2 - y^2 = 2$

$\Rightarrow \quad \frac{81}{49}y^2 - y^2 = 2$

$\Rightarrow \quad \frac{81y^2 - 49y^2}{49} = 2$

$\Rightarrow \quad 32y^2 = 98 \quad \Rightarrow \quad y^2 = \frac{98}{32} = \frac{49}{16}$

$\Rightarrow \quad y = \pm\frac{7}{4}$

put these values in equation (5)

when $y = \frac{7}{4}$

equation (5) $\Rightarrow$

$x = \frac{9}{7}\cdot\frac{7}{4}$

$x = \frac{9}{4}$

$\Rightarrow \quad \left(\frac{9}{4}, \frac{7}{4}\right)$

when $y = -\frac{7}{4}$

equation (5) $\Rightarrow$

$x = \frac{9}{7}\left(-\frac{7}{4}\right)$

$x = -\frac{9}{4}$

$\Rightarrow \quad \left(-\frac{9}{4}, -\frac{7}{4}\right)$

(ii) Now we solve

$$x + y = 0 \quad \ldots\ldots\ldots (6)$$

and $$x^2 - y^2 = 2 \quad \ldots\ldots\ldots (7)$$

from equation (6)

$$x = -y \quad \ldots\ldots\ldots (8)$$

Put in equation (7)

$$(-y)^2 - y^2 = 2$$

$$y^2 - y^2 = 2$$

$$0 = 2 \qquad \text{No solution.}$$

Hence solution set $= \left\{\left(\frac{9}{4}, \frac{7}{4}\right), \left(-\frac{9}{4}, -\frac{7}{4}\right)\right\}$

Q.9 $\mathbf{y^2 - 7 = 2xy \quad ; \quad 2x^2 + 3 = xy.}$

Solution:

Given equations

$$y^2 - 7 = 2xy \quad \ldots\ldots\ldots (1)$$

$$2x^2 + 3y = xy \quad \ldots\ldots\ldots (2)$$

Multiplying equation (1) by 3 and equation (2) by 7 and adding

$$3y^2 - 21 = 6xy$$

$$14x^2 \quad + 21 = 7xy$$

$$14x^2 + 3y^2 \quad = 0$$

$\Rightarrow \quad 14x^2 - 13xy + 3y^2 = 0$

$\Rightarrow \quad 14x^2 - 7xy - 6xy + 3y^2 = 0$

$\Rightarrow \quad 7x(2x - y) - 3y(2x - y) = 0$

$\Rightarrow \quad (2x - y)(7x - 3y) = 0$

Either $2x - y = 0$ or $7x - 3y = 0$

(i) First we solve

$$2x - y = 0 \quad \ldots\ldots\ldots (3)$$

and $$2x^2 + 3 = xy \quad \ldots\ldots\ldots (4)$$

from equation (3)

$$y = 2x \quad \ldots\ldots\ldots (5)$$

Put this value in equation (4)

$$2x^2 + 3 = x\,(2x)$$
$$2x^2 + 3 = 2x^2$$
$$2x^2 - 2x^2 + 3 = 0$$
$$3 = 0 \quad \text{No Solution}$$

(ii) Now we solve

$$7x - 3y = 0 \qquad \ldots\ldots\ldots (6)$$

and $$2x^2 + 3 = xy \qquad \ldots\ldots\ldots (7)$$

from equation (6)

$$7x = 3y$$
$$x = \frac{3y}{7} \qquad \ldots\ldots\ldots (8)$$

Put this value in equation (7)

$$\Rightarrow \quad 2\left(\frac{3y}{7}\right)^2 + 3 = \frac{3y}{7}\,.\,y$$
$$\Rightarrow \quad 2\,.\,\frac{9y^2}{49} + 3 = \frac{3y^2}{7}$$
$$\Rightarrow \quad \frac{18y^2}{49} + 3 = \frac{3y^2}{7}$$
$$\Rightarrow \quad \frac{18y^2 + 147}{49} = \frac{3y^2}{7}$$
$$\Rightarrow \quad 7\,(18y^2 + 147) = 147y^2$$
$$\Rightarrow \quad 126y^2 + 1029 = 147y^2$$
$$\Rightarrow \quad 147y^2 - 126y^2 = 1029$$
$$\Rightarrow \quad 21y^2 = 1029 \quad \Rightarrow \quad y^2 = \frac{1029}{21}$$
$$\Rightarrow \quad y^2 = 49 \quad \Rightarrow \quad y = \pm 7$$

put these values in equation (8)

when $y = 7$

equation (8) $\Rightarrow$

$$x = \frac{3}{7}(7)$$
$$x = -3$$
$$\Rightarrow \quad (3, 7)$$

when $y = -7$

equation (8) $\Rightarrow$

$$x = \frac{3}{7}(-7)$$
$$x = -3$$
$$\Rightarrow \quad (-3, -7)$$

Hence solution set $= \{(3, 7), (-3, -7)\}$

Q.10 $x^2 + y^2 = 5, \quad xy = 2$ **(Lahore Board 2003, Gujranwala Board 2005)**

Solution:

Given equations

$$x^2 + y^2 = 5 \quad \ldots\ldots\ldots (1)$$

and $$xy = 2 \quad \ldots\ldots\ldots (2)$$

from equation (2)

$$y = \frac{2}{x} \quad \ldots\ldots\ldots (3)$$

Put this value in equation (1)

$$\Rightarrow \quad x^2 + \left(\frac{2}{x}\right)^2 = 5$$

$$\Rightarrow \quad x^2 + \frac{4}{x^2} = 5$$

$$\Rightarrow \quad \frac{x^4 + 4}{x^2} = 5$$

$$\Rightarrow \quad x^4 + 4 = 5x^2$$

$$\Rightarrow \quad x^4 - 5x^2 + 4 = 0$$

using quadratic formula

$$x^2 = \frac{-(-5) \pm \sqrt{(-5)^2 - 4\,(1)\,(4)}}{2\,(1)} = \frac{5 \pm \sqrt{25 - 16}}{2}$$

$$= \frac{5 \pm \sqrt{25 - 16}}{2} = \frac{5 \pm 3}{2}$$

$$\Rightarrow \quad x^2 = \frac{5 + 3}{2} \quad \text{or} \quad x^2 = \frac{5 - 3}{2}$$

$$\Rightarrow \quad x^2 = 4 \quad \text{or} \quad x^2 = 1$$

$$\Rightarrow \quad x = \pm 2 \quad \text{or} \quad x = \pm 1$$

Put these values in equation (3)

when $x = 2 \Rightarrow y = \frac{2}{2} = 1 \Rightarrow (2, 1)$

when $x = -2 \Rightarrow y = \frac{2}{-2} = -1 \Rightarrow (-2, -1)$

when $x = 1 \Rightarrow y = \frac{2}{1} = 2 \Rightarrow (1, 2)$

when $x = -1 \Rightarrow y = \frac{2}{-1} = -2 \Rightarrow (-1, -2)$

Hence solution set $= \{(2, 1), (-2, -1), (1, 2), (-1, -2)\}$

WORD PROBLEMS ON QUADRATIC EQUATIONS

Now we solve the problems which, when expressed symbolically, lead to quadratic equations in one or two variables.

In order to solve such problems, we must;

(1) Suppose the unknown quantities to be x or y etc.

(2) Translate the problem into symbols and form the equations satisfying the given conditions.