EXERCISE 9.3

Q.1 Verify that following:

(i) $\sin 60^o \cos 30^o - \cos 60^o \sin 30^o = \sin 30^o$

(ii) $\sin^2\frac{\pi}{6} + \sin^2\frac{\pi}{3} + \tan^2\frac{\pi}{4} = 2$ **(Lahore Board 2010)**

(iii) $2\sin 45^o + \frac{1}{2}\operatorname{cosec} 45^o = \frac{3}{\sqrt{2}}$

(iv) $\sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{3} : \sin^2\frac{\pi}{2} = 1:2:3:4$

Solution:

(i) $\sin 60^o \cos 30^o - \cos 60^o \sin 30^o = \sin 30^o$

L.H.S $= \sin60° \cos30° - \cos60° \sin30°$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{3-1}{4}$

$= \frac{1}{2}$

R.H.S $= \sin 30°$

$= \frac{1}{2}$

$\therefore$ L.H.S. = R.H.S. Hence proved.

(ii) $\sin^2\frac{\pi}{6} + \sin^2\frac{\pi}{3} + \tan^2\frac{\pi}{4} = 2$

L.H.S. $= \sin^2\frac{\pi}{6} + \sin^2\frac{\pi}{3} + \tan^2\frac{\pi}{4}$

$= \sin^2 30^o + \sin^2 60^o + \tan^2 45^o$

$= (\sin 30^o)^2 + (\sin 60^o)^2 + (\tan 45^o)^2$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2$

$= \frac{1}{4} + \frac{3}{4} + 1$

$= \frac{1+3+4}{4} = \frac{8}{4} = 2$

= R.H.S. Hence proved.

(iii) $\mathbf{2\sin 45^o + \frac{1}{2}\operatorname{cosec} 45^o = \frac{3}{\sqrt{2}}}$

$$\text{L.H.S.} = 2\sin 45^o + \frac{1}{2}\operatorname{cosec} 45^o$$

$$= 2\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{2}\frac{1}{\frac{1}{\sqrt{2}}}$$

$$= \frac{2}{\sqrt{2}} + \frac{\sqrt{2}}{2}$$

$$= \frac{4+2}{2\sqrt{2}} = \frac{6}{2\sqrt{2}} = \frac{3}{\sqrt{2}}$$

= R.H.S. Hence proved.

(iv) $\mathbf{\sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{3} : \sin^2\frac{\pi}{2} = 1 : 2 : 3 : 4}$

$$\text{L.H.S.} = \sin^2\frac{\pi}{6} : \sin^2\frac{\pi}{4} : \sin^2\frac{\pi}{3} : \sin^2\frac{\pi}{2}$$

$$= \sin^2 30^o : \sin^2 45^o : \sin2\ 60^o : \sin^2 90^o$$

$$= (\sin 30^o)^2 : (\sin 45^o)^2 : (\sin 60^o)^2 : (\sin 90^o)^2$$

$$= \left(\frac{1}{2}\right)^2 : \left(\frac{1}{\sqrt{2}}\right)^2 : \left(\frac{\sqrt{3}}{2}\right)^2 : (1)^2$$

$$= \frac{1}{4} : \frac{1}{2} : \frac{3}{4} : 1$$

Multiplying throughout by 4

$$= \frac{1}{4} \times 4 : \frac{1}{2} \times 4 : \frac{3}{4} \times 4 : 1 \times 4$$

$$= 1 : 2 : 3 : 4$$

= R.H.S. Hence proved.

Q.2 Evaluate the following:

(i) $\dfrac{\tan\frac{\pi}{3}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{6}}$ (ii) $\dfrac{1-\tan^2\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}}$

Solution:

(i)
$$\frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{6}} = \frac{\tan 60^o-\tan 30^o}{1+\tan 60^o\tan 30^o}$$

$$= \frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+\sqrt{3}\cdot\frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{(\sqrt{3})^2-1}{\sqrt{3}}}{1+1}$$

$$= \frac{\frac{3-1}{\sqrt{3}}}{2}$$

$$= \frac{2}{\sqrt{3}}\times\frac{1}{2}$$

$$\frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{6}} = \frac{1}{\sqrt{3}} \quad \text{Ans.}$$

(ii)
$$\frac{1-\tan^2\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}} = \frac{1-\tan^2 60^o}{1+\tan^2 60^o}$$

$$= \frac{1-(\tan 60^o)^2}{1+(\tan 60^o)^2}$$

$$= \frac{1-(\sqrt{3})^2}{1+(\sqrt{3})^2}$$

$$= \frac{1-3}{1+3} = \frac{-2}{4}$$

$$\frac{1-\tan^2\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}} = \frac{-1}{2}$$

Q.3 Verify the following, when $\theta = 30^o, 45^o$

(i) $\sin 2\theta = 2\sin\theta\cos\theta$ **(Lahore Board 2008)**

(ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

(iii) $\cos 2\theta = 2\text{co}^2\theta - 1$

(iv) $\cos 2\theta = 1 - 2\sin^2\theta$

(v) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Solution:

(i) $\sin 2\theta = 2\sin\theta\cos\theta$

at $\theta = 30^o$

$\sin 2(30^o) = 2\sin 30^o \cos 30^o$

$\sin 60^o = 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

L.H.S. = R.H.S.

at $\theta = 45^o$

$\sin 2(45^o) = 2\sin 45^o \cos 45^o$

$\sin 90^o = 2\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}$

$1 = 2 \cdot \frac{1}{2}$

$1 = 1$

L.H.S. = R.H.S.

(ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

at $\theta = 30^o$

$\cos 2(30^o) = \cos^2 30^o - \sin^2 30^o$

$\cos 60^o = (\cos 30^o)^2 - (\sin 30^o)^2$

$\frac{1}{2} = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$\frac{1}{2} = \frac{3}{4} - \frac{1}{4}$

$\frac{1}{2} = \frac{3-1}{4}$

$\frac{1}{2} = \frac{2}{4}$

$\frac{1}{2} = \frac{1}{2}$

L.H.S. = R.H.S.

at $\theta = 45^o$

$\cos 2(45) = \cos^2 45 - \sin^2 45$

$\cos 90^o = (\cos 45)^2 - (\sin 45^o)^2$

$0 = \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$

$0 = \frac{1}{2} - \frac{1}{2}$

$0 = 0$

L.H.S. = R.H.S.

(iii) $\cos 2\theta = 2\cos^2\theta - 1$

at $\theta = 30^o$

$\cos 2(30^o) = 2\cos^2 30^o - 1$

$\cos 60^o = 2(\cos 30^o)^2 - 1$

$\frac{1}{2} = 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1$

$\frac{1}{2} = 2 \times \frac{3}{4} - 1$

$\frac{1}{2} = \frac{3}{2} - 1$

$\frac{1}{2} = \frac{3-2}{2}$

$\frac{1}{2} = \frac{1}{2}$

L.H.S. = R.H.S.

at $\theta = 45^o$

$\cos 2(45^o) = 2\cos^2 45^o - 1$

$\cos 90^o = 2(\cos 45^o)^2 - 1$

$0 = 2\left(\frac{1}{\sqrt{2}}\right)^2 - 1$

$= 2\left(\frac{1}{2}\right) - 1$

$0 = 1 - 1$

$0 = 0$

L.H.S. = R.H.S.

(iv) $\cos 2\theta = 1 - 2\sin^2\theta$

at $\theta = 30^o$

$\cos 2(30^o) = 1 - 2\sin^2 30°$

$\cos 60^o = 1 - 2(\sin 30^o)^2$

$\frac{1}{2} = 1 - 2\left(\frac{1}{2}\right)^2$

$\frac{1}{2} = 1 - 2\left(\frac{1}{4}\right)$

$\frac{1}{2} = 1 - \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$

L.H.S. = R.H.S.

at $\theta = 45^o$

$\cos 2(45^o) = 1 - 2\sin^2 45°$

$\cos 90^o = 1 - 2(\sin 45^o)^2$

$0 = 1 - 2\left(\frac{1}{\sqrt{2}}\right)^2$

$0 = 1 - 2 \cdot \frac{1}{2}$

$0 = 1 - 1$

$0 = 0$

L.H.S. = R.H.S.

(v) $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

at $\theta = 30^o$

$\tan 2(30^o) = \frac{2\tan 30^o}{1 - \tan^2 30^o}$

at $\theta = 45^o$

$\tan 2(45^o) = \frac{2\tan 45^o}{1 - \tan^2 45^o}$

$$\tan 60^o = \frac{2\tan 30^o}{1-(\tan 30^o)^2} \qquad \tan 90^o = \frac{2\tan 45^o}{1-(\tan 45^o)^2}$$

$$\sqrt{3} = \frac{2\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} \qquad \infty = \frac{2(1)}{1-(1)^2}$$

$$\sqrt{3} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} \qquad \infty = \frac{2}{1-1}$$

$$\sqrt{3} = \frac{2}{\sqrt{3}} \times \frac{3}{2} \qquad \infty = \frac{2}{0}$$

$$\sqrt{3} = \sqrt{3} \qquad \infty = \infty$$

L.H.S. = R.H.S. L.H.S. = R.H.S.

Q.4 Find x, if $\tan^2 45^o - \cos^2 60^o = x\sin 45^o \cos 45^o \tan 60^o$.

Solution:

$$\tan^2 45^o - \cos^2 60^o = x\sin 45^o \cos 45^o \tan 60^o$$

$$(\tan 45^o)^2 - (\cos 60^o)^2 = x\sin 45^o \cos 45^o \tan 60^o$$

$$(1)^2 - \left(\frac{1}{2}\right)^2 = x\left(\frac{1}{\sqrt{2}}\right).\left(\frac{1}{\sqrt{2}}\right).\sqrt{3}$$

$$1-\frac{1}{4} = x\frac{\sqrt{3}}{2}$$

$$\frac{3}{4} = \frac{\sqrt{3}}{2}x$$

$$x = \frac{2\times 3}{4\times\sqrt{3}}$$

$$x = \frac{\sqrt{3}}{2}$$

Q.5 Find the values of the trigonometric functions of the following quadrantal angles.

(i) $-\pi$ (ii) -3π (iii) $\frac{5}{2}\pi$ (iv) $-\frac{9}{2}\pi$

(v) -15π (vi) 1530^o (vii) -2430 (viii) $\frac{235}{2}\pi$ (ix) $\frac{407}{2}\pi$

Solution:

Remember

if angle is in the form of π then the formula is $2k\pi + \theta$ and if angle is in the form of degree then use the formula $2k(180^o) + \theta$ or $k(360°) + \theta$

(i) $-\pi = -2(\pi) + \pi$

$= \pi$

$\sin(-\pi) = \sin(\pi) = 0$

$\cos(-\pi) = \cos(\pi) = -1$

$\tan(-\pi) = \tan(\pi) = 0$

$\cot(-\pi) = \cot(\pi) = \infty$ (undefined)

$\sec(-\pi) = \sec(\pi) = -1$

$\text{cosec}(-\pi) = \text{cosec}(\pi) = \infty$ (undefined)

(ii) $-3\pi = -(2 \,.\, 1 \,.\, \pi + \pi)$ $\boxed{-(2 \,.\, k \,.\, \pi + \theta)}$

$= -2 \,.\, 1 .\, \pi - \pi$

$\theta = -\pi$

$= \pi$ (which is same as π)

$\sin(-3\pi) = \sin(\pi) = 0$; $\text{cosec}(-3\pi) = \text{cosec}\,\pi = \infty$ (undefined)

$\cos(-3\pi) = \cos(\pi) = -1$; $\sec(-3\pi) = \sec(\pi) = -1$

$\tan(-3\pi) = \tan(\pi) = 0$; $\cot(-3\pi) = \cot\pi = \infty$ (undefined)

(iii)

$\frac{5}{2}\pi = 2 \,.\, 1 \,.\, \pi + \frac{\pi}{2}$ $(2 \,.\, k \,.\, .\pi + \theta)$

$\Rightarrow \quad = \frac{\pi}{2}$

$$\boxed{\begin{aligned} \frac{5}{2} \times 180^o &= \frac{450^o}{360^o} = 1.25 \quad k = 1 \\ \theta &= 450^o - 360^o \\ &= 90^o = \frac{\pi}{2} \end{aligned}}$$

$\sin\frac{5}{2}\pi = \sin\left(\frac{\pi}{2}\right) = 1$; $\text{cosec}\,\frac{5}{2}\pi = \text{cosec}\,\frac{\pi}{2} = 1$

$\cos\frac{5\pi}{2} = \cos\frac{\pi}{2} = 0$; $\sec\frac{5\pi}{2} = \sec\frac{\pi}{2} = \infty$ (undefined)

$\tan\frac{5\pi}{2} = \tan\frac{\pi}{2} = \infty$ undefined ; $\cot\frac{5\pi}{2} = \cot\frac{\pi}{2} = 0$

(iv) $\boldsymbol{-\frac{9}{2}\pi}$

$-\frac{9\pi}{2} = -(3(2\pi) - 3\pi/2)$

$= -3(2\pi) + 3\pi/2$

$$\boxed{\begin{aligned} \frac{9}{2} \times 180 &= \frac{810^o}{360^o} = 2.25 \quad k = 3 \\ \theta &= 1080^o - 270 = 810 \end{aligned}}$$

$\Rightarrow \quad = \dfrac{3\pi}{2}$ $\qquad \boxed{= 90^o = \dfrac{\pi}{2}}$

$= \dfrac{3\pi}{2}\left(\text{which is same as } -\dfrac{\pi}{2}\right)$

$\sin\left(-\dfrac{9}{2}\pi\right) = \sin\left(\dfrac{3\pi}{2}\right) = -1$; $\operatorname{cosec}\left(-\dfrac{9}{2}\pi\right) = \operatorname{cosec}\dfrac{3\pi}{2} = -1$

$\cos\left(-\dfrac{9}{2}\pi\right) = \cos\left(\dfrac{3\pi}{2}\right) = 0$; $\sec\left(-\dfrac{9}{2}\pi\right) = \sec\dfrac{3\pi}{2} = \infty$ undefined

$\tan\left(-\dfrac{9}{2}\pi\right) = \tan\left(\dfrac{3\pi}{2}\right) = \infty$ (undefined) ; $\cot\left(-\dfrac{9}{2}\pi\right) = \cot\dfrac{3\pi}{2} = 0$

(v) -15π

$-(2 . k . \pi + \theta)$

$-15\pi = -(8 . 2 . \pi - \pi)$

$= -8 . 2 . \pi + \pi$

$\theta = \pi$

$$\boxed{\begin{array}{l} 15 \times 180^o = \dfrac{2700}{360} = 7.5 \\ k = 8 \end{array}}$$

$\sin(-15\pi) = \sin(\pi) = 0$; $\operatorname{cosec}(-15\pi) = \operatorname{cosec}\pi = \infty$ (undefined)

$\cos(-15\pi) = \cos(\pi) = -1$; $\sec(-15\pi) = \sec\pi = -1$

$\tan(-15\pi) = \tan(\pi) = 0$; $\cot(-15\pi) = \cot\pi = \infty$ (undefined)

(vi) 1530^o

$2 . k . 180^o + \theta$

$1530^o = 2 . 4 . 180^o + 90^o$

$\theta = 90^o$

$$\boxed{\begin{array}{l} \dfrac{1530^o}{360^o} = 4.25 \\ k = 4 \\ \theta = 1530^o - 4(360^o) = 90^o \end{array}}$$

$\sin 1530^o = \sin 90^o = 1$; $\operatorname{cosec} 1530^o = \operatorname{cosec} 90^o = 1$

$\cos 1530^o = \cos 90^o = 0$; $\sec 1530^o = \sec 90^o = \infty$ (undefined)

$\tan 1530^o = \tan 90^o = \infty$ undefined ; $\cot 1530^o = \cot 90^o = 0$

(vii) -2430^o

$-(2 . k . 180^o + \theta)$ $\qquad \boxed{\dfrac{2430^o}{360^o} = 6.75}$

$-2430^o = -(7 \,.\, 2 \,.\, 180^o - 90^o)$

$-2430^o = -7 \,.\, 2 \,.\, 180^o + 90^o$

$\theta = 90^o$

$$\boxed{\begin{aligned} k &= 7 \\ \theta &= 2430^o - 7 \times 360^o = -90^o \end{aligned}}$$

$\sin(-2430^o) = \sin 90^o = 1$; $\operatorname{cosec}(-2430^o) = \operatorname{cosec} 90^o = 1$

$\cos(-2430) = \cos 90^o = 0$; $\sec(-2430^o) = \sec 90^o = \infty$ (undefined)

$\tan(-2430^o) = \tan 90^o = \infty$ (undefined) ; $\cot(-2430^o) = \cot 90^o = 0$

(viii) $\mathbf{\dfrac{235}{2}\pi}$

$2k \,.\, \pi + \theta$

$\dfrac{235}{2}\pi = 2.58 \,.\, \pi + \dfrac{3\pi}{2}$

$\Rightarrow \quad \theta = \dfrac{3\pi}{2}$

$$\boxed{\begin{aligned} &\frac{235}{2} \times 180^o = \frac{21150^o}{360^o} = 58.75 \\ &k = 58 \\ &\theta = 21150^o - 58\,(360^o) = 270^o \end{aligned}}$$

$\sin 235\dfrac{\pi}{2} = \sin\dfrac{3\pi}{2} = -1$; $\operatorname{cosec} 235\dfrac{\pi}{2} = \operatorname{cosec}\dfrac{3\pi}{2} = -1$

$\cos 235\dfrac{\pi}{2} = \cos\dfrac{3\pi}{2} = 0$; $\sec 235\dfrac{\pi}{2} = \sec\dfrac{3\pi}{2} = \infty$

$\tan 235\dfrac{\pi}{2} = \tan\dfrac{3\pi}{2} = \infty$; $\cot 235\dfrac{\pi}{2} = \cot\dfrac{3\pi}{2} = 0$

(ix) $\mathbf{\dfrac{407}{2}\pi}$

$2\,K \,.\, \pi + \theta$

$\dfrac{407}{2}\pi = 2 \,.\, 101\pi + \dfrac{3\pi}{2}$

$\theta = \dfrac{3\pi}{2}$

$$\boxed{\begin{aligned} &407 \times 90^o = \frac{36630}{360^o} = 101.75 \\ &k = 101 \\ &\theta = 36630 - 101\,(360^o) = \frac{3\pi}{2} \end{aligned}}$$

$\sin\dfrac{407}{2}\pi = \sin\dfrac{3\pi}{2} = -1$; $\operatorname{cosec}\dfrac{407}{2}\pi = \operatorname{cosec}\dfrac{3\pi}{2} = -1$

$\cos\dfrac{407}{2}\pi = \cos\dfrac{3\pi}{2} = 0$; $\sec\dfrac{407}{2}\pi = \sec\dfrac{3\pi}{2} = \infty$

$\tan\dfrac{407}{2}\pi = \tan\dfrac{3\pi}{2} = \infty$; $\cot\dfrac{407}{2}\pi = \cot\dfrac{3\pi}{2} = 0$

Q.6 Find the values of the trigonometric functions of the following angles.

(i) 390 (ii) – 330 (iii) 765 (iv) – 675

(v) $\frac{-17}{3}\pi$ (vi) $\frac{13}{3}\pi$ (vii) $\frac{25}{6}\pi$

(viii) $-\frac{71}{6}\pi$ (ix) – 1035

Solution:

(i) 390^o

$2 . k . 180^o + \theta$

$$\boxed{\frac{390^o}{360^o} = 1.08 \quad k = 1}$$

$390^o = 2 . 1 . 180^o + 30^o$

$\theta = 30^o$

$\sin 390^o = \sin 30^o = \frac{1}{2}$; $\text{cosec } 390^o = \text{cosec } 30^o = 2$

$\cos 390^o = \cos 30^o = \frac{\sqrt{3}}{2}$; $\sec 390^o = \sec 30^o = \frac{2}{\sqrt{3}}$

$\tan 390^o = \tan 30^o = \frac{1}{\sqrt{3}}$; $\cot 390^o = \cot 30^o = \sqrt{3}$

(ii) -330^o

$-(2 . k . 180^o + \theta)$

$-330^o = (2 . 1 . 180^o - 30^o)$

$-330^o = -2 . 1. 180^o + 30^o$

$\theta = 30^o$

$\sin(-330^o) = \sin 30^o = \frac{1}{2}$; $\text{cosec}(-330^o) = \text{cosec } 30^o = 2$

$\cos(-330^o) = \cos 30^o = \frac{\sqrt{3}}{2}$; $\sec(-330^o) = \sec 30^o = \frac{2}{\sqrt{3}}$

$\tan(-330^o) = \tan 30^o = \frac{1}{\sqrt{3}}$; $\cot(-330^o) = \cot 30^o = \sqrt{3}$

(iii) 765^o

$2 k . 180^o + \theta$

$$\boxed{\frac{765^o}{360^o} = 2.125 , k = 2 \quad \theta = 765 - 2(360^o) = 45^o}$$

$765^o = 2 . 2 . 180^o + 45^o$

$\Rightarrow \quad \theta = 45^o$

$\sin 765^o = \sin 45^o = \frac{1}{\sqrt{2}}$; $\operatorname{cosec} 765^o = \operatorname{cosec} 45^o = \sqrt{2}$

$\cos 765^o = \cos 45^o = \frac{1}{\sqrt{2}}$; $\sec 765^o = \sec 45^o = \sqrt{2}$

$\tan 765^o = \tan 45^o = 1$; $\cot 765^o = \cot 45^o = 1$

(iv) $\mathbf{-675^o}$

$-(2k \cdot 180^o + \theta)$

$$\boxed{\frac{675}{360^o} = 1.875 \qquad k = 2}$$

$-675^o = -(2 \cdot 2 \cdot 180^o - 45^o)$

$-675^o = -2 \cdot 2 \cdot 180^o + 45^o$

$\Rightarrow \quad \theta = 45^o$

$\sin(-675^o) = \sin 45^o = \frac{1}{\sqrt{2}}$; $\operatorname{cosec}(-675^o) = \operatorname{cosec} 45^o = \sqrt{2}$

$\cos(-675^o) = \cos 45^o = \frac{1}{\sqrt{2}}$; $\sec(-675^o) = \sec 45^o = \sqrt{2}$

$\tan(-675^o) = \tan 45^o = 1$; $\cot(-675^o) = \cot 45^o = 1$

(v) $\mathbf{-17\frac{\pi}{3}}$

$$\boxed{\frac{17}{3} \times 180^o = \frac{1020^o}{360^o} = 2.8 \qquad k = 3}$$

$-(2k \cdot 180^o + \theta)$

$-17\frac{\pi}{3} = -(2 \cdot 3 \cdot 180^o - 60^o)$ $\qquad 1020 - 3(360^o) = 60^o$

$\theta = 60^o$

$\sin\left(\frac{-17}{3}\pi\right) = \sin 60^o = \frac{\sqrt{3}}{2}$; $\sec\left(\frac{-17}{3}\pi\right) = \sec 60^o = \frac{2}{\sqrt{3}}$

$\cos\left(\frac{-17}{3}\pi\right) = \cos 60^o = \frac{1}{2}$; $\sec\left(\frac{-17}{3}\pi\right) = \sec 60^o = 2$

$\tan\left(\frac{-17}{3}\pi\right) = \tan 60^o = \sqrt{3}$; $\cot\left(\frac{-17}{3}\pi\right) = \cot 60^o = \frac{1}{\sqrt{3}}$

(vi) $\mathbf{13\frac{\pi}{3}}$

$2k \cdot 180^o + \theta$

$13\frac{\pi}{3} = 2 \cdot 2 \cdot 180^o + 60^o$

$\theta = 60^o$

$$\boxed{\begin{array}{l} \frac{13}{3} \times 180^o = \frac{780^o}{360^o} = 2.16 \\ k = 2 \\ \theta = 780^o - 2(360^o) = 60^o \end{array}}$$

$\sin\frac{13}{3}\pi = \sin 60^o = \frac{\sqrt{3}}{2}$; $\operatorname{cosec}\frac{13}{3}\pi = \operatorname{cosec} 60^o = \frac{2}{\sqrt{3}}$

$\cos\frac{13}{3}\pi = \cos 60^o = \frac{1}{2}$; $\sec\frac{13}{3}\pi = \sec 60^o = \frac{2}{1}$

$\tan\frac{13}{3}\pi = \tan 60^o = \sqrt{3}$; $\cot\frac{13}{3}\pi = \cot 60^o = \frac{1}{\sqrt{3}}$

(vii) $\mathbf{25\frac{\pi}{6}}$

$2k \cdot 180^o + \theta$

$25\frac{\pi}{6} = 2 \cdot 2 \cdot 180^o + 30^o$

$\Rightarrow \quad \theta = 30^o$

$$\boxed{\begin{array}{l} \frac{25}{6} \times 180^o = \frac{750^o}{360^o} = 2.08 \\ k = 2 \\ 750^o - 2(360^o) = 30^o \end{array}}$$

$\sin\frac{25}{6}\pi = \sin 30^o = \frac{1}{2}$; $\operatorname{cosec}\frac{25}{6}\pi = \operatorname{cosec} 30^o = 2$

$\cos\frac{25}{6}\pi = \cos 30^o = \frac{\sqrt{3}}{2}$; $\sec\frac{25}{6}\pi = \sec 30^o = \frac{2}{\sqrt{3}}$

$\tan\frac{25}{6}\pi = \tan 30^o = \frac{1}{\sqrt{3}}$; $\cot\frac{25}{6}\pi = \cot 30^o = \sqrt{3}$

(viii) $\mathbf{-71\frac{\pi}{6}}$

$-(2k \cdot 180^o + \theta)$

$-71\frac{\pi}{6} = -(2 \cdot 6 \cdot 180^o - 30^o)$

$= -2 \cdot 6 \cdot 180^o + 30^o$

$\Rightarrow \quad \theta = 30^o$

$$\boxed{\begin{array}{l} \frac{71}{6} \times 180^o = \frac{2130^o}{360^o} = 5.91 \\ k = 6 \\ \theta = 2130^o - 6(360^o) \\ \theta = -30^o \end{array}}$$

$\sin\left(-\frac{71}{6}\pi\right) = \sin 30^o = \frac{1}{2}$; $\text{cosec}\left(-\frac{71}{6}\pi\right) = \text{cosec } 30^o = 2$

$\cos\left(-\frac{71}{6}\pi\right) = \cos 30^o = \frac{\sqrt{3}}{2}$; $\sec\left(-\frac{71}{6}\pi\right) = \sec 30^o = \frac{2}{\sqrt{3}}$

$\tan\left(-\frac{71}{6}\pi\right) = \tan 30^o = \frac{1}{\sqrt{3}}$; $\cot\left(-\frac{71}{6}\pi\right) = \cot 30^o = \sqrt{3}$

(ix) – 1035°

$-(2k\,.\,180^o + \theta)$

$-1035° = -(2\,.\,3\,.\,180^o - 45^o)$

$= -2\,.\,3\,.\,180^o + 45^o$

$\Rightarrow \quad \theta = 45^o$

$\boxed{\begin{array}{l} \frac{1035^o}{360^o} = 2.875 \\ k = 3 \\ \theta = 1035^o - 3\,(360^o) \\ \theta = -45^o \end{array}}$

$\sin(-1035^o) = \sin 45^o = \frac{1}{\sqrt{2}}$; $\text{cosec}(-1035^o) = \text{cosec } 45^o = \sqrt{2}$

$\cos(-1035^o) = \cos 45^o = \frac{1}{\sqrt{2}}$; $\sec(-1035^o) = \sec 45^o = \sqrt{2}$

$\tan(-1035^o) = \tan 45^o = 1$; $\cot(-1035^o) = \cot 45^o = 1$

EXERCISE 9.4

Q.1 Prove the identity, state the domain of θ in each case.

$\tan\theta + \cot\theta = \text{cosec}\theta \sec\theta$. **(Gujranwala Board 2005)**

Solution:

$$\begin{aligned} \text{L.H.S.} &= \tan\theta + \cot\theta \\ &= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \\ &= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} = \text{cosec}\,\theta\,.\,\sec\theta \quad \text{R.H.S.} \end{aligned}$$

Domain of $\theta : \theta \in \Re$, but $\theta \neq n\frac{\pi}{2}$, $n \in Z$.

Q.2

$\sec\theta \;\text{cosec}\,\theta \;\sin\theta\cos\theta = 1$

Solution:

$$\begin{aligned} \text{L.H.S.} &= \sec\theta\,\text{cosec}\,\theta\,\sin\theta\cos\theta \\ &= \frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}\,.\,\sin\theta\,.\,\cos\theta \\ &= 1 = \text{R.H.S.} \end{aligned}$$

Domain of $\theta : \theta \in \Re$, but $\theta \neq n\frac{\pi}{2}$, $n \in Z$