sum of n terms of a geometric Series

The formulas to find the sum of n terms of a geometric series is given by

$$S_n = \frac{a_1(1-r^n)}{1-r} \quad \text{if} \quad |r| < 1$$

and

$$S_n = \frac{a_1(r^n-1)}{r-1} \quad \text{if} \quad |r| > 1$$

Infinite Geometric Series

The geometric series which has infinite number of terms is called infinite geometric series. For example,

$$a_1 + a_1r + a_1r^2 + a_1r^3 + \ldots\ldots + a_1r^{n-1} + \ldots\ldots$$

is an infinite geometric series.

The formula to find the sum of infinite terms of a geometric series is given by

$$S_\infty = \frac{a_1}{1-r} \quad \text{if} \quad |r| < 1$$

EXERCISE 6.8

Q.1 Find sum of first 15 terms of geometric sequence, $1, \frac{1}{3}, \frac{1}{9}, \ldots\ldots$

Solution:

Given sequence

$1, \frac{1}{3}, \frac{1}{9}, \ldots\ldots$

Here $a_1 = 1, \quad r = \frac{\frac{1}{3}}{1} = \frac{1}{3}, \quad n = 15$

As $S_n = \frac{a(1-r^n)}{1-r}$

$$S_n = \frac{1\left[1-\left(\frac{1}{3}\right)^{15}\right]}{1-\frac{1}{3}} = \frac{1-\left(\frac{1}{3}\right)^{15}}{\frac{2}{3}}$$

$$= \frac{3}{2}\left[1-\left(\frac{1}{3}\right)^{15}\right] = \frac{3}{2}\left[1-\frac{1}{14348907}\right]$$

$$= \frac{3}{2}\left[\frac{14348907-1}{14348907}\right] = \frac{3}{2}\left[\frac{14348906}{14348907}\right] = \frac{7174453}{4782969}$$

Q.2 Sum to n terms, the series

(i) .2 + .22 + .222 (ii) 3 + 33 + 333 + ... (Lahore Board 2010)

Solution:

(i) .2 + .22 + .222 + ...

$$S_n = .2 + .22 + .222 + \ldots\ldots \text{ n terms}$$
$$= 2\,[.1 + .11 + .111 + \ldots\ldots \text{ n terms}]$$
$$= \frac{2}{9}[.9 + .99 + .999 + \ldots\ldots \text{ n terms}]$$
$$= \frac{2}{9}[(1 - .1) + (1 - .01) + (1 - .001) + \ldots\ldots \text{ n terms}]$$
$$= \frac{2}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \ldots\ldots \text{ n terms}\right]$$
$$= \frac{2}{9}\left[(1 + 1 + \ldots\ldots \text{ n terms}) - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots\ldots \text{ n terms}\right)\right]$$
$$= \frac{2}{9}\left[n - \frac{\frac{1}{10}\left(1 - \frac{1}{10^n}\right)}{1 - \frac{1}{10}}\right]$$
$$= \frac{2}{9}\left[n - \frac{1}{10}\,\frac{1 - \frac{1}{10^n}}{\frac{9}{10}}\right]$$
$$= \frac{2}{9}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$$

(ii) 3 + 33 + 333 + ... (GujranwalaBoard 2007)

$$S_n = 3 + 33 + 333 + \ldots\ldots \text{ n terms}$$
$$= 3\,[1 + 11 + 111 + \ldots\ldots \text{ n terms}]$$
$$= \frac{1}{3}[9 + 99 + 999 + \ldots\ldots \text{ n terms}]$$
$$= \frac{1}{3}[(10 - 1) + (100 - 1) + (1000 - 1) + \ldots\ldots \text{n terms}]$$
$$= \frac{1}{3}[(10 + 100 + 1000 + \ldots\ldots \text{ n terms}) - (1 + 1 + 1 + \ldots\ldots \text{ n times})]$$
$$= \frac{1}{3}\left[\frac{10\,(10^n - 1)}{10 - 1} - n\right]$$
$$= \frac{1}{3}\left[\frac{10}{9}(10^n - 1) - n\right]$$

Q.3 Sum to n terms of series

(i) $\mathbf{1 + (a + b) + (a^2+ab+b^2) + (a^3 + a^2b + ab^2 + b^3) + ...}$ **(Lahore Board 2008)**

(ii) $\mathbf{r + (1 + k)\, r^2 + (1 + k + k^2)\, r^3 + ...}$ **(Lahore Board 2003)**

Solution:

(i) $\mathbf{1 + (a + b) + (a^2+ ab + b^2) + (a^3 +a^2b+ab^2+b^3) + ...+ n \text{ terms}}$

$S_n = 1 + (a + b) + (a^2+ ab + b^2) + (a^3 +a^2b+ab^2+b^3) + ...+ n$ terms

Multiply and dividing by $(a - b)$

$(a - b)\, S_n = a - b + (a - b)(a + b) + (a - b)(a^2+ ab + b^2) + (a - b)(a^3 +a^2b+ab^2+b^3) + ...+ n$ terms

$(a - b)\, S_n = a - b+(a^2 - b^2)+(a^3 - b^3)+(a^4 - b^4) +.....\ n$ terms

$(a - b)\, S_n = [(a + a^2 + a^3 + a^4 + ...\ n \text{ terms}) - (b + b^2 + b^3 + b^4 + ...\ n \text{ terms})$

$$\frac{a(a^n- 1)}{a - 1} - \frac{b(b^n- 1)}{b - 1}$$

$$S_n = \frac{1}{(a - b)}\left[\frac{a(a^n- 1)}{a - 1} - \frac{b(b^n- 1)}{b - 1}\right]$$

$$= \frac{1}{(a - b)}\left[\frac{a(b - 1)(a^n - 1) - b(a - 1)(b^n - 1)}{(a - 1)(b - 1)}\right]$$

$$= \frac{a(b - 1)(a^n - 1) - b(a - 1)(b^n - 1)}{(a - b)(a - 1)(b - 1)}$$

(ii) $\mathbf{r + (1 + k)\, r^2 + (1 + k + k^2)\, r^3 + ...}$

$S_n = r + (1 + k)\, r^2 + (1 + k + k^2)\, r^3 + ...\ n$ terms

Multiply both sides by $(1 - k)$, we get

$(1 - k)\, S_n = r(1 - k) + (1 - k)(1 + k)\, r^2 + (1 - k)(1 + k + k^2)\, r^3 + ...\ n$ terms

$= r(1 - k)\, r+(1 - k^2)\, r^2+(1 - k^3)\, r^3 +.....\ n$ terms

$= r - rk + r^2 - r^2k^2 + r^3 - r^3k^3 + ...\ n$ terms

$= (r + r^2 + r^3 + ...\ n \text{ terms}) - (rk + r^2k^2 + r^3k^3 + ...\ n \text{ terms}]$

$$= \frac{r(r^n - 1)}{r - 1} - \frac{kr(k^n r^n - 1)}{kr - 1}$$

Q.4 Sum the series $\mathbf{2 + (1 - i) + \left(\frac{1}{i}\right) + ...}$ **to 8 terms.**

Solution:

$2 + (1 - i) + \frac{1}{i} + ...$

$a = 2,, \quad n = 8, \quad r = \dfrac{1-i}{2}, \quad S_8 = ?$

As $\quad S_n = \dfrac{a(1-r^n)}{(1-r)}$

$$= \frac{2\left[1 - \frac{(1-i)^8}{2}\right]}{1 - \left(\frac{1-i}{2}\right)}$$

$$= \frac{2\left[1 - \left(\frac{1-i}{2}\right)^8\right]}{\frac{1+i}{2}}$$

$\because (1-i)^2$
$= 1 + i^2 - 2i$
$= 1 - 1 - 2i$
$= -2i$

$$= \frac{4\left[1 - \left(\frac{1-i}{2}\right)^8\right]}{1+i}$$

$$= \frac{4\left[1 - \frac{((1-i)^2)^4}{2^8}\right]}{1+i}$$

$$= \frac{4\left[1 - \frac{(-2i)^4}{2^8}\right]}{1+i}$$

$$= \frac{4\left[1 - \frac{2^4 \cdot i^4}{2^8}\right]}{1+i}$$

$$= \frac{4\left[1 - \frac{+1}{2^4}\right]}{1+i}$$

$$= \frac{4\left[1 - \frac{1}{16}\right]}{1+i}$$

$$= \frac{4\left(\frac{15}{16}\right)}{1+i} = \frac{15}{4(1+i)} \times \frac{1-i}{1-i}$$

$\because \; i^2 = -1$

$$= \frac{15(1-i)}{4(1-i^2)} = \frac{15(1-i)}{4(1+1)} = \frac{15}{8}(1-i)$$

Q.5 Find the sums of the following infinite geometric series.

(i) $\frac{1}{5}+\frac{1}{25}+\frac{1}{125}+\ldots\ldots.$ (ii) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots.$

(iii) $\frac{9}{4}+\frac{3}{2}+1+\frac{2}{3}+\ldots\ldots.$ (iv) $2+1+0.5+\ldots\ldots.$

(v) $4+2\sqrt{2}+2+\sqrt{2}+1+\ldots\ldots.$ (vi) $0.1+0.05+0.025+\ldots\ldots.$

Solution:

(i) $\frac{1}{5}+\frac{1}{25}+\frac{1}{125}+\ldots\ldots.$

Here $a = \frac{1}{5}$, $r = \frac{\frac{1}{25}}{\frac{1}{5}} = \frac{1}{5}$, $S_\infty = ?$

As $S_\infty = \frac{a_1}{1-r}$

$$= \frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{\frac{1}{5}}{\frac{4}{5}} = \frac{1}{5}\times\frac{5}{4} = \frac{1}{4}$$

(ii) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots.$

Here $a_1 = \frac{1}{2}$, $r = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{4}\times 2 = \frac{1}{2}$, $S_\infty = ?$

As $S_\infty = \frac{a_1}{1-r}$

$$= \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

(iii) $\mathbf{\frac{9}{4} + \frac{3}{2} + 1 + \frac{2}{3} + ...}$

Here $a_1 = \frac{9}{4}$, $r = \frac{\frac{3}{2}}{\frac{9}{4}} = \frac{3}{2} \times \frac{4}{9} = \frac{3}{2}$, $S_\infty = ?$

As $S_\infty = \frac{a_1}{1-r} = \frac{\frac{9}{4}}{1-\frac{2}{3}} = \frac{\frac{9}{4}}{\frac{1}{3}} = \frac{9}{4} \times 3 = \frac{27}{4}$

(iv) $\mathbf{2 + 1 + 0.5 + ...}$

Here $a_1 = 2$, $r = \frac{1}{2}$, $S_\infty = ?$

As $S_\infty = \frac{a_1}{1-r}$

$= \frac{2}{1-\frac{1}{2}} = \frac{2}{\frac{1}{2}} = 4$

(v) $\mathbf{4 + 2\sqrt{2} + 2 + \sqrt{2} + 1 + ...}$

Here $a_1 = 4$, $r = \frac{2\sqrt{2}}{4} = \frac{1}{\sqrt{2}}$, $S_\infty = ?$

As $S_\infty = \frac{a_1}{1-r}$

$= \frac{4}{1-\frac{1}{\sqrt{2}}} = \frac{4\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{4\sqrt{2}\left(\sqrt{2}+1\right)}{\left(\sqrt{2}\right)^2 - (1)^2}$

$= \frac{4\left(2+\sqrt{2}\right)}{2-1} = 4\left(2+\sqrt{2}\right)$

(vi) $\mathbf{0.1 + 0.05 + 0.025 + ...}$

Here $a_1 = 0.1$, $r = \frac{0.05}{0.1} = 0.5$, $S_\infty = ?$

As $S_\infty = \frac{a_1}{1-r}$

$= \frac{0.1}{1-.05} = \frac{0.1}{0.5} = 0.2$

Q.6 Find vulgar fractions equivalent to the following recurring decimals.

(i) $1.\dot{3}\dot{4}$ **(ii)** $0.\dot{7}$

(iii) $0.\dot{2}\dot{5}\dot{9}$ **(iv)** $1.\dot{5}\dot{3}$

(v) $1.\dot{1}\dot{5}\dot{9}$ **(vi)** $1.1\dot{4}\dot{7}$

Solution:

(i) $1.\dot{3}\dot{4}$

$$1.\dot{3}\dot{4} = 1.3434343434\ ....$$
$$= 1 + 0.34 + 0.0034 + 0.000034 + ...$$
$$= 1 + \frac{0.34}{1-0.01} \qquad \because \quad a_1 = 0.34, \quad r = \frac{0.0034}{0.34} = 0.01,$$
$$= 1 + \frac{0.34}{1-.01} = 1 + \frac{0.34}{.99} = 1 + \frac{34}{99} = \frac{133}{99}$$

(ii) $0.\dot{7}$

$$0.\dot{7} = 0.777777\ ....$$
$$= 0.7 + 0.07 + 0.007 + 0.0007 + ...$$
$$= \frac{0.7}{1-.1} = \frac{0.7}{.9} = \frac{7}{9} \qquad \because \quad a = 0.7, \quad r = \frac{0.07}{0.7} = 0.1$$

(iii) $0.\dot{2}\dot{5}\dot{9}$

$$0.\dot{2}\dot{5}\dot{9} = 0.259259259259259\ ....$$
$$= 0.259 + 0.000259 + 0.000000259 + ...$$
$$= \frac{0.259}{1-0.001} \qquad \because \quad \text{Here} \quad a = 0.259, \quad r = \frac{0.000259}{0.259} = .001$$
$$= \frac{0.259}{.999} = \frac{259}{999}$$

(iv) $1.\dot{5}\dot{3}$

$$1.\dot{5}\dot{3} = 1.53535353\ ....$$

$= 1 + 0.53 + 0.0053 + 0.000053 + ...$

$= 1 + \dfrac{0.53}{1 - 0.01} \qquad \because \ a_1 = 0.53, \quad r = \dfrac{0.0053}{0.53} = 0.01, \quad n = \infty$

$= \dfrac{0.53}{0.99}$

$= 1 + \dfrac{53}{99} = \dfrac{152}{99}$

(v) $\mathbf{1.\dot{1}\dot{5}\dot{9}}$

$1.\dot{1}\dot{5}\dot{9} = 1.159159159 ....$

$= 0.159 + .000159 + .000000159 + ...$

$= \dfrac{0.159}{1 - 0.001} \qquad \because \ a_1 = 0.159 \quad r = \dfrac{0.000159}{0.159} = 0.001$

$= \dfrac{0.159}{0.999} = \dfrac{159}{999}$

$= \dfrac{53}{333}$

(vi) $\mathbf{1.1\dot{4}\dot{7}}$

$1.1\dot{4}\dot{7} = 1 + .047 + .000047 + ...$

$= 1 + 0.1 + 0.047 + 0.000047 + ...$

$= 1 + 0.1 + \dfrac{0.047}{1 - 0.001}$

$= 1 + 0.1 + \dfrac{0.047}{0.99} \qquad \because \ a_1 = .047 \quad r = \dfrac{0.000047}{0.0047} = 0.001$

$= 1 + 0.1 + \dfrac{47}{990}$

$= 1 + \dfrac{1}{10} + \dfrac{47}{990}$

$= 1 + \dfrac{99 + 47}{990}$

$= 1 + \dfrac{146}{9990} = \dfrac{1136}{990} = \dfrac{568}{495}$

Q.7 Find the sum to infinity of the series;
$r + (1 + k) r^2 + (1 + k + k^2) r^3 + ...$ r and k being proper functions.

Solution:

Given series

$r + (1 + k) r^2 + (1 + k + k^2) r^3 + ...$

$S = r + (1 + k) r^2 + (1 + k + k^2) r^3 + ...$to infinity

multiplying by $(1 - k)$, we get

$$(1 - k) S_n = r (1 - k) + (1 - k) (1 + k) r^2 + (1 - k) (1 + k + k^2) r^3 + ...\text{ to infinity}$$

$$= r (1 - k) r + (1 - k^2) r^2 + (1 - k^3) r^3 + ...\text{ to infinity}$$

$$= r - rk + r^2 - r^2 k^2 + r^3 - r^3 k^3 + ...\text{ to infinity}$$

$$= (r + r^2 + r^3 + ...\ n\ \text{terms}) - (rk + r^2 k^2 + r^3 k^3 + ...\ n\ \text{terms}$$

$$= \frac{r}{1 - r} = \frac{r\,k}{1 - r\,k}$$

$$= \frac{r (1 - kr) - rk (1 - r)}{(1 - r)(1 - rk)} = \frac{r - kr^2 - rk + r^2 k}{(1 - r)(1 - rk)}$$

$$(1 - k) S = \frac{r - rk}{(1 - r)(1 - rk)}$$

$$(1 - k) S = \frac{r (1 - k)}{(1 - r)(1 - rk)}$$

$$S = \frac{r}{(1 - r)(1 - rk)}$$

Q.8 If $y = \frac{x}{2} + \frac{1}{4} x^2 + \frac{1}{8} x^3 + ...$ and $0 < x < 2$ then prove that $x = \frac{2y}{1 + y}$

Solution:

$$y = \frac{x}{2} + \frac{1}{4} x^2 + \frac{1}{8} x^3 + ...\text{ to infinity}$$

Here $a_1 = \frac{x}{2}$, $r = \dfrac{\frac{x^2}{4}}{\frac{x}{2}} = \frac{x^2}{4} \cdot \frac{2}{x} = \frac{x}{2}$

so using $S_\infty = \frac{a_1}{1 - r}$

$$y = \frac{\frac{x}{2}}{1 - \frac{1}{2} x} = \frac{\frac{x}{2}}{\frac{2 - x}{2}} = \frac{x}{2 - x}$$

$$y = \frac{x}{2 - x}$$

$$(2-x)\,y = x$$

$$2y - xy = x$$

$$2y = x + xy$$

$$2y = x\,(1+y)$$

$$x = \frac{2y}{y+1}$$

Q.9 If $y = \frac{2}{3}x + \frac{4}{9}x^2 + \frac{8}{27}x^3 + ...$ and if $0 < x < \frac{3}{2}$ then prove that $x = \frac{3y}{2\,(1+y)}$

Solution:

Given series

$$y = \frac{2}{3}x + \frac{4}{9}x^2 + \frac{8}{27}x^3 + ...$$

Here $a_1 = \frac{2}{3}x$, $\quad r = \dfrac{\frac{4}{9}x^2}{\frac{2}{3}x} = \frac{4x^2}{9} \times \frac{3}{2x} = \frac{2x}{3}$

using $\quad S_\infty = \frac{a_1}{1-r}$

$$y = \frac{\frac{2}{3}x}{1-\frac{2x}{3}} = \frac{\frac{2}{3}x}{\frac{3-2x}{3}} = \frac{2x}{3-2x}$$

$$y = \frac{2x}{3-2x}$$

$$y\,(3-2x) = 2x$$

$$3y - 2xy = 2x$$

$$3y = 2x\,(1+y)$$

$$x = \frac{3y}{2\,(1+y)}$$

Hence proved.

Q.10 A ball is dropped from a height of 27 meters and it rebounds two third of the distance it falls. If it continues to fall in the same way what distance will it travel before coming to rest?

Solution:

Distance traveled in first fall = 27 meters

Height of first rebound $= 27 \times \frac{2}{3} = 12$ m

Height of second rebound $= 18 \times \frac{2}{3}$

Total distance $= 27 + 2\left[\dfrac{18}{1-\frac{2}{3}}\right]$ $\quad \because a_1 = 18 \quad r = \frac{12}{18} = \frac{2}{3}$

$= 27 + 2 \times \dfrac{18}{\frac{1}{3}}$

$= 27 + 108 = 135$ meters

Q.11 What distance will a ball travel before coming to rest if it is dropped from a height of 75 meters and after each fall rebounds $\frac{2}{5}$ of the distance it fell?

Solution:

Distance of first fall $= 75$ meters

Height of first rebound $= 75 \times \frac{2}{5} = 30$ m

Height of second rebound $= 30 \times \frac{2}{5} = 12$ m

Height of third rebound $= 12 \times \frac{2}{5} = \frac{24}{5}$ m

Total distance $= 75 + 2\left[30 + 12 + \frac{24}{5} \ldots \text{ to infinity}\right]$

$= 75 + 2\left[\dfrac{30}{1-\frac{2}{5}}\right]$ $\quad \because a = 30, \quad r = \frac{12}{30} = \frac{2}{5}$

$= 75 + 2\left[\dfrac{30}{\frac{3}{5}}\right]$

$= 75 + 2 \times \frac{150}{3} = 75 + 2(50) = 75 + 100 = 175$ m

Q.12 If $y = 1 + 2x + 4x^2 + 8x^3 + ...$

(i) Show that $x = \frac{y-1}{2y}$

(ii) Find the interval in which the series is convergent

Solution:

(i) $x = \frac{y-1}{2y}$

Given

$y = 1 + 2x + 4x^2 + 8x^3 + ... \infty$

Here $a_1 = 1$, $r = 2x$, using $S\infty = \frac{a_1}{1-r}$

$y = \frac{1}{1-2x} = y(1-2x) = 1$

$y - 2xy = 1$

$y - 1 = 2xy$

$x = \frac{y-1}{2y}$

(ii) $y = 1 + 2x + 4x^2 + 8x^3 + ...$

The given series will be convergent if

$|r| < 1$ $\because r = 2x$

$\Rightarrow |2x| < 1$

$\Rightarrow |x| < \frac{1}{2}$

$\Rightarrow -\frac{1}{2} < x < \frac{1}{2}$

Q.13 If $y = 1 + \frac{x}{2} + \frac{x^2}{4} + ...$

(i) Show that $x = 2\left(\frac{y-1}{y}\right)$

(ii) Find the interval in which series is convergent.

Solution:

(i) $y = 1 + \frac{x}{2} + \frac{x^2}{4} + ...$

Given series

$y = 1 + \frac{x}{2} + \frac{x^2}{4} + ... \infty$

Here $a_1 = 1, \quad r = \frac{x}{2}, \quad$ using $S\infty = \frac{a_1}{1-r}$

$$y = \frac{1}{1-\frac{x}{2}} = \frac{1}{\frac{2-x}{2}}$$

$$y = \frac{2}{2-x}$$

$$y(2-x) = 2$$
$$2y - xy = 2$$
$$2y - 2 = xy$$
$$2(y-1) = xy$$
$$x = \frac{2(y-1)}{y}$$

(ii) $\quad \mathbf{y = 1 + \frac{x}{2} + \frac{x^2}{4} + ...}$

The given series is convergent if

$|r| < 1$

Here $r = \frac{x}{2}$

$\Rightarrow \quad \left|\frac{x}{2}\right| < 1$

$\Rightarrow \quad |x| < 2 \quad \Rightarrow \quad -2 < x < 2$

Q.14 The sum of an infinite geometric series is 9 and sum of sequence of its terms is $\frac{81}{5}$. Find the series.

Solution:

Let the required geometric series is $a + ar + ar^2 + ar^3 + ... \infty$

By the given condition

$a + ar + ar^2 + ar^3 + ... \infty = 9$

or $\quad \frac{a}{1-r} = 9$

or $\quad a = 9(1-r) \qquad .... (1)$

also it is given that

$a^2 + a^2r^2 + a^2r^4 + a^2r^6 + ... \infty = \frac{81}{5}$

or $\quad \frac{a^2}{1-r^2} = \frac{81}{5}$

or $\quad 5a^2 = 81(1-r^2) \qquad .... (2)$

Put equation (1) in equation (2), we get

$5\,[9\,(1-r)]^2 = 81\,(1-r^2)$

$5 \times 81\,(1-r)^2 = 81\,(1-r)\,(1+r)$

$5\,(1-r) = 1+r$

$5-5r = 1+r$

$5r+r-5+1 = 0$

$6r-4 = 0 \Rightarrow r = \frac{4}{6} \Rightarrow \boxed{r = \frac{2}{3}}$

Put $r = \frac{2}{3}$ in equation (1), we get

$a = 9\left(1-\frac{2}{3}\right)$

$a = 9\left(\frac{1}{3}\right) = 3$

so

the required series is

$a + a\,r + a\,r^2 + \ldots\ldots$

i.e. $3 + 3\left(\frac{2}{3}\right) + 3\left(\frac{2}{3}\right)^2 + 3\left(\frac{2}{3}\right)^3 + \ldots\ldots$

or $3 + 2 + \frac{4}{3} + \frac{8}{9} \ldots\ldots$

EXERCISE 6.9

Q.1 A man deposits in a bank Rs. 8 in the first year, Rs. 24 in the second year, Rs. 72 in the third year and so on. Find the amount he will have deposited in the bank by the fifth year.

Solution:

Deposited amount is given by

$8 + 24 + 72 + \ldots\ldots$ which is a G.P

Here $a_1 = 8$, $r = 3$, $n = 5$, $S_5 = ?$

As $S_n = \frac{a_1\,(r^n-1)}{r-1}$

$S_5 = \frac{8\,(3^5-1)}{(3-1)}$

$= \frac{8\,(243-1)}{2} = \text{Rs. } 968$