EXERCISE 9.2

Q.1 Find the signs of the following:

(i) sin 160 (ii) cos 190 (iii) tan 115

(iv) sec 245 (v) cot 80^o (vi) cosec 297^o

(i) sin 160°

Since $\sin\ 160^o$ lies in the IInd quadrant, so its sign is +ve.

(ii) cos 190°

Since $\cos 190^o$ lies in the III quadrant, so its sign is – ve.

(iii) tan 115^o

Since $\tan 115^o$ lies in the II quadrant, so its sign is – ve.

(iv) sec 245^o

Since $\sec 245^o$ lies in III quadrant, so its sign is – ve.

(v) cot 80^o

Since $\cot 80^o$ lies in the I quadrant, so its sign is + ve.

(vi) cosec 297^o

Since $\text{cosec}\ 297^o$ lies in the IV quadrant, so its sign is – ve.

Q.2 Fill in the blanks:

(i) sin (–310°) = ________ sin 310°

(ii) cosec (–75°) = ________ cos 75°

(iii) tan (–182°) = ________ tan 182°

(iv) cot (–137°) = ________ cot 137°

(v) sec (–216°) = ________ sec 216°

(vi) cosec (–15) = ________ cosec 15^o

Solution:

(i) $\sin(-310^o) = \underline{-\text{ ve}}\ \sin 310°$

(ii) $\cos(-75^o) = \underline{+\text{ ve}}\ \cos 75°$

(iii) $\tan(-182^o) = \underline{-\text{ ve}}\ \tan 182°$

(iv) $\cot(-137^o) = \underline{-\text{ ve}}\ \cot 137°$

(v) $\sec(-216^o) = \underline{+\text{ ve}}\ \sec 216°$

(vi) $\text{cosec}(-15^o) = \underline{-\text{ ve}}\ \text{cosec}\ 15^o$

Q.3 In which quadrant are the terminal arms of the angle lies when

(i) $\sin\theta < 0$ **and** $\cos\theta > 0$

(ii) $\cot\theta > 0$ **and** $\text{cosec}\ 0 > 0$

(iii) $\tan\theta < 0$ **and** $\cos\theta > 0$

(iv) $\sec\theta < 0$ **and** $\sin\theta < 0$

(v) $\cot\theta > 0$ **and** $\sin\theta < 0$

(vi) $\cos\theta < 0$ **and** $\tan\theta < 0$

Solution:

(i) $\sin\theta < 0$ **and** $\cos\theta > 0$

$\Rightarrow$ $\sin\theta$ is $-$ ve & $\cos\theta$ is $+$ ve so they lie in IV Quadrant.

(ii) $\cot\theta > 0$ **and** $\text{cosec}\,\theta > 0$ **(Lahore Board 2005)**

As cot is $+$ ve and cosec θ is $+$ ve they lie in I Quadrant.

(iii) $\tan\theta < 0$ **and** $\cos\theta > 0$

Since $\tan\theta$ is $-$ ve and $\cos\theta$ is $+$ ve so they lie in IV Quadrant.

(iv) $\sec\theta < 0$ **and** $\sin\theta < 0$ **(Gujranwala Board 2007)**

As $\sec\theta$ is $-$ ve , $\sin\theta$ is $-$ ve so they lie in III Quadrant.

(v) $\cot\theta > 0$ **and** $\sin\theta < 0$

Since $\cot\theta$ is $+$ ve and $\sin\theta$ is $-$ ve so they lie in III Quadrant.

(vi) $\cos\theta < 0$ **and** $\tan\theta < 0$

Since $\cos\theta$ is $-$ ve and $\tan\theta$ is $-$ ve so they lie in II Quadrant.

Q.4 Find the values of the remaining trigonometric functions.

(i) $\sin\theta = \dfrac{12}{13}$ **and the terminal arm of the angle is in quadrant I.**

(ii) $\cos\theta = \dfrac{9}{41}$ **and the terminal arm of the angle is in quadrant IV.**

(Lahore Board 2007)

(iii) $\cos\theta = -\dfrac{\sqrt{3}}{2}$ **and the terminal arm of the angle is in quadrant III.**

(iv) $\tan\theta = -\dfrac{1}{3}$ **and the terminal arm of the angle is in quadrant II.**

(Gujranwala Board 2004)

(v) $\sin\theta = -\dfrac{1}{\sqrt{2}}$ **and the terminal arm of the angle is not in quadrant III**

(Lahore Board 2007)

Solution:

(i) $\sin\theta = \frac{12}{13}$ **and the terminal arm of the angle is in quad. I.**

$$\sin\theta = \frac{12}{13} \Rightarrow \boxed{\operatorname{cosec}\theta = \frac{13}{12}}$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{12}{13}\right)^2} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{169-144}{169}} = \sqrt{\frac{25}{169}}$$

$\boxed{\cos\theta = \frac{5}{13}}$ (since terminal arm of the angle is in I quadrant. So all trigonometric functions will be +ve.)

$$\boxed{\sec\theta = \frac{13}{5}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{12}{13} \div \frac{5}{13} = \frac{12}{13} \times \frac{13}{5} = \boxed{\frac{12}{5} = \tan\theta}$$

$$\boxed{\cot\theta = \frac{5}{12}}$$

(ii) $\cos\theta = \frac{9}{41}$ **and the terminal arm of the angle is in quadrant IV.**

(Lahore Board 2007)

$$\cos\theta = \frac{9}{41} \Rightarrow \boxed{\sec\theta = \frac{41}{9}}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{9}{41}\right)^2} = \sqrt{1-\frac{81}{1681}} = \sqrt{\frac{1681-81}{1681}} = \sqrt{\frac{1600}{1681}}$$

$\sin\theta = \pm\frac{40}{41}$ (since terminal arm of the angle is in quadrant IV)

$\sin\theta = -\frac{40}{41}$ (so only $\cos\theta$ & $\sec\theta$ are +ve, other four trigonometric ratios will be –ve)

$\Rightarrow$ $\operatorname{cosec}\theta = \frac{-41}{40}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{-40}{41}}{\frac{9}{41}} = \frac{-40}{9}$$

$$\cot\theta = \frac{-9}{40}$$

(iii) $\cos\theta = -\dfrac{\sqrt{3}}{2}$ and the terminal arm of the angle is in quadrant III.

$$\cos\theta = \frac{-\sqrt{3}}{2}$$

$$\Rightarrow \quad \sec\theta = \frac{-2}{\sqrt{3}}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{-\sqrt{3}}{2}\right)^2} = \sqrt{1-\frac{3}{4}} = \sqrt{\frac{4-3}{4}} = \sqrt{\frac{1}{4}} = \pm\frac{1}{2}$$

$\sin\theta = \dfrac{-1}{2}$ (Since terminal arm of the angle is in quad. III. So only $\tan\theta$ and $\cot\theta$ will be +ve, other four will be −ve)

$$\text{cosec}\theta = -2$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{-1}{2}}{\frac{-\sqrt{3}}{2}}$$

$$\tan\theta = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \cot\theta = \sqrt{3}$$

(iv) $\tan\theta = -\dfrac{1}{3}$ and the terminal arm of the angle is in quadrant II.

(Gujranwala Board 2004)

$$\tan\theta = \frac{-1}{3} \quad \Rightarrow \cot\theta = -3$$

$$\Rightarrow \quad \sec\theta = \sqrt{1+\tan^2\theta}$$

$$\sec\theta = \sqrt{1+\frac{1}{9}} = \sqrt{\frac{9+1}{9}} = \sqrt{\frac{10}{9}} = \pm\frac{\sqrt{10}}{3}$$

$\sec\theta = -\dfrac{\sqrt{10}}{3}$ since terminal arm of angle is in quadrant II, so $\sin\theta$, $\text{cosec}\,\theta$ will be +ve and $\sec\theta$ and $\cos\theta$ will be − ve.

$$\cos\theta = \frac{-3}{\sqrt{10}}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{9}{10}} = \sqrt{\frac{10-9}{10}} = \sqrt{\frac{1}{10}} = \frac{1}{\sqrt{10}}$$

$$\text{cosec}\,\theta = \sqrt{10}$$

(v) $\sin\theta = -\dfrac{1}{\sqrt{2}}$ and the terminal arm of the angle is not in quadrant III

(Lahore Board 2007)

$$\sin\theta = \frac{-1}{\sqrt{2}}$$

$$\operatorname{cosec}\theta = -\sqrt{2}$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{1}{2}} = \sqrt{\frac{2-1}{2}} = \frac{1}{\sqrt{2}}$$

$$\cos\theta = \pm\frac{1}{\sqrt{2}}$$

Since terminal arm of angle is not in quadrant III, $\sin\theta$ is $-$ ve, therefore quadrant will be IV, in IV quadrant $\cos\theta$ & $\sec\theta$ will be + ve.

$$\cos\theta = \frac{1}{\sqrt{2}} \qquad \sec\theta = \sqrt{2}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{-1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$$

$$\tan\theta = -1 \quad \Rightarrow \quad \cot\theta = -1$$

Q.5 If $\cot\theta = \frac{15}{8}$ and the terminal arm of angle is not in I quadrant find values find values of $\cos\theta$ & $\operatorname{cosec}\theta$. (Gujranwala Board 2005)

Solution:

$$\cot\theta = \frac{15}{8}$$

$$\Rightarrow \quad \tan\theta = \frac{8}{15}$$

$$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\frac{64}{225}} = \sqrt{\frac{225+64}{225}} = \frac{289}{225}$$

$$\sec\theta = \pm\frac{17}{15}$$

Since terminal arm is not in I quadrant so it is in III quadrant.
In III quadrant $\tan\theta$ and $\cot\theta$ is +ve. All other will be $-$ ve.

$$\text{Sec}\,\theta = \frac{-17}{15}$$

$$\Rightarrow \quad \cos\theta = \frac{-15}{17}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{225}{289}} = \sqrt{\frac{289-225}{289}} = \sqrt{\frac{64}{289}}$$

$$\sec\theta = \pm\frac{8}{17}$$

$$\sin\theta = \frac{-8}{17} \text{ (terminal arm in in III quadrant)}$$

$$\operatorname{cosec}\theta = \frac{-17}{18}$$

Q.6 If $\operatorname{cosec}\theta = \dfrac{m^2+1}{2m}$ and $m > 0$ $\left(0 < \theta < \dfrac{\pi}{2}\right)$, find the value of the remaining trigonometric ratios. (Lahore Board 2005)

Solution:

$$\operatorname{cosec}\theta = \frac{m^2+1}{2m}$$

$$\Rightarrow \quad \sin\theta = \frac{2m}{m^2+1}$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{2m}{m^2+1}\right)^2} = \sqrt{1-\frac{4m^2}{m^4+1+2m^2}}$$

$$= \sqrt{\frac{m^4+1+2m^2-4m^2}{m^4+1+2m^2}}$$

$$\cos\theta = \sqrt{\frac{m^4-2m^2+1}{m^4+1+2m^2}} = \sqrt{\frac{(m^2-1)^2}{(m^2+1)^2}}$$

$$\cos\theta = \frac{m^2-1}{m^2+1} \quad \left(\sin\theta \;\; 0 < \theta < \frac{\pi}{2} \text{ so quadrant is I, all trigonometric ratios are +ve}\right)$$

$$\sec\theta = \frac{m^2+1}{m^2-1}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{2m}{m^2+1}}{\frac{(m^2-1)}{(m^2+1)}} = \frac{2m}{m^2-1}$$

$$\cot\theta = \frac{m^2-1}{2m}$$

Q.7 If $\tan\theta = \dfrac{1}{\sqrt{7}}$ and terminal arm of the angle is not the III quadrant, find the value of $\dfrac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$. (Lahore Board 2010)

Solution:

$$\tan\theta = \frac{1}{\sqrt{7}}$$

$$\Rightarrow \quad \cot\theta = \sqrt{7}$$

$$\sec^2\theta = 1+\tan^2\theta = 1+\frac{1}{7} = \frac{7+1}{7} = \frac{8}{7}$$

$$\sec^2\theta = \frac{8}{7}$$

$$\operatorname{cosec}^2\theta = 1 + \cot^2\theta = 1 + 7$$

$$\operatorname{cosec}^2\theta = 8$$

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}} = \frac{\frac{56-8}{7}}{\frac{56+8}{7}} = \frac{48}{64}$$

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{3}{4}$$

Q.8 If $\cot\theta = \frac{5}{2}$ and the terminal arm of the angle is in the I quadrant, find the value of $\frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta}$. (Lahore Board 2009)

Solution:

$$\cot\theta = \frac{5}{2}$$

$$\Rightarrow \quad \tan\theta = \frac{2}{5}$$

$$\operatorname{cosec}\theta = \sqrt{1 + \cot^2\theta} = \sqrt{1 + \frac{25}{4}} = \sqrt{\frac{4+25}{4}} = \frac{\sqrt{29}}{2}$$

$\Rightarrow \quad \sin\theta = \frac{2}{\sqrt{29}}$ (since terminal arm is in I quad, so all trigonometric functions are +ve)

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{4}{29}} = \sqrt{\frac{29-4}{29}} = \sqrt{\frac{25}{29}}$$

$$\cos\theta = \frac{5}{\sqrt{29}}$$

$$\frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta} = \frac{3\left(\frac{2}{\sqrt{29}}\right) + 4\left(\frac{5}{\sqrt{29}}\right)}{\frac{5}{\sqrt{29}} - \frac{2}{\sqrt{29}}} = \frac{\frac{6}{\sqrt{29}} + \frac{20}{\sqrt{29}}}{\frac{3}{\sqrt{29}}} = \frac{\frac{26}{\sqrt{29}}}{\frac{3}{\sqrt{29}}}$$

$$\frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta} = \frac{26}{3}$$