then $A - a = b - a \Rightarrow A = \dfrac{a+b}{2}$

In general, we can say that a_n is the A.M between a_{n-1} and a_{n+1}

i.e.

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

EXERCISE 6.3

Q.1 Find A.M. between

(i) $3\sqrt{5}$ and $5\sqrt{5}$ (ii) $x-3$ and $x+5$

(iii) $1-x+x^2$ and $1+x+x^2$ (Gujranwala Board 2006, 2007)

Solution:

(i) $3\sqrt{5}$ and $5\sqrt{5}$

Let $a = 3\sqrt{5}$ $b = 5\sqrt{5}$

As $\text{A.M.} = \dfrac{a+b}{2}$

$$= \frac{3\sqrt{5} + 5\sqrt{5}}{2}$$

$$= \frac{8\sqrt{5}}{2} = 4\sqrt{5}$$

(ii) $x-3$ and $x+5$

Let $a = x-3$, $b = x+5$

As $\text{A.M} = \dfrac{a+b}{2}$

$$\text{A.M} = \frac{(x-3)+(x+5)}{2} = \frac{2x+2}{2} = \frac{2(x+1)}{2} = x+1$$

(iii) $1-x+x^2$ and $1+x+x^2$

As $\text{A.M} = \dfrac{a+b}{2}$

$$= \frac{1-x+x^2+1+x+x^2}{2} = \frac{2+2x^2}{2} = \frac{2(1+x^2)}{2} = 1+x^2$$

Q.2 If 5, 8 are two A.M's between a and b, find a and b.
(Gujranwala Board 2005, 2007, Lahore Board 2004, 2011)

Solution:

As 5, 8 are A.M's between a and b

so

a, 5, 8, b are in A.P

$\Rightarrow \quad 5 = \frac{a+8}{2}$

$\Rightarrow \quad 10 = a + 8 \quad \Rightarrow \quad \boxed{a = 2}$

and $\quad 8 = \frac{5+b}{2}$

$16 = 5 + b$

$\Rightarrow \quad \boxed{b = 11}$

Q.3 Find 6 A.M's between 2 and 5. (Gujranwala Board 2003)

Solution:

Let $A_1, A_2, A_3, A_4, A_5, A_6$ are required A.M's between 2 and 5

then

$2, A_1, A_2, A_3, A_4, A_5, A_6, 5$ are in A.P

$a_1 = 2, \quad a_8 = 5$

$\Rightarrow \quad a_1 + 7d = 5$

$2 + 7d = 5 \quad \Rightarrow \quad 7d = 5 - 2 = 3$

$$d = \frac{3}{7}$$

Now

$$A_1 = a_2 = a_1 + d = 2 + \frac{3}{7} = \frac{14+3}{7} = \frac{17}{7}$$

$$A_2 = a_3 = a_1 + 2d = 2 + 2 \cdot \frac{3}{7} = 2 + \frac{6}{7} = \frac{14+6}{7} = \frac{20}{7}$$

$$A_3 = a_4 = a_1 + 3d = 2 + 3 \cdot \frac{3}{7} = 2 + \frac{9}{7} = \frac{14+9}{7} = \frac{23}{7}$$

$$A_4 = a_5 = a_1 + 4d = 2 + 4 \cdot \frac{3}{7} = 2 + \frac{12}{7} = \frac{14+12}{7} = \frac{26}{7}$$

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$$A_5 = a_6 = a_1 + 5d = 2 + 5 \cdot \frac{3}{7} = 2 + \frac{15}{7} = \frac{14+15}{7} = \frac{29}{7}$$

$$A_6 = a_7 = a_1 + 6d = 2 + 6 \cdot \frac{3}{7} = 2 + \frac{18}{7} = \frac{14+18}{7} = \frac{32}{7}$$

Hence required.

$\Rightarrow$ $\frac{17}{7}, \frac{20}{7}, \frac{23}{7}, \frac{26}{7}, \frac{29}{7}, \frac{32}{7}$ are required A.Ms.

Q.4 Find four A.M's between $\sqrt{2}$ and $\frac{12}{\sqrt{2}}$.

Solution:

Let A_1, A_2, A_3, A_4 are required A.Ms

then

$\sqrt{2}$, $A_1, A_2, A_3, A_4, \frac{12}{\sqrt{2}}$ are in A.P

$$a_1 = \sqrt{2},$$

$$a_6 = \frac{12}{\sqrt{2}} \Rightarrow a_1 + 5d = \frac{12}{\sqrt{2}}$$

$$\sqrt{2} + 5d = \frac{12}{\sqrt{2}}$$

$$5d = \frac{12}{\sqrt{2}} - \sqrt{2}$$

$$= \frac{12-2}{\sqrt{2}} = \frac{10}{\sqrt{2}}$$

$$d = \frac{10}{5\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$A_1 = a_2 = a_1 + d = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$$
$$A_2 = a_3 = a_1 + 2d = \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$$
$$A_3 = a_4 = a_1 + 3d = \sqrt{2} + 3\sqrt{2} = 4\sqrt{2}$$
$$A_4 = a_5 = a_1 + 4d = \sqrt{2} + 4\sqrt{2} = 5\sqrt{2}$$

Are required A.Ms.

Q.5 Insert 7 A.M's between 4 and 8.

Solution:

Let $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ are required A.Ms

then

$4, A_1, A_2, A_3, A_4, A_5, A_6, A_7, 8$

Here

$\boxed{a_1 = 4}$ and $a_9 = 8$

$$\Rightarrow a_1 + 8d = 8$$
$$4 + 8d = 8$$
$$4 + 8d = 8$$

$$8d = 4$$

$$\boxed{d = \frac{1}{2}}$$

So,

$$A_1 = a_2 = a_1 + d = 4 + \frac{1}{2} = \frac{9}{2}$$

$$A_2 = a_3 = a_1 + 2d = 4 + 2 \cdot \frac{1}{2} = 5$$

$$A_3 = a_4 = a_1 + 3d = 4 + 3 \cdot \frac{1}{2} = 4 + \frac{3}{2} = \frac{11}{2}$$

$$A_4 = a_5 = a_1 + 4d = 4 + 4 \cdot \frac{1}{2} = 4 + 2 = 6$$

$$A_5 = a_6 = a_1 + 5d = 4 + 5 \cdot \frac{1}{2} = 4 + \frac{5}{2} = \frac{13}{2}$$

$$A_6 = a_7 = a_1 + 6d = 4 + 6 \cdot \frac{1}{2} = 4 + 3 = 7$$

$$A_7 = a_8 = a_1 + 7d = 4 + 7 \cdot \frac{1}{2} = 4 + \frac{7}{2} = \frac{15}{2}$$

Are required A.Ms.

Q.6 Find three A.Ms between 3 and 11.

Solution:

Let A_1, A_2, A_3 are required A.M's

then

$3, A_1, A_2, A_3, 11$ are in A.P

$\Rightarrow$ $a_1 = 3$ and $a_5 = 11$

$$\Rightarrow \quad a_1 + 4d = 11$$

$$\Rightarrow \quad 3 + 4d = 11$$

$$4d = 8$$

$$\boxed{d = 2}$$

So

$$A_1 = a_2 = a_1 + d = 3 + 2 = 5$$

$$A_2 = a_3 = a_1 + 2d = 3 + 2(2) = 7$$

$$A_3 = a_4 = a_1 + 3d = 3 + 3(2) = 9$$

are required A.Ms.

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Q.7 Find n so that $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ may be A.M. between a and b.

Solution:

As we know that the A.M between a and b is

$$A = \frac{a+b}{2}$$

$$\frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \frac{a+b}{2}$$

$$\frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \frac{a+b}{2}$$

$\Rightarrow \quad 2(a^n + b^n) = (a+b)(a^{n-1} + b^{n-1})$

$\Rightarrow \quad 2a^n + 2b^n = a^n + a^{n-1}b + ab^{n-1} + b^n$

$\Rightarrow \quad 2a^n + 2b^n - a^n - b^n = a^{n-1}b + ab^{n-1}$

$\Rightarrow \quad a^n + b^n = a^{n-1}b + ab^{n-1}$

$\Rightarrow \quad a^n - a^{n-1}b = ab^{n-1} - b^n$

$\Rightarrow \quad a^{n-1}(a-b) = b^{n-1}(a-b)$

$\Rightarrow \quad a^{n-1} = b^{n-1}$

$\Rightarrow \quad \frac{a^{n-1}}{b^{n-1}} = 1$

$\Rightarrow \quad \left(\frac{a}{b}\right)^{n-1} = \left(\frac{a}{b}\right)^0$

$\Rightarrow \quad n - 1 = 0$

$\Rightarrow \quad \boxed{n = 1}$

Q.8 Show that sum of n A.Ms between a and b is equal to n times their A.M. (Lahore Board 2010)

Solution:

Let $A_1, A_2, A_3, \ldots\ldots A_n$ are n arithmetic means between a and b \

then

$a, A_1, A_2, A_3, \ldots\ldots\ldots A_n, b$ are in A.P

$\Rightarrow \quad a_1 = a$ and $a_{n+2} = b$

$\Rightarrow \quad a_1 + (n+1)d = b$

$$\Rightarrow \quad a + (n+1)d = b$$
$$(n+1)d = b - a$$
$$d = \frac{b-a}{n+1}$$

Now

$$A_1 = a_2 = a_1 + d = a + \frac{b-a}{n+1}$$

$$A_2 = a_3 = a_1 + 2d = a + 2 \cdot \frac{b-a}{n+1}$$

$$A_3 = a_4 = a_1 + 3d = a + 3 \cdot \frac{b-a}{n+1}$$

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$$A_n = a_{n+1} = a_1 + nd = a + n \cdot \frac{b-a}{n+1}$$

$\therefore$ $A_1 + a_2 + A_3 + \ldots\ldots + A_n$

$$= a + \frac{b-a}{n+1} + a + 2 \cdot \frac{b-a}{n+1} + a + 3 \cdot \frac{b-a}{n+1} + \ldots\ldots + a + n \cdot \frac{b-a}{n+1}$$

$$= na + \frac{b-a}{n+1} + 2\frac{b-a}{n+1} + \ldots\ldots + n \cdot \frac{b-a}{n+1}$$

$$= na + \frac{b-a}{n+1}[1 + 2 + 3 + \ldots\ldots + n]$$

$$= na + \frac{b-a}{n+1} \frac{n(n+1)}{2}$$

$$= na + \frac{n(b-a)}{2} = \frac{2na + nb - na}{2}$$

$$= \frac{na + nb}{2} = n \cdot \frac{a+b}{2} = nA$$

Hence proved

Series

The sum of an indicated number of terms in a sequence is called a series.