(iv) $3x^2 + 5y^2 = 60$ and $9x^2 + y^2 = 124$

Solution:

$3x^2 + 5y^2 = 60$ .... (1) $9x^2 + y^2 = 124$ .... (2)

Multiplying (1) by (3) & Subtracting from (2)

$$\begin{array}{rcl} 9x^2 + \quad y^2 &=& 124 \\ -9x^2 \pm 15y^2 &=& -180 \\ \hline -14y^2 &=& -56 \end{array}$$

$y^2 = 4 \quad \Rightarrow \quad y = \pm 2$

Put in (1)

$9x^2 + 4 = 124$

$9x^2 = 120$

$x^2 = \frac{120}{9} = \frac{40}{3} \qquad x = \pm\sqrt{\frac{40}{3}}$

Hence points of intersection are $\left(\pm\sqrt{\frac{40}{3}}\ \pm 2\right)$

EXERCISE 6.8

Q.1: Find an equation of each of the following with respect to new parallel axes obtained by shifting the origin to the indicated point.

Remember

Solution:

(i) $x^2 + 16y - 16 = 0$ .... (1) $O'(0, 1) \Rightarrow h = 0,\ k = 1$

We know that equations of transformation are

$x = X + h \quad , \quad y = Y + k$

$x = X + 0 \quad , \quad y = Y + 1$ Put in (1)

$X^2 + 16(Y + 1) - 16 = 0$

$X^2 + 16Y + 16 - 16 = 0$

$X^2 + 16Y = 0$ Ans

(ii) $4x^2 + y^2 + 16x - 10y + 37 = 0$ $O'(-2, 5)$

Solution:

$4x^2 + y^2 + 16x - 10y + 37 = 0$ .... (i) , $O'(-2, 5) \Rightarrow h = -2, k = 5$

We know that equations of transformation are

$$x = x + h, \quad y = Y + k$$
$$x = x - 2, \quad y = Y + 5$$

Put in (1)

$$4(X-2)^2 + (Y+5)^2 + 16(X-2) - 10(Y+5) + 37 = 0$$
$$4(X^2 + 4 - 4X) + Y^2 + 25 + 10Y + 16X - 32 - 10Y - 50 + 37 = 0$$
$$4X^2 + 16 - 16X + Y^2 + 16X - 20 = 0$$
$$4X^2 + Y^2 - 4 = 0 \qquad \text{Ans}$$

(iii) $\mathbf{9x^2 + 4y^2 + 18x - 16y - 11 = 0 \qquad O'(-1, 2)}$

Solution:

$9x^2 + 4y^2 + 18x - 16y - 11 = 0$ .... (1) , $O'(-1, 2) \Rightarrow h = -1, k = 2$

We know that equations of transformation are

$$x = X + h \quad , \quad y = Y + k$$
$$x = X - 1 \quad , \quad y = Y + 2$$

Put in (1)

$$9(X-1)^2 + 4(Y+2)^2 + 18(X-1) - 16(Y+2) - 11 = 0$$
$$9(X^2 + 1 - 2x) + 4(Y^2 + 4 + 4Y) + 18X - 18 - 16Y - 32 - 11 = 0$$
$$9X^2 + 9 - 18X + 4Y^2 + 16 + 16Y + 18X - 18 - 16Y - 32 - 11 = 0$$
$$9X^2 + 4Y^2 - 36 = 0 \qquad \text{Ans}$$

(iv) $\mathbf{x^2 - y^2 + 4x + 8y - 11 = 0 \qquad O'(-2, 4)}$

Solution:

$x^2 - y^2 + 4x + 8y - 11 = 0$ .... (1) , $O'(-2, 4) \Rightarrow h = -2, k = 4$

We know that equations of transformations of transformation

$$x = X + h \quad , \quad y = Y + k$$
$$x = X - 2 \quad , \quad y = Y + 4$$

Put in (1)

$$(X-2)^2 - (Y+4)^2 + 4(X-2) + 8(Y+4) - 11 = 0$$
$$X^2 + 4 - 4X - Y^2 - 16 - 8Y + 4X - 8 + 8Y + 32 - 11 = 0$$
$$X^2 - Y^2 + 1 = 0 \qquad \text{Ans.}$$

(v) $\mathbf{9x^2 - 4y^2 + 36x + 8y - 4 = 0 \qquad O'(-2, 1)}$

Solution:

$9x^2 - 4y^2 + 36x + 8y - 4 = 0$ .... (1) $O'(-2, 1) \Rightarrow h = -2 \quad , \quad k = 1$

We know that equations of transformation are

$$x = X + h \quad , \quad y = Y + k$$
$$x = X - 2 \quad , \quad y = Y + 1$$

Put in (1)

$$9(X-2)^2 - 4(Y+1)^2 + 36(X-2) + 8(Y+1) - 4 = 0$$

$9(X^2 + 4 - 4X) - 4(Y^2 + 1 + 2Y) + 36X - 72 + 8Y + 8 - 4 = 0$
$9X^2 + 36 - 36X - 4Y^2 - 4 - 8Y + 36x - 72 + 8Y + 4 = 0$
$9X^2 - 4Y^2 - 36 = 0$ Ans.

Q.2: Find coordinates of the new origin so that first-degree terms are removed from the transformed equation of each of the following. Also find the transformed equation.

(i) $\mathbf{3x^2 - 2y^2 + 24x + 12y + 24 = 0}$

Solution:

$3x^2 - 2y^2 + 24x + 12y + 24 = 0$ ..... (1)

Let the coordinates of the new origin be (h, k)

Equations of transformed axes are $x = X + h$, $y = Y + K$ Put in (i)

$3(X + h)^2 - 2(Y + K)^2 + 24(X + h) + 12(Y + k) + 24 = 0$
$3(X^2 + h^2 + 2Xh) - 2(Y^2 + k^2 + 2Yk) + 24X + 24h + 12Y + 12k + 24 = 0$
$3X^2 + 3h^2 + 6Xh - 2Y^2 - 2k^2 - 4Yk + 24X + 24h + 12Y + 12k + 24 = 0$
$3X^2 - 2Y^2 + X(6h + 24) - Y(4k - 12) + 3h^2 - 2k^2 + 24h + 12k + 24 = 0$ ..... (2)

Now, we remove first-degree terms from the transformed equation

$6h + 24 = 0$, $4k - 12 = 0$
=> $h = -4$ $k = 3$

Thus new origin $O'(h, k) = O'(-4, 3)$

Put $h = -4$, $k = 3$ in (2)

$3X^2 - 2Y^2 + 0 - 0 + 3(16) - 2(9) + 24(-4) + 12(3) + 24 = 0$
$3X^2 - 2Y^2 + 48 - 18 - 96 + 36 + 24 = 0$
$3X^2 - 2Y^2 - 6 = 0$

(ii) $\mathbf{25x^2 + 9y^2 + 50x - 36y - 164 = 0}$

Solution:

$25x^2 + 9y^2 + 50x - 36y - 164 = 0$ .... (1)

Let the coordinates of the new origin be (h, k) equation of transformed axes are $x = X + h$, $y = Y + k$

Put in (1)

$25(X + h)^2 + 9(Y + k)^2 + 50(X + h) - 36(Y + k) - 164 = 0$
$25(X^2 + h^2 + 2Xh) + 9(Y^2 + k^2 + 2Yk) + 50X + 50h - 36Y - 36k - 164 = 0$
$25X^2 + 25h^2 + 50Xh + 9Y^2 + 9k^2 + 18Yk + 50X + 50h - 36Y - 36k - 164 = 0$
$25X^2 + 9Y^2 + (50h + 50)X + (18k - 36)Y + 25h^2$
$+ 9k^2 + 50h - 36k - 164 = 0$ ..... (2)

Now we remove first – degree terms, from the transformed equation

$50h + 50 = 0$ $18k - 36 = 0$

$50h = -50$ $\qquad$ $18k = 36$

$\boxed{h = -1}$ $\qquad$ $\boxed{k = 2}$

New origin is $O'(h, k) = O'(-1, 2)$

Put $h = -1$, $K = 2$ in (2)

$25X^2 + 9Y^2 + 0 + 0 + 25(1) + 9(4) + 50(-1) - 36(2) - 164 = 0$

$25X^2 + 9Y^2 + 25 + 36 - 50 - 72 - 164 = 0$

$25X^2 + 9Y^2 - 225 = 0$

(iii) $\quad$ $\mathbf{x^2 - y^2 - 6x + 2y + 7 = 0}$

Solution:

$x^2 - y^2 - 6x + 2y + 7 = 0 \qquad (1)$

Let the coordinates of the new origin O' be (h, k). The equations of transformation are $x = X + h$, $y = Y + k$.

Put in (1)

$(X + h)^2 - (Y + k)^2 - 6(X + h) + 2(Y + k) + 7 = 0$

$X^2 + h^2 + 2Xh - Y^2 - k^2 - 2Yk - 6X - 6h + 2Y + 2k + 7 = 0$

$X^2 - Y^2 + X(2h - 6) - Y(2k - 2) + h^2 - k^2 - 6h + 2k + 7 = 0 \quad .... (2)$

Now we remove first degrees terms from the transformed equation

$2h - 6 = 0 \qquad 2k - 2 = 0$

$h = \frac{6}{2} \qquad 2k = 2$

$h = 3 \qquad k = 1$

New origin is $O'(h, k) = O'(3, 1)$

Put $h = 3$, $k = 1$ in (2)

$X^2 - Y^2 + 0 - 0 + 9 - 1 - 18 + 2 + 7 = 0$

$X^2 - Y^2 - 1 = 0$

Q.3: **In each of the following, find an equation referred to the new axes obtained by rotation of axes about the origin through the given angle.**

(i) $\quad$ $\mathbf{xy = 1 \ .... (1) \quad , \quad \theta = 45^\circ}$

Solution:

We know that equations of rotation are

$x = X\cos\theta - Y\sin\theta \Rightarrow x = X\cos 45^\circ - Y\sin 45^\circ \Rightarrow x = \frac{X}{\sqrt{2}} - \frac{Y}{\sqrt{2}}$

$y = X\sin\theta + Y\cos\theta \Rightarrow y = X\sin 45^\circ + Y\cos 45^\circ \Rightarrow y = \frac{X}{\sqrt{2}} + \frac{Y}{\sqrt{2}}$

Putting these values in (1) we get

$$\left(\frac{X-y}{\sqrt{2}}\right)\left(\frac{X+y}{\sqrt{2}}\right) = 1$$

$$\frac{X^2-Y^2}{2} = 1 \quad => \quad X^2 - Y^2 = 2 \quad \text{Ans.}$$

(ii) $\mathbf{7x^2 - 8xy + y^2 - 9 = 0}$ **,** $\boldsymbol{\theta}$ **= arc tan 2**

Solution:

$$7x^2 - 8xy + y^2 - 9 = 0 \quad (1) \quad \theta = \tan^{-1} 2$$

$$=> \quad \tan\theta = 2 \quad => \quad \cot\theta = \frac{1}{2}$$

$$\sec\theta = \sqrt{1+\tan^2\theta} \quad , \quad \text{cosec}\,\theta = \sqrt{1+\cot^2\theta}$$

$$= \sqrt{1+4} \quad , \quad = \sqrt{1+\frac{1}{4}}$$

$$\sec\theta = \sqrt{5} \quad , \quad \text{cosec}\,\theta = \frac{\sqrt{5}}{2}$$

$$\cos\theta = \frac{1}{\sqrt{5}} \quad , \quad \sin\theta = \frac{2}{\sqrt{5}}$$

$$x = X\cos - Y\sin\theta \quad , \quad y = X\sin\theta + Y\cos\theta$$

$$= X\frac{1}{\sqrt{5}} - Y\frac{2}{\sqrt{5}} \quad , \quad y = X\frac{2}{\sqrt{5}} + Y\frac{1}{\sqrt{5}}$$

$$x = \frac{X-2Y}{\sqrt{5}} \quad , \quad y = \frac{2X+Y}{\sqrt{5}}$$

Putting these values in (1)

$$7\left(\frac{X-2Y}{\sqrt{5}}\right)^2 - 8\left(\frac{X-2Y}{\sqrt{5}}\right)\left(\frac{2X+Y}{\sqrt{5}}\right) + \left(\frac{2X-Y}{\sqrt{5}}\right)^2 - 9 = 0$$

$$7\left(\frac{(X^2+4Y^2-4XY)}{5}\right) - 8\left(\frac{2X^2+XY-4XY-2Y^2}{5}\right) + \frac{4X^2+Y^2+4XY}{5} - 9 = 0$$

$$7X^2 - 28XY + 28Y^2 - 16X^2 + 24XY + 16Y^2 + 4X^2 + 4XY + Y^2 - 45 = 0$$

$$-5X^2 + 45Y^2 - 45 = 0$$

$$-5(X^2 - 9Y^2 + 9) = 0$$

$$X^2 - 9Y^2 + 9 = 0$$

(iii) $\mathbf{9x^2 + 12xy + 4y^2 - x - y = 0}$ $\quad\quad \boldsymbol{\theta = \text{arc tan}\frac{2}{3}}$

Solution:

$$9x^2 + 12xy + 4y^2 - x - y = 0 \quad\quad (1)$$

$$\theta = \tan^{-1}\frac{2}{3}$$

$$\tan\theta = \frac{2}{3} \quad => \quad \cot\theta = \frac{3}{2}$$

$$\sec\theta = \sqrt{1+\tan^2\theta}$$

$$= \sqrt{1+\frac{4}{9}} = \frac{\sqrt{13}}{3}$$

$$\sec\theta = \frac{\sqrt{13}}{3} \quad => \quad \cos\theta = \frac{3}{\sqrt{13}}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{9}{13}} = \sqrt{\frac{13-9}{13}} = \sqrt{\frac{4}{13}} = \frac{2}{\sqrt{13}}$$

$$x = X\cos\theta - Y\sin\theta \quad => \quad x = X\frac{3}{\sqrt{13}} - Y\frac{2}{\sqrt{13}} = \frac{3X-2Y}{\sqrt{13}}$$

$$y = X\sin\theta + Y\sin\theta \quad => \quad y = X\frac{2}{\sqrt{13}} + Y\frac{3}{\sqrt{13}} \quad y = \frac{2X+3Y}{\sqrt{13}}$$

Putting the values in (1)

$$9\left(\frac{3X-2Y}{\sqrt{13}}\right)^2 + 12\left(\frac{3X-2Y}{\sqrt{13}}\right)\left(\frac{2X+3Y}{\sqrt{13}}\right) + 4\left(\frac{2X+3Y}{\sqrt{13}}\right)^2 - \left(\frac{3X-2Y}{\sqrt{13}}\right)$$
$$-\left(\frac{2X+3Y}{\sqrt{13}}\right) = 0$$

$$9\frac{(9X^2+4Y^2-12XY)}{13} + \frac{12}{13}(6X^2+9XY-4XY-6Y^2) + \frac{4}{13}(4X^2+9Y^2+$$
$$12XY) - \frac{3X-2Y}{\sqrt{13}} - \frac{2X+3Y}{\sqrt{13}} = 0$$

$$81X^2 + 36Y^2 - 108XY + 72X^2 + 108XY - 48XY - 72Y^2 + 16X^2 + 36Y^2 +$$
$$48XY - 3\sqrt{13}\,X + 2\sqrt{13}\,Y - 2\sqrt{13}\,X - 3\sqrt{13}\,Y = 0$$

$$169X^2 - 5\sqrt{13}\,X - \sqrt{13}\,Y = 0$$

Dividing throughout by $\sqrt{13}$

$$13\sqrt{13}\,X^2 - 5X - Y = 0 \quad\quad \text{Ans}$$

Q.4: Find the measure of angle through which the axes be rotated, so that the product term XY is removed from transformed equation. Also find transformation.

$$2x^2 + 6xy + 10y^2 - 11 = 0 \qquad (1)$$

Solution:

$2x^2 + 6xy + 10y^2 - 11 = 0$ ...... (1)

The transformed equations for rotated axes are

$x = X\cos\theta - Y\sin\theta$, $y = X\sin\theta + Y\cos\theta$ Put in (1)

$2(X\cos\theta - Y\sin\theta)^2 + 6(X\cos\theta - Y\sin\theta)(X\sin\theta + Y\cos\theta) + 10(X\sin\theta + Y\cos\theta)^2 - 11 = 0$

$2X^2\cos^2\theta - 2Y^2\sin^2\theta - 4XY\sin\theta\cos\theta + 6(X^2\cos\theta\sin\theta + XY\cos^2\theta$
$- XY\sin^2\theta - Y^2\sin\theta\cos\theta) + 10X^2\sin^2\theta + 10Y^2\cos^2\theta$
$+ 20XY\sin\theta - \cos\theta - 11 = 0$

$2X^2\cos^2\theta + 10X^2\sin^2\theta + 2Y^2\sin^2\theta + 10Y^2\cos\theta^2\theta + XY(6\cos^2\theta$
$- 6\sin^2\theta + 20\sin\theta\cos\theta - 4\cos\theta\sin\theta) + 6X^2\cos\theta\sin\theta - 6Y^2\sin^2\theta = 0$

$X^2(2\cos^2\theta + 10\sin^2\theta + 6\sin\theta\cos\theta) + XY(6\cos^2\theta - 6\sin^2\theta + 20\sin\theta\cos\theta$
$- 4\cos\theta\sin\theta + Y^2(2\sin^2\theta - 6\sin\theta\cos\theta + 10\cos^2\theta) - 11 = 0$ ...... (2)

To remove XY term, we put

$6\cos^2\theta - 6\sin^2\theta + 20\sin\theta\cos\theta - 4\sin\theta\cos\theta = 0$

$3\cos^2\theta - 3\sin^2\theta + 8\sin\theta\cos\theta = 0$

$3 - 3\tan^2\theta - 8\tan\theta = 0$ (Dividing throughout by $\cos^2\theta$)

$3\tan^2\theta + 8\tan\theta - 3 = 0$

$$\tan\theta = \frac{-8 \pm \sqrt{(8)^2 - 4(3)(-3)}}{2(3)} = \frac{8 \pm \sqrt{64 + 36}}{6}$$

$$= \frac{8 \pm \sqrt{100}}{6} = \frac{8 \pm 10}{6}$$

Either

$$\tan\theta = \frac{8 + 10}{6} = \frac{18}{6} \quad \text{or} \quad \tan\theta = \frac{8 - 10}{6} = \frac{-2}{6}$$

$$\tan\theta = 3 \ , \quad \tan\theta = \frac{-1}{3}$$

As θ is taken is 1^{st} quadrant so $\tan\theta = 3$ is only admissible value

$$\sec\theta = \sqrt{1 + \tan^2\theta} = \sqrt{1 + 9} = \sqrt{10}$$

$$\cos\theta = \frac{1}{\sqrt{10}}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{1}{10}} = \sqrt{\frac{10-1}{10}}$$

$$= \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$$

Put in (2)

$$X^2\left[2\left(\frac{1}{\sqrt{10}}\right)^2 + 6\frac{1}{\sqrt{10}}\frac{3}{\sqrt{10}} + 10\left(\frac{3}{\sqrt{10}}\right)^2\right] + XY(0) + Y^2$$

$$\left[2\left(\frac{3}{\sqrt{10}}\right)^2 - 6\frac{3}{\sqrt{10}}\frac{1}{\sqrt{10}} + 10\left(\frac{1}{\sqrt{10}}\right)^2\right] - 11 = 0$$

$$X^2\left[\frac{2}{10} + \frac{18}{10} + \frac{90}{10}\right] + Y^2\left[\frac{18}{10} - \frac{18}{10} + \frac{10}{10}\right] - 11 = 0$$

$$X^2\left[\frac{2+18+90}{10}\right] + Y^2\left[\frac{18-18+10}{10}\right] - 11 = 0$$

$11X^2 + Y^2 - 11 = 0$ Ans.

(ii) $\mathbf{xy + 4x - 3y - 10 = 0}$ **....... (1)**

We know that for rotation at axes

$x = X\cos\theta - Y\sin\theta$, $y = X\sin\theta + Y\cos\theta$ Put in (1)

$(X\cos\theta - Y\sin\theta)(X\sin\theta + Y\cos\theta) + 4(X\cos\theta - Y\sin\theta) - 3(X\sin\theta + Y\cos\theta) - 10 = 0$

$X^2\cos\theta\sin\theta + XY\cos^2\theta - XY\sin^2\theta - Y^2\sin\theta\cos\theta + 4X\cos\theta - 4Y\sin\theta - 3X\sin\theta - 3Y\cos\theta - 10 = 0$

$X^2\cos\theta\sin\theta - Y^2\sin\theta\cos\theta + XY(\cos^2\theta - \sin^2\theta) + X(4\cos\theta - 3\sin\theta) + Y(-4\sin\theta - 3\cos\theta) - 10 = 0$ (2)

To remove XY terms we put

$\cos^2\theta - \sin^2\theta = 0 \Rightarrow \cos 2\theta = 0$

=> $2\theta = \cos^{-1}(0) = 90^\circ$

=> $\theta = 45^\circ$ Put in (2)

$X^2\cos 45^\circ\sin 45^\circ - Y^2\sin 45^\circ\cos 45^\circ + XY(0) + X(4\cos 45^\circ - 3\sin 45^\circ) + Y(-4\sin 45^\circ - 3\cos 45^\circ) - 10 = 0$

$$X^2\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} - Y^2\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} + X\left(4\times\frac{1}{\sqrt{2}} - 3\times\frac{1}{\sqrt{2}}\right) + Y\left(-4\frac{1}{\sqrt{2}} - 3\frac{1}{\sqrt{2}}\right) - 10 = 0$$

$$\frac{1}{2}X^2 - \frac{1}{2}Y^2 + X\left(\frac{1}{\sqrt{2}}\right) + Y\left(\frac{-7}{\sqrt{2}}\right) - 10 = 0$$

$$X^2 - Y^2 + \frac{2}{\sqrt{2}} + X - \frac{14\,Y}{\sqrt{2}} - 20 = 0$$

$X^2 - Y^2 + \sqrt{2}\,X - 7\sqrt{2} - 20 \quad = \quad 0$ Ans.

(iii) $\mathbf{5x^2 - 6xy + 5y^2 - 8 = 0}$

Solution:

$5x^2 - 6xy + 5y^2 - 8 = 0 \qquad (1)$

We know that for rotation of axes

$x = X\cos\theta - Y\sin\theta \quad , \quad y = X\sin\theta + Y\cos\theta$ Put in (1)

$5(X\cos\theta - Y\sin\theta)^2 - 6(X\cos\theta - Y\sin\theta)(X\sin\theta + Y\cos\theta) + 5(X\sin\theta + Y\cos\theta)^2 - 8 = 0$

$5(X^2\cos^2\theta + Y^2\sin^2\theta - 2XY\sin\theta\cos\theta) - 6(X^2\cos\theta\sin\theta + XY\cos^2\theta - XY\sin^2\theta - Y^2\sin\theta\cos\theta) + 5(X^2\sin^2\theta + Y^2\cos^2\theta + 2XY\sin\theta\cos\theta) - 8 = 0$

$5X^2\cos^2\theta + 5Y^2\sin^2\theta - 10XY\sin\theta\cos\theta - 6X^2\cos\theta\sin\theta - 6XY\cos^2\theta + 6XY\sin^2\theta + 6Y^2\sin\theta\cos\theta + 5X^2\sin^2\theta + 5Y^2\cos^2\theta + 10XY\sin\theta\cos\theta - 8 = 0$

$X^2(5\cos^2\theta - 6\cos\theta\sin\theta + 5\sin^2\theta) + XY(-10\cos\theta\sin\theta - 6\cos^2\theta + 6\sin^2\theta + 10\sin\theta\cos\theta) + Y^2(5\sin^2\theta + 6\sin\theta\cos\theta + 5\cos^2\theta) - 8 = 0 \quad ......\ (2)$

To remove XY terms put (2)

$-6\cos^2\theta + 6\sin^2\theta = 0$

$\cos^2\theta = \sin^2\theta \quad \Rightarrow \quad \tan^2\theta = 1$

$\tan\theta = 1$ (θ is taken is 1st Quadrant)

$\theta = 45^\circ$ Put in (2)

$X^2(5(\cos 45^\circ)^2 - 6\cos 45^\circ\sin 45^\circ + 5(\sin 45^\circ)^2) + 0 + Y^2(5(\sin 45^\circ)^2 + 6\sin 45^\circ\cos 45^\circ + 5(\cos 45^\circ)^2) - 8 = 0$

$$X^2\left(5 + \left(\frac{1}{\sqrt{2}}\right)^2 - 6\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} + 5\left(\frac{1}{\sqrt{2}}\right)^2\right) + Y^2\left(5 + \left(\frac{1}{\sqrt{2}}\right)^2 + 6\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} + 5\left(\frac{1}{\sqrt{2}}\right)^2\right)$$

$$X^2\left(\frac{5}{2} - \frac{6}{2} + \frac{5}{2}\right) + Y^2\left(\frac{5}{2} + \frac{6}{2} + \frac{5}{2}\right) - 8 = 0$$

$2X^2 + 8Y^2 - 8 = 0$

$\Rightarrow \quad X^2 + 4Y^2 - 4 = 0$ Ans

EXERCISE 6.9

Q.1: By rotation of axes, eliminates the xy-term in each of the following equations. Identify the conic & find its elements.

(i) $\mathbf{4x^2 - 4xy + y^2 - 6 = 0}$

Solution:

$4x^2 - 4xy + y^2 - 6 = 0 \qquad (1)$

Here $a = 4 \qquad b = 1 \qquad 2h = -4$