Permutation containing $E_1 E_2 = {}^7P_7 = 7! = 5040$

Permutation containing $E_2 E_1 = {}^7P_7 = 7! = 5040$

Permutation in which English book are together $= 5040 + 5040 = 10080$

Total permutation of 8 books $= {}^8P_8 = 8! = 40320$

Permutation in which English books are not together $= 40320 - 10080 = 30240$

Q.13 Find the number of arrangements of 3 books on English and 5 books on Urdu for placing them on a shelf such that the books on the same subjects are together?

Solution:

Let E, U denote English and Urdu books.

Permutation in the form EEEEEUUU $= 5!\,3! = 720$

Permutation in the form UUUEEEEE $= 3!\,5! = 720$

Total permutation $= 720 + 720 = 1440$

Q.14 In how many ways can 5 boys and 4 girls be seated on a bench so that the girls & the boys occupy alternate seats?

Solution:

Let B, G denote boys and girls then

Permutation in the form BGBGBGBGB $= {}^5P_5\,{}^4P_4 = 5!\,4! = 120 \times 24 = 2880$

PERMUTATION OF THINGS NOT ALL DIFFERENT

If there are n_1 alike things of one kind, n_2 alike things of second kind and n_3 alike things of third kind, then the number of permutation of n things taken all at a time is given by

$$\frac{n!}{(n_1)! \times (n_2)! \times (n_3)!} = \binom{n}{n_1, n_2, n_3}$$

CIRCULAR PERMUTATION

The permutation of things which can be represented by the points on a circle are called circular permutation.

EXERCISE 7.3

Q.1 How many arrangements of the letters of the following words, taken all together can be made?

(i) PAKPATTAN (Lahore Board 2007) (ii) PAKISTAN

(iii) MATHEMATICS (Lahore Board 2004) (iv) ASSASSINATION

Solution:

(i) PAKPATTAN

$n = 9, \quad n_1 = 3\,(A), \quad n_2 = 2\,(P), \quad n_3 = 2\,(T)$

so using formula

$$\text{Permutation} = \binom{n}{n_1, n_2, n_3} = \frac{n!}{n_1!\,n_2!\,n_3!} = \frac{9!}{3!\,2!\,2!} = \frac{9.8.7.6.5.3!}{3!\,2.1\,.\,2.1} = 15120$$

(ii) PAKISTAN

Here $n = 8$

$n_1 = 2$ (A)

using formula

$$\text{Permutation} = \binom{n}{n_1, n_2, n_3} = \frac{n!}{n_1!\, n_2!\, n_3!} = \frac{8!}{2!} = \frac{8.7.6.5.4.3.2.1}{2.1} = 20160$$

(iii) MATHEMATICS

Here $n = 11$, $n_1 = 2$ (A), $n_2 = 2$ (M), $n_3 = 2$ (T)

So using formula

$$\text{Permutation} = \binom{n}{n_1, n_2, n_3} = \frac{n!}{n_1!\, n_2!\, n_3!} = \frac{11!}{2!\,2!\,2!} = 4989600$$

(iv) ASSASSINATION

Here $n = 13$, $n_1 = 4$ (S), $n_2 = 3$ (A), $n_3 = 2$ (N), $n_4 = 2$ (I)

So using formula

$$\text{Permutation} = \binom{n}{n_1, n_2, n_3, n_4} = \frac{n!}{n_1!\, n_2!\, n_3!\, n_4!} = \frac{13!}{4!\,3!\,2!\,2!} = 10810800$$

Q.2 How many permutation of the letters of the word PANAMA can be made, if P is to be the first letter in each arrangement? (Gujranwala Board 2007)

Solution:

If we fix P at first place then remaining number of letters $= n = 5$

with $n_1 = 3$ (A), $n_2 = 1$ (N), $n_3 = 1$ (M)

using formula

$$\text{Permutation} = \binom{n}{n_1, n_2, n_3} = \frac{n!}{n_1!\, n_2!\, n_3!} = \frac{5!}{3!\,1!\,1!} = \frac{5!}{3!} = \frac{5.4.3!}{3!} = 20$$

Q.3 How many arrangements of the letters of the word ATTACKED can be made, if each arrangement begins with C and ends with K?

Solution:

If we fix C at start and K at the end then we have to find permutation of remaining 6 letters.

$\Rightarrow$ $n = 6$ with $n_1 = 2$ (A), $n_2 = 2$ (T) using formula

$$\text{Permutation} = \binom{n}{n_1, n_2} = \frac{n!}{n_1!\, n_2!} = \frac{6!}{2!\,2!} = 180$$

Q.4 How many numbers greater than 1000000 can be formed from the digits 0, 2, 2, 2, 3, 4, 4?

Solution:

The numbers greater than 1000000 are of the forms:

$$2\square\square\square\square\square\square = \binom{6}{2,\,2} = \frac{6!}{2!\,2!} = 180$$

$$3\square\square\square\square\square\square = \binom{6}{3,\,2} = \frac{6!}{3!\,2!} = 60$$

$$4\square\square\square\square\square\square = \binom{6}{3} = \frac{6!}{3!} = 120$$

So required numbers $= 180 + 60 + 120 = 360$

Q.5 How many 6–digit numbers can be formed from the digits 2, 2, 3, 3, 4, 4? How many of them lie between 400000 and 430,000?

Solution:

$n = 6, \quad n_1 = 2\ (2), \quad n_2 = 2\ (3), \quad n_3 = 2\ (4)$

So using formula

$$\text{Required permutation} = \binom{n}{n_1,\, n_2,\, n_3} = \binom{6}{2,\,2,\,2} = \frac{6!}{2!\,2!\,2!} = 90$$

(ii) The number lying between 400,000 and 430,000 are of the form $42\square\square\square\square$

$$\text{so required arrangements} = \binom{4}{2,\,1,\,1} = \frac{4!}{2!\,1!\,1!} = \frac{4!}{2!} = 12$$

Q.6 11 members of a club form 4 committees of 3, 4, 2, 2 members so that so that no member is the member of more than one committee. Find the number of committees?

Solution:

Total members $= n = 11$

with $n_1 = 4, \quad n_2 = 3, \quad n_3 = 2, \quad n_4 = 2$

$$\text{So required number of committees} = \binom{11}{4,\,3,\,2,\,2} = \frac{11!}{4!\,3!\,2!\,2!} = 69300$$

Q.7 The D.C.O's of 11 districts meet to discuss the law and order situation in these districts. In how many ways can they be seated a round table, when two particular D.C.O's insist on sitting together?

Solution:

Consider the two D.C.O's as one man. Now the number of men $= 10$

and number of ways that 10 men can be seated at a round table $= 9!$ also the two particular D.C.O's can sit in two different ways so the total number of ways $= 9! \times 2 = 725760$

Q.8 The Governor of Punjab calls a meeting of 12 officers. In how many ways can they be seated at a round table?

Solution:

If we fix seat of one officer then there are 11 seats for 11 officers so

required no. of arrangements $= 11! = 39916800$

Q.9 Fatima invites 14 people to dinner. There are 9 males and 5 females who are seated at two different tables so that guests of one sex sit at one round table and the guests of the other sex at the second table. Find the number of ways in which all guest are seated?

Solution:

If we fix the seat of one male and one female then

the required number of arrangements $= 8! \times 4! = 967680$

Q.10 Find the number of ways in which 5 men and 5 women can be seated at a round table in such a way that no two persons of the same sex sit together?

Solution:

If we fix a seat of one man or one woman then the number of ways in which 5 males and 5 females can be seated $= 5! \times 4! = 2880$.

Q.11 In how many ways can 4 keys by arranged on a circular key ring?

Solution:

The number of keys $= 4$

The number of ways in which 4 keys can be arranged in a circular key ring $= \frac{1}{2} \times 3! = 3$

Q.12 How many necklaces can be made from 6 beads of different colours?

(Gujranwala Board 2007)

Solution:

The number of beads $= 6$

The number of ways in which 6 beads can be arranged in a necklace $= \frac{1}{2} \times 5! = 60$

COMBINATION

A group or a set in which the order is immaterial.

For example the combination of three letters A, B, C taken all at a time is just one set while the permutation of these three A, B, C taken all at a time are 6 i.e.

ABC, ACB, BAC, CBA, CAB, BCA

Then number of combinations of 'n' different objects taken r at a time is denoted by ^nC_r or $C(n, r)$ or $\binom{n}{r}$ and is given by

$$^nC_r = \frac{n!}{r!\,(n-r)!} \quad \text{when } r \leq n$$

Also $^nC_r = {^nC_{n-r}}$