Checking:

Put $x = \frac{-7+\sqrt{17}}{8}$ in equation (1)

$$\sqrt{5\left(\frac{-7+\sqrt{17}}{8}\right)^2 + 7\left(\frac{-7+\sqrt{17}}{8}\right) + 2} - \sqrt{4\left(\frac{-7+\sqrt{17}}{8}\right)^2 + 7\left(\frac{-7+\sqrt{17}}{8}\right) + 18} = \frac{-7+\sqrt{17}}{8} - 4$$

$$\sqrt{5\,(-0.3596)^2 + 7\,(-0.3596) + 2} - \sqrt{4\,(-0.3596)^2 + 7\,(-0.359) + 18} = -0.3596 - 4$$
$$= -4.3596$$

$$\sqrt{-1.8706 + 2} - \sqrt{-1.8706 + 18} = -4.3596$$

$$0.3597 - 4.0161 = -4.3596$$

$$-3.6564 = -4.3596$$

Put, $x = \frac{-7-\sqrt{17}}{8}$ in equation (i)

$$\sqrt{5\left(\frac{-7-\sqrt{17}}{8}\right)^2 + 7\left(\frac{-7-\sqrt{17}}{8}\right) + 2} - \sqrt{4\left(\frac{-7-\sqrt{17}}{8}\right)^2 + 7\left(\frac{-7-\sqrt{17}}{8}\right) + 18} = \frac{-7-\sqrt{17}}{8} - 4$$

$$\sqrt{5(-1.39)^2 + 7(-1.39) + 2} - \sqrt{4(-1.39)^2 + 7(-1.39) + 18} = -1.39 - 4$$

$$\sqrt{5(1.93) - 9.73 + 2} - \sqrt{4(1.93) - 9.73 + 18} = -5.39$$

$$\sqrt{1.92} - \sqrt{15.99} = -5.39$$

$$1.38 - 3.99 = -5.39$$

$$-2.61 = -5.39$$

$$\Rightarrow \quad \text{L.H.S} \neq \text{R.H.S}$$

$\Rightarrow \quad x = \frac{-7 \pm \sqrt{17}}{8}$ are extraneous roots

$$\Rightarrow \quad \text{S.S} = \phi$$

THREE CUBE ROOTS OF UNITY

As we know that square roots of one (unity) are two, 1 and – 1. Similarly cube roots of one (unity) are three and these can be calculated as:

Let ‘x’ be the cube root of unity, then

$$1^{1/3} = x$$

$$\Rightarrow \quad 1 = x^3 \quad \Rightarrow \quad x^3 = 1 \quad \Rightarrow \quad (x)^3 - (1)^3 = 0$$

$\Rightarrow \quad (x-1)(x^2+x+1) = 0$

$\Rightarrow \quad$ Either $x - 1 = 0$ or $x^2 + x + 1 = 0$

$\Rightarrow \quad x = 1$ or $x = \dfrac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

$$= \frac{-1 \pm \sqrt{1-4}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$\Rightarrow$ Cube roots of unity (one) are

$$1, \frac{-1+\sqrt{-3}}{2}, \frac{-1-\sqrt{-3}}{2}$$

or $1, \omega, \omega^2$

There are four properties of cube roots of unity.

(i) Each complex cube root of unity is square of the other.

i.e. if $\dfrac{-1+\sqrt{-3}}{2} = \omega$ then $\dfrac{-1-\sqrt{-3}}{2} = \omega^2$

and if $\dfrac{-1-\sqrt{-3}}{2} = \omega$ then $\dfrac{-1+\sqrt{-3}}{2} = \omega^2$

(ii) The sum of all the three cube roots of unity is zero.

i.e. $1 + \omega + \omega^2 = 0$

(iii) The product of all the three cube roots of unity is unity.

i.e. $1 . \omega . \omega^2 = 1 \Rightarrow \omega^3 = 1$

(iv) For any $n \in z$, ω^n is equivalent to one of the cube roots of unity.

e.g. $\omega^4 = \omega^3 . \omega = 1 . \omega = \omega$

$$\omega^{-5} = \frac{1}{\omega^5} = \frac{1}{\omega^3 . \omega^2} = \frac{1}{1 . \omega^2} = \frac{1}{\omega^2} . \frac{\omega}{\omega} = \frac{\omega}{\omega^3} = \frac{\omega}{1} = \omega$$

FOUR FOURTH ROOTS OF UNITY

Let ‘x’ be the fourth root of unity then

$1^{1/4} = x \Rightarrow x^4 = 1$

$\Rightarrow \quad x^4 - 1 = 0$

$\Rightarrow \quad (x^2)^2 - (1)^2 = 0$

$\Rightarrow \quad (x^2 + 1)(x^2 - 1) = 0$

$\Rightarrow \quad$ Either $x^2 + 1 = 0$ or $x^2 - 1 = 0$

$\Rightarrow \quad x^2 = -1$ or $x^2 = 1$

$\Rightarrow \quad x = \pm\sqrt{-1}$ or $x = \pm\sqrt{1}$

$\Rightarrow \quad x = \pm i$ or $x = \pm 1$

$\Rightarrow$ Four fourth roots of unity are $1, -1, i, -i$.

PROPERTIES OF FOUR FOURTH ROOTS OF UNITY

(i) Sum of all the four fourth roots of unity is zero.

i.e. $1 - 1 + i - i = 0$

(ii) The real fourth roots of unity are additive inverses of each other.

$+1$ and -1 are real fourth roots of unity and $+1 + (-1) = 0 = (-1) + 1$

(iii) Both the complex fourth roots of unity are conjugate of each other.

i and $-i$ are complex fourth roots of unity which are conjugate of each other.

(iv) Product of all the fourth roots of unity is -1.

i.e. $1 \times (-1) \times i \times (-i) = -1$

EXERCISE 4.4

Q.1 Find the three cube roots of: 8, – 8, 27, – 27, 64

Solution:

(i) Cube root of 8.

Let 'x' be the cube root of 8.

$\Rightarrow \quad 8^{1/3} = x$

$\Rightarrow \quad 8 = x^3 \quad \Rightarrow \quad x^3 = 8$

$\Rightarrow \quad x^3 - 8 = 0$

$\Rightarrow \quad (x)^3 - (2)^3 = 0$

$\Rightarrow \quad (x - 2)(x^2 + 2x + 4) = 0$

$\Rightarrow \quad$ Either $x - 2 = 0$ or $x^2 + 2x + 4 = 0$

$\Rightarrow \quad x = 2$ or $x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$

$$= \frac{-2 \pm \sqrt{4 - 16}}{2}$$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm 2\sqrt{-3}}{2}$$

$$= 2\left(\frac{-1 \pm \sqrt{-3}}{2}\right)$$

$$= 2\omega,\ 2\omega^2$$

$\Rightarrow$ cube roots of 8 are $2, 2\omega, 2\omega^2$.

(ii) Cube root of -8.

Let 'x' be the cube root of -8.

$\Rightarrow \quad (-8)^{1/3} = x$

$\Rightarrow \quad -8 = x^3 \quad \Rightarrow \quad x^3 = 8 = 0$

$\Rightarrow \quad (x)^3 + (2)^3 = 0$

$\Rightarrow \quad (x + 2)(x^2 - 2x + 4) = 0$

$\Rightarrow$ Either $x + 2 = 0$ or $x^2 - 2x + 4 = 0$

$\Rightarrow \quad x = -2$ or $x = \dfrac{-(-2) \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$

$$= \frac{+2 \pm \sqrt{4 - 16}}{2}$$

$$= \frac{2 \pm \sqrt{-12}}{2}$$

$$= \frac{2 \pm 2\sqrt{-3}}{2}$$

$$= -2\left(\frac{-2 \pm \sqrt{-3}}{2}\right)$$

$$= -2\omega, \ -2\omega^2$$

$\Rightarrow$ cube roots of -8 are $-2, -2\omega, -2\omega^2$.

(iii) Cube root of 27.

Let 'x' be the cube root of 27.

$\Rightarrow \quad (27)^{1/3} = x \quad \Rightarrow \quad x^3 = 27$

$\Rightarrow \quad x^3 - 27 = 0$

$\Rightarrow \quad (x)^3 - (3)^3 = 0$

$\Rightarrow \quad (x - 3)(x^2 + 3x + 9) = 0$

$\Rightarrow$ Either $x - 3 = 0$ or $x^2 + 3x + 9 = 0$

$\Rightarrow \quad x = 3$ or $x = \dfrac{-3 \pm \sqrt{(3)^2 - 4(1)(9)}}{2(1)}$

$$= \frac{-3 \pm \sqrt{9 - 36}}{2}$$

$$= \frac{-3 \pm \sqrt{-27}}{2}$$

$$= \frac{-3 \pm 3\sqrt{-3}}{2}$$

$$= 3\left(\frac{-1 \pm \sqrt{-3}}{2}\right)$$

$$= 3\omega, \ 3\omega^2$$

$\Rightarrow$ cube roots of 27 are $3, 3\omega, 3\omega^2$.

(iv) Cube root of – 27.

Let 'x' be the cube root of – 27.

$\Rightarrow \quad (-27)^{1/3} = x \quad \Rightarrow \quad -27 = x^3$

$\Rightarrow \quad x^3 + 27 = 0$

$\Rightarrow \quad (x)^3 + (3)^3 = 0$

$\Rightarrow \quad (x+3)(x^2 - 3x + 9) = 0$

$\Rightarrow$ Either $x + 3 = 0$ or $x^2 - 3x + 9 = 0$

$$\Rightarrow \quad x = -3 \quad \text{or} \quad x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9 - 36}}{2}$$

$$= \frac{3 \pm \sqrt{-27}}{2}$$

$$= \frac{3 \pm 3\sqrt{-3}}{2}$$

$$= -3\left(\frac{-1 \pm \sqrt{-3}}{2}\right)$$

$$= -3\omega, \ -3\omega^2$$

$\Rightarrow$ cube roots of -27 are $-3, -3\omega, -3\omega^2$.

(v) Cube root of 64.

Let 'x' be the cube root of 64.

$\Rightarrow \quad (64)^{1/3} = x \quad \Rightarrow \quad 64 = x^3$

$\Rightarrow \quad x^3 - 64 = 0$

$\Rightarrow \quad (x)^3 - (4)^3 = 0$

$\Rightarrow \quad (x-4)(x^2 + 4x + 16) = 0$

$\Rightarrow$ Either $x - 4 = 0$ or $x^2 + 4x + 16 = 0$

$$\Rightarrow \quad x = 4 \quad \text{or} \quad x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(16)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16 - 64}}{2}$$

$$= \frac{-4 \pm \sqrt{-48}}{2}$$

$$= \frac{-4 \pm 4\sqrt{-3}}{2}$$

$$= 4\left(\frac{-1 \pm \sqrt{-3}}{2}\right)$$

$$= 4\omega, \ 4\omega^2$$

$\Rightarrow$ cube roots of 64 are $4, 4\omega, 4\omega^2$.

Q.2 Evaluate:

Solution:

(i) **(Lahore Board 2008, 2011)**

$$
\begin{aligned}
(1+\omega-\omega^2)^8 &= (-\omega^2-\omega^2)^8 \\
&= (-2\omega^2)^8 && \because\ 1+\omega+\omega^2 = 0 \\
&= (-2)^8\,(\omega^2)^8 \\
&= 256\,\omega^{16} = 256\,\omega^{15}\,.\,\omega && \because\ \omega^3 = 1 \\
&= 256\,(\omega^3)^5\,.\,\omega = 256\,(1)^5\,.\,\omega = 256\,\omega
\end{aligned}
$$

(ii) **(Lahore Board 2004, 2006, 2007)**

$$
\begin{aligned}
\omega^{28}+\omega^{29}+1 &= \omega^{27}\,.\,\omega+\omega^{27}\,.\,\omega^2+1 \\
&= (\omega^3)^9\,.\,\omega+(\omega^3)^9\,.\,\omega^2+1 \\
&= (1)^9\,.\,\omega+(1)^9\,.\,\omega^2+1 && \because\ \omega^3 = 1 \\
&= \omega+\omega^2+1 = 0
\end{aligned}
$$

(iii) **(Lahore Board 2006)**

$$
\begin{aligned}
(1+\omega-\omega^2)(1-\omega+\omega^2) &= 1-\omega+\omega^2+\omega-\omega^2+\omega^3-\omega^2+\omega^3-\omega^4 \\
&= 1-\omega^2+\omega^3+\omega^3-\omega^4 \\
&= 1-\omega^2+1+1-\omega^3\,.\,\omega && \because\ 1+\omega+\omega^2 = 0 \\
&= 3-\omega^2-(1)\,.\,\omega && \omega+\omega^2 = -1 \\
&= 3-\omega^2-\omega \\
&= 3-(\omega^2+\omega) = 3-(-1) = 3+1 = 4
\end{aligned}
$$

(iv)

$$
\begin{aligned}
\left(\frac{-1+\sqrt{-3}}{2}\right)^9+\left(\frac{-1-\sqrt{-3}}{2}\right)^7 &= \omega^9+(\omega^2)^7 \\
&= (\omega^3)^3+\omega^{14} = (1)^3+\omega^{12}\,.\,\omega^2 \\
&= 1+(\omega^3)^4\,.\,\omega^2 \\
&= 1+(1)^4\,.\,\omega^2 \\
&= 1+\omega^2 && \because\ 1+\omega+\omega^2 = 0 \\
&= -\omega && 1+\omega^2 = -\omega
\end{aligned}
$$

(v)

$$
\begin{aligned}
\left(-1+\sqrt{-3}\right)^5+\left(-1-\sqrt{-3}\right)^5 &= \left(2\,.\,\frac{-1+\sqrt{-3}}{2}\right)^5+\left(2\,.\,\frac{-1-\sqrt{-3}}{2}\right)^5 \\
&= (2\omega)^5+(2\omega^2)^5 \\
&= (2)^5\,.\,\omega^5+2^5\,.\,(\omega^2)^5 \\
&= 32\,.\,\omega^3\,.\,\omega^2+32\,.\,\omega^{10} \\
&= 32\,.\,(1)\,.\,\omega^2+32\,.\,\omega^9\,.\,\omega \\
&= 32\,\omega^2+32\,.\,(\omega^3)^3\,.\,\omega \\
&= 32\,\omega^2+32\,(1)^3\,.\,\omega \\
&= 32\,\omega^2+32\,\omega && \because\ 1+\omega+\omega^2 = 0 \\
&= 32\,(\omega^2+\omega) && \Rightarrow\ \omega^2+\omega = -1 \\
&= 32\,(-1) = -32
\end{aligned}
$$

Q.3 Show that

Solution:

(i) $x^3 - y^3 = (x - y)(x - \omega y)(x - \omega^2 y)$ **(Gujranwala Board 2004)**

$$\begin{aligned}
\text{R.H.S.} &= (x - y)(x - \omega y)(x - \omega^2 y)\\
&= (x - y)(x^2 - x y \omega^2 - x y \omega + y^2 \omega^3)\\
&= (x - y)(x^2 - x y \omega^2 - x y \omega + y^2) && ¤\ \omega^3 = 1\\
&= (x - y)[x^2 - xy(\omega^2 + \omega) + y^2]\\
&= (x - y)[x^2 - xy(-1) + y^2] && ¤\ 1 + \omega + \omega^2 = 0\\
&= (x - y)(x^2 + xy + y^2) && \omega + \omega^2 = -1\\
&= x^3 - y^3 = \text{L.H.S.}
\end{aligned}$$

Hence proved.

(ii) $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x + \omega y + \omega^2 y)(x + \omega^2 y + \omega z)$ **(Lahore Board 2004)**

$(x + y + z)(x + \omega y + \omega^2 y)(x + \omega^2 y + \omega z)$

$= (x + y + z)\left[x^2 + xy\omega^2 + xz\omega + xy\omega + y^2\omega^3 + yz\omega^2 + xz\omega^2 + yz\omega^4 + z^2\omega^3\right]$

$= (x + y + z)\left[x^2 + y^2\omega^3 + z^2\omega^3 + xy\omega^2 + xy\omega + yz\omega^2 + yz\omega^4 + yz\omega^4 + xz\omega + xz\omega^2\right]$

$= (x + y + z)\left[x^2 + y^2 + z^2 + xy(\omega^2 + \omega) + yz(\omega^2 + \omega^4) + xz(\omega + \omega^2)\right]$

$= (x + y + z)\left[x^2 + y^2 + z^2 + xy(-1) + yz(\omega^2 + \omega) + xz(-1)\right]$ ¤ $\omega^4 = \omega^3 . \omega = \omega$

$= (x + y + z)\left[x^2 + y^2 + z^2 - xy + yz(-1) - xz\right]$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= x^3 + y^3 + z^3 - 3xyz = \text{R.H.S.} \Rightarrow$ Hence proved.

(iii) $(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8) \ldots\ldots 2n$ factors $= 1$ **(Lahore Board 2006)**

$$\begin{aligned}
\text{L.H.S.} &= (1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8) \ldots.\ 2n \text{ factors.}\\
&= (1 + \omega)(1 + \omega^2)(1 + \omega^3 . \omega)(1 + \omega^6 . \omega^2) \ldots.\ 2n \text{ factors.}\\
&= (-\omega^2)(-\omega)(-\omega^2)(-\omega) \ldots.\ 2n \text{ factors}\\
&= \left[(-\omega^2)(-\omega)\right]\left[(-\omega^2)(-\omega)\right] \ldots.\ n \text{ factors}\\
&= \left[\omega^3\right]\left[\omega^3\right] \ldots.\ n \text{ factors}\\
&= 1 . 1 \ldots.\ n \text{ factors}\\
&= (1)^n = 1 = \text{R.H.S.}
\end{aligned}$$

Hence the proof.

Q.4 If ω is a root of $x^2 + x + 1 = 0$ show that its other root is ω^2 and prove that $\omega^3 = 1$.

Solution:

If 'ω' is a root of $x^2 + x + 1 = 0$

$\Rightarrow$ It will satisfy the given equation.

$\Rightarrow$ $\omega^2 + \omega + 1 = 0$ .... (1)

To prove ω^2 is also its root.

We will prove that

$(\omega^2)^2 + \omega^2 + 1 = 0$

i.e. $\omega^4 + \omega^2 + 1 = 0$ .... (2)

Take L.H.S.

$$\begin{aligned}
\Rightarrow \quad \omega^4 + \omega^2 + 1 &= (\omega^2)^2 + \omega^2 + \omega^2 - \omega^2 + 1 \\
&= (\omega^2)^2 + 2\,\omega^2 + (1)^2 - \omega^2 \\
&= (\omega^2 + 1)^2 - \omega^2 \\
&= (\omega^2 + 1 - \omega^2)\,(\omega^2 + 1 + \omega) \qquad \because \text{ from equation (1)} \\
&= (\omega^2 + 1 - \omega^2)\,(0) \\
&= 0 = \text{R.H.S.}
\end{aligned}$$

$\Rightarrow$ ω^2 is also root of $x^2 + x + 1$.

Now to prove $\omega^3 = 1$.

Subtracting equation (1) from equation (2)

$\omega^4 + \omega^2 + 1 - \omega^2 - \omega - 1 = 0$

$\Rightarrow$ $\omega^4 - \omega = 0$

$\Rightarrow$ $\omega\,(\omega^3 - 1) = 0$

As $\omega \neq 0 \quad \Rightarrow \quad \omega^3 - 1 = 0 \quad \Rightarrow \quad \omega^3 = 1$

Hence proved.

Q.5 Prove that complex cube roots of -1 are $\dfrac{1 + \sqrt{3}\,i}{2}$ and $\dfrac{1 - \sqrt{3}\,i}{2}$ and hence prove that $\left(\dfrac{1 + \sqrt{-3}}{2}\right)^9 + \left(\dfrac{1 - \sqrt{-3}}{2}\right)^9 = -2$.

Solution:

Let 'x' be the cube root of -1 then $(-1)^{1/3} = x \;\Rightarrow\; x^3 = -1$.

$\Rightarrow$ $x^3 + 1 = 0$

$\Rightarrow$ $(x + 1)\,(x^2 - x + 1) = 0$

$\Rightarrow$ Either $x + 1 = 0$ or $x^2 - x + 1 = 0$

$\Rightarrow$ $x = -1$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4\,(1)\,(1)}}{2\,(1)}$$

$$= \frac{1 \pm \sqrt{1 - 4}}{2}$$

$$= \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm \sqrt{3}\,i}{2}$$

$\Rightarrow$ cube roots of -1 are -1, $\dfrac{1 \pm \sqrt{3}\,i}{2}$

$\Rightarrow$ complex cube roots of -1 are $\dfrac{1 \pm \sqrt{3}\,i}{2}$ and $\dfrac{1 - \sqrt{3}\,i}{2}$.

Now to prove

$$\left(\frac{1+\sqrt{-3}}{2}\right)^9 + \left(\frac{1-\sqrt{-3}}{2}\right)^9 = -2$$

Take L.H.S.

$$\left(\frac{1+\sqrt{-3}}{2}\right)^9 + \left(\frac{1-\sqrt{-3}}{2}\right)^9 = \left[-\left(\frac{-1-\sqrt{-3}}{2}\right)\right]^9 + \left[-\left(\frac{-1+\sqrt{-3}}{2}\right)\right]^9$$

$$= (-\omega^2)^9 + (-\omega)^9$$

$$= -\omega^{18} - \omega^9 = -(\omega^3)^6 - (\omega^3)^3$$

$$= -(1)^6 - (1)^3$$

$$= -1 - 1 = -2 = \text{R.H.S.}$$

$\Rightarrow$ Hence proved.

Q.6 It 'ω' is a cube root of unity, form an equation whose roots are 2ω and $2\omega^2$. (Lahore Board 2003)

Solution:

Let the required equation is

$x^2 - Sx + P = 0$ ……… (1)

where S = Sum of roots

$= 2\,\omega + 2\,\omega^2 = 2\,(\omega + \omega^2) = 2\,(-1) = -2$ ¤ $1 + \omega + \omega^2 = 0$

$\omega + \omega^2 = -1$

P = Product of roots

$= 2\,\omega \,.\, 2\,\omega^2 = 4\,\omega^3 = 4\,(1) = 4$ ¤ $\omega^3 = 1$

$\Rightarrow$ equation (1) becomes

$x^2 - (-2)\,x + 4 = 0$

$\Rightarrow$ $x^2 + 2x + 4 = 0$

Q.7 Find the four fourth roots of 16, 81; 625.

Solution:

(i) Fourth roots of 16

Let 'x' be the fourth root of 16

$\Rightarrow \quad (16)^{1/4} = x \quad \Rightarrow \quad x^4 = 16$

$\Rightarrow \quad x^4 - 16 = 0$

$\Rightarrow \quad (x^2)^2 - (4)^2 = 0$

$\Rightarrow \quad (x^2 + 4)(x^2 - 4) = 0$

$\Rightarrow$ Either $x^2 + 4 = 0$ or $x^2 - 4 = 0$

$\Rightarrow \quad x^2 = -4 \qquad x^2 = 4$

$\Rightarrow \quad x = \pm\sqrt{-4} \qquad x = \pm\sqrt{4}$

$\Rightarrow \quad x = \pm 2i \qquad x = \pm 2$

$\Rightarrow$ four fourth roots of 16 are $-2, 2, -2i, 2i$.

(ii) Fourth roots of 81

Let 'x' be the fourth root of 81

$\Rightarrow \quad (81)^{1/4} = x \quad \Rightarrow \quad x^4 = 81$

$\Rightarrow \quad x^4 - 81 = 0$

$\Rightarrow \quad (x^2)^2 - (9)^2 = 0$

$\Rightarrow \quad (x^2 + 9)(x^2 - 9) = 0$

$\Rightarrow$ Either $x^2 + 9 = 0$ or $x^2 - 9 = 0$

$\Rightarrow \quad x^2 = -9 \qquad x^2 = 9$

$\Rightarrow \quad x = \pm\sqrt{-9} \qquad x = \pm\sqrt{9}$

$\Rightarrow \quad x = \pm 3i \qquad x = \pm 3$

$\Rightarrow$ four fourth roots of 81 are $3, -3, 3i, -3i$.

(iii) Fourth roots of 625

Let 'x' be the fourth root of 625

$\Rightarrow \quad (625)^{1/4} = x \quad \Rightarrow \quad x^4 = 625$

$\Rightarrow \quad x^4 - 625 = 0$

$\Rightarrow \quad (x^2)^2 - (25)^2 = 0$

$\Rightarrow \quad (x^2 + 25)(x^2 - 25) = 0$

$\Rightarrow$ Either $x^2 + 25 = 0$ or $x^2 - 25 = 0$

$\Rightarrow \quad x^2 = -25 \qquad x^2 = 25$

$\Rightarrow \quad x = \pm\sqrt{-25} \qquad x = \pm\sqrt{25}$

$\Rightarrow \quad x = \pm 5\,i \qquad x = \pm 5$

$\Rightarrow$ four fourth roots of 625 are $5, -5, 5i, -5i$.

Q.8 Solve the following equation:

Solution:

(i) $\mathbf{2x^4 - 32 = 0}$

$2(x^4 - 16) = 0$

$\Rightarrow \quad x^4 - 16 = 0$

$\Rightarrow \quad (x^2)^2 - (4)^2 = 0$

$\Rightarrow \quad (x^2 + 4)(x^2 - 4) = 0$

$\Rightarrow$ Either $x^2 + 4 = 0$ or $x^2 - 4 = 0$

$\Rightarrow \quad x^2 = -4 \qquad x^2 = 4$

$\Rightarrow \quad x = \pm\sqrt{-4} \qquad x = \pm\sqrt{4}$

$\Rightarrow \quad x = \pm 2\,i \qquad x = \pm 2$

$\Rightarrow \quad x = \pm 2, \pm 2i$

Hence the solution set $= \{\pm 2, \pm 2i\}$

(ii) $\mathbf{3y^5 - 243y = 0}$

$3y^5 - 243y = 0$

$3y(y^4 - 81) = 0$

$\Rightarrow$ Either $y = 0$ or $y^4 - 81 = 0$

$\Rightarrow \quad y = 0 \qquad \Rightarrow \quad (y^2)^2 - (9)^2 = 4$

$\Rightarrow \quad (y^2 + 9)(y^2 - 9) = 0$

$\Rightarrow$ Either $y^2 + 9 = 0$ or $y^2 - 9 = 0$

$\Rightarrow \quad y^2 = -9 \qquad y^2 = +9$

$\Rightarrow \quad y = \pm\sqrt{-9} \qquad y = \pm\sqrt{9}$

$\Rightarrow \quad y = \pm 3i \qquad y = \pm 3$

$\Rightarrow \quad y = 0, \pm 3, \pm 3i$

Hence the solution set $= \{0, \pm 3, \pm 3i\}$

(iii) $\mathbf{x^3 + x^2 + x + 1 = 0}$

$x^3 + x^2 + x + 1 = 0$

$\Rightarrow \quad x^2(x + 1) + 1(x + 1) = 0$

$\Rightarrow \quad (x + 1)(x^2 + 1) = 0$

$\Rightarrow$ Either $x + 1 = 0$ or $x^2 + 1 = 0$

$x = -1 \qquad x^2 = -1$

$x = \pm\sqrt{-1}$

$x = \pm i$

Hence the solution set $= \{-1, \pm i\}$

(iv) $\mathbf{5x^5 - 5x = 0}$

$5x^5 - 5x = 0$

$\Rightarrow \quad 5x(x^4 - 1) = 0$

$\Rightarrow$ Either $x = 0$ or $x^4 - 1 = 0$

$\Rightarrow \quad x = 0$ or $(x^2)^2 - (1)^2 = 0$

$(x^2 + 1)(x^2 - 1) = 0$

Either $x^2 + 1 = 0$ or $x^2 - 1 = 0$

$\Rightarrow \quad x^2 = -1$ or $x^2 = 1$

$\Rightarrow \quad x = \pm\sqrt{-1}$ or $x = \pm\sqrt{1}$

$\Rightarrow \quad x = \pm i$ or $x = \pm 1$

Hence the solution set $= \{0, \pm 1, \pm i\}$

POLYNOMIAL FUNCTION

A polynomial in 'x' is an expression of the form $a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$, $a_n \neq 0$ where 'n' is a non–negative integer and the coefficients $a_n, a_{n-1}, \ldots a_1$ and a_0 are real numbers. The highest power of 'x' is called the degree of the polynomial. So the above expression is a polynomial of degree n.

REMAINDER THEOREM

If a polynomial $f(x)$ of degree $n \geq 1$, 'n' is non–negative integer is divided by $x - a$ till no x–term exists in the remainder, then $f(a)$ is the remainder i.e. $f(a) = R$.

FACTOR THEOREM

The polynomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$.