(i) Number of eggs that will be broken out of 7000 $= 7000 \times \frac{9}{7}\%$

$$= 7000 \times \frac{9}{7} \times \frac{1}{100} = 90$$

(ii) Number of eggs that will be broken out of 8400 $= 8400 \times \frac{9}{7}\%$

$$= 8400 \times \frac{9}{7} \times \frac{1}{100} = 108$$

(iii) Number of eggs that will be broken out of 10500 $= 10500 \times \frac{9}{7}\%$

$$= 10500 \times \frac{9}{7} \times \frac{1}{100} = 135$$

MUTUALLY EXCLUSIVE EVENTS

If a sample space $S = \{1, 3, 5, 7, 9\}$ and an event $A = \{1, 3, 5\}$ and another event $B = \{9\}$, then A and B are disjoint sets and they are said to be mutually exclusive events.

EQUALLY LIKELY EVENTS

If two events A and B occur in an experiment then A and B are said to be equally likely events if each one of them has equal number of chances of occurrence.

ADDITION OF PROBABILITIES

If A and B are two events, then the formulas for the addition of probabilities are:

(i) $P(A \cup B) = P(A) + P(B)$, when A and B are disjoint.

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ when A and B are overlapping or $B \subseteq A$.

EXERCISE 7.7

Q.1 If sample spaces = {1, 2, 3, ... 9}, Event A = {2, 4, 6, 8} and Event B = {1, 3, 5} find P (A∪B)

Solution:

Here $S = \{1, 2, 3, \ldots\ldots 9\} \Rightarrow n(S) = 9$

$A = \{2, 4, 6, 8\} \Rightarrow n(A) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{9}$$

Also $B = \{1, 3, 5\} \Rightarrow n(B) = 3$

$$\Rightarrow P(B) = \frac{n(B)}{n(S)} = \frac{3}{9} = \frac{1}{3}$$

As A and B are disjoint or mutually exclusive events. So

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{4}{9} + \frac{1}{3} = \frac{4+3}{9} = \frac{7}{9}$$

Q.2 A box contains 10 red, 30 white, and 20 black marbles. A marble is drawn at random. Find the probability that it is either red or white.

Solution:

Total marbles $= 10 + 30 + 20 = 60$

$\Rightarrow$ $n(S) = 60$

Let A = Event: the drawing marble is red

$\Rightarrow$ $n(A) = 10$

So $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{10}{60} = \dfrac{1}{6}$

Let B = Event: The drawing marble is white

$\Rightarrow$ $n(B) = 30$

So $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{30}{60} = \dfrac{1}{2}$

As A and B are disjoint or mutually exclusive events.

So
$$P(A \cup B) = P(A) + P(B)$$
$$= \frac{1}{6} + \frac{1}{2} = \frac{1+3}{6} = \frac{4}{6} = \frac{2}{3}$$

Q.3 A natural number is chosen out of the first fifty natural numbers. What is the probability that the chosen numbers is a multiple of 3 or of 5?

Solution:

$S = \{1, 2, 3, \ldots\ldots., 50\}$ $\Rightarrow$ $n(S) = 50$

Let A = Event: the chosen number is a multiple of 3

$= \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48\}$

$\Rightarrow$ $n(A) = 16$

So $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{16}{50} = \dfrac{8}{25}$

Let B = Event: The chosen number is a multiple of 5

$= \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$

$\Rightarrow$ $n(B) = 10$

So $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{10}{50} = \dfrac{1}{5}$

Now $A \cap B = \{15, 30, 45\}$ $\Rightarrow$ $n(A \cap B) = 3$

So $P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{3}{50}$

As A and B are not mutually exclusive events. So
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{8}{25} + \frac{1}{5} - \frac{3}{50} = \frac{16 + 10 - 3}{50} = \frac{23}{50}$$

Q.4 A card is drawn from a deck of 52 playing cards. What is the probability that it is a diamond card or an ace?

Solution:

Total cards = 52 $\Rightarrow$ n (S) = 52

Let A = Event: the drawing card is a diamond card

$\Rightarrow$ n (A) = B

so $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{13}{52}$

Let B = Event: the drawing card is an ace card

$\Rightarrow$ n (B) = 4

so $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{4}{52}$

Now $A \cap B$ = The ace of diamond

$\Rightarrow$ $n(A\cap B) = 1$

$\Rightarrow$ $P(A\cap B) = \dfrac{n(A\cap B)}{n(S)} = \dfrac{1}{52}$

As A and B are not mutually exclusive events.

$\Rightarrow$ $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

Q.5 A die is thrown twice. What is the probability that the sum of the number of dots shown is 3 or 11?

Solution:

When two die is thrown twice, the possible outcomes are

$$S = \left\{ \begin{array}{llllll} (1, 1), & (1, 2), & (1, 3), & (1, 4), & (1, 5), & (1, 6) \\ (2, 1), & (2, 2), & (2, 3), & (2, 4), & (2, 5), & (2, 6) \\ (3, 1), & (3, 2), & (3, 3), & (3, 4), & (3, 5), & (3, 6) \\ (4, 1), & (4, 2), & (4, 3), & (4, 4), & (4, 5), & (4, 6) \\ (5, 1), & (5, 2), & (5, 3), & (5, 4), & (5, 5), & (5, 6) \\ (6, 1), & (6, 2), & (6, 3), & (6, 4), & (6, 5), & (6, 6) \end{array} \right\}$$

$\Rightarrow$ n (S) = 36

Let A = Event: sum of the number of dots is 3

$= \{(1, 2), (2, 1)\}$

$\Rightarrow$ n (A) = 2

So $\quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{2}{36} = \dfrac{1}{18}$

Let $\quad B$ = Event: sum of the number of dots is 11

$= \{(5, 6), (6, 5)\}$

$\Rightarrow \quad n(B) = 2$

so $\quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{2}{36} = \dfrac{1}{18}$

As A and B are mutually exclusive events so

$$P(A \cup B) = P(A) + P(B)$$
$$= \frac{1}{18} + \frac{1}{18} = \frac{2}{18} = \frac{1}{9}$$

Q.6 Two dice are thrown. What is the probability that the sum of the number of dots appearing on them is 4 or 6?

Solution:

When two dice are thrown, the possible outcomes are

$$S = \begin{Bmatrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{Bmatrix}$$

$\Rightarrow \quad n(S) = 36$

Let $\quad A$ = Event: sum of the number of dots is 4

$= \{(1, 3), (2, 2), (3, 1)\}$

$\Rightarrow \quad n(A) = 3$

So $\quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{3}{36} = \dfrac{1}{12}$

Let $\quad B$ = Event: sum of the number of dots is 6

$= \{(1, 5), (2, 4), (3,3), (4, 2), (5, 1)\}$

$\Rightarrow \quad n(B) = 5$

so $\quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{5}{36}$

As A and B are mutually exclusive events so

$$P(A \cup B) = P(A) + P(B)$$
$$= \frac{1}{12} + \frac{5}{36} = \frac{3+5}{36} = \frac{8}{36} = \frac{2}{9}$$

Q.7 Two dice are thrown simultaneously. If the event A is that the sum of the number of dots shown is an odd number and the event B is that the number of dots shown on at least one die is 3. Find P (A∪B).

Solution:

When two dice are thrown, then possible outcomes are

$$S = \left\{\begin{array}{cccccc} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{array}\right\}$$

$\Rightarrow$ $n(S) = 36$

Let A = Event: sum of the number of dots is odd

$= \{(1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (3, 6), (4, 1), (4, 3), (4, 5)$
$(5, 2), (5, 4), (5, 6), (6, 1), (6, 3), (6, 5)\}$

$\Rightarrow$ $n(A) = 18$

So $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{18}{36} = \dfrac{1}{2}$

Let B = Event: number of dots shown on at least one die is 3

$= \{(1, 3), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 3), (5, 3), (6, 3)\}$

$\Rightarrow$ $n(B) = 11$

so $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{11}{36}$

Now $A \cap B = \{(2, 3), (3, 2), (3, 4), (3, 6), (4, 3), (6, 3)\}$

$n(A \cap B) = 6$

So $P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{6}{36} = \dfrac{1}{6}$

As A and B are not mutually exclusive events so

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{11}{36} - \frac{1}{6} = \frac{18 + 11 - 6}{36} = \frac{23}{36}$$

Q.8 There are 10 girls and 20 boys in a class. Half of the boys and half of the girls have blue eyes. Find the probability that that one student chosen as monitor is either a girl or has blue eyes.

Solution:

Total number of students $= 10 + 20 = 30$

$\Rightarrow$ $n(S) = 30$

Let A = Event: the chosen student is a girl

$n(A) = 10$

So $P(A) = \frac{n(A)}{n(S)} = \frac{10}{30} = \frac{1}{3}$

Let B = Event: the chosen student has blue eyes

$\Rightarrow$ $n(B) = 15$

so $P(B) = \frac{n(B)}{n(S)} = \frac{15}{30} = \frac{1}{2}$

Let $A\cap B$ = The chosen student is a girl and has blue eyes

$n(A\cap B) = 5$

So $P(A\cap B) = \frac{n(A\cap B)}{n(S)} = \frac{5}{30} = \frac{1}{6}$

As A and B are not mutually exclusive events

So $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$$= \frac{1}{3} + \frac{1}{2} - \frac{1}{6}$$

$$= \frac{2+3-1}{6} = \frac{4}{6} = \frac{2}{3}$$

MULTIPLICATION OF PROBABILITIES

Two events A and B are said to be independent, if the occurrence of any one of them does not influence the occurrence of the other event.

THEOREM

If A and B are two independent events, the probability that both of them occur is equal to the probability of the occurrence of A multiplied by the probability of the occurrence of B. Symbolically, it is denoted as:

$P(A\cap B) = P(A) \cdot P(B)$