$f(61^o) \approx 0.8660 + 0.0087$

$\text{Sin}61^o \approx 0.8747$ Ans.

Q.4 Find the approximate increase in the volume of a cube if the length of its each edge changes from 5 to 5.02

Solution:

Let x be the each edge of cube. Since the length of each edge changes from 5 to 5.02

$\therefore \quad x = 5, dx = \delta x = 5.02 - 5$
$= 0.02$

Volume of cube $= V = x \times x \times x$

$V = x^3$

$dv = 3x^2\, dx$

$dv = 3(5)^2 (0.02)$

$dv = 1.5$ cubic unit

Increase in volume = dv = 1.5 cubic unit

Q.5 Find the approximate increase in the area of a circular disc if its diameter is increased from 44cm to 44.4 cm

Solution:

Let r be the radius of circular disc. Since diameter increased from 44 cm to 44.4 cm so radius is

$\frac{44}{2}$ cm to $\frac{44.4}{2}$ cm

22cm to 22.2 cm

$r = 22\text{cm}, \quad dr = \delta r = 22.2 - 22$
$= 0.2$ cm

$dA = \pi\, d(r^2)$

$dA = 2\pi r dr$

$= 2\pi(22)(0.2)$

$= 27.467 \text{ cm}^2$

$\therefore$ Increase in area $= dA = 27.467\text{cm}^2$ Ans.

EXERCISE 3.2

Q.1 Evaluate the following indefinite integrals

(i) $\int(3x^2 - 2x + 1)dx$

(ii) $\int\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx\ (x > 0)$

(iii) $\int x(\sqrt{x} + 1)dx,\ (x > 0)$

(iv) $\int(2x + 3)^{1/2}dx$

(v) $\int(\sqrt{x} + 1)^2\, dx\ (x > 0)$

(vi) $\int\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx\ (x > 0)$

(vii) $\int \frac{3x + 2}{\sqrt{x}}\, dx\ (x > 0)$

(viii) $\int \frac{\sqrt{y}\,(y + 1)}{y}\, dy,\ (y > 0)$

(ix) $\int \frac{(\sqrt{\theta} - 1)^2}{\sqrt{\theta}}\, d\theta\ (\theta > 0)$

(x) $\int \frac{(1 - \sqrt{x})^2}{\sqrt{x}}\, dx,\ (x > 0)$

(xi) $\int \frac{e^{2x} + e^x}{e^x}dx$

Solution:

(i) $\int(3x^2 - 2x + 1)\, dx$

$= 3\int x^2dx - 2\int xdx + \int dx$

$= \frac{3x^3}{3} - \frac{2x^2}{2} + x + c$

$= x^3 - x^2 + x + c$ Ans.

(ii) $\int\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx\ \ (x > 0)$

$= \int \sqrt{x}\, dx + \int \frac{dx}{\sqrt{x}}$

$= \int x^{1/2}\, dx + \int x^{-1/2}\, dx$

$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= \frac{2}{3}\, x^{3/2} + 2\sqrt{x} + c$ Ans.

(iii) $\int x(\sqrt{x} + 1)dx \quad (x > 0)$

$= \int (x\sqrt{x} + x)\, dx$

$= \int x \,.\, x^{1/2}\, dx + \int xdx$

$$= \int x^{\frac{3}{2}}\,dx + \int x\,dx$$

$$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^2}{2} + c$$

$$= \frac{2}{5}x^{\frac{5}{2}} + \frac{x^2}{2} + c \qquad \text{Ans.}$$

(iv) $\int (2x+3)^{1/2}\,dx$

$$= \frac{1}{2}\int (2x+3)^{1/2}\,2dx$$

$$= \frac{1}{2}\,\frac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{(2x+3)^{\frac{3}{2}}}{3} + c \qquad \text{Ans}$$

(v) $\int (\sqrt{x}+1)^2\,dx \quad (x>0)$

$$= \int\left[(\sqrt{x})^2 + 2(\sqrt{x})(1) + (1)^2\right]dx$$

$$= \int x dx + 2\int x^{1/2}\,dx + \int dx$$

$$= \frac{x^2}{2} + \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} + x + c$$

$$= \frac{x^2}{2} + \frac{2}{3}x^{\frac{3}{2}} + x + c \qquad \text{Ans.}$$

(vi) $\int \left(\sqrt{x} - \frac{1}{x}\right)^2 dx \quad (x>0)$

$$= \int \left[(\sqrt{x})^2 - 2(\sqrt{x})\left(\frac{1}{\sqrt{x}}\right) + \left(\frac{1}{\sqrt{x}}\right)^2\right] dx$$

$$= \int \left(x - 2 + \frac{1}{x}\right) dx$$

$$= \int x dx - 2\int dx + \int \frac{dx}{x}$$

$= \dfrac{x^2}{2} - 2x + \ell nx + c$ Ans.

(vii) $\int \dfrac{3x+2}{\sqrt{x}}\,dx$ **$(x > 0)$**

$= \int \left(\dfrac{3x}{\sqrt{x}} + \dfrac{2}{\sqrt{x}}\right)dx$

$= \int (3x^{1-½} + 2x^{-½})\,dx$

$= 3\int x^{½}\,dx + 2\int x^{-½}dx$

$= \dfrac{3x^{\frac{3}{2}}}{\frac{3}{2}} + \dfrac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= 2x^{\frac{3}{2}} + 4\sqrt{x} + c$ Ans.

(viii) $\int \dfrac{\sqrt{y}\,(y+1)}{y}$ **$(y > 0)$**

$= \int \dfrac{\sqrt{y}\,(y+1)}{\sqrt{y}\,.\,\sqrt{y}}\,dy$

$= \int \left(\dfrac{y}{\sqrt{y}} + \dfrac{1}{\sqrt{y}}\right)dy$

$= \int (y^{1-½} + y^{-½})\,dy$

$= \int y^{½}dy + \int y^{-½}\,dy$

$= \dfrac{y^{\frac{3}{2}}}{\frac{3}{2}} + \dfrac{y^{½}}{\frac{1}{2}} + c$

$= \dfrac{2}{3}y^{\frac{3}{2}} + 2y^{\frac{1}{2}} + c$ Ans.

(ix) $\int \dfrac{(\sqrt{\theta}-1)^2}{\sqrt{\theta}}\,d\theta$ **$(\theta > 0)$** **(Guj. Board 2006)**

$$= \int \frac{(\sqrt{\theta})^2 - 2(\sqrt{\theta})(1) + (1)^2}{\sqrt{\theta}}\, d\theta$$

$$= \int \frac{\theta - 2\sqrt{\theta} + 1}{\sqrt{\theta}}\, d\theta$$

$$= \int \left(\frac{\theta}{\sqrt{\theta}} - \frac{2\sqrt{\theta}}{\sqrt{\theta}} + \frac{1}{\sqrt{\theta}}\right) d\theta$$

$$= \int (\theta^{1-\frac{1}{2}} - 2 + \theta^{-\frac{1}{2}})\, d\theta$$

$$= \int \theta^{\frac{1}{2}}\, d\theta - 2\int d\theta + \int \theta^{-\frac{1}{2}}\, d\theta$$

$$= \frac{\theta^{\frac{3}{2}}}{\frac{3}{2}} - 2\theta + \frac{\theta^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{2}{3}\theta^{\frac{3}{2}} - 2\theta + 2\theta^{\frac{1}{2}} + c \qquad \text{Ans.}$$

(x) $\displaystyle\int \frac{(1-\sqrt{x})^2}{\sqrt{x}}\, dx$ **(x > 0)** **(Lhr. Board 2006)**

$$= \int \frac{(1)^2 - 2(1)(\sqrt{x}) + (\sqrt{x})^2}{\sqrt{x}}\, dx$$

$$= \int \frac{1 - 2\sqrt{x} + x}{\sqrt{x}}\, dx$$

$$= \int \left(\frac{1}{\sqrt{x}} - \frac{2\sqrt{x}}{\sqrt{x}} + \frac{x}{\sqrt{x}}\right) dx$$

$$= \int (x^{\frac{-1}{2}} - 2 + x^{1-\frac{1}{2}})\, dx$$

$$= \int x^{\frac{-1}{2}}\, dx - 2\int dx + \int x^{\frac{1}{2}}\, dx$$

$$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - 2x + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= 2x^{\frac{1}{2}} - 2x + \frac{2}{3}x^{\frac{3}{2}} + c \qquad \text{Ans.}$$

(xi) $\displaystyle\int \frac{e^{2x} + e^{x}}{e^{x}}\, dx$

$$= \int \frac{e^x (e^x + 1)}{e^x} \, dx$$

$$= \int (e^x + 1) \, dx$$

$$= \int e^x \, dx + \int dx$$

$$= e^x + x + c \qquad \text{Ans.}$$

Q.2 Evaluate

(i) $\displaystyle\int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}} \begin{pmatrix} x+a>0 \\ x+b>0 \end{pmatrix}$

(ii) $\displaystyle\int \frac{1 - x^2}{1 + x^2} \, dx$

(iii) $\displaystyle\int \frac{dx}{\sqrt{x+a} + \sqrt{x}} \; (x > 0, a > 0)$

(iv) $\displaystyle\int (a - 2x)^{\frac{3}{2}} \, dx$

(v) $\displaystyle\int \frac{(1 + e^x)^3}{e^x} \, dx$

(vi) $\displaystyle\int \text{Sin}\,(a + b)\, x \, dx$

(vii) $\displaystyle\int \sqrt{1 - \cos 2x} \; dx \; (1 - \cos 2x > 0)$

(viii) $\displaystyle\int (\ell nx) \times \frac{1}{x} \, dx \; (x > 0)$

(ix) $\displaystyle\int \sin^2 x \; dx$

(x) $\displaystyle\int \frac{1}{1 + \cos x} \, dx \left(\frac{-\pi}{2} < x < \pi/2\right)$

(xi) $\displaystyle\int \frac{ax + b}{ax^2 + 2bx + c} dx$

(xii) $\displaystyle\int \cos 3x \sin 2x \, dx$

(xiii) $\displaystyle\int \frac{\cos 2x - 1}{1 + \cos 2x} \, dx, \; (1 + \cos 2x \neq 0)$

(xiv) $\displaystyle\int \tan^2 x \, dx$ **(Lhr. Board 2011)**

Solution:

(i) $\displaystyle\int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}} \begin{pmatrix} x+a>0 \\ x+b>0 \end{pmatrix}$

$$= \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}} \times \frac{\sqrt{x+a} - \sqrt{x+b}}{\sqrt{x+a} - \sqrt{x+b}}$$

$$= \int \frac{\sqrt{x+a} - \sqrt{x+b}}{\sqrt{(x+a)^2} - \sqrt{(x+b)^2}} \, dx$$

$$= \int \frac{\sqrt{x+a} - \sqrt{x+b}}{x + a - (x + b)} \, dx$$

$$= \int \frac{(x+a)^{\frac{1}{2}} - (x+b)^{\frac{1}{2}}}{x + a - x - b} \, dx$$

$$= \frac{1}{a-b}\int[(x+a)^{1/2}-(x-b)^{1/2}]\,dx$$

$$= \frac{1}{a-b}\left[\int(x+a)^{1/2}\,dx-\int(x-b)^{1/2}\,dx\right]$$

$$= \frac{1}{a-b}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}}\right]+c$$

$$= \frac{2}{3(a-b)}\,[(x+b)^{\frac{3}{2}}-(x-b)^{\frac{3}{2}}]+c \qquad \text{Ans.}$$

(ii) $\displaystyle\int\frac{\mathbf{1-x^2}}{\mathbf{1+x^2}}\,\mathbf{dx}$ **(Lhr. Board 2008)**

$$= \int\left(-1+\frac{2}{1+x^2}\right)dx$$

$$\begin{array}{r|l} & \quad 1 \\ 1+x^2 & 1-x^2 \\ & \underline{\mp 1 \mp x^2} \\ & \quad 2 \end{array}$$

$$= -\int dx+2\int\frac{dx}{1+x^2}$$

$$= -x+2\tan^{-1}x+c \qquad \text{Ans.}$$

(iii) $\displaystyle\int\frac{\mathbf{dx}}{\sqrt{\mathbf{x+a}}+\sqrt{\mathbf{x}}}$ **(x > 0, a > 0)**

$$= \int\frac{dx}{\sqrt{x+a}+\sqrt{x}}\times\frac{\sqrt{x+a}-\sqrt{x}}{\sqrt{x+a}-\sqrt{x}}$$

$$= \int\frac{\sqrt{x+a}-\sqrt{x}}{(\sqrt{x+a})^2-(\sqrt{x})^2}\,dx$$

$$= \int\frac{(x+a)^{1/2}-x^{1/2}}{x+a-x}\,dx$$

$$= \frac{1}{a}\int[(x+a)^{1/2}-x^{1/2}]\,dx$$

$$= \frac{1}{a}\left[\int(x+a)^{1/2}\,dx-\int x^{1/2}\,dx\right]$$

$$= \frac{1}{a}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right] + c$$

$$= \frac{2}{3a}\,[(x+a)^{\frac{3}{2}} - x^{\frac{3}{2}}] + c \qquad \text{Ans.}$$

(iv) $\int (a - 2x)^{3/2}\,dx$

$$= \frac{-1}{2}\int (a-2x)^{3/2} - 2\,dx$$

$$= \frac{-1}{2}\,\frac{(a-2x)^{5/2}}{5/2} + c$$

$$= \frac{-1}{5}\,(a-2x)^{5/2} + c \qquad \text{Ans.}$$

(v) $\int \frac{(1+e^x)^3}{e^x}\,dx$

$$= \int \frac{(1)^3 + 3\,(1)^2\,(e^x) + 3(1)\,(e^x)^2 + (e^x)^3}{e^x}\,dx$$

$$= \int \frac{1 + 3e^x + 3e^{2x} + e^{3x}}{e^x}\,dx$$

$$= \int \left(\frac{1}{e^x} + \frac{3e^x}{e^x} + \frac{3e^{2x}}{e^x} + \frac{e^{3x}}{e^x}\right) dx$$

$$= \int (e^{-x} + 3 + 3e^x + e^{2x})\,dx$$

$$= -\int e^{-x} - dx + 3\int dx + 3\int e^x\,dx + \frac{1}{2}\int e^{2x}.\,2dx \;+ \frac{1}{2}\int e^{2x}\,.\,2dx$$

$$= -e^{-x} + 3x + 3e^x + \frac{1}{2}e^{2x} + c \qquad \text{Ans.}$$

(vi) $\int \sin (a+b)\,x dx$

$$= \frac{1}{a+b}\int \sin (a+b)\,x.\;(a+b)\,dx$$

$$= \frac{1}{a+b} \cdot -\cos(a+b)x + c$$

$$= -\frac{1}{a+b}\cos(a+b)x + c \qquad \text{Ans.}$$

(vii) $\int \sqrt{1-\cos 2x}\; dx \;\; (1-\cos 2x > 0)$ **(Lhr. Board 2008, 2009)**

$$= \int \sqrt{2\sin^2 x} \qquad \begin{pmatrix} \because \cos 2x = 1 - 2\sin^2 x \\ 2\sin^2 x = 1 - \cos 2x \end{pmatrix}$$

$$= \sqrt{2} \int \sin x \, dx$$

$$= \sqrt{2}\,(-\cos x) + c$$

$$= -\sqrt{2}\cos x + c \qquad \text{Ans.}$$

(viii) $\int (\ell nx) \times \frac{1}{x}\, dx \;\; (x > 0)$

$$= \frac{(\ell nx)^2}{2} + c \qquad \text{Ans.}$$

(ix) $\int \sin^2 x \, dx$

$$= \int \frac{1-\cos 2x}{2}\, dx \qquad \begin{pmatrix} \because \cos 2x = 1 - 2\sin^2 x \\ 2\sin^2 x = 1 - \cos 2x \\ \sin^2 x = \dfrac{1-\cos 2x}{2} \end{pmatrix}$$

$$= \frac{1}{2}\int (1 - \cos 2x)\, dx$$

$$= \frac{1}{2}\int dx - \frac{1}{2.2}\int \cos 2x \,.\, 2\, dx$$

$$= \frac{1}{2}x - \frac{1}{4}\sin 2x + c \qquad \text{Ans.}$$

(x) $\int \frac{1}{1+\cos x}\, dx \qquad \left(\frac{-\pi}{2} < x < \frac{\pi}{2}\right)$

$$= \int \frac{1}{2\cos^2\frac{x}{2}}\,dx \qquad \left(\begin{aligned} \because\ \cos 2x &= 2\cos^2 x - 1 \\ 2\cos^2 x &= 1 + \cos 2x \\ 2\cos^2\frac{x}{2} &= 1 + \cos x \end{aligned}\right)$$

$$= \int \sec^2\frac{x}{2}\cdot\frac{1}{2}\,dx$$

$$= \tan\frac{x}{2} + c \qquad \text{Ans.}$$

(xi) $\displaystyle\int \frac{\mathbf{ax + b}}{\mathbf{ax^2 + 2bx + c}}\ \mathbf{dx}$

$$= \frac{1}{2}\int \frac{2ax + 2b}{ax^2 + 2bx + c}\ dx$$

$$= \frac{1}{2}\ ln\ |(ax^2 + 2bx + c| + c_1 \qquad \text{Ans.}$$

(xii) $\displaystyle\int \mathbf{\cos 3x \sin 2x\ dx}$

$$= \frac{1}{2}\int 2\cos 3x \sin 2x dx$$

$$= \frac{1}{2}\int [\sin(3x + 2x) - \sin(3x - 2x)]\ dx$$

$$\because\ 2\cos\alpha \sin\beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$$

$$= \frac{1}{2}\int (\sin 5x - \sin x)\ dx$$

$$= \frac{1}{2}\left[\int \sin 5x\ dx - \int \sin x\ dx\right]$$

$$= \frac{1}{2}\left[\frac{1}{5}\int \sin 5x\ .\ 5dx - \int \sin x dx\right]$$

$$= \frac{1}{2}\left(\frac{-\cos 5x}{5} + \cos x\right) + c$$

$$= \frac{-1}{2}\left(\frac{\cos 5x}{5} - \cos x\right) + c \qquad \text{Ans.}$$

(xiii) $\displaystyle\int \frac{\mathbf{\cos 2x - 1}}{\mathbf{1 + \cos 2x}}\ \mathbf{dx,} \qquad \mathbf{(1 + \cos 2x \neq 0)}$

$$= -\int \frac{1 - \cos 2x}{1 + \cos 2x}\ dx$$

$$= -\int \frac{2\sin^2 x}{2\cos^2 x}\ dx$$

$$= -\int \tan^2 x\ dx$$

$= -\int (\sec^2 x - 1)\, dx$

$= -\int \sec^2 x\, dx + \int dx$

$= x - \tan x + c$ Ans.

(xiv) $\int \tan^2 x\, dx$ **(Guj. Board 2005, 2007) (Lhr. Board 2011)**

$= \int (\sec^2 x - 1)\, dx$

$= \int \sec^2 x\, dx - \int dx$

$= \tan x - x + c$ Ans.

EXERCISE 3.3

Evaluate the following integrals.

Q.1 $\int \dfrac{-2x}{\sqrt{4 - x^2}}$

Solution:

$$\int \frac{-2x}{\sqrt{4 - x^2}}\, dx$$

$$= \int (4 - x^2)^{-\frac{1}{2}} - 2x\, dx$$

$$= \frac{(4 - x^2)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{4 - x^2} + c \qquad \text{Ans.}$$

Q.2 $\int \dfrac{dx}{x^2 + 4x + 13}$

Solution:

$$\int \frac{dx}{x^2 + 4x + 13}$$