Solution:

$$y = \tan(P \tan^{-1} x)$$
$$\tan^{-1} y = P \tan^{-1} x$$

Diff. w.r.t. 'x'

$$\frac{1}{1+y^2} \cdot y_1 = P \frac{1}{1+x^2}$$
$$(1+x^2)\, y_1 = P(1+y^2)$$
$$\boxed{(1+x^2)\, y_1 - P(1+y^2) = 0}$$

Hence proved.

EXERCISE 2.6

Q.1: Find $f'(x)$ if

(i) $f(x) = e^{\sqrt{x}-1}$

(ii) $f(x) = x^3 e^{\frac{1}{x}} \; (x \neq 0)$

(iii) $f(x) = ex(1 + \ell nx)$

(iv) $f(x) = \dfrac{e^x}{e^{-x}+1}$

(v) $f(x) = \ell n(e^x + e^{-x})$

(vi) $f(x) = \dfrac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}}$

(vii) $f(x) = \sqrt{\ell n(e^{2x} + e^{-2x})}$

(viii) $f(x) = \ell n\left(\sqrt{e^{2x} + e^{-2x}}\right)$

Solution:

(i) $f(x) = e^{\sqrt{x}-1}$

Diff. w.r.t. 'x'

$$f'(x) = e^{\sqrt{x}-1} \cdot \frac{d}{dx}(\sqrt{x}-1)$$
$$f'(x) = e^{\sqrt{x}-1} \cdot \frac{1}{2} x^{\frac{-1}{2}}$$
$$\boxed{f'(x) = \frac{e^{\sqrt{x}-1}}{2\sqrt{x}}}$$

Ans.

(ii) $f(x) = x^3 e^{\frac{1}{x}} \; (x \neq 0)$

Diff. w.r.t. 'x'

$$f'(x) = x^3 \frac{d}{dx}\left(e^{\frac{1}{x}}\right) + e^{\frac{1}{x}} \frac{d}{dx}(x^3)$$

$$f'(x) = x^3 e^{\frac{1}{x}} . \frac{d}{dx}\left(\frac{1}{x}\right) + e^{\frac{1}{x}} \; 3x^2$$

$$f'(x) = x^3 e^{\frac{1}{x}} \times \frac{-1}{x^2} + 3x^2 e^{\frac{1}{x}}$$

$$f'(x) = -x e^{\frac{1}{x}} + 3x^2 e^{\frac{1}{x}}$$

$$\boxed{f'(x) = xe^{\frac{1}{x}} (3x-1)} \quad \text{Ans.}$$

(iii) $\mathbf{f(x) = e^x (1 + \ell nx)}$

Diff. w.r.t. 'x'

$$f'(x) = e^x \frac{d}{dx} (1 + \ell nx) + (1 + \ell nx) \frac{d}{dx} (e^x)$$

$$f'(x) = e^x . \frac{1}{x} + (1 + \ell nx) e^x$$

$$f'(x) = e^x \left[\frac{1}{x} + (1 + \ell nx) \right]$$

$$\boxed{f'(x) = \frac{e^x [1 + x (1 + \ell nx)]}{x}} \quad \text{Ans}$$

(iv) $\mathbf{f(x) = \dfrac{e^x}{e^{-x} + 1}}$

Diff. w.r.t. 'x'

$$f'(x) = \frac{(e^{-x} + 1)\frac{d}{dx}(e^x) - e^x \frac{d}{dx} (e^{-x} + 1)}{(e^{-x} + 1)^2}$$

$$f'(x) = \frac{(e^{-x} + 1)e^x - e^x . e^{-x} (-1)}{(e^{-x} + 1)^2}$$

$$f'(x) = \frac{e^{-x} .e^x + e^x + e^{x-x}}{(e^{-x} + 1)^2}$$

$$f'(x) = \frac{e^{-x+x} + e^x + e^0}{(e^{-x} + 1)^2}$$

$$f'(x) = \frac{e^0 + e^x + 1}{(e^{-x} + 1)^2}$$

$$f'(x) = \frac{1 + e^x + 1}{(e^{-x} + 1)^2}$$

$$\boxed{f'(x) = \frac{e^x + 2}{(e^{-x}+1)^2}} \qquad \text{Ans.}$$

(v) $\quad \mathbf{f(x) = \ell n\,(e^x + e^{-x})}$

Diff. w.r.t. 'x'

$$f'(x) = \frac{1}{e^x + e^{-x}} \frac{d}{dx}(e^x + e^{-x})$$

$$f'(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$f'(x) = \frac{e^x - \frac{1}{e^x}}{e^x + \frac{1}{e^x}}$$

$$f'(x) = \frac{\frac{e^{2x}-1}{e^x}}{\frac{e^{2x}+1}{e^x}}$$

$$\boxed{f'(x) = \frac{e^{2x}-1}{e^{2x}+1}} \qquad \text{Ans.}$$

(vi) $\quad \mathbf{f(x) = \frac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}}}$

Diff. w.r.t. 'x'

$$f'(x) = \frac{(e^{ax} + e^{-ax})\frac{d}{dx}(e^{ax} - e^{-ax}) - (e^{ax} - e^{-ax})\frac{d}{dx}(e^{ax} + e^{-ax})}{(e^{ax} + e^{-ax})^2}$$

$$f'(x) = \frac{(e^{ax} + e^{-ax})\,(e^{ax}\,.\,a - e^{-ax}(-a)) - (e^{ax} - e^{-ax})(e^{ax}\,.a + e^{-ax} \times -a)}{(e^{ax} + e^{-ax})^2}$$

$$f'(x) = \frac{a(e^{ax} + e^{-ax})\,(e^{ax} + e^{-ax}) - a\,(e^{ax} - e^{-ax})(e^{ax} - e^{-ax})}{(e^{ax} + e^{-ax})^2}$$

$$f'(x) = \frac{a[(e^{ax} + e^{-ax})^2 - (e^{ax} - e^{-ax})^2]}{(e^{ax} + e^{-ax})^2}$$

$$f'(x) = \frac{a[e^{2ax} + e^{-2ax} + 2\,e^{ax}\,e^{-ax} - (e^{2ax} + e^{-2ax} - 2e^{ax}\,.\,e^{-ax})]}{(e^{ax} + e^{-ax})^2}$$

$$f'(x) = \frac{a(e^{2ax} + e^{-2ax} + 2 - e^{2ax} - e{-}2^{ax} + 2)}{(e^{ax} + e^{-ax})^2}$$

$$\boxed{f'(x) = \frac{4a}{(e^{ax} + e^{-ax})^2}} \quad \text{Ans.}$$

(vii) $\mathbf{f(x) = \sqrt{\ell n\, (e^{2x} + e^{-2x})}}$

Diff. w.r.t. 'x'

$$f'(x) = \frac{1}{2}\,[ln(e^{2x} + e^{-2x})]^{-1/2} \cdot \frac{d}{dx}\,[\ell n\,(e^{2x} + e^{-2x})]$$

$$f'(x) = \frac{1}{2\sqrt{\ell n\,(e^{2x} + e^{-2x})}} \cdot \frac{1}{e^{2x} + e^{-2x}} \cdot \frac{d}{dx}\,(e^{2x} + e^{-2x})$$

$$f'(x) = \frac{1}{2\sqrt{\ell n\,(e^{2x} + e^{-2x})}} \cdot \frac{1}{e^{2x} + e^{-2x}} \cdot (e^{2x}.2 + e^{-2x}(-2))$$

$$f'(x) = \frac{1}{2\sqrt{\ell n\,(e^{2x} + e^{-2x})}} \cdot \frac{1}{e^{2x} + e^{-2x}} \cdot 2\,(e^{2x} - e^{-2x})$$

$$\boxed{f'(x) = \frac{e^{2x} - e^{-2x}}{(e^{2x} - e^{-2x})\sqrt{\ell n\,(e^{2x} + e^{-2x})}}} \quad \text{Ans.}$$

(viii) $\mathbf{f(x) = \ell n\left(\sqrt{e^{2x} + e^{-2x}}\right)}$ ***(L.B 2009 (s))***

Diff. w.r.t. 'x'

$$f'(x) = \frac{1}{\sqrt{e^{2x} + e^{-2x}}} \cdot \frac{d}{dx}\left(\sqrt{e^{2x} + e^{-2x}}\right)$$

$$f'(x) = \frac{1}{\sqrt{e^{2x} + e^{-2x}}} \cdot \frac{1}{2}\,(e^{2x} + e^{-2x})^{\frac{-1}{2}} \cdot \frac{d}{dx}(e^{2x} + e^{-2x})$$

$$f'(x) = \frac{1}{2\sqrt{e^{2x} + e^{-2x}}\,\sqrt{e^{2x} + e^{-2x}}} \quad (e^{2x}\,.\,2 + e^{-2x} - 2)$$

$$f'(x) = \frac{2(e^{2x} - e^{-2x})}{2(e^{2x} + e^{-2x})} = \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$$

$$\boxed{f'(x) = \tan h2x} \quad \text{Ans.}$$

Q.2: **Find $\mathbf{\frac{dy}{dx}}$ if**

(i) $\mathbf{y = x^2\, \ell n \sqrt{x}}$

(ii) $\mathbf{y = x\sqrt{\ell n\, x}}$

(iii) $\mathbf{y = \frac{x}{\ell\, nx}}$

(iv) $\mathbf{y = x^2\, \ell n \frac{1}{x}}$

(v) $y = \ell n\sqrt{\dfrac{x^2-1}{x^2+1}}$ **(vi)** $y = \ell n\,(x+\sqrt{x^2+1}\,)$

(vii) $y = \ell n\,(9-x^2)$ ***(L.B 2009)*** **(viii)** $y = e^{-2x}\sin 2x$ ***(L.B 2009 (s))***

(ix) $y = e^{-x}(x^3+2x^2+1)$ ***(L.B 2009)*** **(x)** $y = xe^{\sin x}$

(xi) $y = 5e^{3x-4}$ **(xii)** $y = (x+1)^x$

(xiii) $y = (\ell nx)^{\ell nx}$ **(xiv)** $y = \dfrac{\sqrt{x^2-1}\,(x+1)}{(x^3+1)^{\frac{3}{2}}}$

Solution:

(i) $y = x^2\ell n\sqrt{x}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = x^2\frac{d}{dx}(\ell n\sqrt{x}) + \ell n\sqrt{x}\frac{d}{dx}(x^2)$$

$$\frac{dy}{dx} = x^2\cdot\frac{1}{\sqrt{x}}\cdot\frac{d}{dx}\left(\sqrt{x}\right) + \ell n\sqrt{x}\,.\,2x$$

$$\frac{dy}{dx} = \frac{x^2}{\sqrt{x}}\cdot\frac{1}{2}\,x^{\frac{-1}{2}} + 2x\ \ell n\sqrt{x}$$

$$\frac{dy}{dx} = \frac{x^2}{2\sqrt{x}\,.\,\sqrt{x}} + 2x\ \ell n\sqrt{x}$$

$$\frac{dy}{dx} = \frac{x^2}{2x} + 2x\ \ell n\sqrt{x}$$

$$\boxed{\frac{dy}{dx} = \frac{x}{2} + 2x\ \ell n\sqrt{x}} \qquad \text{Ans.}$$

(ii) $y = x\sqrt{\ell n\,x}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = x\frac{d}{dx}\left(\sqrt{\ell n\,x}\right) + \sqrt{\ell n\,x}\ \frac{d}{dx}\ (x)$$

$$\frac{dy}{dx} = x\,.\,\frac{1}{2}\ (\ell n\,x)^{\frac{-1}{2}}\ .\ \frac{d}{dx}(\ell n\,x) + \sqrt{\ell n\,x}\,.\,1$$

$$\frac{dy}{dx} = \frac{x}{2\sqrt{\ell nx}}\ .\frac{1}{x} + \sqrt{\ell n\,x}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\ell nx}} + \sqrt{\ell n\,x}$$

$$\boxed{\frac{dy}{dx} = \frac{1+2\ell nx}{2\sqrt{\ell nx}}}$$ Ans

(iii) $\mathbf{y = \dfrac{x}{\ell nx}}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{(\ell nx)\frac{d}{dx}(x) - x\frac{d}{dx}(\ell nx)}{(\ell nx)^2}$$

$$\frac{dy}{dx} = \frac{\ell nx \,.\, 1 - x \,.\, \frac{1}{x}}{(\ell nx)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{\ell nx - 1}{(\ell nx)^2}}$$ Ans.

(iv) $\mathbf{y = x^2 \,\ell n \dfrac{1}{x}}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = x^2 \frac{d}{dx}\left(\ell n\frac{1}{x}\right) + \ell n\frac{1}{x}\frac{d}{dx}(x^2)$$

$$\frac{dy}{dx} = x^2 \,.\, \frac{1}{\frac{1}{x}} \cdot \frac{d}{dx}\left(\frac{1}{x}\right) + \ell n\frac{1}{x} \,.\, 2x$$

$$\frac{dy}{dx} = x^3 \,.\, \frac{-1}{x^2} + 2x\,\ell n\frac{1}{x}$$

$$\frac{dy}{dx} = -x + 2x\,\ell n\frac{1}{x}$$

$$\boxed{\frac{dy}{dx} = x\left[2\,\ell n\frac{1}{x} - 1\right]}$$ Ans.

(v) $\mathbf{y = \ell n\sqrt{\dfrac{x^2-1}{x^2+1}}}$

$$y = \frac{1}{2}\,\ell n\left(\frac{x^2-1}{x^2+1}\right)$$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{\frac{x^2-1}{x^2+1}}\cdot\frac{d}{dx}\left(\frac{x^2-1}{x^2+1}\right)$$

$$\frac{dy}{dx} = \frac{x^2+1}{2(x^2-1)}\left[\frac{(x^2+1)\frac{d}{dx}(x^2-1)-(x^2-1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}\right]$$

$$\frac{dy}{dx} = \frac{(x^2+1)2x-(x^2-1)2x}{2(x^2-1)(x^2+1)}$$

$$\frac{dy}{dx} = \frac{2x\,(x^2+1-x^2+1)}{2(x^4-1)}$$

$$\boxed{\frac{dy}{dx} = \frac{2x}{x^4-1}} \quad \text{Ans.}$$

(vi) $\mathbf{y = \ell n\,(x+\sqrt{x^2+1}\,)}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}\ .\ \frac{d}{dx}\,(x+\sqrt{x^2+1}\,)$$

$$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}\ .\ \left[1+\frac{1}{2}(x^2+1)^{\frac{-1}{2}}\ .\ \frac{d}{dx}(x^2+1)\right]$$

$$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}\ .\ \left[1+\frac{1}{2\sqrt{x^2+1}}\ .\ 2x\right]$$

$$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}\ .\ \left[\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right]$$

$$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{x^2+1}}} \quad \text{Ans.}$$

(vii) $\mathbf{y = \ell n\,(9-x^2)}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{1}{9-x^2}\cdot\frac{d}{dx}\,(9-x^2)$$

$$\boxed{\frac{dy}{dx} = \frac{-2x}{9-x^2}} \quad \text{Ans.}$$

(viii) $\mathbf{y = e^{-2x}\sin 2x}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = e^{-2x} \frac{d}{dx}(\sin 2x) + \sin 2x \frac{d}{dx}(e^{-2x})$$

$$\frac{dy}{dx} = e^{-2x} . \cos 2x\ (2) + \sin 2x . e^{-2x} . (-2)$$

$$\boxed{\frac{dy}{dx} = 2e^{-2x}(\cos 2x - \sin 2x)} \quad \text{Ans.}$$

(ix) $\mathbf{y = e^{-x}(x^3 + 2x^2 + 1)}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = e^{-x} \frac{d}{dx}(x^3 + 2x^2 + 1) + (x^3 + 2x^2 + 1)\frac{d}{dx}(e^{-x})$$

$$\frac{dy}{dx} = e^{-x}(3x^2 + 4x) + (x^3 + 2x^2 + 1) . e^{-x}(-1)$$

$$\frac{dy}{dx} = e^{-x}(3x^2 + 4x - x^3 - 2x^2 - 1)$$

$$\frac{dy}{dx} = e^{-x}(-x^3 + x^2 + 4x - 1)$$

$$\boxed{\frac{dy}{dx} = -e^{-x}(x^3 - x^2 - 4x + 1)} \quad \text{Ans.}$$

(x) $\mathbf{y = xe^{\sin x}}$ ***(G.B 2007)***

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = x\frac{d}{dx}(e^{\sin x}) + e^{\sin x}\frac{d}{dx}(x)$$

$$\frac{dy}{dx} = xe^{\sin x}\frac{d}{dx}(\sin x) + e^{\sin x} . 1$$

$$\frac{dy}{dx} = xe^{\sin x} \cos x + e^{\sin x}$$

$$\boxed{\frac{dy}{dx} = e^{\sin x}(x \cos x + 1)} \quad \text{Ans.}$$

(xi) $\mathbf{y = 5e^{3x-4}}$ ***(G.B 2006)***

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = 5\, e^{3x-4} . \frac{d}{dx}(3x - 4)$$

$$\frac{dy}{dx} = 5\, e^{3x-4} . 3$$

$$\boxed{\frac{dy}{dx} = 15\,e^{3x-4}}$$ Ans.

(xii) $\mathbf{y = (x+1)^x}$

Taking '*l*n' on both sides

$$\ell ny = \ell n\,(x+1)^x$$

$$\ell ny = x\,\ell n\,(x+1)$$

Diff. w.r.t. 'x'

$$\frac{1}{y}\cdot\frac{dy}{dx} = x\frac{d}{dx}[\ell n\,(x+1)] + \ell n\,(x+1)\frac{d}{dx}(x)$$

$$\frac{dy}{dx} = y\left[x\cdot\frac{1}{x+1} + \ell n\,(x+1)\right]$$

$$\boxed{\frac{dy}{dx} = (x+1)^x\left[\frac{x+(x+1)\ \ell n\,(x+1)}{x+1}\right]}$$ Ans.

(xiii) $\mathbf{y = (\ell nx)^{\ell nx}}$

Taking '*l*n' on both sides

$$\ell ny = \ell n\,(\ell nx)^{\ell nx}$$

$$\ell ny = \ell nx\ \ell n\,(\ell nx)$$

Diff. w.r.t. 'x'

$$\frac{1}{y}\frac{dy}{dx} = \ell nx\frac{d}{dx}[\ell n\,(\ell nx)] + \ell n\,(\ell nx)\frac{d}{dx}(\ell nx)$$

$$\frac{dy}{dx} = y\left[\ell nx\cdot\frac{1}{\ell nx}\cdot\frac{d}{dx}(\ell nx) + \ell n\,(\ell nx)\cdot\frac{1}{x}\right]$$

$$\frac{dy}{dx} = (\ell nx)^{\ell nx}\left[\frac{1}{x} + \frac{\ell n\,(\ell nx)}{x}\right]$$

$$\boxed{\frac{dy}{dx} = (\ell nx)^{\ell nx}\left[1 + \frac{\ell n\,(\ell nx)}{x}\right]}$$ Ans.

(xiv) $\mathbf{y = \dfrac{\sqrt{x^2-1}\,(x+1)}{(x^3+1)^{\frac{3}{2}}}}$

$$y = \frac{\sqrt{(x+1)\,(x-1)}\,(x+1)}{[(x+1)\,(x^2-x+1)]^{\frac{3}{2}}}$$

$$y = \frac{\sqrt{x+1}\,.\,\sqrt{x-1}\,(x+1)}{(x+1)^{\frac{3}{2}}\,(x^2-x+1)^{\frac{3}{2}}}$$

$$y = \frac{(x-1)^{\frac{1}{2}}}{(x^2-x+1)^{\frac{3}{2}}}$$

Taking 'ℓn' on both sides

$$\ell ny = \ell n\left[\frac{(x-1)^{\frac{1}{2}}}{(x^2-x+1)^{\frac{3}{2}}}\right]$$

$$\ell ny = \ell n\,(x-1)^{\frac{1}{2}} - \ell n\,(x^2-x+1)^{\frac{3}{2}}$$

$$\ell ny = \frac{1}{2}\,\ell n\,(x-1) - \frac{3}{2}\,\ell n\,(x^2-x+1)$$

Diff. w.r.t. 'x'

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{x-1}\cdot\frac{d}{dx}(x-1) - \frac{3}{2}\cdot\frac{1}{x^2-x+1}\frac{d}{dx}\,(x^2-x+1)$$

$$\frac{dy}{dx} = y\left[\frac{1}{2(x-1)} - \frac{3(2x-1)}{2(x^2-x+1)}\right]$$

$$\frac{dy}{dx} = \frac{(x-1)^{\frac{1}{2}}}{(x^2-x+1)^{\frac{3}{2}}}\left[\frac{x^2-x+1-3(2x-1)(x-1)}{2(x-1)(x^2-x+1)}\right]$$

$$\frac{dy}{dx} = \frac{x^2-x+1-3\,(2x^2-3x+1)}{2\sqrt{x-1}\,(x^2-x+1)^{\frac{3}{2}+1}}$$

$$\frac{dy}{dx} = \frac{x^2-x+1-6x^2+9x-3}{2\sqrt{x-1}\,(x^2-x+1)^{\frac{5}{2}}}$$

$$\frac{dy}{dx} = \frac{-5x^2+8x-2}{2\sqrt{x-1}\,(x^2-x+1)^{\frac{5}{2}}}$$

$$\boxed{\frac{dy}{dx} = -\frac{5x^2-8x+2}{2\sqrt{x-1}\,(x^2-x+1)^{\frac{5}{2}}}} \qquad \text{Ans.}$$

Q.3: Find $\frac{dy}{dx}$ if

(i) $y = \cosh 2x$ **(ii)** $y = \sinh 3x$

(iii) $y = \tanh^{-1}(\sin x) -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

(iv) $y = \sinh^{-1}(x^3)$ **(v)** $y = (\ell n \tan hx)$

(vi) $y = \sinh^{-1}\left(\dfrac{x}{2}\right)$

Solution:

(i) $y = \cosh 2x$ ***(L.B 2008)***

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \sinh 2x \cdot \frac{d}{dx}(2x)$$

$$\boxed{\frac{dy}{dx} = 2\sinh 2x} \quad \text{Ans.}$$

(ii) $y = \sinh 3x$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \cosh 3x \cdot \frac{d}{dx}(3x)$$

$$\boxed{\frac{dy}{dx} = 3\cosh 3x} \quad \text{Ans.}$$

(iii) $y = \tanh^{-1}(\sin x) \quad \dfrac{-\pi}{2} < x < \dfrac{\pi}{2}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{1}{1-\sin^2 x} \cdot \frac{d}{dx}(\sin x)$$

$$\boxed{\frac{dy}{dx} = \frac{\cos x}{\cos^2 x} = \frac{1}{\cos x} = \sec x} \quad \text{Ans.}$$

(iv) $y = \sinh^{-1}(x^3)$ ***(L.B 2008), (G.B 2008)***

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+(x^3)^2}} \cdot \frac{d}{dx}(x^3)$$

$$\boxed{\frac{dy}{dx} = \frac{3x^2}{\sqrt{1+x^6}}} \quad \text{Ans.}$$

(v) $y = (\ell n \tan hx)$ ***(L.B 2009 (s))***

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{1}{\tanh x} \cdot \frac{d}{dx}(\tanh x)$$

$$\frac{dy}{dx} = \frac{1}{\tanh x} \cdot \sec h^2 x$$

$$\frac{dy}{dx} = \frac{\frac{1}{\cosh^2 x}}{\frac{\sinh x}{\cosh . x}}$$

$$\frac{dy}{dx} = \frac{1}{\sinh x \cosh x}$$

$$\frac{dy}{dx} = \frac{2}{2 \sinh x \cosh x} = \frac{2}{\sinh 2x}$$

$$\boxed{\frac{dy}{dx} = 2\,\text{cosech} 2x} \qquad \text{Ans.}$$

(vi) $\mathbf{y = \sinh^{-1}\left(\frac{x}{2}\right)}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+\left(\frac{x}{2}\right)^2}} \cdot \frac{d}{dx}\left(\frac{x}{2}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+\frac{x^2}{4}}} \cdot \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{\frac{4+x^2}{4}}} \cdot \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{1}{\frac{\sqrt{4+x^2}}{2}} \cdot \frac{1}{2}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{4+x^2}}} \qquad \text{Ans.}$$

EXERCISE 2.7

Q.1: **Find y_2 if**