Q.4 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution:

Its logical form is $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$. Its truth table is given below.

p	q	r	$q \wedge r$	$p \vee (q \vee r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
F	T	T	T	T	T	T	T
F	F	T	F	F	F	T	F
F	T	F	F	F	T	F	F
T	F	F	F	T	T	T	T
F	F	F	F	F	F	F	F

As entries in the columns of $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are same. So

$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

or

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Hence proved.

EXERCISE 2.6

Binary Relation

Let A and B be two non–empty sets, then any subset of Cartesian product A x B is called a binary relation, or simply a relation from A to B.

Q.1 For A = {1, 2, 3, 4}, find the following relation in A. State the domain and range of each relation. Also draw the graph of each.

(i) $\{(x, y) \mid y = x\}$ **(Lahore Board 2010)**

(ii) $\{(x, y) \mid y + x = 5\}$

(iii) $\{(x, y) \mid x + y < 5\}$ **(Lahore Board 2011)**

(iv) $\{(x, y) \mid x + y > 5\}$ **(Gujranwala Board 2003, Lahore Board 2003)**

Solution:

Given that

$A = \{1, 2, 3, 4\}$

Then

$$A \times A = \{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$$
$$= \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$$

(i) $\{(x, y) \mid y = x\}$

Let $r_1 = \{(x, y) \mid y = x\}$

$= \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

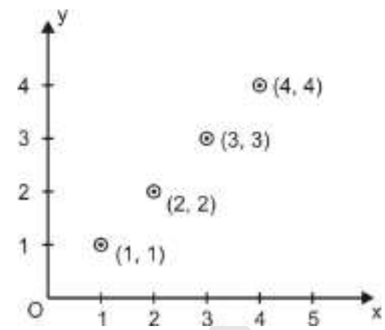
$\therefore$ r_1 is the required relation.

Now

Domain $r_1 = \{1, 2, 3, 4\}$

Range $r_1 = \{1, 2, 3, 4\}$

and graph of r_1 is given



(ii) $\{(x, y) \mid y + x = 5\}$

Let $r_2 = \{(x, y) \mid y + x = 5\}$

$= \{(1, 4), (2, 3), (3, 2), (4, 1)\}$

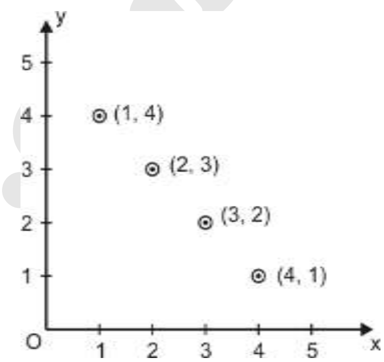
$\Rightarrow$ r_2 is the required relation.

Now

Domain $r_2 = \{1, 2, 3, 4\}$

Range $r_2 = \{4, 3, 2, 1\}$

and its graph is given



(iii) $\{(x, y) \mid x + y < 5\}$

Let $r_3 = \{(x, y) \mid x + y < 5\}$

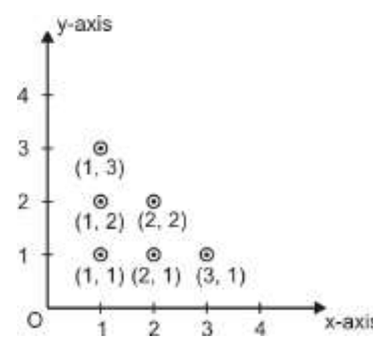
$= \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)\}$

$\therefore$ r_3 is the required relation.

Domain $r_3 = \{1, 2, 3\}$

Range $r_3 = \{1, 2, 3\}$

Its graph is given



(iv) $\{(x, y) \mid x + y > 5\}$

Let $r_4 = \{(x, y) \mid x + y > 5\}$

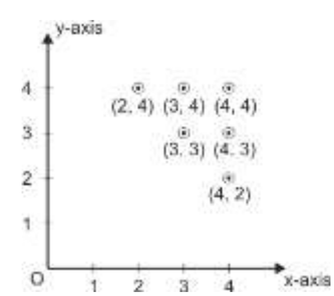
$= \{(2, 4), (3, 3), (3, 4), (4, 2), (4, 3), (4, 4)\}$

$\therefore$ r_4 is the required relation.

Domain $r_4 = \{2, 3, 4\}$

Range $r_4 = \{4, 3, 2\}$

Its graph is given



Q.2 Repeat Q.1 when $A = \Re$, the set of real numbers. Which of the real lines are functions.

(i) $A = \Re$ $\{(x, y) \mid y = x\}$

(ii) $\{(x, y) \mid y + x = 5\}$

(iii) $\{(x, y) \mid x + y < 5\}$

(iv) $\{(x, y) \mid x + y > 5\}$

Solution:

(i) $\{(x, y) \mid y = x\}$

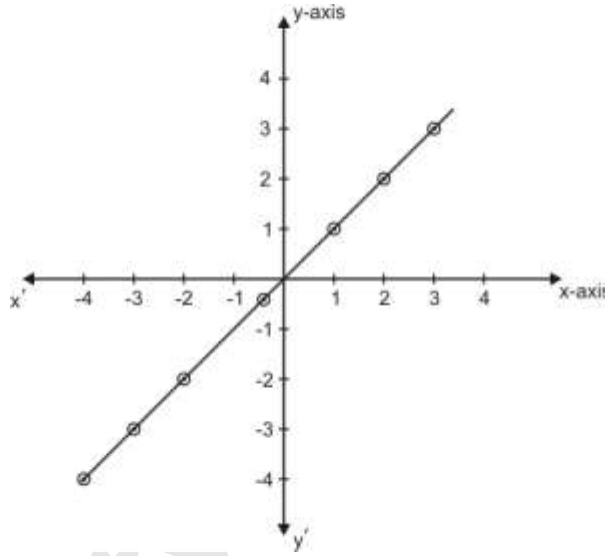
Let r_1 be the required relation, then

$r_1 = \{(x, y) \mid y = x\}$

Dom $r_1 = R$

Range $r_1 = R$

Its graph is given



Its graph will be a straight line and r_1 is a function, because any vertical line will cut it at only one point.

(ii) $\{(x, y) \mid y + x = 5\}$

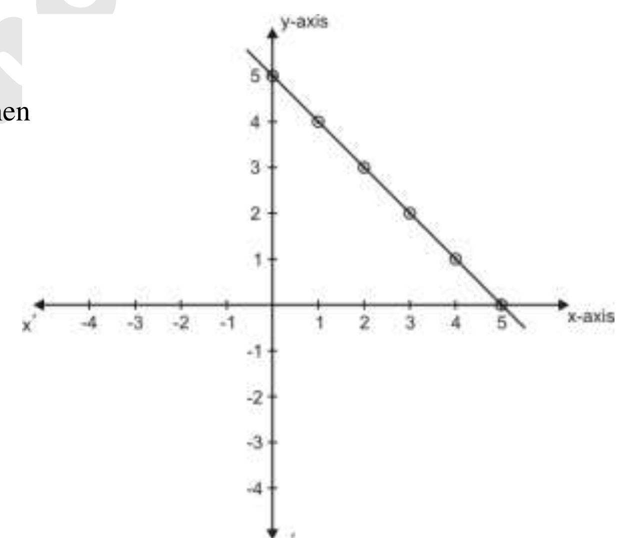
Let r_2 be the required relation, then

$r_2 = \{(x, y) \mid y + x = 5\}$

Dom $r_2 = \Re$

Range $r_2 = \Re$

Its graph will be



Its graph will be a straight line and it is a function because any vertical line will cut it at only one point.

(iii) **$\{(x, y) \mid x + y < 5\}$**

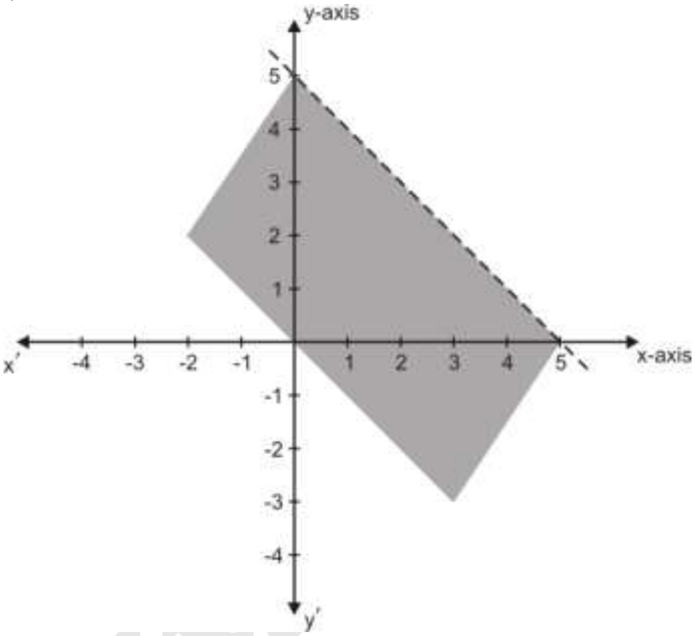
Let r_3 be the required relation,

then $r_3 = \{(x, y) \mid x + y < 5\}$

Dom $r_3 = \mathfrak{R}$

Range $r_3 = \mathfrak{R}$

Its graph is given below



It is not a function because any vertical line does not cut it at only one point.

(iv) **$\{(x, y) \mid x + y > 5\}$**

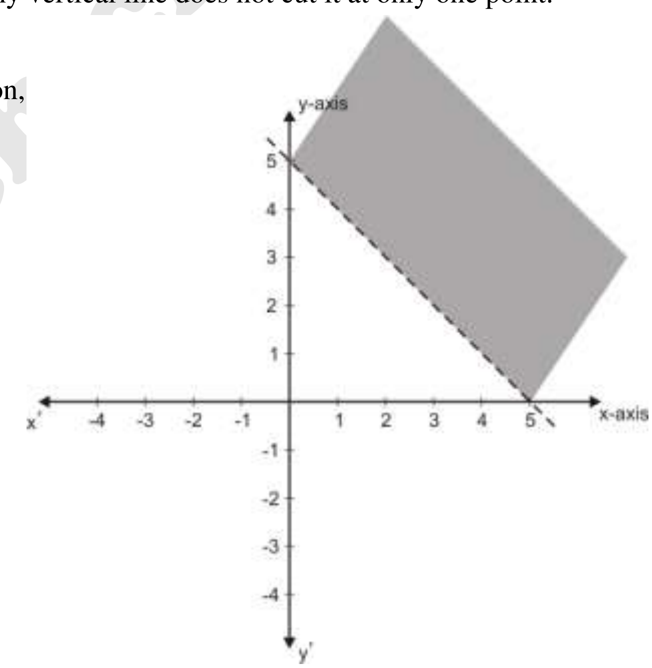
Let r_4 be the required relation,

then $r_4 = \{(x, y) \mid x + y > 5\}$

Dom $r_4 = \mathfrak{R}$ and

Range $r_4 = \mathfrak{R}$

Its graph is given



It is not a function because any vertical line does not cut it at only one point.

Function

A binary relation 'f' is called a function if there is no repetition in the domain of f.

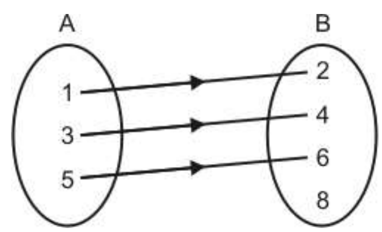
It is written as $f : A \rightarrow B$

Into Function

If a function $f : A \rightarrow B$ is such that Range $f \neq B$ then 'f' is called an Into function.

For example,

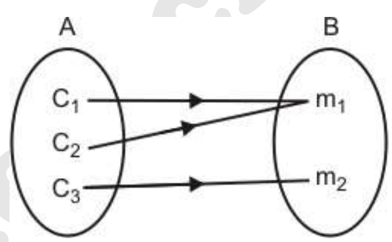
$f = \{(1, 2), (3, 4), (5, 6)\}$



Onto (Surjective) Function

If a function $f : A \rightarrow B$ is such that Range $f = B$ then 'f' is called an Onto (surjective) function. For example

$f : \{(c_1, m_1), (c_2, m_1), (c_3, m_2)\}$

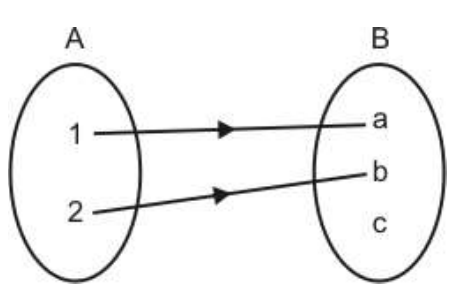


(1 – 1) and Into (Injective) Function

If f is an Into function and there is no repetition in the range of 'f' then 'f' is called an Injective (1 – 1 and into) function.

For example,

$f : \{(1, a), (2, b)\}$

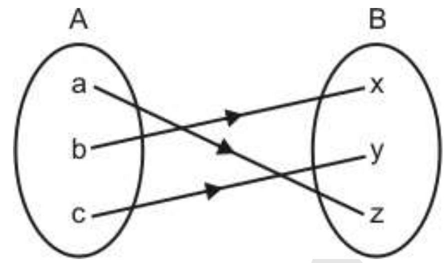


(1 – 1) and Onto (Bijective) Function

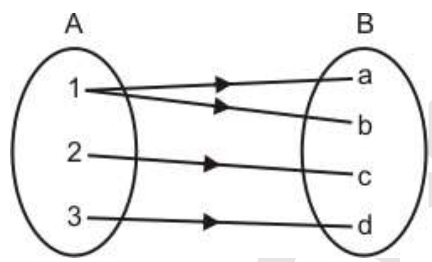
If 'f' is an onto function and there is no repetition in the range of 'f' then 'f' is called a Bijective (1 – 1 and onto) function.

For example,

f : {(a, z), (b, x), (c, y)}



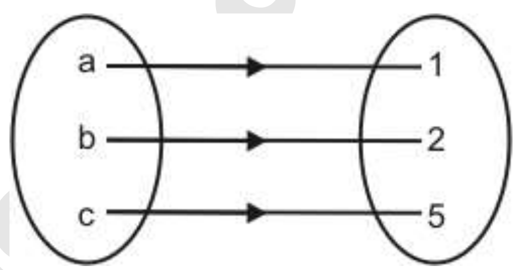
Q.3 Which of the following diagram represents a function and of which type?



= {(1, a), (1, b), (2, c), (3, d)}

It is not a function because there is repetition in its domain.

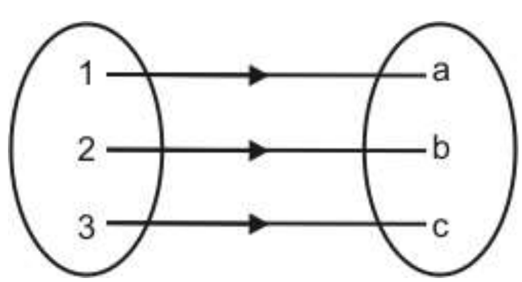
fig (ii)



= {(a, 1), (b, 2), (c, 5)}

It is a function and it is 1 – 1 and onto (Bijective) function.

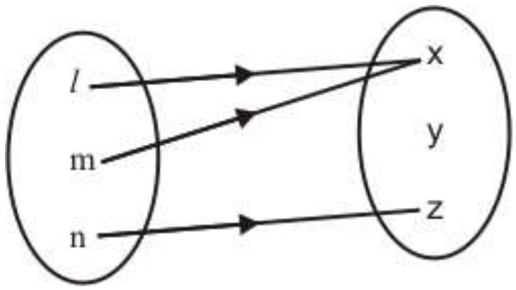
fig. (iii)



= {(1, a), (2, b), (3, c)}

It is also a function and it is $1-1$ and Onto (Bijective) function.

fig. (iv)



$= \{(l, x), (m, x), (n, z)\}$

It is a function and it is an into function.

Inverse of A Function

Inverse of a function or relation can be obtained by interchanging the components of each ordered pair.

Q.4 Find the inverse of each of the following relations. Tell whether each relation and its inverse is a function or not.

(i) $\{(2, 1), (3, 2), (4, 3), (5, 4), (6, 5)\}$

(ii) $\{(1, 3), (2, 5), (3, 7), (4, 9), (5, 11)\}$

(iii) $\{(x, y) \mid y = 2x + 3,\ x \in \Re\}$ **(Gujranwala Board 2003)**

(iv) $\{(x, y) \mid y^2 = 4ax,\ x \geq 0\}$

(v) $\{(x, y) \mid x^2 + y^2 = 9,\ |x| \leq 3,\ |y| \leq 3\}$

Solution:

(i) $\{(2, 1), (3, 2), (4, 3), (5, 4), (6, 5)\}$

Let

$r = \{(2, 1), (3, 2), (4, 3), (5, 4), (6, 5)\}$

There is no repetition in the domain of r.

$\Rightarrow$ r is a function

now its inverse is

$r^{-1} = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

As there is no repetition in the domain of r^{-1}.

$\Rightarrow$ r^{-1} is also a function.

(ii) $\{(1, 3), (2, 5), (3, 7), (4, 9), (5, 11)\}$

Let

$r = \{(1, 3), (2, 5), (3, 7), (4, 9), (5, 11)\}$

As there is no repetition in the domain of r.

$\Rightarrow$ r is a function

now

$r^{-1} = \{(3, 1), (5, 2), (7, 3), (9, 4), (11, 5)\}$

There is no repetition in the domain of r^{-1}.

$\Rightarrow$ r^{-1} is also a function.

(iii) $\{(x, y) \mid y = 2x + 3,\ x \in \Re\}$

Let $r = \{(x, y) \mid y = 2x + 3,\ x \in \Re\}$

'r' is a function because for each value of $x \in \Re$, there is only one value of y.

Now

$$r^{-1} = \{(x, y) \mid x = 2y + 3,\ x \in \Re\}$$
$$= \left\{(x, y) \mid y = \frac{x-3}{2},\ x \in \Re\right\}$$

r^{-1} is also a function because for each value of $x \in \Re$, there is only one value of $y \in \Re$.

(iv) $\{(x, y) \mid y^2 = 4ax, x \geq 0\}$

Let
$$r = \{(x, y) \mid y^2 = 4ax,\ x \geq 0\}$$
$$= \{(x, y) \mid y = \pm\sqrt{4ax},\ x \geq 0\}$$
$$= \{(x, y) \mid y = \pm 2\sqrt{ax},\ x \geq 0\}$$

'r' is not a function because fore each value of x, there are two values of y.

Now

$$r^{-1} = \{(x, y) \mid x^2 = 4ay, x \geq 0\}$$
$$= \left\{(x, y) \mid y = \frac{x^2}{4a},\ x \geq 0\right\}$$

r^{-1} is a function because for each value of x, there is only one value of y.

(v) $\{(x, y) \mid x^2 + y^2 = 9,\ |x| \leq 3,\ |y| \leq 3\}$

Let
$$r = \{(x, y) \mid x^2 + y^2 = 9,\ |x| \leq 3,\ |y| \leq 3\}$$
$$= \left\{(x, y) \mid y = \pm\sqrt{9 - x^2},\ |x| \leq 3,\ |y| \leq 3\right\}$$

'r' is not a function because for each value of x there are two values of y.

Now

$$r^{-1} = \{(x, y) \mid x^2 + y^2 = 9,\ |x| \leq 3,\ |y| \leq 3\}$$
$$= \left\{(x, y) \mid y = \pm\sqrt{9 - x^2},\ |x| \leq 3,\ |y| \leq 3\right\}$$

r^{-1} is not a function because for each value of x, there are two values of y.