$\Rightarrow \quad a_1 = 1$

$a_2 = -3$

$a_3 = 5$

$a_4 = -7$

$a_5 = 9$

$a_6 = -11$

$a_7 = 13$

$a_8 = -15$

$\Rightarrow$ next two terms are $12, -15$.

Arithmetic Progression (A. P)

A sequence $\{a_n\}$ is an Arithmetic sequence or Arithmetic progression (A.P) if $a_n - a_{n-1}$ is the same number for all $n \in N$ and $n > 1$. The difference of two consecutive terms of an A.P is called common difference and is usually denoted by d. $a_n = a_1 + (n-1)d$ is called the nth term or general term of the A.P.

General form of A.P $a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \ldots$

EXERCISE 6.2

Q.1 Write the first four terms of the following arithmetic sequence, if

(i) $a_1 = 5$ and other three consecutive terms are 23, 26, 29

(ii) $a_5 = 17$ and $a_9 = 37$

(iii) $a_7 = 7a_4$ and $a_{10} = 33$

Solution:

(i) $a_1 = 5$ and other three consecutive terms are 23, 26, 29

As the given sequence is arithmetic sequence so $d = 26 - 23 = 3$

and $a_1 = 5$ (given)

$\Rightarrow \quad a_2 = a_1 + d = 5 + 3 = 8$

$a_3 = a_2 + d = 8 + 3 = 11$

$a_4 = a_3 + d = 11 + 3 = 14$

$\Rightarrow$ first four terms of the sequence are 5, 8, 11, 14.

(ii) $a_5 = 17$ and $a_9 = 37$

As $a_5 = 17 \quad \Rightarrow \quad a_1 + 4d = 17 \quad \ldots\ldots\ldots (1)$

$a_9 = 37 \quad \Rightarrow \quad a_1 + 8d = 37 \quad \ldots\ldots\ldots (2)$

Subtracting (1) from (2), we get

$4d = 20$

$\Rightarrow \quad d = 5$

Put $d = 5$ in equation (1), we get

$a_1 + 4(5) = 17$

$a_1 + 20 = 17$

$\boxed{a_1 = -3}$

Now

$a_2 = a_1 + d = -3 + 5 = 2$

$a_3 = a_2 + d = 2 + 5 = 7$

$a_4 = a_3 + d = 7 + 5 = 12$

$\Rightarrow$ first four terms of the sequence are

$-3, 2, 7, 12, ...$

(iii) $3a_7 = 7a_4$ and $a_{10} = 33$

$3a_7 = 7a_4$

$\Rightarrow \quad 3(a_1 + 6d) = 7(a_1 + 3d)$

$3a_1 + 18d = 7a_1 + 21d$

$4a_1 + 3d = 0 \quad (1)$

and $\quad a_{10} = 33$

$\Rightarrow \quad a_1 + 9d = 33 \quad (2)$

from equation (1) $3d = -4a_1$

$9d = -12a_1$

Put $9d = -12a_1$ in equation (2), we get

$a_1 - 12a_1 = 33$

$-11a_1 = 33 \quad \Rightarrow \quad \boxed{a_1 = -3}$

Put $a_1 = -3$ in equation (1), we get

$4(-3) + 3d = 0$

$-12 + 3d = 0 \quad \Rightarrow \quad \boxed{d = 4}$

Now

$a_2 = a_1 + d = -3 + 4 = 1$

$a_3 = a_1 + 2d = -3 + 2(4) = -3 + 8 = 5$

$a_4 = a_1 + 3d = -3 + 3(4) = -3 + 12 = 9$

so first four terms of the sequence are $-3, 1, 5, 9, ...$

Q.2 If $a_{n-3} = 2n - 5$ find nth term of the sequence

Solution:

Given $a_{n-3} = 2n - 5$

Put $n = n + 3$, we get

$a_{n+3-3} = 2(n+3) - 5$

$= 2n + 6 - 5$

$a_n = 2n + 1$

$\Rightarrow$ nth term of the sequence $= a_n = 2n + 1$

Q.3 If the 5th term of an A.P is 16 and 20th term is 46 what is the 12th term.

Solution:

Given

$a_5 = 16 \Rightarrow a_1 + 4d = 16$ .... (1)

$a_{20} = 46 \Rightarrow a_1 + 19d = 46$ .... (2)

subtracting equation (1) from equation (2), we get

$15d = 30$

$\boxed{d = 2}$

Put $d = 2$ in equation (1), we get

$a_1 + 4(2) = 16$

$a_1 + 8 = 16$

$a_1 = 16 - 8$

$\boxed{a_1 = 8}$

so required 12^{th} term $= a_{12} = a_1 + 11d$

$= 8 + 11(2)$

$\boxed{a_{12} = 30}$

Q.4 Find 13th term of the sequence x, 1, 2 – x, 3 – 2x, ...

Solution:

Given sequence

$x, 1, 2 - x, 3 - 2x$ ....

$a_1 = x,$ $d = 1 - x,$

so required 13th term will be

$a_{13} = a_1 + 12d = x + 12(1 - x) = x + 12 - 12x$

$a_{13} = 12 - 11x$

Q.5 Find 18th term of A.P if its 6th term is 19 and 9th term is 31.
(Gujranwala Board 2005)

Solution:

Given

$a_6 = 19 \Rightarrow a_1 + 5d = 19$ .... (1)

$a_9 = 31 \Rightarrow a_1 + 8d = 31$ .... (2)

subtracting equation (1) from equation (2)

$3d = 12$

$\boxed{d = 4}$

Put $d = 4$ in equation (1), we get

$a_1 + 5(4) = 19$

$\boxed{a_1 = -1}$

so 18th term of the A.P is

$a_{18} = a_1 + 17d$

$= -1 + 17(4)$

$= -1 + 68 = 67$

Q.6 Which term of A.P 5, 2, –1, ... is – 85? (Lahore Board 2010)

Solution:

Given sequence

5, 2, –1, ...

$a_1 = 5, \quad d = 2 - 5 = -3, \quad a_n = -85$

$n = ?$

As

$an = a_1 + (n-1)d$

Put values

$-85 = 5 + (n-1)(-3)$

$-85 - 5 = -3n + 3$

$-90 = -3n + 3$

$-90 - 3 = -3n$

$-93 = -3n \Rightarrow n = 31$

$\Rightarrow$ -85 is the 31st term of the A.P

Q.7 Which term of the A.P. $-2, 4, 10, \ldots\ldots$ is 148?

Solution:

Given sequence

$-2, 4, 10, \ldots\ldots$

$a = -2, \quad d = 4-(-2) = 6 \quad a_n = 148 \quad n = ?$

As $a_n = a_1 + (n-1)\,d$

Put values

$148 = -2 + (n-1)(6)$

$148 = -2 + 6n - 6$

$148 = 6n - 8$

$148 + 8 = 6n$

$156 = 6n$

$n = 26$

$\Rightarrow$ 148 is 26^{th} term of the A.P.

Q.8 How many terms are there in the A.P. in which
$a_1 = 11, \quad a_n = 68, \quad d = 3.$

Solution:

Given that

$a_1 = 11, \quad a_n = 68, \quad d = 3$

As

$a_n = a_1 + (n-1)\,d$

$68 = 11 + (n-1)(3)$

$68 = 11 + 3n - 3$

$68 = 8 + 3n$

$68 - 8 = 3n \quad \Rightarrow \quad \boxed{n = 20}$

$\Rightarrow$ 68 is the 20th term of the A.P.

Q.9 If nth term of the A.P. is $3n-1$, find A.P.

(Gujranwala Board 2007, Lahore Board 2007)

Solution:

Given nth term of the A.P is

$a_n = 3n - 1$

Put $n = 1, 2, 3, 4$

$n = 1 \quad \Rightarrow \quad a_1 = 3(1) - 1 = 3 - 1 = 2$

$n = 2 \quad \Rightarrow \quad a_2 = 3(2) - 1 = 5$

$n = 3 \quad \Rightarrow \quad a_3 = 3(3) - 1 = 8$

For $n = 4 \quad a_4 = 3(4) - 1 = 11$

so required A.P is $2, 5, 8, 11, \ldots\ldots$

Q.10 Determine whether (i) – 19 (ii) 2 are the terms of the A.P. 17, 13, 9, ... or not.

Solution:

(i) Given

$a_1 = 17, \quad d = 13 - 17 = -4 \quad a_n = -19 \quad n = ?$

As $a_n = a_1 + (n-1)d$

$-19 = 17 + (n-1)(-4)$

$-19 = 17 - 4n + 4$

$-19 = 21 - 4n$

$4n = 21 + 19 = 40$

$\boxed{n = 10}$

– 19 is 10th term of the A.P.

(ii)

Here $a_1 = 17, \quad d = -4, \quad a_n = 2 \quad n = ?$

As $a_n = a_1 + (n-1)d$

$2 = 17 + (n-1)(-4)$

$2 = 17 - 4n + 4$

$2 = 21 - 4n = 19$

$4n = 21 - 2 = 19$

$n = \frac{19}{4}$ (which is not an integer)

$\Rightarrow$ 2 is not the term of given A.P.

Q.11 If l, m, n are the pth, qth, rth terms of an A.P. show that

(i) $l(q-r) + m(r-p) + n(p-q) = 0$

(ii) $p(m-n) + q(n-l) + r(l-m) = 0$

Solution:

(i) $l(q-r) + m(r-p) + n(p-q) = 0$

As $a_n = a_1 + (n-1)d$

$a_p = a_1 + (p-1)d$

$a_q = a_1 + (q-1)d$

$a_r = a_1 + (r-1)d$

it is given that $a_p = l, \quad a_q = m, \quad a_r = n$

$\Rightarrow$

$l = a_1 + (p-1)d \qquad (1)$

$m = a_1 + (q-1)d \qquad (2)$

$n = a_1 + (r-1)d \qquad (3)$

To prove $l(q-r) + m(r-p) + n(p-q) = o$

Take L.H.S.

$l(q-r) + m(r-p) + n(p-q)$

Put values of l, m, n from equation (1), (2), (3) we get

$[a_1 + (p-1)d](q-r) + [a_1 + (q-1)d](r-p) + [a_1 + (r-1)d](p-q)$

$= (a_1 + pd - d)(q-r) + (a_1 + qd - d)(r-p) + (a_1 + rd - d)(p-q)$

$= a_1q - a_1r + pqd - pdr - dq + dr + a_1r - a_1p + qdr - qdp - dr + dp + a_1q$

$+ rdp - rdq - dp + dq$

$0 = \text{R.H.S.}$

(ii) $\mathbf{p(m-n) + q(n-l) + r(l-m) = 0}$

Now to prove

$p(m-n) + q(n-l) + r(l-m) = 0$

Take its L.H.S.

$p(m-n) + q(n-l) + r(l-m)$

Put values of l, m, n from (1), (2), (3)

$p[a_1 + qd - d - (a_1 + rd - d)] + q[a_1 + rd - d - (a_1 + pd - d)]$

$+ r[a_1 + pd - d - (a_1 + qd - d)]$

$= p[a_1 + qd - d - a_1 - rd + d)] + q[a_1 + rd - d - a_1 - pd + d)]$

$+ r[a_1 + pd - d - a_1 - qd + d)]$

$= p[qd - rd] + q[rd - pd] + r[pd - qd]$

$= pqd - prd + qrd - qpd + rpd - rqd$

$= 0 = \text{R.H.S.}$

Hence proved.

Q.12 Find nth term of the sequence **(Lahore Board 2008)**

$\left(\frac{4}{3}\right)^2, \left(\frac{7}{3}\right)^2, \left(\frac{10}{3}\right)^2 \ldots\ldots.$

Solution:

nth term of the sequence 4, 7, 10,….. is

$a_n = a_1 + (n-1)d$

$= 4 + (n-1)(3)$

$= 4 + 3n - 3$

$an = 3n + 1$

so the nth term of the given sequence is $\left(\frac{3n+1}{3}\right)^2$

Q.13 If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in A.P. Show that $b = \frac{2ac}{a+c}$. (Gujranwala Board 2003)

Solution:

As $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$\Rightarrow \quad \frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$

$$\frac{1}{b} + \frac{1}{b} = \frac{1}{c} + \frac{1}{a}$$

$$\frac{2}{b} = \frac{a+c}{ac} \quad \Rightarrow \quad \boxed{b = \frac{2ac}{a+c}}$$

Q.14 If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in A.P. Show that common difference is $\frac{a-c}{2ac}$.

(Gujranwala Board 2006)

Solution:

As $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$\Rightarrow$

$$d = \frac{1}{b} - \frac{1}{a} \quad \text{......... (1)}$$

and $$d = \frac{1}{c} - \frac{1}{b} \quad \text{......... (2)}$$

Adding equation (1) and (2), we get

$$2d = \frac{1}{b} - \frac{1}{a} + \frac{1}{c} - \frac{1}{b}$$

$$2d = \frac{-c+a}{ac} = \frac{a-c}{ac}$$

$\boxed{d = \frac{a-c}{2ac}}$ Hence proved.

Arithmetic Mean (A.M)

A number A is said to be the A.M between the two numbers a and b if a, A, b are in A.P.

Middle term of three consecutive terms in A.P is the A.M between the extreme terms. If a, A, b are in A.P