Chapter 11

TRIGONOMETRIC FUNCTIONS & THEIR GRAPHS

DOMAIN AND RNAGE OF TRIGONOMETRIC FUNCTION

Function	Domain	Range
$y = \sin x$	$\Re$	$-1 \le y \le 1$
$y = \cos x$	$\Re$	$-1 \le y \le 1$
$y = \tan x$	$\forall\, x \in \Re$, but $x \neq (2n+1)\frac{\pi}{2}$, $n \in z$	$\Re$
$y = \cot x$	$\forall\, x \in \Re$, but $x \neq n\pi$, $n \in z$	$\Re$
$y = \sec x$	$\forall\, x \in \Re$, but $x \neq (2n+1)\pi/2$, $n \in z$	$y \ge 1$ or $y \le -1$
$y = \operatorname{cosec} x$	$\forall\, x \in \Re$, but $x \neq n\pi$, $n \in Z$	$y \ge 1$ or $y \le -1$

Period

Period of a trigonometric function is the smallest +ve number which, when added to the original circular measure of the angle, given the same value of the function.

Formula to Find Period

$f(x) = f(x + P)$

Standard Periods

$\sin x = 2\pi$

$\cos x = 2\pi$

$\sec x = 2\pi$

$\operatorname{cosec} x = 2\pi$

$\tan x = \pi$

$\cot x = \pi$

EXERCISE 11.1

Find the periods of the following functions:

Q.1 sin 3 x (Lahore Board 2005)

Solution:

Given function is $\sin 3x$

Since period of $\sin$ is 2π

Therefore $f(x) = f(x + P)$

$\sin 3x = \sin(3x + 2\pi)$

$\sin 3x = \sin 3\left(x + \frac{2\pi}{3}\right)$

Thus period of $\sin 3x$ is $\frac{2\pi}{3}$

Q.2 cos 2 x

Solution:

Given function is $\cos 2x$

Since period of $\cos$ is 2π

Therefore $f(x) = f(x + P)$

$\cos 2x = \cos(2x + 2\pi)$

$\cos 2x = \cos 2(x + \pi)$

Thus period of $\cos 2x$ is π.

Q.3 tan 4 x (Gujranwala Board 2007)

Solution:

Given function is $\tan 4x$

Since period of $\tan x$ is π

Therefore $f(x) = f(x + P)$

$\tan 4x = \tan(4x + \pi)$

$\tan 4x = \tan 4\left(x + \frac{\pi}{4}\right)$

Thus period of $\tan 4x$ is $\frac{\pi}{4}$.

Q.4 $\cot \frac{x}{2}$

Solution:

Given function is $\cot \frac{x}{2}$

Since period of cot is π

Therefore $f(x) = f(x + P)$

$$\cot \frac{x}{2} = \cot\left(\frac{x}{2} + \pi\right)$$

$$\cot \frac{x}{2} = \cot \frac{1}{2}(x + 2\pi)$$

Thus period of $\cot \frac{x}{2}$ is 2π

Q.5 $\sin \frac{x}{3}$ **(Lahore Board 2006)**

Solution:

Given function is $\sin \frac{x}{3}$

Since period of sin is 2π

Therefore $f(x) = f(x + P)$

$$\sin \frac{x}{3} = \sin\left(\frac{x}{3} + 2\pi\right)$$

$$\sin \frac{x}{3} = \sin \frac{1}{3}(x + 6\pi)$$

Thus period of $\sin \frac{x}{3}$ is 6π

Q.6 $\operatorname{cosec} \frac{x}{4}$

Solution:

Given function is $\operatorname{cosec} \frac{x}{4}$

Since period of cosec is 2π

Therefore $f(x) = f(x + P)$

$$\operatorname{cosec} \frac{x}{4} = \operatorname{cosec}\left(\frac{x}{4} + 2\pi\right)$$

$$\operatorname{cosec}\frac{x}{4} = \operatorname{cosec}\frac{1}{4}(x + 8\pi)$$

Thus period of $\operatorname{cosec}\frac{x}{4}$ is 8π

Q.7 $\sin\frac{x}{5}$ (Lahore Board 2010)

Solution:

Given function is $\sin\frac{x}{5}$

Since period of $\sin x$ is 2π

Therefore $f(x) = f(x + P)$

$$\sin\frac{x}{5} = \sin\left(\frac{x}{5} + 2\pi\right)$$

$$\sin\frac{x}{5} = \sin\frac{1}{5}(x + 10\pi)$$

Thus period of $\sin\frac{x}{5}$ is 10π

Q.8 $\cos\frac{x}{6}$

Solution:

Given function is $\cos\frac{x}{6}$

Since period of $\cos x$ is 2π

$f(x) = f(x + P)$

Therefore $\cos\frac{x}{6} = \cos\left(\frac{x}{6} + 2\pi\right) = \cos\frac{1}{6}(x + 12\pi)$

Thus period of $\cos\frac{x}{6}$ is 12π

Q.9 $\tan\frac{x}{7}$

Solution:

Given function is $\tan\frac{x}{7}$

Since period of $\tan x$ is π

$f(x) = f(x + P)$

Therefore $\tan \frac{x}{7} = \tan\left(\frac{x}{7} + \pi\right)$

$\tan \frac{x}{7} = \tan \frac{1}{7}(x + 7\pi)$

Thus period of $\tan \frac{x}{7}$ is 7π

Q.10 cot 8 x

Solution:

Given function is cot 8 x

Since period of cot x is π

$f(x) = f(x + P)$

Therefore $\cot 8x = \cot(8x + \pi)$

$\cot 8x = \cot 8\left(x + \frac{\pi}{8}\right)$

Thus period of cot 8 x is $\frac{\pi}{8}$

Q.11 sec 9 x

Solution:

Given function is sec 9 x

Since period of sec x is 2π

$f(x) = f(x + P)$

Therefore $\sec 9x = \sec(9x + 2\pi)$

$\sec 9x = \sec 9\left(x + \frac{2\pi}{9}\right)$

Thus period of sec 9 x is $\frac{2\pi}{9}$

Q.12 cosec 10 x

Solution:

Given function is cosec 10 x

Since period of cosec x is 2π

$f(x) = f(x + P)$

Therefore cosec 10 x = cosec (10x + 2π)

cosec 10 x = cosec $10\left(x + \frac{2\pi}{10}\right)$

Thus period of cosec 10 x is $\frac{\pi}{5}$

Q.13 3 sin x

Solution:

Given function is 3 sin x

Since period of sin x is 2 π

f (x) = f (x + P)

Therefore 3 sin x = 3 sin (x + 2π)

Thus period of 3 sin x is 2 π.

Q.14 2 cos x

Solution:

Given function is 2 cos x

Since period of cos x is 2 π

f (x) = f (x + P)

Therefore 2 cos x = 2 cos (x + 2π)

Thus period of 2 cos x is 2 π

Q.15 3 cos $\frac{x}{5}$ (Lahore Board 2008)

Solution:

Given function is 3 cos $\frac{x}{5}$

Since period of cos x is 2 π

f (x) = f (x + P)

Therefore 3 cos $\frac{x}{5}$ = 3 cos $\left(\frac{x}{5} + 2\pi\right)$

3 cos $\frac{x}{5}$ = 3 cos $\frac{1}{5}$ (x + 10 π)

Thus period of 3 cos $\frac{x}{5}$ is 10 π .