Q.5 Prove that $p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$ (Lahore Board 2005)

Solution:

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \vee (\sim p \wedge \sim q)$	$P \wedge q$	$p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$
T	T	F	F	F	T	T	F
T	F	F	F	F	T	F	T
F	T	T	T	F	F	F	T
F	F	T	T	T	T	F	T

As entries in the columns of $p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$ and $p \vee (\sim p \wedge \sim q)$ are same.

$\Rightarrow$ $p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$

Hence proved.

EXERCISE 2.5

Convert the following theorems to logical and prove them by constructing truth tables.

Q.1 $(A \cap B)' = A' \cup B'$ (Lahore Board 2004)

Solution:

Its logical form is $\sim (p \wedge q) = \sim p \vee \sim q$ its truth table is given below

p	q	$p \wedge q$	$\sim (p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

As entries in the columns of $\sim (p \wedge q)$ and $\sim p \vee \sim q$ are same.

$\Rightarrow$ $\sim (p \wedge q) = \sim p \vee \sim q$

$\Rightarrow$ $(A \cap B)' = A' \cup B'$

Q.2 $(A \cup B)' = A' \cap B'$

Solution:

$(A \cup B) \cup C = A \cup (B \cup C)$

Its logical form is $(p \vee q) \vee r = p \vee (q \vee r)$

Its truth table is given below

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$q \vee r$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
F	T	T	T	T	T	T
F	F	T	F	T	T	T
F	T	F	T	T	T	T
T	F	F	T	T	F	T
F	F	F	F	F	F	F

As the entries in columns of $(p \vee q) \vee r$ and $p \vee (q \vee r)$ are same.

$\Rightarrow$ $(p \vee q) \vee r = p \vee (q \vee r)$

$\Rightarrow$ $(A \cup B) \cup C = A \cup (B \cup C)$

Hence proved.

Q.3 $(A \cap B) \cap C = A \cap (B \cap C)$

Solution:

$(A \cap B) \cap C = A \cap (B \cap C)$

Its logical form is $(p \wedge q) \wedge r = p \wedge (q \wedge r)$.

Its truth table is given below.

p	q	r	$p \wedge q$	$(p \wedge q) \wedge r$	$(q \wedge r)$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
F	T	T	F	F	T	F
F	F	T	F	F	F	F
F	T	F	F	F	F	F
T	F	F	F	F	F	F
F	F	F	F	F	F	F

As entries in the columns of $(p \wedge q) \wedge r$ and $p \wedge (q \wedge r)$ are same.

$\Rightarrow$ $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

$\Rightarrow$ $(A \cap B) \cap C = A \cap (B \cap C)$

Q.4 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution:

Its logical form is $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$. Its truth table is given below.

p	q	r	$q \wedge r$	$p \vee (q \vee r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
F	T	T	T	T	T	T	T
F	F	T	F	F	F	T	F
F	T	F	F	F	T	F	F
T	F	F	F	T	T	T	T
F	F	F	F	F	F	F	F

As entries in the columns of $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are same. So

$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

or

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Hence proved.

EXERCISE 2.6

Binary Relation

Let A and B be two non–empty sets, then any subset of Cartesian product $A \times B$ is called a binary relation, or simply a relation from A to B.

Q.1 For $A = \{1, 2, 3, 4\}$, find the following relation in A. State the domain and range of each relation. Also draw the graph of each.

(i) $\{(x, y) \mid y = x\}$ (Lahore Board 2010)

(ii) $\{(x, y) \mid y + x = 5\}$

(iii) $\{(x, y) \mid x + y < 5\}$ (Lahore Board 2011)

(iv) $\{(x, y) \mid x + y > 5\}$ (Gujranwala Board 2003, Lahore Board 2003)

Solution:

Given that

$A = \{1, 2, 3, 4\}$

Then

$$A \times A = \{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$$
$$= \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$$