$(\because \quad |\hat{b}| = 1, \quad |\hat{a}| = 1)$

$\text{Sin}(\alpha - \beta) = \text{Sin}\alpha\,\text{Cos}\beta - \text{Cos}\alpha\,\text{Sin}\beta$ Hence proved

Q.9 If $\underline{a} \times \underline{b} = 0$ and $\underline{a} \cdot \underline{b} = 0$. What conclusion can be drawn about $\underline{a}$ or $\underline{b}$?
(Gujranwala Board 2004, 2007, Lahore Board 2009 (Supply)

Solution:

If $\underline{a} \times \underline{b} = 0$ $\Rightarrow$ (i) $\underline{a}$ and $\underline{b}$ are parallel (ii) Either $\underline{a} = 0$ or $\underline{b} = \underline{0}$

If $\underline{a} \cdot \underline{b} = 0$ $\Rightarrow$ (iii) $\underline{a}$ and $\underline{b}$ are perpendicular (iv) Either $\underline{a} = \underline{0}$ or $\underline{b} = \underline{0}$

This is not possible that $\underline{a}$ and $\underline{b}$ are parallel and perpendicular at the same time

So either $\underline{a} = \underline{0}$ or $\underline{b} = \underline{0}$

$\therefore$ $\underline{a}$ and $\underline{b}$ are null vectors.

EXERCISE 7.5

Q.1 Find the volume of parallelepiped for which the given vectors are three edges.

(i) $\underline{u} = 3\underline{i} + 0\underline{j} + 2\underline{k}$; $\underline{v} = \underline{i} + 2\underline{j} + \underline{k}$; $\underline{w} = 0\underline{i} - \underline{j} + 4\underline{k}$

Solution:

Formula

Volume of parallelepiped $= \underline{u} \cdot (\underline{v} \times \underline{w})$

$$\underline{u} \cdot (\underline{v} \times \underline{w}) = \begin{vmatrix} 3 & 0 & 2 \\ 1 & 2 & 1 \\ 0 & -1 & 4 \end{vmatrix}$$

$$= 3\begin{vmatrix} 2 & 1 \\ -1 & 4 \end{vmatrix} - 0\begin{vmatrix} 1 & 1 \\ 0 & 4 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix}$$

$= 3(8 + 1) - 0 + 2(-1) = 27 - 2 = 25$ cubic units Ans.

(ii) $\underline{u} = \underline{i} - 4\underline{j} - \underline{k}$; $\underline{v} = \underline{i} - \underline{j} - 2\underline{k}$; $\underline{w} = 2\underline{i} - 3\underline{j} + \underline{k}$

Solution:

Volume of parallelepiped $= \underline{u} \cdot (\underline{v} \times \underline{w})$

$$= \begin{vmatrix} 1 & -4 & -1 \\ 1 & -1 & -2 \\ 2 & -3 & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} -1 & -2 \\ -3 & 1 \end{vmatrix} + 4\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix}$$

$= 1(-1-6) + 4(1+4) - 1(-3+2) \quad = -7 + 20 + 1 = 14$ cubic units Ans.

(iii) $\underline{u} = \underline{i} - 2\underline{j} + 3\underline{k}$; $\underline{v} = 2\underline{i} - \underline{j} - \underline{k}$; $\underline{w} = \underline{j} + \underline{k}$

Solution:

Volume of parallelepiped = $\underline{u} \cdot (\underline{v} \times \underline{w})$

$$\underline{u} \cdot (\underline{v} \times \underline{w}) = \begin{vmatrix} 1 & -2 & 3 \\ 2 & -1 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} -1 & -1 \\ 1 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} + 3\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix}$$

$= 1(-1+1) + 2(2-0) + 3(2-0)$

$= 4 + 6 = 10$ cubic units Ans.

Q.2 Verify that $\underline{a} \cdot \underline{b} \times \underline{c} = \underline{b} \cdot \underline{c} \times \underline{a} = \underline{c} \cdot \underline{a} \times \underline{b}$

If $\underline{a} = 3\underline{i} - \underline{j} + 5\underline{k}$, $\underline{b} = 4\underline{i} + 3\underline{j} - 2\underline{k}$, $\underline{c} = 2\underline{i} + 5\underline{j} + \underline{k}$

(Gujranwala Board, 2003, Lahore Board 2007)

Solution:

$$\underline{a} \cdot (\underline{b} \times \underline{c}) = \begin{vmatrix} 3 & -1 & 5 \\ 4 & 3 & -2 \\ 2 & 5 & 1 \end{vmatrix}$$

$= 3(3+10) + 1(4+4) + 5(20-6)$

$= 39 + 8 + 70 = 117$ .... (i)

$$\underline{b} \cdot (\underline{c} \times \underline{a}) = \begin{vmatrix} 4 & 3 & -2 \\ 2 & 5 & 1 \\ 3 & -1 & 5 \end{vmatrix}$$

$= 4(25+1) - 3(10-3) - 2(-2-15)$

$= 104 - 21 + 34 = 117$ .... (ii)

$$\underline{c} \cdot (\underline{a} \times \underline{b}) = \begin{vmatrix} 2 & 5 & 1 \\ 3 & -1 & 5 \\ 4 & 3 & -2 \end{vmatrix}$$

$= 2(2-15) - 5(-6-20) + 1(9+4)$

$$= -26 + 130 + 13 = 117 \quad \text{........ (iii)}$$

From (i), (ii) & (iii) it is verified that

$$\underline{a} \cdot \underline{b} \times \underline{c} = \underline{b} \cdot \underline{c} \times \underline{a} = \underline{c} \cdot \underline{a} \times \underline{b}$$

Q.3 Prove that the vectors $\underline{i} - 2\underline{j} + 3\underline{k}$, $-2\underline{i} + 3\underline{j} - 4\underline{k}$ and $\underline{i} - 3\underline{j} + 5\underline{k}$ are coplaner. (Gujranwala Board 2007)

Solution:

Let $\underline{u} = \underline{i} - 2\underline{j} + 3\underline{k}$, $\underline{v} = -2\underline{i} + 3\underline{j} - 4\underline{k}$, $\underline{w} = \underline{i} - 3\underline{j} + 5\underline{k}$

$$\underline{u} \cdot \underline{v} \times \underline{w} = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}$$

$$= 1\,(15 - 12) + 2(-10 + 4) + 3(6 - 3) \quad = 3 - 12 + 9 \quad = 12 - 12 = 0$$

$$\underline{u} \cdot \underline{v} \times \underline{w} = 0$$

Hence $\underline{u}$, $\underline{v}$, $\underline{w}$ are coplanar.

Q.4 Find the constant α such that the vectors are coplanar.

(i) $\underline{i} - \underline{j} + \underline{k}$, $\underline{i} - 2\underline{j} - 3\underline{k}$ & $3\underline{i} - \alpha\underline{j} + 5\underline{k}$ (Lahore Board 2007, 2009)

Solution:

Let

$$\underline{a} = \underline{i} - \underline{j} + \underline{k}$$
$$\underline{b} = \underline{i} - 2\underline{j} - 3\underline{k}$$
$$\underline{c} = 3\underline{i} - \alpha\underline{j} + 5\underline{k}$$

Since given vectors are coplanar so

$$\underline{a} \cdot \underline{b} \times \underline{c} = 0$$

$$\begin{vmatrix} 1 & -1 & 1 \\ 1 & -2 & -3 \\ 3 & -\alpha & 5 \end{vmatrix} = 0$$

$$1\,(-10 - 3\alpha) + 1\,(5 + 9) + 1\,(-\alpha + 6) = 0$$
$$-10 - 3\alpha + 14 - \alpha + 6 = 0$$
$$-4\alpha + 10 = 0$$
$$4\alpha = 10$$
$$\alpha = \frac{10}{4} = \frac{5}{2} \qquad \text{Ans.}$$

(ii) $\underline{a} = \underline{i} - 2\alpha\underline{j} - \underline{k}$, $\underline{b} = \underline{i} - \underline{j} + 2\underline{k}$, $\underline{c} = \alpha\underline{i} - 2\underline{j} + \underline{k}$

Solution:

Since $\underline{a}$, $\underline{b}$, $\underline{c}$ are coplanar so $\underline{a} \cdot \underline{b} \times \underline{c} = 0$

$$\begin{vmatrix} 1 & -2\alpha & -1 \\ 1 & -1 & 2 \\ \alpha & -2 & 1 \end{vmatrix} = 0$$

$1(-1+4) + 2\alpha(1-2\alpha) - 1(-2+\alpha) = 0$

$3 + 2\alpha - 4\alpha^2 + 2 - \alpha = 0$

$-4\alpha^2 + \alpha + 5 = 0$

$4\alpha^2 - \alpha - 5 = 0$

$4\alpha^2 - 5\alpha + 4\alpha - 5 = 0$

$\alpha(4\alpha - 5) + 1(4\alpha - 5) = 0$

$(4\alpha - 5)(\alpha + 1) = 0$

$4\alpha - 5 = 0 \quad , \quad \alpha + 1 = 0$

$4\alpha = 5 \quad\quad \alpha = -1$

$\alpha = \frac{5}{4} \quad , \quad \alpha = -1$ Ans.

Q.5 Find the value of

(i) $2\underline{i} \times 2\underline{j} \cdot \underline{k}$

Solution:

$2\underline{i} \times 2\underline{j} \cdot \underline{k} = 4(\underline{i} \times \underline{j}) \cdot \underline{k} = 4(\underline{k} \cdot \underline{k}) = 4$

(ii) $3\underline{j} \cdot \underline{k} \times \underline{i}$

Solution:

$3\underline{i} \cdot \underline{k} \times \underline{i} = 3\underline{j} \cdot \underline{j} = 3(1) = 3$ Ans.

(iii) $[\underline{k}\ \ \underline{i}\ \ \underline{j}]$

Solution:

$= \underline{k} \cdot \underline{i} \times \underline{j} = \underline{k} \cdot \underline{k} = 1$ Ans.

(iv) $[\underline{i}\ \ \underline{i}\ \ \underline{k}]$

Solution:

$= \underline{i} \cdot \underline{i} \times \underline{k} = \underline{i} \cdot (-\underline{j}) = -(\underline{i} \cdot \underline{j}) = -0 = 0$ Ans.

Q.5(b) Prove that $\underline{u} \cdot (\underline{v} \times \underline{w}) + \underline{v} \cdot (\underline{w} \times \underline{u}) + \underline{w} \cdot (\underline{u} \times \underline{v}) = 3\underline{u} \cdot (\underline{v} \times \underline{w})$

(Lahore Board, 2011)

Solution:

L.H.S. $\underline{u} \cdot (\underline{v} \times \underline{w}) + \underline{v} \cdot (\underline{w} \times \underline{u}) + \underline{w} \cdot (\underline{u} \times \underline{v})$

We know that

$\underline{u} \cdot (\underline{v} \times \underline{w}) = \underline{v} \cdot (\underline{u} \times \underline{w}) = \underline{w} \cdot (\underline{v} \times \underline{u})$

Putting values in L.H.S.

$= \underline{u} \cdot (\underline{v} \times \underline{w}) + \underline{u} \cdot (\underline{v} \times \underline{w}) + \underline{u} \cdot (\underline{v} \times \underline{w})$

$= \quad 3\underline{u} \cdot (\underline{v} \times \underline{w})$

R.H.S Hence proved

Q.6 Find volume of tetrahedron with the vertices

(i) (0, 1, 2) , (3, 2, 1), (1, 2, 1) & (5, 5, 6)

Solution:

Formula

Volume of tetrahedron when A,B,C, D whose vertices are given $= \frac{1}{6}(\overrightarrow{AB} . \overrightarrow{AC} \times \overrightarrow{AD})$

Let A (0, 1, 2), B (3, 2, 1) . C (1, 2, 1) D (5, 5, 6)

$\overrightarrow{AB} =$ Position vector of B-Position vector of A

$= (3-0)\underline{i} + (2-1)\underline{j} + (1-2)\underline{k}$

$= 3\underline{i} + \underline{j} - \underline{k}$

$\overrightarrow{AC} =$ Position vector of C-Position vector of A

$= (1-0)\underline{i} + (2-1)\underline{j} + (1-2)\underline{k}$

$\overrightarrow{AC} = \underline{i} + \underline{j} - \underline{k}$

$\overrightarrow{AD} =$ Position vector of D-Position vector of A

$= (5-0)\underline{i} + (5-1)\underline{j} + (6-2)\underline{k}$

$\overrightarrow{AD} = 5\underline{i} + 4\underline{j} + 4\underline{k}$

Now

$$\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} = \begin{vmatrix} 3 & 1 & -1 \\ 1 & 1 & -1 \\ 5 & 4 & 4 \end{vmatrix}$$

$= 3(4+4) - 1(4+5) - 1(4-5) = 24 - 9 + 1 = 16$

Volume of tetrahedron $= \frac{1}{6}(16) = \frac{8}{3}$ cubic units Ans.

(ii) (2, 1, 8) , (3, 2, 9), (2, 1, 4) & (3, 3, 10) (Lahore Board 2011)

Solution:

Let A (2, 1, 8), B (3, 2, 9), C (2, 1, 4), D (3, 3, 10)

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (3-2)\underline{i} + (2-1)\underline{j} + (9-8)\underline{k}$

$\overrightarrow{AB} = \underline{i} + \underline{j} + \underline{k}$

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$$
$$= (2-2)\underline{i} + (1-1)\underline{j} + (4-8)\underline{k}$$
$$\overrightarrow{AC} = 0\underline{i} + 0\underline{j} - 4\underline{k}$$
$$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$$
$$= (3-2)\underline{i} + (3-1)\underline{j} + (10-8)\underline{k}$$
$$\overrightarrow{AD} = \underline{i} + 2\underline{j} + 2\underline{k}$$
$$\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & -4 \\ 1 & 2 & 2 \end{vmatrix}$$
$$= 1(0+8) - 1(0+4) + 1(0-0) = 8-4 = 4$$

Volume of tetrahedron $= \frac{1}{6}(\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD}) = \frac{1}{6}(4) = \frac{2}{3}$ cubic units Ans.

Q.7 Find the work done, if the point at which the constant force $\overrightarrow{F} = 4\underline{i} + 3\underline{j} + 5\underline{k}$ is applied to an object, moves from $P_1(3, 1, -2)$ to $P_2(2, 4, 6)$

(Gujranwala Board, 2004)

Solution:

Given $\overrightarrow{F} = 4\underline{i} + 3\underline{j} + 5\underline{k}$

$$\overrightarrow{d} = \overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1}$$
$$= (2-3)\underline{i} + (4-1)\underline{j} + (6+2)\underline{k}$$
$$\overrightarrow{d} = -\underline{i} + 3\underline{j} + 8\underline{k}$$

Work done $= \overrightarrow{F} \cdot \overrightarrow{d}$
$$= (4\underline{i} + 3\underline{j} + 5\underline{k}) \cdot (-\underline{i} + 3\underline{j} + 8\underline{k})$$
$$= -4 + 9 + 40$$
$$= 45 \quad \text{Ans.}$$

Q.8 A particle, acted by constant forces $4\underline{i} + \underline{j} - 3\underline{k}$ and $3\underline{i} - \underline{j} - \underline{k}$ is displacement from A(1, 2,3) to B (5, 4, 1). Find the work done.

Solution:

$$\overrightarrow{F_1} = 4\underline{i} + \underline{j} - 3\underline{k}, \quad \overrightarrow{F_2} = 3\underline{i} - \underline{j} - \underline{k}$$

$$\vec{F} = \vec{F_1} + \vec{F_2} = 7\underline{i} + 0\underline{j} - 4\underline{k},$$

$$\vec{d} = \vec{AB} = \vec{OB} - \vec{OA}$$
$$= (5-1)\underline{i} + (4-2)\underline{j} + (1-3)\underline{k}$$
$$\vec{d} = 4\underline{i} + 2\underline{j} - 2\underline{k}$$

$$\text{Work done} = \vec{F} \cdot \vec{d}$$
$$= (7\underline{i} + 0\underline{j} - 4\underline{k}) \cdot (4\underline{i} + 2\underline{j} - 2\underline{k})$$
$$= 28 + 0 + 8$$
$$= 36 \quad \text{Ans.}$$

Q.9 A particle is displaced from the point A(5,–5,–7) to the point B(6, 2, –2) under the action of constant forces defined by $10\underline{i} - \underline{j} + 11\underline{k}$, $4\underline{i} + 5\underline{j} + 9\underline{k}$ and $-2\underline{i} + \underline{j} - 9\underline{k}$. Show that the total work done by the forces is 102 unit.

Solution:

$$\vec{d} = \vec{AB} = \vec{OB} - \vec{OA}$$
$$= (6-5)\underline{i} + (2+5)\underline{j} + (-2+7)\underline{k}$$
$$\vec{d} = \underline{i} + 7\underline{j} + 5\underline{k}$$
$$\vec{F_1} = 10\underline{i} - \underline{j} + 11\underline{k}, \quad \vec{F_2} = 4\underline{i} + 5\underline{j} + 9\underline{k}, \quad \vec{F_3} = 2\underline{i} + \underline{j} - 9\underline{k}$$

Total forces $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3}$
$$= 10\underline{i} - \underline{j} + 11\underline{k} + 4\underline{i} + 5\underline{j} + 9\underline{k} - 2\underline{i} + \underline{j} - 9\underline{k}$$
$$\vec{F} = 12\underline{i} + 5\underline{j} + 11\underline{k}$$

$$\text{Work done} = \vec{F} \cdot \vec{d}$$
$$= (12\underline{i} + 5\underline{j} + 11\underline{k}) \cdot (\underline{i} + 7\underline{j} + 5\underline{k})$$
$$= 12 + 35 + 55$$
$$= 102 \text{ units} \quad \text{Hence proved} \quad \text{Ans.}$$

Q.10 A force of magnitude 6 units acting parallel to $2\underline{i} - 2\underline{j} + \underline{k}$ displaces, the point of application from (1, 2, 3) to (5, 3, 7). Find the work done.

Solution:

Let $\underline{u} = 2\underline{i} - 2\underline{j} + \underline{k}$

$$|\underline{u}| = \sqrt{(2)^2 + (-2)^2 + (1)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$$

$$\hat{n} = \frac{\underline{u}}{|\underline{u}|} = \frac{2\underline{i} - 2\underline{j} + \underline{k}}{\sqrt{3}}$$

Hence, the force of magnitude 6 units is

$$\vec{F} = 6\hat{n}$$

$$= 6\left(\frac{2\underline{i} - 2\underline{j} + \underline{k}}{3}\right)$$

$$\vec{F} = 4\underline{i} - 4\underline{j} + 2\underline{k}$$

Let

A (1, 2, 3), B (5, 3, 7)

$$\vec{d} = \vec{AB} = \vec{OB} - \vec{OA}$$

$$= (5-1)\underline{i} + (3-2)\underline{j} + (7-3)\underline{k}$$

$$\vec{d} = 4\underline{i} + \underline{j} + 4\underline{k}$$

$$\text{Work done} = \vec{F} \cdot \vec{d}$$

$$= (4\underline{i} - 4\underline{j} + 2\underline{k}) \cdot (4\underline{i} + \underline{j} + 4\underline{k})$$

$$= 16 - 4 + 8 = 20 \quad \text{Ans.}$$

Q.11 A force $\vec{F} = 3\underline{i} + 2\underline{j} - 4\underline{k}$ is applied at the point (1, –1, 2). Find the moment of the force about the point (2, –1, 3)

Solution:

$$\vec{F} = 3\underline{i} + 2\underline{j} - 4\underline{k}$$

Let A (1, –1, 2) and B(2, –1,3)

$$\vec{r} = \vec{BA} = \vec{OA} - \vec{OB}$$

$$= (1-2)\underline{i} + (-1+1)\underline{j} + (2-3)\underline{k}$$

$$\vec{r} = -\underline{i} + 0\underline{j} - \underline{k}$$

$$\text{Required moment } M = \vec{r} \times \vec{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1 & 0 & -1 \\ 3 & 2 & -4 \end{vmatrix}$$

$$= \underline{i}\begin{vmatrix} 0 & -1 \\ 2 & -4 \end{vmatrix} - \underline{j}\begin{vmatrix} -1 & -1 \\ 3 & -4 \end{vmatrix} + \underline{k}\begin{vmatrix} -1 & 0 \\ 3 & 2 \end{vmatrix}$$

$$= \underline{i}\,(0+2) - \underline{j}\,(4+3) + \underline{k}\,(-2-0)$$

$= 2\underline{i} - 7\underline{j} - 2\underline{k}$ Ans.

Q.12 A force $\overrightarrow{F} = 4\underline{i} - 3\underline{k}$ passes through the point A(2, –2, 5). Find the moment of $\overrightarrow{F}$ about point B (1, – 3, 1) **(Lahore Board 2009)**

Solution:

$$\overrightarrow{F} = 4\underline{i} + 0\underline{j} - 3\underline{k}$$

$$\overrightarrow{r} = \overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$$

$$= (2-1)\underline{i} + (-2+3)\underline{j} + (5-1)\underline{k}$$

$$\overrightarrow{r} = \underline{i} + \underline{j} + 4\underline{k}$$

Required moment

$$\overrightarrow{r} \times \overrightarrow{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 1 & 4 \\ 4 & 0 & -3 \end{vmatrix} = \underline{i}\begin{vmatrix} 1 & 4 \\ 0 & -3 \end{vmatrix} - \underline{j}\begin{vmatrix} 1 & 4 \\ 4 & -3 \end{vmatrix} + \underline{k}\begin{vmatrix} 1 & 1 \\ 4 & 0 \end{vmatrix}$$

$$= \underline{i}(-3-0) - \underline{j}(-3-16) + \underline{k}(0-4)$$

$$\overrightarrow{r} \times \overrightarrow{F} = -3\underline{i} + 19\underline{j} - 4\underline{k} \quad \text{Ans.}$$

Q.13 Give a force $\overrightarrow{F} = 2\underline{i} + \underline{j} - 3\underline{k}$ acting at a point A (1,–2,1). Find the moment of $\overrightarrow{F}$ about the point B (2, 0,–2)

Solution:

$$\overrightarrow{F} = 2\underline{i} + \underline{j} - 3\underline{k}$$

$$\overrightarrow{r} = \overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$$

$$= (1-2)\underline{i} + (-2-0)\underline{j} + (1+2)\underline{k}$$

$$\overrightarrow{r} = -\underline{i} - 2\underline{j} + 3\underline{k}$$

Required moment

$$\overrightarrow{r} \times \overrightarrow{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1 & -2 & 3 \\ 2 & 1 & -3 \end{vmatrix}$$

$$= \underline{i}\begin{vmatrix} -2 & 3 \\ 1 & -3 \end{vmatrix} - \underline{j}\begin{vmatrix} -1 & 3 \\ 2 & -3 \end{vmatrix} + \underline{k}\begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix}$$

$$= \underline{i}(6-3) - \underline{j}(3-6) + \underline{k}(-1+4)$$

$$= 3\underline{i} + 3\underline{j} + 3\underline{k} \qquad \text{Ans.}$$

Q.14 Find the moment about A(1, 1, 1) of each of the concurrent forces $\underline{i} - 2\underline{j}$, $3\underline{i} + 2\underline{j} - \underline{k}$, $5\underline{j} + 2\underline{k}$, where P (2, 0, 1) is their point of concurrency.

(Lahore Board 2009)

Solution:

$$\vec{r} = \vec{AP} = \vec{OP} - \vec{OA}$$

$$= (2-1)\underline{i} + (0-1)\underline{j} + (1-1)\underline{k}$$

$$\vec{r} = \underline{i} - \underline{j} + 0\underline{k}$$

$$\vec{F_1} = \underline{i} - 2\underline{j}, \quad \vec{F_2} = 3\underline{i} + 2\underline{j} - \underline{k}, \quad \vec{F_3} = 0\underline{i} + 5\underline{j} + 2\underline{k}$$

$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3}$$

$$= \underline{i} - 2\underline{j} + 3\underline{i} + 2\underline{j} - \underline{k} + 0\underline{i} + 5\underline{j} + 2\underline{k}$$

$$\vec{F} = 4\underline{i} + 5\underline{j} + \underline{k}$$

Moment of force

$$\vec{r} \times \vec{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -1 & 0 \\ 4 & 5 & 1 \end{vmatrix}$$

$$= \underline{i}\begin{vmatrix} -1 & 0 \\ 5 & 1 \end{vmatrix} - \underline{j}\begin{vmatrix} 1 & 0 \\ 4 & 1 \end{vmatrix} + \underline{k}\begin{vmatrix} 1 & -1 \\ 4 & 5 \end{vmatrix}$$

$$= \underline{i}(-1-0) - \underline{j}(1-0) + \underline{k}(5+4)$$

$$\vec{r} \times \vec{F} = -\underline{i} - \underline{j} + 9\underline{k} \qquad \text{Ans.}$$

Q.15 A force $\vec{F} = 7\underline{i} + 4\underline{j} - 3\underline{k}$ is applied at P (1, –2, 3). Find its moment about the point Q (2, 1, 1).

Solution:

$$\vec{F} = 7\underline{i} + 4\underline{j} - 3\underline{k}$$

$$\vec{r} = \vec{QP} = \vec{OD} - \vec{OQ}$$

$$= (1-2)\underline{i} + (-2-1)\underline{j} + (3-1)\underline{k}$$

$$\vec{r} = -\underline{i} - 3\underline{j} + 2\underline{k}$$

Moment of force

$$\vec{r} \times \vec{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1 & -3 & 2 \\ 7 & 4 & -3 \end{vmatrix}$$

$$= \underline{i}\begin{vmatrix} -3 & 2 \\ 4 & -3 \end{vmatrix} - \underline{j}\begin{vmatrix} -1 & 2 \\ 7 & -3 \end{vmatrix} + \underline{k}\begin{vmatrix} -1 & -3 \\ 7 & 4 \end{vmatrix}$$

$$= \underline{i}\,(9-8) - \underline{j}\,(3-14) + \underline{k}\,(-4+21)$$

$$\vec{r} \times \vec{F} = \underline{i} + 11\underline{j} + 17\underline{k} \qquad \text{Ans.}$$