$$\boxed{C = \frac{(a-d)}{(b-d)(c-d)}}$$

Put these values in equation (1), we get

$$\frac{y+a}{(y+b)(y+c)(y+d)} = \frac{\frac{(a-b)}{(c-b)(d-b)}}{y+b} + \frac{\frac{(a-c)}{(b-c)(d-c)}}{y+c} + \frac{\frac{(a-d)}{(b-b)(c-d)}}{y+d}$$

$$\frac{a}{b+c+d} = \frac{a-b}{(c-b)(d-b)(y+b)} + \frac{a-c}{(b-c)(d-c)(y+c)} + \frac{a-d}{(b-d)(c-d)(y+d)}$$

Replacing the neglecting squares, we get

$$\frac{x^2+a^2}{(x^2+b^2)(x^2+c^2)(x^2+d^2)} = \frac{a^2-b^2}{(c^2-b^2)(d^2-b^2)(x^2+b^2)} + \frac{a^2-c^2}{(b^2-c^2)(d^2-c^2)(x^2+c^2)} + \frac{a^2-d^2}{(b^2-d^2)(c^2-d^2)(x^2+d^2)}$$

are required Partial fractions.

EXERCISE 5.2

Resolve the following into Partial fraction.

Q.1 $\dfrac{2x^2-3x+4}{(x-1)^3}$

Solution:

Let

$$\frac{2x^2-3x+4}{(x-1)^3} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} \quad \ldots\ldots\ldots (1)$$

$$\frac{2x^2-3x+4}{(x-1)^3} = \frac{A(x-1)^2 + B(x-1) + C}{(x-1)^3}$$

$$2x^2-3x+4 = A(x-1)^2 + B(x-1) + C \quad \ldots\ldots\ldots (2)$$

$$2x^2-3x+4 = A(x^2+1-2x) + Bx - B + C$$

$$2x^2-3x+4 = Ax^2 + A - 2Ax + Bx - B + C \quad \ldots\ldots\ldots (3)$$

Put $x = 1$ in equation (2), we get

$$2(1)^2 - 3(1) + 4 = A(1-1)^2 + B(1-1) + C$$

$$2 - 3 + 4 = 0 + 0 + C$$

$$3 = C \quad \Rightarrow \quad \boxed{C = 3}$$

Equating coefficients of x^2, x in equation (3), we get

x^2 ; $\boxed{A = 2}$

x ; $-2A + B = -3$

$\Rightarrow -2(2) + B = -3$

$\Rightarrow -4 + B = -3$

$B = 4 - 3$

$\boxed{B = 1}$

Put values of A, B and C in equation (1)

$$\frac{2x^2 - 3x + 4}{(x-1)^3} = \frac{2}{(x-1)} + \frac{1}{(x-1)^2} + \frac{3}{(x-1)^3}$$

are required partial fractions.

Q.2 $\dfrac{\mathbf{5x^2 - 2x + 3}}{\mathbf{(x+2)^2}}$ **(Lahore Board 2005)**

Solution:

Let

$$\frac{5x^2 - 2x + 3}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3} \quad \ldots\ldots\ldots (1)$$

$$\frac{5x^2 - 2x + 3}{(x+2)^2} = \frac{A(x+2)^2 + B(x+2) + C}{(x+2)^3}$$

$5x^2 - 2x + 3 = A(x+2)^2 + B(x+2) + C$ ..... (2)

$5x^2 - 2x + 3 = A(x^2 + 4 + 4x) + Bx + 2B + C$

Put $x = -2$ in equation (2), we get

$5(-2)^2 - 2(-2) + 3 = A(-2+2)^2 + B(-2+2) + C$

$5(4) + 4 + 3 = 0 + 0 + C$

$20 + 7 = C \Rightarrow \boxed{C = 27}$

Equating coefficients of x^2, x, we have

x^2 ; $\boxed{A = 5}$

x ; $4A + B = -2$

$4(5) + B = -2$ ¤ $A = 5$

$20 + B = -2$

$B = -2 - 20$

$\boxed{B = -22}$

Put values of A, B and C in equation (1) we get

$$\frac{5x^2 - 2x + 3}{(x+2)^2} = \frac{5}{x+2} + \frac{-22}{(x+2)^2} + \frac{27}{(x+2)^3}$$

$$= \frac{5}{x+2} - \frac{22}{(x+2)^2} + \frac{27}{(x+2)^3}$$

are required partial fractions.

Q.3 $\dfrac{4x}{(x+1)^2(x-1)}$ **(Lahore Board 2009, Gujranwala Board 2003, 2005)**

Solution:

Let

$$\frac{4x}{(x+1)^2(x-1)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x-1)} \quad \ldots\ldots\ldots (1)$$

$$\frac{4x}{(x+1)^2(x-1)} = \frac{A(x+1)(x-1) + B(x-1) + C(x+1)^2}{(x+1)^2(x-1)}$$

$$4x = A(x+1)(x-1) + B(x-1) + C(x+1)^2 \quad \ldots\ldots\ldots (2)$$

$$4x = A(x^2 - 1) + Bx - B + C(x^2 + 1 + 2x)$$

$$4x = Ax^2 - A + Bx - B + Cx^2 + C + 2Cx \quad \ldots\ldots\ldots (3)$$

Put $x = -1$ in equation (2), we get

$$4(-1) = A(0) + B(-1-1) + C(0)$$

$$-4 = -2B \Rightarrow \boxed{B = 2}$$

Put $x = 1$ in equation (2), we get

$$4(1) = A(0) + B(0) + C(1+1)^2$$

$$4 = C(2)^2 \Rightarrow 4 = 4C \Rightarrow \boxed{C = 1}$$

Equating coefficient of x^2 in equation (3) we get,

x^2 ; $A + C = 0$ ¤ $C = 1$

$A + 1 = 0$

$\boxed{A = -1}$

Put values of A, B and C in equation (1), we have

$$\frac{4x}{(x+1)^2(x-1)} = \frac{-1}{(x+1)} + \frac{2}{(x+1)^2} + \frac{1}{(x-1)}$$

are required partial fractions.

Q.4 $\dfrac{9}{(x+2)^2(x-1)}$

Solution:

Let

$$\frac{9}{(x+2)^2(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2} + \frac{C}{(x-1)} \qquad \ldots\ldots\ldots (1)$$

$$\frac{9}{(x+2)^2(x-1)} = \frac{A(x+2)(x-1) + B(x-1) + C(x+2)^2}{(x+2)^2(x-1)}$$

$$9 = A(x+2)(x-1) + B(x-1) + C(x+2)^2 \qquad \ldots\ldots\ldots (2)$$

$$9 = A(x^2 + x - 2) + Bx - B + C(x^2 + 4 + 4x)$$

$$9 = Ax^2 + Ax - 2A + Bx - B + Cx^2 + 4C + 4Cx \qquad \ldots\ldots\ldots (3)$$

Put $x = -2$ in equation (2), we get

$$9 = A(0) + B(-2-1) + C(0)$$

$$9 = -3B \Rightarrow \boxed{B = -3}$$

Put $x = 1$ in equation (2), we get

$$9 = A(0) + B(0) + C(1+2)^2$$

$$9 = 0 + 0 + C(3)^2$$

$$9 = 9C \Rightarrow \boxed{C = 1}$$

Equating co–efficient of x^2 in equation (3) we get

$$A + C = 0$$

$$A + 1 = 0 \qquad \because \quad C = 1$$

$$\boxed{A = -1}$$

Put values of A, B and C in equation (1) we get

$$\frac{9}{(x+2)^2(x-1)} = \frac{-1}{(x+2)} - \frac{3}{(x+2)^2} + \frac{1}{(x-1)}$$

are required partial fractions.

Q.5 $\dfrac{1}{(x-3)^2(x+1)}$ **(Lahore Board 2007)**

Solution:

Let

$$\frac{1}{(x-3)^2(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x-3)^2} + \frac{C}{(x+1)} \quad \ldots\ldots\ldots (1)$$

$$\frac{1}{(x-3)^2(x+1)} = \frac{A(x-3)(x+1) + B(x+1) + C(x-3)^2}{(x-3)^2(x+1)}$$

$$1 = A(x-3)(x+1) + B(x+1) + C(x-3)^2 \quad \ldots\ldots\ldots (2)$$

$$1 = A(x^2 - 2x - 3) + Bx + B + C(x^2 + 9 - 6x)$$

$$1 = Ax^2 - 2Ax - 3A + Bx + B + Cx^2 + 9C - 6Cx \quad \ldots\ldots\ldots (3)$$

Put $x = 3$ in equation (2), we get

$$1 = A(0) + B(3+1) + C(0)$$

$$1 = 0 + 4B + 0$$

$$1 = 4B \Rightarrow \boxed{B = \frac{1}{4}}$$

Put $x = -1$ in equation (2), we get

$$1 = A(0) + B(0) + C(-1-3)^2$$

$$1 = 0 + 0 + C(-4)^2$$

$$1 = 16C \Rightarrow \boxed{C = \frac{1}{16}}$$

Equating coefficients of x^2 in equation (3) we get

$$A + C = 0$$

$$A + \frac{1}{16} = 0 \qquad \because \quad C = \frac{1}{16}$$

$$\boxed{A = -\frac{1}{16}}$$

Put values of A, B and C in equation (1), we get

$$\frac{1}{(x-3)^2(x+1)} = \frac{-\frac{1}{16}}{x-3} + \frac{\frac{1}{4}}{x-3} + \frac{\frac{1}{16}}{x-3}$$

$$= \frac{1}{16(x-3)} + \frac{1}{4(x-3)^2} + \frac{1}{16(x+1)}$$

Q.6 $\dfrac{x^2}{(x-2)(x-1)^2}$ **(Lahore Board 2011)**

Solution:

Let

$$\frac{x^2}{(x-2)(x-1)^2} = \frac{A}{(x-2)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \quad \ldots\ldots\ldots (1)$$

$$\frac{x^2}{(x-2)(x-1)^2} = \frac{A(x-1)^2 + B(x-2)(x-1) + C(x-2)}{(x-2)(x-1)^2}$$

$$x^2 = A(x-1)^2 + B(x-2)(x-1) + C(x-2) \quad \ldots\ldots\ldots (2)$$

$$x^2 = A(x^2 + 1 - 2x) + B(x^2 - 3x - 2) + Cx - 2C$$

$$x^2 = Ax^2 + A - 2Ax + Bx^2 - 3Bx - 2B + Cx - 2C \quad \ldots\ldots\ldots (3)$$

Put $x = 2$ in equation (2), we get

$$(2)^2 = A(2-1)^2 + B(0) + C(0)$$

$$4 = A(3)^2$$

$$4 = 9A \Rightarrow \boxed{A = \frac{4}{9}}$$

Put $x = 1$ in equation (2), we get

$$(1)^2 = A(0) + B(0) + C(1-2)$$

$$1 = 0 + 0 + C(-1)$$

$$1 = -C \Rightarrow \boxed{C = -1}$$

Equating coefficients of x^2 in equation (3) we get

$$A + B = 1$$

$$\frac{4}{9} + B = 1$$

$$B = 1 - \frac{4}{9}$$

$$\boxed{B = \frac{5}{9}}$$

Put values of A, B and C in equation (1), we get

$$\frac{x^2}{(x-2)(x-1)^2} = \frac{\frac{4}{9}}{(x-2)} + \frac{\frac{5}{9}}{(x-1)} + \frac{-1}{(x-1)^2}$$

$$= \frac{4}{9(x-2)} + \frac{5}{9(x-1)} - \frac{1}{(x-1)^2}$$

Q.7 $\dfrac{1}{(x-1)^2(x+1)}$

Solution:

Let

$$\frac{1}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)} \qquad \ldots\ldots\ldots (1)$$

$$\frac{1}{(x-1)^2(x+1)} = \frac{A(x-1)(x+1) + B(x+1) + C(x-1)^2}{(x-1)^2(x+1)}$$

$$1 = A(x-1)(x+1) + B(x+1) + C(x-1)^2 \qquad \ldots\ldots\ldots (2)$$

$$1 = A(x^2-1) + Bx + B + C(x^2+1-2x)$$

$$1 = Ax^2 - A + Bx + B + Cx^2 + C - 2Cx \qquad \ldots\ldots\ldots (3)$$

Put $x = 1$ in equation (2), we get

$$1 = A(0) + 2B + 0$$

$$1 = 2B \quad \Rightarrow \quad \boxed{B = \frac{1}{2}}$$

Put $x = -1$ in equation (2), we get

$$1 = A(0) + B(0) + C(-1-1)^2$$

$$1 = 0 + 0 + C(-2)^2$$

$$1 = 4C \quad \Rightarrow \quad \boxed{C = \frac{1}{4}}$$

Equating coefficients of x^2 in equation (3), we get

$$A + C = 0$$

$$A + \frac{1}{4} = 0 \qquad \because \quad C = \frac{1}{4}$$

$$\boxed{A = \frac{1}{4}}$$

Put values of A, B and C in equation (1) we get

$$\frac{1}{(x-1)^2(x+1)} = \frac{-\frac{1}{4}}{x-1} + \frac{\frac{1}{2}}{(x-1)^2} + \frac{\frac{1}{4}}{x+1}$$

$$= \frac{-1}{4(x-1)} + \frac{1}{2(x-1)^2} + \frac{1}{4(x+1)}$$

are required partial fraction.

Q.8 $\dfrac{x^2}{(x-1)^3(x+1)}$ **(Gujranwala Board 2003)**

Solution:

Let

$$\frac{x^2}{(x-1)^3(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{(x+1)} \quad \ldots\ldots\ldots (1)$$

$$\frac{x^2}{(x-1)^3(x+1)} = \frac{A(x-1)^2(x+1) + B(x-1)(x+1) + C(x+1) + D(x-1)^3}{(x-1)^3(x+1)}$$

$$x^2 = A(x-1)^2(x+1) + B(x-1)(x+1) + C(x+1) + D(x-1)^3 \quad \ldots\ldots\ldots (2)$$

$$x^2 = A(x^3 - x^2 - x + 1) + Bx^2 - B + Cx + C + Dx^3 - 3Dx^2 + 3Dx - D$$

$$x^2 = Ax^3 - Ax^2 - Ax + A + Bx^2 - B + Cx + C + Dx^3 - 3Dx^2 + 3Dx - D \quad \ldots\ldots\ldots (3)$$

Put $x = 1$ in equation (2), we get

$$(1)^2 = A(0) + B(0) + C(1+1) + D(0)$$

$$1 = 0 + 0 + 2C + 0$$

$$2C = 1 \Rightarrow \boxed{C = \frac{1}{2}}$$

Put $x = -1$ in equation (2), we get

$$(-1)^2 = A(0) + B(0) + C(0) + D(-1-1)^3$$

$$1 = 0 + 0 + 0\,D(-2)^3$$

$$1 = -8D \Rightarrow \boxed{D = -\frac{1}{8}}$$

Equating coefficients of x^3, x^2 in equation (3), we get

x^3 ; $A + D = 0$

$$A - \frac{1}{8} = 0 \qquad \because \quad D = -\frac{1}{8}$$

$$\boxed{A = \frac{1}{8}}$$

x^2 ; $-A + B - 3D = 1$

$$-\frac{1}{8} + B - \left(-\frac{1}{8}\right) = 1 \qquad \because \quad A = \frac{1}{8},\ D = -\frac{1}{8}$$

$$-\frac{1}{8} + B + \frac{3}{8} = 1$$

$$B = 1 + \frac{1}{8} - \frac{3}{8}$$

$$B = \frac{8 + 1 - 3}{8}$$

$$B = \frac{6}{8} \Rightarrow \boxed{B = \frac{3}{4}}$$

Put values of A, B, C and D in equation (1) we get

$$\frac{x^2}{(x-1)^3 (x+1)} = \frac{\frac{1}{8}}{(x-1)} + \frac{\frac{3}{4}}{(x-1)^2} + \frac{\frac{1}{2}}{(x-1)^2} + \frac{-\frac{1}{8}}{(x+1)}$$

$$= \frac{1}{8(x-1)} + \frac{3}{4(x-1)^2} + \frac{1}{2(x-1)^2} - \frac{1}{8(x+1)}$$

are required partial fractions.

Q.9 $\dfrac{x-1}{(x-2)(x+1)^3}$ **(Gujranwala Board 2006)**

Solution:

Let

$$\frac{x-1}{(x-2)(x+1)^3} = \frac{A}{(x-2)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3} \quad \ldots\ldots\ldots (1)$$

$$\frac{x-1}{(x-2)(x+1)^3} = \frac{A(x+1)^3 + B(x-2)(x+1)^2 + C(x-2)(x+1) + D(x-2)}{(x-2)(x+1)^3}$$

$$(x-1) = A(x+1)^3 + B(x-2)(x+1)^2 + C(x-2)(x+1) + D(x-2) \quad \ldots\ldots\ldots (2)$$

$$x - 1 = A(x^3 + 1 + 3x^2 + 3x) + B(x^3 - 3x - 2) + C(x^2 - x + 2) + D(x-2)$$

$$x - 1 = Ax^3 + A + 3Ax^2 + 3Ax + Bx^3 - 3Bx - 2B + Cx^2 - Cx + 2C + Dx - 2D \quad \ldots\ldots (3)$$

Put $x = 2$ in equation (2), we get

$$2 - 1 = A(2+1)^3 + 0 + 0 + 0$$

$$1 = A(3)^3$$

$$1 = 27A \Rightarrow \boxed{A = \frac{1}{27}}$$

Put $x = -1$ in equation (2), we get

$$-1 - 1 = 0 + 0 + 0 + D(-1 - 2)$$

$-2 = D(-3)$

$$\boxed{D = \frac{2}{3}}$$

x^3 ; $A + B = 0$

$$\frac{1}{27} + B = 0 \qquad \text{¤} \quad A = \frac{1}{27}$$

$$\boxed{B = -\frac{1}{27}}$$

x^2 ; $3A + C = 0$

$$3 \cdot \frac{1}{27} + C = 0$$

$$\frac{1}{9} + C = 0 \Rightarrow \boxed{C = -\frac{1}{9}}$$

Put values of A, B, C and D in equation (1) we get

$$\frac{x-1}{(x-2)(x+1)^3} = \frac{\frac{1}{27}}{(x-2)} + \frac{-\frac{1}{27}}{(x+1)} + \frac{-\frac{1}{9}}{(x+1)^2} + \frac{\frac{2}{3}}{(x+1)^3}$$

$$= \frac{1}{27(x-2)} - \frac{1}{27(x+1)} - \frac{1}{9(x+1)^2} + \frac{2}{3(x+1)^3}$$

are required partial fractions.

Q.10 $\dfrac{4x^3}{(x^2-1)(x+1)^2}$ **(Gujranwala Board 2005, Lahore Board 2005)**

Solution:

$$\frac{4x^3}{(x^2-1)(x+1)^2} = \frac{4x^3}{(x+1)(x-1)(x+1)^2} = \frac{4x^3}{(x-1)(x+1)^3}$$

Let

$$\frac{4x^3}{(x-1)(x+1)^3} = \frac{A}{(x-1)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3} \qquad \ldots\ldots\ldots (1)$$

$$\frac{4x^3}{(x-1)(x+1)^3} = \frac{A(x+1)^3 + B(x-1)(x+1)^2 + C(x-1)(x+1) + D(x-1)}{(x-1)(x+1)^3}$$

$$4x^3 = A(x+1)^3 + B(x-1)(x+1)^2 + C(x-1)(x+1) + D(x-1) \qquad \ldots\ldots\ldots (2)$$

$$4x^3 = A(x^3 + 1 + 3x^2 + 3x) + B(x^3 + x^2 - x - 1) + C(x^2 - 1) + D(x-1)$$

$$4x^3 = Ax^3 + A + 3Ax^2 + 3Ax + Bx^3 + Bx^2 - Bx - B + Cx^2 - C + Dx - D \qquad \ldots\ldots\ldots (3)$$

Put $x = 1$ in equation (2), we get

$4(1)^3 = A(1+1)^3 + 0 + 0 + 0$

$4 = (2)^3 A$

$4 = 8A \Rightarrow \boxed{A = \frac{1}{2}}$

Put $x = -1$ in equation (2), we get

$4(-1)^3 = 0 + 0 + 0 + D(-1-1)$

$-4 = -2D \Rightarrow \boxed{D = 2}$

Equating coefficients of x^3, x^2 in equation (3) we get

x^3 ; $A + B = 4$

$\frac{1}{2} + B = 4$ ¤ $A = \frac{1}{2}$

$B = 4 - \frac{1}{2}$

$\boxed{B = \frac{7}{2}}$

x^2 ; $3A + B + C = 0$

$3 \cdot \frac{1}{2} + \frac{7}{2} + C = 0$

$\frac{3}{2} + \frac{7}{2} + C = 0$

$\frac{10}{2} + c = 0$

$5 + C = 0 \Rightarrow \boxed{C = 5}$

Put values of A, B, C and D in equation (1), we get

$$\frac{4x^3}{(x-1)(x+1)^3} = \frac{\frac{1}{2}}{(x-1)} + \frac{\frac{7}{2}}{(x+1)} - \frac{5}{(x+1)^2} + \frac{2}{(x+1)^3}$$

$$= \frac{1}{2(x-1)} + \frac{7}{2(x+1)} - \frac{5}{(x+1)^2} + \frac{2}{(x+1)^3}$$

are required partial fractions.

Q.11 $\dfrac{2x+1}{(x+3)(x-1)(x+2)^2}$

Solution:

Let

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{A}{(x+3)} + \frac{B}{(x-1)} + \frac{C}{(x+2)} + \frac{D}{(x+2)^2} \quad \ldots\ldots\ldots (1)$$

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2}$$

$$= \frac{A(x-1)(x+2)^2 + B(x+3)(x+2)^2 + C(x+3)(x-1)(x+2) + D(x+3)(x-1)}{(x+3)(x-1)(x+2)^2}$$

$$(2x+1) = A(x-1)(x+2)^2 + B(x+3)(x+2)^2 + C(x+3)(x-1)(x+2) + D(x+3)(x-1) \quad (2)$$

$$2x+1 = A(x^3+3x^2-4) + B(x^3+7x^2+16x+12) + C(x^3+4x^2+x-6) + D(x^2+2x-3)$$

$$2x+1 = Ax^3+3Ax^2-4A+Bx^3+7Bx^2+16Bx+12B+Cx^3+4Cx^2+Cx-6C + Dx^2+2Dx-3D \quad (3)$$

Put $x = -3$ in equation (2), we get

$$2(-3)+1 = A(-3-1)(-3+2)^2 + 0 + 0 + 0$$

$$-6+1 = A(-4)(-1)^2$$

$$-5 = -4A \Rightarrow \boxed{A = \frac{5}{4}}$$

Put $x = 1$ in equation (2), we get

$$2(1)+1 = 0 + B(1+3)(1+2)^2 + 0 + 0$$

$$2+1 = B(4)(3)^2$$

$$3 = B(4)(9)$$

$$3 = 36B \Rightarrow B = \frac{3}{36} \Rightarrow \boxed{B = \frac{1}{12}}$$

Put $x = -2$ in equation (2), we get

$$2(-2)+1 = 0 + 0 + 0 + D(-2+3)(-2-1)$$

$$-4+1 = D(1)(-3)$$

$$-3 = -3D \Rightarrow \boxed{D = 1}$$

Equating coefficients of x^3 in equation (3) we get

$$A + B + C = 0$$

$$\frac{5}{4} + \frac{1}{12} + C = 0$$

$$\frac{15+1}{12} + C = 0$$

$$\frac{16}{12} + C = 0$$

$$\frac{4}{3} + C = 0 \Rightarrow \boxed{C = -\frac{4}{3}}$$

Put values of A, B, C and D in equation (1), we get

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{\frac{5}{4}}{(x+3)} + \frac{\frac{1}{12}}{(x-1)} + \frac{-\frac{4}{3}}{(x+2)} + \frac{1}{(x+2)^2}$$

$$= \frac{5}{4(x+3)} + \frac{1}{12(x-1)} - \frac{4}{3(x+2)} + \frac{1}{(x+2)^2}$$

are required partial fraction.

Q.12 $\dfrac{2x^4}{(x-3)(x+2)^2}$

Solution:

$$\frac{2x^4}{(x-3)(x+2)^2} = \frac{2x^4}{(x-3)(x^2+4+4x)} = \frac{2x^4}{x^3+x^2-8x-12}$$

$$\begin{array}{r} 2x-2 \\ x^3+x^2-8x-12 \;\overline{)\;2x^4} \\ 2x^4+2x^3-16x^2-24x \\ - \quad - \quad + \quad + \\ \hline -2x^3+16x^2+24x \\ -2x^3- \;2x^2+16x+24 \\ + \quad + \quad - \quad - \\ \hline 18x^2+8x-24 \end{array}$$

$$\frac{2x^4}{x^3+x^2-8x-12} = 2x-2+\frac{18x^2+8x-24}{x^3+x^2-8x-12}$$

$$= 2x-2+\frac{18x^2+8x-24}{(x-3)(x+2)^2} \quad \ldots\ldots\ldots (1)$$

Let $\dfrac{18x^2+8x-24}{(x-3)(x+2)^2} = \dfrac{A}{(x-3)} + \dfrac{B}{(x+2)} + \dfrac{C}{(x+2)^2}$ ..... (2)

$$\frac{18x^2+8x-24}{(x-3)(x+2)^2} = \frac{A(x+2)^2 + B(x-3)(x+2) + C(x-3)}{(x-3)(x+2)^2}$$

$18x^2 + 8x - 24 = A(x+2)^2 + B(x-3)(x+2) + C(x-3)$ ..... (3)

$18x^2 + 8x - 24 = A(x^2 + 4x + 4) + B(x^2 - x - 6) + C(x-3)$

$18x^2 + 8x - 24 = Ax^2 + 4Ax + 4A + Bx^2 - Bx - 6B + Cx - 3C$ ..... (4)

Put $x = 3$ in equation (3), we get

$18(3)^2 + 8(3) - 24 = A(3+2)^2 + 0 + 0$

$18(9) + 24 - 24 = A(5)^2$

$162 = 25A \Rightarrow \boxed{A = \frac{162}{25}}$

Put $x = -2$ in equation (3), we get

$18(-2)^2 + 8(-2) - 24 = 0 + 0 + C(-2-3)$

$-36 - 16 - 24 = C(-5)$

$32 = -5C \Rightarrow \boxed{B = -\frac{32}{5}}$

Equating coefficients of x^2 in equation (4), we get

$A + B = 18$

$B = 18 - A$

$B = 18 - \frac{162}{25} \Rightarrow \boxed{B = \frac{288}{25}}$

Put values of A, B, C in equation (2), we get

$$\frac{18x^2 + 8x - 24}{(x-3)(x+2)^2} = \frac{162}{25(x-3)} + \frac{288}{25(x+2)} - \frac{32}{5(x+2)^2}$$

equation (1) becomes

$$\frac{4x^3}{(x-3)(x+2)^2} = (2x-2) + \frac{162}{25(x-3)} + \frac{288}{25(x+2)} - \frac{32}{5(x+2)^2}$$

are required partial fractions.

EXERCISE 5.3

Resolve the following into partial fractions.

Q.1 $\dfrac{9x-7}{(x^2+1)(x+3)}$ **(Lahore Board 2004, 2010)**

Solution:

Let

$$\frac{9x-7}{(x^2+1)(x+3)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{(x+3)} \quad \text{..... (1)}$$

$$\frac{9x-7}{(x^2+1)(x+3)} = \frac{(Ax+B)(x+3) + C(x^2+1)}{(x^2+1)(x+3)}$$

$9x - 7 = (Ax + B)(x+3) + C(x^2+1)$ ..... (2)