## EXERCISE 12.6

**Q.1 Solve the following triangles, in which**

**a = 7 , b = 7 , c = 9**

**Solution:**

$$S = \frac{a+b+c}{2} = \frac{7+7+9}{2} = \frac{23}{2} = 11.5$$

$S - a = 11.5 - 7 = 4.5$

$S - b = 11.5 - 7 = 4.5$

$S - c = 11.5 - 9 = 2.5$

Now by half angle formulas

$$\cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}} \qquad \cos\frac{\beta}{2} = \sqrt{\frac{S(S-b)}{ac}} \qquad \cos\frac{\gamma}{2} = \sqrt{\frac{S(S-c)}{ab}}$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{11.5(4.5)}{7 \times 9}} \qquad \cos\frac{\beta}{2} = \sqrt{\frac{11.5(4.5)}{7 \times 9}} \qquad \cos\frac{\gamma}{2} = \sqrt{\frac{11.5 \times 2.5}{49}}$$

$\alpha = 2 \times \cos^{-1}(0.9063)$ $\qquad$ $\beta = 2 \times \cos^{-1}(0.9063)$ $\qquad$ $\gamma = 2 \times \cos^{-1}(0.7659)$

$\boxed{\alpha = 50^\circ}$ $\qquad$ $\boxed{\beta = 50^\circ}$ $\qquad$ $\boxed{\gamma = 80^\circ}$

**Q.2 a = 32 , b = 40 , c = 66 (Lahore Board 2008)**

**Solution:**

$$S = \frac{a+b+c}{2} = \frac{32+40+66}{2} = 69$$

$S - a = 69 - 32 = 37$

$S - b = 69 - 40 = 29$

$S - c = 69 - 66 = 3$

Now by half angle formulas

$$\cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}} \qquad \cos\frac{\beta}{2} = \sqrt{\frac{S(S-b)}{ac}} \qquad \cos\frac{\gamma}{2} = \sqrt{\frac{S(S-c)}{ab}}$$

$$\alpha = 2\cos^{-1}\sqrt{\frac{69 \times 37}{40 \times 66}} \qquad \beta = 2\cos^{-1}\sqrt{\frac{69 \times 29}{32 \times 66}} \qquad \gamma = 2\cos^{-1}\sqrt{\frac{69 \times 3}{32 \times 40}}$$

$\boxed{\alpha = 20^\circ 56'}$ $\qquad$ $\boxed{\beta = 26^\circ 30'}$ $\qquad$ $\boxed{\gamma = 132^\circ 34'}$

**Q.3 a = 28.3 , b = 31.7 , c = 42.8**

**Solution:**

$$S = \frac{a+b+c}{2} = \frac{28.3+31.7+42.8}{2} = 51.4$$

$$S-a = 51.4-28.3 = 23.1$$

$$S-b = 51.4-31.7 = 19.7$$

$$S-c = 51.4-42.8 = 8.6$$

Now by half angle formulas

$$\cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}} \qquad \cos\frac{\beta}{2} = \sqrt{\frac{S(S-b)}{ac}} \qquad \cos\frac{\gamma}{2} = \sqrt{\frac{S(S-c)}{ab}}$$

$$\alpha = 2\cos^{-1}\sqrt{\frac{51.4 \times 23.1}{31.7 \times 42.8}} \qquad \beta = 2\cos^{-1}\sqrt{\frac{51.4 \times 19.7}{28.3 \times 42.8}} \qquad \gamma = 2\cos^{-1}\sqrt{\frac{51.4 \times 8.6}{28.3 \times 31.7}}$$

$\alpha = 41^\circ 23'$ $\beta = 47^\circ 46'$ $\gamma = 90^\circ 51'$

**Q.4 a = 31.9 , b = 56.31 , c = 40.27**

**Solution:**

$$S = \frac{a+b+c}{2} = \frac{31.9+56.31+40.27}{2} = 64.24$$

$$S-a = 64.24-31.9 = 32.34$$

$$S-b = 64.24-56.31 = 7.93$$

$$S-c = 64.24-40.27 = 23.97$$

Now by half angle formulas

$$\cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}} \qquad \cos\frac{\beta}{2} = \sqrt{\frac{S(S-b)}{ab}} \qquad \cos\frac{\gamma}{2} = \sqrt{\frac{S(S-c)}{ab}}$$

$$\alpha = 2\cos^{-1}\sqrt{\frac{64.24 \times 32.34}{56.31 \times 40.27}} \qquad \beta = 2\cos^{-1}\sqrt{\frac{64.24 \times 7.93}{31.9 \times 56.31}} \qquad \gamma = 2\cos^{-1}\sqrt{\frac{64.24 \times 23.97}{31.9 \times 56.31}}$$

$\alpha = 33^\circ 39'$ $\beta = 115^\circ 38'$ $\gamma = 30^\circ 43'$

**Q.5 a = 4584 , b = 5140 , c = 3624**

**Solution:**

$$S = \frac{a+b+c}{2} = \frac{4584+5140+3624}{2} = 6674$$

$$S-a = 6674-4584 = 2090$$

$$S-b = 6674-5140 = 1534$$

$$S-c = 6674-3624 = 3050$$

Now by half angle formulas

$\cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}}$

$\alpha = 2\cos^{-1}\sqrt{\frac{6674 \times 2090}{18627360}}$

$\boxed{\alpha = 60^{o}\,9'}$

$\cos\frac{\beta}{2} = \sqrt{\frac{S(S-b)}{ac}}$

$\beta = 2\cos^{-1}\sqrt{\frac{6674 \times 1534}{16612416}}$

$\boxed{\beta = 76^{o}\,34'}$

$\cos\frac{\gamma}{2} = \sqrt{\frac{S(S-c)}{ab}}$

$\gamma = 2\cos^{-1}\sqrt{\frac{6674 \times 3050}{23561760}}$

$\boxed{\gamma = 43^{o}\,17'}$

**Q.6 Find the smallest angle of the triangle ABC when**
**a = 37.34 , b = 3.24 , c = 35.06**

**Solution:**

By law of cosines

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$2ac\cos\beta = a^2 + c^2 - b^2$

$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$

$\beta = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$

$\beta = \cos^{-1}\left(\frac{(37.34)^2 + (35.06)^2 - (3.24)^2}{2\,(37.34)\,(35.06)}\right)$

$\boxed{\beta = 3^{o}\,39'}$

**Q.7 find the measure of the greatest angle, if sides of triangle are 16, 20, 23.**
**(Lahore Board 2010)**

**Solution:**

$a = 16 , \quad b = 20 , \quad c = 33 , \quad \gamma = ?$

Now by law of cosines

$c^2 = a^2 + b^2 - 2ac\cos\gamma$

$2ac\cos\gamma = a^2 + b^2 - c^2$

$\cos\gamma = \frac{a^2 + b^2 - c^2}{2ac}$

$\gamma = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$

$\gamma = \cos^{-1}\left(\frac{(16)^2 + (20)^2 - (33)^2}{2 \times 16 \times 20}\right)$

$\gamma = 132^{o}\,35'$

**Q.8 The sides of a triangle are $x^2 + x + 1$, $2x + 1$ and $x^2 - 1$. Prove that the greatest angle of the triangle is $120^o$. (Gujranwala Board 2006)**

**Solution:**

$$a = x^2 + x + 1, \quad b = 2x + 1, \quad c = x^2 - 1, \quad \alpha = ?$$

$$S = \frac{a + b + c}{2} = \frac{x^2 + x + 1 + 2x + 1 + x^2 - 1}{2} = \frac{2x^2 + 3x + 1}{2}$$

$$S - a = \frac{2x^2 + 3x + 1}{2} - (x^2 + x + 1)$$

$$S - a = \frac{2x^2 + 3x + 1 - 2x^2 - 2x - 2}{2} = \frac{x - 1}{2}$$

By half angle formula

$$\cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}}$$

$$\alpha = 2\cos^{-1}\sqrt{\frac{\frac{2x^2 + 3x + 1}{2}\left(\frac{x-1}{2}\right)}{(2x+1)(x^2-1)}}$$

$$= 2\cos^{-1}\sqrt{\frac{(2x^2 + 2x + x + 1)(x-1)}{4(2x+1)(x-1)(x+1)}}$$

$$= 2\cos^{-1}\sqrt{\frac{[2x(x+1) + (x+1)](x-1)}{4(2x+1)(x-1)(x+1)}}$$

$$= 2\cos^{-1}\sqrt{\frac{(2x+1)(x+1)(x-1)}{4(2x+1)(x-1)(x+1)}}$$

$$\alpha = 2\cos^{-1}\left(\frac{1}{2}\right) = 2 \times 60^o = 120^o$$

Hence proved.

**Q.9 The measures of the sides of a triangular plot are 413, 214 and 375 meters. Find the measure of the corner angles of the plot.**

**Solution:**

$a = 413\ ,\ b = 214\ ,\ c = 375$

$S = \frac{a+b+c}{2} = \frac{413+214+375}{2} = 501$

$S - a = 501 - 413 = 88$

$S - b = 501 - 214 = 287$

$S - c = 501 - 375 = 126$

Now by half angle formulas

$\cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}}$

$\alpha = 2\cos^{-1}\sqrt{\frac{501 \times 88}{214 \times 375}}$

$\boxed{\alpha = 84^\circ 20'}$

$\cos\frac{\beta}{2} = \sqrt{\frac{S(S-b)}{ac}}$

$\beta = 2\cos^{-1}\sqrt{\frac{501 \times 287}{413 \times 375}}$

$\boxed{\beta = 31^\circ 2'}$

$\cos\frac{\gamma}{2} = \sqrt{\frac{S(S-c)}{ab}}$

$\gamma = 2\cos^{-1}\sqrt{\frac{501 \times 126}{413 \times 214}}$

$\boxed{\gamma = 64^\circ 38'}$

**Q.10 Three villages A, B, C are connected by straight roads 6km, 9km, 13km. What angles these roads makes with each other.**

**Solution:**

Let, $a = 6\ ,\ b = 9\ ,\ c = 13$

$S = \frac{a+b+c}{2} = \frac{6+9+13}{2} = 14$

$S - a = 14 - 6 = 8$

$S - b = 14 - 9 = 5$

$S - c = 14 - 13 = 1$

Now by half angle formulas

$\cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}}$

$\alpha = 2\cos^{-1}\sqrt{\frac{14 \times 8}{117 \times 375}}$

$\boxed{\alpha = 23^\circ 51'}$

$\cos\frac{\beta}{2} = \sqrt{\frac{S(S-b)}{ac}}$

$\beta = 2\cos^{-1}\sqrt{\frac{14 \times 5}{78}}$

$\boxed{\beta = 37^\circ 21'}$

$\cos\frac{\gamma}{2} = \sqrt{\frac{S(S-c)}{ab}}$

$\gamma = 2\cos^{-1}\sqrt{\frac{14 \times 1}{54}}$

$\boxed{\gamma = 118^\circ 46'}$

**Area of Triangle**

**Case 1:** When two sides & included angle is given