Q.7 $\gamma = 90^o$, $\beta = 50^o\,10'$, $c = 0.832$

Solution:

Since $\alpha + \beta + \gamma = 180^o$

$\alpha + 50^o\,10' + 90^o = 180^o$

$\boxed{\alpha = 39^o\,50'}$

$\sin\alpha = \frac{a}{c}$

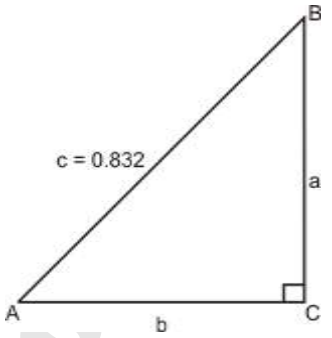
$\sin 39^o\,50' = \frac{a}{0.832}$

$\Rightarrow$ $\boxed{a = 0.5329}$

$\cos\alpha = \frac{a}{b}$

$\cos 39^o\,50' = \frac{0.5329}{b}$

$\boxed{c = 0.6939}$



EXERCISE 12.3

Q.1 **A vertical pole is 8m high and the length of its shadow is 6m. What is the angle of elevation of the sun at that moment?**

Solution:

Let the required angle be $= \theta$

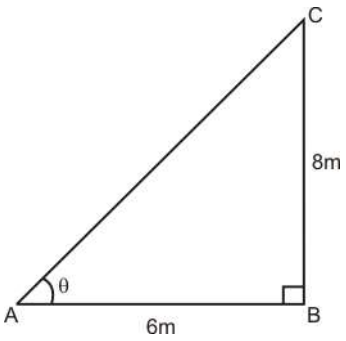
We know that

$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$\tan\theta = \frac{8}{6}$

$\theta = \tan^{-1}\left(\frac{8}{6}\right)$

$\theta = 53^o\,7'$



Q.2 A man 18dm tall observes that the angle of elevation of the top of a tree at a distance of 12m from him is 32^o. What is the height of the tree?

Solution:

Let the required height of the tree be = x + 18

we know that

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

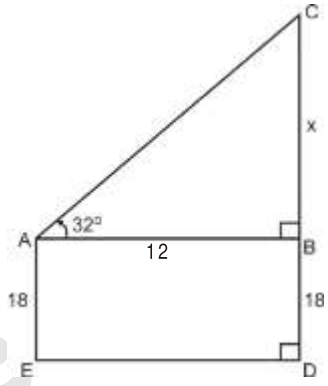
$$\tan 32^o = \frac{x}{12}$$

$$x = 7.50 \text{ m}$$

Required height = x + 18

= 7.50 + 1.8 m (18dm = 1.8m)

= 9.3m



Q.3 At the top of a cliff 80m high, the angle of depression of a boat is 12^o. How far is the boat from the cliff?

Solution:

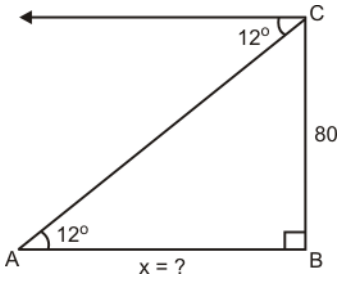
Let the boat from the cliff be = x

we know that

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan 12^o = \frac{80}{x}$$

$$x = 376.37 \text{ m}$$



Q.4 A ladder leaning against a vertical wall makes an angle of 24^o with the wall. Its foot is 5m from the wall. Find its length.

Solution:

Let length of ladder be = l

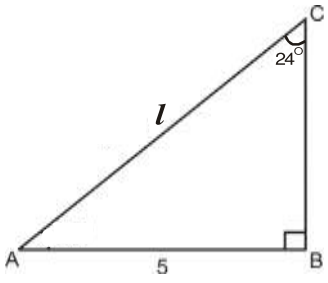
we know that

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\sin 24^o = \frac{5}{l}$$

$$l = \frac{5}{\sin 24°}$$

$$l = 12.30\text{m}$$



Q.5 A kite is flying at a height of 67.2 m is attached to a fully stretched string inclined at an angle of 55° to the horizontal. Find the length of the string.

Solution:

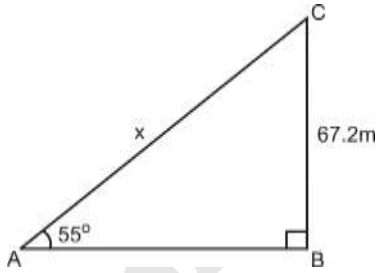
Let the string of the kite be = x

we know that

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\sin 55^o = \frac{67.2}{x}$$

$$x = 82 \text{ m}$$



Q.6 When the angle between the ground and the sun is 30°, flag pole casts a shadow of 40m long. Find the height of the top of flag. (Lahore Board 2008)

Solution:

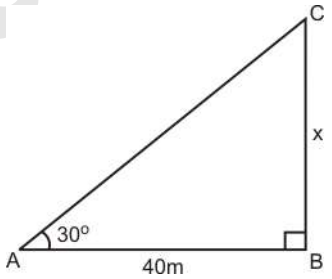
Let the height of the top of the flag be = x

we know that

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan 30^o = \frac{x}{40}$$

$$x = 23.09 \text{ m}$$



Q.7 A plane flying directly above a post 6000m away from an anti–aircraft observes the gun at an angle of depression of 27°. Find the height of the plane.

Solution:

Let the height of plane be = x

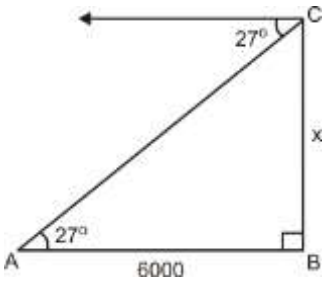
we know that

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan\theta = \frac{x}{6000}$$

$$\tan 27^o = \frac{x}{6000}$$

$$x = 3057.15 \text{ m}$$



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Q.8 A man on the top of a 100m high light house tower is in line with two ships on the same side of it, whose angle of depression from the man are 17^o and 19^o respectively. Find the distance between the ships.

Solution:

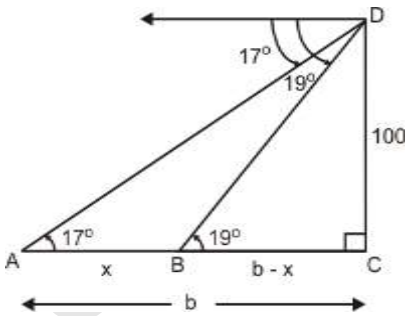
Let the distance between the ships be = x

First consider ΔDAC

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan 17^o = \frac{100}{b}$$

$\Rightarrow$ b = 327.08 m



now we consider ΔDBC

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan 19^o = \frac{100}{b-x}$$

$$0.344 = \frac{100}{327-x}$$

$$112.59 - 0.344x = 100$$

$$12.59 = 0.344x$$

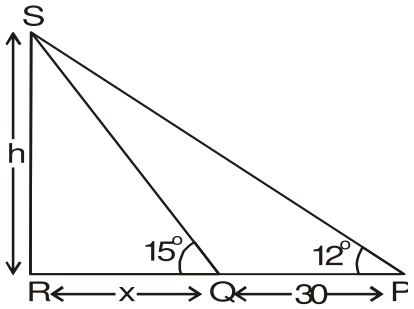
$$\frac{12.59}{0.344} = x$$

$$x = 36.58 \text{ m}$$

Q.9 P and Q are two points in line with a tree. If the distance between P and Q be 30m and the angle of elevation of the top of the tree at P and Q be 12^o and 15^o respectively. Find the height of tree.

Solution:

Let the height of the tree be = h



First consider ΔSQR

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan 15^o = \frac{h}{x}$$

$\Rightarrow$ $\boxed{h = x \tan 15°}$ ..... (1)

Now we consider ΔSPR

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan 12 = \frac{h}{x+30}$$

$\Rightarrow$ $\boxed{h = (x + 30)\tan 12}$ ..... (2)

Using eq. (1) and (2).

$x \tan 15° = x \tan 12° + 30 \tan 12°$

$x(\tan 15 - \tan 12°) = 30 \tan 12°$

$0.055x = 6.38$

$\boxed{x = 115.94}$ put in eq. (1)

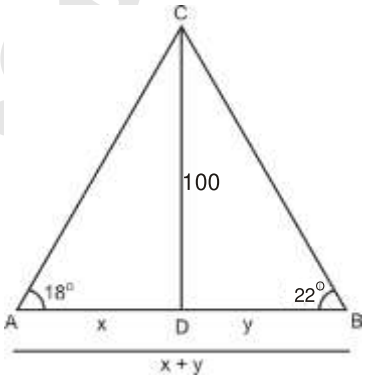
$h = 115.94 \tan 15°$

$h = 31.066$ m

Q.10 Two men are on the opposite sides of a 100m high tower. If the measure of the angles of the angles of elevation of the top of the tower are 18^o & 22^o respectively. Find the distance between them.

Solution:

Let the distance between the two men = x + y



Consider ΔCAD

$\tan 18^o = \frac{100}{x}$

$x = \frac{100}{\tan 18^o}$

$x = 307.77$

Consider ΔBCD

$\tan 22^o = \frac{100}{y}$

$y = \frac{100}{\tan 22^o}$

$y = 247.51$

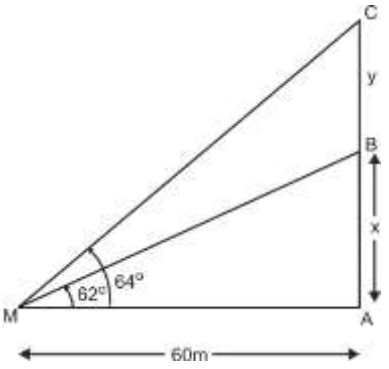
Required distance between them = x + y

= 307.77 + 247.51 = 555.28 m

Q.11 A man standing 60m away from a tower notices that the angle of elevation of the top and the bottom of a flag staff on the top of the tower are 64^o and 62^o respectively. Find the length of the flag staff.

Solution:

Let the length of the flag staff be = y



First consider ΔBMA

$\tan 62^o = \frac{x}{60}$

$x = 112.84$

Now consider ΔCMA

$\tan 64^o = \frac{x + y}{60}$

$60^o \times \tan 64^o = 112.84 + y$

$123 = x + 112.84$

$x = 10.2$ m

Q.12 The angle of elevation of the top of 60m high tower from a point A, on the same level as the foot of the tower is 25^o. Find the angle of elevation of the top of the tower from a point B, 20m nearer to A from the foot of the tower.

Solution:

Let the required angle of elevation be $= \theta$

First consider ΔDAC

$$\tan 25^o = \frac{60}{x+20} \Rightarrow x + 20 = \frac{60}{\tan 25^\circ}$$

$$x = 128.67 - 20$$

$$x = 108.67$$

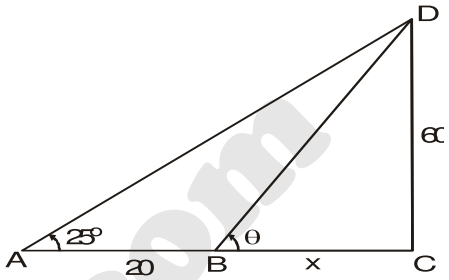
Now we consider ΔDBC

$$\tan\theta = \frac{60}{x}$$

$$\tan\theta = \frac{60}{108.67}$$

$$\tan\theta = \frac{60}{108.67}$$

$$\theta = \tan^{-1}\left(\frac{60}{108.67}\right) = 28^o\,54'$$



Q.13 Two buildings A and B are 100m apart. The angle of elevation from the top of the building A to the top of the building B is 20^o. the angle of elevation from the base of the building B to the top of the building A is 50^o. Find the height of the building B.

Solution:

Let the required height of building B be $= x + y$

We know that $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$$\tan 20^o = \frac{y}{100}$$

$$y = 36.39$$

Now consider ΔABC

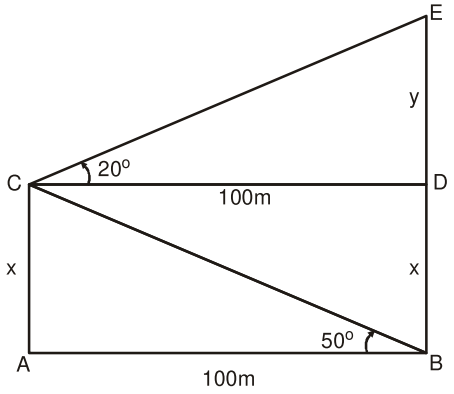
$$\tan 50^o = \frac{x}{100}$$

$$x = 119.2$$

required height of building B $= x + y$

$= 119.2 + 36.39$

$= 115.6$



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Q.14 A window washer is working in a hotel building at a distance of 20m from the building finds angle of elevation of the worker to be of 30^o. The worker climbs up 12m and the observer moves 4m further away from the building. Find the new angle of elevation of the worker.

Solution:

Let the required angle of elevation $= \theta$

First consider ΔABC

$$\tan 30^o = \frac{x}{20}$$

$$\Rightarrow \quad x = 11.55m$$

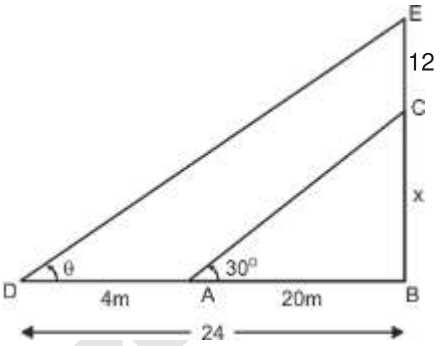
Now we consider ΔDBE

$$\tan \theta = \frac{12 + x}{24}$$

$$\tan \theta = \frac{12 + 11.55}{24}$$

$$\theta = \tan^{-1}(0.98125)$$

$$\theta = 44^o\, 27'$$



Q.15 A man standing on a bank of canal observes that the measure of the angle of elevation of a tree on the other side of the canal is 60^o. On retreating 40m from the bank, he finds the measure of elevation of tree is 30o. find the height of the tree and width of the canal.

Solution:

Let height of tree = h & width of canal = x

First consider ΔBAC

$$\tan 30^o = \frac{h}{x + 40}$$

$$h = (x + 40)\tan 30° \quad \ldots\ldots\ldots (1)$$

Now we consider ΔBDC

$$\tan 60^o = \frac{h}{x} \Rightarrow h = x \tan 60° \quad \ldots\ldots\ldots (2)$$

From eq. (1) and (2).

$(x + 40)\tan 30° = x \tan 60°$

$x \tan 30° + 40 \tan 30° = x \tan 60°$

$x \tan 60° - x \tan 30° = 40 \tan 30°$

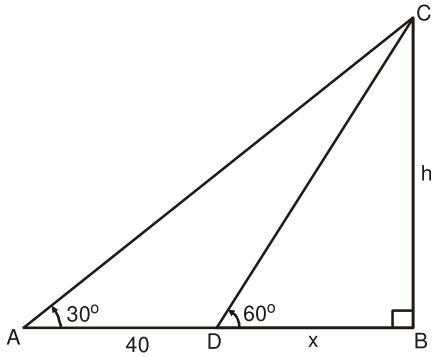
$x(\tan 60° - \tan 30°) = 40 \tan 30°$

$1.15x = 23.09$

$\boxed{x = 20}$ put in (2).

$h = 20 \tan 60°$

$\boxed{h = 34.64}$



OBLIQUE TRIANGLE:

A triangle, which is not right, is called an oblique triangle.

For the solution of oblique triangles we can use law of sines, law of cosines, law of tangents.

1. The Law of Cosine

In any triangle ABC, with usual notations

(i) $a^2 = b^2 + c^2 - 2bc \cos \alpha$

(ii) $b^2 = a^2 + c^2 - 2ac \cos \beta$

(iii) $c^2 = a^2 + b^2 - 2ab \cos \gamma$

2. Law of Sines

In any triangle ABC, with usual notations

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

3. The Law of Tangents

In any triangle ABC, with usual notations

(i) $\dfrac{a-b}{a+b} = \dfrac{\tan \frac{\alpha - \beta}{2}}{\tan \frac{\alpha + \beta}{2}}$ (ii) $\dfrac{b-c}{b+c} = \dfrac{\tan \frac{\beta - \gamma}{2}}{\tan \frac{\beta + \gamma}{2}}$

(iii) $\dfrac{c-a}{c+a} = \dfrac{\tan \frac{\gamma - \alpha}{2}}{\tan \frac{\gamma + \alpha}{2}}$

4. Half angle Formulas

In any triangle ABC, with usual notation

(i) $\sin \frac{\alpha}{2} = \sqrt{\frac{(S-b)(S-c)}{bc}}$	(i) $\cos \frac{\alpha}{2} \sqrt{\frac{S(S-a)}{bc}}$	(i) $\tan \frac{\alpha}{2} = \sqrt{\frac{(S-b)(S-c)}{S(S-a)}}$
(ii) $\sin \frac{\beta}{2} = \sqrt{\frac{(S-a)(S-c)}{ac}}$	(ii) $\cos \frac{\beta}{2} \sqrt{\frac{S(S-b)}{ac}}$	(ii) $\tan \frac{\beta}{2} = \sqrt{\frac{(S-a)(S-c)}{S(S-b)}}$
(iii) $\sin \frac{\gamma}{2} = \sqrt{\frac{(S-a)(S-b)}{ab}}$	(iii) $\cos \frac{\gamma}{2} \sqrt{\frac{S(S-c)}{ab}}$	(iii) $\tan \frac{\gamma}{2} = \sqrt{\frac{(S-a)(S-b)}{S(S-c)}}$

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