Chapter 7

PERMUTATION, COMBINATION AND PROBABILITY

FACTORIAL NOTATION

Let n be a positive integer. Then the product n (n – 1) (n – 2) ……. 3 . 2 . 1 is denoted by n! or $\underline{|n}$ and read as n factorial.

i.e. $n! = n(n-1)(n-2) \ldots\ldots 3 . 2 . 1$

and $n! = n(n-1)!$ where $0! = 1$

EXERCISE 7.1

Q.1 Evaluate each of the following:

(i) 4! (ii) 6! (iii) $\frac{8!}{7!}$ (iv) $\frac{10!}{7!}$ (v) $\frac{11!}{4!\,7!}$ (vi) $\frac{6!}{3!\,3!}$ (vii) $\frac{8!}{4!\,2!}$

(viii) $\frac{11!}{2!\,4!\,5!}$ (ix) $\frac{9!}{2!\,(9-2)!}$ (x) $\frac{15!}{15!\,(15-15)!}$ (xi) $\frac{3!}{0!}$ (xii) 4! 0! 1!

Solution:

(i) $4! = 4.3.2.1 = 24$

(ii) $6! = 6.5.4.3.2.1 = 720$

(iii) $\frac{8!}{7!} = \frac{8.7.6.5.4.3.2.1}{7.6.5.4.3.2.1} = 8$

(iv) $\frac{10!}{7!} = \frac{10.9.8.7.6.5.4.3.2.1}{7.6.5.4.3.2.1} = 720$

(v) $\frac{11!}{4!\,7!} = \frac{11.10.9.8.7.6.5.4.3.2.1}{4.3.2.1\ 7.6.5.4.3.2.1} = 330$

(vi) $\frac{6!}{3!\,3!} = \frac{6.5.4.3.2.1}{3.2.1.3.2.1} = 20$

(vii) $\dfrac{8!}{4!\,2!} = \dfrac{8.7.6.5.4.3.2.1}{4.3.2.1.2.1} = 840$

(viii) $\dfrac{11!}{2!\,4!\,5!} = \dfrac{11.10.9.8.7.6.5!}{2.1.4.3.2.1.5!} = 6930$

(ix) $\dfrac{9!}{2!\,(9-2)!} = \dfrac{9!}{2!\,7!} = \dfrac{9.8.7!}{2.1.7!} = 36$

(x) $\dfrac{15!}{15!\,(15-15)!} = \dfrac{15!}{15!\,0!} = \dfrac{15!}{15!.1} = 1$

(xi) $\dfrac{3!}{0!} = \dfrac{3.2.1}{1} = 6$

(xii) $4!\,0!\,1! = 4.3.2.1.1.1 = 24$

Q.2 Write each of the following in factorial form:

(i) $\mathbf{6.5.4}$ **(ii)** $\mathbf{12, 11, 10}$ **(iii)** $\mathbf{20.19.18.17}$

(iv) $\dfrac{\mathbf{10.9}}{\mathbf{2.1}}$ **(v)** $\dfrac{\mathbf{8.7.6}}{\mathbf{3.2.1}}$ **(vi)** $\dfrac{\mathbf{52.51.50.49}}{\mathbf{4.3.2.1}}$

(vii) $\mathbf{n\,(n-1)\,(n-2)}$ **(viii)** $\mathbf{(n+2)\,(n+1)\,(n)}$ **(ix)** $\dfrac{\mathbf{(n+1)\,(n)\,(n-1)}}{\mathbf{3.2.1}}$

(x) $\mathbf{n\,(n-1)\,(n-2)\,\ldots\ldots.\,(n-r+1)}$

Solution:

(i) $6.5.4 = \dfrac{6.5.4.3.2.1}{3!} = \dfrac{6!}{3!}$

(ii) $12.11.10 = \dfrac{12.11.10.9!}{9!} = \dfrac{12!}{9!}$

(iii) $20.19.18.17 = \dfrac{20.19.18.17.16!}{16!} = \dfrac{20!}{16!}$

(iv) $\dfrac{20.9}{2.1} = \dfrac{10.9.8!}{2.8!} = \dfrac{10!}{2!\,8!}$

(v) $\dfrac{8.7.6}{3.2.1} = \dfrac{8.7.6.5!}{3.2.1.5!} = \dfrac{8!}{3!\,5!}$

(vi) $\dfrac{52.51.50.49}{4.3.2.1} = \dfrac{52.51.50.49.48!}{4.3.2.1.\,48!} = \dfrac{52!}{4!\,48!}$

(vii) $n\,(n-1)\,(n-2) = \dfrac{n\,(n-1)\,(n-2)\,(n-3)!}{(n-3)!} = \dfrac{n!}{(n-3)!}$

(viii) $(n+2)\,(n+1)\,(n) = \dfrac{(n+2)\,(n+1)\,n\,(n-1)!}{(n-1)!} = \dfrac{(n+2)!}{(n-1)!}$

(ix) $\dfrac{(n+1)(n)(n-1)}{3.2.1} = \dfrac{(n+1)\,n\,(n-1)(n-2)!}{3.2.1.(n-2)!} = \dfrac{(n+1)!}{3!\,(n-2)!}$

(x) $n(n-1)(n-2)\ldots\ldots(n-r+1) = \dfrac{n(n-1)(n-2)\ldots\ldots(n-r+1)(n-r)!}{(n-r)!} = \dfrac{n!}{(n-r)!}$

PERMUTATION

An ordering (arrangement) of n objects is called a permutation of the objects.

EXPLANATION

Think of three places as shown $\boxed{}\ \boxed{}\ \boxed{}$. Since we can write any one of three vertices A, B, C at first place, so it is written in 3 different ways as shown $\boxed{3}\ \boxed{}\ \boxed{}$.

Now two vertices are left. So, corresponding to each way of writing at first place, there are two ways of writing at second place as shown $\boxed{3}\ \boxed{2}\ \boxed{}$.

Now just one vertex is left. So, we can write at third place only one vertex in one way as shown $\boxed{3}\ \boxed{2}\ \boxed{1}$.

$\Rightarrow$ The total number of possible ways is the product $3.2.1. = 6$.

FUNDAMENTAL PRINCIPLE OF COUNTING

Suppose A and B are two events. The first event A can occur in P different ways. After A has occurred, B can occur in q different ways. The number of ways that the two events can occur is the product p.q.

THEOREM

A permutation of n different objects taken $r\ (\leq n)$ at a time is an arrangement of the r objects. Generally it is denoted by ${}^nP_r$ or $P(n, r)$ where

$$^nP_r = n(n-1)(n-2)\ldots\ldots(n-r+1) = \frac{n!}{(n-r)!}$$

EXERCISE 7.2

Q.1 Evaluate the following:

(i) $^{20}P_3$ (ii) $^{16}P_4$ (iii) $^{12}P_5$ (Lahore Board 2006) (iv) $^{10}P_7$ (v) $^{9}P_8$

Solution:

Using formula $\;^nP_r = \dfrac{n!}{(n-r)!}$

(i) $^{20}P_3 = \dfrac{20!}{(20-3)!} = \dfrac{20.19.18.17!}{17!} = 6840$