EXERCISE 7.8

Q.1 The probability that a person A will be alive 15 years hence is $\frac{5}{7}$ and the probability that another person B will be alive 15 years hence $\frac{7}{9}$. Find the probability that both will be alive 15 years hence.

Solution:

Let A = Event: person A will be alive for 15 years given that $P(A) = \frac{5}{7}$

Let B = Event: person B will be alive for 15 years given that $P(B) = \frac{7}{9}$

So $P(A \cap B) = P(A) . P(B) = \frac{5}{7}\,\frac{7}{9} = \frac{5}{9}$

Q.2 A die is rolled twice: Event E_1 is the appearance of even number of dots and event E_2 is the appearance of more than 4 dots prove that

$P(E_1 \cap E_2) = P(E_1)\,P(E_2)$

Solution:

When a die is rolled twice, the possible outcomes are

$$S = \left\{\begin{array}{cccccc} (1, 1), & (1, 2), & (1, 3), & (1, 4), & (1, 5), & (1, 6) \\ (2, 1), & (2, 2), & (2, 3), & (2, 4), & (2, 5), & (2, 6) \\ (3, 1), & (3, 2), & (3, 3), & (3, 4), & (3, 5), & (3, 6) \\ (4, 1), & (4, 2), & (4, 3), & (4, 4), & (4, 5), & (4, 6) \\ (5, 1), & (5, 2), & (5, 3), & (5, 4), & (5, 5), & (5, 6) \\ (6, 1), & (6, 2), & (6, 3), & (6, 4), & (6, 5), & (6, 6) \end{array}\right\}$$

$\Rightarrow$ $n(S) = 36$

Let E_1 = Event of appearance of even number of dots

$= \{(2, 2), (2, 4), (2, 6), (4, 2), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)\}$

$\Rightarrow$ $n(E_1) = 9$

So $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{36} = \frac{1}{4}$

Let E_2 = Event of appearance of more than 4 dots

$= \{(5, 5), (5, 6), (6, 5), (6, 6)\}$

$\Rightarrow$ $n(E_2) = 4$

so $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

Now $E_1 \cap E_2 = \{(6, 6)\}$

$\Rightarrow$ $n(E_1 \cap E_2) = 1$

So $P(E_1 \cap E_2) = \dfrac{n(E_1 \cap E_2)}{n(S)} = \dfrac{1}{36}$ .... (1)

and $P(E_1) . P(E_2) = \dfrac{1}{4} \cdot \dfrac{1}{9} = \dfrac{1}{36}$ .... (2)

From (1) and (2).

It is clear that

$P(E_1 \cap E_2) = P(E_1) . P(E_2)$

Hence proved.

Q.3 Determine the probability of getting 2 heads in two successive tosses of a balanced coin.

Solution:

When two coins are tossed the possible outcomes are

$S = \{HH, HT, TH, TT\}$

$\Rightarrow$ $n(S) = 4$

Let A = Event: getting two heads

$\Rightarrow$ $n(A) = 1$

So $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{1}{4}$

Q.4 Two coins are tossed twice each. Find the probability that the head appears on the first toss and the same faces appear in the two tosses.

Solution:

Possible outcomes in first and second toss are same

i.e. $S = \{HH, HT, TH, TT\}$

$\Rightarrow$ $n(S) = 4$

Let A = Event: head appears in the first toss

$= \{TH, HT\}$

$\Rightarrow$ $n(A) = 2$

So $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{2}{4} = \dfrac{1}{2}$

Let B = Event: the same faces appear in the second toss

$= \{HH, TT\}$

$\Rightarrow$ $n(B) = 2$

So $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{2}{4} = \dfrac{1}{2}$

So $P(A \cap B) = P(A) . P(B)$

$$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

Q.5 Two cards are drawn from a deck of 52 playing cards. If one card is drawn and replaced before drawing the second card, find the probability that both the cards are aces.

Solution:

Total cards = 52 $\Rightarrow$ $n(S) = 52$

Let A = Event: drawing card is an ace

$\Rightarrow$ $n(A) = 4$

So $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{4}{52} = \dfrac{1}{13}$

Let B = Event: drawing card is an ace

$\Rightarrow$ $n(B) = 4$

So $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{4}{52} = \dfrac{1}{13}$

and $P(A \cap B) = P(A) . P(B) = \dfrac{1}{13} \cdot \dfrac{1}{13} = \dfrac{1}{169}$

Q.6 Two cards from a deck of 52 playing cards are drawn in such a way that card is replaced after the first draw. Find the probability in the following cases:

(i) first card is king and second is queen.

(ii) both the cards are faced cards i.e. king, queen, jack.

Solution:

Total cards = 52 $\Rightarrow$ $n(S) = 52$

(i) Let A = Event: the first card is king

$\Rightarrow$ $n(A) = 4$

So $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{4}{52} = \dfrac{1}{13}$

Let B = Event: second card is a queen

$\Rightarrow$ $n(B) = 4$

So $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{4}{52} = \dfrac{1}{13}$

$\therefore$ $P(A \cap B) = P(A) . P(B)$

$$= \frac{1}{13} \cdot \frac{1}{13} = \frac{1}{169}$$

(ii) Let

A = Event: the first card is faced card

$\Rightarrow$ $n(A) = 12$

So $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{12}{52} = \dfrac{3}{13}$

Let B = Even: the second card is also faced card

$n(B) = 12$

So $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{12}{52} = \dfrac{3}{13}$

$\therefore$ $P(A \cap B) = P(A) \,.\, P(B)$

$$= \frac{3}{13} \cdot \frac{3}{13} = \frac{9}{169}$$

Q.7 Two dice are thrown twice. What is the probability that sum of the dots shown in the first throw is 7 and that of the second throw is 11?

Solution:

When two dice are thrown twice, the possible outcomes are

$$S = \left\{ \begin{array}{cccccc} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{array} \right\}$$

$\Rightarrow$ $n(S) = 36$

Let A = Event: sum of dots is 7

$= \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$\Rightarrow$ $n(A) = 6$

So $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{6}{36} = \dfrac{1}{6}$

Let B = Event: sum of dots is 11

$= \{(5,6), (6,5)\}$

$\Rightarrow$ $n(B) = 2$

So $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{2}{36} = \dfrac{1}{18}$

$\therefore$ $P(A \cap B) = P(A) \,.\, P(B)$

$$= \frac{1}{6} \cdot \frac{1}{18} = \frac{1}{108}$$

Q.8 Find the probability that the sum of dots appearing in two successive throws of two dice is every time 7.

Solution:

The possible outcomes are

$$S = \left\{ \begin{array}{cccccc} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{array} \right\}$$

$\Rightarrow$ $n(S) = 36$

Let A = Event: sum of dots is 7

$= \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

$\Rightarrow$ $n(A) = 6$

So $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{6}{36} = \dfrac{1}{6}$

Let B = Event: sum of dots in second throw is 7

$= \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

$\Rightarrow$ $n(B) = 6$

So $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{6}{36} = \dfrac{1}{6}$

$\therefore$ $P(A \cap B) = P(A) \, . \, P(B)$

$$= \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

Q.9 A fair die is thrown twice. Find the probability that a prime number of dots appear in the first throw and the number of dots in the second throw is less than 5.

Solution:

We have same outcomes in the first and second throw.

When a die is throw the possible number of dots are

$S = \{1, 2, 3, 4, 5, 6\}$ $\Rightarrow$ $n(S) = 6$

Let A = Event: prime dots appear

$= \{2, 3, 5\}$

$\Rightarrow$ $n(A) = 3$

So $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{3}{6} = \dfrac{1}{2}$

Let B = Event: number of dots are less than 5

$= \{1, 2, 3, 4\}$

$\Rightarrow$ $n(B) = 4$

So $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{4}{6} = \dfrac{2}{3}$

$\therefore$ $P(A \cap B) = P(A) \cdot P(B)$

$$= \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$$

Q.10 A bag containing 8 red, 5 white and 7 black balls. 3 balls are drawn from the bag. What is the probability that the first ball is red, the second ball is white and the third ball is black, when every time the ball drawn is replaced?

Solution:

Total balls $= 8 + 5 + 7 = 20$

$\Rightarrow$ $n(S) = 20$

Let A = Event: drawing ball is red

$\Rightarrow$ $n(A) = 8$

So $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{8}{20} = \dfrac{2}{5}$

Let B = Event: drawing ball is white

$\Rightarrow$ $n(B) = 5$

So $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{5}{20} = \dfrac{1}{4}$

Let C = Event: drawing ball is black

$\Rightarrow$ $n(C) = 7$

So $P(C) = \dfrac{n(C)}{n(S)} = \dfrac{7}{20}$

$\therefore$ $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

$$= \frac{2}{5} \cdot \frac{1}{4} \cdot \frac{7}{20} = \frac{7}{200}$$