$$\frac{\delta y}{\delta z} = \frac{a\delta z}{\delta z(az-b)^{1+7}}\left[-7+\frac{(-7)(-7-1)}{2!}\cdot\frac{a\delta z}{az-b}+\dots\dots\dots\right]$$

$$\frac{\delta y}{\delta z} = \frac{a}{(az-b)^{8}}\left[-7+\frac{(-7)(-7-1)}{2!}\cdot\frac{a\delta z}{az-b}+\dots\dots\dots\right]$$

Taking limit $\delta z \to 0$

$$\lim_{\delta z\to 0}\frac{\delta y}{\delta z} = \lim_{\delta z\to 0}\frac{a}{(az-b)^{8}}\left[-7+\frac{(-7)(-7-1)}{2!}\cdot\frac{a\delta z}{az-b}+\dots\dots\dots\right]$$

$$\frac{dy}{dz} = \frac{a}{(az-b)^{8}}(-7)$$

$$\boxed{\frac{d}{dz}\left[\frac{1}{(az-b)^{7}}\right] = \frac{-7a}{(az-b)^{8}}}$$ Ans.

EXERCISE 2.3

Q.1 Differentiate w.r.t. 'x'
$x^4 + 2x^3 + x^2$

Solution:

Let $y = x^4 + 2x^3 + x^2$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(x^4) + 2\frac{d}{dx}(x^3) + \frac{d}{dx}(x^2)$$

$$\frac{dy}{dx} = 4x^{4-1}\cdot\frac{d}{dx}(x) + 2.3\,x^{3-1}\cdot\frac{d}{dx}(x) + 2x^{2-1}\cdot\frac{d}{dx}(x)$$

$$\frac{dy}{dx} = 4x^3\,.\,1 + 6x^2\,.\,1 + 2x\,.\,1$$

$$\boxed{\frac{dy}{dx} = 4x^3 + 6x^2 + 2x}$$ Ans.

Q.2 $x^{-3} + 2x^{-3/2} + 2$

Solution:

Let $y = x^{-3} + 2x^{-3/2} + 2$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(x^{-3}) + 2\frac{d}{dx}(x^{-3/2}) + \frac{d}{dx}(2)$$

$$\frac{dy}{dx} = -3x^{-3-1} \cdot \frac{d}{dx}(x) + 2 \cdot \frac{-3}{2} x^{\frac{3}{2}-1} \cdot \frac{d}{dx}(x) + 0$$

$$\frac{dy}{dx} = -3x^{-4} \cdot 1 - 3x^{-5/2} \cdot 1$$

$$\boxed{\frac{dy}{dx} = -3\left(\frac{1}{x^4} + \frac{1}{x^{5/2}}\right)} \quad \text{Ans.}$$

Q.3 $\mathbf{\dfrac{a + x}{a - x}}$ ***(Lhr. Board 2008, 2010, 2011)***

Solution:

Let $y = \dfrac{a + x}{a - x}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{a + x}{a - x}\right)$$

$$= \frac{(a - x)\frac{d}{dx}(a + x) - (a + x)\frac{d}{dx}(a - x)}{(a - x)^2}$$

$$= \frac{(a - x)(0 + 1) - (a + x)(0 - 1)}{(a - x)^2}$$

$$= \frac{a - x + a + x}{(a - x)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{2a}{(a - x)^2}} \quad \text{Ans.}$$

Q.4 $\mathbf{\dfrac{2x - 3}{2x + 1}}$ ***(L.B 2008)***

Solution:

Let $y = \dfrac{2x - 3}{2x + 1}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2x - 3}{2x + 1}\right)$$

$$\frac{dy}{dx} = \frac{(2x + 1)\frac{d}{dx}(2x - 3) - (2x - 3)\frac{d}{dx}(2x + 1)}{(2x + 1)^2}$$

$$\frac{dy}{dx} = \frac{(2x+1) . 2 - (2x-3) . 2}{(2x+1)^2}$$

$$\frac{dy}{dx} = \frac{2[2x+1-2x+3]}{(2x+1)^2}$$

$$\frac{dy}{dx} = \frac{2(4)}{(2x+1)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{8}{(2x+1)^2}} \quad \text{Ans.}$$

Q.5 (x – 5)(3 – x)

Solution:

Let $y = (x-5)(3-x)$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx}[(x-5)(3-x)]$$

$$\frac{dy}{dx} = (x-5)\frac{d}{dx}(3-x) + (3-x)\frac{d}{dx}(x-5)$$

$$\frac{dy}{dx} = (x-5)(-1) + (3-x)(1)$$

$$\frac{dy}{dx} = -x + 5 + 3 - x$$

$$\boxed{\frac{dy}{dx} = 8 - 2x} \quad \text{Ans.}$$

Q.6 $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$

Solution:

Let $y = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$

$$y = (\sqrt{x})^2 + \left(\frac{1}{\sqrt{x}}\right)^2 - 2(\sqrt{x})\left(\frac{1}{\sqrt{x}}\right)$$

$$y = x + \frac{1}{x} - 2$$

$$y = x + x^{-1} - 2$$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}(x^{-1}) - \frac{d}{dx}(2)$$

$$\frac{dy}{dx} = 1 + (-1)\, x^{-1-1} \cdot \frac{d}{dx}(x) - 0$$

$$\frac{dy}{dx} = 1 - x^{-2} \cdot 1$$

$$\frac{dy}{dx} = 1 - \frac{1}{x^2}$$

$$\boxed{\frac{dy}{dx} = \frac{x^2 - 1}{x^2}} \qquad \text{Ans.}$$

Q.7 $\dfrac{\mathbf{(1 + \sqrt{x})\,(x - x^{3/2})}}{\mathbf{\sqrt{x}}}$

Solution:

$$y = \frac{(1 + \sqrt{x})\,(x - x^{3/2})}{\sqrt{x}}$$

$$y = \frac{(1 + \sqrt{x})\,(x - x\sqrt{x})}{\sqrt{x}}$$

$$y = \frac{x(1 + \sqrt{x})\,(1 - \sqrt{x})}{\sqrt{x}}$$

$$y = \frac{\sqrt{x} \times \sqrt{x}\,\left[(1)^2 - (\sqrt{x})^2\right]}{\sqrt{x}}$$

$$y = \sqrt{x}\,(1 - x)$$

$$y = \sqrt{x} - x\sqrt{x}$$

$$y = x^{1/2} - x^{3/2}$$

Diff. w.r.t 'x'

$$\frac{dy}{dx} = \frac{dy}{dx}(x^{1/2}) - \frac{d}{dx}(x^{3/2})$$

$$\frac{dy}{dx} = \frac{1}{2}x^{1/2-1} \cdot \frac{d}{dx}(x) - \frac{3}{2}x^{3/2-1} \cdot \frac{d}{dx}(x)$$

$$\frac{dy}{dx} = \frac{1}{2}x^{1/2-1} \cdot 1 - \frac{3}{2}x^{3/2-1} \cdot 1$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} - \frac{3\sqrt{x}}{2}$$

$$\boxed{\frac{dy}{dx} = \frac{1 - 3x}{2\sqrt{x}}} \qquad \text{Ans.}$$

Q.8 $\dfrac{(x^2+1)^2}{x^2-1}$ ***(L.B 2007) (G.B 2007)***

Solution:

$$y = \frac{(x^2+1)^2}{x^2-1}$$

Diff. w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{(x^2+1)^2}{x^2-1}\right]$$

$$\frac{dy}{dx} = \frac{(x^2-1)\frac{d}{dx}(x^2+1)^2 - (x^2+1)^2 \cdot \frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{(x^2-1)\cdot 2(x^2+1)\cdot \frac{d}{dx}(x^2+1) - (x^2+1)^2 \cdot 2x}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{2(x^2-1)(x^2+1)\cdot 2x - 2x(x^2+1)^2}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{2x(x^2+1)\,[2(x^2-1)-(x^2+1)]}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{2x(x^2+1)(2x^2-2-x^2-1)}{(x^2-1)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{2x(x^2+1)(x^2-3)}{(x^2-1)^2}} \quad \text{Ans.}$$

Q.9 $\dfrac{x^2+1}{x^2-3}$ ***(L.B 2009)***

Solution:

Let $y = \dfrac{x^2+1}{x^2-3}$

Diff. w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2+1}{x^2-3}\right)$$

$$\frac{dy}{dx} = \frac{(x^2-3)\frac{d}{dx}(x^2+1) - (x^2+1)\frac{d}{dx}(x^2-3)}{(x^2-3)^2}$$

$$\frac{dy}{dx} = \frac{(x^2-3)\cdot 2x - (x^2+1)\cdot 2x}{(x^2-3)^2}$$

$$\frac{dy}{dx} = \frac{2x(x^2-3-x^2-1)}{(x^2-3)^2}$$

$$\frac{dy}{dx} = \frac{2x(-4)}{(x^2-3)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{-8x}{(x^2-3)^2}} \quad \text{Ans.}$$

Q.10 $\dfrac{\sqrt{1+x}}{\sqrt{1-x}}$

Solution:

Let $y = \dfrac{\sqrt{1+x}}{\sqrt{1-x}}$

$$y = \left(\frac{1+x}{1-x}\right)^{1/2}$$

Diff. w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1+x}{1-x}\right)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}-1} \cdot \frac{d}{dx}\left(\frac{1+x}{1-x}\right)$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-1/2}\left[\frac{(1-x)\frac{d}{dx}(1+x)-(1+x)\frac{d}{dx}(1-x)}{(1-x)^2}\right]$$

$$\frac{dy}{dx} = \frac{1}{2}\frac{(1+x)^{-1/2}}{(1-x)^{-1/2}}\left[\frac{(1-x)(0+1)-(1+x)(0-1)}{(1-x)^2}\right]$$

$$\frac{dy}{dx} = \frac{1-x+1+x}{2(1+x)^{1/2}(1-x)^{-1/2+2}}$$

$$\frac{dy}{dx} = \frac{2}{2\sqrt{1+x}\,(1-x)^{3/2}}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{1+x}\,(1-x)^{3/2}}} \quad \text{Ans.}$$

Q.11 $\dfrac{2x-1}{\sqrt{x^2+1}}$

Solution:

Let $y = \dfrac{2x-1}{\sqrt{x^2+1}}$

Diff. w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2x-1}{\sqrt{x^2+1}}\right)$$

$$\frac{dy}{dx} = \frac{\left(\sqrt{x^2+1}\right)\frac{d}{dx}(2x-1) - (2x-1)\frac{d}{dx}\left(\sqrt{x^2+1}\right)}{\left(\sqrt{x^2+1}\right)^2}$$

$$\frac{dy}{dx} = \frac{\left(\sqrt{x^2+1}\right)2 - (2x-1)\,.\,\frac{1}{2}(x^2+1)^{-1/2}\,.\,2x}{x^2+1}$$

$$\frac{dy}{dx} = \frac{2\sqrt{x^2+1} - \frac{x(2x-1)}{\sqrt{x^2+1}}}{x^2+1}$$

$$\frac{dy}{dx} = \frac{\frac{2(x^2+1)\ \ 2x^2+x}{\sqrt{x^2+1}}}{x^2+1}$$

$$\frac{dy}{dx} = \frac{2x^2+2-2x^2+x}{(x^2+1)\sqrt{x^2+1}}$$

$$\boxed{\frac{dy}{dx} = \frac{x+2}{(x^2+1)^{3/2}}} \text{ Ans.}$$

Q.12 $\sqrt{\frac{a-x}{a+x}}$ ***(L.B 2004)***

Solution:

Let $y = \sqrt{\frac{a-x}{a+x}}$

$$y = \left(\frac{a-x}{a+x}\right)^{1/2}$$

Diff. w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{a-x}{a+x}\right)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{a-x}{a+x}\right)^{-1/2}\,.\,\frac{d}{dx}\left(\frac{a-x}{a+x}\right)$$

$$\frac{dy}{dx} = \frac{1}{2}\frac{(a-x)^{-1/2}}{(a+x)^{-1/2}}\left[\frac{(a+x)\frac{d}{dx}(a-x) - (a-x)\frac{d}{dx}(a+x)}{(a+x)^2}\right]$$

$$\frac{dy}{dx} = \frac{(a+x)(0-1)-(a-x)(0+1)}{2(a-x)^{1/2}(a+x)^{\frac{-1}{2}+2}}$$

$$\frac{dy}{dx} = \frac{-a-x-a+x}{2\sqrt{a-x}\,(a+x)^{3/2}}$$

$$\frac{dy}{dx} = \frac{-2a}{2\sqrt{a-x}\,(a+x)^{3/2}}$$

$$\boxed{\frac{dy}{dx} = \frac{-a}{\sqrt{a-x}\,(a+x)^{3/2}}} \quad \text{Ans.}$$

Q.13 $\dfrac{\sqrt{x^2+1}}{\sqrt{x^2-1}}$

Solution:

Let $y = \dfrac{\sqrt{x^2+1}}{\sqrt{x^2-1}}$ $\quad y = \left(\dfrac{x^2+1}{x^2-1}\right)^{1/2}$

Diff. w.r.t 'x'.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{x^2+1}{x^2-1}\right)^{1/2} \cdot \frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\frac{(x^2+1)^{-1/2}}{(x^2-1)^{-1/2}}\left[\frac{(x^2-1)\frac{d}{dx}(x^2+1)-(x^2+1)\frac{d}{dx}(x^2-1)}{(x^2-1)^2}\right]$$

$$\frac{dy}{dx} = \frac{(x^2-1)\,.\,2x-(x^2+1)\,.\,2x}{2(x^2+1)^{1/2}(x^2-1)^{-1/2+2}}$$

$$\frac{dy}{dx} = \frac{2x(x^2-1-x^2-1)}{2\sqrt{x^2+1}\,(x^2-1)^{3/2}}$$

$$\boxed{\frac{dy}{dx} = \frac{-2x}{\sqrt{x^2+1}\,(x^2-1)^{3/2}}} \quad \text{Ans.}$$

Q.14 $\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

Solution:

Let $y = \dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

$$y = \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} \times \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}$$

$$y = \frac{\left(\sqrt{1+x}-\sqrt{1-x}\right)^2}{\left(\sqrt{1+x}\right)^2-\left(\sqrt{1-x}\right)^2}$$

$$y = \frac{\left(\sqrt{1+x}\right)^2+\left(\sqrt{1-x}\right)^2-2\left(\sqrt{1+x}\right)\left(\sqrt{1-x}\right)}{1+x-(1-x)}$$

$$y = \frac{1+x+1-x-2\sqrt{(1+x)(1-x)}}{1+x-1+x}$$

$$y = \frac{2-2\sqrt{1-x^2}}{2x}$$

$$y = \frac{2\left(1-\sqrt{1-x^2}\right)}{2x}$$

$$y = \frac{1-\sqrt{1-x^2}}{x}$$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1-\sqrt{1-x^2}}{x}\right)$$

$$\frac{dy}{dx} = \frac{x\frac{d}{dx}\left(1-\sqrt{1-x^2}\right)-\left(1-\sqrt{1-x^2}\right)\frac{d}{dx}(x)}{x^2}$$

$$\frac{dy}{dx} = \frac{x\,.\frac{-1}{2}(1-x^2)^{-1/2}\,.-2x-\left(1-\sqrt{1-x^2}\right).\,1}{x^2}$$

$$\frac{dy}{dx} = \frac{\frac{x^2}{\sqrt{1-x^2}}-1+\sqrt{1-x^2}}{x^2}$$

$$\frac{dy}{dx} = \frac{\frac{x^2-\sqrt{1-x^2}+1-x^2}{\sqrt{1-x^2}}}{x^2}$$

$$\boxed{\frac{dy}{dx} = \frac{1-\sqrt{1-x^2}}{x^2\sqrt{1-x^2}}} \qquad \text{Ans.}$$

Q.15 $\mathbf{\dfrac{x\sqrt{a+x}}{\sqrt{a-x}}}$

Solution:

Let $y = \frac{x\sqrt{a+x}}{\sqrt{a-x}}$ $\quad y = x\left(\frac{a+x}{a-x}\right)^{1/2}$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left[x\left(\frac{a+x}{a-x}\right)^{1/2}\right]$$

$$= x\frac{d}{dx}\left(\frac{a+x}{a-x}\right)^{1/2} + \left(\frac{a+x}{a-x}\right)^{1/2} . \frac{d}{dx}(x)$$

$$\frac{dy}{dx} = x . \frac{1}{2}\left(\frac{a+x}{a-x}\right)^{-1/2} . \frac{d}{dx}\left(\frac{a+x}{a-x}\right) + \frac{\sqrt{a+x}}{\sqrt{a-x}}$$

$$\frac{dy}{dx} = \frac{x}{2}\ \frac{(a+x)^{-1/2}}{(a-x)^{-1/2}}\left[\frac{(a-x)\frac{d}{dx}(a+x) - (a+x)\frac{d}{dx}(a-x)}{(a-x)^2}\right] + \frac{\sqrt{a+x}}{\sqrt{a-x}}$$

$$\frac{dy}{dx} = \frac{x\,[(a-x)(0+1) - (a+x)(0-1)]}{2(a+x)^{1/2}(a-x)^{\frac{-1}{2}+2}} + \frac{\sqrt{a+x}}{\sqrt{a-x}}$$

$$\frac{dy}{dx} = \frac{x(a-x+a+x)}{2\sqrt{a+x}\,(a-x)^{3/2}} + \frac{\sqrt{a+x}}{\sqrt{a-x}}$$

$$\frac{dy}{dx} = \frac{2ax}{2\sqrt{a+x}\,(a-x)^{3/2}} + \frac{\sqrt{a+x}}{\sqrt{a-x}}$$

$$\frac{dy}{dx} = \frac{ax}{\sqrt{a+x}\,(a-x)^{3/2}} + \frac{\sqrt{a+x}}{\sqrt{a-x}}$$

$$\frac{dy}{dx} = \frac{ax + (a+x)(a-x)}{\sqrt{a+x}\,(a-x)^{3/2}}$$

$$\frac{dy}{dx} = \frac{ax + a^2 - x^2}{\sqrt{a+x}\,(a-x)^{3/2}}$$

$$\boxed{\frac{dy}{dx} = \frac{a^2 + ax - x^2}{\sqrt{a+x}\,(a-x)^{3/2}}} \quad \text{Ans.}$$

Q.16 If $y = \sqrt{x} - \frac{1}{\sqrt{x}}$, show that $2x\frac{dy}{dx} + y = 2\sqrt{x}$. ***(L.B 2004)***

Solution:

Let $y = \sqrt{x} - \frac{1}{\sqrt{x}}$

$$y = x^{1/2} - x^{-1/2}$$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(x^{1/2}) - \frac{d}{dx}(x^{-1/2})$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} - \left(\frac{-1}{2}\right)x^{-3/2}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} + \frac{1}{2x^{3/2}}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} + \frac{1}{2x\sqrt{x}}$$

Taking $2x\frac{dy}{dx} + y = 2x\left(\frac{1}{2\sqrt{x}} + \frac{1}{2x\sqrt{x}}\right) + \sqrt{x} - \frac{1}{\sqrt{x}}$

$$= \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}} + \sqrt{x} - \frac{1}{\sqrt{x}}$$

$$= \sqrt{x} + \sqrt{x}$$

$\boxed{2x\frac{dy}{dx} + y = 2\sqrt{x}}$ Hence proved.

Q.17 If $y = x^4+2x^2+2$, prove that $\frac{dy}{dx} = 4x\sqrt{y-1}$. ***(G.B 2006)(L.B 2009(S) (L.B 2007)***

Solution:

Let $y = x^4 + 2x^2 + 2$

Diff. w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(x^4) + 2\frac{d}{dx}(x^2) + \frac{d}{dx}(2)$$

$$= 4x^3 + 2(2x) + 0$$

$$\frac{dy}{dx} = 4x^3 + 4x$$

Taking,

$$4x\sqrt{y-1} = 4x\sqrt{x^4 + 2x^2 + 2 - 1}$$

$$= 4x\sqrt{x^4 + 2x^2 + 1}$$

$$= 4x\sqrt{(x^2+1)^2}$$

$$= 4x(x^2+1)$$

$$= 4x^3 + 4x$$

$\frac{dy}{dx} = 4x\sqrt{y-1}$ Hence proved.