For example,

2, 4, 6, 8, …….. is a sequence

2 + 4 + 6 + 8 + …… is a series

The formula to find the sum of first n terms of an arithmetic series is

$$S_n = \frac{n}{2}(a_1 + a_n)$$

or $$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

EXERCISE 6.4

Q.1 Find the sum of all the integral multiples of 3 between 4 and 97.

Solution:

Integral multiples between 4 and 97 are

6, 9, 12, 15, ... 96

$a_1 = 6, \quad d = 9 - 6 = 3, \quad a_n = 96, \quad n = ?$

As $a_n = a_1 + (n-1)d$

$96 = 6 + (n-1)3$

$96 - 6 = 3n - 3$

$90 = 3n - 3$

$3n = 90 + 3$

$n = 31$

Now using formula

$$S_n = \frac{n}{2}\{2a_1 + (n-1)d\}$$

$$= \frac{31}{2}\{2(6) + (31-1)(3)\}$$

$$= \frac{31}{2}\{12 + 90\}$$

$$= \frac{31}{2}(102) = 1581$$

Q.2 Sum of the series

(i) $-3 + (-1) + 1 + 3 + 5 + \ldots\ldots\ldots\ldots a_{16}$ **(Lahore Board 2007)**

(ii) $\frac{3}{\sqrt{2}} + 2\sqrt{2} + \frac{5}{\sqrt{2}} + \ldots\ldots + a_{13}$

(iii) $1.11, + 1.41 + 1.71 + \ldots\ldots\ldots + a_{10}$

(iv) $-8 - \frac{7}{2} + 1 + \ldots\ldots\ldots + a_{11}$

(v) $(x - a) + (x + a) + (x + 3a) + \ldots\ldots\ldots$ n terms

(vi) $\frac{1}{1-\sqrt{x}} + \frac{1}{1-x} + \frac{1}{1+\sqrt{x}} \ldots\ldots\ldots$ n terms

(vii) $\frac{1}{1+\sqrt{x}} + \frac{1}{1-x} + \frac{1}{1-\sqrt{x}} + \ldots\ldots$ n terms

Solution:

(i) $-3 + (-1) + 1 + 3 + 5 + \ldots\ldots\ldots\ldots a_{16}$

$a_1 = -3, \quad d = -1-(-3) = 2, \quad n = 16, \quad S_n = ?$

using

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{16}{2}\{2(-3) + (16-1)(2)\}$$

$$= 8\{-6 + 30\}$$

$$= 192$$

(ii) $\frac{3}{\sqrt{2}} + 2\sqrt{2} + \frac{5}{\sqrt{2}} + \ldots\ldots + a_{13}$

$a_1 = \frac{3}{\sqrt{2}}, \quad d = 2\sqrt{2} - \frac{3}{\sqrt{2}} = \frac{4-3}{\sqrt{2}} = \frac{1}{\sqrt{2}} \quad , \quad n = 13, \quad S_n = ?$

using

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{13}{2}\left\{2\left(\frac{3}{\sqrt{2}}\right) + (13-1)\left(\frac{1}{\sqrt{2}}\right)\right\}$$

$$= \frac{13}{2}\left\{\frac{6}{\sqrt{2}} + \frac{12}{\sqrt{2}}\right\}$$

$$S_{13} = \frac{13}{2} \cdot \frac{18}{\sqrt{2}} = \frac{117}{\sqrt{2}}$$

(iii) $\mathbf{1.11, + 1.41 + 1.71 + ... + a_{10}}$

$a_1 = 1.11, \quad d = 1.41 - 1.11 = 0.30 \quad n = 10, \quad S_n = ?$

using

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{10}{2}\{2(1.11) + (10-1)(0.30)\}$$

$$S_{10} = 5\{2.22 + 2.70\}$$

$$= 24.6 \quad \text{Ans.}$$

(iv) $\mathbf{-8 - \frac{7}{2} + 1 + ... + a_{11}}$

Here $a_1 = -8 \quad d = 1 - \left(-3\frac{1}{2}\right) = 1 + \frac{7}{2} = \frac{9}{2}, \quad n = 11, \quad S_n = ?$

As $S_n = \frac{n}{2}\{2a_1 + (n-1)d\}$

$$= \frac{11}{2}\left\{2(-8) + (11-1)\left(\frac{9}{2}\right)\right\}$$

$$= \frac{11}{2}\{-16 + 45\}$$

$$= \frac{11}{2}\{29\} = 159.5$$

(v) $\mathbf{(x - a) + (x + a) + (x + 3a) + ... \text{ n terms}}$

$a_1 = x - a, \quad d = x + a - (x - a) = 2a, \quad n = n, \quad S_n = ?$

As $S_n = \frac{n}{2}\{2a_1 + (n-1)d\}$

$$= \frac{n}{2}\{2(x-a) + (n-1)(2a)\}$$

$$= n[(x-a) + (n-1)a]$$

$$= n[x - a + na - a]$$

$$= n[x + na - 2a]$$

$$= n(x + (n-2)a)$$

(vi) $\mathbf{\dfrac{1}{1-\sqrt{x}} + \dfrac{1}{1-x} + \dfrac{1}{1+\sqrt{x}}}$ **....... n terms**

$a_1 = \dfrac{1}{1-\sqrt{x}}, \quad d = \dfrac{1}{1-x} - \dfrac{1}{1-\sqrt{x}} = \dfrac{1-(1-\sqrt{x})}{1-x} = \dfrac{-\sqrt{x}}{1-x}$

$n = n, \quad S_n = ?$ ¤ $1 - x = (1+\sqrt{x})(1-\sqrt{x})$

As $S_n = \dfrac{n}{2}\{2a_1 + (n-1)d\}$

$$= \frac{n}{2}\left\{\frac{2}{1-\sqrt{x}} + (n-1)\left(\frac{-\sqrt{x}}{1-x}\right)\right\}$$

$$= \frac{n}{2}\left[\frac{2}{1-\sqrt{x}} - \frac{(n-1)\sqrt{x}}{1-x}\right]$$

$$= \frac{n}{2}\left\{\frac{2(1+\sqrt{x}) - (n-1)\sqrt{x}}{1-x}\right\}$$ ¤ $1 - x = (1+\sqrt{x})(1-\sqrt{x})$

$$= \frac{n}{2}\left\{\frac{2 + 2\sqrt{x} - n\sqrt{x} + \sqrt{x}}{1-x}\right\}$$

$$= \frac{n}{2}\left\{\frac{2 + 3\sqrt{x} - n\sqrt{x}}{1-x}\right\}$$

$$= \frac{n}{2}\left\{\frac{2 + (3-n)\sqrt{x}}{1-x}\right\}$$

(vii) $\mathbf{\dfrac{1}{1+\sqrt{x}} + \dfrac{1}{1-x} + \dfrac{1}{1-\sqrt{x}}}$ **+ ...... n terms**

$a_1 = \dfrac{1}{1+\sqrt{x}}, \quad d = \dfrac{1}{1-x} - \dfrac{1}{1+\sqrt{x}} = \dfrac{1-(1-\sqrt{x})}{1-x} = \dfrac{-\sqrt{x}}{1-x}$

$n = n, \quad S_n = ?$

As $S_n = \dfrac{n}{2}\{2a + (n-1)d\}$

$$= \frac{n}{2}\left\{2\frac{1}{1+\sqrt{x}} + (n-1)\frac{\sqrt{x}}{1-x}\right\}$$

$$= \frac{n}{2}\left\{\frac{2}{1+\sqrt{x}} + \frac{(n-1)\sqrt{x}}{1-x}\right\}$$

$$= \frac{n}{2}\left\{\frac{2\left(1-\sqrt{x}\right)+(n-1)\sqrt{x}}{\left(1-\sqrt{x}\right)(1-x)}\right\}$$

$$= \frac{n}{2}\left\{\frac{2-2\sqrt{x}+n\sqrt{x}-\sqrt{x}}{1-x}\right\}$$

$$= \frac{n}{2}\left\{\frac{2-3\sqrt{x}+n\sqrt{x}}{1-x}\right\}$$

$$= \frac{n}{2}\left\{\frac{2+(n-3)\sqrt{x}}{1-x}\right\}$$

Q.3 How many terms of the series

(i) $-7+(-5)+(-3)+\ldots\ldots\ldots\ldots$ amount to 65

(ii) $-7+(-4)+(-1)+\ldots\ldots\ldots$ amount to 114

Solution:

(i) $-7+(-5)+(-3)+\ldots\ldots\ldots$

Here $a_1 = -7 \quad d = -5-(-7) = -5+7 = 2, \quad S_n = 65, \quad n = ?$

As $S_n = \frac{n}{2}\{2a_1+(n-1)d\}$

$$65 = \frac{n}{2}\{2(-7)+(n-1)(2)\}$$

$$65 = \frac{n}{2}\{-14+2n-2\}$$

$$65 = \frac{n}{2}(2n-16)$$

$$65 = n(n-8)$$

$$65 = n^2-8n \quad \Rightarrow \quad n^2-8n-65 = 0$$

$$n^2-13n+5n-65 = 0$$

$$n(n-13)+5(n-13) = 0$$

$$(n-13)(n+5) = 0$$

$\Rightarrow \quad n = 13 \quad \text{or} \quad n = -5 \quad$ (not possible)

$$\Rightarrow \boxed{n = 13}$$

(ii) $-7 + (-4) + (-1) + \text{.........}$ amount to 114

$a_1 = -7 \quad d = -4-(-7) = -+7 = 3, \quad S_n = 114, \quad n = ?$

As $S_n = \frac{n}{2}\{2a_1 + (n-1)d\}$

$$114 = \frac{n}{2}\{2(-7) + (n-1)(3)\}$$

$$114 = \frac{n}{2}[-14 + 3n - 3]$$

$$114 = \frac{n}{2}[3n - 17]$$

$$228 = 3n^2 - 17n$$

$$3n^2 - 17n - 228 = 0$$

$$3n^2 - 36n + 19n - 228 = 0$$

$$3n(n-12) - 19(n-12) = 0$$

$$(3n-19)(n-12) = 0$$

$n = 12$ or $n = \frac{-19}{3}$ (not possible)

$\Rightarrow \boxed{n = 12}$

Q.4 Sum the series

(i) $3 + 5 - 7 + 9 + 11 - 13 + 15 + 17 - 19 + \text{......}$ to $3n$ terms.

(Lahore Board 2007, 2009)

(ii) $1 + 4 - 7 + 10 + 13 - 16 + 19 + 22 - 25 + \text{.......}$ to $3n$ terms

Solution:

(i) $3 + 5 - 7 + 9 + 11 - 13 + 15 + 17 - 19 + \text{......} +$ to $3n$ terms.

It can be written as

$(3 + 5 - 7) + (9 + 11 - 13) + (15 + 17 - 19) + \ldots\ldots$ n terms

$1 + 7 + 13 + \ldots\ldots$ n terms

Now $a_1 = 1, \quad d = 7 - 1 = 6, \quad n = n, \quad S_n = ?$

As $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$$= \frac{n}{2}[2(1) + (n-1)(6)]$$

$$S_n = \frac{n}{2}[2 + 6n - 6]$$

$$= \frac{n}{2} [6n - 4]$$

$$= n(3n - 2)$$

(ii) $1 + 4 - 7 + 10 + 13 - 16 + 19 + 22 - 25 + \ldots\ldots$ to $3n$ terms

$1 + 4 - 7 + 10 + 13 - 16 + 19 + 22 - 25 + \ldots\ldots$ to $3n$ terms

It can be written as

$(1 + 4 - 7) + (10 + 13 - 16) + (19 + 22 - 25) + \ldots\ldots$ n terms

$-2 + 7 + 16 + \ldots\ldots$ n terms

Now $a_1 = -2, \quad d = 7 - (-2) = 9, \quad n = n, \quad S_n = ?$

As $S_n = \frac{n}{2} \{2a_1 + (n-1)d\}$

$$= \frac{n}{2} \{2(-2) + (n-1)(9)\}$$

$$= \frac{n}{2} \{-4 + 9n - 9\}$$

$$= \frac{n}{2} [9n - 13]$$

Q.5 Find sum of 20 terms of the series whose rth term is $3r + 1$

(Gujranwala Board 2007)

Solution:

Given that

$a_r = 3r + 1$

Put $r = 1, 2, 3$

$r = 1 \Rightarrow a_1 = 3(1) + 1 = 4$

$r = 2 \Rightarrow a_2 = 3(2) + 1 = 7$

$r = 3 \Rightarrow a_3 = 3(3) + 1 = 10$

$\Rightarrow$ Series will be

$4 + 7 + 10 + \ldots\ldots$

$\Rightarrow a_1 = 4, \quad d = 7 - 4 = 3, \quad n = 20, \quad S_n = ?$

As $S_n = \frac{n}{2} [2a_1 + (n-1)d]$

$$= \frac{20}{2} [2(4) + (20-1)(3)]$$

$S_{20} = 10[8 + 57] = 650$

Q.6 If $S_n = n(2n-1)$ then find the series.

Solution:

Given

Put $n = 1, 2, 3, 4$

$n = 1 \Rightarrow S_1 = 1(2(1)-1) = 1$

$n = 2 \Rightarrow S_2 = 2(2(2)-1) = 2(4-1) = 6$

$n = 3 \Rightarrow S_3 = 3(2(3)-1) = 3(6-1) = 15$

$n = 4 \Rightarrow S_4 = 4(2(4)-1) = 4(8-1) = 28$

Now $a_1 = S_1 = 1$

$a_2 = S_2 - S_1 = 6 - 1 = 5$

$a_3 = S_3 - S_2 = 15 - 6 = 9$

$a_4 = S_4 - S_3 = 28 - 15 = 13$

so, the required series is

$1 + 5 + 9 + 13 + ...$

Q.7 The ratio of the sum of n terms of two series in A.P is $3n + 2 : n + 1$. Find the ratio of their 8th term. (Lahore Board 2004, 2006)

Solution:

Let two A.Ps with a, d and a', d'

$$\Rightarrow \quad \frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2a' + (n-1)d']} = \frac{3n+2}{n+1}$$

$$\Rightarrow \quad \frac{2a + (n-1)d}{2a' + (n-1)d'} = \frac{3n+2}{n+1} \quad \text{……… (1)}$$

To find the ratio of their 8^{th} terms

i.e. $\dfrac{a_8}{a'_8} = \dfrac{a + 7d}{a' + 7d}$

we will put $n = 15$ in equation (1) we get

$$\frac{2a + (15-1)d}{2a' + (15-1)d'} = \frac{3(15)+2}{15+1}$$

$$\Rightarrow \quad \frac{2a + 14d}{2a' + 14d'} = \frac{47}{16}$$

$$\frac{a + 7d}{a' + 7d'} = \frac{47}{16}$$

Q.8 If S_2, S_3, S_5 are sum of 2n, 3n, 5n terms of the A.P. Show that $S_5 = 5(S_3 - S_2)$ (Lahore Board 2011)

Solution:

As $S_n = \frac{2n}{2}$ [2a + (2n – 1) d}

It is given that

$S_2 = \frac{2n}{2}[2a + (2n-1)d]$ ……… (1)

$S_3 = \frac{3n}{2}[2a + (3n-1)d]$ ……… (2)

$S_5 = \frac{5n}{2}[2a + (5n-1)d]$ ……… (3)

Now from (1), (2)

$$S_3 - S_2 = \frac{3n}{2}[2a_1 + (3n-1)d] - \frac{2n}{2}[2a + (2n-1)d]$$

$$= \frac{n}{2}[6a + (9n-3)d] - \frac{n}{2}[4a + (4n-2)d]$$

$$= \frac{n}{2}[6a + 9nd - 3d] - \frac{n}{2}[4a + 4nd - 2d]$$

$$= \frac{n}{2}[6a + 9nd - 3d - 4a - 4nd + 2d]$$

$$= \frac{n}{2}[2a + 5nd - d]$$

$$= \frac{n}{2}[2a + (5n-1)d]$$

$$5(S_3 - S_2) = \frac{5n}{2}[2a + (5n-1)d]$$

$$= S_5 \quad \text{from equation (3)}$$

Hence proved.

Q.9 Obtain the sum of all the integers in the first 1000 integers which are neither divisible by 5 nor 2.

Solution:

The integers not divisible by 5 and 2

$\underline{1 + 3 + 7 + 9} + \underline{11 + 13 + 17 + 19} + \underline{21 + 23 + 27 + 29} + \ldots\ldots + \underline{991 + 993 + 997 + 999}$

$= 20 + 60 + 100 + \ldots.. + 3980$ which is A.P

$a_1 = 20, \quad d = 60 - 20 = 40, \quad a_n = 3980$

so using

$a_n = a_1 + (n-1)d$

$3980 = 20 + (n-1)40$

$3980 = 20 + 40n - 40$

$3980 = 60 + 40n$

$3980 - 60 = 40n \Rightarrow n = 100$

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$= \frac{100}{2}[2(20) + (100-1)40]$

$= 50[40 + 3960]$

$S_{100} = 50(4000) = 200000$

Q.10 S_8 and S_9 are sum of the first eight and nine terms of an A.P. Find S_9 if $50S_9 = 63S_8$ and $a_1 = 2$

Solution:

It is given that

$50S_9 = 63S_8$

$50 \cdot \frac{9}{2}\left[2a_1 + (9-1)d\right] = 63 \cdot \frac{8}{2}\left[2a_1 + (8-1)d\right]$

$25 \times 9\left[2(2) + 8d\right] = 63 \times 4\left[2(2) + 7d\right]$

$225(4 + 8d) = 252(4 + 7d)$

$900 + 1800d = 1008 + 1764d$

$900 - 1008 + 1800d - 1764d = 0$

$-108 + 36d = 0$

$36d = 108$

$d = \frac{108}{36} \Rightarrow \boxed{d = 3}$

So $S_9 = \frac{9}{2}[2a_1 + (9-1)d]$

$= \frac{9}{2}[2(2) + 8(3)]$

$= \frac{9}{2}[4 + 24] = \frac{9}{2} \times 28 = 126$

Q.12 The sum of S_9 and S_7 is 203 and $S_9 - S_7 = 49$ and S_7 and S_9 being sum of 7 or 9 terms of an A.P. respectively. Determine the series.

Solution:

Given that

$S_9 + S_7 = 203$ .... (1)

$S_9 - S_7 = 49$ .... (2)

Adding (1) and (2), we get

$2S_n = 252 \Rightarrow S_9 = 126$

Subtract (2) from (1), we get

$2S_7 = 154 \Rightarrow S_7 = 77$

$S_9 = 126$

$\Rightarrow \quad \frac{9}{2}\left[2a_1 + (9-1)d\right] = 126$

$\Rightarrow \quad \frac{9}{2}[2a + 8d] = 126$

$\Rightarrow \quad 9(a + 4d) = 126$

$\Rightarrow \quad a + 4d = 14$ .... (3)

As $\quad S_7 = 77$

$\Rightarrow \quad \frac{7}{2}[2a_1 + 6d] = 77$

$\Rightarrow \quad 7(a + 3d) = 77$

$\Rightarrow \quad a + 3d = 11$ .... (4)

Subtracting (4) from (3)

$d = 14 - 11 = 3$

Put $d = 3$ in (3)

$a + 4(3) = 14$

$a + 12 = 14 \Rightarrow a = 2$

so $\quad a_1 = 2$

$a_2 = a_1 + d = 2 + 3 = 5$

$a_3 = a_1 + 2d = 2 + 2(3) = 8$

$\Rightarrow$ required series will be $2 + 5 + 8 + ...$

Q.13 S_7 and S_9 are the sums of the first seven and nine terms of an A.P. respectively if $\frac{S_9}{S_7} = \frac{18}{11}$ and $a_7 = 20$ find the series.

Solution:

Given that

$$\frac{S_9}{S_7} = \frac{18}{11}$$

$$\Rightarrow \quad \frac{\frac{9}{2}(2a_1 + 8d)}{\frac{7}{2}(2a_1 + 6d)} = \frac{18}{11}$$

$$\Rightarrow \quad \frac{9\,(a_1 + 4d)}{7\,(a_1 + 3d)} = \frac{18}{11}$$

$$\Rightarrow \quad 126\,(a_1 + 3d) = 99\,(a_1 + 4d)$$

$$\Rightarrow \quad 126a_1 + 378d = 99a_1 + 396d$$

$$\Rightarrow \quad 27a_1 - 18d = 0$$

$$\Rightarrow \quad 3a_1 - 2d = 0 \qquad \ldots\ldots\ldots (1)$$

Also given that

$$a_7 = 20 \quad \Rightarrow \quad a_1 + 6d = 20 \qquad \ldots\ldots\ldots (2)$$

Put $a_1 = 20 - 6d$ from (2) in (1)

$$3\,(20 - 6d) - 2d = 0$$

$$60 - 18d - 2d = 0$$

$$60 - 20d \quad \Rightarrow \quad \boxed{d = 3}$$

Put $d = 3$ in (1), we get

$$3a_1 - 2\,(3) = 0$$

$$3a_1 - 6 = 0 \quad \Rightarrow \quad 3a_1 = 6 \quad \Rightarrow \quad \boxed{a_1 = 2}$$

so $\quad a_2 = a_1 + d = 2 + 3 = 5$

$$a_3 = a_1 + 2d = 2 + 2\,(3) = 8$$

so the required series is

$$2 + 5 + 8 + \ldots\ldots..$$

Q.14 The sum of three numbers in an A.P. is 24 and their product is 440. Find the numbers. (Lahore Board 2005, 2010)

Solution:

Let the required numbers are $a - d, a, a + d$

then as given

$a - d + a + a + d = 24$

$\Rightarrow \quad 3a = 24 \quad \Rightarrow \quad \boxed{a = 8}$

and $\quad (a - 8)(a)(a + d) = 440$

$\Rightarrow \quad a(a^2 - d^2) = 440$

$\Rightarrow \quad 8(64 - d^2) = 440 \qquad$ ¤ $a = 8$

$\Rightarrow \quad 64 - d^2 = 55$

$\Rightarrow \quad 64 - 55 = d^2 \quad \Rightarrow \quad d^2 = 9 \quad \Rightarrow \quad d = \pm 3$

when $a = 8$ and $d = 3$

then numbers are

$a - d = 8 - 3 = 5$

$a = 8$

$a + d = 8 + 3 = 11$

when $a = 8$ and $d = -3$

then numbers are

$a - d = 8 + 3 = 11$

$a = 8$

$a + d = 8 - 3 = 5$

so required numbers are

5, 8, 11 or 11, 8, 5.

Q.15 Find four numbers in A.P. whose sum is 32 and the sum of whose squares is 276.

Solution:

Let the required numbers are $a - 3d$, $a - d$, $a + d$, $a + 3d$ then it is given that

$a - 3d + a - d + a + d + a + 3d = 32$

$4a = 32 \quad \Rightarrow \quad \boxed{a = 8}$

Also given that

$$(a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 276$$

$$4a^2 = 20d^2 = 276$$

$$a^2 = 5d^2 = 69$$

$$5d^2 = 69 - a^2 = 69 - (8)^2 = 69 - 64 \qquad ¤ \quad a = 8$$

$$5d^2 = 5 \quad \Rightarrow \quad \boxed{d = \pm 1}$$

when $a = 8, \quad d = 1$

$$a - 3d = 8 - 3(1) = 5$$

$$a - d = 8 - 1 = 7$$

$$a + d = 8 + 1 = 9$$

$$a + 3d = 8 + 3(1) = 11$$

when $a = 8, \ d = -1$

$$a - 3d = 8 - 3(-1) = 11$$

$$a - d = 8 - (-1) = 9$$

$$a + d = 8 - 1 = 7$$

$$a + 3d = 8 + 3(-1) = 5$$

so required numbers are

5, 7, 9, 11 or 11, 9, 7, 5

Q.16 Find the five numbers in A.P. whose sum is 25 and the sum of whose squares is 135. (Lahore Board 2003)

Solution:

Let the required numbers are $a - 2d, \ a - d, \ a, \ a + d, \ a + 2d$ then it is given that

$$a - 2d + a - d + a + a + d + a + 2d = 25$$

$$5a = 25 \quad \Rightarrow \quad \boxed{a = 5}$$

also it is given that

$$(a - 2d)^2 + (a - d)^2 + a^2 + (a + d)^2 + (a + 2d)^2 = 135$$

$$a^2 + 4d^2 - 4da + a^2 + d^2 - 2ad + a^2 + a^2 + d^2 + 2ad + a^2 + 4d^2 + 4ad = 135$$

$$5a^2 + 10d^2 = 135$$

$$a^2 + 2d^2 = 27$$

$$2d^2 = 27 - a^2 = 27 - (5)^2 = 27 - 25 = 2$$

$$d^2 = 1 \quad \Rightarrow \quad \boxed{d = \pm 1}$$

when $a = 5, \ d = 1$

$a - 2d = 5 - 2(1) = 3$

$a - d = 5 - 1 = 4$

$a = 5$

$a + d = 5 + 1 = 6$

$a + 2d = 5 + 2(1) = 7$

when $a = 5,\ d = -1$

$a - 2d = 5 - 2(-1) = 7$

$a - d = 5 - (-1) = 6$

$a = 5$

$a + d = 5 + (-1) = 4$

$a + 2d = 5 + 2(-1) = 3$

so the required numbers are

3, 4, 5, 6, 7 or 7, 6, 5, 4, 3.

Q.17 The sum of the 6^{th} and 8^{th} terms of an A.P. is 40 and the product of 4^{th} and 7^{th} terms is 220. Find the A.P.

Solution:

Given that

$a_6 + a_8 = 40$

$\Rightarrow\quad a + 5d + a + 7d = 40$

$\Rightarrow\quad 2a + 12d = 40$

$\Rightarrow\quad a + 6d = 20 \qquad \ldots\ldots\ldots (1)$

Also it is given that

$a_4 \,.\, a_7 = 220$

$\Rightarrow\quad (a + 3d)(a + 6d) = 220$

$\Rightarrow\quad (a + 3d)(20) = 220$ from (1)

$\Rightarrow\quad a + 3d = \dfrac{220}{20}$

$\Rightarrow\quad a + 3d = 11 \qquad \ldots\ldots\ldots (2)$

Subtracting (2) from (1), we get

$3d = 9 \quad \Rightarrow \quad \boxed{d = 3}$

Put $d = 3$ in (1), we get

$$a + 6(3) = 20$$

$$a + 18 = 20 \Rightarrow \boxed{a = 2}$$

$\Rightarrow$ $a_1 = 2$

$a_2 = a_1 + d = 2 + 3 = 5$

$a_3 = a_1 + 2d = 2 + 2(3) = 8$

$\Rightarrow$ required A.P. is 2, 5, 8, …….

Q.18 If a^2, b^2 and c^2 are in A.P. show that $\dfrac{1}{b+c}, \dfrac{1}{c+a}, \dfrac{1}{a+b}$ are in A.P.

Solution:

If a^2, b^2, c^2 are in A.P.

$\Rightarrow$ $b^2 - a^2 = c^2 - b^2$

To show that $\dfrac{1}{b+c}, \dfrac{1}{c+a}, \dfrac{1}{a+b}$ are in A.P. we will prove that

$$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

or $$\frac{b+c-(c+a)}{(c+a)(b+c)} = \frac{c+a-(a+b)}{(a+b)(c+a)}$$

or $$\frac{b+c-c-a}{b+c} = \frac{c+a-a-b}{a+b}$$

or $$\frac{b-a}{b+c} = \frac{c-b}{a+b}$$

or $(b-a)(b+a) = (c-b)(c+b)$

or $b^2 - c^2 = c^2 - b^2$ (given)

Hence $\dfrac{1}{b+c}, \dfrac{1}{c+a}, \dfrac{1}{a+b}$ are in A.P.

EXERCISE 6.5

Q.1 A man deposits in a bank Rs. 10 in the first month Rs. 15 in the second month, Rs. 20 in the third month and so on. Find how much he will have deposited in the bank by 9th months.

Solution:

Deposited amount is

10, 15, 20, ... which is A.P

Here $a_1 = 10$, $d = 15 - 10 = 5$, $n = 9$

We find S_9