EXERCISE 12.4

Q.1 Solve triangle ABC if

$\beta = 60^o$, $\gamma = 15^o$, $b = \sqrt{6}$ **(Gujranwala Board 2007)**

Solution:

Since $\alpha + \beta + \gamma = 180^o$

$\alpha + 60^o + 15^o = 180^o$

$\alpha = 180^o - 60^o - 15^o$

$\boxed{\alpha = 105}$

Now by law of sines

$$\frac{a}{\sin\alpha} = \frac{a}{\sin\beta}$$

$$\frac{a}{\sin 105^o} = \frac{\sqrt{6}}{\sin 60^o} \Rightarrow a = \frac{\sqrt{6}}{\sin 60^o} \times \sin 105^o$$

$\boxed{\alpha = 2.732}$

again

$$\frac{c}{\sin\gamma} = \frac{b}{\sin\beta}$$

$$\frac{c}{\sin 15^o} = \frac{\sqrt{6}}{\sin 60^o} \Rightarrow c = \frac{\sqrt{6}}{\sin 60^o} \times \sin 15^o$$

$\boxed{a = 0.7320}$

Q.2 $\beta = 52^o$, $\gamma = 89^o\,35'$, $a = 89.35$

Solution:

Since $\alpha + \beta + \gamma = 180^o$

$\alpha + 52^o + 89^o\,35' = 180^o$

$\alpha = 180^o - 52^o - 89^o\,35'$

$\boxed{\alpha = 38^o\,25'}$

Now by law of sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

$$\Rightarrow \quad \frac{89.35}{\sin 38^o 25'} = \frac{b}{\sin 52^o}$$

$$b = \frac{89.35}{\sin 38^o 25'} \times \sin 52^o$$

$$\boxed{b = 113.18}$$

again

$$\frac{c}{\sin \gamma} = \frac{a}{\sin \alpha}$$

$$\frac{c}{\sin 89^o 35'} = \frac{89.35}{\sin 38^o 25'}$$

$$c = \frac{89.35}{\sin 38^o 25'} \times \sin 89^o 35'$$

$$\boxed{c = 143.79}$$

Q.3 $b = 125$, $\gamma = 53^o$, $\alpha = 47^o$

Solution:

Since $\alpha + \beta + \gamma = 180^o$

$47^o + \beta + 35^o = 180^o$

$\beta = 180^o - 53^o - 47^o$

$$\boxed{\beta = 80^o}$$

Now by law of sines

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta}$$

$$\frac{a}{\sin 47^o} = \frac{125}{\sin 80^o}$$

$$a = \frac{125}{\sin 80^o} \times \sin 47^o$$

$$\boxed{a = 92.8}$$

and $$\frac{c}{\sin \gamma} = \frac{b}{\sin \beta}$$

$$\frac{c}{\sin 53^o} = \frac{125}{\sin 80^o}$$

$$c = \frac{125}{\sin 80^o} \times \sin 53^o$$

$$\boxed{c = 101}$$

Q.4 $c = 16.1$, $\alpha = 53^o\, 45'$, $\gamma = 74^o\, 32'$

Solution:

Since $\alpha + \beta + \gamma = 180^o$

$42^o\, 45' + \beta + 74^o\, 32' = 180^o$

$\beta = 180^o - 42^o\, 45' - 74^o\, 32'$

$\boxed{\beta = 62^o\, 43'}$

Now by law of sines

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$\frac{a}{\sin 42^o\, 45'} = \frac{16.1}{\sin 74^o\, 32'}$$

$$a = \frac{16.1}{\sin 74^o\, 32'} \times \sin 42^o\, 45'$$

$\boxed{a = 11.3}$

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\frac{b}{\sin 62^o\, 43'} = \frac{16.1}{\sin 74^o\, 32'}$$

$$b = \frac{16.1}{\sin 74^o\, 32'} \times \sin 62^o\, 43'$$

$\boxed{b = 14.8}$

Q.5 $a = 53$, $\beta = 88^o\, 36'$, $\gamma = 31^o\, 54'$

(Gujranwala Board 2006,Lahore Board 2007)

Solution:

Since $\alpha + \beta + \gamma = 180^o$

$\alpha + 88^o\, 36' + 31^o\, 54' = 180^o$

$\alpha = 180^o - 88^o\, 36' - 31^o\, 54'$

$\boxed{\alpha = 59^o\, 30'}$

Now by law of sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

$$\frac{53}{\sin 59^o\,30'} = \frac{b}{\sin 88^o\,36'}$$

$$b = \frac{53}{\sin 59^o\,30'} \times \sin 88^o\,36'$$

$$\boxed{b = 61.49}$$

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\frac{61.49}{\sin 88^o\,36'} = \frac{c}{\sin 31^o\,54'}$$

$$c = \frac{61.49}{\sin 88^o\,36'} \times \sin 31^o\,54'$$

$$\boxed{c = 32.5}$$

EXERCISE 12.5

Solve the triangle ABC, in which

Q.1 $b = 59$, $c = 34$, and $\alpha = 52^o$ (Gujranwala Board 2007)

Solution:

Using law of cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$a^2 = (95)^2 + (34)^2 - 2\,(95)\,(34)\cos 52^o$$

$$= 9025 + 1156 - 3977$$

$$a^2 = 6204$$

$$\boxed{a = 78.76}$$

$$\because\ b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$2ac\cos\beta = a^2 + c^2 - b^2$$

$$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\because\ \cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$$

Now $\beta = \cos^{-1}\left[\dfrac{a^2 + c^2 - b^2}{2ac}\right]$

$$\beta = \cos^{-1}\left[\frac{(78.76)^2 + (34)^2 - (95)^2}{2\,(78.76)\,(34)}\right]$$

$$\boxed{\beta = 71^o\,53'}$$

so $\gamma = 180^o - \beta - \alpha$

$$= 180^o - 71^o\,53' - 52^o$$

$$\boxed{\gamma = 56^o\,7'}$$