$\sin\left(-\frac{71}{6}\pi\right) = \sin 30^o = \frac{1}{2}$ ; $\text{cosec}\left(-\frac{71}{6}\pi\right) = \text{cosec } 30^o = 2$

$\cos\left(-\frac{71}{6}\pi\right) = \cos 30^o = \frac{\sqrt{3}}{2}$ ; $\sec\left(-\frac{71}{6}\pi\right) = \sec 30^o = \frac{2}{\sqrt{3}}$

$\tan\left(-\frac{71}{6}\pi\right) = \tan 30^o = \frac{1}{\sqrt{3}}$ ; $\cot\left(-\frac{71}{6}\pi\right) = \cot 30^o = \sqrt{3}$

**(ix) – 1035°**

$-(2k \cdot 180^o + \theta)$

$-1035° = -(2 \cdot 3 \cdot 180^o - 45^o)$

$= -2 \cdot 3 \cdot 180^o + 45^o$

$\Rightarrow \quad \theta = 45^o$

$$\boxed{\begin{array}{l} \frac{1035^o}{360^o} = 2.875 \\ k = 3 \\ \theta = 1035^o - 3(360^o) \\ \theta = -45^o \end{array}}$$

$\sin(-1035^o) = \sin 45^o = \frac{1}{\sqrt{2}}$ ; $\text{cosec}(-1035^o) = \text{cosec } 45^o = \sqrt{2}$

$\cos(-1035^o) = \cos 45^o = \frac{1}{\sqrt{2}}$ ; $\sec(-1035^o) = \sec 45^o = \sqrt{2}$

$\tan(-1035^o) = \tan 45^o = 1$ ; $\cot(-1035^o) = \cot 45^o = 1$

## EXERCISE 9.4

**Q.1 Prove the identity, state the domain of $\theta$ in each case.**

**$\tan\theta + \cot\theta = \text{cosec}\theta \sec\theta$.** **(Gujranwala Board 2005)**

**Solution:**

$$\begin{aligned} \text{L.H.S.} &= \tan\theta + \cot\theta \\ &= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \\ &= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} = \text{cosec}\,\theta \cdot \sec\theta \quad \text{R.H.S.} \end{aligned}$$

Domain of $\theta : \theta \in \Re$, but $\theta \neq n\frac{\pi}{2}$, $n \in Z$.

**Q.2**

**$\sec\theta \ \text{cosec}\theta \ \sin\theta\cos\theta = 1$**

**Solution:**

$$\begin{aligned} \text{L.H.S.} &= \sec\theta\,\text{cosec}\,\theta\sin\theta\cos\theta \\ &= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} \cdot \sin\theta \cdot \cos\theta \\ &= 1 = \text{R.H.S.} \end{aligned}$$

Domain of $\theta : \theta \in \Re$, but $\theta \neq n\frac{\pi}{2}$, $n \in Z$

**Q.3** $\cos\theta + \tan\theta \sin\theta = \sec\theta$

**Solution:**

$$\text{L.H.S.} = \cos\theta + \tan\theta \,.\, \sin\theta$$
$$= \cos\theta + \frac{\sin\theta}{\cos\theta} \,.\, \sin\theta$$
$$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$$
$$= \frac{1}{\cos\theta} = \sec\theta = \text{R.H.S.}$$

Domain of $\theta : \theta \in \Re$, but $\theta \neq (2n+1)\frac{\pi}{2}, n \in z$

**Q.4** $\text{cosec}\,\theta + \tan\theta \,.\, \sec\theta = \text{cosec}\,\theta \sec^2\theta$.

**Solution:**

$$\text{L.H.S.} = \text{cosec}\,\theta + \tan\theta \sec\theta$$
$$= \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$$
$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos^2\theta}$$
$$= \frac{1}{\cos^2\theta\sin\theta}$$
$$= \text{cosec}\,\theta \sec^2\theta = \text{R.H.S.}$$

Domain of $\theta \quad \theta \in \Re$, but $\theta \neq n\frac{\pi}{2}, \quad n \in Z$

**Q.5** $\sec^2\theta - \text{cosec}^2\theta = \tan^2\theta - \cot^2\theta$

**Solution:**

$$\text{L.H.S.} = \sec^2\theta - \text{cosec}^2\theta$$
$$= 1 + \tan^2\theta - (1 + \cot^2\theta)$$
$$= 1 + \tan^2\theta - 1 - \cot^2\theta$$
$$= \tan^2\theta - \cot^2\theta$$
$$= \text{R.H.S.}$$

Domain of $\theta : \theta \in R$ but $\phi \neq \frac{n\pi}{2}, n \in z$

**Q.6** $\cot^2\theta - \cos^2\theta = \cot^2\theta\cos^2\theta$

**Solution:**

$$\begin{aligned}
\text{L.H.S.} &= \cot^2\theta - \cos^2\theta \\
&= \frac{\cos^2\theta}{\sin^2\theta} - \frac{\cos^2\theta}{1} \\
&= \frac{\cos^2\theta - \sin^2\theta\cos^2\theta}{\sin^2\theta} \\
&= \frac{\cos^2\theta\,(1 - \sin^2\theta)}{\sin^2\theta} \\
&= \frac{\cos^2\theta}{\sin^2\theta}\cos^2\theta \\
&= \cot^2\theta\cos^2\theta \\
&= \text{R.H.S.}
\end{aligned}$$

Domain of $\theta$, $\theta \in \Re$ but $\theta \neq n\pi$, $n \in z$

**Q.7** $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

**Solution:**

$$\begin{aligned}
\text{L.H.S.} &= (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) \\
&= (\sec\theta)^2 - (\tan\theta)^2 = \sec^2\theta - \tan^2\theta \\
&= 1 + \tan^2\theta - \tan^2\theta \\
&= 1 = \text{R.H.S.}
\end{aligned}$$

Domain of $\theta = \Re$.

**Q.8** $2\cos^2\theta - 1 = 1 - 2\sin^2\theta$.

**Solution:**

$$\begin{aligned}
\text{L.H.S.} &= 2\cos^2\theta - 1 \\
&= 2(1 - \sin^2\theta) - 1 \\
&= 2 - 2\sin^2\theta - 1 \\
&= 1 - 2\sin^2\theta = \text{R.H.S.}
\end{aligned}$$

Domain of $\theta = \Re$

**Q.9** $\cos^2\theta - \sin^2\theta = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$.

**Solution:**

$$\text{R.H.S.} = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}.$$

$$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos\theta^2}} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$= \cos^2\theta - \sin^2\theta = \text{L.H.S.}$$

Domain of $\theta$ : but $\theta \neq (2n+1)\frac{\pi}{2}$ , $n \in Z$

**Q.10** $\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \dfrac{\cot\theta - 1}{\cot\theta + 1}$

**Solution:**

$$\text{R.H.S.} = \frac{\cot\theta - 1}{\cot\theta + 1}$$

$$= \frac{\frac{\cos\theta}{\sin\theta} - 1}{\frac{\cos\theta}{\sin\theta} + 1} = \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \text{L.H.S.}$$

Domain of $\theta$ ; $\theta \in \Re$ but $\theta \neq n\pi + \frac{3\pi}{4}$ , $n \in Z$

**Q.11** $\dfrac{\sin\theta}{1+\cos\theta} + \cot\theta = \operatorname{cosec}\theta$

**Solution:**

$$\text{L.H.S.} = \frac{\sin\theta}{1+\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos\theta\,(1+\cos\theta)}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{\sin^2\theta + \cos^2\theta + \cos\theta}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{(1+\cos\theta)}{\sin\theta\,(1+\cos\theta)} = \frac{1}{\sin\theta} = \operatorname{cosec}\theta$$

$$= \text{R.H.S.}$$

Domain of $\theta : \theta \in \Re$ but $\theta \neq n\pi$ , $n \in Z$

**Q.12** $\dfrac{\cot^2\theta - 1}{1+\cot^2\theta} = 2\cos^2\theta - 1$

**Solution:**

$$\text{L.H.S.} = \frac{\cot^2\theta - 1}{1+\cot^2\theta}$$

$$= \frac{\frac{\cos^2\theta}{\sin^2\theta} - 1}{1 + \frac{\cos^2\theta}{\sin^2\theta}} = \frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta + \cos^2\theta}$$

$$= \frac{\cos^2\theta - (1 - \cos^2\theta)}{1}$$

$$= \cos^2\theta - 1 + \cos^2\theta$$

$$= 2\cos^2\theta - 1 = \text{R.H.S.}$$

Domain of $\theta$ : $\theta \in \Re$ but $\theta \neq n\pi$ , $n \in Z$

**Q.13** $\mathbf{\frac{1 + \cos\theta}{1 - \cos\theta} = (\text{cosec}\,\theta + \cot\theta)^2}$

**Solution:**

$$\text{R.H.S.} = (\text{cosec}\,\theta + \cot\theta)^2$$

$$= \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1 + \cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1 + \cos\theta)^2}{\sin^2\theta} = \frac{(1 + \cos\theta)^2}{1 - \cos^2\theta}$$

$$= \frac{(1 + \cos\theta)^2}{(1 - \cos\theta)(1 + \cos\theta)}$$

$$= \frac{1 + \cos\theta}{1 - \cos\theta} = \text{L.H.S.}$$

Domain of $\theta$ : $\theta \in \Re$ but $\theta \neq n\pi$ , $n \in Z$

**Q.14** $\mathbf{(\sec\theta - \tan\theta)^2 = \frac{1 - \sin\theta}{1 + \sin\theta}}$

**Solution:**

$$\text{L.H.S.} = (\sec\theta - \tan\theta)^2$$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$$= \left(\frac{1 - \sin\theta}{\cos\theta}\right)^2 = \frac{(1 - \sin\theta)^2}{\cos^2\theta}$$

$$= \frac{(1 - \sin\theta)^2}{1 - \sin^2\theta} = \frac{(1 - \sin\theta)^2}{(1 - \sin\theta)(1 + \sin\theta)}$$

$$= \frac{1 - \sin\theta}{1 + \sin\theta} = \text{R.H.S.}$$

Domain of $\theta$ , $\theta \in \Re$ but $\theta \neq (2n+1)\frac{\pi}{2}$, $n \in Z$

**Q.15** $\dfrac{2\tan\theta}{1 + \tan^2\theta} = 2\sin\theta\cos\theta$

**Solution:**

$$\text{L.H.S.} = \frac{2\tan\theta}{1 + \tan^2\theta}$$

$$= \frac{2\dfrac{\sin\theta}{\cos\theta}}{1 + \dfrac{\sin^2\theta}{\cos^2\theta}} = \frac{2\sin\theta}{\cos\theta} \div \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}$$

$$= \frac{2\sin\theta}{\cos\theta} \times \frac{\cos^2\theta}{1} = 2\sin\theta\cos\theta$$

$$= \text{R.H.S.}$$

Domain of $\theta$ , $\theta \in \Re$ but $\theta \neq (2n+1)\frac{\pi}{2}$, $n \in Z$

**Q.16** $\dfrac{1 - \sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1 + \sin\theta}$

**Solution:**

$$\text{L.H.S.} = \frac{1 - \sin\theta}{\cos\theta}$$

$$= \frac{1 - \sin\theta}{\cos\theta} \times \frac{1 + \sin\theta}{1 + \sin\theta}$$

$$= \frac{1 - \sin^2\theta}{\cos\theta(1 + \sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta(1 + \sin\theta)}$$

$$= \frac{\cos\theta}{1 + \sin\theta} = \text{R.H.S.}$$

Domain of $\theta$ : $\theta \in \Re$ but $\theta \neq (2n+1)\frac{\pi}{2}$, $n \in Z$

**Q.17** $(\tan\theta + \cot\theta)^2 = \sec^2\theta\,\mathrm{cosec}^2\theta$

**Solution:**

$$\begin{aligned}
\text{L.H.S.} &= (\tan\theta + \cot\theta)^2 \\
&= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2 \\
&= \left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)^2 \\
&= \frac{1}{\cos^2\theta\sin^2\theta} = \sec^2\theta\,\mathrm{cosec}^2\theta \\
&= \text{R.H.S.}
\end{aligned}$$

Domain of $\theta : \theta \in \Re$ but $\theta \neq n\frac{\pi}{2}$ , $n \in Z$

**Q.18** $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$ **(Lahore Board 2008)**

**Solution:**

$$\begin{aligned}
\text{L.H.S.} &= \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} \\
&= \frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1} && \because\ 1 + \tan^2\theta = \sec^2\theta \\
&= \frac{\tan\theta + \sec\theta - (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}{\tan\theta - \sec\theta + 1} && 1 = \sec^2\theta - \tan^2\theta \\
&= \frac{(\sec\theta + \tan\theta)\big(1 - (\mathrm{Sec}\,\theta - \tan\theta)\big)}{\tan\theta - \sec\theta + 1} \\
&= \frac{(\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)}{\tan\theta - \sec\theta + 1} \\
&= \tan\theta + \sec\theta = \text{R.H.S.}
\end{aligned}$$

Domain of $\theta : \theta \in \Re$ but $\theta \neq (2n+1)\frac{\pi}{2}$

**Q.19** $\dfrac{1}{\mathrm{cosec}\,\theta - \cot\theta} - \dfrac{1}{\sin\theta} = \dfrac{1}{\sin\theta} - \dfrac{1}{\mathrm{cosec}\,\theta + \cot\theta}$ **(Gujranwala Board 2005)**

**Solution:**

$$\begin{aligned}
\text{L.H.S.} &= \frac{1}{\mathrm{cosec}\,\theta - \cot\theta} - \frac{1}{\sin\theta} \\
&= \frac{1}{\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}} - \frac{1}{\sin\theta} \\
&= \frac{\sin\theta}{1 - \cos\theta} - \frac{1}{\sin\theta} = \frac{\sin^2\theta - 1 + \cos\theta}{(1 - \cos\theta)\sin\theta}
\end{aligned}$$

$$= \frac{(1-\cos^2\theta)-(1-\cos\theta)}{(1-\cos\theta)\sin\theta}$$

$$= \frac{(1-\cos\theta)(1+\cos\theta)-(1-\cos\theta)}{(1-\cos\theta)\sin\theta}$$

$$= \frac{(1-\cos\theta)(1+\cos\theta-1)}{(1-\cos\theta)\sin\theta}$$

$$= \frac{\cos\theta}{\sin\theta} = \cot\theta$$

$$\text{R.H.S.} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

$$= \frac{1}{\sin\theta} - \frac{1}{\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}}$$

$$= \frac{1}{\sin\theta} - \frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{1+\cos\theta-\sin^2\theta}{(\sin\theta)(1+\cos\theta)}$$

$$= \frac{(1+\cos\theta)-(1-\cos\theta)(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$= \frac{(1+\cos\theta)(1-1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\cos\theta}{\sin\theta} = \cot\theta$$

Hence L.H.S. = R.H.S.

Domain of $\theta : \theta \in \Re$ but $\theta \neq n\pi$ , $n \in Z$

**Q.20 $\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(1+\sin\theta\cos\theta)$ (Lahore Board 2004)**

**Solution:**

$$\text{L.H.S.} = \sin^3\theta - \cos^3\theta$$

$$= (\sin\theta)^3 - (\cos\theta)^3$$

$$\boxed{\because\ a^3 - b^3 = (a-b)(a^2+ab+b^2)}$$

$$= (\sin\theta-\cos\theta)(\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta)$$

$$= (\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)$$

$$= \text{R.H.S.}$$

Domain of $\theta = \Re$.

**Q.21** $\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

**Solution:**

L.H.S. $= \sin^6\theta - \cos^6\theta$

$= (\sin^2\theta)^3 - (\cos^2\theta)^3$ $\boxed{|a^2 - b^2| = (a+b)(a-b)}$

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta)^2 + (\cos^2\theta)^2 + \sin^2\theta\cos^2\theta]$

Adding and subtracting $2\sin^2\theta\cos^2\theta$.

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta + \sin^2\theta\cos^2\theta]$

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta + \cos^2\theta)^2 - \sin^2\theta\cos^2\theta]$

$= (\sin^2\theta - \cos^2\theta)\left((1)^2 - (\sin\theta\cos\theta)^2\right)$

$= (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

$=$ R.H.S.

Domain of $\theta = \Re$.

**Q.22** $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$. **(Gujranwala, Lahore Board 2007)**

**Solution:**

L.H.S. $= \sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$\boxed{a^3 + b^3 = (a+b)(a^2 + b^2 - ab)}$

$= (\sin^2\theta + \cos^2\theta)\left((\sin^2\theta)^2 + (\cos^2\theta)^2 - \sin^2\theta\cos^2\theta\right)$

$= 1(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$

Adding & subtracting $2\sin^2\theta\cos^2\theta$

$= (\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta) - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta$

$= 1 - 3\sin^2\theta\cos^2\theta$

$=$ R.H.S.

Domain of $\theta = \Re$.

**Q.23** $\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta} = 2\sec^2\theta$

**Solution:**

$$\textbf{L.H.S.} = \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$$

$$= \frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta}$$

$$= 2\sec^2\theta = \text{R.H.S.}$$

Domain of $\theta : \theta \in \Re$ but $\theta \neq (2n+1)\frac{\pi}{2}$ , $n \in Z$

**Q.24** $$\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} + \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \frac{2}{1-2\sin^2\theta}$$

**Solution:**

$$\text{L.H.S.} = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} + \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$

$$= \frac{(\cos\theta+\sin\theta)^2+(\cos\theta-\sin\theta)^2}{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}$$

$$= \frac{\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$$

$$= \frac{2\cos^2\theta+2\sin^2\theta}{1-\sin^2\theta-\sin^2\theta} = \frac{2(\cos^2\theta+\sin^2\theta)}{1-2\sin^2\theta}$$

$$= \frac{2}{1-2\sin^2\theta} = \text{R.H.S.}$$

Domain of $\theta : \theta \in R$ but $\theta \neq (2n+1)\frac{\pi}{4}$, $n \in Z$