

SHORT QUESTIONS

15.1 Does the induced emf in a circuit depend on the resistance of the circuit? Does the induced current depend on the resistance of the circuit?

Ans. As we know that according to Faraday's law of electromagnetic induction.

“Induced emf in a circuit is directly proportional to the negative of rate of change of magnetic flux.

$$\varepsilon = -N \frac{\Delta\phi}{\Delta t}$$

From this equation we see that induced emf depends on the rate of change of magnetic flux and induced emf does not depend upon the resistance of the circuit but induced current depends on the resistance because.

$$I = \frac{E}{R}$$

This shows that induced current is inversely proportional to resistance i.e., if resistance of conductor is less then current will be more and vice versa.

15.2 A square loop of wire is moving through a uniform magnetic field. The normal to the loop is oriented parallel to the magnetic field. Is an emf induced in the loop? Give a reason for your answer?

Ans. There will be no induced emf produced in the loop because we know that according to Faraday's law of electromagnetic induction.

$$\varepsilon = -N \frac{\Delta\phi}{\Delta t}$$

Here, $\frac{\Delta\phi}{\Delta t} = 0$

i.e., rate of change of magnetic flux is zero because normal of loop is oriented parallel to the magnetic field.

According to the relation

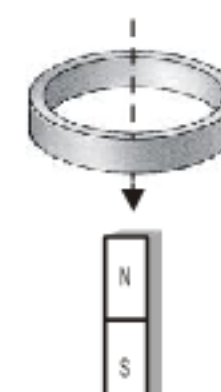
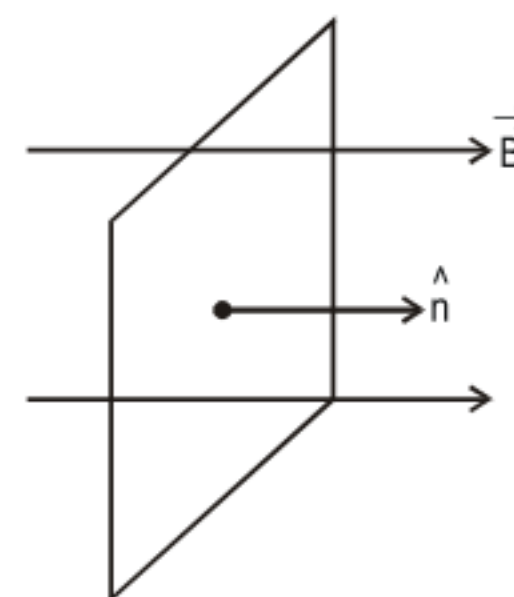
$$\varepsilon = -N \frac{\Delta\phi}{\Delta t}$$

$$\varepsilon = -N (0)$$

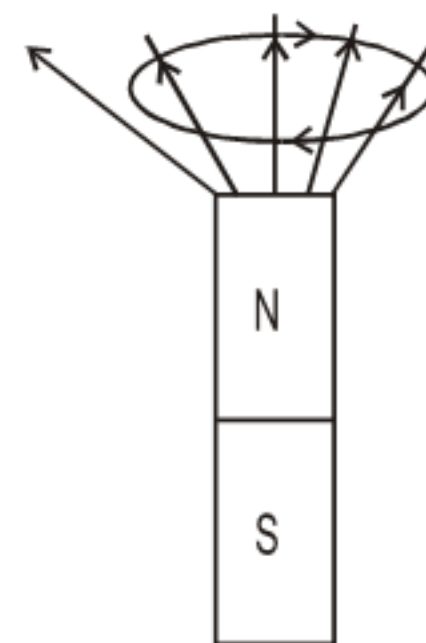
$$\varepsilon = 0$$

Hence no induced emf is produced.

15.3 A light metallic ring is released from above into a vertical bar magnet (figure). Viewed for above, does the current flow clockwise or anticlockwise in the ring?



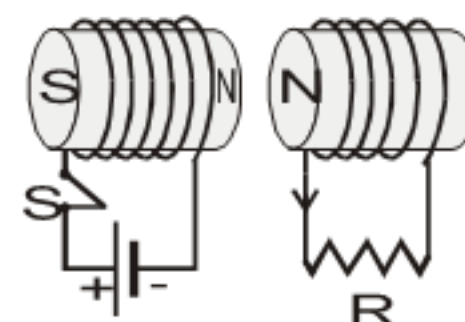
Ans. According to Lenz law the direction of induced current is opposite to the cause which produces it, therefore when the metallic ring is released from above into the bar magnet, the magnetic flux is changed in the ring and an induced emf is produced in it and hence North Pole is developed in the ring towards the north pole of the bar magnet. As view above, the current flows in clockwise direction.



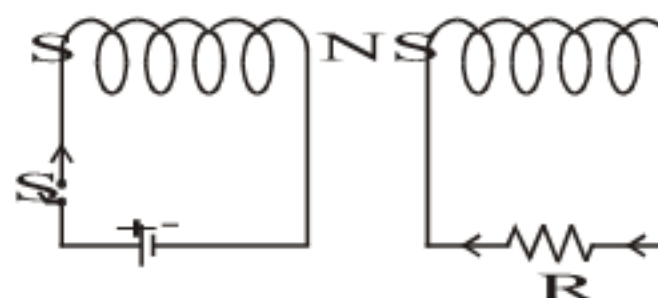
15.4 What is the direction of the current through resistor R in figure? As switch S is

(a) Closed (b) Opened

Ans. (a) When switch is closed, the current in the circuit increases from zero to maximum. During this interval, magnetic flux in the second coil increases from zero to maximum and an induced current is produced in it. The side of current carrying coil facing the other coil becomes North Pole so the current in the other coil must flow in anticlockwise direction shown.



(b) However when switch is opened, the current in the circuit decreases from maximum to zero and the flux links with other coil decrease and induced current is produced in reverse direction as shown in figure.

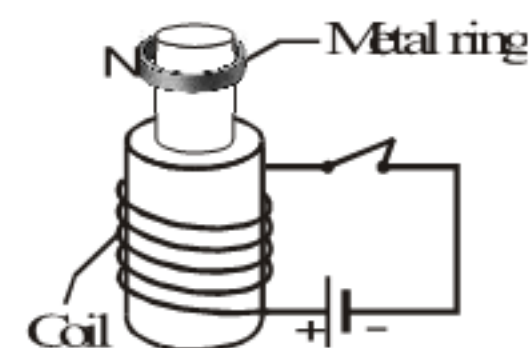


15.5 Does the induced emf always act to decrease the magnetic flux through a circuit?

Ans. No, because according to Lenz's law "the induced emf is always such as to oppose the cause which produces it" therefore if magnetic flux increases then induced emf will act in such a way to decrease the magnetic flux and if magnetic flux decreases then induced emf will act to increase the magnetic flux. So induced emf does not always act to decrease the magnetic flux.

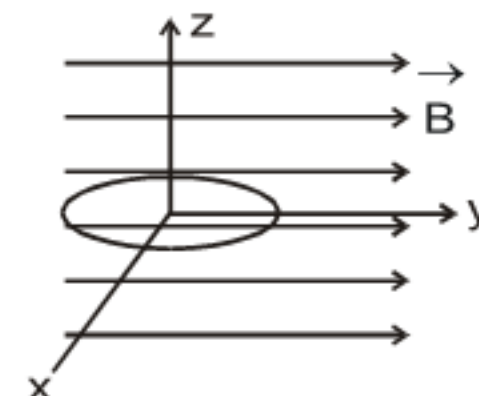
15.6 When the switch in the circuit is closed, a current is established in the coil and the metal ring jumps upward figure. Why? Describe what would happen to the ring if the battery polarity were reversed?

Ans. When the switch in the circuit is closed, the current is setup in the coil. Magnetic flux changes through the metallic ring and an induced emf is produced in it. The face of ring opposite to the coil develops similar poles of magnet and experiences repulsion from the side of coil and the ring jumps up. If the polarity of the battery is reversed then the ring will jump upward also.



15.7 The figure shows a coil of wire in the xy-plane with a magnetic field directed along the Y-axis. Around which of the three co-ordinate axes should the coil be rotated in order to generate an emf and a current in the coil?

Ans. An emf and current in the coil is generated when it is rotated along x-axis. No change of flux takes place along y and z-axis because the coil is parallel to the magnetic field \vec{B} all the time.



15.8 How would you position a flat loop of wire in a changing magnetic field so that there is no emf induced in the loop?

Ans. If the plane of flat loop of wire is placed parallel to the magnetic field \vec{B} , then there is no flux changed through it and no emf is induced in the flat loop.

In this case, the angle between magnetic field \vec{B} and vector area \vec{A} is 90° therefore

$$\Delta\phi = \vec{B} \cdot \vec{A}$$

$$\Delta\phi = BA \cos \theta$$

$$\Delta\phi = BA \cos 90^\circ$$

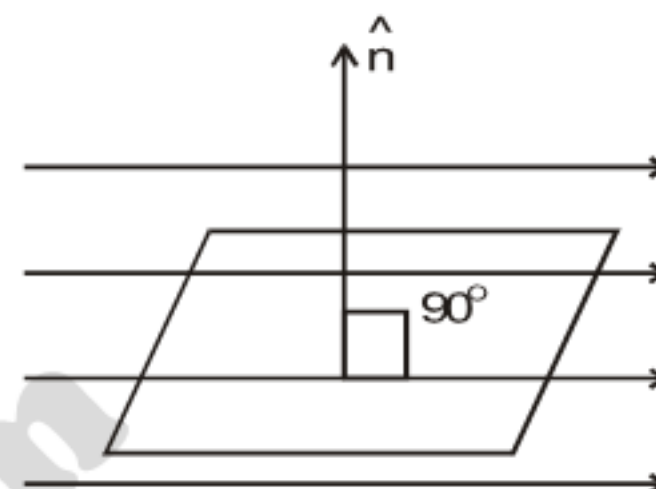
$$\Delta\phi = BA(0)$$

$$\Delta\phi = 0$$

Since $\varepsilon = -N \frac{\Delta\phi}{\Delta t}$

$$\varepsilon = -N \frac{(0)}{\Delta t}$$

$$\varepsilon = 0$$



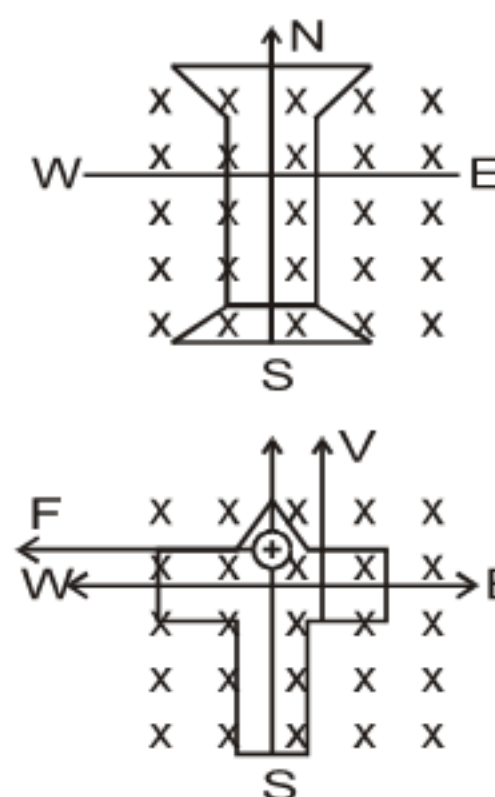
So no emf is induced in the flat loop of wire.

15.9 In a certain region the earth's magnetic field points vertically down. When a plane flies due north, which wingtip is positively charged?

Ans. As we know that the magnetic force on moving charge particle in uniform magnetic field is

$$\vec{F} = q(\vec{V} \times \vec{B})$$

The direction of this force can be found by right hand rule therefore when plane flies due north then according to right hand rule magnetic force will act towards west therefore its west wingtip become positive charge.



15.10 Show that ε and $\frac{\Delta\phi}{\Delta t}$ have same units.

Ans. As $\varepsilon = \frac{W}{q}$, ε weber/sec. where

$$\varepsilon = \frac{J}{C} = \text{Volt} \quad \dots\dots (i)$$

$$e = \frac{\Delta\phi}{\Delta t} = \frac{\text{Weber}}{\text{Sec.}} = \frac{N \times m}{A \times \text{Sec.}}$$

But $N.m = J$ and $A \times \text{Sec.} = C$

$$\text{Thus } \frac{\Delta\phi}{\Delta t} = \frac{\text{Weber}}{\text{Sec.}} = \frac{N \times m}{A \times \text{Sec.}} = J/C \quad \dots\dots (ii)$$

From eq. (i) and (ii)

ε and $\frac{\Delta\phi}{\Delta t}$ have the same units.

15.11 When an electric motor, such as an electric drill, is being used, does it also act as a generator? If so what is the consequence of this?

Ans. Yes, when electric motor is running, its armature is rotating in a magnetic field. A torque acts on the armature and at the same time, magnetic flux is changing through the armature which produces an induced emf. But this emf is back emf.

15.12 Can a D.C motor be turned into a D.C generator? What changes are required to be done?

Ans. Yes, if battery from D.C motor is removed and connect these terminals to an external circuit. Now if the coil (armature) of the motor is rotated by some mechanical means, then D.C motor is converted into D.C generator.

15.13 Is it possible to change both the area of the loop and the magnetic field passing through the loop and still not have an induced emf in the loop?

Ans. As we know that

$$\phi = BA \Rightarrow B = \frac{\phi}{A}$$

If ϕ remain constant

$$B = \frac{\text{Constant}}{A} \Rightarrow B \propto \frac{1}{A}$$

$$BA = \text{Constant}$$

If magnetic field B and vector area A are changed in such a way that the product BA remains constant then the change in flux is zero therefore

$$\Delta\phi = 0$$

Then according to Faraday's law

$$\varepsilon = - \frac{N \Delta\phi}{\Delta t}$$

$$\varepsilon = - \frac{N(0)}{\Delta t}$$

$$\varepsilon = 0$$

Hence no emf is induced in the loop.

15.14 Can an electric motor be used to drive an electric generator with the output from the generator being used to operate the motor?

Ans. No, it is not possible because if it is possible then it will be a self-operating system without getting energy from some external source and this is against the law of conservation of energy.

15.15 A suspended magnet is oscillating freely in a horizontal plane. The oscillations are strongly damped when a metal plate is placed under the magnet. Explain why this occurs?

Ans. The oscillating magnet produce change of magnetic flux close to it. The metal plate placed under it experiences the change of magnetic flux. As a result an induced emf is produced in the metal plate due to the change in magnetic flux. According to Lenz law, induced current opposes its cause which are the oscillation of the magnet. So the oscillation of the magnet are strongly damped.

15.16 Four unmarked wires emerge from a transformer. What steps would you take to determine the turns ratio?

Ans. There are two steps for checking the four unknown wires.

- (1) Separate two coils into primary and secondary coil by checking continuity of wires by using ohm-meter.
- (2) Apply alternating voltage of known value V_p to one of the coil and the voltage across the other coil is measure by using voltmeter as V_s . Then by putting the values of V_p and V_s in

$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$

We can find the turn ratio. If the reading of voltmeter is less than input, then it is a step down transformer and if the reading of voltmeter is greater than input so it is a step up transformer.

15.17 (a) Can a step-up transformer increase the power level?

(b) In a transformer, there is no transfer of charge from the primary to the secondary. How is, then the power transferred?

Ans. (a) No, a step up transformer does not increase power level.

$$\text{As } P = VI$$

Hence a step up transformer increases V by decreasing I and hence $P = VI$ remains constant, otherwise it will against law of conservation of energy.

(b) The two coils of the transformer are magnetically linked i.e., the change of flux through one coil is linked with other coil and induced emf is produced. Power is transferred due to magnetic flux linkage.

15.18 When the primary of a transformer is connected to A.C. mains the current in it?

(a) Is very small if the secondary circuit is open, but.

(b) Increase when the secondary circuit is closed. Explain these facts.

Ans. (a) As for a transformer

$$\text{Input power} = \text{Output power}$$

$$V_p I_p = V_s I_s$$

When secondary circuit is open, then $P_{\text{out}} (VI) = 0$, so input power must be zero or very small. So input current I_p is very small in primary coil.

(b) However, when load is applied to secondary coil, greater power output is needed. Since output power = input power. So greater current is required in primary to equalize the power in the secondary coil.

PROBLEMS WITH SOLUTIONS

PROBLEM 15.1

An emf of 0.45 V is induced between the ends of a metal bar moving through a magnetic field of 0.22T. What field strength would be needed to produce an emf of 1.5 V between the ends of the bar, assuming that all other factors remain the same?

Data

$$\text{Induced emf} = \varepsilon_1 = 0.45 \text{ V}$$

$$\text{Magnetic field} = B_1 = 0.22 \text{ T}$$

$$\text{Induced emf} = \varepsilon_2 = 1.5 \text{ V}$$

To Find

$$\text{Magnetic field} = B_2 = ?$$

SOLUTION

By formula

$$\varepsilon = VBL \sin \theta$$

For 1st case

$$\varepsilon_1 = VB_1L \sin \theta \quad \dots\dots (i)$$

And for 2nd case

$$\varepsilon_2 = VB_2L \sin \theta \quad \dots\dots (ii)$$

Divided eq (i) by (ii)

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{VB_1L \sin \theta}{VB_2L \sin \theta}$$

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{B_1}{B_2}$$

$$B_2 = \frac{B_1 \times \varepsilon_2}{\varepsilon_1}$$

$$\begin{aligned} B_2 &= \frac{0.22 \times 1.5}{0.45} \\ &= 0.73 \text{ T} \end{aligned}$$

Result

$$\text{Magnetic field} = B_2 = 0.73 \text{ T}$$

PROBLEM 15.2

The flux density B in a region between the pole faces of a horseshoe magnet is 0.5 Wbm^{-2} directed vertically downward. Find the emf induced in a straight wire 5.0 cm long perpendicular to B when it is moved in direction at an angle of 60° with the horizontal with a speed of 100 cms^{-1} .

Data

$$\begin{aligned}\text{Flux density} &= B = 0.5 \text{ Wb/m}^2 \\ \text{Length of wire} &= L = 5 \text{ cm} = 0.05 \text{ m} \\ \text{Angle} &= \theta = 60^\circ \\ \text{Speed} &= V = 100 \text{ cm/s} \\ &= 1 \text{ m/s}\end{aligned}$$

To Find

$$\text{Induced emf} = \varepsilon = ?$$

SOLUTION

By formula

$$\varepsilon = VBL \sin \theta$$

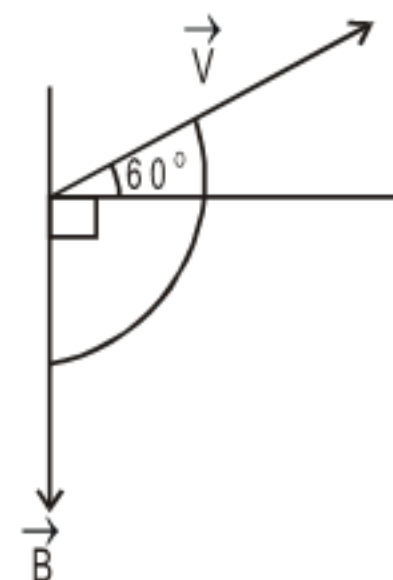
Angle between \vec{V} and \vec{B}

$$\begin{aligned}\theta &= 90^\circ + 60^\circ \\ &= 150^\circ\end{aligned}$$

$$\text{So, } \varepsilon = 1 \times 0.5 \times 0.05 \sin 150^\circ$$

$$\varepsilon = 0.0125 \text{ volt}$$

$$\text{or } \varepsilon = 1.25 \times 10^{-2} \text{ volt}$$

**Result**

$$\text{Induced emf} = \varepsilon = 1.25 \times 10^{-2} \text{ volt}$$

PROBLEM 15.3

A coil of wire has 10 loops. Each loop has an area of $1.5 \times 10^{-3} \text{ m}^2$. A magnetic field is perpendicular to the surface of each loop at all times. If the magnetic field is changed from 0.05 T to 0.06 T in 0.1 s , find the average emf induced in the coil during this time.

Data

$$\begin{aligned}\text{Number of loops} &= N = 10 \\ \text{Area of each loop} &= A = 1.5 \times 10^{-3} \text{ m}^2 \\ \text{Initial magnetic field} &= B_1 = 0.05 \text{ T} \\ \text{Final magnetic field} &= B_2 = 0.06 \text{ T}\end{aligned}$$

$$\begin{aligned}\text{Change in magnetic field} &= B = B_2 - B_1 \\ &= 0.06 - 0.05 \\ &= 0.01 \text{ T}\end{aligned}$$

$$\text{Time taken} = \Delta t = 0.1 \text{ sec.}$$

To Find

$$\text{Average induced emf in the coil} = \varepsilon = ?$$

SOLUTION

According to Faraday's law

$$\varepsilon = N \frac{\Delta\phi}{\Delta t}$$

$$\begin{aligned}\text{As } \Delta\phi &= \vec{B} \cdot \vec{A} \\ &= \Delta B A \cos 0^\circ\end{aligned}$$

$$\text{But } \Delta\phi = BA$$

$$\boxed{\varepsilon = -N \frac{BA}{\Delta t}}$$

$$\left(\begin{array}{l} \because \vec{B} \text{ is perpendicular to surface of loop} \\ \text{i.e., } \vec{B} \text{ and } \vec{A} \text{ are parallel} \end{array} \right)$$

Putting the values

$$= -10 \times \frac{0.01 \times 1.5 \times 10^{-3}}{0.1}$$

$$\varepsilon = 1.5 \times 10^{-3} \text{ volt}$$

Result

Average induced emf in the coil

$$\varepsilon = 1.5 \times 10^{-3} \text{ volt}$$

PROBLEM 15.4

Circular coil has 15 turns of radius 2 cm each. The plane of the coil lies at 40° to a uniform magnetic field of 0.2 T. If the field is increased to 0.5 T in 0.2 s, find the magnitude of the induced emf.

Data

$$\text{Number of turns} = N = 15$$

$$\begin{aligned}\text{Radius of coil} &= r = 2 \text{ cm} \\ &= 0.02 \text{ m}\end{aligned}$$

$$\text{Angle b/w plane of coil and magnetic field} = \theta = 40^\circ$$

$$\text{Initial magnetic field} = B_1 = 0.2 \text{ T}$$

$$\text{Final magnetic field} = B_2 = 0.5 \text{ T}$$

$$\begin{aligned}\text{Change in magnetic field} &= B = B_2 - B_1 \\ &= 0.5 - 0.2 \\ &= 0.3 \text{ T}\end{aligned}$$

$$\text{Time taken} = \Delta t = 0.2 \text{ sec.}$$

To Find

$$\text{Magnitude of induced emf} = \varepsilon = ?$$

SOLUTION

By formula

$$\varepsilon = -N \frac{\Delta\phi}{\Delta t} \quad \dots\dots (i)$$

$$\begin{aligned}\text{But } \Delta\phi &= \vec{B} \cdot \vec{A} \\ &= BA \cos \theta\end{aligned}$$

Where θ is the angle b/w the vector area \vec{A} and magnetic field B i.e.

$$\begin{aligned}\theta &= 90^\circ - 40^\circ \\ &= 50^\circ\end{aligned}$$

$$\begin{aligned}\text{So, } \Delta\phi &= BA \cos 50^\circ \\ \Delta\phi &= BA (0.64)\end{aligned}$$

Therefore,

$$\varepsilon = \frac{-N BA (0.64)}{\Delta t}$$

$$= \frac{15 \times 0.3 \times 3.14 \times (0.02)^2 \times 0.64}{0.2}$$

$$\left(\begin{array}{l} \text{Since } A = \pi r^2 \\ = 3.14 \times (0.02)^2 \end{array} \right)$$

$$\text{So, } \varepsilon = 0.018 \text{ volt}$$

$$\varepsilon = 1.8 \times 10^{-2} \text{ volt}$$

Result

$$\text{Magnitude of induced emf} = \varepsilon = 1.8 \times 10^{-2} \text{ volt}$$

PROBLEM 15.5

Two coils are placed side by side. An emf of 0.8 V is observed in one coil when the current is changing at the rate of 200 As^{-1} in the other coil. What is the mutual inductance of the coils?

Data

$$\text{emf in one coil} = \varepsilon_s = 0.8 \text{ V}$$

$$\text{Rate of current} = \frac{\Delta I_p}{\Delta t} = 200 \text{ A/S}$$

To Find

$$\text{Mutual inductance} = M = ?$$

SOLUTION

By formula

$$\varepsilon_s = -M \left(\frac{\Delta I_p}{\Delta t} \right)$$

$$M = \frac{\varepsilon_s}{\left(\frac{\Delta I_p}{\Delta t} \right)}$$

$$\begin{aligned} \text{So, } M &= \frac{0.8}{200} \\ &= 4 \times 10^{-3} \text{ H} \\ M &= 4 \text{ mH} \end{aligned}$$

Result

$$\text{Mutual inductance b/w the coils} = M = 4 \text{ mH}$$

PROBLEM 15.6

A pair of adjacent coils has a mutual inductance of 0.75 H. If the current in the primary changes from 0 to 10 A in 0.025 s, what is the average induced emf in the secondary? What is the change in flux in it if the secondary has 500 turns?

Data

$$\text{Mutual inductance} = M = 0.75 \text{ H}$$

$$\text{Initial current} = I_1 = 0 \text{ A}$$

$$\text{Final current} = I_2 = 10 \text{ A}$$

$$\begin{aligned} \text{Change in current} &= \Delta I = I_2 - I_1 \\ &= 10 - 0 \\ &= 10 \text{ A} \end{aligned}$$

$$\text{Time taken} = \Delta t = 0.025 \text{ Sec}$$

$$\text{Number of turns} = N = 500$$

To Find

$$\text{Average induced emf in secondary coil} = \varepsilon_s = ?$$

$$\text{Change in flux} = \Delta \phi = ?$$

SOLUTION

For average induced emf

$$\begin{aligned}\varepsilon_s &= M \left(\frac{\Delta I_p}{\Delta t} \right) \\ &= 0.75 \left(\frac{10}{0.025} \right)\end{aligned}$$

$$\varepsilon_s = 300 \text{ volt}$$

For change in flux

$$\varepsilon_s = N_s \frac{\Delta \phi}{\Delta t}$$

$$\Delta \phi = \frac{\varepsilon_s \times \Delta t}{N_s}$$

Putting the values

$$\begin{aligned}\Delta \phi &= 300 \times \frac{0.025}{500} \\ &= 0.015 \text{ Wb} \\ \Delta \phi &= 1.5 \times 10^{-2} \text{ Wb}\end{aligned}$$

Result

$$\text{Average induced emf in secondary} = \varepsilon_s = 300 \text{ volt}$$

$$\text{Change in flux} = \Delta \phi = 1.5 \times 10^{-2} \text{ Wb}$$

PROBLEM 15.7

A solenoid has 250 turns and its self inductance is 2.4 mH. What is flux through each turn when the current is 2A? What is induced emf when current changes at 20 As^{-1} ?

Data

$$\text{Number of turns} = N = 250$$

$$\begin{aligned}\text{Self inductance} &= L = 2.4 \text{ mH} \\ &= 2.4 \times 10^{-3} \text{ H}\end{aligned}$$

$$\text{Current} = I = 2 \text{ A}$$

$$\text{Rate of current} = \frac{\Delta I}{\Delta t} = 20 \text{ A/S}$$

To Find

$$\text{Flux through each turn} = \phi = ?$$

$$\text{Induced emf} = \varepsilon = ?$$

SOLUTION

For flux through each turn

$$L = \frac{N\phi}{I}$$

$$\phi = \frac{L \times I}{N}$$

$$= \frac{2.4 \times 10^{-3} \times 2}{250}$$

$$= 0.0192 \times 10^{-3}$$

$$= 1.92 \times 10^{-5} \text{ Wb}$$

For induced emf

$$\varepsilon = L \frac{\Delta I}{\Delta t}$$

$$\varepsilon = 2.4 \times 10^{-3} \times 20$$

$$= 48 \times 10^{-3} \text{ volt}$$

$$= 48 \text{ mV}$$

Result

$$\text{Flux through each turn} = \phi = 1.92 \times 10^{-5} \text{ Wb}$$

$$\text{Induced emf} = \varepsilon = 48 \text{ m volt}$$

PROBLEM 15.8

A solenoid of length 8.0 cm and cross-sectional area 0.5 cm² has 520 turns. Find the self-inductance of the solenoid when the core is air. If the current in the solenoid increases through 1.5 A in 0.2 s, find the magnitude of induced emf in it. ($\mu_0 = 4\pi \times 10^{-7} \text{ WbA}^{-1}\text{m}^{-1}$)

Data

$$\text{Length of solenoid} = l = 8.0 \text{ cm}$$

$$= 0.08 \text{ m}$$

$$\text{Area of cross-section of solenoid} = A = 0.5 \text{ cm}^2$$

$$= 0.5 \times 10^{-4} \text{ m}^2$$

$$\text{Number of turns} = N = 520$$

$$\text{Change in Current} = \Delta I = 1.5 \text{ A}$$

$$\text{Time taken} = \Delta t = 0.2 \text{ Sec}$$

To Find

$$\text{Self inductance of the solenoid} = L = ?$$

$$\text{Magnitude of induced emf} = \varepsilon = ?$$

SOLUTION

For self inductance of the solenoid when the core is air,

$$L = \mu_0 n^2 l A$$

But $n = \frac{N}{l}$ = Number of turns per unit length

So,
$$L = \mu_0 \left(\frac{N}{l} \right)^2 l A$$

$$L = \mu_0 \frac{N^2 A}{l}$$

Putting the values

$$\begin{aligned} L &= 4\pi \times 10^{-7} \times \frac{(520)^2 \times 0.5 \times 10^{-4}}{0.08} \\ &= 21226400 \times 10^{-7-4} \\ &= 2.12 \times 10^{-4} \text{ H} \end{aligned}$$

And for magnitude of induced emf

$$\begin{aligned} \varepsilon &= L \frac{\Delta I}{\Delta t} \\ &= 2.12 \times 10^{-4} \times \frac{1.5}{0.2} \\ &= 15.9 \times 10^{-4} \text{ V} \\ &= 1.59 \times 10^{-3} \text{ V} \end{aligned}$$

Result

Self inductance of the solenoid = $L = 2.12 \times 10^{-4} \text{ H}$

Magnitude of induced emf = $\varepsilon = 1.59 \times 10^{-3} \text{ volt}$

PROBLEM 15.9

When current through a coil changes from 100 mA to 200 mA in 0.005 s, an induced emf of 40 mV is produced in the coil. (a) What is the self-inductance of the coil? (b) Find the increase in the energy stored in the coil.

Data

$$\begin{aligned} \text{Initial current} &= I_i = 100 \text{ mA} \\ &= 100 \times 10^{-3} \text{ A} = 0.1 \text{ A} \\ \text{Final current} &= I_f = 200 \text{ mA} \\ &= 200 \times 10^{-3} \text{ A} = 0.2 \text{ A} \end{aligned}$$

$$\begin{aligned}\text{Change in current} &= \Delta I = I_f - I_i \\ &= 0.2 - 0.1 = 0.1 \text{ A}\end{aligned}$$

$$\text{Time} = \Delta t = 0.005 \text{ sec.}$$

$$\begin{aligned}\text{Induced emf} &= \varepsilon = 40 \text{ mV} \\ &= 40 \times 10^{-3} \text{ V}\end{aligned}$$

To Find

$$(a) \quad \text{Self inductance of the coil} = L = ?$$

$$(b) \quad \text{Increase in energy stored} = \Delta U_m = ?$$

SOLUTION

$$(a) \quad \text{For self inductance}$$

$$\varepsilon = L \frac{\Delta I}{\Delta t}$$

$$L = \frac{\varepsilon \times \Delta t}{\Delta I}$$

Putting the values

$$L = \frac{40 \times 10^{-3} \times 0.005}{0.1}$$

$$L = 2 \times 10^{-3} \text{ H}$$

$$\text{or } L = 2 \text{ mH}$$

$$(b) \quad \text{For increase in energy stored}$$

$$\Delta U_m = \frac{1}{2} L (I_f^2 - I_i^2)$$

$$\Delta U_m = \frac{1}{2} \times 2 \times 10^{-3} [(200 \times 10^{-3})^2 - (100 \times 10^{-3})^2]$$

$$= 1 \times 10^{-3} [40000 \times 10^{-6} - 10000 \times 10^{-6}]$$

$$= 1 \times 10^{-3} \times 30000 \times 10^{-6}$$

$$\Delta U_m = 0.03 \times 10^{-3} \text{ J}$$

$$\Delta U_m = 0.03 \text{ mJ}$$

Result

$$(a) \quad \text{Self inductance of the coil} = L = 2 \text{ mH}$$

$$(b) \quad \text{Increase in energy stored} = \Delta U_m = 0.03 \text{ mJ}$$

PROBLEM 15.10

Like any field, the earth's magnetic field stores energy. Find the magnetic energy stored in a space where strength of earth's field is 7×10^{-5} T, if the space occupies an area of 10×10^8 m² and has a height of 750 m.

Data

$$\begin{aligned}\text{Earth's magnetic field} &= B = 7 \times 10^{-5} \text{ T} \\ \text{Area} &= A = 10 \times 10^8 \text{ m}^2 \\ \text{Height above the earth} &= h = 750 \text{ m}\end{aligned}$$

To Find

$$\text{Magnetic energy stored} = U_m = ?$$

SOLUTION

By formula

$$U_m = \frac{1}{2} \frac{B^2}{\mu_0} (Al)$$

$$\begin{aligned}\text{But } \mu_0 &= 4\pi \times 10^{-7} \text{ Wb/Am} \\ U_m &= \frac{1}{2} \times \frac{(7 \times 10^{-5})^2}{4\pi \times 10^{-7}} \times 10 \times 10^8 \times 750 \\ &= 14629.7 \times 10^{-10+8+7} \\ &= 14629.7 \times 10^5 \\ U_m &= 1.46 \times 10^9 \text{ J}\end{aligned}$$

Result

$$\text{Magnetic energy stored} = U_m = 1.46 \times 10^9 \text{ J}$$

PROBLEM 15.11

A square coil of side 16 cm has 200 turns and rotates in a uniform magnetic field of magnitude 0.05 T. If the peak emf is 12V, what is the angular velocity of coil?

Data

$$\begin{aligned}\text{Length of square coil} &= l = 16 \text{ cm} \\ \text{Area of the coil} &= A = 16 \times 16 \\ &= 256 \text{ cm}^2 \\ &= 256 \times 10^{-4} \text{ m}^2 \\ \text{Number of turns} &= N = 200 \\ \text{Magnitude of magnetic field} &= B = 0.05 \text{ T} \\ \text{Peak emf} &= \varepsilon_0 = 12 \text{ V}\end{aligned}$$

To Find

$$\text{Angular velocity} = \omega = ?$$

SOLUTION

$$\text{Using } \varepsilon = N\omega AB \sin \theta$$

For peak value $\theta = 90^\circ$

$$\varepsilon_0 = B\omega NA$$

$$\omega = \frac{\varepsilon_0}{BNA}$$

$$\begin{aligned}\omega &= \frac{12}{0.05 \times 200 \times 256 \times 10^{-4}} \\ &= 4.68 \times 10^{-3+4} \\ &= 4.68 \times 10^1 \\ &= 46.8 \text{ rad/s} \\ \omega &= 47 \text{ rad/sec}\end{aligned}$$

Result

$$\text{Angular velocity of the coil} = \omega = 47 \text{ rad/s}$$

PROBLEM 15.12

A generator has a rectangular coil consisting of 360 turns. The coil rotates at 420 rev per min in 0.14 T magnetic fields. The peak value of emf produced by the generator is 50 V. If the coil is 5.0 cm wide; find the length of the side of the coil.

Data

$$\text{Number of turns of coil} = N = 360$$

$$\begin{aligned}\text{Angular velocity} &= \omega = 420 \text{ rev/min} \\ &= 420 \times \frac{2\pi}{60} \text{ rad/s} \\ &= 14\pi \text{ rad/sec}\end{aligned}$$

$$\text{Magnetic field} = B = 0.14 \text{ T}$$

$$\text{Peak emf} = \varepsilon_0 = 50 \text{ V}$$

$$\begin{aligned}\text{Width of the coil} &= b = 5.0 \text{ cm} \\ &= 0.05 \text{ m}\end{aligned}$$

To Find

$$\text{Length of the coil} = l = ?$$

SOLUTION

By formula

$$\varepsilon_0 = B\omega NA$$

But $A = l \times b$

$$\varepsilon_0 = B\omega N (l \times b)$$

$$l = \frac{\varepsilon_0}{B\omega Nb}$$

$$= \frac{50}{0.14 \times 14\pi \times 360 \times 0.05}$$

$$l = 0.45 \text{ m}$$

or $l = 45 \text{ cm}$

Result

Length of the coil $= l = 45 \text{ cm}$

PROBLEM 15.13

It is desired to make an a.c generator that can produce an emf of 5 kV with 50 Hz frequency. A coil of area 1 m^2 consisting of 200 turns is used as armature. What should be the magnitude of the magnetic field in which the coil rotates?

Data

$$\begin{aligned} \text{Peak emf} &= \varepsilon_0 = 5 \text{ KV} \\ &= 5 \times 1000 \\ &= 5000 \text{ Volt} \end{aligned}$$

$$\text{Frequency} = f = 50 \text{ Hz}$$

$$\text{Area of coil} = A = 1 \text{ m}^2$$

$$\text{Number of turns} = N = 200$$

To Find

Magnitude of magnetic field $= B = ?$

SOLUTION

By formula

$$\varepsilon_0 = B\omega NA$$

$$B = \frac{\varepsilon_0}{\omega NA}$$

But $\omega = 2\pi f$

$$B = \frac{\varepsilon_0}{2\pi f NA}$$

$$\begin{aligned}
 B &= \frac{5000}{2(3.14) \times 50 \times 200 \times 1} \\
 &= 0.0796 \\
 &= 0.08\text{T}
 \end{aligned}$$

Result

Magnitude of magnetic field = $B = 0.08\text{T}$

PROBLEM 15.14

The back emf in a motor is 120 V when the motor is turning at 1680 rev per min. What is the back emf when the motor turns 3360 rev per min?

Data

$$\begin{aligned}
 \text{Back emf} &= \varepsilon_1 = 120\text{V} \\
 \text{Initial angular velocity} &= \omega_i = 1680 \text{ rev/min} \\
 \text{Final angular velocity} &= \omega_f = 3360 \text{ rev/min}
 \end{aligned}$$

To Find

$$\text{Back emf when motor turns 3360 rev/min} = \varepsilon_2 = ?$$

SOLUTION

According to formula

$$\varepsilon = B\omega NA$$

For 1st case

$$\varepsilon_1 = B\omega_1 NA \sin \theta \quad \dots\dots (i)$$

And for the 2nd case

$$\varepsilon_2 = B\omega_2 NA \sin \theta \quad \dots\dots (ii)$$

Divide eq (i) by (ii)

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{B\omega_1 NA \sin \theta}{B\omega_2 NA \sin \theta}$$

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{\omega_1}{\omega_2}$$

$$\boxed{\varepsilon_2 = \frac{\varepsilon_1 \times \omega_2}{\omega_1}}$$

Putting the values

$$\varepsilon_2 = \frac{120 \times 3360}{1680}$$

$$\varepsilon_2 = 240 \text{ V}$$

Result

Back emf when motor turns 3360 rev/min = $\varepsilon_2 = 240 \text{ V}$

PROBLEM 15.15

A D.C motor operates at 240 V and has resistance of 0.5Ω . When motor is running at normal speed, the armature current is 15 A. Find the back emf in armature.

Data

$$\text{Voltage of D.C motor} = V = 240 \text{ volt}$$

$$\text{Resistance} = r = 0.5 \Omega$$

$$\text{Armature current} = I = 15 \text{ A}$$

To Find

$$\text{Back emf in the armature} = \varepsilon = ?$$

SOLUTION

By formula

$$V = \varepsilon + Ir$$

$$\varepsilon = V - Ir$$

$$= 240 - 15 \times 0.5$$

$$\varepsilon = 232.5 \text{ Volt}$$

Result

$$\text{Back emf in the armature} = \varepsilon = 232.5 \text{ V}$$

PROBLEM 15.16

A copper ring has a radius of 4.0 cm and resistance of $1.0 \text{ m}\Omega$. A magnetic field is applied over the ring, perpendicular to its plane. If the magnetic field increases from 0.2 T to 0.4 T in a time interval of $5 \times 10^{-3} \text{ s}$, what is the current in the ring during this interval?

Data

$$\text{Radius of copper ring} = r = 4.0 \text{ cm}$$

$$= 0.04 \text{ m}$$

$$\text{Resistance of ring} = R = 5 \text{ m}\Omega$$

$$= 5 \times 10^{-3} \Omega$$

$$\begin{aligned}
 \text{Initial magnetic field} &= B_1 = 0.2 \text{ T} \\
 \text{Final magnetic field} &= B_2 = 0.4 \text{ T} \\
 \text{Change in magnetic field} &= B = B_2 - B_1 \\
 &= 0.4 - 0.2 \\
 &= 0.2 \text{ T} \\
 \text{Time interval} &= \Delta t = 5 \times 10^{-3} \text{ sec.}
 \end{aligned}$$

To Find

$$\text{Current in the ring} = I = ?$$

SOLUTION

As we know that

$$V = IR \quad \text{or} \quad \varepsilon = IR$$

$$\boxed{I = \frac{\varepsilon}{R}} \quad \dots\dots (i)$$

$$\begin{aligned}
 \text{But} \quad \varepsilon &= \frac{\Delta\phi}{\Delta t} \quad \text{and} \quad \Delta\phi = \vec{B} \cdot \vec{A} \\
 &= \Delta B A \cos 0^\circ \\
 \Delta\phi &= \Delta B A
 \end{aligned}$$

$$\text{As} \quad A = \pi r^2$$

$$\text{So,} \quad \Delta\phi = B \times \pi r^2$$

$$\begin{aligned}
 \text{So,} \quad \varepsilon &= \frac{B \times \pi r^2}{\Delta t} \\
 &= \frac{0.2 \times 3.14 \times (0.04)^2}{5 \times 10^{-3}}
 \end{aligned}$$

$$= 2.0 \times 10^{3-4}$$

$$\varepsilon = 0.201 \text{ volt}$$

Putting in eq. (i)

$$\begin{aligned}
 I &= \frac{0.201}{1 \times 10^{-3}} \\
 &= 0.201 \times 10^3 \\
 I &= 201 \text{ A}
 \end{aligned}$$

Result

$$\text{Current in the ring} = I = 201 \text{ A}$$

PROBLEM 15.17

A coil of 10 turns and 35 cm^2 area is in a perpendicular magnetic field of 0.5 T. The coil is pulled out of the field in 1.0 s. Find the induced emf in the coil as it is pulled out of the field?

Data

Number of turns	=	$N = 10$
Area of the coil	=	$A = 35 \text{ cm}^2$ $= 35 \times 10^{-4} \text{ m}^2$
Magnetic field	=	$B = 0.5 \text{ T}$
Time	=	$\Delta t = 1.0 \text{ Sec}$

To Find

Induced emf in the coil = $\varepsilon = ?$

SOLUTION

By formula

$$\varepsilon = N \frac{\Delta\phi}{\Delta t}$$

$$\Delta\phi = BA$$

So,

$$\varepsilon = N \frac{BA}{\Delta t}$$

$$= \frac{10 \times 0.5 \times 35 \times 10^{-4}}{1.0}$$

$$= 175 \times 10^{-4}$$

$$\varepsilon = 1.75 \times 10^{-2} \text{ volt}$$

Result

Induced emf in the coil = $\varepsilon = 1.75 \times 10^{-2} \text{ V}$

PROBLEM 15.18

An ideal step down transformer is connected to main supply of 240 V. It is desired to operate a 12 V, 30 W lamp. Find the current in the primary and the transformation ratio?

Data

Primary voltage	=	$V_p = 240 \text{ V}$
Secondary voltage	=	$V_s = 12 \text{ V}$
Power output	=	$P_s = 30 \text{ watt}$

To Find

$$\text{Current in the primary} = I_p = ?$$

$$\text{Transformer ratio} = \frac{N_s}{N_p} = ?$$

SOLUTION

For an ideal transformer

$$\text{Power output} = \text{Power input}$$

$$V_p I_p = V_s I_s$$

$$I_p = \frac{V_s I_s}{V_p}$$

But

$$P_s = V_s I_s$$

$$I_s = \frac{P_s}{V_s} = \frac{30}{12}$$

$$I_s = 2.5 \text{ A}$$

$$\begin{aligned} \text{So, } I_p &= \frac{12 \times 2.5}{240} \\ &= 0.125 \text{ Amp} \end{aligned}$$

For transformer ratio

$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$

$$\text{Therefore } \frac{N_s}{N_p} = \frac{12}{240}$$

$$\frac{N_s}{N_p} = \frac{1}{20}$$

Result

$$\text{Current in the primary coil} = I_p = 0.125 \text{ Amp}$$

$$\text{Transformer ratio} = \frac{N_s}{N_p} = \frac{1}{20}$$