EXERCISE 4.8

Solve the following system of equations:

Q.1 $\mathbf{2x - y = 4; \quad 2x^2 - 4xy - y^2 = 6}$

Solution:

$$2x^2 - 4xy - y^2 = 6 \qquad \ldots\ldots\ldots (1)$$

$$2x - y = 4$$

$$\Rightarrow \quad y = 2x - 4 \qquad \ldots\ldots\ldots (2)$$

Put equation (2) in equation (1)

$$\Rightarrow \quad 2x^2 - 4x(2x - 4) - (2x - 4)^2 = 6$$

$$\Rightarrow \quad 2x^2 - 8x^2 + 16x - (4x^2 + 16 - 16x) = 6$$

$$\Rightarrow \quad -6x^2 + 16x - 4x^2 - 16 + 16x - 6 = 0$$

$$\Rightarrow \quad -10x^2 + 32x - 22 = 0$$

$$\Rightarrow \quad -2(5x^2 - 16x + 11) = 0$$

$$\Rightarrow \quad 5x^2 - 16x + 11 = 0$$

$$\Rightarrow \quad 5x^2 - 11x - 5x + 11 = 0$$

$$\Rightarrow \quad x(5x - 11) - 1(5x - 11) = 0$$

$$\Rightarrow \quad (5x - 11)(x - 1) = 0$$

$$\Rightarrow \quad \text{Either} \quad 5x - 11 = 0 \quad \text{or} \quad x - 1 = 0$$

$$\Rightarrow \quad x = \frac{11}{5} \quad \text{or} \quad x = 1$$

Put $x = \frac{11}{5}$ and $x = 1$ in equation (2)

when $x = \frac{11}{5}$ then

$$y = 2\left(\frac{11}{5}\right) - 4$$

$$= \frac{22}{5} - 4$$

$$= \frac{22 - 20}{5}$$

$$y = \frac{2}{5}$$

$$\Rightarrow \quad \left(\frac{11}{5}, \frac{2}{5}\right)$$

when $x = 1$ then

$$y = 2(1) - 4$$

$$y = 2 - 4$$

$$y = -2$$

$$\Rightarrow \quad (1, -2)$$

Hence the solution set $= \left\{(1, -2), \left(\frac{11}{5}, \frac{2}{5}\right)\right\}$

Q.2 $\quad \mathbf{x + y = 5, \quad x^2 + 2y^2 = 17}$

Solution:

$$x^2 + 2y^2 = 17 \qquad \ldots\ldots\ldots (1)$$

and $\quad x + y = 5$

$\Rightarrow \quad x = 5 - y \qquad \ldots\ldots\ldots (2)$

Put $x = 5 - y$ in equation (1)

$\Rightarrow \quad (5 - y)^2 + 2y^2 = 17$

$\Rightarrow \quad 25 + y^2 - 10y + 2y^2 = 17$

$\Rightarrow \quad 3y^2 - 10y + 25 - 17 = 0$

$\Rightarrow \quad 3y^2 - 10y + 8 = 0$

$\Rightarrow \quad 3y^2 - 6y - 4y + 8 = 0$

$\Rightarrow \quad 3y(y - 2) - 4(y - 2) = 0$

$\Rightarrow \quad (y - 2)(3y - 4) = 0$

$\Rightarrow \quad$ Either $\; y - 2 = 0 \quad$ or $\quad 3y - 4 = 0$

$\Rightarrow \quad y = 2 \quad$ or $\quad y = \dfrac{4}{3}$

Put $y = 2$ and $y = \dfrac{4}{3}$ in equation (2)

when $y = 2$ then

$\Rightarrow \quad x = 5 - 2$

$\Rightarrow \quad x = 3$

$\Rightarrow \quad (3, 2)$

when $y = \dfrac{4}{3}$ then

$\Rightarrow \quad x = 5 - \dfrac{4}{3}$

$\Rightarrow \quad x = \dfrac{15 - 4}{3} = \dfrac{11}{3}$

$\Rightarrow \quad \left(\dfrac{11}{3}, \dfrac{4}{3}\right)$

Hence the solution set $= \left\{(3, 2), \left(\dfrac{11}{3}, \dfrac{4}{3}\right)\right\}$

Q.3 $\quad \mathbf{3x + 2y = 7; \quad 3x^2 - 2y^2 = 25.}$

Solution:

$$3x^2 = 25 + 2y^2 \qquad \ldots\ldots\ldots (1)$$

and $\quad 3x + 2y = 7$

$\quad 3x = 7 - 2y$

$\Rightarrow \quad x = \dfrac{7 - 2y}{3} \qquad \ldots\ldots\ldots (2)$

Put value of x from equation (2) in equation (1)

$\Rightarrow \quad 3\left(\dfrac{7-2y}{3}\right)^2 = 25 + 2y^2$

$\Rightarrow \quad \dfrac{3(7-2y)^2}{9} = 25 + 2y^2$

$\Rightarrow \quad \dfrac{49 + 4y^2 - 28y}{3} = 25 + 2y^2$

$\Rightarrow \quad 49 + 4y^2 - 28y = 3(25 + 2y^2)$

$\Rightarrow \quad 49 + 4y^2 - 28y = 75 + 6y^2$

$\Rightarrow \quad 6y^2 + 75 - 49 - 4y^2 + 28y = 0$

$\Rightarrow \quad 2y^2 + 28y + 26 = 0$

$\Rightarrow \quad 2(y^2 + 14y + 13) = 0$

$\Rightarrow \quad 2(y^2 + 14y + 13) = 0$

$\Rightarrow \quad y^2 + 14y + 13 = 0$

$\Rightarrow \quad y^2 + 13y + y + 13 = 0$

$\Rightarrow \quad y(y + 13) + 1(y + 13) = 0$

$\Rightarrow \quad (y + 13) + (y + 1) = 0$

$\Rightarrow$ Either $y + 13 = 0$ or $y + 1 = 0$

$\Rightarrow \quad y = -13$ or $y = -1$

Put $y = -13$ and $y = -1$ in equation (2)

when $y = -13$

then $x = \dfrac{7 - 2(-13)}{3}$

$= \dfrac{7 + 26}{3} = \dfrac{33}{3} = 11$

$\Rightarrow \quad (11, -13)$

when $y = -1$

then $x = \dfrac{7 - 2(-1)}{3}$

$= \dfrac{7 + 2}{3} = \dfrac{9}{3} = 3$

$\Rightarrow \quad (3, -1)$

Hence the solution set $= \{(3, -1)\ (11, -13)\}$

Q.4 $\quad \mathbf{x + y = 5, \quad \dfrac{2}{x} + \dfrac{3}{y} = 2, \quad x \neq 0, \quad y \neq 0.}$

Solution:

$\dfrac{2}{x} + \dfrac{3}{y} = 2 \qquad \ldots\ldots\ldots (1)$

and $x + y = 5$

$\Rightarrow \quad y = 5 - x \quad \ldots\ldots\ldots (2)$

Put $y = 5 - x$ from equation (2) in equation (1)

$$\frac{2}{x} + \frac{3}{5-x} = 2$$

$$\Rightarrow \quad \frac{2(5-x) + 3x}{x(5-x)} = 2$$

$$\Rightarrow \quad 2(5-x) + 3x = 2x(5-x)$$

$$\Rightarrow \quad 10 - 2x + 3x = 10x - 2x^2$$

$$\Rightarrow \quad 10 - 2x + 3x - 10x + 2x^2 = 0$$

$$\Rightarrow \quad 2x^2 - 9x + 10 = 0$$

$$\Rightarrow \quad 2x^2 - 5x - 4x + 10 = 0$$

$$\Rightarrow \quad x(2x-5) - 2(2x-5) = 0$$

$$\Rightarrow \quad (2x-5)(x-2) = 0$$

$\Rightarrow$ Either $2x - 5 = 0$ or $x - 2 = 0$

$\Rightarrow \quad x = \frac{5}{2}$ or $x = 2$

Put $x = \frac{5}{2}$ and $x = 2$ in equation (2)

when $x = \frac{5}{2}$

$$\Rightarrow \quad y = 5 - \frac{5}{2}$$

$$\Rightarrow \quad y = \frac{10-5}{2}$$

$$\Rightarrow \quad y = \frac{5}{2}$$

$$\Rightarrow \quad \left(\frac{5}{2}, \frac{5}{2}\right)$$

when $x = 2$

$$\Rightarrow \quad y = 5 - 2$$

$$\Rightarrow \quad y = 3$$

$$\Rightarrow \quad (2, 3)$$

Hence the solution set $= \left\{ (2, 3), \left(\frac{5}{2}, \frac{5}{2}\right) \right\}$

Q.5 $\mathbf{x + y = a + b, \quad \frac{a}{x} + \frac{b}{y} = 2}$ **(Lahore Board 2003, 2005)**

Solution:

$$\frac{a}{x} + \frac{b}{y} = 2 \qquad \text{......... (1)}$$

and $x + y = a + b$

$$y = a + b - y \qquad \text{......... (2)}$$

Put $y = a + b - x$ from equation (2) in equation (1)

$$\frac{a}{x} + \frac{b}{a + b - x} = 2$$

$$\frac{a(a + b - x) + bx}{x(a + b - x)} = 2$$

$$a(a + b - x) + bx = 2x(a + b - x)$$

$$a^2 + ab - ax + bx = 2ax + 2bx - 2x^2$$

$$2x^2 - 2ax - 2bx + a^2 + ab - ax + bx = 0$$

$$2x^2 - 3ax - bx + a^2 + ab = 0$$

$$2x^2 - (3a + b)x + a^2 + ab = 0$$

Comparing with $ax^2 + bx + c = 0$, we have

$$a = 2, \quad b = -(3a + b), \quad c = a^2 + ab$$

$$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\left[-(3a + b)\right] \pm \sqrt{\left[-(3a + b)\right]^2 - 4(2)(a^2 + ab)}}{2(2)}$$

$$= \frac{(3a + b) \pm \sqrt{(3a + b)^2 - 8(a^2 + ab)}}{4}$$

$$= \frac{(3a + b) \pm \sqrt{(9a^2 + b^2 + 6ab) - 8a^2 - 8ab}}{4}$$

$$= \frac{(3a + b) \pm \sqrt{9a^2 + b^2 + 6ab - 8a^2 - 8ab}}{4}$$

$$= \frac{(3a + b) \pm \sqrt{a^2 + b^2 - 2ab}}{4}$$

$$x = \frac{(3a+b) \pm \sqrt{(a-b)^2}}{4} = \frac{(3a+b) \pm (a-b)}{4}$$

$\Rightarrow \quad x = \frac{(3a+b)+(a-b)}{4}$ or $x = \frac{(3a+b)-(a-b)}{4}$

$\Rightarrow \quad x = \frac{3a+b+a-b}{4}$ or $x = \frac{3a+b-a+b}{4}$

$\Rightarrow \quad x = \frac{4a}{4}$ or $x = \frac{2a+2b}{4}$

$\Rightarrow \quad x = a$ or $x = \frac{2(a+b)}{4}$

or $x = \frac{a+b}{2}$

Now put these values in equation (2)

when $x = a$ equation (2) $\Rightarrow$

$\Rightarrow \quad y = a + b - a$

$\Rightarrow \quad y = b$

$\Rightarrow \quad (a, b)$

when $x = \frac{a+b}{2}$ equation (2) $\Rightarrow$

$\Rightarrow \quad y = a + b - \frac{a+b}{2}$

$= \frac{2a+2b-a-b}{2}$

$= \frac{a+b}{2}$

$\Rightarrow \quad \left(\frac{a+b}{2}, \frac{a+b}{2}\right)$

Hence the solution set $= \left\{(a, b), \left(\frac{a+b}{2}, \frac{a+b}{2}\right)\right\}$

Q.6 $\mathbf{3x + 4y = 25, \ \frac{3}{x} + \frac{4}{y} = 2}$

Solution:

$\frac{3}{x} + \frac{4}{y} = 2$ ……… (1)

and $3x + 4y = 25$

$\Rightarrow \quad 4y = 25 - 3x$

$y = \frac{25-3x}{4}$ ……… (2)

Put value of 'y' from equation (2) in equation (1)

$$\Rightarrow \quad \frac{3}{x} + \frac{4}{\frac{25-3x}{4}} = 2$$

$$\Rightarrow \quad \frac{3}{x} + \frac{16}{25-3x} = 2$$

$$\Rightarrow \quad \frac{3(25-3x)+16x}{x(25-3x)} = 2$$

$$\Rightarrow \quad 3(25-3x) + 16x = 2x(25-3x)$$

$$\Rightarrow \quad 75 - 9x + 16x = 50x - 6x^2$$

$$\Rightarrow \quad 6x^2 - 50x + 75 - 9x + 16x = 0$$

$$\Rightarrow \quad 6x^2 - 43x + 75 = 0$$

$$\Rightarrow \quad x = \frac{-(-43) \pm \sqrt{(-43)^2 - 4(6)(75)}}{2(6)}$$

$$= \frac{43 \pm \sqrt{1849-1800}}{12} = \frac{43 \pm \sqrt{49}}{12} = \frac{43 \pm 7}{12}$$

$$\Rightarrow \quad x = \frac{43+7}{12} \quad \text{or} \quad x = \frac{43-7}{12}$$

$$\Rightarrow \quad x = \frac{50}{12} \quad \text{or} \quad x = \frac{36}{12}$$

$$\Rightarrow \quad x = \frac{25}{6} \quad \text{or} \quad x = 3$$

Put these values in equation (2)

when $x = \frac{25}{6}$ equation (2) $\Rightarrow$

$$\Rightarrow \quad y = \frac{25 - 3\left(\frac{25}{6}\right)}{4}$$

$$\Rightarrow \quad y = \frac{25 - \frac{25}{2}}{4}$$

$$\Rightarrow \quad \frac{\frac{50-25}{2}}{4}$$

$$\Rightarrow \quad y = \frac{25}{2}\,\frac{1}{4} = \frac{25}{8}$$

$$\Rightarrow \quad \left(\frac{25}{6}, \frac{25}{8}\right)$$

when $x = 3$ equation (2) $\Rightarrow$

$$\Rightarrow \quad y = \frac{25-3(3)}{4}$$

$$\Rightarrow \quad y = \frac{25-9}{4}$$

$$\Rightarrow \quad = \frac{16}{4} = 4$$

$$\Rightarrow \quad (3, 4)$$

Hence the solution set $= \left\{ (3, 4), \left(\frac{25}{6}, \frac{25}{8}\right) \right\}$

Q.7 $(x-3)^2 + y^2 = 5, \quad 2x = y + 6$

Solution:

$(x-3)^2 + y^2 = 5 \quad \ldots\ldots\ldots (1)$

and $2x = y + 6$

$\Rightarrow \quad y = 2x - 6 \quad \ldots\ldots\ldots (2)$

Put value of 'y' from equation (2) in equation (1)

$\Rightarrow \quad (x-3)^2 + (2x-6)^2 = 5$

$\Rightarrow \quad x^2 + 9 - 6x + 4x^2 + 36 - 24x - 5 = 0$

$\Rightarrow \quad 5x^2 - 30x + 40 = 0$

$\Rightarrow \quad 5(x^2 - 6x + 8) = 0$

$\Rightarrow \quad x^2 - 6x + 8 = 0$

$\Rightarrow \quad x^2 - 4x - 2x + 8 = 0$

$\Rightarrow \quad x(x-4) - 2(x-4) = 0$

$\Rightarrow \quad (x-4)(x-2) = 0$

$\Rightarrow$ Either $x - 4 = 0$ or $x - 2 = 0$

$\Rightarrow \quad x = 4$ or $x = 2$

Put these values in equation (2)

when $x = 4$ equation (2) $\Rightarrow$

$y = 2(4) - 6 = 8 - 6 = 2$

$\Rightarrow \quad (4, 2)$

when $x = 2$ equation (2) $\Rightarrow$

$y = 2(2) - 6 = 4 - 6 = -2$

$\Rightarrow \quad (2, -2)$

Hence the solution set $= \{(4, 2), (2, -2)\}$

Q.8 $(x+3)^2 + (y-1)^2 = 5, \quad x^2 + y^2 + 2x = 9$

Solution:

Let $x^2 + y^2 + 2x = 9 \quad \ldots\ldots\ldots (1)$

and $(x+3)^2 + (y-1)^2 = 5$

$\Rightarrow \quad x^2 + 9 + 6x + y^2 + 1 - 2y = 5$

$\Rightarrow \quad x^2 + y^2 + 6x - 2y + 10 = 5$

$\Rightarrow \quad x^2 + y^2 + 6x - 2y = -5 \quad \ldots\ldots\ldots (2)$

Subtracting equation (1) from equation (2)

$x^2 + y^2 + 6x - 2y = -5$

$$\begin{array}{r} x^2 + y^2 + 2x = 9 \\ - \quad - \quad - \qquad - \\ \hline 4x - 2y = -14 \end{array}$$

$2(2x - y) = -14$

$2x - y = -7$

$\Rightarrow \quad y = 2x + 7 \quad \ldots\ldots\ldots (3)$

Put value of y from equation (3) in equation (1)

$x^2 + (2x + 7)^2 + 2x = 9$

$\Rightarrow \quad x^2 + 4x^2 + 49 + 28x + 2x - 9 = 0$

$\Rightarrow \quad 5x^2 + 30x + 40 = 0$

$\Rightarrow \quad 5(x^2 + 6x + 8) = 0$

$\Rightarrow \quad x^2 + 6x + 8 = 0$

$\Rightarrow \quad x^2 + 4x + 2x + 8 = 0$

$\Rightarrow \quad x(x + 4) + 2(x + 4) = 0$

$\Rightarrow \quad (x + 4)(x + 2) = 0$

$\Rightarrow$ Either $x + 4 = 0$ or $x + 2 = 0$

$\Rightarrow \quad x = -4$ or $x = -2$

Put these values in equation (3)

when $x = -4$ equation (3) $\Rightarrow$

$y = 2(-4) + 7$

$y = -8 + 7$

$y = -1$

$\Rightarrow \quad (-4, -1)$

when $x = -2$ equation (3) $\Rightarrow$

$y = 2(-2) + 7$

$y = -7 + 7$

$y = 3$

$\Rightarrow \quad (-2, 3)$

Hence the solution set $= \{(-4, -1), (-2, 3)\}$

Q.9 $\mathbf{x^2 + (y + 1)^2 = 18, \quad (x + 2)^2 + y^2 = 21}$

Solution:

Given equations

$x^2 + (y + 1)^2 = 18$ and $(x + 2)^2 + y^2 = 21$

$x^2 + y^2 + 1 + 2y = 18$

$x^2 + y^2 + 2y - 17 = 0 \quad \ldots\ldots\ldots (1)$

$x^2 + 4 + 4x + y^2 - 21 = 0$

$x^2 + y^2 + 4x - 17 = 0 \quad \ldots\ldots\ldots (2)$

Subtracting equation (1) from equation (2)

$$\begin{array}{rcl} x^2 + y^2 + 4x - 17 & = & 0 \\ x^2 + y^2 - 17 + 2y & = & 0 \\ - \quad - \qquad\quad + \quad - & & \\ \hline 4x \qquad\quad - 2y & = & 0 \end{array}$$

$$2\,(2x - y) = 0$$

$$2x - y = 0$$

$$\Rightarrow \quad y = 2x \qquad \ldots\ldots\ldots (3)$$

Put value of y in equation (3) in equation (1)

$$\Rightarrow \quad x^2 + (2x)^2 + 2\,(2x) - 17 = 0$$

$$x^2 + 4x^2 + 4x - 17 = 0$$

$$5x^2 + 4x - 17 = 0$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4\,(5)\,(-17)}}{2\,(5)} = \frac{-4 \pm \sqrt{16 + 340}}{10}$$

$$= \frac{-4 \pm \sqrt{356}}{10} = \frac{-4 \pm 2\sqrt{89}}{10} = \frac{2\left(-2 \pm \sqrt{89}\right)}{10} = \frac{-2 \pm \sqrt{89}}{5}$$

$$\Rightarrow \quad x = \frac{-2 \pm \sqrt{89}}{5} \quad \text{or} \quad x = \frac{-2 - \sqrt{89}}{5}$$

Put these values in equation (3)

when $x = \dfrac{-2 \pm \sqrt{89}}{5}$ equation (3) $\Rightarrow$

$$y = 2\left(\frac{-2 + \sqrt{89}}{5}\right)$$

$$y = \frac{-4 + 2\sqrt{89}}{5}$$

$$\Rightarrow \quad \left(\frac{-2 + \sqrt{89}}{5}, \frac{-4 + 2\sqrt{89}}{5}\right)$$

when $x = \dfrac{-2 - \sqrt{89}}{5}$ equation (3) $\Rightarrow$

$$y = 2\left(\frac{-2 - \sqrt{89}}{5}\right)$$

$$y = \frac{-4 - 2\sqrt{89}}{5}$$

$$\Rightarrow \quad \left(\frac{-2 - \sqrt{89}}{5}, \frac{-4 - 2\sqrt{89}}{5}\right)$$

Hence the solution set $= \left\{\left(\dfrac{-2 + \sqrt{89}}{5}, \dfrac{-4 + 2\sqrt{89}}{5}\right), \left(\dfrac{-2 - \sqrt{89}}{5}, \dfrac{-4 - 2\sqrt{89}}{5}\right)\right\}$

Q.10 $x^2 + y^2 + 6x = 1, \quad x^2 + y^2 + 2(x + y) = 3$

Solution:

Given equations

$x^2 + y^2 + 6x = 1$ and $x^2 + y^2 + 2(x + y) = 3$

$x^2 + y^2 + 6x - 1 = 0 \quad \ldots\ldots\ldots (1)$ $\qquad x^2 + y^2 + 2x + 2y - 3 = 0 \quad \ldots\ldots\ldots (2)$

Subtracting equation (1) from equation (2)

$$\begin{array}{rrrrrl} x^2 & + y^2 & + 2x & + 2y & - 3 & = 0 \\ x^2 & + y^2 & + 6x & & - 1 & = 0 \\ - & - & - & & + & \\ \hline & & - 4x & + 2y & - 2 & = 0 \end{array}$$

$2(-2x + y - 1) = 0$

$-2x + y - 1 = 0$

$y = 2x + 1 \quad \ldots\ldots\ldots (3)$

Put $y = 2x + 1$ in equation (1)

$\Rightarrow \quad x^2 + (2x + 1)^2 + 6x - 1 = 0$

$\Rightarrow \quad x^2 + 4x^2 + 1 + 4x + 6x - 1 = 0$

$\Rightarrow \quad 5x^2 + 10x = 0$

$\Rightarrow \quad 5x(x + 2) = 0 \quad \Rightarrow \quad x(x + 2) = 0$

$\Rightarrow \quad$ Either $x = 0$ or $x + 2 = 0 \quad \Rightarrow \quad x = -2$

Put $x = 0$ and $x = -2$ in equation (3)

when $x = 0$ equation (3) $\Rightarrow$

$y = 1$

$\Rightarrow \quad (0, 1)$

when $x = -2$ equation (3) $\Rightarrow$

$y = 2(-2) + 1$

$y = -4 + 1 = -3$

$\Rightarrow \quad (-2, -3)$

Hence the solution set $= \{(0, 1), (-2, -3\}$

BOTH THE EQUATIONS ARE QUADRATIC IN TWO VARIABLES

The equations in this case are classified as:

(i) Both the equations contain only x^2 and y^2 terms

To solve these equations we eliminate one of x^2 or y^2, which yields two linear equations.

Example: $2x^2 + y^2 = 13, \quad x^2 + y^2 = 9$

Subtracting: $x^2 = 4 \quad \Rightarrow \quad x = \pm 2$

$\Rightarrow \quad$ two linear equations $x = 2$ and $x = -2$.

(ii) One of the equations is homogeneous in x and y

If every term in an equation is of the same degree then it is called homogeneous equation.

For example $x^2 - 3xy + 2y^2 = 0$ is homogeneous in x and y.

We shall factorize its L.H.S. and get two linear equations as

$x^2 - 3xy + 2y^2 = 0$ gives

$(x - y)(x - 2y) = 0$

$\Rightarrow$ $x - y = 0$ and $x - 2y = 0$ are the two linear equations.

(iii) Both equations are non–homogeneous

To solve these equations we eliminate the constants and then get a homogeneous equation.

For example: $y^2 - 2xy = 7$ .... (1) $2x^2 - xy = -3$ .... (2)

Multiply (1) by 3 and (2) by 7 and adding to eliminate constants, gives

$\Rightarrow$ $14x^2 - 13xy + 3y^2 = 0$ $\Rightarrow$ $(2x - y)(7x - 3y) = 0$ linear factors.

EXERCISE 4.9

Q.1 $2x^2 = 6 + 3y^2$; $3x^2 - 5y^2 = 7$.

Solution:

Given equations

$2x^2 = 6 + 3y^2$ .... (1)

$3x^2 - 5y^2 = 7$ .... (2)

$\Rightarrow$ $2x^2 - 3y^2 = 6$

Multiplying equation (1) by (3) and equation (2) by (2) and subtracting

$$\begin{array}{rcl} 6x^2 - 9y^2 &=& 18 \\ 6x^2 - 10y^2 &=& 14 \\ - \quad + & & - \\ \hline y^2 &=& -4 \end{array}$$

$y^2 = 4$

$\Rightarrow$ $y = \pm 2$