$\Rightarrow \quad 48r(3r-1) - 25(3r-1) = 0$

$\Rightarrow \quad (3r-1)(48r-25) = 0$

$\Rightarrow \quad r = \frac{1}{3} \quad \text{or} \quad r = \frac{25}{48}$

When $a = \frac{1}{3},\ r = \frac{1}{3}$ then

$$\frac{a}{r} = \frac{\frac{1}{3}}{\frac{1}{3}} = 1, \quad a = \frac{1}{3}, \quad ar = \frac{1}{3}\,\frac{1}{3} = \frac{1}{9}$$

When $a = \frac{1}{3}, \quad r = \frac{25}{48}$

$$\frac{a}{r} = \frac{\frac{1}{3}}{\frac{25}{48}} = \frac{16}{25}, \quad a = \frac{1}{3}, \quad ar = \frac{1}{3}\,\frac{25}{48} = \frac{25}{144}$$

So the required numbers are

$$1, \frac{1}{3}, \frac{1}{9} \quad \text{or} \quad \frac{16}{25}, \frac{1}{3}, \frac{25}{144}$$

FORMULAE FOR THE SUMS

$$\sum_{k=1}^{n} 1 = n$$

$$1 + 2 + 3 + \ldots\ldots + n = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

EXERCISE 6.11

Sum the following series upto n terms.

Q.1 $\quad \mathbf{1 \times 1 + 2 \times 4 + 3 \times 7 + \ldots\ldots}$

Solution:

a_n of $1, 2, 3, \ldots\ldots$ is n ¤ $a_n = a_1 + (n-1)d$

and a_n of $1, 4, 7, \ldots\ldots$ is $1 + (n-1)(+3) = 3n - 2$ so nth term of the given series is

$T_n = n(3n-2) = 3n^2 - 2n \quad \Rightarrow \quad T_k = 3k^2 - 2k$

Let S_n is the required sum then

$$S_n = T_1 + T_2 + \dots\dots + T_n$$
$$= \sum_{k=1}^{n} T_k$$
$$= \sum_{k=1}^{n} (3k^2 - 2k)$$
$$= \sum_{k=1}^{n} 3k^2 - \sum_{k=1}^{n} 2k$$
$$= 3\sum_{k=1}^{n} k^2 - 2\sum_{k=1}^{n} k$$
$$= 3\,\frac{n(n+1)(2n+1)}{6} - 2\,\frac{n(n+1)}{2}$$
$$= \frac{n(n+1)(2n+1)}{2} - \frac{2n(n+1)}{2}$$
$$= \frac{n(n+1)}{2}[2n+1-2]$$
$$= \frac{n(n+1)}{2}[2n-1]$$
$$= \frac{n(n+1)(2n-1)}{2}$$
$$= \frac{n}{2}(2n^2 - n + 2n - 1)$$
$$= \frac{n}{2}(2n^2 + n - 1)$$

Q.2 $1 \times 3 + 3 \times 6 + 5 \times 9 + \dots\dots$

Solution:

a_n of $1, 3, 5, \dots\dots$ is $1 + (n-1)(2) = 2n - 1$

a_n of $3, 6, 9, \dots\dots$ is $3 + (n-1)(3) = 3n$

so nth term of the given series is

$T_n = (2n-1)(3n) = 6n^2 - 3n \quad \Rightarrow \quad T_k = 6k^2 - 3k$

Let S_n be the required sum

$\Rightarrow \quad S_n = T_1 + T_2 + T_3 + \ldots\ldots + T_n$

$$= \sum_{k=1}^{n} T_k$$

$$= \sum_{k=1}^{n} (6k^2 - 3k)$$

$$= 6 \sum_{k=1}^{n} k^2 - 3 \sum_{k=1}^{n} k$$

$$= 6 \frac{n(n+1)(2n+1)}{6} - 3 \frac{n(n+1)}{2}$$

$$= \frac{2n(n+1)(2n+1)}{2} - \frac{3n(n+1)}{2}$$

$$= \frac{n(n+1)}{2} [4n + 2 - 3] = \frac{n(n+1)(4n-1)}{2}$$

Q.3 $1 \times 4 + 2 \times 7 + 3 \times 10 + \ldots\ldots\ldots\ldots$

Solution:

a_n of $1, 2, 3, \ldots\ldots$ is n

a_n of $4, 7, 10, \ldots\ldots$ is $4 + (n-1)(3) = 3n + 1$

nth term of the given series is

$T_n = n(3n+1) = 3n^2 + n$

and $T_k = k(3k+1) = 3k^2 + k$

Let S_n be the required sum

$\Rightarrow \quad S_n = T_1 + T_2 + \ldots\ldots + T_n$

$$= \sum_{k=1}^{n} T_k$$

$$= \sum_{k=1}^{n} (3k^2 + k)$$

$$= \sum_{k=1}^{n} 3k^2 + \sum_{k=1}^{n} k$$

$$= 3 \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k$$

$$= 3\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)(2n+1)}{2} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}[2n+1+1]$$

$$= \frac{n(n+1)(2n+2)}{2} = \frac{2n(n+1)(n+1)}{2} = n(n+1)^2$$

Q.4 $3 \times 5 + 5 \times 9 + 7 \times 13 + \dots\dots\dots$

Solution:

a_n of 3, 5, 7, ……. is $3 + (n-1)(2) = 2n + 1$

a_n of 5, 9, 13, ……. is $5 + (n-1)(4) = 4n + 1$

so nth term of the given series is

$T_n = (2n+1)(4n+1) = 8n^2 + 6n + 1$

and $T_k = 8k^2 + 6k + 1$

Let S_n be the required sum

$\Rightarrow$ $S_n = T_1 + T_2 + T_3 + \dots\dots + T_n$

$$= \sum_{k=1}^{n} T_k$$

$$= \sum_{k=1}^{n} (8k^2 + 6k + 1)$$

$$= \sum_{k=1}^{n} 8k^2 + \sum_{k=1}^{n} 6k + \sum_{k=1}^{n} 1$$

$$= 8\sum_{k=1}^{n} k^2 + 6\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$= 8\frac{n(n+1)(2n+1)}{6} + 6\frac{n(n+1)}{2} + n$$

$$= \frac{4n(n+1)(2n+1)}{3} + 3n(n+1) + n$$

$$= \frac{4n(n+1)(2n+1)}{3} + 3n^2 + 3n + n$$

$$= \frac{n(4n+4)(2n+1)}{3} + 3n^2 + 4n$$

$$= n\left[\frac{(4n+4)(2n+1)+9n+12}{3}\right]$$

$$= \frac{n}{3}[8n^2 + 12n + 4 + 9n + 12]$$

$$= \frac{n}{3}[8n^2 + 21n + 16]$$

Q.5 $\mathbf{1^2 + 3^2 + 5^2 + \ldots\ldots\ldots\ldots}$ **(Lahore Board 2006)**

Solution:

a_n of 1, 3, 5, ……. is $1 + (n-1)(2) = 2n - 1$

nth term of the given series is

$T_n = (2n-1)^2 = 4n^2 - 4n + 1$

and $T_k = 4k^2 - 4k + 1$

Let S_n be the required sum

$\Rightarrow$ $S_n = T_1 + T_2 + T_3 + \ldots\ldots + T_n$

$$= \sum_{k=1}^{n} T_k$$

$$= \sum_{k=1}^{n} (4k^2 - 4k + 1)$$

$$= \sum_{k=1}^{n} 4k^2 - \sum_{k=1}^{n} 4k + \sum_{k=1}^{n} 1$$

$$= 4\sum_{k=1}^{n} k^2 - 4\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$= 4\frac{n(n+1)(2n+1)}{6} - 4\frac{n(n+1)}{2} + n$$

$$= \frac{2n(n+1)(2n+1)}{3} - 2n(n+1) + n$$

$$= n\left[\frac{2n(n+1)(2n+1)}{3} - 2(n+1) + 1\right]$$

$$= n\frac{2(n+1)(2n+1) - 6(n+1) + 3}{3}$$

$$= \frac{n}{3}[4n^2 + 6n + 2 - 6n - 6 + 3]$$

$$= \frac{n}{3}[4n^2 - 1]$$

Q.6 $2^2 + 5^2 + 8^2 + ...$

Solution:

a_n of 2, 5, 8, ……. is $2 + (n-1)(3) = 3n - 1$

nth term of the given series is

$T_n = (3n-1)^2 = 9n^2 - 6n + 1$

and $T_k = 9k^2 - 6k + 1$

Let S_n be the required sum

$\Rightarrow$ $S_n = T_1 + T_2 + T_3 + ... + T_n$

$$= \sum_{k=1}^{n} T_k$$

$$= \sum_{k=1}^{n} 9k^2 - 6k + 1$$

$$= \sum_{k=1}^{n} 9k^2 - \sum_{k=1}^{n} 6k + \sum_{k=1}^{n} 1$$

$$= 9\sum_{k=1}^{n} k^2 - 6\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$= \frac{9n(n+1)(2n+1)}{6} - 6\frac{n(n+1)}{2} + n$$

$$= \frac{3n(n+1)(2n+1)}{2} - 3n(n+1) + n$$

$$= \frac{n}{2}[3(n+1)(2n+1) - 6(n+1) + 2]$$

$$= \frac{n}{2}[6n^2 + 9n + 3 - 6n - 6 + 2]$$

$$= \frac{n}{2}[6n^2 + 3n - 1]$$

Q.7 $2 \times 1^2 + 4 \times 2^2 + 6 \times 3^2 + ...$

Solution:

a_n of 2, 4, 6, ……. is $2 + (n-1)(2) = 2n$

a_n of 1, 2, 3, ……. is n

nth term of the given series is

$T_n = 2n(n)^2 = 2n^3$

and $T_k = 2k^3$

Let S_n be the required sum

$\Rightarrow S_n = T_1 + T_2 + T_3 + ... + T_n$

$$= \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} 2k^3$$

$$= 2 \sum_{k=1}^{n} k^3 = 2\left[\frac{n(n+1)}{2}\right]^2$$

$$= 2\,\frac{n^2(n+1)^2}{4} = \frac{n^2(n+1)^2}{2}$$

Q.8 $\mathbf{3 \times 2^2 + 5 \times 3^2 + 7 \times 4^2 + ...}$

Solution:

a_n of 3, 5, 7, ... is $3 + (n-1)(2) = 2n + 1$

a_n of 2, 3, 4, ... is $n + 1$

nth term of the given series is

$T_n = (2n+1)(n+1)^2 = 2n^3 + 5n^2 + 4n + 1$

and $T_k = 2k^3 + 5k^2 + 4k + 1$

Let S_n be the required sum

$\Rightarrow S_n = T_1 + T_2 + T_3 + ... + T_n$

$$= \sum_{k=1}^{n} T_k$$

$$= \sum_{k=1}^{n} (2k^3 + 5k^2 + 4k + 1)$$

$$= \sum_{k=1}^{n} 2k^3 + \sum_{k=1}^{n} 5k^2 + \sum_{k=1}^{n} 4k + \sum_{k=1}^{n} 1$$

$$= 2\sum_{k=1}^{n} k^3 + 5\sum_{k=1}^{n} k^2 + 4\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$= 2\left[\frac{n(n+1)}{2}\right]^2 + 5\,\frac{n(n+1)(2n+1)}{6} + 4\,\frac{n(n+1)}{2} + n$$

$$= \frac{n^2(n+1)^2}{2} + \frac{5n(n+1)(2n+1)}{6} + 2n(n+1) + n$$

$$= \frac{n^2(n^2+2n+1)}{2} + \frac{5n(2n^2+3n+1)}{6} + 2n(n+1) + n$$

$$= \frac{n}{6}\left[3n(n^2+2n+1) + 5(2n^2+3n+1) + 12(n+1) + 6\right]$$

$$= \frac{n}{6}\left[3n^3 + 6n^2 + 3n + 10n^2 + 15n + 5 + 12n + 12 + 6\right]$$

$$= \frac{n}{6}\left[3n^3 + 16n^2 + 30n + 23\right]$$

Q.9 $2 \times 4 \times 7 + 3 \times 6 \times 10 + 4 \times 8 \times 13 + \ldots\ldots$

Solution:

a_n of 2, 3, 4, ……. is $n + 1$

a_n of 4, 6, 8, ……. is $4 + (n-1)2 = 2n + 2$

a_n of 7, 10, 13, ……. is $7 + (n-1)(3) = 3n + 4$

nth term of the given series is

$T_n = (n+1)(2n+2)(3n+4) = 6n^3 + 20n^2 + 22n + 8$

and $T_k = 6k^3 + 20k^2 + 22k + 8$

Let S_n be the required sum

$\Rightarrow$ $S_n = T_1 + T_2 + T_3 + \ldots\ldots + T_n$

$$= \sum_{k=1}^{n} T_k$$

$$= \sum_{k=1}^{n} (6k^3 + 20k^2 + 22k + 8)$$

$$= \sum_{k=1}^{n} 6k^3 + \sum_{k=1}^{n} 20k^2 + \sum_{k=1}^{n} 22k + \sum_{k=1}^{n} 8$$

$$= 6\sum_{k=1}^{n} k^3 + 20\sum_{k=1}^{n} k^2 + 22\sum_{k=1}^{n} k + \sum_{k=1}^{n} 8$$

$$= 6\frac{n^2(n+1)^2}{4} + 20\frac{n(n+1)(2n+1)}{6} + 22\frac{n(n+1)}{2} + 8n$$

$$= \frac{3n^2(n^2+1+2n)}{2} + \frac{10n(n+1)(2n+1)}{3} + 11n(n+1) + 8n$$

$$= n\left[\frac{3n(n^2+1+2n)}{2} + \frac{10(2n^2+3n+1)}{3} + 11(n+1) + 8\right]$$

$$= n\left[\frac{9n(n^2+1+2n) + 20(2n^2+3n+1) + 66(n+1) + 48}{6}\right]$$

$$= \frac{n}{6}\left[9n^3 + 9n + 18n^2 + 40n^2 + 60n + 20 + 66n + 66 + 48\right]$$

$$= \frac{n}{6}\left[9n^3 + 58n^2 + 135n + 134\right]$$

Q.10 $1 \times 4 \times 6 + 4 \times 7 \times 10 + 7 \times 10 \times 14 + \dots\dots$

Solution:

a_n of 1, 4, 7, ……. is $1 + (n-1)^3 = 3n - 2$

a_n of 4, 7, 10, ……. is $4 + (n-1)(3) = 3n + 1$

a_n of 6, 10, 14, ……. is $6 + (n-1)(4) = 4n + 2$

nth term of the given series is

$T_n = (3n-2)(3n+1)(4n+2) = 36n^3 + 6n^2 - 14n - 4$

and $T_k = 36k^3 + 6k^2 - 14k - 4$

Let S_n be the required sum

$\Rightarrow$ $S_n = T_1 + T_2 + T_3 + \dots\dots + T_n$

$$= \sum_{k=1}^{n} T_k$$

$$= \sum_{k=1}^{n} (36k^3 + 6k^2 - 14k - 4)$$

$$= 36\sum_{k=1}^{n} k^3 + 6\sum_{k=1}^{n} k^2 - 14\sum_{k=1}^{n} k - 4\sum_{k=1}^{n} k$$

$$= 36\frac{n^2(n+1)^2}{4} + 6\frac{n(n+1)(2n+1)}{6} - 14\frac{n(n+1)}{2} - 4n$$

$$= 9n^2(n^2+1+2n) + n(2n^2+3n+1) - 7n(n+1) - 4n$$

$$= n\left[9n^3 + 9n + 18n^2 + 2n^2 + 3n + 1 - 7n - 7 - 4\right]$$

$$= n\left[9n^3 + 20n^2 + 5n - 10\right]$$

Q.11 1 + (1 + 2) + (1 + 2 + 3) +..........

Solution:

nth term of the given series is

$$T_n = 1 + 2 + 3 + \ldots\ldots. n = \frac{n(n+1)}{2} = \frac{1}{2}(n^2 + n)$$

and $T_k = \frac{1}{2}(k^2 + k)$

Let S_n be the required sum

$$\Rightarrow \quad S_n = T_1 + T_2 + T_3 + \ldots\ldots. + T_n$$

$$= \sum_{k=1}^{n} T_k$$

$$= \sum_{k=1}^{n} \frac{1}{2}(k^2 + k)$$

$$= \frac{1}{2}\sum_{k=1}^{n} k^2 + \frac{1}{2}\sum_{k=1}^{n} k$$

$$= \frac{1}{2}\,\frac{n(n+1)(2n+1)}{6} + \frac{1}{2}\,\frac{n(n+1)}{2}$$

$$= \frac{1}{2}\,\frac{n(n+1)}{2}\left[\frac{2n+1}{3} + 1\right]$$

$$= \frac{n(n+1)}{4}\left[\frac{2n+1+3}{3}\right]$$

$$= \frac{n(n+1)}{4}\left(\frac{2n+4}{3}\right) = \frac{n(n+1)(n+2)}{6}$$

Q.12 $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots\ldots$

Solution:

nth term of the given series is

$$T_n = 1^2 + 2^2 + 3^2 + \ldots\ldots. n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{1}{6}(2n^3 + 3n^2 + n)$$

and $\quad T_k = \frac{1}{2}(2k^3 + 3k^2 + k)$

Let S_n be the required sum

$$\Rightarrow \quad S_n = T_1 + T_2 + T_3 + \ldots\ldots + T_n$$

$$= \sum_{k=1}^{n} T_k$$

$$= \sum_{k=1}^{n} \frac{1}{6}(2k^3 + 3k^2 + k)$$

$$= \frac{1}{6}\left[2\sum_{k=1}^{n} k^3 + 3\sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k\right]$$

$$= \frac{1}{6}\left[2\frac{n^2(n+1)^2}{4} + 3\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{1}{6}\left[\frac{n^2(n+1)^2}{2} + \frac{n(n+1)(2n+1)}{2} + \frac{n(n+1)}{2}\right]$$

$$= \frac{n(n+1)}{12}\left[n(n+1) + (2n+1) + 1\right]$$

$$= \frac{n(n+1)}{12}\left[n^2 + n + 2n + 1 + 1\right]$$

$$= \frac{n(n+1)}{12}[n^2 + 3n + 2]$$

$$= \frac{n(n+1)}{12}[n^2 + 2n + n + 2]$$

$$= \frac{n(n+1)}{12}\left[(n+1)(n+2)\right]$$

$$= \frac{n(n+1)^2(n+2)}{12}$$

Q.13 $\quad 2 + (2 + 5) + (2 + 5 + 8) + \ldots\ldots$

Solution:

nth term of the given series is

$$T_n = 2 + 5 + 8 + \ldots\ldots$$

Using $S_n = \left[2a_1 + (n-1)d\right] \quad ¤ \; a_1 = 2, \; d = 3$

$$T_n = \frac{n}{2}\left[2(2) + (n-1)(3)\right]$$

$$= \frac{n}{2}[3n + 1] = \frac{1}{2}(3n^2 + n)$$

Let S_n be the required sum

$\Rightarrow$ $S_n = T_1 + T_2 + T_3 + \ldots\ldots + T_n$

$$= \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} \frac{1}{2}(3k^2 + k)$$

$$= \frac{1}{2}\left[3\sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k\right]$$

$$= \frac{1}{2}\left[2\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{2} + \frac{n(n+1)}{2}\right]$$

$$= \frac{1}{2}\,\frac{n(n+1)}{2}[2n+1+1]$$

$$= \frac{1}{4}n(n+1)(2n+2)$$

$$= \frac{n(n+1)(n+1)}{2} = \frac{n(n+1)^2}{2}$$

Q.14 Sum the series.

(i) $\mathbf{1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots\ (2n-1)^2 - (2n)^2}$

(ii) $\mathbf{1^2 - 3^2 + 5^2 - 7^2 + \ldots\ldots\ (4n-3)^2 - (4n-1)^2}$

(iii) $\mathbf{\frac{1^2}{1} + \frac{1^2 + 2^2}{2} + \frac{1^2 + 2^2 + 3^2}{3} + \ldots\ldots + \text{to } n \text{ terms}}$

Solution:

(i) $\mathbf{1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots\ (2n-1)^2 - (2n)^2}$

As $T_n = (2n-1)^2 - (2n)^2 = 4n^2 - 4n + 1 - 4n^2$

$= -4n + 1$

$T_k = -4k + 1$

Let S_n be the required sum

$\Rightarrow$ $S_n = T_1 + T_2 + T_3 + \ldots\ldots + T_n$

$$= \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} (-4k + 1)$$

$$= -4\sum_{k=1}^{n} + \sum_{k=1}^{n} 1$$

$$= -4\frac{n(n+1)}{2} + n = -2n(n+1) + n$$

$$= -2n^2 - 2n + n = -2n^2 - n = -n(2n+1)$$

(ii) $\mathbf{1^2 - 3^2 + 5^2 - 7^2 + \ldots\ldots\ (4n-3)^2 - (4n-1)^2}$

$$T_n = (4n-3)^2 - (4n-1)^2 = 16n^2 - 24n + 9 - 16n^2 + 8n - 1$$
$$= -16n + 8$$
$$T_k = -16k + 8$$

Let S_n be the required sum

$$\Rightarrow \quad S_n = T_1 + T_2 + T_3 + \ldots\ldots + T_n$$
$$= \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} (-16k + 8)$$
$$= -16 \sum_{k=1}^{n} k + 8 \sum_{k=1}^{n} 1$$
$$= -16\frac{n(n+1)}{2} + 8n$$
$$= -8n(n+1) + 8n = -8n^2 - 8n + 8n = -8n^2$$

(iii) $\mathbf{\dfrac{1^2}{1} + \dfrac{1^2+2^2}{2} + \dfrac{1^2+2^2+3^2}{3} + \ldots\ldots}$ **to n terms**

$$T_n = \frac{1}{n}[1^2 + 2^2 + 3^2 + \ldots\ldots + n^2]$$
$$= \frac{1}{n}\left[\frac{n(n+1)(2n+1)}{6}\right] = \frac{1}{6}(2n^2 + 3n + 1)$$
$$T_k = \frac{1}{6}(2k^2 + 3k + 1)$$

Let S_n be the required sum

$$S_n = T_1 + T_2 + T_3 + \ldots\ldots + T_n$$
$$= \sum_{k=1}^{n} T_k = \frac{1}{6}\sum_{k=1}^{n} (2k^2 + 3k + 1)$$
$$= \frac{1}{6}\left[2\sum_{k=1}^{n} k^2 + 3\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1\right]$$
$$= \frac{1}{6}\left[2\frac{n(n+1)(2n+1)}{6} + 3\frac{n(n+1)}{2} + n\right]$$
$$= \frac{1}{6}\left[\frac{2n(n+1)(2n+1) + 9n(n+1) + 6n}{6}\right]$$
$$= \frac{n}{36}\left[2(2n^2 + 3n + 1) + 9(n+1) + 6\right]$$
$$= \frac{n}{36}[4n^2 + 15n + 17]$$

Q.15 Find the sum to n terms of the series whose nth terms are given

(i) $3n^2 + n + 1$ **(ii) $n^2 + 4n + 1$**

Solution:

(i) $3n^2 + n + 1$

$T_n = 3n^2 + n + 1$

$T_k = 3k^2 + k + 1$

Let S_n be the required sum

$\Rightarrow$ $S_n = T_1 + T_2 + T_3 + \ldots\ldots + T_n$

$$= \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} (3k^2 + k + 1)$$

$$= 3\sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$= 3\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} + n$$

$$= \frac{n(n+1)(2n+1)}{2} + \frac{n(n+1)}{2} + \frac{2n}{2}$$

$$= \frac{n}{2}\left[(n+1)(2n+1) + n + 1 + 2\right]$$

$$= \frac{n}{2}\left[2n^2 + n + 2n + 1 + n + 3\right]$$

$$= \frac{n}{2}\left[2n^2 + 4n + 4\right] = n(n^2 + 2n + 2)$$

(ii) $n^2 + 4n + 1$

As $T_n = n^2 + 4n + 1$

$T_k = k^2 + 4k + 1$

Let S_n be the required sum

$\Rightarrow$ $S_n = T_1 + T_2 + T_3 + \ldots\ldots + T_n$

$$= \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} (k^2 + 4k + 1)$$

$$= \sum_{k=1}^{n} k^2 + 4\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$= \frac{n(n+1)(2n+1)}{6} + 4\frac{n(n+1)}{2} + n$$

$$= \frac{n(n+1)(2n+1)}{6} + 2n(n+1) + n$$

$$= \frac{n}{6}\left[(n+1)(2n+1) + 12(n+1) + 6\right]$$

$$= \frac{n}{6}[2n^2 + 3n + 1 + 12n + 12 + 6]$$

$$= \frac{n}{6}[2n^2 + 15n + 19]$$

Q.16 Given nth terms of the series. Find the sum to 2n terms.

(i) $\mathbf{3n^2 + 2n + 1}$ **(ii)** $\mathbf{n^3 + 2n + 3}$

Solution:

(i) $\mathbf{3n^2 + 2n + 1}$

As $T_n = 3n^2 + 2n + 1$

$T_k = 3k^2 + 2k + 1$

Let S_n be the sum of n terms

Then $S_n = T_1 + T_2 + T_3 + \ldots\ldots + T_n$

$$= \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} (3k^2 + 2k + 1)$$

$$= 3\sum_{k=1}^{n} k^2 + 2\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$= 3\frac{n(n+1)(2n+1)}{6} + 2\frac{n(n+1)}{2} + n$$

$$= \frac{n(n+1)(2n+1)}{2} + \frac{2n(n+1)}{2} + n$$

$$= \frac{n}{2}\left[(n+1)(2n+1) + 2(n+1) + 2\right]$$

$$= \frac{n}{2}\left[2n^2 + 3n + 1 + 2n + 2 + 2\right]$$

$$= \frac{n}{2}\left[2n^2 + 5n + 5\right] \quad \ldots\ldots\ldots (1)$$

To find sum up to $2n$ terms

$\Rightarrow$ put $n = 2n$ in (1)

$$S_{2n} = \frac{2n}{2}\left[2(2n)^2 + 5(2n) + 5\right]$$

$$= n[8n^2 + 10n + 5]$$

(ii) $n^3 + 2n + 3$

As $T_n = n^3 + 2n + 3$

$T_k = k^3 + 2k + 3$

Let S_n be the sum of n terms

$\Rightarrow$ $S_n = T_1 + T_2 + T_3 + \ldots\ldots + T_n$

$$= \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} (k^3 + 2k + 3)$$

$$= \sum_{k=1}^{n} k^3 + 2\sum_{k=1}^{n} k + 3\sum_{k=1}^{n} 1$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + 2\,\frac{n(n+1)}{2} + 3n$$

$$= \frac{n^2(n+1)^2}{4} + n(n+1) + 3n$$

$$= \frac{n}{4}\left[n(n^2 + 1 + 2n) + 4(n+1) + 12\right]$$

$$= \frac{n}{4}\left[n^3 + n + 2n^2 + 4n + 4 + 12\right]$$

$$= \frac{n}{4}[n^3 + 2n^2 + 5n + 16] \qquad \ldots\ldots\ldots (1)$$

To find sum of $2n$ terms of the given series

Put $n = 2n$ in (1), we get

$$S_{2n} = \frac{2n}{4}\left[(2n)^3 + 2(2n)^2 + 5(2n) + 16\right]$$

$$= \frac{n}{2}\left[8n^3 + 8n^2 + 10n + 16\right]$$

$$= n(4n^3 + 4n^2 + 5n + 8)$$