(IV) Circle

Equations of tangent in different forms

(i) Point form:

Equation of tangent to the circle at (x_1, y_1) is $xx_1 + yy_1 = a^2$

(ii) Slope form:

Equation of tangent is terms of slope 'm' is $y = mx \pm a\sqrt{1+m^2}$

$(\therefore \quad c^2 = a^2(1+m^2))$

Equations of Normal

(i) Parobola $y^2 = 4ax$ is at (x_1, y_1)

$$y - y_1 = \frac{-y_1}{2a}(x - x_1)$$

(ii) Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at (x_1, y_1) is

$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$$

(iii) Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at (x_1, y_1) is

$$\frac{xa^2}{x_1} + \frac{yb^2}{y_1} = a^2 + b^2$$

EXERCISE 6.7

Q.1: Find equations of tangent and normal to each of the following at the indicated point.

(i) $y^2 = 4ax$ at $(at^2, 2at)$

Solution:

Equation of tangent at $(at^2, 2at)$ is

$$yy_1 = 2a(x + x_1)$$
$$y(2at) = 2a(x + at^2)$$
$$2ayt = 2ax + 2a^2t^2$$
$$2ayt = 2a(x + at^2)$$
$$yt = x + at^2$$

And equation of normal at $(at^2, 2at)$ is

$$y - y_1 = \frac{-y_1}{2a}(x - x_1)$$
$$y - 2at = \frac{-2at}{2a}(x - at^2)$$

$$y - 2at = -tx + at^3$$
$$tx + y - 2at - at^3 = 0$$

(ii) $\mathbf{\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1}$ **at $(\mathbf{a \cos\theta, b \sin\theta})$**

Solution:

Equation of tangent $\dfrac{xx_1}{a^2} + \dfrac{yy_1}{b^2} = 1$

$$\frac{x(a\cos\theta)}{a^2} + \frac{y(b\sin\theta)}{b^2} = 1$$
$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$

and equation of normal

$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$$
$$\frac{a^2x}{a\cos\theta} - \frac{b^2y}{b\sin\theta} = a^2 - b^2$$
$$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2 \quad ax\sec\theta - by\,\mathrm{cosec}\theta = a^2 - b^2$$

(iii) $\mathbf{\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1}$ **at $(\mathbf{a\sec\theta, b\tan\theta})$**

Solution:

Equation of tangent

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$
$$\frac{x\,a\sec\theta}{a^2} - \frac{y\,b\tan\theta}{b^2} = 1$$
$$\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$$

And equation of normal

$$\frac{xa^2}{x_1} + \frac{yb^2}{y_1} = a^2 + b^2$$
$$\frac{xa^2}{a\sec\theta} + \frac{yb^2}{b\tan\theta} = a^2 + b^2$$
$$\frac{xa}{\sec\theta} + \frac{yb}{\tan\theta} = a^2 + b^2$$

OR $\quad x\,a\cos\theta + y\,b\cot\theta = a^2 + b^2$

Q.2: Write equation of the tangent to the given conic at the indicated point

(i) $3x^2 = -16y$ at the points whose ordinate is -3

Solution:

$3x^2 = -16y$ .... (1)

Put $y = -3$ in

$3x^2 = -16(-3)$

$3x^2 = 48 \implies x^2 = 16$

$\implies x = \pm 4$

Hence points are

$(4, -3)$ & $(-4, -3)$

Now diff. (1) w.r.t 'x'

$$6x = -16\frac{dy}{dx}$$

$$\frac{6x}{-16} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-3}{8}x$$

$$m = \text{Slope} = \frac{dy}{dx}\Big|_{(4,-3)} = \frac{-3}{8}(4) = \frac{-3}{2}$$

$$\text{Also} \quad m = \frac{dy}{dx}\Big|_{(-4,-3)} = \frac{-3}{8}(4) = \frac{3}{2}$$

$\therefore$ Equation of tangent at $(4, -3)$ is

$y - y_1 = m(x - x_1)$

$y + 3 = \frac{-3}{2}(x - 4)$

$2y + 6 = -3x + 12$

$3x + 2y = 6$

$3x + 2y - 6 = 0$

Equation of tangent at $(-4, -3)$ is

$y - y_1 = m(x - x_1)$

$y + 3 = \frac{+3}{2}(x + 4)$

$2y + 6 = 3x + 12$

$3x - 2y = -6$

$3x - 2y + 6 = 0$

(ii) $3x^2 - 7y^2 = 20$ at points where $y = -1$.

Solution:

$3x^2 - 7y^2 = 20$ .... (1)

Put $y = -1$ in (1)

$3x^2 - 7(-1)^2 = 20$

$3x^2 = 20 + 7$

$3x^2 = 27 \implies x^2 = 9 \implies x = \pm 3$

Thus the required points on the conic are $(3, -1)$ & $(-3, -1)$

Now diff (1) w.r.t. 'x' we have

$$6x - 14y\frac{dy}{dx} = 0$$

$$14\frac{dy}{dx} = 6x$$

$$\frac{dy}{dx} = \frac{6x}{14y} = \frac{3x}{7y}$$

Now $m = \text{Slope} = \frac{dy}{dx}\Big|_{(3,-1)} = \frac{9}{-7}$ Also $m = \frac{dy}{dx}\Big|_{(-3,-1)} = \frac{9}{7}$

Therefore equation of tangent at (3, – 1) is

$$y - y_1 = m(x - x_1)$$

$$y + 1 = \frac{-9}{7}(x - 3)$$

$$7y + 7 = -9x + 27$$

$$9x + 7y = 20$$

$$9x + 7y - 20 = 0 \quad \text{Ans}$$

Equation of tangent at (– 3, –1)

$$y - y_1 = m(x - x_1)$$

$$y + 1 = \frac{9}{7}(x + 3)$$

$$7y + 7 = 9x + 27$$

$$9x - 7y = -20$$

$$9x - 7y + 20 = 0 \quad \text{Ans}$$

(iii) $3x^2 - 7y^2 + 2x - y - 48 = 0$, at point where $x = 4$

Solution:

$$3x^2 - 7y^2 + 2x - y - 48 = 0 \quad \text{..... (1)}$$

Put $x = 4$ in (1)

$$3(4)^2 - 7y^2 + 2(4) - y - 48 = 0$$

$$48 - 7y^2 + 8 - y - 48 = 0$$

$$\Rightarrow \quad -7y^2 - y + 8 = 0 \quad \Rightarrow \quad 7y^2 + y - 8 = 0$$

$$7y^2 + 8y - 7y - 8 = 0$$

$$y(7y + 8) - 1(7y + 8) = 0$$

$$(7y + 8)(y - 1) = 0$$

Either

$$7y + 8 = 0 \quad , \quad y - 1 = 0$$

$$y = \frac{-8}{7} \quad , \quad y = 1$$

Therefore, required points on the conic are $(4, \frac{-8}{7})$ & $(4, 1)$

Now diff. (1) w.r.t. 'x' $\quad 6x - 14y\frac{dy}{dx} + 2 - \frac{dy}{dx} = 0$

$$(-14y - 1)\frac{dy}{dx} = -6x - 2$$

$$\frac{dy}{dx} = \frac{6x+2}{14y+1}$$

$$m = \frac{dy}{dx}\Big|_{(4,\,1)} = \frac{6(4)+2}{14(1)+1} = \frac{26}{15} \quad \text{Also } m = \frac{dy}{dx}\Big|_{(4,\,-\frac{8}{7})} = \frac{6(4)+2}{14\left(\frac{-8}{7}\right)+1} = \frac{26}{-15}$$

Equation of tangent at (4, 1) is

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \frac{26}{15}(x-4)$$
$$15y - 15 = 26x - 104$$
$$26x - 15y - 89 = 0 \quad \text{Ans}$$

Equation of tangent at $(4, \frac{-8}{7})$ is

$$y - y_1 = m(x - x_1)$$
$$y + \frac{8}{7} = \frac{-26}{15}(x-4)$$
$$105y - 120 = -182x + 728$$
$$182x + 105y - 608 = 0 \quad \text{Ans}$$

Q.3: Find equations of the tangents to each of the following through the given point

(i) $x^2 + y^2 = 25$, through (7, – 1)

Solution:

$$x^2 + y^2 = 25 \Rightarrow r = 5$$

We know that condition of tangency for the circle is

$$c^2 = r^2(1+m^2)$$
$$c^2 = 25(1+m^2)$$
$$\Rightarrow c = \pm 5\sqrt{1+m^2}$$

Let the required equation of tangent be

$$y = mx + c \quad .... (1) \quad \text{Putting value of C in (1)}$$
$$y = mx \pm 5\sqrt{1+m^2} \quad .... (2)$$

Since tangent line passes through point (7, – 1), therefore

$$-1 = 7m \pm 5\sqrt{1+m^2}$$
$$\pm 5\sqrt{1+m^2} = 7m + 1 \quad \text{Squaring}$$
$$25(1+m^2) = (7m+1)^2$$
$$25 + 25m^2 = 49m^2 + 1 + 14m$$
$$-24m^2 - 14m + 24 = 0$$
$$12m^2 + 7m - 12 = 0$$
$$12m^2 + 16m - 9m - 12 = 0$$
$$4m(3m+4) - 3(3m+4) = 0$$
$$(3m+4)(4m-3) = 0$$
$$m = \frac{-4}{3} \qquad m = \frac{3}{4}$$

with $m = \frac{-4}{3}$ ...... (2) becomes

$$y = -\frac{4}{3}x \pm 5\sqrt{1+\frac{16}{9}}$$

$$= -\frac{4}{3}x \pm 5\ \frac{5}{3}$$

$$3y = -4x \pm 25$$

$$4x + 3y \pm 25 = 0$$

with $m = \frac{3}{4}$ ...... (2) becomes

$$y = \frac{3x}{4} \pm 5\sqrt{1+\frac{9}{16}}$$

$$= \frac{3x}{4} \pm \frac{25}{4}$$

$$4y = 3x \pm 25$$

$$3x - 4y \pm 25 = 0$$

(ii) $\mathbf{y^2 = 12x}$ **through (1, 4)**

Solution:

$y^2 = 12\,x$

As standard form is

$y^2 = 4ax$

$4a = 12 \quad \Rightarrow \quad a = 3$

Let $y = mx + c$ ..... (1) be the required equation of tangent. For Parabola we know that condition of tangency is $c = \frac{a}{m} = \frac{3}{m}$ put in (1)

$$y = mx + \frac{3}{m} \text{ .... (2)}$$

Since tangent line passes through point (1, 4)

So (2) becomes

$$4 = m + \frac{3}{m} \quad \Rightarrow \quad 4m = m^2 + 3$$

$$m^2 - 4m + 3 = 0$$

$$(m-1)(m-3) = 0$$

$m = 1 \quad , \quad m = 3$ Put in (1)

$y = x + 3 \quad \& \quad y = 3x + \frac{3}{m}$

$x - y + 3 = 0 \qquad y = 3x + \frac{3}{3}$

$$y = 3x + 1$$

$$3x - y + 1 = 0 \quad \text{Ans}$$

(iii) $\mathbf{x^2 - 2y^2 = 2}$ **through (1, – 2)**

Solution:

$$x^2 - 2y^2 = 2$$

$$\frac{x^2}{2} - \frac{y^2}{1} = 1$$

$\Rightarrow \quad a^2 = 2 \quad , \quad b^2 = 1$

For hyperbola, we know that condition of tangent is

$$c^2 = a^2m^2 - b^2$$

$$\Rightarrow \quad c^2 = 2m^2 - 1 \quad \Rightarrow \quad c = \pm\sqrt{2m^2-1}$$

Let $y = mx + c$ be tangent to the given hyperbola then $y = mx \pm \sqrt{2m^2-1}$ (1)

Since (1) passes through (1, – 2) (1) becomes

$$-2 = m \pm \sqrt{2m^2-1}$$

$$-2 - m = \pm\sqrt{2m^2-1} \qquad \text{Squaring}$$

$$4 + m^2 + 4m = 2m^2 - 1$$

$$2m^2 - 1 - m^2 - 4m - 4 = 0$$

$$m^2 - 4m - 5 = 0$$

$$\Rightarrow \quad (m-5)(m+1) = 0$$

$$\Rightarrow \quad m = 5, \quad m = -1$$

Putting values of m in (1) we get

$$y = 5x \pm \sqrt{2(25)-1} \quad , \qquad y = -x \pm \sqrt{2-1}$$

$$y = 5x \pm \sqrt{49} \quad , \qquad = -x \pm 1$$

$$y = 5x \pm 7 \quad , \qquad y + x \pm 1 = 0$$

$5x - y \pm 7 = 0$ Ans

Q.4: Find equations of normal to the Parabola $y^2 = 8x$, which are parallel to the line $2x + 3y = 10$.

Solution:

$y^2 = 8x$ ...... (1) $\qquad 2x + 3y = 10$ .... (2)

Diff. (1) w.r.t. 'x' $\qquad m_2$ = Slope of line

$$2y\frac{dy}{dx} = 8 \qquad = \frac{-\text{coeff of } x}{\text{coeff of } y}$$

$$\frac{dy}{dx} = \frac{8}{2y} = \frac{4}{y} \qquad = -\frac{2}{3}$$

$$m_1 = \frac{dy}{dx} = \frac{4}{y}$$

$$m_1 = \text{Slope of normal} = \frac{-y}{4}$$

Since normal and given line are Parallel

$$m_1 = m_2$$

$$\frac{-y}{4} = \frac{-2}{3} \quad \Rightarrow \quad y = \frac{8}{3} \qquad \text{Put in (1)}$$

$$\left(\frac{8}{3}\right)^2 = 8x$$

$$\frac{64}{9 \times 8} = x \qquad \Rightarrow \qquad x = \frac{8}{9}$$

Required point $\left(\frac{8}{9}, \frac{8}{3}\right)$

with $y = \frac{8}{3}$, m_1 become

$$m_1 = -\frac{8}{3} \times \frac{1}{4} = \frac{-2}{3}$$

Required equation of normal at $\left(\frac{8}{9}, \frac{8}{3}\right)$ is

$$y - y_1 = m(x - x_1)$$
$$y - \frac{8}{3} = \frac{-2}{3}\left(x - \frac{8}{9}\right)$$
$$3y - 8 = -2\left(\frac{9x - 8}{9}\right)$$
$$27y - 72 = -18x + 16$$
$$18x + 27y - 88 = 0$$

Q.5: Find equations of tangents to the ellipse $\frac{x^2}{4} + y^2 = 1$, which are parallel to the line $2x - 4y + 5 = 0$.

Solution:

$$\frac{x^2}{4} + \frac{y^2}{1} = 1 \qquad\qquad 2x - 4y + 5 = 0$$

$$\Rightarrow \quad a^2 = 4 \quad , \quad b^2 = 1 \qquad\qquad m = \frac{-\text{coeff of } x}{\text{coeff of } y} = \frac{-2}{-4} = \frac{1}{2}$$

We know that condition of tangent for ellipse is

$$c^2 = a^2m^2 + b^2$$
$$c^2 = 4m^2 + 1$$
$$c = \pm\sqrt{4m^2 + 1}$$

Since tangent is parallel to line $2x - 4y + 5 = 0$

$\therefore$ Slope is also $m = \frac{1}{2}$

$$c = \pm\sqrt{4\frac{1}{4} + 1} = \pm\sqrt{2}$$

Let the equation of required tangent by

$$y = mx + c$$

$$y = \frac{1}{2}x \pm \sqrt{2}$$
$$2y = x \pm 2\sqrt{2}$$
$$x - 2y \pm 2\sqrt{2} = 0 \quad \text{Ans}$$

Q.6: Find equations of the tangents to the conics $9x^2 - 4y^2 = 36$ Parallel to $5x - 2y + 7 = 0$.

Solution:

$$9x^2 - 4y^2 = 36$$
$$\frac{x^2}{4} - \frac{y^2}{9} = 1 \quad \text{(Dividing by 36)}$$
$$\Rightarrow \quad a^2 = 4 \quad , \quad b^2 = 9$$
$$5x - 2y + 7 = 0$$
$$m = \text{slope of line} = \frac{5}{2}$$

For hyperbola, we know that

$$c^2 = a^2m^2 - b^2$$
$$c^2 = 4m^2 - 9$$

Since tangent and given line are parallel so their slopes are same. Thus $m = \frac{5}{2}$

$$c^2 = 4\left(\frac{25}{4}\right) - 9 \qquad c^2 = 16 \qquad \Rightarrow \qquad c = \pm 4$$

Let $y = mx + c$ be the required equation of the tangent then $y = \frac{5}{2}x \pm 4$

$$2y = 5x \pm 8$$
$$5x - 2y \pm 8 = 0 \quad \text{Ans.}$$

Q.7: Find equations of common tangents to the given conics.

(i) $x^2 = 80y$ & $x^2 + y^2 = 81$

Solution:

$$x^2 = 80y \quad .... (1) \qquad x^2 + y^2 = 81 \quad ...... (2)$$

Let $y = mx + c$ (3) be the required common tangent. Let a be radius of circle then (2) becomes $a^2 = 81$ Put in (1)

$$x^2 = 80 \ (mx + c)$$
$$x^2 - 80\,mx - 80c = 0$$

For equal roots, we know that Disc = 0

$$b^2 - 4ac = 0$$
$$(-80\,m)^2 - 4(1)(-80\,c) = 0$$
$$80(80\,m^2 + 4c) = 0$$

$$80m^2 + 4c = 0 \qquad c = -20m^2$$

Condition of tangency for circle is $c^2 = a^2(1+m^2)$ (4)

$$(-20m^2)^2 = 81(1+m^2)$$

$$400m^4 = 81 + 81m^2$$

$$400m^4 - 81m^2 - 81 = 0$$

By Quadratic Formula

$$m^2 = \frac{-(-81) \pm \sqrt{(-81)^2 - 4(400)(-81)}}{2(400)}$$

$$= \frac{81 \pm \sqrt{136161}}{800} = \frac{9}{16}$$

$$m = \pm\frac{3}{4}$$

$$\therefore \quad c = -20\left(\frac{9}{16}\right) = \frac{-45}{4}$$

Putting values of m & c in $y = mx + c$

$$y = \pm\frac{3}{4}x - \frac{45}{4}$$

$$4y = \pm 3x - 45$$

$$\pm 3x - 4y - 45 = 0 \quad \text{Ans.}$$

(ii) $\mathbf{y^2 = 16x \quad \& \quad x^2 = 2y}$

Solution:

$$y^2 = 16x \quad \ldots (1) \qquad x^2 = 2y \quad \ldots (2)$$

$$y^2 = 4ax$$

$$4a = 16$$

$$\boxed{a = 4}$$

We know that condition of tangency for Parabola is $c = \dfrac{a}{m}$

$$c = \frac{4}{m}$$

Let $y = mx + c \quad \ldots\ldots (3)$ be required tangent

then $y = mx + \dfrac{4}{m}$ Putting value of y in (2)

$$x^2 = 2\left(mx + \frac{4}{m}\right) \quad \Rightarrow \quad mx^2 = 2m^2x + 8$$

$$mx^2 - 2m^2x - 8 = 0$$

For equal roots, we know that Disc = 0

i.e; $b^2 - 4ac = 0$

$$(-2m^2)^2 - 4(m)(-8) = 0$$

$$4m^4 + 32m = 0$$

$$4m(m^3 + 8) = 0$$

$$m = 0 \quad , \quad m^3 = -8 \quad , \quad m = -2$$

Equation of tangent is

$$y = mx + c$$

$$y = -2x + \frac{4}{-2}$$

$$y = -2x - 2$$

$$\boxed{2x + y + 2 = 0} \qquad \text{Ans.}$$

Q.8: Find the points of intersection of the given conics.

(i) $\dfrac{x^2}{18} + \dfrac{y^2}{8} = 1 \quad \& \quad \dfrac{x^2}{3} - \dfrac{y^2}{3} = 1$

Solution:

$$\frac{x^2}{18} + \frac{y^2}{8} = 1 \qquad \& \qquad \frac{x^2}{3} - \frac{y^2}{3} = 1$$

$$8x^2 + 18y^2 = 144 \qquad x^2 - y^2 = 3 \quad \text{........ (2)}$$

$$4x^2 + 9y^2 = 72 \text{ .... (1)} \qquad \text{(Dividing by 2)}$$

Multiplying Eq. (2) by 9 & add in (1)

$$9x^2 - 9y^2 = 27$$

$$\underline{4x^2 + 9y^2 = 72}$$

$$13x^2 = 99$$

$$x^2 = \frac{99}{13} \quad \Rightarrow \quad x = \pm\sqrt{\frac{99}{13}}$$

Put in (2)

$$\frac{99}{13} - y^2 = 3$$

$$\frac{99}{13} - 3 = y^2$$

$$\frac{99 - 39}{13} = y^2$$

$$y^2 = \frac{60}{13} \quad \Rightarrow \quad y = \pm\sqrt{\frac{60}{13}}$$

Points of intersection are $\left(\pm\sqrt{\frac{99}{13}}\ ,\ \ \pm\sqrt{\frac{60}{13}}\right)$ Ans.

(ii) $\mathbf{x^2 + y^2 = 8}$ **&** $\mathbf{x^2 - y^2 = 1}$

Solution:

$x^2 + y^2 = 8$ .... (1) $\qquad x^2 - y^2 = 1$ .... (2)

Adding (1) & (2)

$$x^2 + y^2 = 8$$
$$\underline{x^2 - y^2 = 1}$$
$$2x^2 = 9 \quad \Rightarrow \quad x^2 = \frac{9}{2} \quad \Rightarrow \quad x = \pm\frac{3}{\sqrt{2}}$$

Put in (1) $\quad \frac{9}{2} + y^2 = 8$

$$y^2 = 8 - \frac{9}{2}$$
$$y^2 = \frac{16-9}{2} = \frac{7}{2}$$
$$y = \pm\sqrt{\frac{7}{2}}$$

Hence points of intersection are $\left(\pm\frac{3}{\sqrt{2}}, \pm\sqrt{\frac{7}{2}}\right)$ Ans

(iii) $\mathbf{3x^2 - 4y^2 = 12}$ **&** $\mathbf{3y^2 - 2x^2 = 7}$

Solution:

$3x^2 - 4y^2 = 12$ .... (1)

$3y^2 - 2x^2 = 7$ .... (2)

Multiplying equation (1) by (2) & (2) by 3 and adding

$$6x^2 - 8y^2 = 24$$
$$\underline{-6x^2 + 9y^2 = 21}$$
$$y^2 = 45 \quad \Rightarrow \quad y = \pm\sqrt{45}$$

Put in (2)

$$-2x^2 + 3(45) = 7$$
$$-2x^2 + 135 = 7$$
$$135 - 7 = 2x^2$$
$$128 = 2x^2$$
$$x^2 = 64 \quad \Rightarrow \quad x = \pm 8$$

Hence points of intersection are

$(\pm 8\ ,\ \ \pm\sqrt{45})$ Ans.

(iv) $\mathbf{3x^2 + 5y^2 = 60}$ **and** $\mathbf{9x^2 + y^2 = 124}$

Solution:

$3x^2 + 5y^2 = 60$ .... (1) $\quad 9x^2 + y^2 = 124$ .... (2)

Multiplying (1) by (3) & Subtracting from (2)

$$\begin{array}{rcl} 9x^2 + \quad y^2 & = & 124 \\ -9x^2 \pm 15y^2 & = & -180 \\ \hline -14y^2 & = & -56 \end{array}$$

$y^2 = 4 \qquad \Rightarrow \qquad y = \pm 2$

Put in (1)

$9x^2 + 4 = 124$

$9x^2 = 120$

$x^2 = \dfrac{120}{9} = \dfrac{40}{3} \qquad x = \pm\sqrt{\dfrac{40}{3}}$

Hence points of intersection are $\left(\pm\sqrt{\dfrac{40}{3}}\ \pm 2\right)$

EXERCISE 6.8

Q.1: Find an equation of each of the following with respect to new parallel axes obtained by shifting the origin to the indicated point.

Remember

$x = X + h$

$y = Y + k$

Solution:

(i) $x^2 + 16y - 16 = 0$ .... (1) $\quad O'(0, 1) \Rightarrow h = 0, k = 1$

We know that equations of transformation are

$x = X + h \quad , \quad y = Y + k$

$x = X + 0 \quad , \quad y = Y + 1 \qquad$ Put in (1)

$X^2 + 16(Y + 1) - 16 = 0$

$X^2 + 16Y + 16 - 16 = 0$

$X^2 + 16Y = 0 \qquad$ Ans

(ii) $\mathbf{4x^2 + y^2 + 16x - 10y + 37 = 0} \qquad \mathbf{O'(-2, 5)}$

Solution:

$4x^2 + y^2 + 16x - 10y + 37 = 0$ .... (i) , $O'(-2, 5) \Rightarrow h = -2, k = 5$

We know that equations of transformation are