$18x^2 + 8x - 24 = A(x+2)^2 + B(x-3)(x+2) + C(x-3)$ ……… (3)

$18x^2 + 8x - 24 = A(x^2 + 4x + 4) + B(x^2 - x - 6) + C(x-3)$

$18x^2 + 8x - 24 = Ax^2 + 4Ax + 4A + Bx^2 - Bx - 6B + Cx - 3C$ ……… (4)

Put $x = 3$ in equation (3), we get

$18(3)^2 + 8(3) - 24 = A(3+2)^2 + 0 + 0$

$18(9) + 24 - 24 = A(5)^2$

$162 = 25A \Rightarrow \boxed{A = \frac{162}{25}}$

Put $x = -2$ in equation (3), we get

$18(-2)^2 + 8(-2) - 24 = 0 + 0 + C(-2-3)$

$-36 - 16 - 24 = C(-5)$

$32 = -5C \Rightarrow \boxed{B = -\frac{32}{5}}$

Equating coefficients of x^2 in equation (4), we get

$A + B = 18$

$B = 18 - A$

$B = 18 - \frac{162}{25} \Rightarrow \boxed{B = \frac{288}{25}}$

Put values of A, B, C in equation (2), we get

$$\frac{18x^2 + 8x - 24}{(x-3)(x+2)^2} = \frac{162}{25(x-3)} + \frac{288}{25(x+2)} - \frac{32}{5(x+2)^2}$$

equation (1) becomes

$$\frac{4x^3}{(x-3)(x+2)^2} = (2x-2) + \frac{162}{25(x-3)} + \frac{288}{25(x+2)} - \frac{32}{5(x+2)^2}$$

are required partial fractions.

EXERCISE 5.3

Resolve the following into partial fractions.

Q.1 $\mathbf{\dfrac{9x - 7}{(x^2+1)(x+3)}}$ **(Lahore Board 2004, 2010)**

Solution:

Let

$$\frac{9x-7}{(x^2+1)(x+3)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{(x+3)} \quad \text{……… (1)}$$

$$\frac{9x-7}{(x^2+1)(x+3)} = \frac{(Ax+B)(x+3) + C(x^2+1)}{(x^2+1)(x+3)}$$

$9x - 7 = (Ax+B)(x+3) + C(x^2+1)$ ……… (2)

$9x - 7 = Ax^2 + 3Ax + Bx + 3B + Cx^2 + C$ ……… (3)

Put $x = -3$ in equation (2), we get

$9(-3) - 7 = 0 + C[(-3)^2 + 1]$

$-27 - 7 = 0 + C(9 + 1)$

$-34 = 10C \Rightarrow \boxed{C = \frac{-17}{5}}$

Equating coefficients of x^2, x in equation (3) we get

$x^2 \quad ; \quad A + C = 0$

$A - \frac{17}{5} = 0$

$\Rightarrow \boxed{A = \frac{17}{5}}$

$x \quad ; \quad 3A + B = 9$

$3\left(\frac{17}{5}\right) + B = 9$

$\frac{51}{5} + B = 9$

$B = 9 - \frac{51}{5}$

$= \frac{45 - 51}{5}$

$\boxed{B = \frac{-6}{5}}$

Put values of A, B, C in equation (1) we get

$$\frac{9x-7}{(x^2+1)(x+3)} = \frac{\frac{17}{5}x - \frac{6}{5}}{x^2+1} - \frac{\frac{17}{5}}{x+3}$$

$$= \frac{17x - 6}{5(x^2+1)} - \frac{17}{5(x+3)}$$

are required partial fractions.

Q.2 $\mathbf{\dfrac{1}{(x^2+1)(x+1)}}$ **(Lahore Board 2003, 2006)**

Solution:

Let

$$\frac{1}{(x^2+1)(x+1)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{(x+1)} \quad \text{……… (1)}$$

$$\frac{1}{(x^2+1)(x+1)} = \frac{(Ax+B)(x+1) + C(x^2+1)}{(x^2+1)(x+1)}$$

$1 = (Ax + B)(x + 1) + C(x^2 + 1)$ ……… (2)

$1 = Ax^2 + Ax + Bx + B + Cx^2 + C$ ……… (3)

Put $x = -1$ in equation (2), we get

$$1 = 0 + C\left((-1)^2 + 1\right)$$

$$1 = C(1+1)$$

$$1 = 2C \Rightarrow \boxed{C = \frac{1}{2}}$$

Equating coefficients of x^2 and x in equation (3) we get

$$x^2 \quad ; \quad A + C = 0$$

$$A = -C$$

$$\boxed{A = -\frac{1}{2}}$$

Equating coefficient of x

$$x \quad ; \quad A + B = 0$$

$$B = -A$$

$$\boxed{B = \frac{1}{2}}$$

Put values of A, B and C in equation (1) we get

$$\frac{1}{(x^2+1)(x+1)} = \frac{-\frac{1}{2}x + \frac{1}{2}}{(x^2+1)} + \frac{\frac{1}{2}}{(x+1)}$$

$$= \frac{-x+1}{2(x^2+1)} + \frac{1}{2(x+1)}$$

are required partial fractions.

Q.3 $\mathbf{\dfrac{3x+7}{(x^2+4)(x+3)}}$

Solution:

Let

$$\frac{3x+7}{(x^2+4)(x+3)} = \frac{Ax+B}{(x^2+4)} + \frac{C}{(x+3)} \quad \text{......... (1)}$$

$$\frac{3x+7}{(x^2+4)(x+3)} = \frac{(Ax+B)(x+3) + C(x^2+4)}{(x^2+4)(x+3)}$$

$$3x + 7 = (Ax+B)(x+3) + C(x^2+4) \quad \text{......... (2)}$$

$$3x + 7 = Ax^2 + 3Ax + Bx + 3B + Cx^2 + 4C \quad \text{......... (3)}$$

Put $x = -3$ in equation (2), we get

$$3(-3) + 7 = 0 + C\left((-3)^2 + 4\right)$$
$$-9 + 7 = C(9 + 4)$$
$$-2 = 13C \Rightarrow \boxed{C = -\frac{2}{13}}$$

Equating coefficients of x^2 and x in equation (3) we get

x^2 ; $A + C = 0$

$$A = -C$$
$$\boxed{A = \frac{2}{13}}$$

x ; $3A + B = 3$

$$B = 3 - 3A$$
$$= 3 - 3 \cdot \frac{2}{13}$$
$$= 3 - \frac{6}{13}$$
$$= \frac{39 - 6}{13}$$
$$\boxed{B = \frac{33}{13}}$$

Put values of A, B and C in equation (1) we get

$$\frac{3x + 7}{(x^2 + 4)(x + 3)} = \frac{\frac{2}{13}x + \frac{33}{13}}{x^2 + 4} + \frac{-\frac{2}{13}}{x + 3}$$
$$= \frac{2x + 33}{13(x^2 + 4)} - \frac{2}{13(x + 3)}$$

are required partial fractions.

Q.4 $\dfrac{\mathbf{x^2 + 15}}{\mathbf{(x^2 + 2x + 5)(x - 1)}}$

Solution:

Let

$$\frac{x^2 + 15}{(x^2 + 2x + 5)(x - 1)} = \frac{Ax + B}{(x^2 + 2x + 5)} + \frac{C}{(x - 1)} \quad \ldots\ldots\ldots (1)$$

$$\frac{x^2 + 15}{(x^2 + 2x + 5)(x - 1)} = \frac{(Ax + B)(x - 1) + C(x^2 + 2x + 5)}{(x^2 + 2x + 5)(x - 1)}$$

$$x^2 + 15 = (Ax + B)(x - 1) + C(x^2 + 2x + 5) \quad \ldots\ldots\ldots (2)$$

$$x^2 + 15 = Ax^2 - Ax + Bx - B + Cx^2 + 2Cx + 5C \quad \ldots\ldots\ldots (3)$$

Put $x = 1$ in equation (2), we get

$(1)^2 + 15 = 0 + C\left((1)^2 + 2(1) + 5\right)$

$1 + 15 = C(1 + 2 + 5)$

$16 = 8C \Rightarrow \boxed{C = 2}$

Equating coefficients of x^2, and x in equation (3), we get

$x^2 \quad ; \quad A + C = 1$

$A = 1 - C$

$A = 1 - 2$

$\boxed{A = 1}$

$x \quad ; \quad -A + B + 2C = 0$

$B = A - 2C$

$B = -1 - 2(2) = -1 - 4$

$\boxed{B = -5}$

Put values of A, B and C in equation (1) we get

$$\frac{x^2 + 15}{(x^2 + 2x + 5)(x - 1)} = \frac{-x - 5}{x^2 + 2x + 5} + \frac{2}{(x - 1)}$$

are required partial fraction.

Q.5 $\mathbf{\dfrac{x^2}{(x^2 + 4)(x + 2)}}$

Solution:

Let

$$\frac{x^2}{(x^2 + 4)(x + 2)} = \frac{Ax + B}{(x^2 + 4)} + \frac{C}{(x + 2)} \qquad \text{......... (1)}$$

$$\frac{x^2}{(x^2 + 4)(x + 2)} = \frac{(Ax + B)(x + 2) + C(x^2 + 4)}{(x^2 + 4)(x + 2)}$$

$$x^2 = (Ax + B)(x + 2) + C(x^2 + 4) \qquad \text{......... (2)}$$

$$x^2 = Ax^2 + 2Ax + Bx + 2B + Cx^2 + 4C \qquad \text{......... (3)}$$

Put $x = -2$ in equation (2), we get

$(-2)^2 = 0 + C\left((-2)2 + 4\right)$

$4 = 0 + C(4 + 4)$

$4 = 8C \Rightarrow \boxed{C = \frac{1}{2}}$

Equating coefficients of x^2 and x in equation (3), we get

$x^2 \quad ; \quad A + C = 1$

$$A = 1 - C = 1 - \frac{1}{2}$$

$$\boxed{A = \frac{1}{2}}$$

$x \quad ; \quad 2A + B = 0$

$$2\left(\frac{1}{2}\right) + B = 0$$

$$1 + B = 0$$

$$\boxed{B = -1}$$

Put values of A, B and C in equation (1) we get

$$\frac{x^2}{(x^2+4)(x+2)} = \frac{\frac{1}{2}x - 1}{x^2+4} + \frac{\frac{1}{2}}{x+2}$$

$$= \frac{x-2}{2(x^2+4)} + \frac{1}{2(x+2)}$$

are required partial fraction.

Q.6 $\mathbf{\dfrac{x^2+1}{x^3+1}}$ **(Lahore Board 2003)**

Solution:

Let

$$\frac{x^2+1}{x^3+1} = \frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2-x+1)} \quad \ldots\ldots\ldots (1)$$

$$\Rightarrow \quad \frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{A(x^2-x+1) + (Bx+C)(x+1)}{(x+1)(x^2-x+1)}$$

$$x^2 + 1 = A(x^2-x+1) + (Bx+C)(x+1) \quad \ldots\ldots\ldots (2)$$

$$x^2 + 1 = Ax^2 - Ax + A + Bx^2 + Bx + Cx + C \quad \ldots\ldots\ldots (3)$$

Put $x = -1$ in equation (2), we get

$$(-1)^2 + 1 = A\left((-1)2 - (-1) + 1\right)$$

$$1 + 1 = A(1 + 1 + 1)$$

$$2 = 3A \Rightarrow \boxed{A = \frac{2}{3}}$$

Equating coefficients of x2 and x in equation (3), we get

$$x^2 \quad ; \quad A + B = 1$$

$$\frac{2}{3} + B = 1$$

$$B = 1 - \frac{2}{3} = \frac{1}{3}$$

$$\boxed{B = \frac{1}{3}}$$

$$x \quad ; \quad -A + B + C = 0$$

$$-\frac{2}{3} + \frac{1}{3} + C = 0$$

$$-\frac{1}{3} + C = 0$$

$$\boxed{C = \frac{1}{3}}$$

Put values of A, B and C in equation (1), we get

$$\frac{x^2 + 1}{x^3 + 1} = \frac{\frac{2}{3}}{x + 1} + \frac{\frac{1}{3}x + \frac{1}{3}}{x^2 - x + 1}$$

$$= \frac{2}{3(x + 1)} + \frac{x + 1}{3(x^2 - x + 1)}$$

are required partial fraction.

Q.7 $\displaystyle \mathbf{\frac{x^2 + 2x + 2}{(x^2 + 3)(x + 1)(x - 1)}}$

Solution:

Let

$$\frac{x^2 + 2x + 2}{(x^2 + 3)(x + 1)(x - 1)} = \frac{Ax + B}{(x^2 + 3)} + \frac{C}{(x + 1)} + \frac{D}{(x - 1)} \quad \text{.........} (1)$$

$$\frac{x^2 + 2x + 2}{(x^2 + 3)(x + 1)(x - 1)} = \frac{(Ax + B)(x + 1)(x - 1) + C(x^2 + 3)(x - 1) + D(x^2 + 3)(x + 1)}{(x^2 + 3)(x + 1)(x - 1)}$$

$$x^2 + 2x + 2 = (Ax + B)(x + 1)(x - 1) + C(x^2 + 3)(x - 1) + D(x^2 + 3)(x + 1) \quad \ldots\ldots (2)$$

$$x^2 + 2x + 2 = Ax^3 - Ax + Bx^2 - B + Cx^3 - Cx^2 + 3Cx - 3C + Dx^3 + Dx^2 + 3Dx + 3D$$

Put $x = -1$ in equation (2), we get

$$(-1)^2 + 2(-1) + 2 = 0 + C\left((-1)^2 + 3\right)(-1 - 1) + 0$$

$$1 - 2 + 2 = C(1 + 3)(-2)$$

$$1 = -8C \quad \Rightarrow \quad \boxed{C = -\frac{1}{8}}$$

Put $x = 1$ in equation (2), we get

$$(1)^2 + 2(1) + 2 = 0 + 0 + D((1)^2 + 3)(1 + 1)$$

$$1 + 2 + 2 = D(1 + 3)(2)$$

$$5 = 8D \quad \Rightarrow \quad \boxed{D = \frac{5}{8}}$$

Equating coefficients of x^3 and x^2 in equation (3), we get

x^3 ; $A + C + D = 0$

$$A = -C - D$$

$$= \frac{1}{8} - \frac{5}{8} = \frac{-4}{8}$$

$$\boxed{A = -\frac{1}{2}}$$

x^2 ; $B + D - C = 1$

$$B = 1 - D + C = 1 - \frac{5}{8} - \frac{1}{8}$$

$$\boxed{B = \frac{1}{4}}$$

Put values of A, B, C, D in equation (1), we get

$$\frac{x^2 + 2x + 2}{(x^2 + 3)(x + 1)(x - 1)} = \frac{-\frac{1}{2}x + \frac{1}{4}}{(x^2 + 3)} + \frac{-\frac{1}{8}}{(x + 1)} + \frac{\frac{5}{8}}{(x - 1)}$$

$$= \frac{1 - 2x}{4(x^2 + 3)} - \frac{1}{8(x + 1)} + \frac{5}{8(x - 1)}$$

are required partial fraction.

Q.8 $\dfrac{\mathbf{1}}{\mathbf{(x-1)^2\,(x^2+2)}}$ **(Gujranwala Board 2006)**

Solution:

Let

$$\frac{1}{(x-1)^2\,(x^2+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{Cx+D}{(x^2+2)} \qquad \ldots\ldots\ldots (1)$$

$$\frac{1}{(x-1)^2\,(x^2+2)} = \frac{A\,(x-1)\,(x^2+2) + B\,(x^2+2) + (Cx+D)\,(x-1)^2}{(x-1)^2\,(x^2+2)}$$

$$1 = A\,(x-1)\,(x^2+2) + B\,(x^2+2) + (Cx+D)\,(x-1)^2 \qquad \ldots\ldots\ldots (2)$$

$$1 = A\,(x^3 - x^2 + 2x - 2) + B\,(x^2+2) + (Cx+D)\,(x^2+1-2x)$$

$$1 = Ax^3 - Ax^2 + 2Ax - 2A + Bx^2 + 2B + Cx^3 + Cx - 2Cx^2 + Dx^2 + D - 2Dx \qquad \ldots\ldots (3)$$

Put $x = 1$ in equation (2), we ge

$$1 = 0 + B\left((1)^2 + 2\right) + 0$$

$$1 = B\,(1+2)$$

$$1 = 3B \quad \Rightarrow \quad \boxed{B = \frac{1}{3}}$$

Equating coefficients of x^3, x^2, and x in equation (3), we get

$$1 = 0 + B\left((1)^2 + 2\right) + 0$$

$$1 = B\,(1+2)$$

$$1 = 3B \quad \Rightarrow \quad \boxed{B = \frac{1}{3}}$$

Equating coefficients of x^3, x^2, and x in equation (3), we get

$$x^3 \quad ; \quad A + C = 0 \qquad \ldots\ldots\ldots (i)$$

$$x^2 \quad ; \quad -A + B - 2C + D = 0 \qquad ¤ \quad B = \frac{1}{3}$$

$$-A - 2C + D = -\frac{1}{3} \qquad \ldots\ldots\ldots (ii)$$

$$x \quad ; \quad 2A + C - 2D = 0 \qquad \ldots\ldots\ldots (iii)$$

from (i) $A = -C$

Put in (ii) and (iii).

$$-(-C) - 2C + D = \frac{1}{3}$$

$-C + D = \frac{1}{3}$ .... (iv)

and $2(-C) + C - 2D = 0$

$-2C + C - 2D = 0$

$-C - 2D = 0$ .... (v)

Subtracting (iv) from (v)

$$\begin{array}{l} -C - 2D = 0 \\ -C + \;\; D = -\frac{1}{3} \\ + \quad - \qquad + \\ \hline \quad -3D = 0 + \frac{1}{3} \end{array}$$

$-3D = \frac{1}{3} \Rightarrow \boxed{D = -\frac{1}{9}}$

Put $D = -\frac{1}{9}$ in equation (v)

$-C - 2\left(-\frac{1}{9}\right) = 0$

$-C + \frac{2}{9} = \Rightarrow \boxed{C = \frac{2}{9}}$

Put this value in equation (i) we get

$\boxed{A = -C = \frac{-2}{9}}$

Putting values of A, B, C and D in equation (1) we get

$$\frac{1}{(x-1)^2(x^2+2)} = \frac{-\frac{2}{9}}{x-1} + \frac{\frac{1}{3}}{(x-1)^2} + \frac{\frac{2}{9}x - \frac{1}{9}}{(x^2+2)}$$

$$= \frac{-2}{9(x-1)} + \frac{1}{3(x-1)^2} + \frac{2x-1}{9(x^2+2)}$$

are required partial fraction.

Q.9 $\dfrac{x^4}{1 - x^4}$

Solution:

$$\frac{x^4}{1 - x^4} = \frac{-x^4}{x^4 - 1}$$

By division

$$\begin{array}{r|l} & \quad -1 \\ x^4 - 1 & \overline{-x^4} \\ & -x^4 + 1 \\ & + \quad - \\ \hline & \quad -1 \end{array}$$

$$\Rightarrow \quad \frac{-x^4}{x^4 - 1} = -1 - \frac{1}{x^4 - 1} \qquad \text{.........} (1)$$

$$\frac{1}{x^4 - 1} = \frac{1}{(x^2 - 1)(x^2 + 1)} = \frac{1}{(x + 1)(x - 1)(x^2 + 1)}$$

Let

$$\frac{1}{(x + 1)(x - 1)(x^2 + 1)} = \frac{A}{x + 1} + \frac{B}{x - 1} + \frac{Cx + D}{x^2 + 1} \qquad \text{.........} (2)$$

$$\frac{1}{(x + 1)(x - 1)(x^2 + 1)} = \frac{A(x - 1)(x^2 + 1) + B(x + 1)(x^2 + 1) + (Cx + D)(x + 1)(x - 1)}{(x + 1)(x - 1)(x^2 + 1)}$$

$$1 = A(x - 1)(x^2 + 1) + B(x + 1)(x^2 + 1) + (Cx + D)(x + 1)(x - 1) \qquad \text{.........} (3)$$

$$1 = A(x^3 - x^2 + x - 1) + B(x^3 + x^2 + x + 1) + (Cx + D)(x^2 - 1)$$

$$1 = Ax^3 - Ax^2 + Ax - A + Bx^3 + Bx^2 + Bx + B + Cx^3 - Cx - Cx + Dx^2 - D \qquad \text{.........} (4)$$

Put $x = 1$ in equation (3), we get

$$1 = A(-1 - 1)\left((-1)^2 + 1\right) + 0 + 0$$

$$1 = A(-2)(1 + 1)$$

$$1 = 4A \quad \Rightarrow \quad \boxed{A = -\frac{1}{4}}$$

Put $x = -1$ in equation (3), we get

$$1 = 0 + B(1 + 1)\left((1)^2 + 1\right) + 0$$

$1 = B(2)(1+1)$

$1 = 4B \quad \Rightarrow \quad \boxed{B = \frac{1}{4}}$

Equating coefficients of x^3 and x^2 in equation (4) we get

$x^3 \quad ; \quad A + B + C = 0$

$-\frac{1}{4} + \frac{1}{4} + C = 0$

$\Rightarrow \quad \boxed{C = 0}$

$x^2 \quad ; \quad -A + B + D = 0$

$\frac{1}{4} + \frac{1}{4} + D = 0$

$\boxed{D = -\frac{1}{2}}$

Put values of A, B, C, D in equation (2) we get

$$\frac{1}{(1-x)(1+x)(1+x^2)} = \frac{-\frac{1}{4}}{x+1} + \frac{\frac{1}{4}}{x-1} + \frac{(0)\,x - \frac{1}{2}}{x^2+1}$$

$$= \frac{-1}{4(x+1)} + \frac{1}{4(x-1)} - \frac{1}{2(x^2+1)}$$

Equation (1) becomes

$$\frac{-x^4}{x^4-1} = -1 - \left[\frac{-1}{4(x+1)} + \frac{1}{4(x-1)} - \frac{1}{2(x^2+1)}\right]$$

$$= -1 + \frac{1}{4(x+1)} - \frac{1}{4(x-1)} + \frac{1}{2(x^2+1)}$$

are required partial fractions.

Q.10 $\mathbf{\dfrac{x^2 - 2x + 3}{x^4 + x^2 + 1}}$

Solution:

$$\frac{x^2-2x+3}{x^4+x^2+1} = \frac{x^2-2x+3}{(x^2+x+1)(x^2-x+1)}$$

Let

$$\frac{x^2-2x+3}{(x^2+x+1)(x^2-x+1)} = \frac{Ax+B}{(x^2+x+1)} + \frac{Cx+D}{(x^2-x+1)} \quad \ldots\ldots\ldots (1)$$

$$\frac{x^2-2x+3}{(x^2+x+1)(x^2-x+1)} = \frac{(Ax+B)(x^2-x+1)(Cx+D)(x^2+x+1)}{(x^2+x+1)(x^2-x+1)}$$

$$x^2 - 2x + 3 = (Ax+B)(x^2+x+1)(Cx+D)(x^2-x+1)$$

$$x^2 - 2x + 3 = Ax^3 - Ax^2 + Ax + Bx^2 - Bx + B + Cx^3 + Cx^2 + Cx + Dx^2 + Dx + D \quad \ldots\ldots\ldots (2)$$

Equating coefficients of x^3, x^2, x, x and constant term in equation (2), we get

x^3 ; $A + C = 0$ (i)

x^2 ; $-A + B + C + D = 1$ (ii)

x ; $A - B + C + D = -2$ (iii)

cons ; $B + D = 3$ (iv)

Put $A + C = 0$ in equation (iii), we get

$-B + D = -2$ (v)

adding equation (iv) and (v), we get

$2D = 1 \quad \Rightarrow \quad \boxed{D = \frac{1}{2}}$

Put $D = \frac{1}{2}$ in equation (iv), we get

$B + \frac{1}{2} = 3$

$B = 3 - \frac{1}{2} = \frac{6-1}{2}$

$\boxed{B = \frac{5}{2}}$

Put $B + D = 3$ in equation (ii), we get

$-A + C + 3 = 1$

$-A + C = -2$ (vi)

adding equation (vi) and (i)

$2C = -2$

$\boxed{C = -1}$

Put $C = -1$ in equation (i), we get

$A - 1 = 0$

$\boxed{A = 1}$

Put values of A, B, C, and D in equation (1), we get

$$\frac{x^2 - 2x + 3}{(x^2 - x + 1)(x^2 + x + 1)} = \frac{x + \frac{5}{2}}{x^2 + x + 1} + \frac{-x + \frac{1}{2}}{x^2 - x + 1}$$

$$= \frac{(2x + 5)}{2(x^2 + x + 1)} - \frac{2x - 1}{2(x^2 - x + 1)}$$

are required partial fraction.