$\Rightarrow \quad x^2 = \dfrac{4 \pm 2\sqrt{3}}{2}$

$\Rightarrow \quad x^2 = 2 \pm \sqrt{3}$

$\Rightarrow \quad x = \pm\sqrt{2 \pm \sqrt{3}}$

$\Rightarrow \quad$ Either $x + 1 = 0$ or $x - 1 = 0$

$\Rightarrow \quad x = -1$ or $x = 1$

Hence the solution set $= \left\{ -1, 1, \pm\sqrt{2 \pm \sqrt{3}} \right\}$

## TYPE V: RADICAL EQUATIONS

Equations involving radical expressions of the variable are called radical equations. To solve a radical equation, we first obtain an equation free from radicals. Every solution of radical equation is also a solution of the radical free equation but the new equation have solutions that are not solutions of the original radical equation. Such extra solutions are called **extraneous roots**.

There are four types of radical equations.

(i) The equations of the form: $l\,(ax^2 + bx) + m\sqrt{ax^2 + bx + c} = 0$

(ii) The equations of the form: $\sqrt{x + a} + \sqrt{x + b} = \sqrt{x + c}$

(iii) The equations of the form: $\sqrt{ax^2 + bx + c} + \sqrt{px^2 + qx + r} = \sqrt{1x^2 + mx + n}$

(iv) The equations of the form: $\sqrt{ax^2 + bx + c} + \sqrt{px^2 + qx + r} = mx + n$

## EXERCISE 4.3

Solve the following equation:

**Q.1** $\mathbf{3x^2 + 2x - \sqrt{3x^2 + 2x - 1} = 3}$

**Solution:**

$3x^2 + 2x - \sqrt{3x^2 + 2x - 1} = 3 \quad \ldots\ldots\ldots (1)$

Put $\sqrt{3x^2 + 2x - 1} = y \quad \ldots\ldots\ldots (2)$

$\Rightarrow \quad 3x^2 + 2x - 1 = y^2$

$\Rightarrow \quad 3x^2 + 2x = y^2 + 1$

$\Rightarrow$ equation (1) becomes

$y^2 + 1 - y = 3$

$\Rightarrow \quad y^2 - y + 1 - 3 = 0$

$\Rightarrow \quad y^2 - y - 2 = 0$

$\Rightarrow \quad y^2 - 2y + y - 2 = 0$

$\Rightarrow \quad y\,(y - 2) + 1\,(y - 2) = 0$

$\Rightarrow \quad (y-2)(y+1) = 0$

$\Rightarrow \quad$ Either $y-2 = 0$ or $y+1 = 0$

$\Rightarrow \quad y = 2$ or $y = -1$

Put $y = 2$ and $y = -1$ in equation (2)

$\Rightarrow \quad \sqrt{3x^2+2x-1} = 2$

$\Rightarrow \quad 3x^2+2x-1 = 4$

$\Rightarrow \quad 3x^2+2x-1-4 = 0$

$\Rightarrow \quad 3x^2+2x-5 = 0$

$\Rightarrow \quad 3x^2+5x-3x-5 = 0$

$\Rightarrow \quad x(3x+5)-1(3x+5) = 0$

$\Rightarrow \quad (3x+5)(x-1) = 0$

$\Rightarrow \quad$ Either $3x+5 = 0$ or $x-1 = 0$

$\Rightarrow \quad x = \frac{-5}{3}$ or $x = 1$

$\Rightarrow \quad \sqrt{3x^2+2x-1} = -1$

$\Rightarrow \quad 3x^2+2x-1 = 1$

$\Rightarrow \quad 3x^2+2x-1-1 = 0$

$\Rightarrow \quad 3x^2+2x-2 = 0$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{(2)^2-4(3)(-2)}}{2(3)}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{4+24}}{6}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{28}}{6}$

$\Rightarrow \quad x = \frac{-2 \pm 2\sqrt{7}}{6}$

$\Rightarrow \quad x = \frac{-1 \pm \sqrt{7}}{3}$

$\Rightarrow \quad x = 1, \ -\frac{5}{3}, \ \frac{-1 \pm \sqrt{7}}{3}$

Checking:

Put $x = 1$ in equation (1)

$$3(1)^2+2(1)-\sqrt{3(1)^2+2(1)-1} = 3$$

$$3+2-\sqrt{3+2-1} = 3$$

$$3 = 3$$

$\Rightarrow \quad$ L.H.S. = R.H.S.

Put $x = -\frac{5}{3}$ in equation (1)

$$3\left(-\frac{5}{3}\right)^2+2\left(-\frac{5}{3}\right)-\sqrt{3\left(-\frac{5}{3}\right)^2+2\left(-\frac{5}{3}\right)-1} = 3$$

$$3 = 3$$

$\Rightarrow$ L.H.S. = R.H.S.

Put $x = \dfrac{-1+\sqrt{7}}{3}$ in equation (1)

$$3\left(\frac{-1+\sqrt{7}}{3}\right)^2 + 2\left(\frac{-1+\sqrt{7}}{3}\right) - \sqrt{3\left(\frac{-1+\sqrt{7}}{3}\right)^2 + 2\left(\frac{-1+\sqrt{7}}{3}\right) - 1} = 3$$

$\Rightarrow$ $1 \neq 3$

$\Rightarrow$ L.H.S. $\neq$ R.H.S.

Put $x = \dfrac{-1-\sqrt{7}}{3}$ in equation (1)

$$3\left(\frac{-1-\sqrt{7}}{3}\right)^2 + 2\left(\frac{-1-\sqrt{7}}{3}\right) - \sqrt{3\left(\frac{-1-\sqrt{7}}{3}\right)^2 + 2\left(\frac{-1-\sqrt{7}}{3}\right) - 1} = 3$$

$1 \neq 3$

$\Rightarrow$ L.H.S. $\neq$ R.H.S.

On checking, we find that $\dfrac{-1\pm\sqrt{7}}{3}$ are extraneous roots.

Hence the solution set $= \left\{1, \dfrac{-5}{3}\right\}$

**Q.2** $\mathbf{x^2 - \frac{x}{2} - 7 = = x - 3\sqrt{2x^2 - 3x + 2}}$

**Solution:**

$$x^2 - \frac{x}{2} - 7 = = x - 3\sqrt{2x^2 - 3x + 2}$$

$$\Rightarrow \quad \frac{2x^2 - x - 14}{2} = x - 3\sqrt{2x^2 - 3x + 2}$$

$$\Rightarrow \quad 2x^2 - x - 14 = 2x - 6\sqrt{2x^2 - 3x + 2}$$

$$\Rightarrow \quad 2x^2 - x - 14 - 2x + 6\sqrt{2x^2 - 3x + 2} = 0$$

$$\Rightarrow \quad 2x^2 - 3x - 14 + 6\sqrt{2x^2 - 3x + 2} = 0 \qquad \ldots\ldots\ldots (1)$$

Put $\sqrt{2x^2 - 3x + 2} = y$ ……… (2)

Then $2x^2 - 3x + 2 = y^2$

$2x^2 - 3x = y^2 - 2$

Equation (1) becomes

$$y^2 - 2 - 14 + 6y = 0$$
$$\Rightarrow \quad y^2 + 6y - 16 = 0$$
$$\Rightarrow \quad y^2 + 8y - 2y - 16 = 0$$
$$\Rightarrow \quad y(y+8) - 2(y+8) = 0$$
$$\Rightarrow \quad (y+8)(y-2) = 0$$
$$\Rightarrow \quad \text{Either } y + 8 = 0 \quad \text{or} \quad y - 2 = 0$$
$$\Rightarrow \quad y = -8 \quad \text{or} \quad y = 2$$

Put $y = -8$ and $y = 2$ in equation (2)

$$\Rightarrow \quad \sqrt{2x^2 - 3x + 2} = -8$$
$$\Rightarrow \quad 2x^2 - 3x + 2 = 64$$
$$\Rightarrow \quad 2x^2 - 3x + 2 - 64 = 0$$
$$\Rightarrow \quad 2x^2 - 3x + -62 = 0$$
$$\Rightarrow \quad x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-62)}}{2(2)}$$
$$\Rightarrow \quad x = \frac{3 \pm \sqrt{9 + 496}}{4}$$
$$\Rightarrow \quad x = \frac{3 \pm \sqrt{505}}{4}$$

$$\Rightarrow \quad \sqrt{2x^2 - 3x + 2} = 2$$
$$\Rightarrow \quad 2x^2 - 3x + 2 = 0$$
$$\Rightarrow \quad 2x^2 - 3x + 2 - 4 = 0$$
$$\Rightarrow \quad 2x^2 - 3x + 2 - 4 = 0$$
$$\Rightarrow \quad 2x^2 - 4x + x - 2 = 0$$
$$\Rightarrow \quad 2x(x-2) + 1(x-2) = 0$$
$$\Rightarrow \quad (x-2)(2x+1) = 0$$
$$\Rightarrow \quad \text{Either } x - 2 = 0 \quad \text{or} \quad 2x + 1 = 0$$
$$\Rightarrow \quad x = 2 \quad \text{or} \quad x = -\frac{1}{2}$$

$$x = -\frac{1}{2}, 2, \frac{3 \pm \sqrt{505}}{4}$$

Checking:

Put $x = -\frac{1}{2}$ in L.H.S. of equation (1)

$$2\left(-\frac{1}{2}\right)^2 - 3\left(-\frac{1}{2}\right) - 14 + 6\sqrt{2\left(-\frac{1}{2}\right)^2 - 3\left(-\frac{1}{2}\right) + 2}$$
$$= 2\left(\frac{1}{4}\right) + \frac{3}{2} - 14 + 6\sqrt{2\left(\frac{1}{4}\right) + \frac{3}{2} + 2} = 0 = \text{R.H.S.}$$

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Put x = 2 in L.H.S. of equation (1)

$$2(2)^2 - 3(2)^2 - 14 + 6\sqrt{2(2)^2 - 3(2) + 2} = 0 = \text{R.H.S.}$$

Put $x = \dfrac{3+\sqrt{505}}{4}$ in L.H.S. of equation (1)

$$2\left(\frac{3+\sqrt{505}}{4}\right)^2 - 3\left(\frac{3+\sqrt{505}}{4}\right) - 14 + 6\sqrt{2\left(\frac{3+\sqrt{505}}{4}\right)^2 - 3\left(\frac{3+\sqrt{505}}{4}\right) + 2} \neq 0$$

$\Rightarrow$ L.H.S. $\neq$ R.H.S.

Put $x = \dfrac{3-\sqrt{505}}{4}$ in L.H.S. of equation (1)

$$2\left(\frac{3-\sqrt{505}}{4}\right)^2 - 3\left(\frac{3-\sqrt{505}}{4}\right) - 14 + 6\sqrt{2\left(\frac{3-\sqrt{505}}{4}\right)^2 - 3\left(\frac{3-\sqrt{505}}{4}\right) + 2} \neq 0$$

$\Rightarrow$ L.H.S. $\neq$ R.H.S.

On checking, we find that $\dfrac{3\pm\sqrt{505}}{4}$ are extraneous roots.

Hence the solution set $= \left\{-\dfrac{1}{2},\ 2\right\}$

**Q.3** $\mathbf{\sqrt{2x+8} + \sqrt{x+5} = 7}$

**Solution:**

$$\sqrt{2x+8} + \sqrt{x+5} = 7 \qquad \ldots\ldots\ldots (1)$$

Squaring on both sides

$$\left(\sqrt{2x+8} + \sqrt{x+5}\right)^2 = (7)^2$$

$\Rightarrow$ $\left(\sqrt{2x+8}\right)^2 + \left(\sqrt{x+5}\right)^2 + 2\left(\sqrt{2x+8}\right)\left(\sqrt{x+5}\right) = 49$

$\Rightarrow$ $2x + 8 + x + 5 + 2\sqrt{2x+8\ (x+5))} = 49$

$\Rightarrow$ $3x + 13 + 2\sqrt{2x^2 + 10x + 8x + 40} = 49$

$\Rightarrow$ $2\sqrt{2x^2 + 18x + 40} = 49 - 3x - 13$

$\Rightarrow$ $2\sqrt{2x^2 + 18x + 40} = 36 - 3x$

$\Rightarrow$ $-2\sqrt{2x^2 + 18x + 40} = 3x - 36$

Squaring again

$$\left(-2\sqrt{2x^2 + 18x + 40}\right)^2 = (3x - 36)^2$$

$\Rightarrow$ $4(2x^2 + 18x + 40) = 9x^2 + 1296 - 216x$

$\Rightarrow$ $8x^2 + 72x + 160 = 9x^2 + 1296 - 216x$

$\Rightarrow \quad 9x^2 + 1296 - 216x - 8x^2 - 72x - 160 = 0$

$\Rightarrow \quad x^2 - 288x + 1136 = 0$

$\Rightarrow \quad x^2 - 284x - 4x + 1136 = 0$

$\Rightarrow \quad x(x - 284) - 4(x - 284) = 0$

$\Rightarrow \quad (x - 284)(x - 4) = 0$

$\Rightarrow$ Either $x - 284 = 0$ or $x - 4 = 0$

$\Rightarrow \quad x = 284$ or $x = 4$

Checking:

Put $x = 284$ in L.H.S. of equation (1)

$\sqrt{2(284) + 8} + \sqrt{284 + 5} = 41 \neq$ R.H.S.

Put $x = 4$ in L.H.S. of equation (1)

$\sqrt{2(4) + 8} + \sqrt{4 + 5} = 7 =$ R.H.S.

$\Rightarrow \quad x = 284$ is an extraneous root.

Hence the solution set $= \{4\}$

**Q.4** $\mathbf{\sqrt{3x + 4} = 2 + \sqrt{2x - 4}}$

**Solution:**

$\sqrt{3x + 4} = 2 + \sqrt{2x - 4}$

$\sqrt{3x - 4} - \sqrt{2x - 4} = 2 \qquad \ldots\ldots\ldots (1)$

Squaring both sides

$\left(\sqrt{3x + 4} - \sqrt{2x - 4}\right)^2 = (2)^2$

$\Rightarrow \quad \left(\sqrt{3x + 4}\right)^2 + \left(\sqrt{2x - 4}\right)^2 - 2\sqrt{3x + 4}\sqrt{2x - 4} = 4$

$\Rightarrow \quad 3x + 4 + 2x - 4 - 2\sqrt{(3x + 4)(2x - 4)} = 4$

$\Rightarrow \quad 5x - 2\sqrt{6x^2 - 12x + 8x - 16} = 4$

$\Rightarrow \quad 5x - 4 = 2\sqrt{6x^2 - 4x - 16}$

Squaring again

$\Rightarrow \quad 25x^2 + 16 - 40x = 4(6x^2 - 4x - 16)$

$\Rightarrow \quad 25x^2 + 16 - 40x = 24x^2 - 16x - 64$

$\Rightarrow \quad 25x^2 + 16 - 40x - 24x^2 + 16x + 64 = 0$

$\Rightarrow \quad x^2 - 24x + 80 = 0$

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$\Rightarrow \quad x^2 - 20x - 4x + 80 = 0$

$\Rightarrow \quad x(x - 20) - 4(x - 20) = 0$

$\Rightarrow$ Either $x - 20 = 0$ or $x - 4 = 0$

$\Rightarrow \quad x = 20$ or $x = 4$

Put $x = 20$ in L.H.S. of equation (1)

$\sqrt{3(20) + 4} - \sqrt{2(20) - 4} = 2 = \text{R.H.S.}$

Put $x = 4$ in L.H.S. of equation (1)

$\sqrt{3(4) + 4} - \sqrt{2(4) - 4} = 2 = \text{R.H.S.}$

None of them is extraneous root.

Hence the solution set $= \{4, 20\}$

**Q.5** $\mathbf{\sqrt{x + 7} + \sqrt{x + 2} = \sqrt{6x + 13}}$ **(Gujranwala Board 2006)**

**Solution:**

$\sqrt{x + 7} + \sqrt{x + 2} = \sqrt{6x + 13}$ ......... (1)

Squaring on both sides

$(\sqrt{x + 7})^2 + (\sqrt{x + 2})^2 + 2\sqrt{x + 7}\sqrt{x + 2} = (\sqrt{6x + 13})^2$

$\Rightarrow \quad x + 7 + x + 2 + 2\sqrt{(x + 7)(x + 2)} = 6x + 13$

$\Rightarrow \quad 2x + 9 + 2\sqrt{x^2 + 2x + 7x + 14} = 6x + 13$

$\Rightarrow \quad 2\sqrt{x^2 + 9x + 14} = 6x + 13 - 2x - 9$

$\Rightarrow \quad 2\sqrt{x^2 + 9x + 14} = 4x + 4$

$\Rightarrow \quad 2\sqrt{x^2 + 9x + 14} = 2(2x + 2)$

$\Rightarrow \quad \sqrt{x^2 + 9x + 14} = 2x + 2$

Squaring again

$\Rightarrow \quad x^2 + 9x + 14 = (2x + 2)^2$

$\Rightarrow \quad x^2 + 9x + 14 = 4x^2 + 4 + 8x$

$\Rightarrow \quad 4x^2 + 4 + 8x - x^2 - 9x - 14 = 0$

$\Rightarrow \quad 3x^2 - x - 10 = 0$

$\Rightarrow \quad 3x^2 - 6x + 5x - 10 = 0$

$\Rightarrow \quad 3x(x - 2) + 5(x - 2) = 0$

$\Rightarrow \quad (x - 2) + (3x + 5) = 0$



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$\Rightarrow$ Either $x - 2 = 0$ or $3x + 5 = 0$

$\Rightarrow$ $x = 2$ or $x = \frac{-5}{3}$

$\Rightarrow$ $x = 2$ and $x = \frac{-5}{3}$

Checking:

Put $x = 2$ in equation (1)

$$\sqrt{(2) + 7} + \sqrt{2 + 2} = \sqrt{6(2) + 13}$$

$$5 = 5$$

$\Rightarrow$ L.H.S. = R.H.S.

Put $x = \frac{-5}{3}$ in equation (1)

$$\sqrt{\frac{-5}{3} + 7} + \sqrt{\frac{-5}{3} + 2} = \sqrt{6\left(\frac{-5}{3}\right) + 13}$$

$$\frac{5}{\sqrt{3}} \neq \sqrt{3}$$

$\Rightarrow$ L.H.S. $\neq$ R.H.S.

$\Rightarrow$ $x = -\frac{5}{3}$ is an extraneous root.

Hence the solution set = $\{2\}$

**Q.6** $\sqrt{\mathbf{x^2 + x + 1}} - \sqrt{\mathbf{x^2 + x - 1}} = \mathbf{1}$

**Solution:**

$$\sqrt{x^2 + x + 1} - \sqrt{x^2 + x - 1} = 1 \qquad \text{......... (1)}$$

Let $\sqrt{x^2 + x + 1} = a$ and $\sqrt{x^2 + x - 1} = b$

Now $a^2 - b^2 = (x^2 + x + 1) - (x^2 + x - 1) = x^2 + x + 1 - x^2 - x + 1$

$$a^2 - b^2 = 2 \qquad \text{......... (2)}$$

The given equation can be written as

$$a - b = 1 \qquad \text{......... (3)}$$

Dividing equation (2) by equation (3)

$$\frac{a^2 - b^2}{a - b} = \frac{2}{1}$$



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$$\frac{(a+b)(a-b)}{(a-b)} = 2$$

$$a + b = 2 \qquad \ldots\ldots\ldots (4)$$

adding (3) and (4)

$$\Rightarrow \quad 2a = 3 \quad \Rightarrow \quad a = \frac{3}{2}$$

$$\Rightarrow \quad \sqrt{x^2 + x + 1} = \frac{3}{2}$$

$$\Rightarrow \quad x^2 + x + 1 = \frac{9}{4}$$

$$\Rightarrow \quad 4x^2 + 4x + 4 = 9$$

$$\Rightarrow \quad 4x^2 + 4x + 4 - 9 = 0$$

$$\Rightarrow \quad 4x^2 + 4x - 5 = 0$$

$$\Rightarrow \quad x = \frac{-4 \pm \sqrt{(4)^2 - 4(4)(-5)}}{2(4)}$$

$$= \frac{-4 \pm \sqrt{16 + 80}}{8} = \frac{-4 \pm \sqrt{96}}{8} = \frac{-4 \pm 4\sqrt{6}}{8} = \frac{-1 \pm \sqrt{6}}{2}$$

Checking

Put $x = \frac{-1 \pm \sqrt{6}}{2}$ in L.H.S. of equation (1)

$$\Rightarrow \quad \sqrt{\left(\frac{-1+\sqrt{6}}{2}\right)^2 + \left(\frac{-1+\sqrt{6}}{2}\right) + 1} - \sqrt{\left(\frac{-1+\sqrt{6}}{2}\right)^2 + \frac{-1+\sqrt{6}}{2} - 1} = 1 = \text{R.H.S.}$$

Put $x = \frac{-1 - \sqrt{6}}{2}$ in L.H.S. of equation (1)

$$\Rightarrow \quad \sqrt{\left(\frac{-1-\sqrt{6}}{2}\right)^2 + \left(\frac{-1-\sqrt{6}}{2}\right) + 1} - \sqrt{\left(\frac{-1-\sqrt{6}}{2}\right)^2 + \frac{-1-\sqrt{6}}{2} - 1} = 1 = \text{R.H.S.}$$

Hence the solution set $= \left\{\frac{-1 \pm \sqrt{6}}{2}\right\}$

**Q.7** $\sqrt{x^2 + 2x - 3} + \sqrt{x^2 + 7x - 8} = \sqrt{5(x^2 + 3x - 4)}$

**Solution:**

$$\sqrt{x^2 + 2x - 3} + \sqrt{x^2 + 7x - 8} = \sqrt{5(x^2 + 3x - 4)} \qquad \ldots\ldots\ldots (1)$$

$$\Rightarrow \quad \sqrt{x^2 + 3x - x - 3} + \sqrt{x^2 + 8x - x - 8} = \sqrt{5(x^2 + 4x - x - 4)}$$

$\Rightarrow \quad \sqrt{x(x+3)-1(x+3)}+\sqrt{x(x+8)-1(x+8)} = \sqrt{5(x+4)(x-1)}$

$\Rightarrow \quad \sqrt{(x+3)(x-1)}+\sqrt{(x+8)(x-1)} = \sqrt{5(x+4)(x-1)}$

$\Rightarrow \quad \sqrt{(x+3)(x-1)}+\sqrt{(x+8)(x-1)}-\sqrt{5(x+4)(x-1)} = 0$

$\Rightarrow \quad \sqrt{(x-1)}\left[\sqrt{x+3}+\sqrt{x+8}-\sqrt{5(x+4)}\right] = 0$

$\Rightarrow$ Either $\sqrt{x-1} = 0$ or $\sqrt{x+3}+\sqrt{x+8}-\sqrt{5(x+4)} = 0$

$\Rightarrow$ $x-1 = 0$ or $\sqrt{x+3}+\sqrt{x+8} = \sqrt{5(x+4)}$

$\Rightarrow$ $\boxed{x = 1}$ or Squaring on both sides

$\Rightarrow \quad x+3+x+8+2\sqrt{x+3}\sqrt{x+8} = 5(x+4)$

$\Rightarrow \quad 2x+11+2\sqrt{x^2+8x+3x+24} = 5x+20$

$\Rightarrow \quad 2\sqrt{x^2+11x+24} = 5x+20-2x-11$

$\Rightarrow \quad 2\sqrt{x^2+11x+24} = 3x+9$

Squaring again

$\Rightarrow \quad \left(2\sqrt{x^2+11x+24}\right)^2 = (3x+9)^2$

$\Rightarrow \quad 4(x^2+11x+24) = 9x^2+81+54x$

$\Rightarrow \quad 4x^2+44x+96 = 9x^2+81+54x$

$\Rightarrow \quad 9x^2+81+54x-4x^2-44x-96 = 0$

$\Rightarrow \quad 5x^2+10x-15 = 0$

$\Rightarrow \quad 5(x^2+2x-3) = 0$

$\Rightarrow \quad x^2+2x-3 = 0$

$\Rightarrow \quad x^2+3x-x-3 = 0$

$\Rightarrow \quad x(x+3)-1(x+3) = 0$

$\Rightarrow \quad (x+3)(x-1) = 0$

$\Rightarrow$ Either $x+3 = 0$ or $x-1 = 0$

$\Rightarrow$ $x = -3$ or $x = 1$

Checking:

Put $x = 1$ in equation (1)

$$\sqrt{(1)^2+2(1)-3}+\sqrt{(1)^2+7(1)-8} = \sqrt{5\left((1)^2+3(1)-4\right)}$$

$$0+0 = 0$$



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$\Rightarrow$ L.H.S. = R.H.S.

Put x = – 3 in equation (1)

$$\sqrt{(-3)^2 + 2(-3) - 3} + \sqrt{(-3)^2 + 7(-3) - 8} = \sqrt{5\left((-3)^2 + 3(-3) - 4\right)}$$

$$0 + \sqrt{-20} = \sqrt{-20}$$

$\Rightarrow$ L.H.S. = R.H.S.

Hence the solution set = $\{1, -3\}$

**Q.8** $\sqrt{2x^2 - 5x - 3} + 3\sqrt{2x + 1} = \sqrt{2x^2 + 25x + 12}$

**Solution:**

$$\sqrt{2x^2 - 5x - 3} + 3\sqrt{2x+1} = \sqrt{2x^2 + 25x + 12} \quad \text{......... (1)}$$

$\Rightarrow \sqrt{2x^2 - 6x + x - 3} + 3\sqrt{2x+1} = \sqrt{2x^2 + 24x + x + 12}$

$\Rightarrow \sqrt{2x(x-3) + 1(x-3)} + 3\sqrt{2x+1} = \sqrt{2x(x+12) + 1(x+12)}$

$\Rightarrow \sqrt{(2x+1)(x-3)} + 3\sqrt{2x+1} = \sqrt{(2x+1)(x+12)}$

$\Rightarrow \sqrt{(2x+1)(x-3)} + 3\sqrt{2x+1} - \sqrt{(2x+1)(x+12)} = 0$

$\Rightarrow \sqrt{2x+1}\left[\sqrt{x-3} + 3 - \sqrt{x+12}\right] = 0$

Either $\sqrt{2x+1} = 0$

$\Rightarrow 2x + 1 = 0$

$\Rightarrow 2x = -\frac{1}{2}$

or $\sqrt{x-3} + 3 - \sqrt{x+12} = 0$

$\sqrt{x-3} + 3 = \sqrt{x+12}$

Squaring on both sides

$x - 3 + 9 + 6\sqrt{x-3} = x + 12$

$6\sqrt{x-3} = x + 12 - x + 3 - 9$

$6\sqrt{x-3} = 6$

$\sqrt{x-3} = 1$

Squaring again

$x - 3 = 1$

$x = 4$

$\Rightarrow x = -\frac{1}{2}, 4$

Checking:

Put $x = -\frac{1}{2}$ in equation (1)

$$\sqrt{2\left(-\frac{1}{2}\right)^2 - 5\left(-\frac{1}{2}\right) - 3} + 3\sqrt{2\left(-\frac{1}{2}\right) + 1} = \sqrt{2\left(-\frac{1}{2}\right)^2 - 25\left(-\frac{1}{2}\right) + 12}$$

$$0 + 0 = 0$$

$\Rightarrow$ L.H.S. = R.H.S.

Put x = 4 in equation (1)

$\sqrt{2(4)^2 - 5(4) - 3} + 3\sqrt{2(4) + 1} = \sqrt{2(4) + 25(4) + 12}$

$3 + 9 = 12$

$\Rightarrow$ L.H.S. = R.H.S.

Hence the solution set = $\left\{-\frac{1}{2}, 4\right\}$

**Q.9** $\sqrt{3x^2 - 5x + 2} + \sqrt{6x^2 - 11x + 5} = \sqrt{5x^2 - 9x + 4}$ **(Lahore Board 2003)**

**Solution:**

$\sqrt{3x^2 - 5x + 2} + \sqrt{6x^2 - 11x + 5} = \sqrt{5x^2 - 9x + 4}$ ......... (1)

$\Rightarrow \sqrt{3x^2 - 3x - 2x + 2} + \sqrt{6x^2 - 6x - 5x + 5} = \sqrt{5x^2 - 5x - 4x + 4}$

$\Rightarrow \sqrt{3x(x-1) - 2(x-1)} + \sqrt{6x(x-1) - 5(x-1)} = \sqrt{5x(x-1) - 4(x-1)}$

$\Rightarrow \sqrt{(x-1)(3x-2)} + \sqrt{(x-1)(6x-5)} = \sqrt{(x-1)(5x-4)}$

$\Rightarrow \sqrt{(x-1)(3x-2)} + \sqrt{(x-1)(6x-5)} - \sqrt{(x-1)(5x-4)} = 0$

$\Rightarrow \sqrt{x-1}\left[\sqrt{3x-2} + \sqrt{6x-5} - \sqrt{5x-4}\right] = 0$

$\Rightarrow$ Either $\sqrt{3x-2} + \sqrt{6x-5} - \sqrt{5x-4} = 0$ or $\sqrt{x-1} = 0$

$\Rightarrow \sqrt{3x-2} + \sqrt{6x-5} = \sqrt{5x-4}$ or $x - 1 = 0$

or $x = 1$

Squaring on both sides

$3x - 2 + 6x - 5 + 2\sqrt{3x-2}\sqrt{6x-5} = 5x - 4$

$\Rightarrow 9x - 7 + 2\sqrt{(3x-2)(6x-5)} = 5x - 4$

$\Rightarrow 2\sqrt{18x^2 - 15x - 12x + 10} = 5x - 4 - 9x + 7$

$\Rightarrow 2\sqrt{18x^2 - 27x + 10} = -4x + 3$

$\Rightarrow -2\sqrt{18x^2 - 27x + 10} = 4x - 3$

Squaring again

$\Rightarrow 4(18x^2 - 27x + 10) = 16x^2 + 9 - 24x$

$\Rightarrow 72x^2 - 108x + 40 = 16x^2 + 9 - 24x$

$\Rightarrow 72x^2 - 108x + 40 - 16x^2 - 9 + 24x = 0$

$\Rightarrow 56x^2 - 84x + 31 = 0$

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$$\Rightarrow \quad x = \frac{+84 \pm \sqrt{(-84)^2 - 4(56)(31)}}{2(56)}$$

$$\Rightarrow \quad = \frac{84 \pm \sqrt{7056 - 6944}}{112} = \frac{84 \pm \sqrt{112}}{112}$$

$$\Rightarrow \quad = \frac{84 \pm 4\sqrt{7}}{112} = \frac{21 \pm \sqrt{7}}{28}$$

$$\Rightarrow \quad x = 1, \quad \frac{21 \pm \sqrt{7}}{28}$$

Checking:

Put $x = 1$ in equation (1)

$$\sqrt{3(1)^2 - 5(1) + 2} + \sqrt{6(1)^2 - 11(1) + 5} = \sqrt{5(1)^2 - 9(1) + 4}$$

$$0 + 0 = 0$$

$\Rightarrow$ L.H.S. = R.H.S.

Put $x = \frac{21 + \sqrt{7}}{28}$ in equation (1)

$$\sqrt{3\left(\frac{21+\sqrt{7}}{28}\right)^2 - 5\left(\frac{21+\sqrt{7}}{28}\right) + 2} + \sqrt{6\left(\frac{21+\sqrt{7}}{28}\right)^2 - 11\left(\frac{21+\sqrt{7}}{28}\right) + 5}$$

$$= \sqrt{5\left(\frac{21+\sqrt{7}}{28}\right)^2 - 9\left(\frac{21+\sqrt{7}}{28}\right) + 4}$$

$$\sqrt{-0.083} + \sqrt{-0.010} \neq \sqrt{-0.035}$$

$\Rightarrow$ L.H.S. $\neq$ R.H.S.

Put $x = \frac{21 - \sqrt{7}}{28}$ in equation (1)

$$\sqrt{3\left(\frac{21-\sqrt{7}}{28}\right)^2 - 5\left(\frac{21-\sqrt{7}}{28}\right) + 2} + \sqrt{6\left(\frac{21-\sqrt{7}}{28}\right)^2 - 11\left(\frac{21-\sqrt{7}}{28}\right) + 5}$$

$$= \sqrt{5\left(\frac{21-\sqrt{7}}{28}\right)^2 - 9\left(\frac{21-\sqrt{7}}{28}\right) + 4}$$

L.H.S. $\neq$ R.H.S.

$\Rightarrow$ $x = \frac{21 \pm \sqrt{7}}{28}$ are extraneous roots .

Hence the solution set = $\{1\}$



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**Q.10** $(x + 4)(x + 1) = \sqrt{x^2 + 2x - 15} + 3x + 31$

**Solution:**

$$(x + 4)(x + 1) = \sqrt{x^2 + 2x - 15} + 3x + 31$$

$$\Rightarrow \quad x^2 + x + 4x + 4 = \sqrt{x^2 + 2x - 15} + 3x + 31$$

$$\Rightarrow \quad x^2 + 5x + 4 - \sqrt{x^2 + 2x - 15} - 3x - 31 = 0$$

$$\Rightarrow \quad x^2 + 2x - 27 - \sqrt{x^2 + 2x - 15} = 0 \quad \text{......... (1)}$$

Put $\sqrt{x^2 + 2x - 15} = y$ ......... (2)

Then $x^2 + 2x - 15 = y^2$

$$x^2 + 2x = y^2 + 15$$

Equation (1) becomes

$$y^2 + 15 - 27 - y = 0$$

$$\Rightarrow \quad y^2 - y - 12 = 0$$

$$\Rightarrow \quad y^2 - 4y + 3y - 12 = 0$$

$$\Rightarrow \quad y(y - 4) + 3(y - 4) = 0$$

$$\Rightarrow \quad (y - 4)(y + 3) = 0$$

$\Rightarrow$ Either $y - 4 = 0$ or $y + 3 = 0$

$\Rightarrow$ $y = 4$ or $y = -3$

Put $y = 4$ and $y = -3$ in equation (2)

| | |
|---|---|
| $\sqrt{x^2 + 2x - 15} = 4$ | $\sqrt{x^2 + 2x - 15} = -3$ |
| $\Rightarrow x^2 + 2x - 15 = 16$ | $\Rightarrow x^2 + 2x - 15 = 9$ |
| $\Rightarrow x^2 + 2x - 15 - 16 = 0$ | $\Rightarrow x^2 + 2x - 15 - 9 = 0$ |
| $\Rightarrow x^2 + 2x - 31 = 0$ | $\Rightarrow x^2 + 2x - 24 = 0$ |



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$$\Rightarrow \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-31)}}{2(1)}$$
$$= \frac{-2 \pm \sqrt{4 + 24}}{2}$$
$$= \frac{-2 \pm \sqrt{128}}{2}$$
$$= \frac{-2 \pm 8\sqrt{2}}{2}$$
$$= -1 \pm 4\sqrt{2}$$

$$\Rightarrow \quad x^2 + 6x - 4x - 24 = 0$$
$$\Rightarrow \quad x(x + 6) - 4(x + 6) = 0$$
$$\Rightarrow \quad (x + 6)(x - 4) = 0$$
$\Rightarrow$ Either $x + 6 = 0$ or $x - 4 = 0$
$\Rightarrow$ $x = -6$ or $x = 4$

$$x = -6, 4, -1 \pm 4\sqrt{2}$$

Checking:

Put $x = -6$ in L.H.S. of equation (1)

$$(-6)^2 + 2(-6) - 27 - \sqrt{(-6)^2 + 2(-6) - 15} - 6 \neq 0$$

$\Rightarrow$ L.H.S. $\neq$ R.H.S.

Put $x = 4$ in L.H.S. of equation (1)

$$(4)^2 + 2(4) - 27 - \sqrt{(4)^2 + 2(4) - 15} - 6 \neq 0$$

$\Rightarrow$ L.H.S. $\neq$ R.H.S.

Put $x = -1 + 4\sqrt{2}$ in L.H.S. of equation (1)

$$\left(-1 + 4\sqrt{2}\right)^2 + 2\left(-1 + 4\sqrt{2}\right) - 27 - \sqrt{\left(-1 + 4\sqrt{2}\right)^2 + 2\left(-1 + 4\sqrt{2}\right) - 15} = 0 = \text{R.H.S.}$$

$\Rightarrow$ L.H.S. = R.H.S.

Put $x = -1 - 4\sqrt{2}$ in L.H.S. of equation (1)

$$\left(-1 - 4\sqrt{2}\right)^2 + 2\left(-1 - 4\sqrt{2}\right) - 27 - \sqrt{\left(-1 - 4\sqrt{2}\right)^2 + 2\left(-1 - 4\sqrt{2}\right) - 15} = 0 = \text{R.H.S.}$$

$\Rightarrow$ L.H.S. = R.H.S.

$\Rightarrow$ $x = -6$, and $x = 4$ are extraneous roots.

Hence the solution set $= \left\{-1 \pm 4\sqrt{2}\right\}$

**Q.11** $\sqrt{3x^2 - 2x + 9} + \sqrt{3x^2 - 2x - 4} = 13$

**Solution:**

$$\sqrt{3x^2 - 2x + 9} + \sqrt{3x^2 - 2x - 4} = 13 \quad \text{......... (1)}$$

Let $\sqrt{3x^2 - 2x + 9} = a$ and $\sqrt{3x^2 - 2x - 4} = b$

Now $a^2 - b^2 = (3x^2 - 2x + 9) - (3x^2 - 2x - 4)$

$= 3x^2 - 2x + 9 - 3x^2 + 2x + 4$

$a^2 - b^2 = 13$ ......... (2)

Equation (1) can be written as

$a + b = 13$ ......... (3)

Dividing (2) by (3)

$$\frac{a^2 - b^2}{a + b} = \frac{13}{13}$$

$$\frac{(a + b)(a - b)}{a + b} = 1$$

$a - b = 1$ ......... (4)

Adding (3) and (4)

$2a = 14 \Rightarrow a = 7$

$\Rightarrow \sqrt{3x^2 - 2x + 9} = 7$

$\Rightarrow 3x^2 - 2x + 9 = 49$

$\Rightarrow 3x^2 - 2x + 9 - 49 = 0$

$\Rightarrow 3x^2 - 2x - 40 = 0$

$\Rightarrow 3x^2 - 12x + 10x - 40 = 0$

$\Rightarrow 3x(x - 4) + 10(x - 4) = 0$

$\Rightarrow (x - 4)(3x + 10) = 0$

$\Rightarrow$ Either $x - 4 = 0$ or $3x + 10 = 0$

$\Rightarrow x = 4$ or $x = \frac{-10}{3}$

Checking:

Put $x = 4$ in L.H.S. of equation (1)

$\sqrt{3(4)^2 - 2(4) + 9} + \sqrt{3(4)^2 - 2(4) - 4}$

$= 7 + 6 = 13 =$ R.H.S. $\Rightarrow$ L.H.S. $=$ R.H.S.

Put $x = -\frac{10}{3}$ in L.H.S. of equation (1)

$$\sqrt{3\left(-\frac{10}{3}\right)^2 - 2\left(-\frac{10}{3}\right) + 9} + \sqrt{3\left(-\frac{10}{3}\right)^2 - 2\left(-\frac{10}{3}\right) - 4} = 7 + 6 = 13 = \text{R.H.S.}$$



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$\Rightarrow$ L.H.S. = R.H.S.

Hence the solution set $= \left\{4, -\dfrac{10}{3}\right\}$

**Q.12** $\sqrt{5x^2 + 7x + 2} - \sqrt{4x^2 + 7x + 18} = x - 4$ **(Gujranwala Board 2003)**

**Solution:**

$$\sqrt{5x^2 + 7x + 2} - \sqrt{4x^2 + 7x + 18} = x - 4 \quad \text{......... (1)}$$

Let $\sqrt{5x^2 + 7x + 2} = a$ and $\sqrt{4x^2 + 7x + 18} = b$

Then $a^2 - b^2 = (5x^2 + 7x + 2) - (4x^2 + 7x + 18)$

$= 5x^2 + 7x + 2 - 4x^2 - 7x - 18$

$$a^2 - b^2 = x^2 - 16 \quad \text{......... (2)}$$

Equation (1) can be written as

$$a - b = x - 4 \quad \text{......... (3)}$$

Divide (2) by (3)

$$\frac{a^2 - b^2}{a - b} = \frac{x^2 - 16}{x - 4}$$

$$\frac{(a - b)(a + b)}{(a - b)} = \frac{(x + 4)(x - 4)}{(x - 4)}$$

$$a + b = x + 4 \quad \text{......... (4)}$$

Adding (3) and (4)

$2a = 2x$

$\Rightarrow$ $a = x$

$\Rightarrow$ $\sqrt{5x^2 + 7x + 2} = x$

$\Rightarrow$ $5x^2 + 7x + 2 = x^2$

$\Rightarrow$ $5x^2 - x^2 + 7x + 2 = 0$

$\Rightarrow$ $4x^2 + 7x + 2 = 0$

$$\Rightarrow \quad x = \frac{-7 \pm \sqrt{(7)^2 - 4(4)(2)}}{2(4)}$$

$$= \frac{-7 \pm \sqrt{49 - 32}}{8} = \frac{-7 \pm \sqrt{17}}{8}$$

$$\Rightarrow \quad x = \frac{-7 + \sqrt{17}}{8}, \quad \frac{-7 - \sqrt{17}}{8}$$

Checking:

Put $x = \dfrac{-7+\sqrt{17}}{8}$ in equation (1)

$$\sqrt{5\left(\frac{-7+\sqrt{17}}{8}\right)^2 + 7\left(\frac{-7+\sqrt{17}}{8}\right) + 2} - \sqrt{4\left(\frac{-7+\sqrt{17}}{8}\right)^2 + 7\left(\frac{-7+\sqrt{17}}{8}\right) + 18} = \frac{-7+\sqrt{17}}{8} - 4$$

$$\sqrt{5\,(-0.3596)^2 + 7\,(-0.3596) + 2} - \sqrt{4\,(-0.3596)^2 + 7\,(-0.359) + 18} = -0.3596 - 4$$

$$= -4.3596$$

$$\sqrt{-1.8706 + 2} - \sqrt{-1.8706 + 18} = -4.3596$$

$$0.3597 - 4.0161 = -4.3596$$

$$-3.6564 = -4.3596$$

Put, $x = \dfrac{-7-\sqrt{17}}{8}$ in equation (i)

$$\sqrt{5\left(\frac{-7-\sqrt{17}}{8}\right)^2 + 7\left(\frac{-7-\sqrt{17}}{8}\right) + 2} - \sqrt{4\left(\frac{-7-\sqrt{17}}{8}\right)^2 + 7\left(\frac{-7-\sqrt{17}}{8}\right) + 18} = \frac{-7-\sqrt{17}}{8} - 4$$

$$\sqrt{5(-1.39)^2 + 7(-1.39) + 2} - \sqrt{4(-1.39)^2 + 7(-1.39) + 18} = -1.39 - 4$$

$$\sqrt{5(1.93) - 9.73 + 2} - \sqrt{4(1.93) - 9.73 + 18} = -5.39$$

$$\sqrt{1.92} - \sqrt{15.99} = -5.39$$

$$1.38 - 3.99 = -5.39$$

$$-2.61 = -5.39$$

$\Rightarrow$ L.H.S $\neq$ R.H.S

$\Rightarrow$ $x = \dfrac{-7 \pm \sqrt{17}}{8}$ are extraneous roots

$\Rightarrow$ S.S $= \phi$

**THREE CUBE ROOTS OF UNITY**

As we know that square roots of one (unity) are two, 1 and – 1. Similarly cube roots of one (unity) are three and these can be calculated as:

Let ‘x’ be the cube root of unity, then

$$1^{1/3} = x$$

$\Rightarrow$ $1 = x^3$ $\Rightarrow$ $x^3 = 1$ $\Rightarrow$ $(x)^3 - (1)^3 = 0$