EXERCISE 2.3

Q.1 Verify the commutative properties of union and intersection for the following pair of sets.

(i) $A = \{1, 2, 3, 4, 5\}, \quad B = \{4, 6, 8, 10\}$

(ii) N, Z

(iii) $A = \{x \mid x \in R \wedge x \geq 0\} \quad B = R$

Solution:

(i) $A = \{1, 2, 3, 4, 5\} \quad B = \{4, 6, 8, 10\}$

$$A \cup B = , 2, 3, 4, 5\} \cup \{4, 6, 8, 10\}$$

$$= \{1, 2, 3, 4, 5, 6, 8, 10\}$$

$$B \cup A = \{4, 6, 8, 10\} \cup \{1, 2, 3, 4, 5\}$$

$$= \{1, 2, 3, 4, 5, 6, 8, 10\}$$

$\therefore \quad A \cup B = B \cup A$

$$A \cap B = \{1, 2, 3, 4, 5\} \cap \{4, 6, 8, 10\}$$

$$= \{4\}$$

$$B \cap A = \{4, 6, 8, 10\} \cap \{1, 2, 3, 4, 5\}$$

$$= \{4\}$$

$\therefore \quad A \cap B = B \cap A$

Commutative properties of union and intersection are verified.

(ii) N, Z

$N \cup Z = Z$

and $Z \cup N = Z$

$\Rightarrow \quad N \cup Z = Z \cup N$

Now

$N \cap Z = N \quad$ and $\quad Z \cap N = N$

$\Rightarrow \quad N \cap Z = Z \cap N$

Commutative properties of union and intersection are verified.

(iii) $A = \{x \mid x \in R \wedge x \geq 0\} \quad B = R$

Commutative property of union

$$A \cup B = \{x \mid x \in R \wedge x \geq 0\} \cup R$$

$$= R$$

$$(B \cup A) = R \cup \{x \mid x \in R \wedge x \geq 0\}$$
$$= R$$
$$A \cap B = \{x \mid x \in R \wedge x \geq 0\} \cap R$$
$$= \{x \mid x \in R \wedge x \geq 0\}$$
$$B \cap A = R \cap \{x \mid x \in R \wedge x \geq 0\}$$
$$= \{x \mid x \in R \wedge x \geq 0\}$$

$\Rightarrow$ Commutative properties of union and intersection are verified.

Q.2 Verify the properties for the sets A, B, C given below:

(i) Associativity of Union $A \cup (B \cup C) = (A \cup B) \cup C$

(a) $A = \{1, 2, 3, 4\}$ $B = \{3, 4, 5, 6, 7, 8\}$ $C = \{5, 6, 7, 9, 10\}$

(b) $A = \phi$, $B = \{0\}$, $C = \{0, 1, 2\}$

(c) N, Z, Q

(ii) Associativity of intersection $A \cap (B \cap C) = (A \cap B) \cap C$

(a) $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7, 8\}$, $C = \{5, 6, 7, 9, 10\}$

(b) $A = \phi$, $B = \{0\}$, $C = \{0, 1, 2\}$

(c) N, Z, Q

(iii) Distributivity of union over intersection (Lahore Board 2005)

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(a) $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7, 8\}$, $C = \{5, 6, 7, 9, 10\}$

(b) $A = \phi$, $B = \{0\}$, $C = \{0, 1, 2\}$

(c) N, Z, Q

(iv) Distributivity of intersection over union

$A \cup C\ (B \cap C) = (A \cup B) \cap (A \cup C)$

(a) $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7, 8\}$, $C = \{5, 6, 7, 9, 10\}$

(b) $A = \phi$ $B = \{0\}$ $C = \{0, 1, 2\}$, **(c)** N, Z, Q

Solution:

(i) Associativity of union

(a) $A = \{1, 2, 3, 4\}$ $B = \{3, 4, 5, 6, 7, 8\}$ $C = \{5, 6, 7, 9, 10\}$

i.e. $(A \cup B) \cup C = A \cup (B \cup C)$

L.H.S.

$$(A \cup B) \cup C = (\{1, 2, 3, 4\} \cup \{3, 4, 5, 6, 7, 8\}) \cup \{5, 6, 7, 9, 10\}$$
$$= \{1, 2, 3, 4, 5, 6, 7, 8\} \cup \{5, 6, 7, 9, 10\}$$
$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$\text{R.H.S} = A \cup (B \cup C)$$
$$= \{1, 2, 3, 4\} \cup (\{3, 4, 5, 6, 7, 8\} \cup \{5, 6, 7, 9, 10\})$$
$$= \{1, 2, 3, 4\} \cup \{3, 4, 5, 6, 7, 8, 9, 10\}$$
$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$\Rightarrow$ $(A \cup B) \cup C = (A \cup B) \cup C$

(b) $\mathbf{A = \phi \quad , \quad B = \{0\} \quad , \quad C = \{0, 1, 2\}}$

To show $(A \cup B) \cup C = A \cup (B \cup C)$

L.H.S.

$$(A \cup B) \cup C = (\phi \cup \{0\}) \cup \{0, 1, 2\}$$
$$= \{0\} \cup \{0, 1, 2\}$$
$$= \{0, 1, 2\}$$

R.H.S.

$$A \cup (B \cup C) = \phi \cup (\{0\} \cup \{0, 1, 2\})$$
$$= \phi \cup \{0, 1, 2\}$$
$$= \{0, 1, 2\}$$

$\Rightarrow$ $(A \cup B) \cup C = A \cup (B \cup C)$

(c) **N, Z, Q**

To show $(N \cup Z) \cup Q = N \cup (Z \cup Q)$

$(N \cup Z) \cup Q = Z \cup Q = Q$

$N \cup (Z \cup Q) = N \cup Q = Q$

(ii) **Associativity of intersection**

(a) $\mathbf{A = \{1, 2, 3, 4\} \quad , \quad B = \{3, 4, 5, 6, 7, 8\} \quad , \quad C = \{5, 6, 7, 9, 10\}}$

Associativity of intersection is

$(A \cap B) \cap C = A \cap (B \cap C)$

L.H.S.

$$(A \cap B) \cap C = (\{1, 2, 3, 4\} \cap \{3, 4, 5, 6, 7, 8\}) \cap \{5, 6, 7, 9, 10\}$$
$$= \{3, 4\} \cap \{5, 6, 7, 9, 10\}$$
$$= \{\ \}$$

$A \cap (B \cap C) = \{1, 2, 3, 4\} \cap \{5, 6, 7\} = \{\ \}$

R.H.S.

$$A \cap (B \cap C) = \{1, 2, 3, 4\} \cap (\{3, 4, 5, 6, 7, 8\} \cap \{5, 6, 7, 9, 10\})$$
$$= \{1, 2, 3, 4\} \cap \{5, 6, 7\}$$
$$= \{\ \}$$

$(A \cap B) \cap C = A \cap (B \cap C)$

(b) $\mathbf{A = \phi \quad , \quad B = \{0\} \quad , \quad C = \{0, 1, 2\}}$

To show $(A \cap B) \cap C = A \cap (B \cap C)$

L.H.S.

$$\begin{aligned}(A \cap B) \cap C &= (\phi \cap \{0\}) \cap \{0, 1, 2\} \\ &= \{\ \ \} \cap \{0, 1, 2\} \\ &= \{\ \ \}\end{aligned}$$

R.H.S.

$$\begin{aligned}A \cap (B \cap C) &= \{\ \ \} \cap (\{0\} \cap \{0, 1, 2\}) \\ &= \{\ \ \} \cap \{0\} \\ &= \{\ \ \}\end{aligned}$$

$\Rightarrow \quad (A \cap B) \cap C = A \cap (B \cap C)$

(c) **N, Z, Q**

To show $(N \cap Z) \cap Q = N \cap (Z \cap Q)$

L.H.S.

$$(N \cap Z) \cap Q = N \cap Q = N$$

R.H.S.

$$N \cap (Z \cap Q) = N \cap Z = N$$

$\Rightarrow \quad (N \cap Z) \cap Q = N \cap (Z \cap Q)$

(iii) **Distributivity of Union over intersection**

(a) $\mathbf{A = \{1, 2, 3, 4\} \quad , \quad B = \{3, 4, 5, 6, 7, 8\} \quad , \quad C = \{5, 6, 7, 9, 10\}}$

Distributivity of Union over intersection

i.e. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S.

$$\begin{aligned}A \cup (B \cap C) &= \{1, 2, 3, 4\} \cup (\{3, 4, 5, 6, 7, 8\} \cap \{5, 6, 7, 9, 10\}) \\ &= \{1, 2, 3, 4\} \cup \{5, 6, 7\} \\ &= \{1, 2, 3, 4, 5, 6, 7\}\end{aligned}$$

R.H.S.

$$\begin{aligned}(A \cup B) \cap (A \cup C) &= (\{1, 2, 3, 4\} \cup \{3, 4, 5, 6, 7, 8\}) \cap (\{1, 2, 3, 4\} \cup \{5, 6, 7, 9, 10\}) \\ &= \{1, 2, 3, 4, 5, 6, 7, 8\} \cap \{1, 2, 3, 4, 5, 6, 7, 9, 10\}) \\ &= \{1, 2, 3, 4, 5, 6, 7\}\end{aligned}$$

$\Rightarrow \quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $\mathbf{A = \phi \quad , \quad B = \{0\} \quad , \quad C = \{0, 1, 2\}}$

To show $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S.

$$A \cup (B \cap C) = \{\ \} \cup (\{0\} \cap \{0, 1, 2\})$$
$$= \{\ \} \cup \{0\} = \{0\}$$

R.H.S.

$$(A \cup B) \cap (A \cup C) = (\{\ \} \cup \{0\}) \cap (\{\ \} \cup \{0, 1, 2\})$$
$$= \{0\} \cap \{0, 1, 2\} = \{0\}$$

$\Rightarrow$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(c) **N, Z, Q**

To show $N \cup (Z \cap Q) = (N \cup Z) \cap (N \cup Q)$

L.H.S.

$$N \cup (Z \cap Q) = N \cup Z = Z$$

R.H.S.

$$(N \cup Z) \cap (N \cup Q) = Z \cap Q = Z$$

$\Rightarrow$ $N \cup (Z \cap Q) = (N \cup Z) \cap (N \cup Q)$

(iv) **Distributivity of intersection over union.**

(a) $\mathbf{A = \{1, 2, 3, 4\} \quad , \quad B = \{3, 4, 5, 6, 7, 8\} \quad , \quad C = \{5, 6, 7, 9, 10\}}$

Distributivity of intersection over union

i.e. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

To show

L.H.S.

$$A \cap (B \cup C) = \{1, 2, 3, 4\} \cap (\{3, 4, 5, 6, 7, 8\} \cup \{5, 6, 7, 9, 10\})$$
$$= \{1, 2, 3, 4\} \cap \{3, 4, 5, 6, 7, 8, 9, 10\}$$
$$= \{3, 4\}$$

R.H.S.

$$(A \cap B) \cup (A \cap C) = (\{1, 2, 3, 4\} \cap \{3, 4, 5, 6, 7, 8\}) \cup (\{1, 2, 3, 4\} \cap \{5,6,7,9,10\})$$
$$= \{3, 4\} \cup \{\ \}$$
$$= \{3, 4\}$$

$\Rightarrow$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(b) $\mathbf{A = \phi \quad , \quad B = \{0\} \quad , \quad C = \{0, 1, 2\}}$

To show $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S.

$$A \cap (B \cup C) = \{\ \} \cap (\{0\} \cup \{0, 1, 2\})$$
$$= \{\ \} \cap \{0, 1, 2\} = \{\ \}$$

R.H.S.

$$(A \cap B) \cup (A \cap C) = (\{\ \} \cap \{0\}) \cup (\{\ \} \cup \{0, 1, 2\})$$
$$= \{\ \} \cup \{\ \}$$
$$= \{\ \}$$

$\Rightarrow \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(c) N, Z, Q

To show $N \cap (Z \cup Q) = (N \cap Z) \cup (N \cap Q)$

L.H.S.

$$N \cap (Z \cup Q) = N \cap Z = N$$

R.H.S.

$$(N \cap Z) \cup (N \cap Q) = N \cup N = N$$

$\Rightarrow \quad N \cap (Z \cup Q) = (N \cap Z) \cup (N \cap Q)$

Q.3 Verify De–Morgan's Laws for the following sets

$\mathbf{U = \{1, 2, 3, \ldots\ldots 20\}, \quad A = \{2, 4, 6, \ldots\ldots 20\}, \quad B = \{1, 3, 5, \ldots\ldots 19\}}$.

Solution:

De Morgan's Laws are

$$(A \cup B)' = A' \cap B'$$
$$(A \cap B)' = A' \cup B'$$

$$A \cup B = \{2, 4, 6, \ldots\ldots. 20\} \cup \{1, 3, 5, \ldots\ldots., 19\}$$
$$= \{1, 2, 3, \ldots\ldots. 20\}$$

$$(A \cup B)' = U - (A \cup B)$$
$$= U - \{1, 2, 3, \ldots\ldots. 20\}$$
$$= \{1, 2, 3, \ldots\ldots., 20\} - \{1, 2, 3, \ldots\ldots. 20\}$$
$$= \{\ \} \qquad \ldots\ldots\ldots (1)$$

Now, $A' = U - A$

$= \{1, 2, 3, \ldots\ldots., 20\} - \{2, 4, 6, \ldots\ldots., 20\}$

$= \{1, 3, 5, \ldots\ldots., 19\}$

$B' = U - B$

$= \{1, 2, 3, \ldots\ldots., 20\} - \{1, 3, 5, \ldots\ldots., 19\}$

$= \{2, 4, 6, \ldots\ldots., 20\}$

$A' \cap B' = \{1, 3, 5, \ldots\ldots., 19\} \cap \{2, 4, 6, \ldots\ldots., 20\}$

$= \{\ \}$ ……… (2)

From equations (1) and (2) it is clear that

$(A \cup B)' = A' \cap B'$

Now $A \cap B = \{2, 4, 6, \ldots\ldots, 20\} \cap \{1, 3, 5, \ldots\ldots., 19\}$

$= \{\ \}$

$(A \cap B)' = U - (A \cap B)$

$= \{1, 2, 3, \ldots\ldots., 20\} - \{\ \}$

$= \{1, 2, 3, \ldots\ldots., 20\}$ ……… (3)

$A' = U - A$

$= \{1, 2, 3, \ldots\ldots., 20\} - \{2, 4, 6, \ldots\ldots., 20\}$

$= \{1, 3, 5, \ldots\ldots., 19\}$

$B' = U - B$

$= \{1, 2, 3, \ldots\ldots., 20\} - \{1, 3, 5, \ldots\ldots., 19\}$

$= \{2, 4, 6, \ldots\ldots., 20\}$

$A' \cup B' = \{1, 3, 5, \ldots\ldots., 19\} \cup \{2, 4, 6, \ldots\ldots., 20\}$

$= \{1, 2, 3, 4, \ldots\ldots., 20\}$ ……… (4)

From equations (3) and (4) it is clear that $(A \cup B)' = A' \cup B'$.

Q.4 Let U = the set of all the English alphabet.

$A = \{x \mid x$ is a vowel$\}$, $B = \{y \mid y$ is a consonant$\}$

Verify De–Morgan's Laws for these sets.

Solution:

De–Morgan's Law are

$(A \cup B)' = A' \cap B'$ and

$(A \cap B)' = A' \cup B'$

$$A \cup B = \{x \mid x \text{ is a vowel}\} \cup \{y \mid y \text{ is a consonant}\}$$
$$= \text{The set of the English alphabet.}$$

$$(A \cup B)' = U - A \cup B$$
$$= \text{The set of English alphabet} - \text{The set of English alphabet}$$
$$= \{\ \} \qquad \text{......... (1)}$$

$$A' = U - A$$
$$= \text{The set of English alphabet} - \{x \mid x \text{ is a vowel}\}$$
$$= \{y \mid y \text{ is a consonant}\}$$

$$B' = U - B$$
$$= \text{The set of English alphabet} - \{y \mid y \text{ is a consonant}\}$$
$$= \{x \mid x \text{ is a vowel}\}$$

$$A' \cap B' = \{y \mid y \text{ is a consonant}\} \cap \{x \mid x \text{ is a vowel}\}$$
$$= \{\ \} \qquad \text{......... (2)}$$

From equations (1) and (2) $(A \cup B)' = A' \cap B'$.

Now $A \cap B = \{x \mid x \text{ is a vowel}\} \cap \{y \mid y \text{ is a consonant}\}$
$$= \{\ \}$$

$$(A \cap B)' = U - A \cap B$$
$$= \text{The set of English alphabet} - \{\ \}$$
$$= \text{The set of English alphabet} \qquad \text{......... (1)}$$

$$A' = U - A$$
$$= \text{The set of English alphabet} - \{x \mid x \text{ is a vowel}\}$$
$$= \{y \mid y \text{ is a consonant}\}$$

$$B' = U - B$$
$$= \text{The set of English alphabet} - \{y \mid y \text{ is a consonant}\}$$
$$= \{x \mid x \text{ is a vowel}\}$$

$$A' \cup B' = \{y \mid y \text{ is a consonant}\} \cup \{x \mid x \text{ is a vowel}\}$$
$$= \text{The set of English alphabet} \qquad \text{......... (2)}$$

From equations (1) and (2).

It is clear that $(A \cap B)' = A' \cap B'$.

Q.5 With help of Venn Diagram, verify the two distributive laws in the following sets w.r.t. union and intersection.

(i) $A \subseteq B$, $A \cap C = \phi$ and B and C are overlapping.

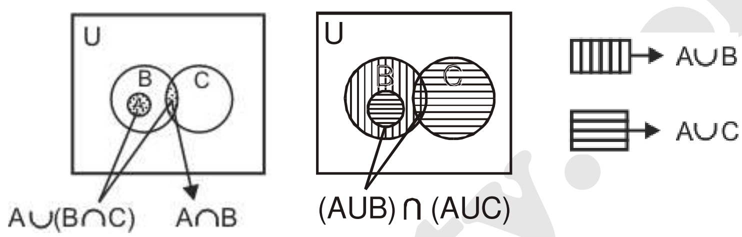
(ii) A and B are overlapping, B and C are overlapping but A and C are disjoint.

Solution:

(i) $A \subseteq B$, $A \cap C = \phi$ and B and C are overlapping

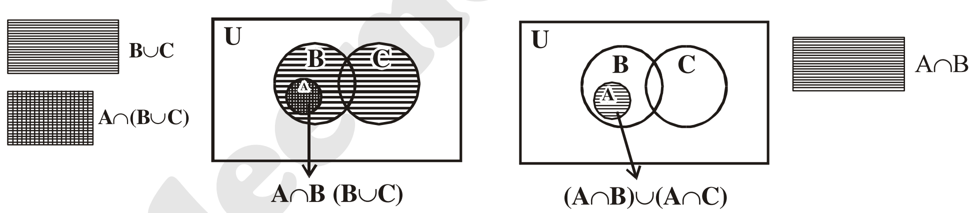
Distributivity of union over intersection

i.e. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$



Distributivity of intersection over union i.e.

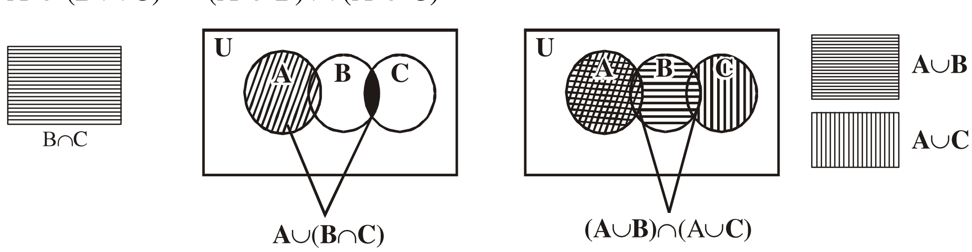
$A \cap (B \cup C) = (A \cap B) \cup (A \cup C)$



(ii) A and B are overlapping, B and C are overlapping, but A and C are disjoint.

Distributivity of union over intersection

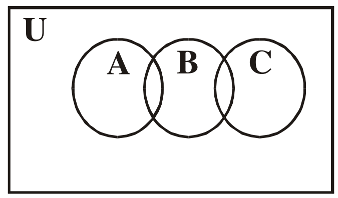
i.e. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$



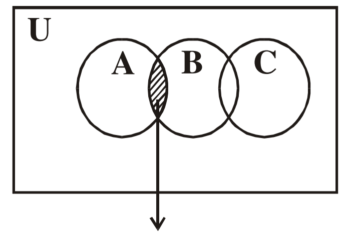
Distributivity of intersection over union.

i.e. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$\rightarrow B \cup C$



$A \cap (B \cup C)$



$(A \cap B) \cup (A \cap C)$

Q.6 Taking any set, say $A = \{1, 2, 3, 4, 5\}$ verify the following

(i) $A \cup \phi = A$ (ii) $A \cup A = A$ (iii) $A \cap A = A$

Solution:

(i) $A \cup \phi = A$

$A = \{1, 2, 3, 4, 5\}$

$A \cup \phi = \{1, 2, 3, 4, 5\} \cup \{\ \}$

$= \{1, 2, 3, 4, 5\}$

$= A$

$\Rightarrow$ $A \cup \phi = A$

(ii) $A \cup A = A$

$A\ \{1, 2, 3, 4, 5\}$

$A \cup A = \{1, 2, 3, 4, 5\} \cup \{1, 2, 3, 4, 5\}$

$= \{1, 2, 3, 4, 5\} = A$

$\Rightarrow$ $A \cup A = A$

(iii) $A \cap A = A$

$A = \{1, 2, 3, 4, 5\}$

$A \cap A = \{1, 2, 3, 4, 5\} \cap \{1, 2, 3, 4, 5\}$

$= \{1\ 2, 3, 4, 5\} = A$

$\Rightarrow$ $A \cap A = A$

Q.7 If $U = \{1, 2, 3, 4, 5, \ldots\ldots 20\}$, $A = \{1, 3, 5, \ldots\ldots 19\}$ verify the following

(i) $A \cup A' = U$ (ii) $A \cap U = A$ (iii) $A \cap A' = \phi$

Solution:

(i) $A \cup A' = U$

$A' = U - A = \{1, 2, 3, 4, 5, \ldots\ldots 20\} - \{1, 3, 5, \ldots\ldots 19\}$

$= \{2, 4, 6, 8, \ldots\ldots 20\}$

$\Rightarrow$ $A \cup A' = \{1, 3, 5, \ldots\ldots 19\} \cup \{2, 4, 6, \ldots\ldots 20\}$

$= \{1, 2, 3, 4, 5, \ldots\ldots 20\} = U$

$\Rightarrow$ $A \cup A' = U$

(ii) $\mathbf{A \cap U = A}$

$A \cap U = \{1, 3, 5, \ldots\ldots 19\} \cap \{1, 2, 3, 4, 5, \ldots\ldots 20\}$

$= \{1, 3, 5, \ldots\ldots 19\} = A$

$\Rightarrow \quad A \cap U = A$

(iii) $\mathbf{A \cap A' = \phi}$

$A' = U - A$

$= \{1, 2, 3, 4, 5, \ldots\ldots 20\} - \{1, 3, 5, \ldots\ldots 19\}$

$= \{2, 4, 6, \ldots\ldots 20\}$

$A \cap A' = \{1, 3, 5, \ldots\ldots 19\} \cap \{2, 4, 6, \ldots\ldots 20\}$

$= \{\ \} = \phi$

$\Rightarrow \quad A \cap A' = \phi$

Q.8 From suitable properties of union and intersection deduce the following results:

(i) $\mathbf{A \cap (A \cup B) = A \cup (A \cap B)}$ **(Lahore Board 2007, 2010)**

(ii) $\mathbf{A \cup (A \cap B) = A \cap (A \cup B)}$ **(Gujranwala Board 2003)**

Solution:

(i) $\mathbf{A \cap (A \cup B) = A \cup (A \cap B)}$

L.H.S. $= A \cap (A \cup B)$

$= (A \cap A) \cup (A \cap B)$ Using distributive law

$= A \cup (A \cap B)$ ¤ $A \cap A = A$

$=$ R.H.S.

(ii) $\mathbf{A \cup (A \cap B) = A \cap (A \cup B)}$

L.H.S. $= A \cup (A \cap B)$

$= (A \cup A) \cap (A \cup B)$ by distributive law

$= A \cap (A \cup B)$ ¤ $A \cup A = A$

$=$ R.H.S.

Q.9 Using Venn Diagram, verify the following results.

(i) $\mathbf{A \cap B' = A \text{ iff } A \cap B = \phi}$

(ii) $\mathbf{(A - B) \cup B = A \cup B}$

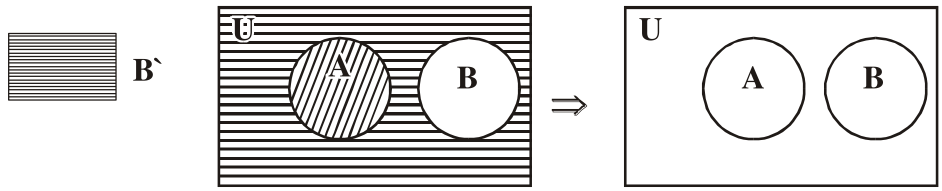
(iii) $\mathbf{(A - B) \cap B = \phi}$

(iv) $\mathbf{A \cup B = A \cup (A' \cap B')}$

Solution:

(i) $\mathbf{A \cap B' = A \text{ iff } A \cap B = \phi}$

i.e. by Venn diagram

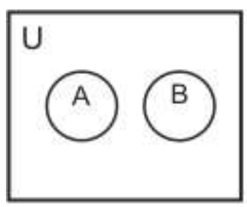


A∩B` = A **A∩B = φ**

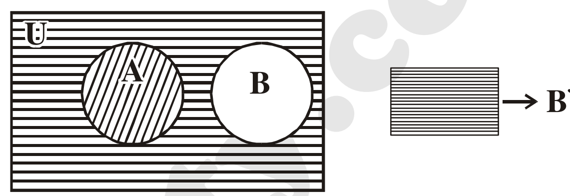
on contrary;

Suppose $A \cap B = \phi$

i.e.



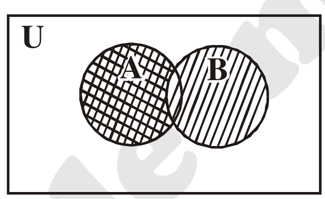
A∩B = φ



A∩B` = A

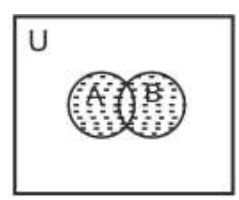
$\Rightarrow$ $A \cap B' = A$ if $A \cap B = \phi$

(ii) $\mathbf{(A - B) \cup B = A \cup B}$



(A-B) ∪ B

And

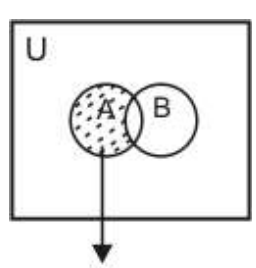


A∪B

From above figures it is clear that

$(A - B) \cup B = A \cup B$

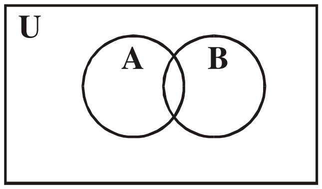
(iii) $\mathbf{(A - B) \cap B = \phi}$



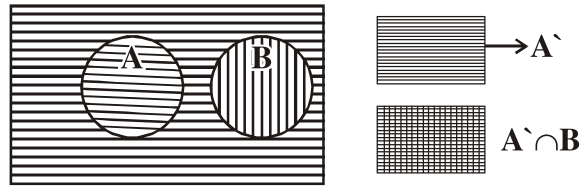
A – B

It is clear from above figure that $(A - B) \cap B = \phi$

(iv) $\mathbf{A \cup B = A \cup (A' \cap B)}$



A∪B



A∪(A`∩B)

In above two figures the shaded portion is same.

$\Rightarrow \quad A \cup B = A \cup (A' \cap B)$

INDUCTIVE AND DEDUCTIVE LOGIC

Induction

The way of drawing conclusions on the basis of a few basic experiments or observations is called induction.

Deduction

The way of drawing conclusions by accepting some well known facts is called deduction.

Proposition

A declarative statement which may be true or false but not both is called proposition.

Aristotelian and non–Aristotelian Logics:

Deductive logic in which every statement is regarded as true or false and there is no other possibility, is called Aristotelian logic.

Logic in which there is scope for a third or fourth possibility is called non–Aristotelian logic.

Symbolic Logic

Symbol	How to be read	Symbolic expression	How to be read
~ (Negation)	not	$\sim p$	Not p
$\wedge$ (Conjunction)	and	$p \wedge q$	p and q
$\vee$ (Disjunction)	or	$p \vee q$	p or q
$\rightarrow$ (Conditional)	if …. then implies	$p \rightarrow q$	p implies q
$\leftrightarrow$ (Biconditional)	if and only if	$p \leftrightarrow q$	p if and only if q or p is equivalent to q